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DELAYED AGREEMENTS AND NON-EXPECTED UTILITY

by

Chaim Fershtman*^R

Zvi Safra**

and

Daniel Vincent***

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*Tel-Aviv Department of Economics and Department of Managerial Economics and Decision Sciences, Kellogg Graduate School of Management, Northwestern University, 2001 Sheridan Road, Evanston, Illinois 60208.

**Recanati School of Management, Tel Aviv University, Tel Aviv and Department of Economics, University of Pennsylvania.

***Department of Managerial Economics and Decision Sciences, Kellogg Graduate School of Management, Northwestern University, 2001 Sheridan Road, Evanston, Illinois 60208.

1. Introduction

It is common in the analysis of economic environments to consider the effects of making contingent plans. Typically, the ability to formulate a contingent strategy before a particular state of Nature is realized enhances an agent's choice set and, if it does not strictly improve welfare, usually does not reduce it. We analyze a game situation in which two agents may interact strategically either before or after a random move of Nature. Since the strategy choices may be contingent on the state, players do not, by moving earlier, lose any ability to exploit the information that knowledge of the state may impart. The standard analysis, in the context of expected utility maximizing agents, suggests that offering the players the opportunity to decide earlier has no welfare effects. Alternatively stated, if agents can make contingent plans in an expected utility framework, the act of waiting to observe the state of Nature provides no value to the players.

The objective of this paper is to demonstrate that such a result is highly dependent on the assumption of expected utility agents. Expected utility requires strong restrictions on the form of preferences over lotteries, restrictions which in effect imply the irrelevance of the order of moves described above. Since a change in the order of moves induces a change in the character of the lotteries which face an agent, a more general class of preferences may suggest that such a change will not be innocuous (for a recent survey, see Machina, 1987). In particular, if preferences under uncertainty are such that a player strictly prefers to wait until Nature moves before he makes his move, even if players are impatient, that is, even in the presence of economic reasons which encourage an earlier rather than a later agreement, we may witness an incentive for a player to delay coming to an agreement.

The paper describes a strategic situation in which players may play a simultaneous move game either before or after a move of Nature. The structure is such that if the players were expected utility maximizers, they would be indifferent over the order of play.[1] However, if at least one of the players is a non-expected utility maximizer, for example, if player one has preferences over lotteries which exhibit Allais paradox type of behavior (Allais, 1953), such a player may strictly prefer to wait before playing the game. It is known that the Allais paradox violates the independence axiom and therefore affects evaluation over compound lotteries. Since our example contains a component game with a Nash equilibrium in mixed strategies--different compound lotteries are created by different strategy choices. The general form of the Allais paradox known as the common consequence effect has the feature that if the possibility of a bad common outcome can be eliminated, a non-expected utility maximizer will become <u>less</u> willing to bear risk. By waiting until after Nature moves, before playing the component game, player one forces himself into a position of lower risk tolerance. In equilibrium, then, player two is forced to offer a more attractive equilibrium mixing.

The result has implications for both non-expected utility theory and for economic game theory. In one-person decision problems with non-expected utility, the problem of time consistency often suggests that decision-makers

