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INFORMATION STRUCTURES AND THE DELEGATION OF MONITORING

by

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ABSTRACT

We explore a situation in which the (risk neutral) owner of a firm cannot commit ex-ante to monitor his (risk averse) workers. If the monitoring technology is capable of accusing a diligent worker of shirking (a "false positive"), the worker shirks in equilibrium with positive probability even if monitoring is costless. The owner alleviates this problem by delegating monitoring to a supervisor. Delegation is most effective if the monitoring technology is capable of producing false positives. Nevertheless, the owner may still benefit from delegation when false positives are impossible by contracting with a risk neutral supervisor.

1. INTRODUCTION

We explore a situation in which the (risk neutral) owner of a firm cannot commit ex-ante to monitor his (risk averse) workers. The most general result is that shirking occurs in equilibrium with positive probability if monitoring is costly and there exist limits on the fines that a worker can be coerced to pay.

The fact that commitment is important in the provision of incentives is well established. We go one step further by studying how different monitoring technologies affect the role of commitment. A stronger result than the one stated above holds if the monitoring technology is capable of accusing a diligent worker of shirking, a circumstance that I call a "false positive". In this situation, the probability that the worker shirks is strictly positive in equilibrium even when monitoring costs are zero.

A situation in which false positives are impossible no matter what the owner does is also studied. In this case, our result that shirking occurs in equilibrium even if monitoring is costless disappears. This highlights the fact that commitment plays a different role depending on the monitoring technology.

We also study the efficacy to the owner of a firm of hiring a supervisor to monitor his workers. As stated above, if the owner cannot commit ex-ante to a monitoring intensity, shirking must occur in equilibrium. In this case, delegating the monitoring task to a third agent may be beneficial, even if the supervisor's effort is unobservable to the owner, as it is assumed here. This result provides an additional explanation for the separation of ownership and control in the modern corporation: once an owner delegates monitoring, his only task is to choose the wage contracts of the supervisors and workers.

I present two results on the efficacy of delegation as a commitment device. First, I show that delegation of the monitoring task is extremely effective when the monitoring technology can produce false positives. In this case, delegation allows the owner, in equilibrium, to implement a zero probability of shirking, even when monitoring is costly. Second, I present an advantage of delegation not yet exploited in the extant literature: when there exist institutional constraints that limit the efficacy of wage contracts as incentive tools, delegation can be used to overcome these constraints. To show this we study a monitoring technology in which false positives are impossible.

A more precise description of the model and the results mentioned above, together with some examples, is provided next.

In this model, the worker has only two choices: either he shirks (exerts no effort) or he is diligent. Denote by p the probability that the worker is diligent.

For simplicity, we assume that contracts on output are not feasible.[1] We also assume that the firm cannot be sold to the worker.[2] One way to justify this assumption is to consider the case where output is an intermediate good without value to the worker.

Instead of output, it is assumed that contracts can be written on a verifiable signal produced via monitoring. Examples of such a signal are a videotape or the number of typographical errors in a document. That signal takes only two values, "high" and "low". A low (respectively, high) signal suggests that the worker shirked (respectively, was diligent). A realization of the signal is a false positive if it is low but the worker was diligent. The intensity of monitoring is parametrized by an unidimensional choice variable θ which belongs to a closed interval $[\underline{\theta},\overline{\theta}]$. If $\theta = \underline{\theta}$, the signal is uninformative about the worker's effort. If $\theta = \overline{\theta}$, the signal is perfectly informative. More intense monitoring is more costly. If the monitoring technology is capable of producing false positives, we say it is a "false positives (FP) monitoring technology". The probability of false positives occurring decreases with θ. If false positives are impossible for any θ, we say that the monitoring technology is a "no-false positives (NFP) monitoring technology". In this situation only "false negatives" can occur. A realization of the signal is a false negative if it is high but the worker shirked. The probability of a false negative occurring also decreases with θ.

The FP and NFP monitoring technologies can be illustrated by means of two examples. The following example illustrates a FP technology: the effort devoted by a teacher (worker) is evaluated by means of student interviews at the end of the course. If only one interview is conducted, a false positive may occur, i.e., a teacher that was actually diligent may be accused of shirking. As the number of student interviews increases (i.e., as the monitoring effort of the school's owner increases), the probability of a false positive occurring decreases.

The following is an example of a NFP monitoring technology. The worker is a secretary. His job is to type letters for his boss, the owner. If the secretary is diligent, there will be no typographical errors. The owner chooses how many letters will be inspected. If many letters are inspected, a typographical error will very likely be found if the secretary shirked. Note that a letter with a typographical error is verifiable evidence proving beyond doubt that the secretary shirked. In other words, false positives are impossible.

Finally, the difference between the wage paid to an agent when the signal is high and the wage paid when the signal is low is the "wage spread".

Shapiro and Stiglitz (1982), Singh (1985), Sparks (1986) and others have derived conditions on optimal monitoring in principal-agent models. In all these papers, the owner can commit to a monitoring strategy ex-ante. This requires that either θ or its cost is publicly observable, either ex-ante or ex-post, so that contracts can be signed on θ. However, this seems unlikely to be the case. Just as the worker's effort is likely to be unobservable to the owner, the owner's monitoring effort is likely to be unobservable to the worker. This is particularly the case if the owner chooses his monitoring intensity at the same time or after the worker chooses his effort level.

Commitment is precluded in our model by assuming that the owner's monitoring intensity is not observable by anyone except himself. Formally, this amounts to assuming that the owner and the worker make their monitoring and effort choices simultaneously, in ignorance of the other's choice. I call this the "no commitment" (NC) game. For comparison purposes, I analyze the case in which the owner <u>can</u> commit to a monitoring intensity; I call this case the "full commitment" (FC) game.

The NC game is a two stage game. In the first stage, the owner offers a contract stipulating what wage will be paid for each possible outcome of the monitoring signal, and the worker accepts or rejects the contract. In the second stage, provided the worker accepts the contract, owner and worker simultaneously choose their monitoring and effort levels, respectively. We study the (subgame) perfect Nash equilibria of this game. To illustrate this, consider the FP example given above: if the principal is unable to commit to the number of students that will be interviewed, the situation illustrates the FP-NC game. One more example: consider the situation used above to illustrate a NFP monitoring technology. If the letters are inspected when the secretary is absent, it seems unlikely that the owner could commit ex-ante to how many letters will be inspected. This corresponds to the NFP-NC case. If the secretary were able to observe the number of letters inspected, and he was able to take the owner to court if he discovered that the owner did not inspect as many letters as she promised, then we would be back to the full-commitment situation, that is, to the NFP-FC case.

No matter which monitoring technology is considered, in the NC game $p < 1$ in equilibrium: if $p = 1$ the owner's best response is to set $\theta = \underline{\theta}$, since θ is costly. But the worker's best response to $\theta = \underline{\theta}$ is to set $p = 0$. On the other hand, in the FC game $p = 1$ in equilibrium (as long as monitoring is not too costly), since the owner can commit to a monitoring intensity such that the worker prefers to be diligent.

