The Center for Mathematical Studies in Economics and Management Science

Discussion Paper No. 849

The Volume and Composition of Trade Between Rich and Poor Countries

by

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September 1989

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* I am grateful to Larry Jones, Robert E. Lucas, Jr., and Kiminori Matsuyama for helpful comments. This research was supported by National Science Foundation Grant SES-8606755.

Abstract

North-South trade is studied using a characteristics model of quality differentiation. In it, the South produces a low-quality spectrum of goods and the North a high-quality spectrum. An increase in the North's population decreases the relative wage there, shifts the spectrum of Northern products downward, and contracts the spectrum of Southern products. An increase in the North's human capital decreases its unit labor-input requirement for each good and increases its relative wage. If the net effect is to improve the South's terms of trade, then the spectrum of Northern products expands, the spectrum of Southern products contracts, and the volume of trade grows; otherwise, these effects are reversed. Similar results hold for changes in the South.

Introduction

In his classic article on the product cycle, Vernon (1966) discussed the reasons certain types of new goods would be invented and first produced in the countries with the highest per capita income, and only later produced in less advanced countries. More recently, Krugman (1979), Dollar (1986), Jensen and Thursby (1986, 1987), and Grossman and Helpman (1989) have studied the product cycle using formal models with many common features: Innovation in the more advanced region, hereafter called the North, takes the form of the introduction of new product varieties; imitation or reverse engineering is required in the less developed region, the South, before a new good can be produced there; and preferences over product varieties have the symmetric, constant-elasticity form introduced in Dixit and Stiglitz (1977). Hence trade is motivated by a preference for diversity, and growth is represented by an increase in the total number of varieties produced in each region. The focus in these papers is on how the relative wage rate and the economic incentives for innovation and imitation depend on costs, population sizes, market structure, and other factors.

These models capture many interesting and important features of the development and diffusion of new products, but there are also some features of North-South trade that they miss.[1] First, they predict that consumption patterns will be the same in both regions. The higher relative wage in the North simply leads to a proportionately higher per capita level of consumption of each variety. That is, the per capita consumption vector in the North is simply a scalar multiple of that in the South. This is clearly at odds with the facts. Typically, new goods are not immediately exported to the South: they are too expensive and there is little or no demand for them there. In fact, one of Vernon's main points was precisely that an important reason for initially locating production in the North is to be close to the output market.

Second, these models predict that a new good, once introduced, is never dropped from production. Since preferences are symmetric over goods, variety is valued and there is an incentive to produce as many varieties as possible. This, too, is unrealistic. Many new products are higher-quality versions of older goods, and as the cost of the newer version falls, demand for the older one disappears.

Third, these models specify that the labor used in manufacturing is equally productive in both regions. A wage differential between the two regions arises only because the South cannot produce a new good until it has engaged in reverse engineering. That is, labor in the North earns a higher wage only because the North has monopoly power in the production of some goods. Again, this description is at odds with the facts. Multinational corporations can and do send engineers all over in the world, and the average wage differential between rich and poor countries has little to do with limited access to blueprints in the latter. Instead, it is due in large part to differences in average labor quality between the two regions. These differences also explain a great deal about the location of various production activities. Low-skill labor has a comparative advantage in carrying out simpler, more standardized activities, and higher-skill labor in carrying out more sophisticated and more novel activities.

Many of these objections are addressed in a recent paper by Flam and Helpman (1987), where a quite different model of the product cycle is studied. There are two commodities, a homogenous good and a quality-differentiated product, with each agent consuming exactly one unit of the latter. All households have identical preferences, so the distribution of income across households determines the distribution of demand across quality levels of the differentiated product. The exogenously given distribution of labor endowments across households and the endogenously determined relative wage rate determine the distribution of income within and between regions. Finally, the technology gives Northern labor a comparative advantage in the production of higher-quality varieties of the differentiated product. Flam and Helpman focus on the case where in equilibrium the North specializes in the production of high-quality varieties of the differentiated product, the South produces low-quality varieties of the differentiated good and the homogenous good, and Northern labor has a higher relative wage.

This model has many attractive features. First, because the distribution of income differs in the two regions, consumption patterns differ. Second, the model produces a very appealing picture of the product cycle as labor productivity rises: higher-quality varieties are continuously introduced in the North, where they are first produced and consumed; consumption of each new variety spreads gradually to the South; production of intermediate-quality varieties shifts from North to South; and the lowest-quality varieties produced in the South are continuously dropped. Finally, the wage differential between North and South arises not because of monopoly power in the North, but because of differences in labor productivity.

In the present paper, a model of North-South trade is developed that is similar in spirit to the one studied by Flam and Helpman, but uses the Lancasterian (1966) characteristics model of new product introduction developed in Stokey (1988). In this formulation, new products are taken to be better than old ones in the sense of providing more characteristics, and each agent consumes variable quantities of all products. Hence there is demand for many product varieties even if all households are identical.

The main features of the model are as follows. There are two regions, North and South. Any commodity can be produced in either region, and all commodities are tradeable at zero cost. Each region has a fixed number of identical, immobile households, and all households (in both regions) have identical preferences. All of the households within a region have the same level of human capital, and each household inelastically supplies one unit of labor, the only factor of production. The underlying production technology is the same for the two regions, and it displays constant returns to scale. Gains from trade arise because households in the two regions have different levels of human capital and the unit labor requirement for producing any good depends on the level of human capital of the labor used.

The main results of the paper are as follows. In Theorem 1, existence of a competitive equilibrium is established. It is shown that in any equilibrium the South produces a spectrum of lower-quality goods, the North produces a spectrum of higher-quality goods, and there is a gap between the highest-quality good produced by the South and the lowest-quality good produced by the North. All of the quality levels produced in the South are consumed domestically and exported. Of the quality levels produced in the North, all are consumed domestically but only those at the lower end of the spectrum are exported. Since households within each region are identical, it is immediate that they all benefit from free trade.

In Theorem 2 it is shown that under two additional restrictions on preferences, the competitive equilibrium is unique. The two preliminary lemmas used to prove this theorem deal with substitution and income effects, and are drawn upon extensively in the rest of the analysis.

Next, the effects of population size are studied. In Theorem 3 it is shown that an increase in the relative population size in the North decreases the relative wage rate there. In Theorem 4 it is shown that this, in turn, implies that the spectrum of goods produced by the South contracts at the upper end, and the spectrum produced by the North expands at the lower end. The highest quality level consumed by the South increases, and the highest quality level consumed (and produced) by the North decreases. It is also shown that if there are some low-skill households in the North or some high-skill households in the South, these groups are made worse off by free trade.

The effects of changes in the two levels of human capital are then studied. In Theorem 5 it is shown that the relative wage in the North increases with the level of human capital in the North and decreases with the level in the South. The effects on the volume and composition of trade are then studied. It is shown that two quite different patterns are possible, depending on the relative effect of the change in human capital on the labor input requirements for higher- and lower-quality goods. If an increase in Northern human capital causes a proportionate reduction in the labor input requirements for all of the goods it produces, then that reduction is not completely offset by the increase in the Northern wage. Consequently, the South enjoys a favorable shift in its terms of trade. This expands the spectrum of goods exported by the North at both the high- and low-quality ends, and increased competition from imports contracts the spectrum of goods produced by the South at the high-quality end. The volume of trade grows and welfare is enhanced in both countries.

The effects are quite different if an increase in Northern human capital reduces the labor input requirements for goods at the high end of its quality spectrum, those produced for domestic consumption only, but leaves unchanged the labor requirements for goods at the low end, its exports. In this case, the increase in the Northern wage means that the South suffers an unfavorable shift in its terms of trade. The spectrum of goods exported by the North contracts at both the high- and low-quality ends, and reduced competition from exports expands the spectrum of goods produced by the South at the high-quality end. The volume of trade shrinks and the South suffers a decrease in welfare. The North is better off, however, since both its production opportunities and its terms of trade have improved.