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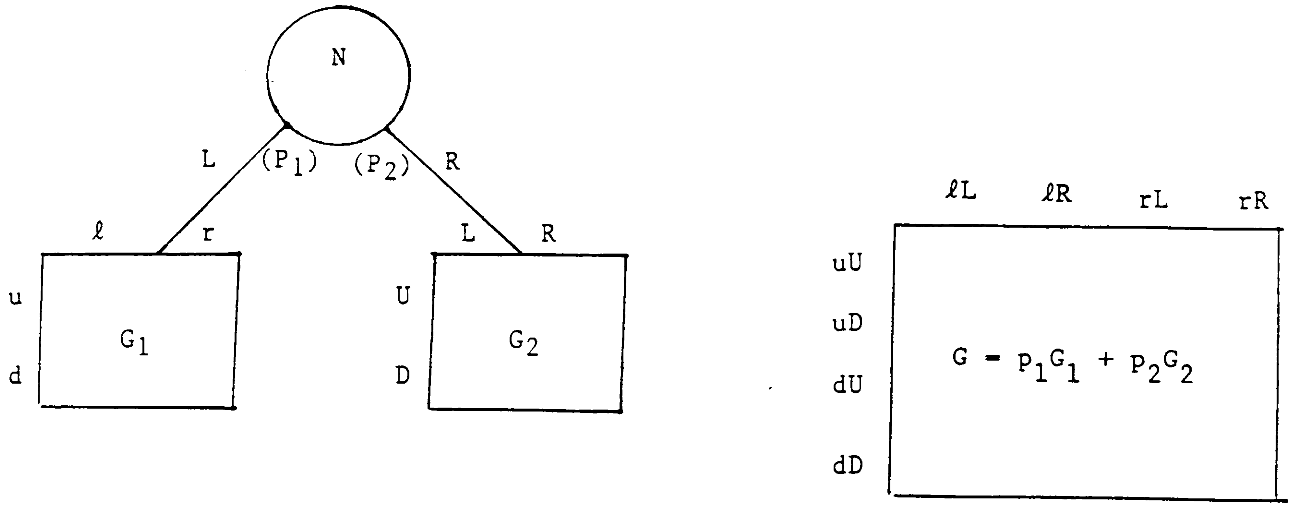
[1]Other papers that examine the integration of game theory and non-expected utility include Crawford (1988), Karni and Safra (1986),(1989), and Dekel, Segal, and Safra (1989).

have a strict preference to be able to commit themselves earlier rather than later.[2] This example shows that in a strategic setting, such a preference may be reversed. In economic games, specifically dynamic games, it is of interest to determine when players may agree to participate in a trade. In labor economics the reason for occurrence of strikes and delays in reaching agreements is still unclear. This problem is often referred to in the literature as the Hicks' paradox (Hicks, 1953) (see also Kennan (1986) and Hart (1989)). The general question of why we witness delays in bargaining is a characteristic problem. While our framework is different from a standard dynamic bargaining game, the result may shed some light on the issue of timing of agreements.

2. The Case of Expected Utility Maximizer

Consider the strategic situations in Figures 1a and 1b. In 1a, Nature moves left (with probability p_1) or right, after which two players participate in a simultaneous move game. The payoffs depend on the move of nature: if nature moves left, they play game G_1; otherwise they play G_2. It is assumed that both G_1 and G_2 possess a unique Nash equilibrium. Figure 1b describes an alternative situation in which players may meet before Nature moves and play, instead, a simultaneous game, G, in contingent strategies. A strategy choice of a player commits him to a play in each state of Nature.

[2] For more on the time consistency problem of a non-expected utility, see Karni and Safra (1989), Machina (1989).



1a 1b

Figure 1

The two structures can be integrated by the following extensive form game. In period one, players decide independently whether to meet before Nature moves and to play the simultaneous game G or to postpone play until after the state is realized. If both agree to meet, G is played and players are committed to their strategy choices--Nature moves and payoffs are realized according to these strategies. If either or both decide not to meet, then Nature moves and players are forced to play either G_1 or G_2. We will refer to this outcome as 'delay' to agreement.

It is well-known that if players are expected utility maximizers, the outcome of the game does not depend on which situation the players are in. In either case, the unique Nash equilibrium of each subgame determines the ex ante distribution of outcomes. If the players do meet earlier rather than later the unique equilibrium in contingent strategies simply reproduces the Nash equilibrium strategies of each component game. Obviously, with expected utility preferences, players are indifferent between being able to contract (i.e., play the contingent strategies) earlier or being forced to wait until after the move of Nature. And, if there are economic reasons for coming to an agreement earlier, players will strictly prefer to do so. For example, suppose that players' payoffs are discounted from the time of agreement. In such a situation, both players, if they are expected utility maximizers, will strictly prefer the opportunity to play the contingent strategy game.

