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COMMUNICATION EQUILIBRIA WITH LARGE STATE SPACES

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Abstract: A definition of communication equilibrium of games for which players may have arbitrary [rather than finite] type spaces is examined. The revelation principle is proven, and the set of equilibria is compared with the sets of strategy and action correlated equilibria. The equilibrium correspondence is shown to be discontinuous with respect to the information structure of the game, in contrast with previous continuity results for strategy and action correlated equilibrium.

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1. Introduction

Some recent progress in game theory has been made on characterizing the set of outcomes which follow from the common knowledge of rationality of players. Aumann (1974, 1987) first explained that there exist plausible equilibria of nonstochastic games which are not Nash equilibria. Players may rationally base their actions on a correlation device, which is a set of signals not defined *a priori* as part of the game. If the signals of different players are correlated, then their resulting actions may also be correlated, leading to outcomes which are not Nash equilibria. A correlated equilibrium is then a set of beliefs about the actions of players which can be sustained by some correlation device.

The use of a correlation device can be extended to games with uncertainty and asymmetric information about payoffs. Since the information of players is generally a source of correlation of actions, which is permitted in a Bayesian-Nash equilibrium, the focus is on equilibria which relies on an additional correlation device. One question which arises is to what extent this correlation device may depend on the prior information of players. Two extreme cases have been studied previously by this author. An equilibrium which relies only on an additional correlation device that is independent of the prior information of players is known as a strategy correlated equilibrium [Cotter (1989a)], while permitting the use of any correlation device leads to an action correlated equilibrium [Cotter (1989c)].

In this paper an equilibrium concept which falls between strategy and action correlated equilibrium is constructed. A communication equilibrium relies on a correlation device which depends only on the information of players which is freely and rationally provided by them. This equilibrium allows for all possible forms of direct and noisy communication between players, and generalizes communication equilibrium as defined in the mechanism design literature [e.g., Myerson (1983)] by allowing players to have an arbitrary [rather than finite] set of possible states of information. One of the most important results in mechanism design, the revelation principle, is proven for this more general model. In addition, any strategy correlated equilibrium is shown to be a communication equilibrium with a "deaf" mediator, and any communication equilibrium is an action correlated equilibrium.

The model in this paper is presented in Section 2, along with a brief review of equilibrium concepts which depend on correlation but not communication. The definition of communication equilibrium, along with a proof of the revelation principle, is given in Section 3, and an alternate model with parallel results is presented in Section 4. Section 5 compares communication equilibria with other equilibrium concepts. The continuity of the communication equilibrium correspondence is examined in Section 6.

2. The model

Consider a game with uncertainty and a finite set of players $I \equiv \{1,\ldots,I\}$. To economize notation I denotes both the set and the number of players, and $i \in I$ is a generic player. Each player i has the following characteristics.

action space	A_i, a compact metric space.
privately observed type	$t_i \in T_i$, a complete separable metric space.
payoff function	$u_i: T \times A \to \mathfrak{R}$, where $A = \prod_{i \in I} A_i$ and $T = \prod_{i \in I} T_i$.

For any metric space X, let $\Delta(X)$ be the set of probability[1] measures on X with the usual topology of weak convergence. By Theorems II.6.2 and II.6.4 of Parthasarathy

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[1]When no other qualification is stated, all measures are Borel, and measurability of sets and functions refers to Borel measurability.

(1967), $\Delta(X)$ is a compact [resp. separable[metric space if and only if X is compact [resp. separable]. The information of players about the types of others is given by an information structure $\upsilon \in \Delta(T)$. Let υ_i be the marginal of υ on T_i. The assumptions about the payoff function are straightforward.

Assumption 2.1: For each i,

(a) the mapping $t \rightarrow u_i(\cdot,a)$ is measurable for each $a \in A$,

(b) the mapping $a \rightarrow u_i(t,\cdot)$ is continuous for each $t \in T$,

(c) the mapping $t \rightarrow \sup_{a \in A} |u_i(\cdot,a)|$ is integrable.

