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SEARCH AND PRICE DISPERSION IN AN INFLATIONARY ECONOMY *

by

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ABSTRACT

We present a simple overlapping generations search model of an inflationary economy in which money is the only store of value and identify an inflation based endogenously determined search cost. The latter reflects the fact that resources required for future consumption are random and are thus exposed to excessive erosion. We analyze the effect of the constant inflation rate on the equilibrium real price distribution, output and welfare. An increase in the rate of inflation increases production efficiency and may increase welfare.

Introduction

We present a simple overlapping generations equilibrium search model in which money is the only store of value and the only medium of exchange. In this framework we study the effect of a constant, fully anticipated inflation on the search equilibrium and in particular on equilibrium price dispersion, output and welfare.

As in all overlapping generations models, consumers receive their endowment at birth and use it to finance consumption throughout their lives. In the present model, the economy produces a competitively supplied numeraire good and a "search good" about whose price consumers are imperfectly informed and which is therefore characterized by price dispersion.

When the rate of inflation is positive, resources allocated to consumption in the consumer's second period are subject to inflationary erosion. This real cost of inflation in a dynamic setting has, of course, been extensively explored in the literature, most notably in models of Baumol-Tobin and transactions costs effects (Baumol (1952),Tobin (1965), Helpman and Sadka (1979), Jovanovich (1982), Rotemberg (1984), Sidrauski (1967), Stockman (1981), Romer (1986), Grossman (1982), Grossman and Weiss (1983), Kimborough (1986)). Here the presence of market imperfections in the model makes it possible to identify a novel real effect. Because a young consumer perceives the price of the search good in her second period as a random variable, she on average allocates either too few or excessive reources for its future consumption. In either case, her endowment is subjected to more inflationary depreciation than is "necessary", i.e. than would be the case if future prices were perfectly predictable. This inflation based, endogenously determined "search cost" leads to the allocation of fewer real resources for future consumption on the part of young consumers. In equilibrium this cost is reflected by firms' pricing decisions, affecting the equilibrium distribution of real prices, output and welfare in complex ways. In particular, it is shown that an increase in the rate of inflation decreases the maximum real price in the market and leads to the exit of inefficient firms. Simulation studies reveal a tendency for a decrease in the average real price of the search good and a surprising increase in consumers' welfare associated with moderate increases in inflation.

The preceding effects are a consequence of market imperfections in our model. If markets were perfect, real prices would always be equated with real marginal production costs, and would therefore be independent of the rate of inflation. Consequently, consumers' welfare would be unambiguously reduced by increased inflation. This is the source of the contrast between the welfare predictions of our model and those of most of the Baumol-Tobin and transaction costs literature which emphasizes the loss of welfare imposed by accelerated inflation, deriving from distortions of the flow of consumption associated with increased costs of managing financial assets. Because the existing literature assumes perfect markets, consumers cannot be "compensated" by firms for the increased expense of future consumption. Here, because firms enjoy a measure of monopoly power, the increased cost of future consumption is internalized by firms in equilibrium, leading to "compensatory" changes in real prices which can increase welfare.

Ben Abou (1988) presents an alternative approach to the analysis of search in an inflationary economy based on the "menu cost" models developed by Akerloff and Yellen (1985), Fischer (1977, 1983), Mankiw (1985), Taylor (1979) Blanchard (1983, 1986) and others. Price adjustments are costly, consequently occur infrequently and are staggered across sellers. This friction on the supply side of the market coupled with imperfect information on the demand side is shown by Ben Abou to provide a novel source of price dispersion.

The remainder of the paper is organized as follows. Section 1 presents the model and the equilibrium concept. The analysis of the equilibrium is carried out in two stages. In section 2 we analyze the equilibrium for the case of a zero inflation rate. Section 3 expands the analysis to include positive rates of inflation. Section 4 provides some simulation studies of the equilibrium. Section 5 concludes.

1. A Model of Dynamic Price Dispersion with Overlapping Generations

Two different products are produced in the economy, a competitively supplied numeraire good and a "search good", which is characterized by imperfect information on the part of consumers. The description of the search good market is similar to that of Albrecht and Axell (1984).

Consumers:

A new cohort of identical consumers enters the economy in each period. Each consumer lives for two periods and receives a monetary endowment in the first period of her life. We assume that each consumer entering the market at period t receives a monetary endowment of $M(1+\pi)^t$ where M is the endowment of the first generation and $\pi \geq 0$ is a given constant. While marginal utility from the numeraire good is constant a consumer demands only a single unit of the search good at each period of her life.[1] The first unit of the search good is more valuable to the consumer than the second unit. Specifically each consumer's utility function is assumed to be:

$$u = c + k_1 \alpha + k_2 \beta$$

where

$$k_i = \begin{cases} 1 & \text{if the i'th unit of the search good is purchased} \\ 0 & \text{otherwise} \end{cases}$$

where $\alpha > \beta$ are exogenously given parameters and $c = c_1 + c_2$, where c_i is the quantity of the numeraire good consumed at the i^{th} period of

[1]The assumption of unitary demand is characteristic of most of the price dispersion literature. A notable excetion is Reinganum (1979).

life. Consumers are imperfectly informed about the price of the search good. In each period of her life a consumer costlessly receives a single price quotation from a randomly selected firm. She can buy at that price or "search" for a lower price by waiting for a new price quotation in the following period (if there is one). For simplicity we deny consumers the option of recalling price quotations from previous periods.