Finally, dynamic issues are considered, and it is shown that the present model displays a product cycle with the same appealing qualitative features as the one in Flam and Helpman.

The rest of the paper is organized as follows. The commodity space, preferences, and technology are described in section 1; existence of a competitive equilibrium is proved in section 2; and uniqueness of the equilibrium is established in section 3. In section 4 the effect of population size is examined, and in section 5 changes in the human capital levels are studied. In section 6 dynamics are considered, and in section 7 conclusions are drawn and ideas for further research are discussed.

1. The Commodity Space, Preferences, and Technology

There is a continuum of goods indexed by $z \geq 0$ and a continuum of characteristics indexed by $\xi \geq 0$. A goods allocation is represented by a piecewise continuous density, $x(z)$, $z \geq 0$. The good of quality z

provides one unit of each of the characteristics $\xi \in [0,z]$, so higher-index goods are better in the sense that they provide more characteristics, and the allocation of goods x contains the allocation of characteristics q given by

$$(1) \qquad q(\xi) = \int_{\xi}^{\infty} x(z)dz, \qquad \xi \geq 0.$$

Note that (1) defines a one-to-one mapping between allocations of goods and of characteristics, with $q'(\xi) = - x(\xi)$.

Each household's preferences over allocations of characteristics q are additively separable and symmetric:

$$U(q) = \int_0^{\infty} u[q(\xi)]d\xi.$$

The function u will be restricted as follows.

<u>Assumption 1</u>. u is twice continuously differentiable, strictly increasing, and strictly concave, with $u(0) = 0$ and $u'(0) < \infty$.

It is important that $u'(0)$ be finite, so that equilibria will involve zero consumption of some characteristics, and hence of some goods.

It is useful to begin with a characterization of the solution to the household's maximization problem. Lemmas 1 and 2 state the main facts about that problem.

<u>Lemma 1</u>. Let u satisfy Assumption 1; and let $p(z)$ be twice continuously differentiable, strictly increasing, and convex, with $p(0) = 0$ and $\lim_{z\to\infty} p'(z) = +\infty$. Then for any $y > 0$, the problem

$$\max_{x} \int_0^\infty u\left[\int_\xi^\infty x(z)dz\right]d\xi \tag{2a}$$

$$\text{s.t.} \quad \int_0^\infty p(z)x(z)dz - y \le 0, \tag{2b}$$

has a unique solution, x^*. This solution is described by values $\lambda > 0$ and $Z > 0$ such that the allocation of characteristics q^* associated with x^* satisfies

$$u'[q^*(z)] - \lambda p'(z) \le 0, \quad \text{all} \quad 0 \le z, \quad \text{with equality if } z \le Z; \tag{3a}$$

$$q^*(z) \begin{cases} > 0, & 0 \le z < Z, \\ = 0, & z \ge Z. \end{cases} \tag{3b}$$

The optimal allocation of goods x^* satisfies

$$x^*(z) \begin{cases} > 0, & \text{if} \quad 0 \le z < Z \quad \text{and} \quad p''(z) > 0, \\ = 0, & \text{otherwise}. \end{cases}$$

Lemma 2. Let u satisfy Assumption 1; let $\hat{p}$ be given; and let p be the greatest convex function such that $p(0) = 0$ and $p \leq \hat{p}$. If p satisfies the hypotheses of Lemma 1, then the solution to (2) is the same for p and for $\hat{p}$.

The first result, which is Lemma 1 in Stokey (1988), is established by applying a standard variational argument to (2). Under the stated assumptions on p, the first-order condition holds with equality on $[0,Z]$ and with inequality on $[Z,+\infty)$. Differentiating (3a) for $0 \leq z < Z$ and (3b) for $z \geq Z$, one finds that

$$x^*(z) = -q^{*\prime}(z) = \begin{cases} -\lambda p''(z)/u''[q^*(z)], & 0 \leq z < Z, \\ 0, & z \geq Z, \end{cases}$$

so the stated claim about x^* holds.[2] Lemma 2 follows from the observation that if x^* is the solution to (2) for the prices p, then demand is positive only on intervals where p is strictly convex. Since $p = \hat{p}$ along any such interval,

$$\int_0^\infty \hat{p}(z)x^*(z)dz = \int_0^\infty p(z)x^*(z)dz.$$

Since $p \leq \hat{p}$, the stated claim is immediate. The fact that there is no demand for goods in intervals where p is linear will be used later to determine which commodities are produced.

Next consider the technology. Labor is the only factor of production, and units are chosen so that the supply of labor, which is inelastic, is one unit per household in each region. The technology displays constant returns to scale, and the quantity of labor required to produce one unit of any good depends on the level of human capital of the labor used. Specifically, the technology is described by a function $c(z,k)$, where for each $k \geq 0$, the function $c(\cdot,k)$ describes the unit labor input requirements, for workers with human capital k, for all goods. Thus, given the (common) level of human capital, k, of the labor being used, the total labor time required to produce the goods allocation x is

$$\int_0^\infty c(z,k)x(z)dz.$$

The function c describes the technology in both the North and the South, but labor in the two regions may have different levels of human capital.

Two types of restrictions will be placed on c. First, it will be assumed that for each fixed k, the unit cost function $c(\cdot,k)$ has the following shape: It passes through the origin, is strictly increasing, is linear over an interval near the origin, and is strictly convex elsewhere. In addition, it will be assumed that increases in the level of human capital have the following effect on unit costs: The cost of every good falls, the costs of higher-quality goods fall relatively more, and the region of linearity expands. Formally, the following assumption will be used.

Assumption 2. The function c is twice continuously differentiable, with

$$c_1(z,k) > 0, \quad c_2(z,k) < 0, \qquad \text{all } z,\ k \geq 0;$$

$$c(0,k) = 0, \quad \lim_{z\to\infty} c_1(z,k) = +\infty, \qquad \text{all } k \geq 0;$$

and there is a continuously differentiable, strictly increasing function $\sigma(k)$ such that for each k, $c(\cdot,k)$ is linear on $[0,\sigma(k)]$ and strictly convex on $(\sigma(k),+\infty)$. For $\hat{k} > k$, the ratio $c(z,k)/c(z,\hat{k})$ is strictly increasing for $z > \sigma(k)$.

This assumption could be weakened substantially. Suppose that $\hat{c}$ is the underlying technology, and for each k let $c(\cdot,k)$ be the greatest convex function less than $\hat{c}(\cdot,k)$ that passes through the origin. By Lemma 2, it suffices if c satisfies Assumption 2.

Since $x = -q'$, it follows from an integration by parts that for any allocation x containing the characteristics q,

$$\int_0^\infty c(z,k)x(z)dz = \int_0^\infty c_1(z,k)q(z)dz.$$

Hence $c_1(\cdot,k)$ can be interpreted as the unit cost function for characteristics, in the sense that the cost of producing any goods allocation is simply the cost of producing the characteristics it contains. Assumption 2 ensures that the unit costs of characteristics increase without bound as $z \to \infty$.

2. Existence of a Competitive Equilibrium

In this section, the existence and qualitative properties of a competitive equilibrium will be established. Let $h = (h^n, h^s)$ denote the number of households in the North and the South respectively, and let $k = (k^n, k^s)$ denote the levels of human capital. Then given h and k, a competitive equilibrium with free trade must specify quantities x^n, x^s for consumption and y^n, y^s for production per household in each region; wage rates w^n, w^s for each region; and prices p for all goods.