The standard method for defining an equilibrium for this game has been to transform the game into a nonstochastic game in behavioral [or distributional] strategies. Let $S_i = \{s_i:T_i \rightarrow \Delta(A_i) \mid s_i$ is measurable$\}$ be the set of player i's behavioral strategies and $S^p = \prod_{i \in I} S_i$. Using the convention that for any $s \in S^p$ [resp. $a \in A$, $t \in T$[, s_{-i} [resp. a_{-i}, t_{-i}[is the profile of strategies [resp. actions, types] of players other than i, player i's payoff function in the transformed game is the expected payoff function $U_i:S^p \rightarrow \Re$, where

$$U_i(s_i,s_{-i}) = \int_T \int_{A_{-i}} \int_{A_i} u_i(t_i,t_{-i},a_i,a_{-i}) s_i(t_i)(da_i) s_{-i}(t_{-i})(da_{-i}) \upsilon(dt). \tag{1}$$

Most definitions of equilibrium for Bayesian games have been constructed by applying standard equilibrium concepts to the transformed game. A *Bayesian-Nash equilibrium* (BNE) [Milgrom and Weber (1985), Radner and Rosenthal (1982)] is a Nash equilibrium for the transformed game, i.e., $s^* \in S^p$ such that for each i and $s_i \in S_i$, $U_i(s^*) \geq U_i(s_i,s^*_{-i})$. A *strategy correlated equilibrium* (SCE) [Cotter (1989b)] is a correlated equilibrium [Aumann (1987)] for the transformed game, i.e., a probability distribution $\eta \in \Delta(S^p)$ such that for each i and measurable $\delta_i:S_i \rightarrow S_i$,

$$\int_{S^p} U_i(s)\eta(ds) \geq \int_{S^p} U_i(\delta_i(s_i),s_{-i})\eta(ds). \tag{2}$$

As explained by Cotter (1989c), an SCE only allows correlation based on devices or signals which are independent of the type space T. Let $S = \{s:T \rightarrow \Delta(A) \mid s$ is measurable$\}$ be the set of *joint strategies.* Permitting correlation based on arbitrary leads to an *action correlated equilibrium* (ACE), which is a joint behavioral strategy $s \in S$ such that for each i and measurable function $\alpha_i T_i \times A_i \rightarrow A_i$,

$$\int_{T\times A} u_i(t,a)s(t)(da)\upsilon(d\omega) \geq \int_{T\times A} u_i(t,\alpha_i(t_i,a_i),a_{-i})s(t)(da)\upsilon(d\omega). \tag{3}$$

An ACE uses the correlation device s which "knows" t. The device recommends an action to each player, with the profile of recommendations to all players following the probability distribution $s(t)$. Each player i optimally chooses to follow the recommendation given t_i, his own recommended action, and the belief that all other players will follow their recommendations.

Note that any BNE or SCE generates a joint behavioral strategy, so they are also ACEs.

3. Representation of communication strategies and equilibria

The problem with the definition of an ACE is that it does not explain how the correlation device comes to "know" t. Such a correlation device is best interpreted as a profile of common beliefs about the behavior of players. If, however, the correlation device is interpreted as an explicit mechanism, then it should also satisfy the condition that it somehow receives t from the players. Imposing such a condition leads to a communication equilibrium, which is defined below.

According to Myerson (1983), a mediator is a device which collects information about the state of nature via confidential messages from each player, then transmits a recommended action to each player based on the messages. Each player then takes an action based on the mediator's suggestion. There is no mechanism to insure that players will transmit all of their information correctly or follow the mediator's recommendation. Therefore an equilibrium must satisfy incentive compatibility conditions which takes account of the players' ability to deceive or disobey the mediator.