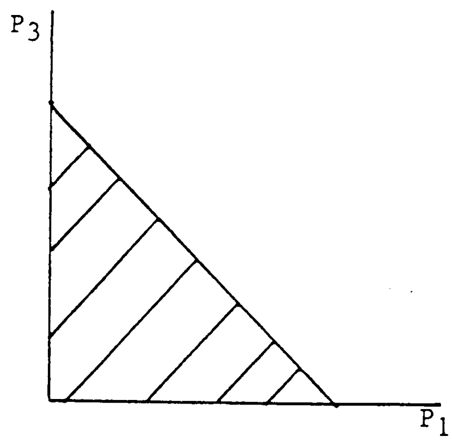
3. Some Remarks on Non-linear Utilities[3]

Since the original presentation of the Allais paradox (Allais,1953) the expected utility framework has been extensively challenged by economists as well as researchers from other disciplines. The expected utility framework assumes linearity in the probabilities, that is, the utility from a lottery (x,p) can be represented by a functional $V(x,p) = \Sigma p_i U(x_i)$. However, the Allais paradox as well as other experimental evidence suggests that agents making decisions under uncertainty exhibit a systematic violation of this linearity.

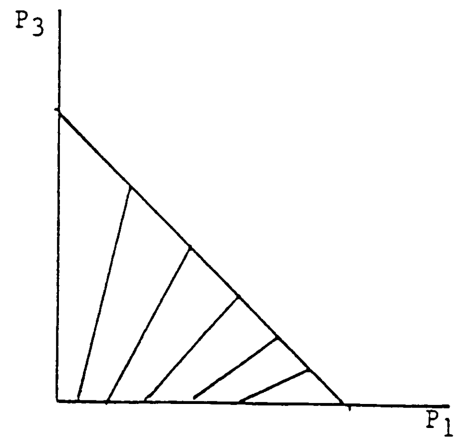
The linearity in probabilities can be represented graphically by considering the set of all possible lotteries on the fixed payoffs $x_1 < x_2 < x_3$. Observe that every such lottery can be described by a pair (p_1,p_3) (with $p_2 = 1 - p_1 - p_3$). The set of all such lotteries is just the unit simplex in Figure 2. Since the prizes of these lotteries are fixed the individual preferences are a map of upward sloping indifference curves; an

[3] This section relies heavily on Machina's (1987) survey. We present here only the technique that is essential for the understanding of our analysis in the next section.

increase of p_1 must be offset by an increase of p_3. A northwestern movement



(2a) Expected Utiilty indifference curves



(2b) Indifference curves with "fanning out"

Figure 2

in this triangle represents a move towards stochastically dominating lotteries. It is generally assumed that such a move is always preferred by any decision-maker--whether or not she is an expected utility maximizer. When an individual maximizes expected utility, her preferences in the (p_1,p_3) plane are linear with slope $[U(x_2)-U(x_1)]/[U(x_3)-U(x_2)]$. Notice that expected utility implies not just linear indifference curves but parallel curves as shown in Figure 2a.

A deviation from the expected utility framework implies that the indifference curves may no longer be either parallel or straight lines. The Allais paradox itself can be explained simply by relaxing the assumption of parallel indifference curves and allowing them to "fan out" as in Figure 2b, that is, higher level indifference curves are also steeper.

4. Non-expected Utility Players--An Example of Delay

Let us now reconsider the problem presented in Section 2 to illustrate that a delay in such games may occur when at least one of the players is a non-expected utility maximizer. For simplicity, we analyze the example with specific numerical entries. It will be clear that the argument holds for any similar class of prizes.

Fix G_1 to be the trivial game, $G_1 = (0,0)$ and G_2 to be the bi-matrix game in Figure 3

G_2 =

Player One		Player Two: ℓ	r
	u	5,0	0,3
	d	0,3	3,0

Figure 3

Thus, if Nature chooses left, then both players get 0. If right, then they play the game G_2. The outcomes in the matrix can be considered as monetary payoffs.