We denote by p_t^c the nominal price of the numeraire good at period t and by p_t^s the nominal price paid for the search good at period t. Clearly if prices of the search good are non-degenerately distributed, p_t^s is unknown to the individual consumers prior to period t. Given her utility function, the decision problem of a consumer at her second period is whether to buy the search good at the offered price or to spend the balance of her endowment on the numeraire good. The optimal decision depends on whether a unit of the search good has previously been purchased. If it has, the second period maximization problem is:

$$\text{Max} \quad c_2 + k_2\beta$$
$$\text{s.t.}$$
$$c_2\, p_2^c + p_2^s = m$$

where m is the cash held over to the second period. If the search good was not purchased at the first period, a similar optimization problem is solved with k_1 and α substitutng for k_2 and β.

At their first period, consumers decide whether to buy the search good at the price offered and on the quantity of the numeraire good to purchase. m, the amount of cash allocated to the next period is determined by this decision.

From the above framework it is evident that at every period there is a variety of consumer types in the market. First, as in all overlapping generations models, young and old consumers exist in the market simultaneously. However, the old consumers are not identical. They may be differentiated on the basis of whether they have previously consumed a unit of the search good or not and according to the amount of the endowment spent in their first period. We let T_t denote the set of types of consumers at period t.

Firms:

There is a continuum of firms producing the numeraire good and the search good using a single input denoted by ℓ. Each unit of the numeraire good is produced using one unit of input. The search good is produced by a second set of firms using λ units of the input. We assume that λ is uniformly distributed over $[0,a]$[2] and let μ be the measure of consumers per firm when all firms are operative.

[2]Even if there exist firms for which $\lambda > a$, it is clear from our analysis that these must be inoperative in equilibrium.

Government:

In this model the government plays a simple role. First, the government owns the input ℓ which it sells to firms at a constant real price, i.e. $p_t^{\ell} = p_{t-1}^{\ell}(1+\pi)$. At each period it collects all the firms' profits as taxes, issues new fiat money sufficient to keep the nominal money supply growing at the rate π and distributes the sum, $M(1+\pi)^t$, to newly born consumers.

Equilibrium:

We seek to characterize a symmetric steady-state equilibrium in which all prices inflate at the constant rate π. In equilibrium no consumer or firm can benefit by unilaterally changing its behavior, i.e., its search rule or price.

For a specific value of π, let $G_\pi(\tau)$ be the stationary distribution of consumer types, $F_\pi(\tau)$ the stationary distribution of real prices for the search good, $\bar{p}_\pi(\tau)$ the reservation price of a consumer of type τ (i.e., a consumer of type τ buys iff she observes a price not exceeding $\bar{p}_\pi(\tau)$), $m_\pi(\tau)$ the balance of the endowment with which a consumer of type τ begins her second period, μ_π the measure of consumers per operative firm, and p_π^c the real price of the numeraire. An equilibrium is defined as

$$< p_\pi^c,\ F_\pi(p^s),\ G_\pi(\tau), \mu_\pi,\ \bar{p}_\pi(\tau),\ m_\pi(\tau) >$$

such that:

(i) Given p^c_π and $F_\pi(\cdot)$, the optimal choice for each type of consumer is $\tilde{p}_\pi(\tau)$ and $m_\pi(\tau)$.

(ii) Given $F_\pi(\cdot)$ and $G_\pi(\cdot)$, consumers' behavior as specified by $(\tilde{p}_\pi(\tau), m_\pi(\tau))$ results in the distribution of types $G_\pi(\tau)$.

(iii) Given $G_\pi(\tau)$ and $\tilde{p}_\pi(\tau)$, the price distribution $F_\pi(p)$ is the outcome of profit maximizing behavior by firms, i.e., given its marginal cost no firm can benefit by changing its price.

(iv) Given the number of firms choosing to be operative, μ_π is the measure of consumers per firm of each new cohort.

(v) The price of the numeraire good equals its marginal cost.

To simplify notation, the index π is suppressed when no ambiguity results. To facilitate the exposition, the model will be analyzed in two stages. As it is easier to derive the equilibrium when $\pi = 0$, we analyze this case first, in section 2. In section 3 we describe the additional complexities involved when $\pi > 0$ and complete the derivation of the equilibrium for the general case.

2. Equilibrium with an Inflation Rate of Zero

In the case of zero inflation, consumers of every generation receive an identical nominal endowment of M at the beginning of their first period. The price of the government owned input is constant and normalized to be equal to one. Since there is no erosion of cash transferred from the first to the second period and utility is additive with no discounting, consumers are indifferent between buying the numeraire at the first or second period.[3] It is therefore an undominated strategy for the consumer to allocate sufficient resources to enable the purchase of the search good in her second period. Thus at any period there are three types of consumers in the market. The first type, denoted by y, is a young consumer during her first period in the market. The second type, denoted by 0^+, is an old consumer who has already bought one unit of the good when young. The third type, denoted by 0^-, is an old consumer who has not yet purchased a unit.

At any date there are three reservation prices, one corresponding to each type of consumer. By assumption $\bar{p}(0^+) = \beta$ and $\bar{p}(0^-) = \alpha$.

Lemma 1: $\bar{p}(y) = E(p)$ where $E(p)$ is the mean of $F(\cdot)$.

Proof: Consider the decision problem of a young consumer. If she buys at a price, p, she becomes a consumer of type 0^+ in the next period with the reservation price β. In this case her expected utility is:

$$(2.1) \qquad u_1(p) = M + \alpha - p + \int_0^\beta (\beta - x)dF(x).$$

[3]This assumption is made to simplify the analysis. We elaborate on this point in section 5.

Here, $\int_0^{\beta} xdF(x)$ is the expected expenditure on the search good in the second period. Therefore $M - p - \int_0^{\beta} xdF(x)$ is the expected amount of the numeraire good that is consumed in the two periods.