In this section Myerson's definition is extended to games for which the type and action spaces of players need not be finite, and the mediator and players may transmit arbitrary messages to each other. Define a *communication game* to be profiles of message spaces $X = (X_1,\ldots,X_I)$ and $M = (M_1,\ldots,M_I)$, with each X_i and M_i complete separable metric spaces, and a measurable *communication function* $r:X \to \Delta(M)$. In a communication game, player i observes t_i and sends a message $x_i \in X_i$ to the communication function, following the distribution $\sigma_i(t_i) \in \Delta(X_i)$. The communication function receives the messages $x \equiv (x_1,\ldots,x_I)$ and sends a profile of messages $m = (m_1,\ldots,m_I) \in M$ to the players, following the distribution $r(x) \in \Delta(M)$. Player i then receives the message m_i and chooses a mixed strategy $\delta_i(t_i,m_i) \in \Delta(A_i)$. Let $\Sigma_i(X_i) = \{\sigma_i:T_i \to \Delta(X_i) \mid \sigma_i \text{ is measurable}\}$ and $D_i = \{\delta_i:T_i \times M_i \to \Delta(A_i) \mid \delta_i \text{ is measurable}\}$ be the sets of *messages functions* and *reaction functions* for player i respectively.

Note that the above definition of communication equilibrium permits arbitrary forms of communication. Special cases include noiseless as well as noisy communication between individual players and groups of players.

The following technical result will be needed.

Lemma 3.1: Let Y be a complete separable metric space and let $\phi:T \to \Delta(Y)$ be measurable. Then for any measurable function $b:Y \to \mathfrak{R}$, the function $t \to \int_Y b(y)\phi(t)(dy)$ is measurable.

Proof: Let $\{b^n\}$ be a sequence of simple functions on Y which increase to b. For each n, the function $t \to \int_Y b^n(y)\phi(t)(dy)$ is measurable by Theorem 3.2(b) of Cotter

(1989c). For each t, the sequence $\{\int_Y b^n(y)\phi(t)(dy)\}$ converges to $\int_Y b(y)\phi(t)(dy)$ by the monotone convergence theorem, proving the result. ∴

The next result states that the combination of messages, recommendations, and reactions is mathematically tractable.

Theorem 3.2: For each i let $(\sigma_i, \delta_i) \in \Sigma_i(X_i) \times D_i$. *Then there exists a unique profile of players' actions* $s \in S$ *that is the outcome of the communication game, such that for any* $f: T \times A \to \mathfrak{R}$ *satisfying Assumption 2.1,*

$$\int_T \int_A f(t,a) s(t)(da) \upsilon(dt)$$

$$= \int_T \int_X \int_M \int_A f(t,a) \delta_1(t_1, m_1)(da_1) \ldots \delta_I(t_I, m_I)(da_I) r(x)(dm) \sigma_1(t_1)(dx_1) \ldots \sigma_I(t_I)(dx_I) \upsilon(dt). \quad (4)$$

Proof: The first step is to show that the right-hand side of (4) is defined. The function $(t,m) \to \int_A f(t,a)\delta_1(t_1,m_1)(da_1)\ldots\delta_I(t_I,m_I)(da_I)$ is measurable since each δ_i is measurable. By Lemma 3.1, $(t,x) \to \int_M \int_A f(t,a)\delta_1(t_1,m_1)(da_1)\ldots\delta_I(t_I,m_I)(da_I)r(x)(dm)$ is measurable, so the right-hand side of (4) is defined by repeated use of Lemma 3.1.

The right-hand side of (4) defines a real function $\phi: L[C(A)] \to \mathfrak{R}$, where $L[C(A)] = \{f: T \to C(A) \mid f$ is measurable with respect to the supremum norm on $C(A)\}$, and ϕ is easily verified to be linear and continuous with respect to the norm topology on $L[C(A)]$. Letting $M(A)$ be the space of signed measures on A with the weak convergence topology, $\phi \in L[C(A)]^* = \{s: T \to M(A) \mid s$ is measurable with respect to the weak topology on $M(A)\}$. Therefore there exists $s: T \to M(A)$ satisfying (4). To show s maps into $\Delta(A)$, note that (4) is nonnegative for nonnegative f, and for a.e. t,