If the players decide on their strategies before nature moves, then they face the 2 × 2 matrix as in Figure Four. If player 1 plays up and player 2 plays left, then player 1 faces the lottery that gives him 0 with probability p_1 and 5 with probability p_2. Player 2 gets 0 with probability one.

		Player 2	
		ℓ	r
G ≡ Player 1	u	$(0,p_1;5,p_2)$, 0	0, $(0,p_1;3,p_2)$
	d	0, $(0,p_1;3,p_2)$	$(0,p_1;3,p_2)$, 0

Figure 4

We assume that both players have a preference relation in the space of lotteries (or probability distributions) that are represented by continuous functions V_i $(i = 1,2)$, which are also monotone with respect to the order induced by first order stochastic dominance. Let player 2 be an expected utility maximizer, which implies that V_2 is "linear in the probabilities" (this assumption is not necessary).

It is clear that in both games G and G_2, there is no Nash equilibrium in pure strategies. It is also clear that in both games if player 1 plays a mixed strategy $(\gamma, 1 - \gamma)$ in which the weight γ of playing up is larger than 1/2, then player 2 will choose to play right. Similarly, if $\gamma < 1/2$, then player 2 will play left. Hence, equilibrium requires that player 1 play (1/2,1/2).[4]

Consider now the equilibrium mixed strategies of player 2. For this we need to specify more precisely the shape of V_1. We assume that V_1 satisfies

[4]Observe that the argument for $\gamma = 1/2$ holds whether or not player two is an expected utility maximizer.

the betweeness property (Chew (1981), Dekel (1986), Fishburn (1983)): $V_1(A) = V_1(B) \Rightarrow V_1(\lambda A + (1 - \lambda)B) = V_1(B)$, for all lotteries A, B and $\lambda \in [0,1]$, where $\lambda A + (1 - \lambda)B$ is the lottery that yields the outcome of A with the probabilities of A, multiplied by λ, and the outcomes of B with B's probabilities, multiplied by $(1 - \lambda)$. This implies that in the triangle representation (described in Section 3) player 1's indifference curves are straight lines (not necessarily parallel to each other). We further assume that player 1's indifference curves "fan out." The betweeness property clearly implies that player 1 will choose to mix exactly when he is indifferent between his pure strategies (assuming, as stated above, that he uses the usual reduction method to evaluate mixed strategies (see Dekel, Safra and Segal (1989)and Segal (1989)).

Consider the game G_2. If player 2 plays the mixed strategy $(\delta, 1 - \delta)$ (δ for left, $1 - \delta$ for right), then player 1 has the choice between the lotteries $A_1(\delta) = (5,\delta;0, 1 - \delta)$, if he plays up, and $A_2(\delta) = (0,\delta;3, 1 - \delta)$, if he plays bottom. Let α be the equilibrium mixing of the second player such that a mixing of α makes the first player indifferent between $A_1(\alpha)$ and $A_2(\alpha)$. That is,

$$A_1(\alpha) = (0,1-\alpha;5,\alpha) \sim_1 (0,\alpha;\ 3,\ 1-\alpha) = A_2(\alpha) \tag{1}$$

When $\delta < \alpha$ we have $V_1(A_2(\delta)) > V_1(A_2(\alpha)) = V_1(A_1(\alpha)) > V_1(A_1(\delta))$ (inequalities follow from the monotonicity of V_1), hence bottom is chosen. If $\delta > \alpha$ then, similarly, $V_1(A_1(\delta)) > V_1(A_2(\delta))$, and hence top is chosen. This implies that the unique equilibrium in G_2 has player 1 play the mixed strategy $\gamma = 1/2$ and player 2 play the mixed strategy $\delta = \alpha$. With these strategies, the game yields player one the lottery

$$(0,p_1;A_1(\alpha),p_2/2;A_2(\alpha),p_2/2)$$

which reduces to

$$\begin{aligned} D_a &= (0,\ p_1 + (1-\alpha)p_2/2 + \alpha p_2/2;\ 3,\ (1-\alpha)p_2/2;\ 5,\alpha p_2/2) \\ &= (0,(1+p_1)/2;\ 3,(1-\alpha)p_2/2;5,\alpha p_2/2) \end{aligned} \tag{2}$$