The main result for the FP-NC case is that shirking must occur in equilibrium, even when monitoring is costless. The intuition is simple: the owner likes false positives to occur because they give him an excuse to pay a low wage. Suppose the owner believes that the worker is being diligent. Then the owner's best response is to not monitor, thereby maximizing the probability that a false positive will occur. Note that this argument is independent of how costly monitoring is.

We also study the efficacy to the owner of hiring a supervisor to monitor his workers.

Consider what happens under a FP Monitoring technology. Suppose that the owner chooses to reward both the worker and supervisor when the signal is high. Then, if the worker is diligent with probability one, the supervisor's optimal strategy is to monitor the worker intensely, so as to avoid false positives. Given that the supervisor is monitoring the worker intensely, the worker's best reply is to be diligent.

This result supports the following suggestion by Melumad and Mookherjee (1987,2):[3]

> The non-coincidence of principal [in our model, owner] and agent's [supervisor's] interests may in fact be beneficial in some contexts. Specifically, the principal may have difficulty in committing to desirable long run policies with respect a second set of agents [the worker in our model], owing to its own opportunistic propensities to deviate ex post from past promises. . . .Faced with a problem of commitment, the principal may more credibly delegate responsibility to an agent [supervisor] with private preferences different from its own.

Under the FP monitoring technology, the owner is able to introduce such a divergence of interests between himself and the supervisor: the owner likes false positives whereas the supervisor tries to avoid them. On the other hand, under the NFP monitoring technology, such a divergence of interest cannot be induced: if the owner rewards the supervisor for a high signal, the supervisor shirks so as to maximize the probability that a false negative occurs (no matter what the worker does). And yet, delegation may still be beneficial to the owner, as I will explain next.

By giving a very large reward to the supervisor if he catches the worker shirking, the owner motivates the supervisor to monitor intensely even if the worker shirks with very low probability. In fact, the owner can induce the worker to set p arbitrarily close to 1 by offering the supervisor "arbitrarily large prizes with arbitrarily small probability". However, this is only beneficial to the owner if the supervisor is not very risk averse. Otherwise, the supervisor's expected wage grows faster than the benefits derived from a higher p.

This suggests an additional advantage of delegation: in the game without delegation, the wage spread serves two purposes: (1) it induces the worker to be diligent, and (2) it induces the owner himself to monitor the worker. Existing limits on the fines that a worker can be coerced to pay obstruct the realization of both objectives. By delegating to a supervisor, the owner has access to two instruments, the worker's wage and the supervisor's wage. The two instruments can be directed to the two objectives independently, which allows the owner to free one of the objectives from the existing limits on fines: the supervisor can be rewarded with a large prize if he catches the worker shirking.

A potential problem arises under the FP monitoring technology: given the incentive contract that we propose (reward both the worker and the supervisor when the signal is high), two equilibria exist in the second stage game. In one of these subgame equilibria, both the worker and the supervisor are diligent, as we explained above. Another subgame equilibrium exists in which both the worker and the supervisor shirk. We show, however, that the supervisor strictly prefers the first of these subgame equilibria, and therefore one (the owner) need not worry about the worker and supervisor agreeing to play the subgame equilibrium in which they shirk, instead of the one in which they are diligent. Other authors have worried about a similar problem, for instance, Antle (1982) and Demski and Sappington (1984).

It is important to note that delegating monitoring to a third agent does not introduce a reporting problem in our model. Indeed, we assume that the monitoring signal is publicly observable, so that the supervisor does not need to report the realization of the signal to the owner. The supervisor merely monitors. Also, I assume away the possibility that any two of the players might form a coalition and play non-equilibrium strategies.

This paper is organized as follows: We outline the model in Section 2, Section 3 is devoted to the false positive (FP) monitoring technology and is divided into three subsections: the first one deals with the full commitment (FC) game, the second one with the no commitment (NC) game, the third one with the delegation game (DG). Section 4 is devoted to the no-false positives (NFP) monitoring technology and is divided in the same manner. In Section 5 we discuss extensions for further research and conclude our analysis.

2. THE MODEL

The firm is composed of a risk neutral owner and a risk averse worker. The owner is the residual claimant. The worker exerts unobservable effort $a \in \{s,d\}$ where s stands for "shirk" and d for "diligent". This effort generates cash flow $\pi > 0$ when $a = d$ and zero when $a = s$. The worker has a Von Neumann-Morgenstern utility function that is additively separable in money income and effort: $U(m,a) = u(m) - V(a)$, where

<u>Assumption 1</u>: the function $u(\cdot)$ satisfies: $u(0) = 0$, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$;

and V is the disutility of effort. Let $V(d) = v > 0$ and $V(s) = 0$. The worker obtains utility $T \geq 0$ (for sure) if he leaves the firm.

Contracts on cash flow cannot be signed, for reasons discussed in the introduction. However, a monitoring technology that provides society with a publicly observable and verifiable signal Y is available. Contracts contingent on this signal, and on this signal alone, can be written. The owner will spend resources on monitoring according to a cost function $C(\cdot):[\underline{\theta},\overline{\theta}] \to \mathbb{R}_+$ where θ is a measure of monitoring intensity, and $C(\cdot)$ satisfies

<u>Assumption 2</u>: $C'(\theta) > 0$ for all $\theta > \underline{\theta}$, $C'(\underline{\theta}) = C(\underline{\theta}) = 0$, $C'(\overline{\theta}) = \infty$, and $C''(\cdot) > 0$.

For analytical simplicity, assume the signal can take up only two values: H (for "high") or L (for "low"). The probabilities of these outcomes depend on the monitoring intensity, θ, and the effort exerted by the worker, a. The realization of H and L will have the natural interpretation, H indicating that the worker was diligent, L indicating that the worker shirked, because the monitoring technologies specified below satisfy the Monotone Likelihood Ratio Condition (MLRC).[4]

Two monitoring technologies are considered. The first one allows for the possibility of <u>false positives</u>, i.e., that the realization of the signal is L when the worker was actually diligent. In the second information technology, only false negatives may occur. The two monitoring technologies are formalized in the following way.

Case 1: False positives (FP) monitoring technology.

Assumptions: 1-a) $[\underline{\theta},\overline{\theta}] = [(1/2),1]$

1-b) $\Pr(Y = H|a=d) = \Pr(Y = L|a=s) = \theta$.

Thus, as θ increases, both the probability of false positives and the probability of false negatives decreases, that is, the signal becomes more informative.

Case 2: No-false positives (NFP) monitoring technology

Assumptions: 2-a) $[\underline{\theta},\overline{\theta}] = [0,1]$

2-b) $\Pr(Y = L|a=d) = 0$; $\Pr(Y = L|a=s) = \theta$.

Thus, the probability of a false positive is 0 for all θ. The probability of a false negative decreases as θ increases, and therefore the signal becomes more informative.

It is easy to check that the MLRC holds for both the FP and NFP monitoring technologies.