Definition. A competitive equilibrium consists of $(x^n, x^s, y^n, y^s, w^n, w^s, p)$ such that

(4) $$x^i = \underset{x}{\operatorname{argmax}} \int_0^\infty u\left[\int_\xi^\infty x(z)dz\right]d\xi$$

$$\text{s.t.} \quad \int_0^\infty p(z)x(z)dz - w^i \leq 0, \qquad i = n,s;$$

(5) $$y^i = \underset{y}{\operatorname{argmax}} \int_0^\infty [p(z) - w^i c(z,k^i)]y(z)dz, \qquad i = n,s;$$

(6) $$\int_0^\infty c(z,k^i)y^i(z)dz - 1 \leq 0, \qquad i = n,s;$$

(7) $$h^n[x^n(z) - y^n(z)] + h^s[x^s(z) - y^s(z)] \leq 0, \qquad \text{all } z \geq 0.$$

Conditions (4) and (5) specify utility-maximizing and profit-maximizing behavior, and (6) and (7) specify market clearing for each type of labor and each good.

Notice that in equilibrium, produced goods are priced at the unit cost of the minimum-cost supplier, and nonproduced goods have prices no greater than the unit cost of the minimum-cost supplier. That is, (5) is equivalent to the requirement that

$$p(z) \le w^i c(z,k^i), \quad \text{with equality if} \quad y^i(z) > 0, \quad \text{all} \quad z \ge 0, \quad i = n,s.$$

Hence the prices of nonproduced goods are, within some range, indeterminate. This indeterminacy can be resolved as follows.

Without loss of generality, normalize prices by letting $w^s = 1$, and let $w = w^n$ denote the relative wage in the North. For each $w > 0$, define $\pi(\cdot,w)$ to be the greatest convex function such that

$$\pi(z,w) \le \min\{wc(z,k^n),\ c(z,k^s)\}, \quad \text{all} \quad z \ge 0.$$

It follows immediately from Lemma 2 that if (x^n,x^s,y^n,y^s,w,p) is a competitive equilibrium, then so is $(x^n,x^s,y^n,y^s,w,\pi(\cdot,w))$. Attention here will be confined to equilibria in which prices are given by $\pi(\cdot,w)$.

Consider first the case where workers in the two regions have equal levels of human capital: $k^n = k^s = \kappa$. Let $w = 1$; note that $\pi(z,1) = c(z,\kappa)$, all z; and let x^* be the unique solution to the consumer's problem. Clearly the quantities $x^i = y^i = x^*$, $i = n,s$, together with the specified wages and prices, constitute a competitive equilibrium. Clearly, too, there are an infinite number of other equilibria, with the same consumption allocations, prices, and relative wage, and with any production levels $\hat{y}^i$ satisfying the market-clearing conditions (6) and (7). The multiplicity of equilibria occurs because patterns of production are indeterminate when there is no comparative advantage. It will be assumed from this point on that the two regions have different levels of human capital and are labeled so that $k^s < k^n$.

The existence of a competitive equilibrium can be established by looking at how demands in each region depend on the relative wage. For any $w > 0$, define the functions $g^i(\cdot,w)$, $i = n,s$, to be the per capita demands for goods in the two regions, if the relative wage in the North is w and goods prices are given by π. That is,

$$g^i(\cdot,w) = \underset{x}{\operatorname{argmax}} \int_0^\infty u\left[\int_\xi^\infty x(z)dz\right]d\xi$$

$$\text{s.t.} \int_0^\infty \pi(z,w)x(z)dz \le \begin{cases} w, & \text{if } i = n, \\ 1, & \text{if } i = s. \end{cases}$$

By Lemma 1, g^n and g^s are well defined.

Next, define $\underline{w}$ to be the highest relative wage such that the North is the minimum-cost producer of all goods. That is,

(8) $$\underline{w} = \max\{w \geq 0: \ wc(\cdot,k^n) \leq c(\cdot,k^s)\}.$$

Since $k^s < k^n$, it follows from Assumption 2 that $c(z,k^s)/c(z,k^n)$ is strictly increasing in z. Hence by L'Hopital's rule,

$$\underline{w} = \lim_{z \to 0} c(z,k^s)/c(z,k^n) = c_1(0,k^s)/c_1(0,k^n) > 1.$$

Looking ahead, it is clear that $w > \underline{w}$ in any competitive equilibrium.

If $w > \underline{w}$, then Assumption 2 implies that $c(\cdot,k^s)$ and $wc(\cdot,k^n)$ cross once, as shown in Figure 1, and there exists a unique line tangent to both curves. Define the values $m(w)$ and $M(w)$ in terms of this tangent line, as shown. The price function $\pi(\cdot,w)$ is then composed of three segments: it coincides with $c(\cdot,k^s)$ on $[0,m(w)]$; it is linear on $(m(w),M(w))$, lying strictly below both cost curves; and it coincides with $wc(\cdot,k^n)$ on $[M(w),+\infty)$.

The functions m and M are continuous and strictly increasing for $w > \underline{w}$. To see this, refer to Figure 1. A higher relative wage in the North shifts the $wc(\cdot,k^n)$ curve upward, so the new tangent line must have a steeper slope and the point $m(w)$ moves to the right. To see that $M(w)$ also shifts to the right, apply a similar argument to the curves $c(\cdot,k^s)/w$ and $c(\cdot,k^n)$. Note that $m(w)$ and $M(w)$ approach zero as $w \to \underline{w}$ and increase without bound as $w \to \infty$.

The next lemma establishes that for all sufficiently high relative wage rates in the North, the South reverts to autarky.

Lemma 3. Let u and c satisfy Assumptions 1 and 2 respectively. Then there exists $\bar{w} > \underline{w}$ such that $w \geq \bar{w}$ implies $g^s(z,w) = 0$, all $z > m(w)$.

Proof. Consider the maximization problem in (2) with prices given by $c(\cdot,k^s)$ and income equal to unity. By Lemma 1 this problem has a unique solution, call it x^a; let q^a, λ^a, Z^a be the associated values satisfying (3). Thus, x^a is the South's demand for goods in autarky, and the interval $[0,Z^a]$ contains all the goods for which there is positive demand. Define $\bar{w}$ by $m(\bar{w}) = Z^a$; since m is strictly increasing, this is possible. Then

$$\pi_1(z,\bar{w}) = \begin{cases} c_1(z,k^s), & 0 \leq z \leq m(\bar{w}), \\ c_1[m(\bar{w}),k^s] = \bar{w}c_1[M(\bar{w}),k^n], & m(\bar{w}) < z < M(\bar{w}), \\ \bar{w}c_1(z,k^n), & M(\bar{w}) \leq z. \end{cases}$$

Hence q^a, λ^a, Z^a satisfy (3) at the prices $\pi(\cdot,\bar{w})$, so by Lemma 1, $g^s(z,\bar{w}) = x^a(z)$, all z. In particular, $g^s(z,\bar{w}) = 0$, all $z > Z^a$. For $w > \bar{w}$, clearly the solution to the consumer's problem is unchanged. □

Theorem 1 draws on this lemma to establish that a competitive equilibrium exists and that every equilibrium involves a positive volume of international trade. The proof will make use of the functions ℓ^n and ℓ^s defined by

$$\ell^i(w) = \int_0^{m(w)} c(z,k^s)g^i(z,w)dz, \qquad i = n,s.$$

The interpretation is that $\ell^i(w)$ is the derived demand (per capita) in region i for the services of Southern labor, when world prices for goods are those consistent with a relative wage of w. Note that there is a positive volume of trade if and only if $\ell^n(w) > 0$ and $\ell^s(w) < 1$.