Now consider the game, G. If player two plays the mixed strategy $(\delta,1-\delta)$, then player one faces the lottery $B_1(\delta)$ if he plays up or $B_2(\delta)$ if he plays down. Let β be the equilibrium mixing of player two such that

$$B_1(\beta) = (0,1-\beta+p_1\beta;5,p_2\beta) \sim_1 (0,\beta+(1-\beta)p_1;\ 3,\ p_2(1-\beta)) = B_2(\beta) \tag{3}$$

As before, G has a unique Nash equilibrium in which player 1 plays $\gamma = 1/2$ and player 2 plays $\delta = \beta$.Given these strategies, player one is faced with the lottery,

$$(B_1(\beta),1/2;\ B_2(\beta),1/2),$$

which reduces to

$$\begin{aligned} D_b &= (0,\ \beta p_1/2 + (1-\beta)/2 + \beta/2 + (1-\beta)p_1/2;\ 3,(1-\beta)p_2/2,5,\beta p_2/2) \\ &= (0,(1+p_1)/2;3,(1-\beta)p_2/2;5,\beta p_2/2). \end{aligned} \tag{4}$$

The question now arises about the relationship between α and β. It is clear that if player one is also an expected utility maximizer, then $\alpha = \beta$.[5]

Proposition 1: When the preferences of player one exhibit fanning out, $\beta < \alpha$ and $\alpha < \beta+(1-\beta)p_1$.

Proof: Suppose that $\alpha \leq \beta$. Compare the indifference curves of player one through $B_2(\beta)$ and $A_2(\alpha)$ respectively.

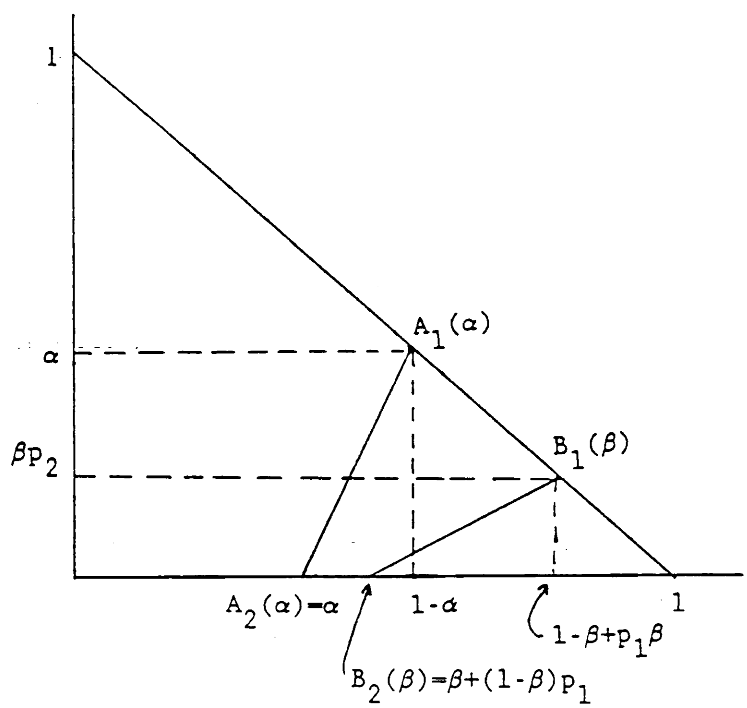


Figure 5

[5] One can easily prove this claim by showing that $\alpha = \beta$ is derived by equating the slopes of the indifference curves.