If she does not buy at p, her expected utility is

$$(2.2) \qquad u_2(p) = M + \int_0^{\alpha}(\alpha - x)dF(x) = M + \alpha - E(p)$$

where $M - E(p)$ is the expected quantity of the numeraire good consumed.

Thus the reservation price $\bar{p}(y)$ must satisfy: $u_1(\bar{p}(y)) = u_2(\bar{p}(y))$, yielding:

$$\bar{p}(y) = E(p) - \int_0^{\beta}(\beta - x)dF(x).$$

which implies:

$$(2.3) \qquad \bar{p}(y) \leq E(p).$$

It is also clear that $\bar{p}(y) \geq \beta$. If not, a consumer of type y observing p', $\bar{p}(y) < p' < \beta$, fails to buy a unit. Since she is then willing to pay up to α in her next period, and moreover is certain to encounter a price not exceeding α, this consumer's utility is $M + \alpha - E(p)$. By purchasing at $p' < \beta$ her utility is increased to $M + \alpha + \beta$

$= E(p) - p'$. Thus $\tilde{p}(y) \geq \beta$ as claimed. Together with (2.3) this implies that

$$\tilde{p}(y) = E(p).$$

□

Suppose $F(p)$ is not degenerate. Then there must exist a positive measure of firms whose price strictly exceeds $E(p)$. Therefore, by Lemma 1, a positive measure of young consumers observes a price exceeding $\tilde{p}(y)$ and fails to buy a unit. Thus there is a positive measure of old consumers of type 0^- in the market in each period. A similar argument establishes the existence of a positive measure of consumers of type 0^+ in each period.

LEMMA 2: Any price dispersion equilibrium is characterized by exactly 3 prices, $p_1 > p_2 > p_3$ where:

$$p_1 = \alpha, \quad p_2 = E(p), \quad p_3 = \beta.$$

PROOF: See Appendix 1.

Since $p_1 = \tilde{p}(0^+) = \alpha$, all firms are operative in this equilibrium. Denote by γ_i the proportion of operative firms whose price is p_i, $i = 1,2,3$. Since $p_2 = \tilde{p}(y) = E(p)$ we have: $p_2 = \gamma_1 p_1 + \gamma_2 p_2 + \gamma_3 p_3$.

This gives:

$$(2.4) \qquad p_2 = E(p) = \frac{\gamma_1 \alpha + \gamma_3 \beta}{1 - \gamma_2}.$$

Given the distribution of prices as determined by γ_i, $i = 1,2,3$, the individual firm optimally chooses a price as a function of its marginal cost, λ.

Let us determine the demand associated with each of the three prices. No consumer of type y or 0^+ accepts p_1. The probability of observing p_1 is γ_1. Thus the demand per firm associated with p_1 is $\mu\gamma_1$, where $\mu = \mu_0$, the measure of consumers per firm when $\pi = 0$. Consumers of each type accept p_3. Thus the demand per firm associated with p_3 is 2μ. Finally consumers of both type y and type 0^- accept p_2. The measure of these two types per firm is $\mu + \gamma_1\mu$ so the demand per firm associated with p_3 is $(1+\gamma_1)\mu$. Let $\Pi(p,\lambda)$ be the profit of a firm whose price is p and whose cost is λ:

Thus we have:

$$(2.7.1) \qquad \Pi(\alpha,\lambda) = \mu\gamma_1(p_1 - \lambda)$$

$$(2.7.2) \qquad \Pi(E(p),\lambda) = (1+\gamma_1)\mu(p_2 - \lambda)$$

$$(2.7.3) \qquad \Pi(\beta,\lambda) = 2\mu(p_3 - \lambda).$$

Define λ_3 by the relation $\Pi(p_3,\lambda_3) = \Pi(p_2, \lambda_3)$. λ_3 is the productivity index such that a firm whose production cost is λ_3 is indifferent between charging p_3 and p_2. It is easy to verify that firms for whom $\lambda < \lambda_3$ strictly prefer p_3 to p_2 while firms for whom $\lambda > \lambda_3$ strictly prefer p_2 to p_3. Similarly define λ_2 by the relation $\Pi(p_2, \lambda_2) = \Pi(p_1, \lambda_2)$. One can verify that firms for whom $\lambda > \lambda_2$ strictly prefer p_1 to p_2 and conversely for firms with $\lambda < \lambda_2$. λ_3 and λ_2 are illustrated in figure 1.

Figure 1 about here

As we require rational expectations, $\lambda_3 = \gamma_3/\alpha$. Similarly, $\lambda_2 = (\gamma_2 + \gamma_3)/\alpha$. In equilibrium $\pi(p_3, \lambda_3) = \pi(p_2, \lambda_3)$ and $\pi(p_2, \lambda_2) = \pi(p_1, \lambda_2)$. Substituting the above gives:

(2.8) $$\gamma_1^2 p_1 = (1+\gamma_1)(p_2 - p_1(\gamma_2 + \gamma_3)).$$

(2.9) $$2(p_3 - \gamma_3 p_1) = (1 + \gamma_1)(p_2 - \gamma_3 p_1).$$

Equations (2.8), (2.9) and the equilibrium prices as described in Lemma 2 fully characterize the price dispersion equilibrium. Some simulated solutions of the equilibrium are presented in Appendix 2.

3. Price Dispersion Equilibria with a Positive Rate of Inflation

We now let $\pi > 0$ such that the endowment of generation t is $M(1+\pi)^t$ and the government inflates the price of the input it owns by π such that $p_{t+1}^{\ell} = (1+\pi)p_t^{\ell}$. Since ℓ is the only input, such a change in price implies that firms' marginal costs inflate at the rate π.