Theorem 1. Let u and c satisfy Assumptions 1 and 2 respectively. Then there exists a competitive equilibrium, all equilibria have relative wage rates that lie in the interval $(\underline{w},\bar{w})$, and all equilibria have a positive volume of trade.[3]

Proof. Assumptions 1 and 2 ensure that ℓ^n and ℓ^s are continuous functions; clearly $\ell^n(w) = \ell^s(w) = 0$, all $w \le \underline{w}$; and Lemmas 1 and 3 respectively establish that $\ell^n(w) > 0$ and $\ell^s(w) = 1$, all $w > \bar{w}$. Hence there exists at least one value $w \in (\underline{w},\bar{w})$ such that

$$(9) \qquad \frac{h^n}{h^s}\ell^n(w) + \ell^s(w) = 1.$$

Solutions to (9) correspond to competitive equilibria. To see this, note that (4) holds for prices $\pi(\cdot,w)$ and demands $g^i(\cdot,w)$. Define the y^i's by

$$h^n y^n(z) = \begin{cases} 0, & z \in [0, M(w)), \\ h^n g^n(z,w) + h^s g^s(z,w), & z \in [M(w), +\infty); \end{cases}$$

$$h^s y^s(z) = \begin{cases} h^n g^n(z,w) + h^s g^s(z,w), & z \in [0, m(w)], \\ 0, & z \in (m(w), +\infty). \end{cases}$$

It is clear that (5) and (7) hold, and it is straightforward to show that (9) implies that (6) also holds. Hence every solution to (9) corresponds to a competitive equilibrium. Since all solutions to (9) have $w \in (\underline{w}, \bar{w})$, all have $\ell^n(w) > 0$ and $\ell^s(w) < 1$. Hence all equilibria have a positive volume of international trade. □

It follows from Lemma 1 that in a competitive equilibrium with relative wage w, there are upper bounds $Z^i(w)$, $i = n,s$, on the quality levels consumed in the two regions. And since the North has a higher relative wage, $Z^n(w) > Z^s(w)$. Hence goods of quality $z \in (Z^n(w), +\infty)$ are not produced. Moreover, it follows from Lemma 2 that there is no demand for goods along linear portions of $\pi(\cdot, w)$. Hence goods of quality $z \in [0, \sigma(k^s))$ and $z \in (m(w), M(w))$ are not produced. Thus, in a competitive equilibrium with relative wage w, goods of quality $z \in [\sigma(k^s), m(w)]$ are produced by the South and those of quality $z \in [M(k), Z^n(w)]$ by the North. All of goods produced by the South are both consumed domestically and exported. Of the goods produced by the North, all are consumed domestically, but only the lower quality levels, $[M(w), Z^s(w)]$, are exported. As in Dornbusch, Fischer and Samuelson (1977), the two regions specialize in the production of disjoint sets of goods, with the regions of specialization dictated by Ricardian comparative advantage. And as in Flam and Helpman (1987), there is a gap between the highest quality produced by the South and the lowest produced by the North.

Since households are identical within each region, a move from the autarky equilibrium to any of the free trade equilibria benefits all households. To see this, simply note that for each region, the only effect of free trade is to reduce the prices of imported consumption goods.

3. Uniqueness of the Equilibrium

Uniqueness of the equilibrium will be established by showing that ℓ^n and ℓ^s are monotone functions. This requires two additional restrictions on preferences.

The first requirement is that the price elasticity of demand for each characteristic exceed unity.

Assumption 3. $\quad -u'(q)/qu''(q) > 1, \qquad \text{all} \quad q \geq 0.$

To see why this assumption is needed, consider the effect of an increase in w on the demand for domestic goods in the South. An increase in w raises the price of imported goods in the South; equivalently, it raises the price of characteristics that are contained only in imported goods. Hence the income effect reduces demand for all characteristics and the substitution effect shifts demand toward the characteristics contained in domestic goods. Assumption 3 ensures that demand is sufficiently elastic so that the substitution effect outweighs the income effect.

It follows immediately from Assumption 3 that if the price of a characteristic rises, with the marginal utility of income unchanged, total expenditure on that characteristic falls. To see this, define $\varphi = u'^{-1}$. Then $\varphi(v)$ is the quantity of characteristic z consumed if $\lambda p'(z) = v$, and $v\varphi(v)/\lambda$ is total expenditure on characteristic z. Under Assumption 3, $v\varphi(v)$ is strictly decreasing:

$$(10) \qquad \frac{d[v\varphi(v)]}{dv} = \varphi(v)\left[1 + \frac{u'[\varphi(v)]}{u''[\varphi(v)]\varphi(v)}\right] < 0.$$

Lemma 4 draws on this fact to show that if the prices of characteristics on some interval rise, with other prices unchanged, and if income rises or is unchanged, then the marginal utility of income falls.

<u>Lemma 4</u>. Let u satisfy Assumptions 1 and 3. Let p, $\hat{p}$ satisfy the hypotheses of Lemma 1, and assume that for some $0 \leq a < b \leq \infty$ with $a < \hat{Z}$, $\hat{p}'(z) = p'(z)$ for $z \notin (a,b)$, and $\hat{p}'(z) > p'(z)$ for $z \in (a,b)$. Let $0 < y < \hat{y}$. Let x be the unique solution to (2) for the prices p and income y, define q by (1), and let $\lambda > 0$ and $Z > 0$ be the values for which (3) holds. Define $\hat{x}$, $\hat{q}$, $\hat{\lambda}$, and $\hat{Z}$ analogously. Then $\hat{\lambda} < \lambda$.

<u>Proof</u>. Suppose to the contrary that $\hat{\lambda} \geq \lambda$. Then $\hat{\lambda}\hat{p}'(z) \geq \lambda p'(z)$, all z, with strict inequality on (a,b). Hence $\hat{Z} \leq Z$. Let $\varphi = u'^{-1}$. It follows from (3a) and (10) that

$$\hat{\lambda}\hat{p}'(z)\hat{q}(z) = \hat{\lambda}\hat{p}'(z)\varphi[\hat{\lambda}\hat{p}'(z)] \le \lambda p'(z)\varphi[\lambda p'(z)] = \lambda p'(z)q(z), \quad 0 \le z \le \hat{Z},$$

with strict inequality on $(a,b) \cap [0,\hat{Z}]$. It then follows from the two budget constraints that

$$\hat{\lambda}\hat{y} = \hat{\lambda}\int_0^{\hat{Z}} \hat{p}'(z)\hat{q}(z)dz < \lambda\int_0^{Z} p'(z)q(z)dz = \lambda y,$$

a contradiction. □

The second additional restriction on preferences is that the income elasticity of demand for each good be positive.

<u>Assumption 4</u>. $-u'(q)/u''(q)$ is increasing in q, for all $q \ge 0$.

A utility function that satisfies Assumptions 1, 3 and 4 is

$$u(q) = [(q + a)^{1-1/\gamma} - a^{1-1/\gamma}]/(1 - 1/\gamma), \quad \text{where} \quad a > 0 \quad \text{and} \quad \gamma > 1.$$

It follows immediately from the additively separable form of preferences that the income elasticity of demand for all characteristics is positive; Assumption 4 is substantially stronger. Under it, the following result holds.

<u>Lemma 5</u>. Let u satisfy Assumptions 1 and 4, let p satisfy the hypotheses of Lemma 1, and let $0 \le a < b$. Let $\lambda > 0$, assume

$u'(0) > \lambda p'(b)$, define q by

$$(11) \qquad u'[q(z)] = \lambda p'(z), \qquad a \le z \le b,$$

and define x by (1). Let $\hat{\lambda} < \lambda$, and define $\hat{q}$ and $\hat{x}$ analogously. Then $x(z) \le \hat{x}(z)$, $a \le z \le b$, with strict inequality where $p"(z) > 0$.