$$\left| \int_A f(t,a) s(t)(da) \right|$$

$$= \left| \int_X \int_M \int_A f(t,a)\delta_1(t_1,m_1)(da_1)\ldots\delta_I(t_I,m_I)(da_I) r(x)(dm) \sigma_1(t_1)(dx_1)\ldots\sigma_I(t_I)(dx_I) \right|$$

$$\leq \int_X \int_M \int_A \sup_{a \in A} |f(t,a)| \, \delta_1(t_1,m_1)(da_1) \ldots \delta_I(t_I,m_I)(da_I) r(x)(dm) \sigma_1(t_1)(dx_1) \ldots \sigma_I(t_I)(dx_I)$$

$$= \sup_{a \in A} |f(t,a)| \tag{5}$$

proving the result. ∴

Corollary 3.3: Equation (4) *holds for any Borel-measurable function* $f{:}T \times A \rightarrow \mathfrak{R}$.

Proof: Follows from Lemmas 3.1 and 3.2. ∴

Replacing f with u_i in (4) yields the payoff function $V_i{:}\prod_{i \in I} (\Sigma_i(X_i) \times D_i) \rightarrow \mathfrak{R}$. A *communication equilibrium* (CE) is a communication game $\{(X_1,\ldots,X_I),(M_1,\ldots,M_I),r^*\}$ and a profile of messages and reactions $((\sigma_1^*,\delta_1^*),\ldots,(\sigma_I^*,\delta_I^*)) \in \prod_{i \in I} (\Sigma_i(X_i) \times D_i)$ which is a Nash equilibrium for the game where each player's payoff function is V_i and strategy set is $\Sigma_i(X_i) \times D_i$.

One well-known problem with this definition of CE is that permitting arbitrary messages and reactions creates an intractably large set of possible equilibria. For any CE there exist a continuum of communication games with equilibria which differ only in the message structure used by the players, though the information conveyed by the players' messages is the same. The solution to this problem is the well-known revelation principle, which states that for any CE, there exists an equivalent CE in which each player accurately transmits her information to the communication function and obeys its recommendation. Define a *direct revelation game* to be a communication game for which $X_i = T_i$ and $M_i = A_i$ for each i. For each i define the truthful message function $\bar{\sigma}_i \in \Sigma_i(T_i)$ such that for a.e. t_i, $\bar{\sigma}_i(t_i)$ is the point mass on t_i, and the obedience function $\bar{\delta}_i \in D_i$ such that $\bar{\delta}_i(t_i,\alpha_i)$ is the point mass on α_i for a.e. t_i and all α_i.

Earlier proofs of the revelation principle have assumed a finite number of player types and a finite action space for each player.

Theorem 3.4: For any CE which yields the joint behavioral strategy $s^* \in S$, the direct revelation game with communication function s^* has the CE $((\bar{\delta}_1,\bar{\sigma}_1),\ldots,(\bar{\delta}_I,\bar{\sigma}_I))$.

Proof: Let $((\delta_1^*,\sigma_1^*),\ldots,(\delta_I^*,\sigma_I^*))$ be a CE for the communication game $\bar{r}^* \in S$ be the joint behavioral strategy generated by (r^*,δ^*,σ^*) as given in Theorem 3.2. Consider the direct revelation game for which the communication function is $\bar{r}^*$. Suppose player i were to play (δ_i,σ_i) while every other player j played $(\bar{\delta}_j,\bar{\sigma}_j)$. Without loss of generality suppose $\delta_i{:}T_i \times A_i \rightarrow A_i$ and $\sigma_i{:}T_i \rightarrow T_i$. The resulting payoff to player i would be

$$\int_T\int_A u_i(t,\delta_i(t_i,a_i),a_{-i})\bar{r}^*(\sigma_i(t_i),t_{-i})(da)\upsilon(dt) \tag{6}$$

which equals, by Corollary 3.3,

$$\int_T\int_X\int_M\int_A u_i(t,\delta_i(t_i,a_i),a_{-i})\delta^*_i(t_i,m_i)(da_i)\delta^*_{-i}(t_{-i},m_{-i})(da_{-i})r(x)(dm)\sigma^*_i(\sigma_i(t_i))(dx_i)\sigma^*_{-i}(t_{-i})(dx_{-i})\upsilon(dt) \tag{7}$$