Given the production technology of the numeraire good, $p_t^c = p_t^{\ell} = (1+\pi)^t$ in equilibrium (by the normalization of the input's real price to 1). Let $p(t)$ be the vector of nominal prices for the search good at period t and let $\rho(t)$ be the corresponding vector of real prices. Since we seek to characterize a steady state equilibrium, i.e., $\rho(t) = \rho$, we require that $\mathbf{p(t)} = (1+\pi)^t \rho(t)$.

The presence of a positive rate of inflation introduces additional complexities into the consumers' decision calculus. When prices are dispersed, the amount of cash required to finance the purchase of the search good in a consumer's second period is a random variable. When the rate of inflation is zero, cash left to the second period and not spent on the search good does not lose any of its purchasing power with respect to the numeraire good. Therefore it involves no loss to retain the maxium amount of cash which might be required to pay for the search good. When the rate of inflation is positive, however, such is not the case. The value of any cash left to the second period in excess of the realized purchase price of the search good is eroded in proportion to the rate of inflation. Therefore the amount of cash a consumer retains forms part of her optimization problem. Since a consumer is unable to buy at a price exceeding her cash allotment, this in turn restricts the real prices which sellers may charge in equilibrium.

Our earlier analysis revealed that when $\pi = 0$, price dispersion is characterized by three prices, one corresponding to each type τ, $\tau \in \{y, 0^+, 0^-\}$. Our purpose now is to derive a corresponding equilibrium in real prices for $\pi > 0$, i.e., an equilibrium in which precisely three real prices are charged in each period, $\rho_1 > \rho_2 > \rho_3$ such that

$$\rho_1 = \frac{\bar{p}_t(0^-)}{(1+\pi)^t},\ \rho_2 = \frac{\bar{p}_t(y)}{(1+\pi)^t},\ \rho_3 = \frac{\bar{p}_t(0^+)}{(1+\pi)^t},$$

(where $\bar{p}_t(\tau)$ is the nominal reservation price of a type τ consumer at period t).

Let m_t^+ and m_t^- denote the nominal balance retained by a consumer of type 0^+ and 0^- at the beginning of period t. Observe that a young consumer at time t anticipates the set of **nominal** prices charged at t+1 to be $\rho_i(1+\pi)^{t+1}$, i = 1,2,3. The following lemmatta serves to characterize three price equilibria of the type proposed above.

LEMMA 3: $m_t^- \in \{0, \rho_1(1+\pi)^t, \rho_2(1+\pi)^t, \rho_3(1+\pi)^t\}$.

PROOF: See Appendix 3.

Recall that in a three price equilibrium, only consumers of type 0^- buy at the highest price. A consequence of this is the following:

LEMMA 4: In equilibrium it must be the case that $m_t^- = \rho_1(1+\pi)^t$.

PROOF: Lemma 3 establishes that m_t^- is never in excess of this amount. Thus suppose $m_t^- < \rho_1(1+\pi)^t$; in this case a consumer of type 0^- is unable to buy at the highest price charged, contradicting the assumption that consumers of type 0^- buy at ρ_1. □

Lemma 4 states that a dispersed price equilibrium cannot exist if a consumer of type 0^- does not retain real resources (in terms of the price level at her second period) in the amount ρ_1, sufficiently to finance the purchase of the search good at any price charged in the market. In order to have a dispersed price equilibrium it needs to be the case that this constitutes optimal behavior on her part. The following lemma establishes the conditions under which this is the case.

LEMMA 5: The following conditions are necessary and sufficient to guarantee that it is optimal for a consumer of type 0^- to hold $m_t^- = (1+\pi)^t\rho_1$ at period t.

$$(3.1) \qquad \rho_1(\gamma_1 + \pi) \leq a - \gamma_2\rho_2 - \gamma_3\rho_3.$$

(3.2) $$(\rho_1 - \rho_2) \le \frac{\gamma_1 \alpha}{\pi + \gamma_1}.$$

(3.3) $$(\gamma_1 + \gamma_2)\alpha - (\rho_1 - \rho_3)(\gamma_1 + \pi) - \gamma_2(\rho_2 - \rho_3) \ge 0.$$

Proof: See Appendix 4.

Equation (3.1) guarantees that the consumer prefers to retain $\rho_1(1+\pi)^t$ than not to hold any cash for the second period. (3.2) guarantees that she prefers to hold $\rho_1(1+\pi)^t$ than to hold $\rho_2(1+\pi)^t$ and (3.3) guarantees that she prefers to hold $\rho_1(1+\pi)^t$ than $\rho_3(1+\pi)^t$.

In our three-price equilibrium, it must be the case that a consumer of type 0^+ has sufficient reources to buy at ρ_3. The following lemma provides the condition under which this behavior is optimal.

LEMMA 6: In equilibrium, $\rho_3 \le \dfrac{\beta\gamma_3}{\pi+\gamma_3}$.

PROOF: Consumers of type 0^+ buy if and only if they observe ρ_3. It follows that $m_t^+ \ge \rho_3(1+\pi)^t$; otherwise a consumer of this type would never have enough cash to buy at any price. Also, m_t^+ cannot exceed $\rho_3(1+\pi)^t$ since by assumption type 0^+ is unwilling to pay more than this amount for a unit. Thus $m_t^+ = \rho_3(1+\pi)^t$. The incremental utility from retaining m_t^+ is then $\beta\gamma_3 + (1-\gamma_3)\rho_3$. In equilibrium the latter must be at least as large as the incremental utility from spending m_t^- on the numeraire good at t-1, i.e., $\rho_3(1+\pi)$. This gives the condition

$$\rho_3(1+\pi) \leq \beta\gamma_3 + (1-\gamma_3)\rho_3.$$

Simplifying gives the condition in the lemma. □

LEMMA 7: In equilibrium,

$$\rho_2 = \frac{(\gamma_1 + \pi)\rho_1 - \rho_3\pi + \beta\gamma_3}{1 - \gamma_2} \tag{3.4}$$