<u>Proof</u>. Since u is strictly concave, it follows immediately that $\hat{q}(z) > q(z)$, all $a \le z \le b$. Differentiating (11) with respect to z and using (1), one finds that

$$- u"[q(z)]x(z) = \lambda p"(z), \qquad a \le z \le b,$$

and similarly for the hatted values. If $p"(z) = 0$, then $x(z) = \hat{x}(z) = 0$. If $p"(z) > 0$, then $x(z)$, $\hat{x}(z) > 0$, and it follows from (11) and Assumption 4 that

$$\frac{\hat{x}(z)}{x(z)} = \frac{\hat{\lambda}p"(z)/u"[\hat{q}(z)]}{\lambda p"(z)/u"[q(z)]} = \frac{u'[\hat{q}(z)]/u"[\hat{q}(z)]}{u'[q(z)]/u"[q(z)]} > 1. \quad \square$$

The next result establishes that if preferences satisfy Assumptions 1, 3 and 4, then the competitive equilibrium is unique. The idea behind the proof is as follows. A higher relative wage in the North implies that the characteristics produced by the North are relatively more expensive. Since the price elasticity of demand for characteristics exceeds unity, total expenditures on those characteristics is then lower. Because the income elasticity of demand for all goods is positive, the demand for the South's goods--whose prices are unchanged--is then higher. Hence the derived demand for the labor services of the South is strictly increasing in w.

Theorem 2. Let u satisfy Assumptions 1, 3 and 4, and let c satisfy Assumption 2. Then the competitive equilibrium is unique.

Proof. By Theorem 1, it suffices to show that ℓ^n and ℓ^s are strictly increasing on $(\underline{w},\bar{w})$. That is, it suffices to show that for $w \in (\underline{w},\bar{w})$ and $z \in [0,m(w)]$, the functions $g^i(z,w)$, $i = n,s$, are increasing in w and are strictly increasing on a subinterval.

For any $w > 0$, it follows from Lemma 1 that there exists $\lambda^i(w) > 0$ and $Z^i(w) > 0$, $i = n,s$, such that the allocation of characteristics $q^i(\cdot,w)$ associated $g^i(\cdot,w)$ satisfies

(12a) $$u'[q^i(z,w)] \le \lambda^i(w)\pi_1(z,w), \quad 0 \le z, \text{ with equality if } z \le Z^i(w);$$

(12b) $$q^i(z,w) \begin{cases} > 0, & \text{if } 0 \le z < Z^i(w), \\ = 0, & \text{if } z \ge Z^i(w); \end{cases}$$

(12c) $$\int_0^\infty \pi_1(z,w)q^i(z,w)dz = \begin{cases} w, & \text{if } i = n, \\ 1, & \text{if } i = s. \end{cases}$$

Moreover, for $w < \bar{w}$, it follows that $M(w) < Z^s(w) < Z^n(w)$. Since $\pi_1(z,w) = wc_1(z,k^n)$, all $z \geq M(w)$, it follows from Lemma 4 that λ^n and λ^s are strictly decreasing on $(\underline{w},\bar{w})$.

Choose any $w, \hat{w} \in (\underline{w},\bar{w})$, with $w < \hat{w}$. Since m is an increasing function,

$$\pi_1(z,w) = \pi_1(z,\hat{w}) = c_1(z,k^s), \quad 0 \leq z \leq m(w).$$

Since $\lambda^i(w) > \lambda^i(\hat{w})$, it then follows from (12a) and Lemma 5 that $g^i(z,w) \leq g^i(z,\hat{w})$, all $0 \leq z \leq m(w)$, with strict inequality wherever $c_{11}(z,k^s) > 0$. Hence ℓ^n and ℓ^s are strictly increasing on $(\underline{w},\bar{w})$. □

Under the hypotheses of Theorem 2, then, the competitive equilibrium is unique. The equilibrium prices and quantities depend on the exogenously given parameters $h = (h^n,h^s)$ and $k = (k^n,k^s)$, the population sizes and human capital levels in the two region. Since the production technology displays constant returns to scale, however, it is clear that only the relative size of the two populations matters. Let $\gamma = h^n/h^s$. In the next two sections, the effects of the parameters γ and k on equilibrium prices and quantities will be studied.

To do this, it is useful to define Π, G^i, and L^i, $i = n,s$, to be the functions describing equilibrium prices, demands for goods, and derived demands for Southern labor services, as functions of γ and k. These parameters may take on any values with $0 < \gamma$ and $0 \le k^s < k^n$.

For each (γ,k), let $\omega(\gamma,k)$ denote the unique equilibrium relative wage. Then define $\Pi(\cdot;\gamma,k)$ to be the greatest convex function such that

$$\Pi(z;\gamma,k) \le \min\{\omega(\gamma,k)c(z,k^n),\ c(z,k^s)\}, \quad \text{all} \quad z \ge 0.$$

Define $m(\gamma,k)$ and $M(\gamma,k)$, as before, by the unique tangent to $c(\cdot,k^s)$ and $\omega(\gamma,k)c(\cdot,k^n)$. Also, for each region define the per capita demand for goods and the per capita derived demand for labor services of the South as before:

$$G^i(\cdot;\gamma,k) = \underset{x}{\text{argmax}} \int_0^\infty u\left[\int_\xi^\infty x(z)dz\right]d\xi$$

$$\text{s.t.} \quad \int_0^\infty \Pi(z;\gamma,k)x(z)dz \le \begin{cases} \omega(\gamma,k), & \text{if} \quad i = n, \\ 1, & \text{if} \quad i = s; \end{cases}$$

and

$$L^i(\gamma,k) = \int_0^{m(\gamma,k)} c(z,k^s)G^i(z;\gamma,k)dz, \quad i = n,s.$$

Next, note that by Lemma 1, for any (γ,k) there exist functions $\Lambda^i(\gamma,k) > 0$ and $Z^i(\gamma,k) > 0$, $i = n,s$, such that the allocation of characteristics $Q^i(\cdot;\gamma,k)$ associated with $G^i(\cdot;\gamma,k)$ satisfies

$$\text{(13a)} \qquad u'[Q^i(z;\gamma,k)] \le \Lambda^i(\gamma,k)\Pi_1(z;\gamma,k), \quad 0 \le z,$$

$$\text{with equality if } z \le Z^i(\gamma,k);$$

$$\text{(13b)} \qquad Q^i(z;\gamma,k) \begin{cases} > 0, & 0 \le z < Z^i(\gamma,k), \\ = 0, & z \ge Z^i(\gamma,k); \end{cases}$$

$$\text{(13c)} \qquad \int_0^\infty \Pi_1(z;\gamma,k)Q^i(z;\gamma,k)dz = \begin{cases} \omega(\gamma,k), & \text{if } i = n, \\ 1, & \text{if } i = s. \end{cases}$$

Moreover, Theorem 1 implies that $M(\gamma,k) < Z^s(\gamma,k) < Z^n(\gamma,k)$, all (γ,k).

Finally, note that (9) implies that

$$\text{(14)} \qquad \gamma L^n(\gamma,k) + L^s(\gamma,k) = 1, \quad \text{all } (\gamma,k).$$

4. Population Size

In this section the effects of changes in the relative population size will be analyzed. Throughout this section, the levels of human capital in the two regions will be taken as fixed, and k will be suppressed as an argument of all functions.

First it will be shown that the relative wage in the North is a strictly decreasing function of the relative size of the Northern population. The idea behind this result is as follows. Suppose the Northern population increases. If the relative wage in the North were to rise, then the characteristics produced only by the North would become relatively more expensive. Since the price elasticity of demand for characteristics exceeds unity, total expenditures on these characteristics would decrease, and demand for Southern goods would increase. This is incompatible with equilibrium. Hence the relative wage in the North must fall.

<u>Theorem 3</u>. Under the hypotheses of Theorem 2, ω is strictly decreasing in γ, with $\lim_{\gamma\to 0}\omega(\gamma) = \bar{w}$ and $\lim_{\gamma\to\infty}\omega(\gamma) = \underline{w}$ (where $\bar{w}$ is defined in the proof of Lemma 3 and $\underline{w}$ in (8)).