Three different games are studied. The first game is referred to as FC for "full commitment" game. In it, the owner's choice of θ is observed by the worker before he chooses his effort level. The owner can therefore commit to a level of monitoring intensity before production starts. The order of moves is the following: at the beginning of the game, the owner offers the worker a contract $w = (h,\ell)$ that stipulates the wage h that the worker will receive if the realized signal is H, and the wage ℓ that the worker will receive if the realized signal is ℓ. An element of the pair w is denoted by w_i. Then the worker decides whether to sign the contract. This decision is represented by the binary variable $x \in \{A,R\}$, where R stands for "reject the contract" and A stands for "accept the contract". The game ends if the worker rejects the contract. If the worker accepts the contract, then the owner chooses θ in $[\underline{\theta},\overline{\theta}]$. Next, the worker observes θ and then chooses whether to shirk, $a = s$, or to be diligent, $a = d$. Finally, at the end of the period, cash flow and the signal are realized, and wage is paid. The extensive form of this game is depicted in Figure 1.

In the second game, referred to as NC for "no commitment" game, the true monitoring intensity cannot be observed by anyone. That makes it impossible for the owner to commit to a certain level of θ. As before, at the beginning of the period the owner offers a contract to the worker, which he accepts or rejects. If the worker accepts, the worker's effort decision and the owner's monitoring intensity decision are made, in effect, simultaneously. These simultaneous decisions are made in the "second stage of the game". Denote by p the probability that the worker is diligent i.e., p is the probability that the worker sets $a = d$. When the owner chooses a wage schedule in the first period, he is basically choosing a second stage equilibrium $\langle\theta,p\rangle$ subject to the constraint that the worker's expected utility be at least T. The extensive form of this game is depicted in Figure 2.

The <u>payoffs</u> of the players in the FC and NC games are the following:
The owner's payoff is:

$$U^{o}(p,\theta,x,w) = \begin{cases} p\pi - E(w_i|\theta,p) - C(\theta) & \text{if } x = A \\ 0 & \text{if } x = R, \end{cases} \tag{2.1}$$

where $E(\cdot|\theta,p)$ denotes expectation conditional on p and θ.
The worker's is:

$$U^{w}(p,\theta,x,w) = \begin{cases} E(u(w_i)\,|\,\theta,p) - pv & \text{if } x = A \\ T & \text{if } x = R. \end{cases} \tag{2.2}$$

The following is assumed throughout:

Assumption 3: there exists $x > 0$ in $\mathbb{R}$ such that $u(x) \geq v + T$.

Assumption 4: $\pi > u^{-1}(v+T)$.

These assumptions imply that in the first best world without moral hazard, the equilibrium would entail hiring the worker, having him being diligent with probability one, and paying him a wage $u^{-1}(v+T)$. This is a necessary but not sufficient condition for the existence of equilibria in which the worker is hired in the games studied, as we will see.

The following Lemma will be often used in the remainder of this paper.

Lemma 2.1: If the owner's payoff, $U^{o}(p,\theta,h,\ell)$, is positive in the FC or the NC equilibrium, then p must be positive.

Proof: Suppose $p = 0$. If the worker does not accept the contract then $U^{o} = 0$, done. Assume that the worker accepts the contract. Then $U^{w} = E(u(w_i)|\theta,p{=}0) \geq T$ (since $pv = 0$). By concavity of $u(\cdot)$, we have $E(w_i|\theta,p{=}0) \geq u^{-1}(T) \geq 0$ since $u(0) = 0$ and $T \geq 0$. Then $U^{o} = -E(w_i|\theta,0) - C(\theta) \leq 0$, a contradiction. Q.E.D.

In the third game, referred to as DG for "delegation game", the owner delegates monitoring to a third player, the supervisor. The supervisor has a Von Neumann-Morgenstern utility function that is additively separable in money income and monitoring intensity θ: $U^s(m,\theta) = u_s(m) - C(\theta)$, where $u_s(\cdot)$ is strictly increasing and concave. Thus, the supervisor has the same disutility of θ function as the owner. This captures the idea that owner and supervisor have access to the same monitoring technology and have the same preferences over monitoring effort.

The order of moves in the DG game is the following: first, the owner writes a contract $\underline{w} = (w,w_s)$ that stipulates a wage schedule for the worker, $w=(h,\ell)$, and a wage schedule for the supervisor, $w_s = (h_s,\ell_s)$. Wage h_s (respectively, ℓ_s) is paid to the supervisor if the realized signal is H (respectively, L). An element of w_s is denoted by w_{si}. Second, the worker decides whether to accept or reject the contract. This choice is denoted by the binary variable $x \in \{R,A\}$. Third, if the worker decides to accept, the supervisor decides whether to accept or reject; his decision is denoted by $x_s \in \{R,A\}$. Let $\underline{x} = (x,x_s)$. If either the worker or the supervisor reject their contacts the game ends, the worker receives $T \geq 0$, the supervisor receives T_s, and the owner receives 0. Assumptions on T_s are given below. Fourth, the supervisor chooses $\theta \in [\underline{\theta},\overline{\theta}]$ and, simultaneously, the worker chooses $a \in \{d,s\}$ according to his strategy $p \in [0,1]$. Finally, chance chooses the signal according to the probabilities specified by the monitoring technology, given θ and a. The extensive form of this game is depicted in Figure 3.

The players' <u>payoffs</u> in the DG game are given next. The worker's payoff is given in (2.2). The supervisor's payoff is:

$$U^s(p,\theta,\underline{x},\underline{w}) = \begin{cases} E[u_s(w_{si})|\theta,p] - C(\theta) & \text{if } \underline{x} = (A,A) \\ T_s & \text{otherwise.} \end{cases} \tag{2.3}$$

The owner's payoff is:

$$U^o(p,\theta,\underline{x},\underline{w}) = \begin{cases} p\pi - E(w_i + w_{si}|\theta,p) & \text{if } \underline{x} = (A,A) \\ 0 & \text{otherwise.} \end{cases} \tag{2.4}$$

So that the DG game is non-trivial, we assume the following:

Assumption 5: There exists x such that $u_s(x) \geq T_s$.

To study delegation as a commitment device, we have to make sure that delegation is not advantageous to the owner when commitment to θ is feasible. If commitment is feasible and the owner does not delegate the monitoring task (FC game), the owner sets (θ,w) to maximize his payoff given the worker's best reply to (θ,w). Let (θ_f,w_f) be the solution to this maximization problem. The monitoring costs are $C(\theta_f)$. If, on the other hand, the owner delegates monitoring to a supervisor, and the supervisor can commit to θ, the optimal contract between owner and supervisor entails the supervisor setting $\theta = \theta_f$ and the supervisor receiving a fixed wage ω that covers the supervisor's opportunity utility T_s plus monitoring disutility $C(\theta_f)$. In other words, $\omega = u_s^{-1}[T_s + C(\theta_f)]$. To ensure that under full commitment to θ the owner does not gain by delegating the monitoring task, we need to assume $\omega \geq C(\theta_f)$. The following is a sufficient condition for $\omega \geq C(\theta_f)$:

<u>Assumption 6</u>: $u_s^{-1}[T_s+C(\theta)] \geq C(\theta)$ for all θ.