PROOF: As derived in the proof of Lemma 6 the expected incremental utility from allocating m^+ is:

$$\beta\gamma_3 + (1-\gamma_3)\rho_3 - \rho_3(1+\pi). \tag{3.5}$$

Consider the decision problem of a buyer of type y. Given the real price distribution $F(\rho)$ her expected utility from purchasing the search good at the real price ρ_2 is her utility from purchasing the good, i.e., $a - \rho_2$ plus the expected utility associated with allocating the optimal m^+ as given by (3.5). If she does not buy, her expected utility is the one associated with the optimal m^- and is given by:

$$(3.6) \qquad \alpha + \gamma_2(\rho_1 - \rho_2) + \gamma_3(\rho_1 - \rho_3) - \rho_1(1+\pi).$$

In equilibrium she must be indifferent between the two options. Simplification completes the proof.

□

With the aid of the preceding lemmatta, an equilibrium may be fully characterized for 'moderate' rates of inflation.

THEOREM 1: For sufficiently small $\pi > 0$, an equilibrium price distribution is given by:

$$(3.7) \qquad \rho_1 = \alpha(1+\pi)^{-1}$$

$$(3.8) \qquad \rho_2 = [\frac{\alpha}{1+\pi}(\gamma_1 + \pi) + \frac{\beta\gamma_3^2}{\gamma_3+\pi}](1 - \gamma_2)^{-1}$$

$$(3.9) \qquad \rho_3 = \beta\gamma_3(\gamma_3 + \pi)^{-1}$$

and the conditions (2.8), (2.9) when ρ_i replaces p_i, i = 1,2,3.

PROOF: A consumer of type 0^- optimality leaves real reserves of $\rho_1(1+\pi)$ iff conditions (3.1),(3.2) and (3.3) are satisfied. It is immediately verifiable that for sufficiently small $\pi > 0$, ρ_1 as specified by (3.7) satisfies these conditions. Thus all firms which charge $\alpha(1+\pi)^{-1}$ make sales to consumers of type 0^+. Any firm which charges more than this price makes no sales to type 0^+ consumers because the latter's resources are insufficient to finance a purchase at this price.

It must be shown that the optimal m^+ is $\rho_3(1+\pi)$. We first verify that m^+ is not $\rho_1(1+\pi)$ or $\rho_2(1+\pi)$. The consumer will clearly not leave reserves in either of these amounts unless the expected incremental utility from buying a second unit at this price is positive. If she buys a second unit at ρ_1, her incremental utility is $\beta - \alpha < 0$. Thus she will never leave real reserves in the amount $\rho_1(1+\pi)$. Now, note that for sufficiently small π, $\rho_2(1+\pi) > \beta$, when ρ_2 is given by (3.8). Thus for small π, she will never leave real reserves in the amount $\rho_2(1+\pi)$. It remains to verify that the incremental utility from leaving real reserves in the amount $\rho_3(1+\pi)$ is not less than leaving zero reserves. This is guaranteed by the fact that ρ_3 as given by (3.9) satisfies the condition of lemma 5. Thus, only sellers whose price is not greater than ρ_3 make sales to consumers of type 0^+.

Given ρ_1 and ρ_3, ρ_2 as given by (3.8) satisfies condition (3.6). Therefore ρ_2 is the reservation price of type y consumers. Thus sellers whose price exceeds ρ_2 make no sales to type y consumers.

The analysis of firms' optimal behavior is now analogous to that of section 2. Note that the number of consumers per operative firm is $\mu(\pi) = \frac{\mu a}{\rho_1} = \mu(1+\pi)$. That is, firms charging ρ_3 sell to each type of consumer and so face a demand of $2\mu(\pi)$; firms charging ρ_2 sell to type y and type 0^- consumers and so face a demand of $(1+\gamma_1)\mu\ (\pi)$ while firms charging ρ_1 sell only to type 0^- consumers and so face a demand of $\mu(\pi)\gamma_1$. Defining λ_3 and λ_2 as in section 2, then, the conditions (2.8) and (2.9) complete the characterization of the equilibrium. □

Theorem 1 establishes an explicit dependence of the equilibrium prices and the proportion of firms charging each price on the steady state rate of inflation. This equilibrium has several notable characteristics. First, it obtains only if the rate of inflation is not "too high". The intuition behind this is not hard to see. In equilibrium, it must be optimal for young consumers to reject ρ_1. However, **all** nominal prices are known to increase by **a** proportion of π in the following period. If π is sufficiently high, the anticipated erosion of real balances makes it preferable to buy immediately at ρ_1 even if it were certain that an inflated low price (i.e., $\rho_3(1+\pi)$ or $\rho_2(1+\pi)$) will be received in the following period.

Next, note that old consumers of type 0^+ receive positive surplus from paying ρ_1. These consumers would be willing to pay more (up to a) **ex post**. Nevertheless, given the equilibrium allocation of resources for second-period consumption, any seller charging more than ρ_1 makes no sales because no old consumer retains money in excess of this amount. That is, the cost of allocating resources to the future drives a wedge between the consumer's ex ante reservation price and her ex post reservation price. A similar observation pertains to ρ_3. In fact, one can immediately notice that for $\pi > 0$, $\rho_3 < \beta(1+\pi)^{-1}$. If a young consumer who has already purchased a unit were certain of buying a second unit, she would be willing to allocate up to $\beta(1+\pi)^{-1}$ which would then be the equilibrium value of ρ_3. Since there is a positive probability of $1-\gamma_3$ that a second unit will not be purchased, the difference $\rho_3 - \beta(1+\pi)^{-1}$ constitutes a "premium" which this type of consumer must be offered in order to be willing to hold resources for the second period. Finally, observe that as π goes to zero, the equilibrium prices specified by (3.7)-(3.9) approach the (unique) equilibrium prices which obtain in an inflationless world.