<u>Proof</u>. Choose $\gamma < \hat{\gamma}$, and suppose that $\omega(\gamma) \le \omega(\hat{\gamma})$. Then it would follow from Lemma 4 that $\Lambda^i(\gamma) \ge \Lambda^i(\hat{\gamma})$, $i = n,s$. It would also follow that $m(\gamma) \le m(\hat{\gamma})$, so that

$$\Pi_1(z;\gamma) = \Pi_1(z;\hat{\gamma}) = c_1(z,k^s), \quad \text{all } 0 \le z \le m(\gamma).$$

It would then follow from (13a) and Lemma 5 that $G^i(z;\gamma) \le G^i(z;\hat{\gamma})$, all $0 \le z \le m(\gamma)$, $i = n,s$. It would then follow that

$$L^i(\gamma) = \int_0^{m(\gamma)} c(z,k^s)G^i(z;\gamma)dz \le \int_0^{m(\hat{\gamma})} c(z,k^s)G^i(z;\hat{\gamma})dz = L^i(\hat{\gamma}), \quad i = n,s,$$

contradicting the fact that (14) must hold for γ and for $\hat{\gamma}$. Hence $\gamma < \hat{\gamma}$ implies $\omega(\gamma) > \omega(\hat{\gamma})$.

As $\gamma \to 0$, (14) implies that $L^s(\gamma) \to 1$. As shown in the proof of Lemma 3, this implies that $\omega(\gamma) \to \bar{w}$. As $\gamma \to \infty$, (14) implies that $L^n(\gamma) \to 0$. By the definition of $\underline{w}$, this implies that $\omega(\gamma) \to \underline{w}$. □

The interpretation of the limiting values is straightforward. The first says that as the relative population size in the North approaches zero, the relative wage in the North rises just enough to drive to zero demand by the representative Southerner for Northern goods. The second says that as the relative population size in the North grows without bound, the relative wage in the North falls just enough to drive to zero demand by the representative Northerner for Southern goods.

The next result draws on Theorem 3 to look at patterns of production and consumption in both regions and at patterns of trade. It shows that a larger relative population size in the North has the following effects. The spectrum of goods produced by the North, all of which are consumed domestically, expands at both ends. The spectrum of Northern exports also expands at both ends. The spectrum of goods produced by the South, all of which are consumed domestically and exported, contracts at the upper end.

Theorem 4. Under the hypotheses of Theorem 2, the functions m and M are strictly decreasing in γ, and Z^s and Z^n are strictly increasing in γ.

Proof. Since ω is strictly decreasing in γ, it follows immediately from the definitions of m and M that both are decreasing in γ.

Next consider Z^s. Let $\gamma < \hat{\gamma}$; then $\omega(\gamma) > \omega(\hat{\gamma})$, $m(\gamma) > m(\hat{\gamma})$, and $M(\gamma) > M(\hat{\gamma})$. Lemma 4 then implies that $\Lambda^s(\gamma) < \Lambda^s(\hat{\gamma})$, and Lemma 5 implies that $G^s(z;\gamma) > G^s(z;\hat{\gamma})$, for all $0 \le z \le m(\hat{\gamma})$. The budget constraint for the South then implies that $G^s(z;\gamma) < G^s(z;\hat{\gamma})$, for all $z \ge M(\gamma)$. By the argument used to prove Lemma 5, this happens if and only if $\Lambda^s(\gamma)\omega(\gamma) > \Lambda^s(\hat{\gamma})\omega(\hat{\gamma})$. This inequality in turn implies that

$$(15) \quad c_1[Z^s(\gamma),k^n] = u'(0)/\Lambda^s(\gamma)\omega(\gamma) < u'(0)/\Lambda^s(\hat{\gamma})\omega(\hat{\gamma}) = c_1[Z^s(\hat{\gamma}),k^n].$$

Since $c(\cdot,k^n)$ is strictly convex, it follows that $Z^s(\gamma) < Z^s(\hat{\gamma})$.

An analogous argument shows that Z^n is also increasing in γ. To see this, take the domestic wage in the North to be unity, the relative wage in the South to be $1/\omega$, and world prices for goods to be Π/ω. Let $\gamma < \hat{\gamma}$, so that $1/\omega(\gamma) < 1/\omega(\hat{\gamma})$ and $M(\gamma) > M(\hat{\gamma})$, and note that Lemmas 4 and 5 still apply to the Northern household's problem stated in terms of the renormalized prices. Let μ^n be the multiplier for the problem so stated. Then Lemma 4 implies that $\mu^n(\gamma) > \mu^n(\hat{\gamma})$, so that

$$c_1[Z^n(\gamma),k^n] = u'(0)/\mu^n(\gamma) < u'(0)/\mu^n(\hat{\gamma}) = c_1[Z^n(\hat{\gamma}),k^n].$$

Hence $Z^n(\gamma) < Z^n(\hat{\gamma})$. □

Since an increase in the relative size of the Northern population causes the North's relative wage to fall, it makes Southern goods less competitive. Hence the spectrum of goods produced by the South contracts at the upper end and the spectrum produced by the North expands at the lower end. Note that the lower endpoint of the spectrum produced by the South, $\sigma(k^S)$, is unchanged.

The lower relative wage in the North reduces import prices in the South, causing Southern households to decrease their consumption of domestic goods. Since their incomes are unchanged and prices of imported goods have fallen, their consumption of imported goods must rise. Part of this increase takes the form of an expansion of the upper endpoint of the spectrum of goods imported by the South. Note that the volume of imports per capita in the South, measured in units of Northern labor, increases.

An increase in the relative size of the population in the North raises the relative wage in the South, increasing the price of imports from the South, with domestic wages and prices in the North unchanged. Hence Northern households shift expenditures away from imports and toward domestic goods. Note that the volume of imports per capita in the North, measured in units of Southern labor, decreases.

These arguments also describe the effects of a move from autarky to free trade. For the South this change is an increase in γ, so it shifts downward the upper endpoint of the spectrum of goods produced in the South, leaving the lower endpoint unchanged. Consumption in the South shifts towards higher-quality goods, as imports displace consumption of some domestic goods. In the North, a move from autarky to free trade is a decrease in γ, so it shifts upward both endpoints of the set of goods produced in the North. Consumption in the North shifts towards lower-quality goods, as imports displace consumption of some domestic goods.

Finally, it is evident that an increase in the relative size of the Northern population decreases Northern welfare and increases Southern welfare. This fact also implies that in a world with populations of fixed size, and with a small number of high-skill workers in the South and a small number of low-skill workers in the North, a shift from autarky to free trade reduces the welfare of the small groups in each region.

5. Human Capital Levels

In this section the effects of changes in the levels of human capital in the two regions will be studied. Throughout the section, the relative size of the Northern population will be taken as fixed, and γ will be suppressed as an argument of all functions.

First it will be shown that the relative wage in the North is strictly increasing in the Northern level of human capital and strictly decreasing in the Southern level. The idea behind this result is as follows. Suppose an increase in Northern human capital were to reduce the North's relative wage. Then the characteristics produced only by the North would become relatively cheaper. Since the price elasticity of demand for characteristics exceeds unity, total expenditures on these characteristics would increase, and demand for Southern goods would fall. This is incompatible with equilibrium.

Theorem 5. Under the hypotheses of Theorem 2, ω is strictly increasing in k^n and strictly decreasing in k^s.

Proof. Note that (14) implies that any change in k must produce changes of opposite sign in L^n and L^s. To show that ω is strictly increasing in k^n, it will be shown that the contrary would imply that L^n and L^s are both strictly decreasing in k^n.