<u>Example</u>: Suppose the supervisor is risk neutral, i.e., $u_s(x) \equiv x$. Then Assumption 6 requires $T_s \geq 0$.

The following Lemma will be used later on.

<u>Lemma 2.2</u>: If the owner's DG equilibrium payoff is positive then $p > 0$.

<u>Proof</u>: If U^o is positive it must be the case that $\underline{x} = (A,A)$. Suppose that $p = 0$. This implies $U^w(0,\theta,(A,A),\underline{w}) = E(u(w_i|\theta,0) \geq T$ and $U^s(u_s(w_{si}|\theta,0) - C(\theta) \geq T_s$. Since $u(0) = 0$, $T \geq 0$, and u is concave, we have $E(w_i|\theta,0) \geq u^{-1}(T) \geq 0$. By concavity of u_s we have $E(w_{si}|\theta,0) \geq u_s^{-1}[C(\theta)+T_s] \geq C(\theta) \geq 0$ by Assumption 6. Therefore, if $p = 0$ the owner's payoff is $U^o = - E(w_i+w_{si}|\theta,0) \leq 0$, contradiction.

Q.E.D.

3. THE FALSE POSITIVES MONITORING TECHNOLOGY

We analyze the FC, NC and DG games in subsections 3.1, 3.2, and 3.3, respectively.

3.1 The full commitment (FC) game

The payoffs of the players for this monitoring technology are given in (2.1) and (2.2) with

$E(g(w_i)|\theta,p) = p[\theta g(h) + (1-\theta)g(\ell)] + (1-p)[(1-\theta)g(h) + \theta g(\ell)]$ for any function g.

A strategy for the owner is given by a wage contract $w \in \mathbb{R}^2$ and a function $\theta(\cdot)$: $\mathbb{R}^2 \to [\frac{1}{2},1]$, which describes the response of the owner to a given w. A strategy for the worker is a pair of functions, one function $x(\cdot):\mathbb{R}^2 \to \{R,A\}$, describing his response to a given w, and the other function $p(\cdot):\mathbb{R}^2 \times [\frac{1}{2},1] \to [0,1]$, describing his response to a given (w,θ). A subgame perfect equilibrium is a 4-tuple $\langle p(\cdot),\theta(\cdot),x(\cdot),w^*\rangle$ such that, for all $w \in \mathbb{R}^2$,

(i) $p(w,\theta) \in \operatorname{argmax}_{p \in [0,1]} U^w(p,\theta,A,w)$ for all θ.

(ii) $\theta(w) \in \operatorname{argmax}_{\theta \in [\frac{1}{2},1]} U^o(p(w,\theta),\theta,A,w)$.

(iii) $x(w) = A$ if and only if $U^w(p(w,\theta(w)),\theta(w),A,w) \geq T$.[5]

(iv) $w^* \in \operatorname{argmax}_{w' \in \mathbb{R}^2} U^o[p(w',\theta(w')),\theta(w'),x(w'),w']$.

We assume that the owner chooses θ after the worker accepts the contract (rather than choosing θ and w simultaneously). This makes the strategy sets in the FC and NC games as similar as possible. This is done to highlight the fact that the differences between the equilibria in the FC and NC games come, exclusively, from the owner not being able to commit to θ before the worker chooses his strategy.

To find an equilibrium, solve the problem recursively, starting from the last move. Thus, the worker solves

$$\max_{p \in [0,1]} \quad p[\theta u(h) + (1-\theta)u(\ell)] + (1-p)[(1-\theta)u(h) + \theta u(\ell)] - pv,$$

which yields a reaction function:

$$p^r(w,\theta) = \begin{cases} 0 & \text{if } (2\theta-1)[u(h)-u(\ell)] < v \\ [0,1] & \text{if } (2\theta-1)[u(h)-u(\ell)] = v \\ 1 & \text{if } (2\theta-1)[u(h)-u(\ell)] > v. \end{cases} \tag{3.1.1}$$

From (3.1.1) we see that, for any w satisfying $u(h) - u(\ell) \geq v$, the worker is indifferent between shirking and not shirking if θ is set equal to:

$$\theta^c(w) \equiv (1/2)\ [1 + \frac{v}{u(h) - u(\ell)}]. \tag{3.1.2}$$

Since it is feasible for the owner to commit to the monitoring intensity $\theta^c(w)$, for large enough π (given $u(\cdot)$, $C(\cdot)$, T, and v) the equilibrium involves the worker setting $p = 1$ and the owner setting $\theta = \theta^c(w)$:

Proposition 3.1.1: In a FC equilibrium, for π large enough, $p = 1$ and $\theta = \theta^c(w)$.

Proof: Once the owner and the worker have signed the contract w, if $p[w,\theta^c(w)]$ was less than 1 (say, $p[w,\theta^c(w)] = 1 - \delta$, $\delta > 0$), the owner would gain by deviating to $\theta^c(w) + \varepsilon$ where ε is arbitrarily small but positive. The gains from such deviation are $\delta\pi$, a fixed positive number, whereas the costs $C[\theta^c(w)] - C[\theta^c(w) + \varepsilon]$ can be made arbitrarily small. If $\theta = \theta^c(w) + \varepsilon$, the owner gains by deviating to $\theta' = \theta^c(w) + \varepsilon/2$, since the worker still sets $p = 1$ whereas the monitoring costs are smaller. If $\theta < \theta^c(w)$, then the worker sets $p = 0$ and the owner gains π by increasing θ to $\theta^c(w) + \varepsilon$, at a cost $C(\theta^c(w)+\varepsilon) - C(\theta)$, which is less than π for π large enough. Q.E.D.

The issue of existence of an equilibrium is addressed in Olivella (1989). A sufficient condition for existence is that the worker's utility function be bounded above.

3.2 The no-commitment (NC) game

The owner's and the worker's payoffs are the same as in Section 3.1 (FC), and given by (2.1) and (2.2), respectively.

A strategy for the owner is composed of two objects: a wage contract $w = (h,\ell) \in \mathbb{R}^2$ and a function $\theta(\cdot):\mathbb{R}^2 \to [1/2,1]$ describing what his choice of monitoring intensity is going to be for a given wage contract. A strategy for the worker is a pair of functions: a function $p(\cdot):\mathbb{R}^2 \to [0,1]$ and a function $x(\cdot):\mathbb{R}^2 \to \{R,A\}$, describing the worker's response to a given wage contract. A subgame perfect equilibrium, denoted by $\langle w^*,x(\cdot),\theta(\cdot),p(\cdot)\rangle$, is a 4-tuple satisfying, for all $w \in \mathbb{R}^2$,

(i) $p(w) \in \underset{p\in[0,1]}{\text{argmax}} \; U^w[p,\theta(w),A,w]$,

(ii) $\theta(w) \in \underset{\theta\in[1/2,1]}{\text{argmax}} \; U^o[p(w),\theta,A,w]$,

(iii) $x(w) = A$ if and only if $U^w(p(w),\theta(w),A,w) \geq T$,

(iv) $w^* \in \underset{w' \in \mathbb{R}^2}{\text{argmax}} \; U^o[p(w'),\theta(w'),x(w'),w']$.