We do not yet know if there are other three-price equilibrium distributions. Even if there are, however, the preceding "continuity" of our equilibrium's characteristics with those of the unique inflationless distribution establishes it as a "natural" equilibrium choice.

Note that the highest real price charged in equilibrium is a decreasing function of π. An immediate consequence of this is the following:

THEOREM 2: Given the equilibrium described in theorem 1, an increase in the inflation rate implies that the least efficient firms exit from the market.

PROOF: Only firms with $\lambda \leq \rho_1$ produce. Since ρ_1 is a decreasing function of π, the theorem is proved.

□

Since each equilibrium real price for the search good depnds on π, the rate of inflation affects price distribution, output and welfare in complex ways. Changes in real prices affect the proportion of firms charging each price, and the averge efficiency of operative firms, affecting the number of individuals which consume a second unit of the search good. Production of the numeraire good depends in turn on the preceding and on the equilibrium intertemporal allocation of money, which is itself a function of π. An important issue concerns the overall impact of the change in π on welfare.

Let $Q(\pi)$ be the probability that a randomly selected consumer buys a second unit of the search good

$$Q(\pi) = \gamma_3(\pi)(\gamma_2(\pi) + \gamma_3(\pi))$$

where $\gamma_i(\pi)$ is the equilibrium value of γ_i for the inflation rate π.

Let $C(\pi)$ be the expected equilibrium lifetime consumption of the numeraire good of a randomly selected consumer

$$(3.10) \qquad C(\pi) = [M - \rho_1(\pi)(1+\pi)]\gamma_1(\pi) + [M - \rho_3(\pi)(1+\pi) - \rho_2(\pi)]\gamma_2(\pi) + \\ + [M - \rho_3(\pi)(1+\pi) - \rho_3(\pi)]\gamma_3(\pi) + (\gamma_2(\pi) + \gamma_3(\pi))(\gamma_1(\pi) + \\ + \gamma_2(\pi))\rho_3(\pi) + \gamma_1(\pi)(\rho_1(\pi) - \rho_2(\pi))\gamma_2(\pi) + \gamma_1(\pi)(\rho_1(\pi) - \\ - \rho_3(\pi)\gamma_3(\pi)$$

where $\rho_i(\pi)$ are the equilibrium real prices for the inflation rate π.

The first term on the r.h.s. of (3.10) is the real expenditure on the numeraire in the first period of life if ρ_1 is observed, consisting of the endowment less m^-, an event which occurs with probability $\gamma_1(\pi)$. The second and third terms are interpreted analogously for ρ_2 and ρ_3 respectively. Thus the sum of the first three terms is the expected consumption of the numeraire good in the first period. The fourth term is the second-period consumption of the numeraire of a consumer of type 0^+ who fails to purchase a second unit of the search good - an event which occurs with probability $(\gamma_2 + \gamma_3)(\gamma_1 + \gamma_2)$. In this case, the eroded value of $m^+ = \rho_3$ is spent on the numeraire. The last two terms is the second-period consumption of the numeraire by a consumer of type 0^- which purchases a (first) unit of the search good at ρ_2 and ρ_3 respectively. Thus the sum of the last three terms is the expected second period consumption of the numeraire.

Substituting the above in the consumers' utility function yields the expected utility of a randomly selected consumer:

$$(3.11) \qquad V(\pi) = C(\pi) + \alpha + Q(\pi)\beta.$$

THEOREM 3: An increase of the rate of inflation may increase welfare.

□

PROOF: The theorem is proved by the numerical examples reported in the following section. □

4. Simulations

In this section we present some numerical examples of the effects of inflation on the equilibrium of our model. These are reported in tables 1 and 2.

TABLE 1: $\alpha = 2$, $\beta = 1.5$, $M = 3$

π	0	1%	2%	3%	4%	5%	6%	7%
E	1.629	1.609	1.58	1.571	1.554	1.54	1.528	1.52
V	3.454	3.466	3.478	3.488	3.497	3.5	3.507	3.505
C	0.704	0.729	0.756	0.784	0.814	0.847	0.883	0.925
Q	0.5	0.491	0.48	0.469	0.454	0.437	0.416	0.386
γ_1	.227	.215	.203	.191	.178	.164	.149	.130
γ_2	.126	.158	.192	.228	.267	.311	.361	.424
γ_3	.646	.626	.604	.580	.553	.524	.489	.444

TABLE 2: $\alpha = 1$, $\beta = 0.8$, $M = 3$

π	0	2%	3%	5%	7%	8%	10%
E	.840	.819	.809	.792	.777	.771	.764
W	3.189	3.203	3.210	3.223	3.233	3.236	3.238
C	1.709	1.734	1.747	1.775	1.806	1.824	1.866
Q	.600	.587	.579	.559	.532	.515	.464
γ_1	.189	.168	.157	.134	.109	.095	.062
γ_2	.0703	.126	.155	.219	.292	.334	.442
γ_3	.740	.705	.686	.646	.598	.56	.494

Symbols:

π	the rate of inflation
E	the average real price of the search good
W	welfare
C	output of the numeraire
Q	proportion of consumers who consume 2 units of the search good
γ_i	proportion of firms charging ρ_i for the search good

Tables 1 and 2 confirm that changes in π have a considerable impact on the shape of the equilibrium price distribution. This, in turn, affects the output and relative prices of the two types of goods and consumers' welfare. (In each case it was ascertained that π is sufficiently small for the equilibrium to exist (cf. the proof of Theorem 1)).