This is the payoff that would result in the original game if player i chose the message function $\sigma^*_i(\sigma_i(\cdot))$ and the reaction function induced by the composition of δ_i and δ^*_i. Since the original game is a CE, (7) is dominated by

$$\int_T\int_A\int_A u_i(t,a)\delta^*_i(t_i,m_i)(da_i)\delta^*_{-i}(t_{-i},m_{-i})(da_{-i})r^*(\sigma^*_i(\sigma_i(t_i)),\sigma^*_i(t_{-i}))(dm)\upsilon(dt)$$

$$= \int_T\int_A u_i(t,a)\bar{r}^*(t)(da)\upsilon(dt) \tag{8}$$

which is the payoff in the direct revelation game if player i chooses (δ^*_i,σ^*_i). This completes the proof. $\therefore$

A consequence of the revelation principle is that every CE can be identified with the joint strategy it generates, as given by Lemma 3.2. Therefore the set of CE is a subset of S.

4. Communication equilibria and the partition model

The results of the previous section are now stated in terms of the partition model of a game with incomplete information. In the partition model a single underlying probability space $(\Omega,\mathcal{F},\mu)$ models all prior uncertainty in the game, where Ω is a complete separable metric space and $\mathcal{F}$ its Borel sets. Each player has the following characteristics:

action space A_i, a compact metric space. Let $A = \prod_{i\in I} A_i$.

information field $\mathcal{G}_i$, a sub-σ-field of $\mathcal{F}$.

payoff function $u_i:\Omega \times A \to \mathfrak{R}$.

The assumptions regarding u_i are analogous to Assumption 2.1.

Assumption 4.1: For each i,

(a) the mapping $\omega \to u_i(\omega,a)$ is measurable for each $a \in A$,

(b) the mapping $a \to u_i(\omega,a)$ is continuous for each $\omega \in \Omega$,

(c) the mapping $\omega \to \sup_{a\in A} |u_i(\omega,a)|$ is integrable.

The advantage of the partition model over the type model described previously is that each player's information is a parameter $\mathcal{G}_i$ which is distinct from the underlying uncertainty in the game. Given a communication game $\{(X_1,\dots,X_I),(M_1,\dots,M_I),r\}$, player i chooses a message function $\sigma_i \in \Sigma_i(\mathcal{G}_i;X_i) = \{\sigma_i:\Omega \to \Delta(X_i) \mid \sigma_i$ is $\mathcal{G}_i$-measurable$\}$ and a deviation function $\delta_i \in D_i(\mathcal{G}_i) = \{\delta_i:\Omega \times M_i \to \Delta(A_i) \mid \delta_i$ is $\mathcal{G}_i \times \mathcal{M}_i$-measurable$\}$. A communication equilibrium for this game is a profile of messages and deviations $((\sigma_1^*,\delta_1^*),\dots,(\sigma_I^*,\delta_I^*)) \in \prod_{i\in I} (\Sigma_i(\mathcal{G}_i;X_i) \times D_i(\mathcal{G}_i))$ such that for any i and any $(\sigma_i,\delta_i) \in \Sigma_i(\mathcal{G}_i;X_i) \times D_i(\mathcal{G}_i)$,

$$\int_\Omega\int_X\int_M\int_A u_i(\omega,a)\delta_i^*(\omega,m_i)(da_i)\delta_{-i}^*(\omega,m_{-i})(da_{-i})r^*(x)(dm)\sigma_i^*(\omega)(dx_i)\sigma_{-i}^*(\omega)(dx_{-i})\mu(d\omega)$$

$$\geq \int_{\Omega}\int_{X}\int_{M}\int_{A} u_i(\omega,a)\delta_i(\omega,m_i)(da_i)\delta^*_{-i}(\omega,m_{-i})(da_{-i})r^*(x)(dm)\sigma_i(\omega)(dx_i)\sigma^*_{-i}(\omega)(dx_{-i})\mu(d\omega) \quad (9)$$

The expressions in Equation (9) are defined by Theorem 3.2.