Finally, let $p^* = p(w^*)$ and $\theta^* = \theta(w^*)$.

The equilibrium is found recursively, starting from the last (simultaneous) moves. The owner and the worker face the decision of simultaneously choosing θ and p, respectively. To find the Nash equilibria of this subgame, the wage contract inherited from the first stage is taken as fixed. First, the reaction function of the worker to the owner's choice of θ is found. Second, the reaction function of the owner to the worker's choice of p is calculated. Finally, the equilibria are found where these two reaction functions meet in $[1/2,1]\times[0,1]$. Enough assumptions on the monitoring cost function $C(\cdot)$ have been made so that a unique equilibrium, $\langle p,\theta \rangle$, exists for any given wage schedule w. To find an equilibrium to the entire game, the following first stage problem has to be solved: maximize the owner's payoff by choosing a wage schedule w, a probability of shirking p, and a monitoring intensity θ subject to 1) a (voluntary participation) constraint that insures that the worker accepts the contract, and 2) an (incentive compatibility) constraint ensuring that the pair (p,θ) is the second stage equilibrium generated by the wage choice.[6] It turns out that one can say a lot about the equilibrium of the game without solving that first stage problem.

The reaction function of the worker, $p^r(w,\theta)$, and of the owner, $\theta^r(w,p)$, for fixed w are found next.

For given w, the worker solves

$$\max_{p\in[0,1]} \quad p[\theta u(h) + (1-\theta)u(\ell)] + (1-p)[\theta u(\ell) + (1-\theta)u(h)] - pv,$$

which yields the same reaction function as in Section 3.1 and repeated here:

$$p^r(w,\theta) = \begin{cases} 0 & \text{if } (2\theta-1)[u(h)-u(\ell)] < v \\ [0,1] & \text{if } (2\theta-1)[u(h)-u(\ell)] = v \\ 1 & \text{if } (2\theta-1)[u(h)-u(\ell)] > v. \end{cases} \tag{3.2.1}$$

Recall that the level of monitoring that leaves the worker indifferent between shirking and being diligent is, as long as $u(h)-u(\ell) \geq v$,

$$\theta^c(w) = (1/2)\ [1 + \frac{v}{u(h) - u(\ell)}]. \tag{3.2.2}$$

The owner's reaction function $\theta^r(w,p)$ is found by solving:

$$\max_{\theta\in[1/2,1]} \quad p\pi - p[\theta h + (1-\theta)\ell] - (1-p)[(1-\theta)h + \theta\ell] - C(\theta),$$

The first order conditions are:

$$(1 - 2p)(h - \ell) - C'(\theta) \begin{cases} > 0 \text{ then } \theta = 1 \\ = 0 \text{ then } \theta \in [1/2,1] \\ < 0 \text{ then } \theta = 1/2. \end{cases} \tag{3.2.3}$$

Hence, using Assumption 2,

$$\theta^r(w,p) = \begin{cases} 1/2 & \text{if } (1\text{-}2p)(h\text{-}\ell) \le 0 \\ [C']^{-1}[(1\text{-}2p)(h\text{-}\ell)] & \text{if } (1\text{-}2p)(h\text{-}\ell) > 0. \end{cases} \tag{3.2.4}$$

For any w, it is possible to define a wage-contingent subgame equilibrium. The following definitions and proposition characterize such equilibria. Proposition 3.2.2 is the most important result in this section.

Definition 3.2.1: Fix $w = (h,\ell)$. A wage contingent equilibrium, written as w-equilibrium, is a pair of real numbers $[p^*(w),\theta^*(w)]$ such that $\theta^*(w) \in \theta^r(w,p^*(w))$ and $p^*(w) \in p^r(w,\theta^*(w))$.

Proposition 3.2.1: A unique w-equilibrium $[\theta^*(w),p^*(w)]$ exists for any $w = (h,\ell)$.

Proof: Suppose that there exist two w-equilibria for some w, $(p,\theta) \neq (p',\theta')$. If $p = p'$, then $\theta = \theta'$ by (3.2.4) (recall that C′ is strictly increasing). Without loss of generality, suppose $p > p'$. Then $p > 0$ and therefore $(2\theta-1)(u(h) - u(\ell)) \ge v$ by (3.2.1). This implies $h > \ell$ and $\theta > ½$. This implies, by (3.2.4), $\theta = [C']^{-1}[(1-2p)(h-\ell)]$. Since $p' < p$, we have $(1-2p')(h-\ell) > (1-2p)(h-\ell) > 0$ and

$\theta' = [C']^{-1}[(1-2p')(h-\ell)] > [C']^{-1}[(1-2p)(h-\ell)] = \theta$, since C' is strictly increasing. Hence $\theta' > \theta$ and therefore $(2\theta'-1)[u(h)-u(\ell)] > v$ implying, by (3.2.1), $p' = 1$. This contradicts $p' < p$. Q.E.D.

Proposition 3.2.2: For any subgame perfect equilibrium of the entire game, the probability of shirking, $1 - p^*(w^*)$, is larger than or equal to 1/2, even if $C'(\theta) = 0$ for all θ.

Proof: Suppose $p > 1/2$. Then (3.2.1) implies $\theta > 1/2$ and $h > \ell$. Therefore, $(1-2p)(h-\ell) - C'(\theta) < 0$ even if $C'(\theta) = 0$ for all θ. This implies, by (3.2.3), $\theta = 1/2$, contradiction. Q.E.D.

Intuitively, lowering θ is beneficial to the owner when $a = d$: the probability of having to pay $(h-\ell)$ decreases as θ decreases. Lowering θ harms the owner when $a = s$: he has to pay $(h-\ell)$ with a higher probability. If $p > 1/2$ (i.e., if d is more probable than s), the benefits of lowering θ outweigh the costs, due to the symmetry of the monitoring technology. Thus, the owner lowers θ down to $\underline{\theta}$ if $p > 1/2$, to which the best response of the worker is to set $p = 0$.

To show that this result hinges on the existence of false positives, in section 4 we will turn to a situation in which false positives are impossible. First, we study delegation as a way to solve this severe moral hazard problem.

3.3 The delegation game

The owner delegates monitoring to a supervisor. The worker's payoff is given in (2.1), the supervisor's payoff is given in (2.3), and the owner's payoff is given in (2.4), with

$E(g(w_i)|\theta,p) = p[\theta g(h) + (1-\theta)g(\ell)] + (1-p)[(1-\theta)g(h) + \theta g(\ell)]$ for any function g.