In particular, our numerical examples exhibit the following effects. Increases in π are accompanied by an increased concentration of firms at the middle price, ρ_2, and a reduction of the average real price charged for the search good. Thus the relative average price of the search good decreases.[4] Nevertheless output of the search good (i.e. $1+Q$) decreases. This is due to the increased concentratin of firms charging the middle price which decreases the proportion of old consumers who succeed in locating a second unit at the low price. Output of the numeraire, C, increases as average expenditure on the first unit of the search good and the number of repeat purchases of the search good both decrease. Finally welfare, V, is seen to increase in almost all cases, proving Theorem 2. Moreover, we would like to emphasize that the preceding effect is not restricted to some pathological example; all the numerical experiments we have conducted associated increased welfare with increased inflation when the latter is sufficiently low.

[4]Various empirical studies have established a relationship between inflation and relative price variability; Cukierman (1979), Cukierman and Vachtel (1982), Vining and Elwertowski (1976), Hercowitz (1981), Pagan, Hall and Trivedi (1983), Parks (1978).

Concluding Remarks

We have presented a simple general equilibrium model of search and price dispersion in an inflating economy. The link between inflation and real price dispersion in our model derives from uncertainty about the amount of resources required for future consumption on the one hand and the depreciation of these resources on the other. In particular, as the rate of inflation increases, the increased penalty associated with carrying over resources in excess of what is required for actual future consumption restricts the set of real prices which may be profitably charged. Surprisingly, the resulting change in the real price distribution may be accompanied by an increase in consumers' welfare.

Two main simplifying assumptions are used in our analysis. First, consumers are assumed to have no time preference with respect to the consumption of the numeraire. Second, there is an absence of alternative, interest bearing assets. Relaxing either of these assumptions should not change the main theme of the paper. The mechanism which drives our analysis is the internalization by firms of the inflation based search cost associated with uncertainty about future prices. This effect will not disappear if the timing of numeraire consumption is important or if the cost of allocating excessive resources to the future includes incurring excessive transaction costs (e.g., "trips to the bank") as in Romer (1986), Rottemberg (1984) or Jovanovich (1982), for example.

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APPENDIX 1: Proof of Lemma 2

We first observe that there are at most three different prices charged in equilibrium, $p_1 > p_2 > p_3$, corresponding to the reservation prices of the three consumer types. By the above, $\bar{p}(0^+) > \bar{p}(y) \geq \bar{p}(0^-)$. Any firm charging a price p, $\bar{p}(y) < p < \bar{p}(0^+)$ sells only to consumers of type 0^+. It could therefore increase its profit by increasing its price to $\bar{p}(0^+)$. This proves that no prices between $\bar{p}(y)$ and $\bar{p}(0^+)$ are charged in equilibrium. An analogous argument eliminates prices between $\bar{p}(y)$ and $\bar{p}(0^-)$, prices below $\bar{p}(0^-)$ and prices above $\bar{p}(0^+)$.

We now show that an equilibrium price distribution in which only two prices are charged does not exist.

Suppose the contrary and let the two prices be $x_1 > x_2$. By the preceding arguments, $x_1 = \alpha > x_2 = \beta$. Obviously all firms whose production cost is less than α are active in the market. There must exist a positive measure of firms whose marginal cost is less than β which optimally charge β. Let this measure be λ^*; that is, each active firm for which $\lambda \leq \lambda^*$ optimally charges β and each firm for which $\lambda > \lambda^*$ optimally charges α.

What is the reservation price of a young consumer? If she buys at α, her expected utility is M. If she postpones for a period, her expected utility is $M + \alpha - E(p) > M$ where $E(p) = \gamma_1\alpha + \gamma_2\beta$ where γ_1 and γ_2 are the proportion of firms which charge α and β

respectively. Thus only old consumers who did not buy when they were young buy at α. The measure of these consumers is $\gamma_1\mu$. Thus the profit from charging α when the marginal cost is λ^* is

$$\Pi(\alpha,\lambda^*) \equiv \mu\gamma_1(\alpha-\lambda^*).$$

Similarly, firms which charge β sell to all their customers. Thus

$$\Pi(\beta,\lambda^*) = 2\mu(\beta - \lambda^*).$$

The equilibrium conditions are:

$$\Pi(\beta,\lambda^*) = \Pi(\alpha,\lambda^*)$$

(A.1)

$$\lambda^* = \gamma_2 \quad , \quad \gamma_1 + \gamma_3 = 1.$$

Substituting, we obtain the condition

(A.2) $$\gamma_1^2 - 2\gamma_1 - 2(\beta-1) = 0.$$

Solving for γ_1, we obtain:

$$\gamma_1 = 2 \pm \frac{\sqrt{8\beta-4}}{2}.$$

For $\beta < 1$, each solution of (A.2) yields $\gamma_1 > 1$. Thus, no two price equilibrium exists.

APPENDIX 2: EQUILIBRIUM SIMULATION FOR THE SEARCH MODEL WITHOUT INFLATION

α	1	1	1	1	1	1
β	.55	0.6	0.7	0.8	0.9	0.95
γ_1	.206	.26	.25	.19	.1	0.05
γ_2	.668	.47	.21	.07	.013	0.003
γ_3	.126	.27	.54	.74	.887	0.947
E(p)	.829	.795	.797	.84	.91	0.952

APPENDIX 3: PROOF OF LEMMA 3

A young consumer will never retain cash in excess of the price she expects to pay for a unit of the search good in the following period because this cash could buy more of the competitve good in her first period. Thus a young consumer at period t-1 will not retain more than the maximum nominal price at t, $\rho_1(1+\pi)^t$. By assumtion, no nominal price between $\rho_2(1+\pi)^t$ and $\rho_1(1+\pi)^t$ will be osbserved at t.