Choose $k = (k^n, k^s)$ and $\hat{k} = (\hat{k}^n, k^s)$, with $k^s < k^n < \hat{k}^n$. Suppose that $\omega(k) \geq \omega(\hat{k})$. Then it would follow that $m(k) > m(\hat{k})$, so that

$$\Pi_1(z,k) = \Pi_1(z,\hat{k}) = c_1(z,k^s), \quad \text{all} \quad 0 \leq z \leq m(\hat{k}).$$

Moreover, it would follow from Lemma 4 that $\Lambda^i(k) \leq \Lambda^i(\hat{k})$, $i = n,s$, and from (13a) and Lemma 5 that $G^i(z,k) \geq G^i(z,\hat{k})$, all $0 \leq z \leq m(\hat{k})$, $i = n,s$. Hence

$$L^i(k) = \int_0^{m(k)} c(z,k^s)G^i(z,k)dz > \int_0^{m(\hat{k})} c(z,k^s)G^i(z,\hat{k})dz = L^i(\hat{k}), \quad i = n,s,$$

contradicting the fact that (14) must hold for k and for $\hat{k}$.

To show that ω is strictly decreasing in k^s, renormalize wages and prices so that the wage in the North is unity, the relative wage in the South is $1/\omega$, and goods prices are Π/ω; define the derived demand for Northern labor services; and use an argument exactly analogous to the one above. □

Note that an increase in either country's human capital necessarily improves domestic welfare. With the domestic wage normalized to unity, the increase reduces the prices of both domestic goods (because unit labor input requirements fall, with the wage unchanged) and imports (because the wage falls, with input requirements unchanged). To determine the effects on patterns of production, consumption, and trade, and on welfare in the country with unchanged human capital, the changes in Λ^n and Λ^s must be determined.

Suppose there is an increase in the North's human capital, with the South's unchanged. Let $k = (k^n, k^s)$ and $\hat{k} = (\hat{k}^n, k^s)$, with $k^n < \hat{k}^n$. Since world prices are unchanged for Southern goods, it follows from Lemma 5 that $L^i(k) \lesseqqgtr L^i(\hat{k})$ as $\Lambda^i(k) \gtreqqless \Lambda^i(\hat{k})$. Hence, in order to maintain equilibrium in the Southern labor market, the multipliers Λ^n and Λ^s must move in opposite directions, or else both remain unchanged. There are two possibilities, which can be analyzed by considering two extreme cases.

First, suppose that the increase in k^n causes the unit costs of all goods produced by the North to fall by the same percentage:

$$c(z,\hat{k}^n)/c(z,k^n) = 1/\rho, \quad \text{all} \quad z \geq 0, \quad \text{where} \quad \rho > 1.$$

(This is incompatible with Assumption 2, but can be thought of as a limiting case.) Suppose that the relative wage in the North were to rise by the factor ρ. Then all goods prices would be unchanged, income in the South would be unchanged, and income in the North would be higher by ρ. Hence by Lemma 4, Λ^s would be unchanged and Λ^n would be lower. As noted above, this would violate the market-clearing condition (14). By the argument in the proof of Theorem 2, a lower relative wage is needed to clear markets. Hence $1 < \omega(\hat{k})/\omega(k) < \rho$. Let $\theta = \omega(\hat{k})/\omega(k)$. Then

$$(16) \qquad \omega(\hat{k})c(z,\hat{k}^n) = \theta\omega(k)c(z,k^n)/\rho < \omega(k)c(z,k^n), \quad \text{all} \quad z \geq 0.$$

This fact has several consequences. First, it implies that m and M fall, so the spectrum of goods produced by the South contracts at the upper end and the spectrum produced by the North expands at the lower end. Second, it implies that in the South, all import prices fall. Since its income and the prices of its domestic goods are unchanged, the South is better off. Third, it implies, by Lemma 4, that Λ^s rises. Hence by Lemma 5, the South decreases its consumption of each domestic good. Budget balance for the South then implies that its expenditure on imports must rise. Since the prices of all imports have fallen, the volume of imports rises. This also implies that Z^s rises, so the spectrum of goods imported by the South expands at the upper end. Fourth, the fact that Λ^s rises implies that Λ^n falls. Hence by Lemma 5, the North buys more of each Southern good. Finally, note that (13a) must hold for k and for $\hat{k}$. Moreover, (16) implies that

$$\Lambda^n(\hat{k})\Pi_1(z;\hat{k}) = \Lambda^n(k)\Pi_1(z;k^n)\alpha\theta/\rho, \quad \text{all} \quad z \geq M(k),$$

where $\alpha = \Lambda^n(\hat{k})/\Lambda^n(k) < 1$. Hence Lemma 5 implies that the North buys more of each domestic good, and an inequality like (15) implies that Z^n rises: the spectrum of goods produced by the North for domestic consumption expands at the upper end.

Alternatively, suppose that the increase in k^n leaves costs unchanged on $[0, Z^s(k)]$ and reduces them by the factor $1/\rho$ on $(Z^s(k), +\infty)$. (Again, this is incompatible with Assumption 2, but may be thought of as a limiting case.) By Theorem 5, the relative wage must rise to restore equilibrium, and this has a number of consequences. First, it implies that m and M increase, so the spectrum of goods produced by the South expands at the upper end and the spectrum produced by the North contracts at the lower end. Second, it implies that for the South, import prices rise, with income and the prices of domestic goods unchanged. Hence welfare falls in the South. Third, by Lemma 4, it implies that Λ^s falls. Hence by Lemma 5, the South increases its consumption of each domestic good. Budget balance for the South then implies that expenditures on imports fall. Since import prices have risen, consumption of imports falls and Z^s falls, so the spectrum of goods imported by the South contracts at the upper end. Fourth, since Λ^s falls, Λ^n rises, and by Lemma 5, the North consumes less of each Southern good. Hence the North increases expenditures on domestic goods, and Z^n increases, so the spectrum of goods produced by the North for domestic consumption expands at the top of its range.

As these two examples illustrate, the effects of an increase in the level of human capital in the North depend on how costs are affected by the change. In the first example, the South enjoys a favorable shift in its terms of trade, the volume of trade grows, and more intense price competition from imports reduces the range of higher-quality goods produced by the South. In the second example, the South suffers an unfavorable shift in its terms of trade, the volume of trade declines, and reduced price competition from imports causes the South to begin producing some higher-quality goods. These two examples illustrate two extremes. In general, changes in k^n will affect costs in a less extreme way, so the overall impact on production, consumption, trade, and Southern welfare may go in either direction.[4]

An increase in the level of human capital in the South that reduces the costs of all goods by a constant factor $1/\rho$, where $\rho > 1$, is completely analogous to the first example above. To see this, it is convenient to renormalize prices so that the Northern wage is unity. Then the relative wage in the South increases, but by less than ρ. This has several effects. First, it implies that m and M rise, so the spectrum of goods produced by the South expands at the upper end and the spectrum produced by the North contracts at the lower end. Note that the lower endpoint of the Southern spectrum also increases, since σ is increasing in k^s. Second, it implies that in the North, all import prices fall. Since its income and the prices of its domestic goods are unchanged, Northern welfare increases. Third, the North decreases its consumption of each domestic good, so the highest-quality good produced and consumed in the North, Z^n, decreases. The North increases its consumption of all imports, however. Finally, the South buys more of all goods, and the upper endpoint z^s of the spectrum of Southern imports increases.

The second example above used the fact that the North produces some goods for domestic consumption only. For the South there is no analogue, since all of the goods it produces are consumed in both regions.

6. Equilibrium Dynamics: The Product Cycle

A rigorous analysis of dynamic issues is beyond the scope of this paper, but it is interesting to speculate on the qualitative properties of the product cycle this model is likely to generate. Assume that there are no international credit markets. Since any such loans would be pure consumption loans, this assumption is not unreasonable. Then trade flows must be balanced at each point in time, and the dynamic equilibrium can be found by solving for a sequence of competitive equilibria of the type studied above.