A strategy for the owner is a vector $\underline{w}$ in $\mathbb{R}^4$. A strategy for the supervisor is a pair of functions, $\theta(\underline{w}):\mathbb{R}^4 \to [\underline{\theta},\overline{\theta}]$, and $x_s(\underline{w}):\mathbb{R}^4 \to \{A,R\}$. A strategy for the worker is also a pair of functions, $p(\underline{w}):\mathbb{R}^4 \to [0,1]$, and $x(\underline{w}):\mathbb{R}^4 \to \{A,R\}$. A subgame perfect equilibrium of the game is a 5-tuple $<p^*(\cdot),\theta^*(\cdot),x_s^*(\cdot),x^*(\cdot),\underline{w}^*>$ satisfying, for all $\underline{w} \in \mathbb{R}^4$,

i) $p^*(\underline{w}) \in \underset{p \in [0,1]}{\operatorname{argmax}} \; U^w(p,\theta^*(\underline{w}),(A,A),\underline{w})$.

ii) $\theta^*(\underline{w}) \in \underset{\theta \in [\underline{\theta},\overline{\theta}]}{\operatorname{argmax}} \; U^s(p^*(\underline{w}),\theta,(A,A),\underline{w})$.

iii) $x_s*(\underline{w}) = A$ if and only if $U^s(p^*(\underline{w}),\theta^*(\underline{w}),(A,A),\underline{w}) \geq T_s$.[7]

iv) $x^*(\underline{w}) = A$ if and only if $U^w(p^*(\underline{w}),\theta^*(\underline{w}),(A,A),\underline{w}) \geq T$.

v) $\underline{w}^* \in \underset{\underline{w}' \in \mathbb{R}^4}{\operatorname{argmax}} \; U^o(p^*(\underline{w}'),\theta^*(\underline{w}'),\underline{x}^*(\underline{w}'),\underline{w}')$.

Finally, let $p^* = p^*(\underline{w}^*)$ and $\theta^* = \theta^*(\underline{w}^*)$.

An equilibrium is found in the following way: first, the reaction functions $p^r(\underline{w},\theta)$ and $\theta^r(\underline{w},p)$ of worker and supervisor (respectively) are calculated, for fixed $\underline{w}$. Second, the owner maximizes his payoff by choosing $\underline{w}$, θ and p subject to the constraint that p belong to $p^r(\underline{w},\theta)$ and θ belong to $\theta^r(\underline{w},p)$ and the constraint that, given p, θ, and $\underline{w}$, both the worker and the supervisor accept the contract.

The reaction function of the worker is found by solving, for each $\underline{w}$, the following problem:

$$\max_{p \in [0,1]} \quad U^w[p,\theta,A,A,\underline{w}] = p[\theta u(h) + (1-\theta)u(\ell)] + (1-p)[(1-\theta)u(h) + \theta u(\ell)] - pv.$$

The solution is:

$$p^r(\underline{w},\theta) = \begin{cases} 0 & \text{if } (2\theta-1)[u(h)-u(\ell)] < v \\ [0,1] & \text{if } (2\theta-1)[u(h)-u(\ell)] = v \\ 1 & \text{if } (2\theta-1)[u(h)-u(\ell)] > v. \end{cases} \tag{3.3.1}$$

The supervisor's reaction function is found by solving

$$\max_{\theta \in [½,1]} \quad U^s[p,\theta,A,A,\underline{w}] = p[\theta u_s(h_s) + (1-\theta)u_s(\ell_s)] + (1-p)[(1-\theta)u_s(h_s) + \theta u_s(\ell_s)] - C(\theta).$$

The first order conditions are:

$$(2p - 1)[u_s(h_s)-u_s(\ell_s)] - C'(\theta) \begin{cases} < 0 \text{ then } \theta = 1/2 \\ = 0 \text{ then } \theta \in [½,1] \\ > 0 \text{ then } \theta = 1. \end{cases} \tag{3.3.2}$$

Hence, using $C'(1/2) = 0$, $C'(1) = \infty$, and $C''(\theta) > 0$, we have

$$\theta^r(\underline{w},p) = \begin{cases} 1/2 & \text{if } (2p-1)[u_s(h_s)-u_s(\ell_s)] \leq 0 \\ [C']^{-1}\{[u_s(h_s)-u_s(\ell_s)](2p-1)\} & \text{otherwise.} \end{cases} \tag{3.3.3}$$

Since the owner can always ensure himself a payoff of zero (by offering the worker and the supervisor unacceptable contracts), in any interesting case the owner obtains a positive payoff. Assume a DG equilibrium exists with positive payoff U^{o*} for the owner. Then, such an equilibrium can be found by solving:

$$\max_{p\in[0,1],\theta\in[\frac{1}{2},1],\underline{w}\in\mathbb{R}^4} \quad p\pi - p[\theta(h+h_s)+(1-\theta)(\ell+\ell_s)] - (1-p)[(1-\theta)(h+h_s)+\theta(\ell+\ell_s)] \tag{3.3.4}$$

subject to:

(i) $p \in p^r(\underline{w},\theta)$

(ii) $\theta \in \theta^r(\underline{w},p)$

(iii) $p[\theta u(h) + (1-\theta)u(\ell)] + (1-p)[(1-\theta)u(h) + \theta u(\ell)] - pv \geq T$

(iv) $p[\theta u_s(h_s)+(1-\theta)u_s(\ell_s)] + (1-p)[(1-\theta)u_s(h_s)+\theta u_s(\ell_s)] - C(\theta) \geq T_s$,

where (iii) and (iv) ensure that the worker and the supervisor accept the contract. Given our assumption that $U^{o*} > 0$, constraint (ii) can be replaced by

(ii′) $(2p-1)[u_s(h_s)-u_s(\ell_s)] = C'(\theta)$.

To see this, note that $U^{o*} > 0$ implies $p^* > 0$ by Lemma 2.2, and therefore $\theta^* > 1/2$, by (3.3.1). Therefore, by (3.3.3), $\theta^r(\underline{w},p) = [C']^{-1}\{[u_s(h_s)-u_s(\ell_s)](2p-1)\}$, and therefore (ii) holds if and only if (ii′) holds.

We will relax constraint (i) by replacing it with

(i′) $(2\theta-1)[u(h)-u(\ell)] \geq v$.

Too see that (i) implies (i') under the assumption that $U^{o*} > 0$, note that $(2\theta-1)[u(h)-u(\ell)] < v$ implies $p = p^r(\underline{w},\theta) = 0$ by (i) and (3.3.1), which contradicts $U^o > 0$. We will show later on that the solution to the relaxed problem satisfies all the constraints in (3.3.4). The relaxed problem is:

$$\max_{p\epsilon[0,1],\theta\epsilon[\frac{1}{2},1],\underline{w}\epsilon\mathbb{R}^4} \; p\pi - p[\theta(h+h_s)+(1-\theta)(\ell+\ell_s)] - (1-p)[(1-\theta)(h+h_s)+\theta(\ell+\ell_s)] \tag{3.3.5}$$

subject to:

(i') $(2\theta-1)[u(h)-u(\ell)] \geq v$

(ii') $(2p-1)[u_s(h_s)-u_s(\ell_s)] = C'(\theta)$

(iii) $p[\theta u(h)+(1-\theta)u(\ell)] + (1-p)[(1-\theta)u(h)+\theta u(\ell)] - pv \geq T$

(iv) $p[\theta u_s(h_s)+(1-\theta)u_s(\ell_s)] + (1-p)[(1-\theta)u_s(h_s)+\theta u_s(\ell_s)] - C(\theta) \geq T_s$.