Consider x, $\rho_2(1+\pi)^t < x < \rho_1(1+\pi)^t$. With probability 1, at least the amount $x - \rho_2(1+\pi)^t$ will not be spent on the search good because no firm charges a price between $\rho_2(1+\pi)^t$ and $\rho_1(1+\pi)^t$ at t. Thus it cannot be the case that $\rho_2(1+\pi)^t < m_t^+ < \rho_1(1+\pi)^t$. Similarly, it cannot be the case that $\rho_3(1+\pi)^t < m_t^+ < \rho_2(1+\pi)^t$. Obviously $m_t^+ <$

$\rho_3(1+\pi)^t$ since at least the latter amount is required to buy a unit at t. This completes the proof. □

APPENDIX 4: PROOF OF LEMMA 5

By Lemma 4, $m_t^- = \rho_1(1+\pi)^t$. With probability γ_1 the consumer encounters the highest price in the market and spends all of m_t^- on the search good. The incremental utility deriving from retaining m_t^- is then $\alpha - \rho_1(1+\pi)$. With probability γ_2 the second highest price is observed in which case $m_t^- - \rho_2(1+\pi)^t$ is spent on the search good and the real expenditure on the competitive good at t is $\rho_1 - \rho_2$. Then the incremental utility deriving from retaining m_t^- is $\alpha + \rho_1 - \rho_2 - \rho_1(1+\pi)$. Similarly, with probability γ_3 the increment utility from retaining m_t^- is $\alpha + \rho_1 - \rho_3 - \rho_1(1+\pi)$. Thus the **expected** incremental real utility from retaining $\rho_1(1+\pi)^t$ and denoted by v_1 is

(A.4.1) $$v_1 = \alpha + \gamma_2(\rho_1 - \rho_2) + \gamma_3(\rho_1 - \rho_3) - \rho_1(1+\pi).$$

Let v_2 and v_3 denote the expected incremental utility deriving from the retention of cash in the amounts $\rho_2(1+\pi)^t$ and $\rho_3(1+\pi)^t$ respectively. Analogously to the above we can derive:

(A.4.2.) $$v_2 = (\gamma_2 + \gamma_3)\alpha + \gamma_3(\rho_2 - \rho_3) - \rho_2(1+\pi)$$

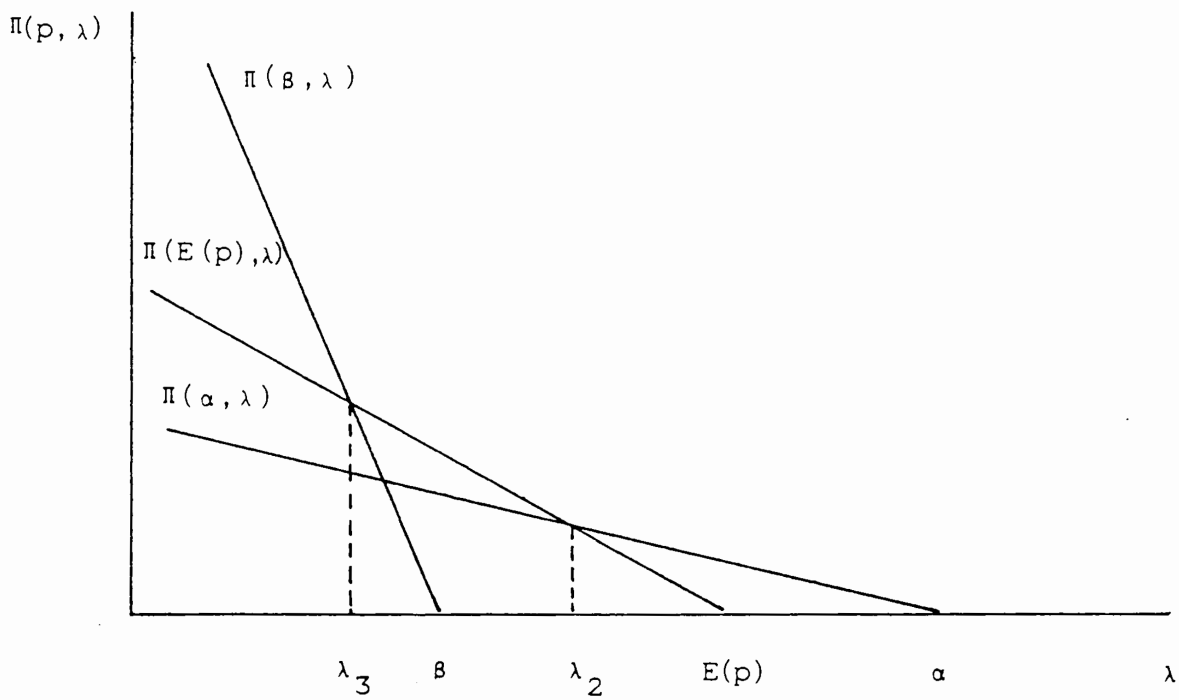
(A.4.3.) $v_3 = \gamma_3 a - \rho_3(1+\pi)$.

Thus, m_t^- is optimally $\rho_1(1+\pi)^t$ iff:

(A.4.4.) $v_1 \geq \text{Max}\ \{0, v_2, v_3\}$.

Substituting in (A.4.4.) and simplifying gives the conditions specified in the lemma.

$\square$



$\Pi(p, \lambda)$
$\Pi(\beta, \lambda)$
$\Pi(E(p), \lambda)$
$\Pi(\alpha, \lambda)$
λ_3
β
λ_2
$E(p)$
α
λ

FIGURE 1