Since $\alpha \leq \beta$, (1) and (3) imply that $V_1(A_2(\alpha)) > V_1(B_2(\beta))$. The slope of the indifference curve through $A_2(\alpha)$ is $\alpha/(1-2\alpha)$ while the slope of the curve through $B_2(\beta)$ is $\beta/(1-2\beta)$. Fanning out implies that higher utility indifference curves are steeper. Thus,

$$\alpha/(1-2\alpha) > \beta/(1-2\beta) \tag{5}$$

Simplifying yields a contradiction--$\alpha > \beta$. Similarly, one can prove that $\alpha < \beta+(1-\beta)p_1$ //

Proposition 2: If the indifference curves of player one exhibit fanning out, there is a delay in reaching agreement as the first player strictly prefers to wait until the realization of the move of nature.

Proof: Comparing the lotteries D_a and D_b ((2) and (4)) yields that since $\alpha > \beta$, D_a stochastically dominates D_b and thus, $V_1(D_a) > V_1(D_b)$. //

Remark: Since the inequality is strict, it is clear that even when player one is impatient--that is, even if his utility is defined both on the equilibrium lotteries and on the date in which the strategies are decided upon, then as long as his impatience is sufficiently small, the above result continues to hold and he prefers to wait to the second period.

In the context of decision theory and non-expected utility, this result is interesting since, typically, in one person decision problems, it is advantageous to be able to commit to decisions earlier rather than later. This preference generally arises because of time consistency problems.

Suppose that an individual realizes that the actions that maximize his utility today are such that at some future point he would like to change them. Unless the player can commit himself to a sequence of actions, the requirement that his own decisions be time consistent enters into the problem as a constraint, thus reducing the possibility set. A non-expected utility maximizing player would generally wish to commit to an original plan and not to wait for the realization of the state of nature. The example above illustrates that this intuition does not carry over to a game context.

Observe that in both G and G_2, the first player plays the mixed strategy(.5,.5). Thus there is no issue of time consistency on his part even though he is the non-expected utility maximizer. It is the second player (the expected utility maximizer) who changes his equilibrium strategy between G and G_2. The change of strategy occurs however because some of the uncertainty is resolved and there is now a different mixing that makes the first player indifferent between playing up and down. In particular, once the game G_2 is played the uncertainty regarding the state of nature is resolved and the possibility of getting (0,0) is eliminated. Our assumptions regarding player one's preferences imply that the possibility of being at G affects his evaluation of lotteries associated with the game G_2. Thus the mixing that makes him indifferent between up and down are no longer the same. Once the uncertainty concerning game G_1 is resolved, player one (in accordance with behaviour observed in tests of the Allais paradox) becomes more reluctant to bear the risk embodied in game G_2. Player two is thus obliged to offer a more favorable gamble in the equilibrium of that game.

It is important to note however that once we compare the games before and after the revelation of the state of nature we do so by analyzing the way the player evaluates these alternatives at the outset of the game, that is, period zero. Thus even if we consider the case of waiting for the move of nature before playing, the evaluation of this possibility takes into account the choice of G_1, (0,0). But in evaluating the equilibrium outcome, player one realizes that from the point of view of period zero, the equilibrium mixing α (in G_2) implies a lottery that stochastically dominates the equilibrium lottery of the game, G (that is, the mixing β). He thus decides to wait another period in order to force the second player to play the mixed strategy, α.

5. Concluding Remarks

The timing of agreements is important in many economic environments -- an earlier agreement frequently means that gains from trade may be enjoyed sooner or for a longer period. Nevertheless, we often observe apparently costly delays to agreement. The explanation for such delays typically encounters a paradox. Hicks articulated the puzzle with respect to strikes -- if a theory can predict the outcome of a strike which is costly to both parties, how can the theory also explain the failure of the parties to agree to the same outcome before incurring these costs (Kennan (1986)? The example in this paper illustrates that even when there are no asymmetries of information, if agents are not expected utility maximizers but behave in accordance with preferences commonly observed in tests of the Allais paradox, they may exhibit a strict preference for delaying agreement. The Allais paradox may help to shed light on the Hicks paradox.

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