With respect to a partition model, a direct revelation game is a communication game for which $X_i = \Omega$ and $M_i = A_i$ for each i. For each i define the truthful message function $\bar{\sigma}_i^{\mathcal{G}_i} \in \Sigma_i(\mathcal{G}_i;\Omega)$ such that for a.e. $\bar{\sigma}_i^{\mathcal{G}_i}(\omega) = P[\cdot \mid \mathcal{G}_i](\omega)$[2], and obedience $\bar{\delta}_i^{\mathcal{G}_i} \in D_i(\mathcal{G}_i)$ such that $\bar{\delta}_i^{\mathcal{G}_i}(t_i,\alpha_i)$ is the point mass on α_i for a.e. ω and all α_i.

For any partition model there exists an equivalent type model. For each i let $(T_i,\mathcal{T}_i)$ be a copy of $(\Omega,\mathcal{G}_i)$, and $\mathcal{Z} = \prod_{i\in I} Z_i$. Assume without loss of generality that $\mathcal{F} = \bigvee_{j\in I} \mathcal{G}_j$, the smallest σ-field containing each $\mathcal{G}_i$. By Lemma 4.1 of Cotter (1989b), there exists a unique measure υ on $(T,\mathcal{T})$ and a set isomorphism $\Phi:\mathcal{G} \to \mathcal{T}$ such that $\upsilon(B) = \mu(\Phi^{-1}(G))$ for $G \in \mathcal{G}$ and for each i and $G_i \in \mathcal{G}_i$, $\Phi(G_1 \cap \ldots \cap G_I) = G_1 \times \ldots \times G_I$. There does not generally exists a corresponding point function from Ω to T, but there do exist [Cotter (1989b, Lemma 4.2, Corollary 4.3, Lemma 4.4, Theorem 4.5)] isomorphisms $\mathbf{T}:L[\Omega;C(A)] \to L[T;C(A)]$, $\mathbf{T}:\Sigma_i(\mathcal{G}_i;X_i) \to \Sigma_i(X_i)$, and $\mathbf{T}:D_i(\mathcal{G}_i) \to D_i$ which preserve players' expected payoff functions. It is also easy to verify that $\mathbf{T}(\bar{\sigma}_i^{\mathcal{G}_i}) = \bar{\sigma}$ and $\mathbf{T}(\bar{\delta}_i^{\mathcal{G}_i}) = \bar{\delta}$. Therefore Theorem 3.4, the revelation principle, applies to the partition model.

5. Comparisons with other equilibria

The sets of BNE, ACE, and CE are subsets of the joint behavioral strategy space S. While an SCE is not an element in S, it does generate a joint strategy. This mapping from SCEs to joint strategies is many-to-one [Cotter (1989b), Example 2.2].

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[2]Since Ω is a complete separable metric space, regular conditional probabilities such as the latter expression exist [Parthasarathy (1967, Theorem V.8.1)].

In this section the sets of joint behavioral strategies generated by these four equilibrium concepts are compared.

Theorem 5.1: Define the sets BNE, CE, and ACE $\subset S$ *to be the sets of* BNEs, CEs, *and* ACEs *respectively, and let SCE* $\subset S$ *be the set of joint behavioral strategies generated by the set of* SCEs. *Then BNE* $\subset$ *SCE* $\subset$ *CE* $\subset$ *ACE.*

Proof: By Theorem 4.4 of Cotter (1989c), BNE $\subset$ SCE $\subset$ ACE. In addition, CE $\subset$ ACE since by Lemma 6.2 of Cotter (1989c), any ACE is a CE without the communication of types to the communication function. It remains only to be shown that SCE $\subset$ CE.