Suppose that both populations grow at the same rate, so that the ratio γ is constant over time. Assume that both regions experience increases in the level of human capital, and that the level of human capital in the North always exceeds that in the South. Finally, assume that the upper and lower boundaries of produced goods in both region shift upward over time, and that the quality level of the highest-quality good consumed by the South also increases over time. This would seem to be the most likely pattern for growth.[5]

In this case, high-quality products are continuously introduced in the North, and the life cycle of a typical product is as follows. Each new product is first introduced in the North, where it is (exclusively) produced and consumed. After a time lag, the South begins to import the product. After another time lag, the product temporarily disappears from world markets, as it moves into the gap between the lowest-quality good produced in the North and the highest-quality produced by the South. Later still, the South starts producing the good, and it is consumed in both regions. Finally, the good again disappears from production, this time permanently, as it is displaced by higher-quality products.

It is also interesting to speculate about the long-run effects of free trade on human capital accumulation. Suppose that human capital accumulation is stimulated by the production of high-quality goods. This might occur because of learning by doing, for example. Then free trade will speed up human capital accumulation in the North and slow it down in the South. It does not follow, however, that the South would be better off under autarky. Whether this is so will depend on the size of the short-term benefits to trade, and on how much benefit, due to more rapidly falling prices, the South enjoys from the North's faster growth. The South seems most likely to suffer if the regions are initially quite similar, since then the short-term gains from trade are almost nil, but the effects on learning may be quite substantial.

7. Conclusions

The model analyzed above displays the basic features of trade between rich and poor countries discussed in the introduction. First, consumption patterns are different in the two regions, with goods at the highest end of the produced quality spectrum consumed only in the North. Second, as the level of human capital in the South increases over time, low-quality goods drop out of production. Third, the wage differential between the two regions arises not from monopoly power in the North over access to blueprints, but from a difference in the level of skill of the two labor forces. These were also features of the model studied by Flam and Helpman (1987), and the similarity of the results obtained here, in a model with very different features, suggests that these conclusions may be robust to a fairly wide range of specifications of quality differentials.

A fruitful area for further research will be to use models of quality differentiation to address dynamic issues. This will require making specific assumptions about the factors affecting population growth and human capital accumulation, but the analysis thus far suggests that such models may be quite tractable and very interesting.

Footnotes

[1]Although all of the authors mentioned above pay tribute to Vernon, their models of the product cycle--like the one in the present paper--have little to do with the issues discussed by him. Vernon stressed that the demand for certain types of consumer goods, especially labor-saving goods, is greatest in the highest-income countries, and that familiarity with the tastes of potential buyers is a critical factor in successful product innovation. Hence it is proximity to the product market, not the availability of engineers or the skill level of the workforce, that determines where new products are developed and first produced. After the product and the production process are standardized, production facilities are moved to less developed countries, to take advantage of lower labor costs. In Vernon's view, however, the latter location decision is made by producers in the more advanced countries, and it has nothing to do with reverse engineering in the low-wage countries or with skill differentials betweeen the two regions.

[2]The left and right derivatives of q^* differ at $z = Z$, so $x^*(Z)$ is not determined. This is unimportant, however, since x^* always appears in an integral. For notational simplicity, the convention $x^*(Z) = 0$ is adopted here.

[3]The existence of an equilibrium also follows from a minor variation on the much more general result proved in Theorem 1 of Jones (1984). The proof for the particular model under study here is substantially simpler, however, and also provides qualitative information about the equilibrium.

[4]Flam and Helpman (1987) also obtain the possibility of two, qualitatively different responses to a productivity increase in the North, depending on whether the increase in the North's relative wage is or is not big enough to offset the productivity change.

[5]In a one-country context, Assumption 2 ensures that the lower boundary, $\sigma(k_t)$, increases over time if k_t increases. In Stokey (1988, Lemma 2), it is shown that the upper boundary also increases if for any $\hat{k} > k$, the ratio $c_1(z,k)/c_1(z,\hat{k})$ is strictly increasing for $z > \sigma(k)$. The latter condition is a strengthening of Assumption 2.

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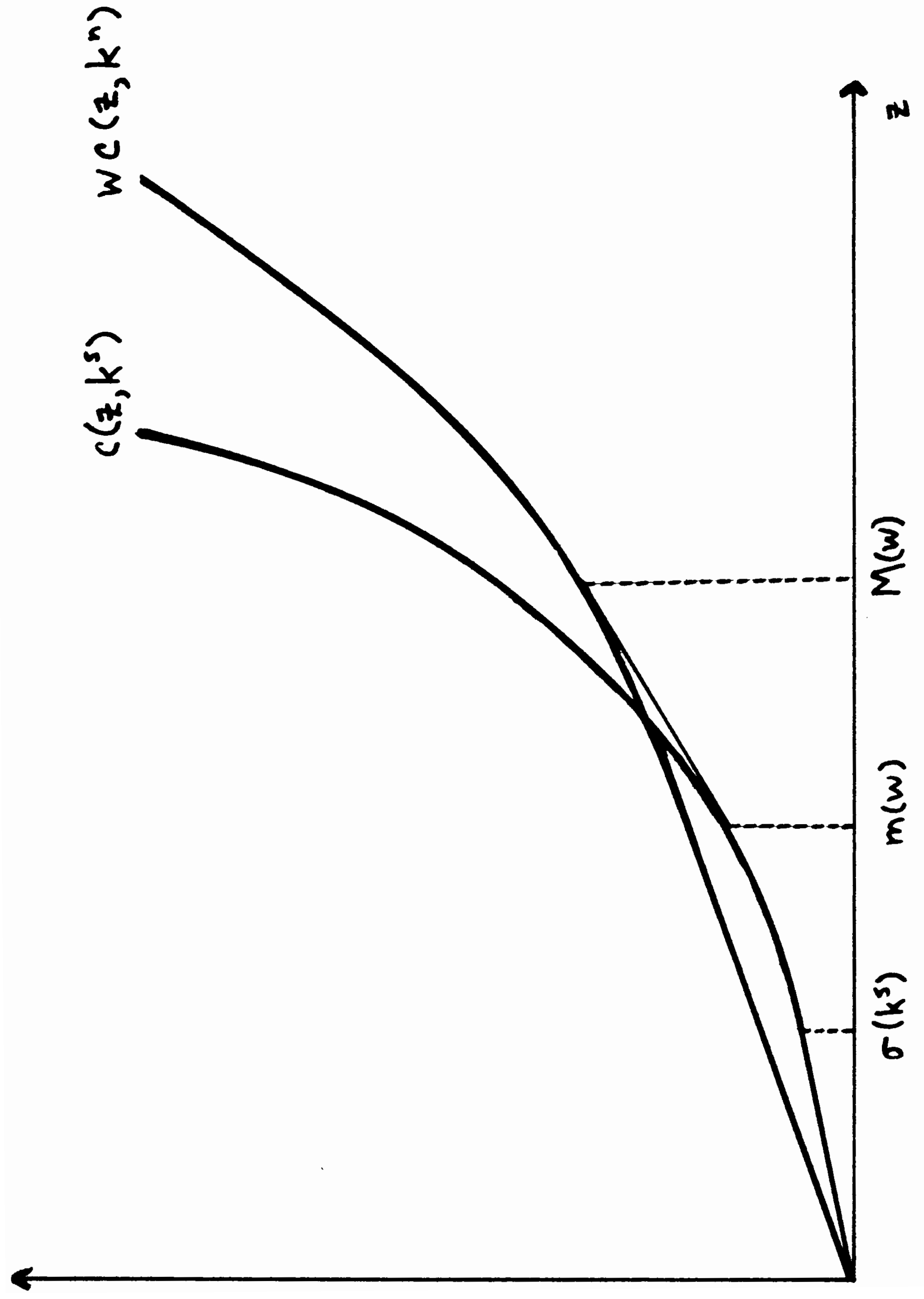

Figure 1