The issue of existence of a solution to (3.3.5) is addressed next.

Lemma 3.3.1: The following are sufficient conditions for existence of a solution $(p^*,\theta^*,\underline{w}^*)$ to (3.3.5) with $U^{o*} = U^o(p^*,\theta^*,(A,A),\underline{w}^*) > 0$:

(1) $\lim_{x \to \infty} u(x) = \bar{u}$.

(2) $\lim_{x \to \infty} u_s(x) = \bar{u}_s$.

(3) $\lim_{\theta \to 1} C(\theta) = \infty$.

(4) π, $\bar{u}$, and $\bar{u}_s$ are large enough for given v, T, and T_s.

Proof: See Olivella (1989).

We assume henceforth that all the conditions in Lemma 3.3.1 hold. We characterize the solution of (3.3.5) in the following Proposition:

<u>Proposition 3.3.1</u>: If π is large enough, a solution $(p^*, \theta^*, \underline{w}^*)$ to (3.3.5), satisfies:

(1) $h_s{}^* > \ell_s{}^*$,

(2) $p^* = 1$, and

(3) $\theta^* = \theta^c(w^*)$ as given in (3.1.2).

<u>Proof</u>: See the Appendix.

Intuitively, if it were the case that $h_s < \ell_s$, constraint (ii') would imply $p \leq 1/2$, and therefore we would run into the same double moral hazard problem as in the NC game. The owner therefore chooses $h_s > \ell_s$. Constraint (i') is binding, and hence $\theta = \theta^c(w)$, because otherwise the risk borne by the worker could be reduced by appropriately adjusting h and ℓ. Finally, if $p < 1$, the owner can adjust h_s and ℓ_s in such a way that the risk borne by the supervisor is reduced, and at the same time p is increased.

For π large enough, the solution to (3.3.5) satisfies all the constraints in the original maximization problem (3.3.4): the only constraint of (3.3.4) relaxed in (3.3.5) is $p \in p^r(\underline{w}, \theta)$. By Proposition 3.3.1, this constraint is satisfied, since $\theta^* = \theta^c(w^*)$ and therefore $p^r[\underline{w}^*, \theta^c(w^*)] = [0,1]$. Hence, as (3.3.5) is a relaxation of (3.3.4), every solution of (3.3.4) must be a solution of (3.3.5).

In other words, $(p^*, \theta^*) = [1, \theta^c(w)]$ is an equilibrium of the second stage game that supervisor and worker play.

Next question is whether other subgame equilibria exist given $\underline{w} = \underline{w}^*$. The answer is yes.

Proposition 3.3.2: Let $\underline{w}^*$ be the solution contract to (3.3.4). For π large enough, exactly two subgame equilibria exist in the subgame played between supervisor and worker once the contract $\underline{w}^*$ is signed. One of them is $(p^*,\theta^*) = [1,\theta^c(w^*)]$, and the other is $(p^0,\theta^0) = (0,½)$.

Proof: By constraints (i) and (ii) of (3.3.4) and Proposition 3.3.1, $(p^*,\theta^*) = [1,\theta^c(w^*)]$ is a subgame equilibrium. We show now that (0,½) is another subgame equilibria. Suppose $p = 0$. Then $\theta^r(\underline{w}^*,0) = 1/2$, since $h_s^* > \ell_s^*$ by Proposition 3.3.1. Suppose $\theta = 1/2$. Then $p^r(\underline{w}^*,½) = 0$. Done.

Finally, prove that no other subgame equilibrium exists given $\underline{w} = \underline{w}^*$. By Proposition 3.3.1, $(p^*,\theta^*,\underline{w}^*) = (1,\theta^c(\underline{w}^*),\underline{w}^*)$ is feasible in (3.3.5). Therefore, by constraint (ii') in (3.3.5), $[u_s(h_s^*)-u_s(\ell_s^*)] = C'[\theta^c(w^*)]$. Suppose that another subgame equilibrium (p',θ') exists. Note that $\theta^r(\underline{w},p)$ is a singleton for all $(\underline{w},p)$. Therefore, assume $0 < p' < 1$ (otherwise either $(p',\theta') = (p^*,\theta^*)$ or $(p',\theta') = (p^0,\theta^0)$). Then, by (3.3.1), it must be the case that $\theta' = \theta^c(w^*)$ as given in (3.1.2). However, since $p' < 1$, we have $(2p'-1)[u_s(h_s^*)-u_s(\ell_s^*)] < u_s(h_s^*)-u_s(\ell_s^*) = C'[\theta^c(w^*)]$. Therefore, either $(2p'-1)[u_s(h_s^*)-u_s(\ell_s^*)] \leq 0$, and then $\theta' = 1/2$ (by (3.3.1)), or $(2p'-1)[u_s(h_s^*)-u_s(\ell_s^*)] > 0$ and $\theta' = [C']^{-1}\{(2p'-1)[u_s(h_s^*)-u_s(\ell_s^*)]\}$ $< [C']^{-1}\{C'[\theta^c(w^*)]\} = \theta^c(w^*)$. In either case, $\theta' < \theta^c(w^*)$, contradiction. Q.E.D.

Given that two subgame equilibria exist for $\underline{w} = \underline{w}^*$, the question arises of whether the equilibrium in Proposition 3.3.1 is "renegotiation-proof" (RN),[8] that is, whether $(p^*, \theta^*) = (1, \theta^c(w^*))$ is <u>not</u> dispreferred by both supervisor and worker to the other subgame equilibrium (0,1/2). Equilibria that are not RN are relatively implausible: suppose that both the worker and the supervisor prefer $(p^0, \theta^0) = (0, \frac{1}{2})$ to $(p^*, \theta^*) = (1, \theta^c(w^*))$ (one of them strictly). Then the worker and supervisor could agree to play the equilibrium (p^0, θ^0), instead of the equilibrium (p^*, θ^*) that the owner prefers they play. We show now that this is not the case.

<u>Proposition 3.3.3</u>: For $\underline{w} = \underline{w}^*$ as given in Proposition 3.3.1, the supervisor strictly prefers the subgame equilibrium $(p^*, \theta^*) = [1, \theta^c(w^*)]$ to the subgame equilibrium $(p^0, \theta^0) = (0, \frac{1}{2})$.

<u>Proof</u>: Since $\theta^* = \theta^r(\underline{w}^*, p^*)$, and $\theta^r(\underline{w}, p)$ is a singleton for all $(\underline{w}, p)$, $U^s(p^*, \theta^*, (A,A), \underline{w}^*) > U^s(p^*, \theta^0, (A,A), \underline{w}^*)$. Since the derivative $[\partial U^s/\partial p](\underline{w}^*) = [u_s(h_s^*) - u_s(\ell_s^*)](2\theta - 1) \geq 0$ for all (θ, p), and $p^* = 1 > 0 = p^0$, we have $U^s(p^*, \theta^0, (A,A), \underline{w}^*) > U^s(p^0, \theta^0, (A,A), \underline{w}^*)$.