Let $\eta^* \in \Delta(S^p)$ be an SCE generating $s^* \in S$. Consider the communication game with $M_i = S_i$ and $r^*(x) \equiv \eta^*$ for all x. Since the mediation function does not depend on x, the communication of players is irrelevant. Let δ_i^* be defined by $\delta_i^*(t_i,m_i) = m_i(t_i)$. The profile $((\bar{\sigma}_1,\delta_1^*),\ldots,(\bar{\sigma}_I,\delta_I^*))$ generates the joint behavioral strategy s^*. Suppose player i were to choose any other (σ_i,δ_i). Then it would be choosing a strategy outside the profile η^*, while the profiles of other players would be unaffected since r^* does not respond to players' messages. Therefore player i's payoff would be reduced since η^* is an SCE. This completes the proof. $\therefore$

The proof of Theorem 5.1 shows that an SCE is a CE with a "deaf" mediator, i.e., a communication function r^* which does not depend on x. Conversely, if r^* is a CE which does not depend on x, then the definition of CE reduces to that of SCE.

Corollary 5.2: The set of CE is nonempty.

Proof: The set of SCE is nonempty by Corollary 4.9 of Cotter (1989a), so the result follows from Theorem 5.1. $\therefore$

5. Continuity of the equilibrium correspondence

It is useful to know the extent to which the equilibria of the game depends on its underlying characteristics. Let G be the set of parameters of the game, and $\Delta(T \times A)$ the set of possible outcomes, where each $(s,\mu) \in S \times \Delta(T)$ is identified with the corresponding measure on $T \times A$ [Cotter (1989c, Theorem 3.2)]. The dependence of the game on its parameters can be posed in terms of the equilibrium correspondence $\xi : G \rightarrow\rightarrow \Delta(T \times A)$. The most important such properties are upperhemicontinuity and nonempty-valuedness. Though many possible definitions of G exist, in this section attention will be restricted to $G = \Delta(T)$, the set of information structures μ on T, with the weak topology. Consider a sequence $\{\mu^n\} \subset \Delta(T)$. The action equilibrium correspondence was shown to be continuous [Cotter (1989c, Theorem 5.2)] provided each player's payoff function is continuous on $T \times A$. Unfortunately, this result does not hold for communication equilibria.

Example 5.1: Consider a two person game. Their action sets are $A_1 = \{U,D\}$ and $A_2 = \{L,R\}$, and type spaces are $T_1 = T_2 = [0,1]$. The payoff functions, which does not depend on the type space, is

x	L	R
U	(1,1)	(0,0)
D	(0,0)	(2,2)

Let $s \in S$ be given by

$$s(t_1,t_2) = \begin{cases} (U,L) & \text{if } t_1 = t_2 \\ (D,R) & \text{if } t_1 \quad t_2 \end{cases}$$

Given the profile of signals received by the communication device, the messages sent to the players are always obeyed regardless of the probability distribution on T.

If player 1 receives U, she knows that player 2 received L and should optimally play U, while if player 1 receives D, she knows that player 2 received R and should optimally play D. Similarly, player 2 will always optimally choose to obey the communication device's recommendation.

Now let $\{\mu^n\}$ be a sequence of probability distributions on T, where μ^n is given by the density function $f^n(t_1,t_2) = 1/[(t_1-t_2)^n(1-n)(1-2n)]$. It is easily verified that μ^n converges weakly to μ, which is the uniform distribution on the diagonal of T. For each n and any t_i observed by player i, the conditional probability that player $-i$ will transmit $\tau_{-i} = t_i$ is zero. Therefore $s \in \xi(\mu^n)$. However, $s \notin \xi(\mu)$, since if player i observes t_i, the conditional probability that player $-i$ will transmit $\tau_{-i} = t_i$ is one, so player i should transmit anything but t_i. Therefore with the weak topology on $\Delta(T)$, ξ is not upperhemicontinuous for *any* topology on S. $\therefore$

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