Q.E.D.

One of the most important results is given in the next Proposition:

<u>Proposition 3.3.4</u>: Provided that π is large enough, the owner prefers the DG equilibrium of Proposition 3.3.1 to the NC equilibrium.

Proof: Denote the NC equilibrium by $(p^{\sim},\theta^{\sim},w^{\sim})$. By the envelope theorem, $\partial U^{o\sim}/\partial\pi = p^{\sim}$. By Proposition 3.2.1, $p^{\sim} \leq 1/2$. On the other hand, $\partial U^{o*}/\partial\pi = p^{*} = 1$ for all $\pi > \pi'$ for some π', by Proposition 3.3.1. Therefore, for π large enough and above π', $U^{o*} > U^{o\sim}$. Q.E.D.

Intuitively, if the owner does not delegate the monitoring task, he cannot implement $p > 1/2$. The owner's expected gain, gross of wage outlays, is at most $\frac{1}{2}\pi$. By delegating and rewarding the supervisor when the signal is high, the owner implements $p = 1$, and therefore his expected gain gross of wage outlay is π. If π is large enough, the additional gross gain ($\frac{1}{2}\pi$) more than covers the expected supervisor's wage outlay. The owner is therefore better off.

Note that two assumptions are crucial here: 1) private contracts between supervisor and owner are not enforceable, and 2) the signal is publicly observable. If (1) was relaxed, the owner would privately propose to the supervisor not to monitor, offering him a utility payment of $T_S + \varepsilon$ no matter how the signal comes out. This would generate monitoring cost savings, plus increasing the probability of a false positive to 1/2. If (2) were relaxed, the supervisor would always announce that the signal is high so as to obtain h_S.[9]

The existence of false positives plays an important role in the efficacy of delegation as a commitment device. To show this, we turn our attention in Section 4 to a monitoring technology in which false positives are impossible.

4. THE NO-FALSE POSITIVES MONITORING TECHNOLOGY

The procedures to find an equilibrium for each game structure (FC,NC,DG) under the NFP monitoring technology are analogous to the respective procedures followed in Section 3. We therefore summarize the results found for this monitoring technology. The details of the analysis can be followed in Olivella (1989).

Recall that according to the No-False Positives (NFP) monitoring technology, the probability of a false positive (i.e., of $Y = L$ when $a = d$) is zero and the probability of a false negative (a high signal occurring when the worked shirked) is $1 - \theta$, and θ ranges between 0 and 1.

4.1 The full commitment game

The payoffs of owner and worker are given in (2.1) and (2.2) with $E(g(w_i)|\theta,p) = pg(h) + (1-p)[\theta g(\ell) + (1-\theta)g(h)]$ for any function g. It turns out that in this game the first best outcome, in which $\theta=0$, $x=A$, $p=1$ and the worker is paid $u^{-1}(v+T)$, can be approximated arbitrarily closely - but not attained. Making ℓ negative enough induces the worker to be diligent even if θ is very low. Moreover, since $p = 1$, he bears no risk. However, the first best itself cannot be attained, and therefore the FC/NFP game does not have an equilibrium.

To avoid this nonexistence problem it is assumed that a upper bound exists to the fine ($-\ell$) that a worker can be forced to pay. For simplicity we assume that the lower bound is zero. We show that this lower bound is binding in equilibrium. We also show that a sufficient condition for $p^* = 1$ in equilibrium is that $\pi > u^{-1}(v+T) + C[v/(v+T)]$

4.2 The no-commitment Game

The same upper bound on fines as in the FC game is imposed so that the equilibria in these two games can be compared. Putting a lower bound on ℓ turns out to be necessary to ensure existence of an equilibrium in the NC game as well.

The following Proposition summarizes our results:

Proposition 4.2.1: If $<p(\cdot),\theta(\cdot),x(\cdot),w^*>$ is a NC equilibrium, and if π is large enough, then the owner's equilibrium payoff U^{o*} is positive and

(1) the worker randomizes between the options of shirking and not shirking, i.e., $0 < p^* < 1$;

(2) the marginal increase in the owner's expected payoff induced by an increase in p (holding everything else fixed) is positive, namely, $[\partial U^o/\partial p](p^*,\theta^*,A,w^*) = \pi - (h^*-\ell^*)\theta^* > 0$.

(3) the constraint on the size of fines is binding, i.e. $\ell^* = 0$.

Part (1) is the most important in the Proposition. It implies that the worker shirks with positive probability in equilibrium. This result hinges on the assumption that monitoring is costly. Otherwise, $p^* = 1$ in equilibrium. In this sense, we say that the moral hazard problem is more severe under the FP monitoring technology than under the NFP monitoring technology, since under the former $p^* \leq 1/2$ even if monitoring is costless.

4.3 The delegation (DG) game

We impose the same upper bound on the fines that a worker can be coerced to pay as in the previous games. This makes any comparisons between the three games meaningful. That upper bound on $-\ell$ is necessary for existence of an equilibrium. The main result is given in the following Proposition:

Proposition 4.3.1: If the supervisor is risk neutral (with $u_S(x) \equiv x$) and $T_S = 0$, a tuple $(p', \theta', \underline{w}')$ exists that satisfies the following:
1) $(p', \theta', \underline{w}')$ is feasible in the owner's first stage problem, that is, (p', θ') constitutes a subgame equilibrium given that the wage schedule is $\underline{w}'$ and given $(p', \theta', \underline{w}')$, both the worker and the supervisor accept the contract.
2) The owner is better off playing the DG game and implementing $(p', \theta', \underline{w}')$ than in the NC equilibrium.

Intuitively, by offering the supervisor arbitrarily large prices for catching the worker shirking, i.e., by choosing ℓ_S sufficiently large, the owner is able to implement a subgame equilibrium in the second stage in which the supervisor sets $\theta = \theta^c(w)$ even if p is very close to 1.[10]

Proposition 4.3.1 does not ensure existence of an equilibrium in the DG game. In fact, it is possible that no equilibrium exists in the DG game given that risk sharing is not worsen by offering large prizes with small probability (since the supervisor is risk neutral). The point we want to make is that, if an equilibrium exists, it must convey a payoff for the owner even higher than the obtained with the tuple $(p', \theta', \underline{w}')$, and therefore that the owner is better off by delegating the monitoring task.

One way to ensure existence is to impose the following constraint: $\ell_S \leq \bar{\ell}$. In that case Proposition 4.3.1 is still valid if one assumes that $\bar{\ell}$ is large enough. This upper bound on ℓ_S could be justified on the grounds that the court system does not accept contracts that stipulate a prize that exceeds the value of the firm.

Finally, we have to make sure that the conditions in Proposition 4.3.1 do not violate Assumption 6. This assumption states: $u^{-1}[C(\theta)+T_S] \geq C(\theta)$ for all θ. Since $T_S = 0$ and the supervisor is risk neutral, $u^{-1}[C(\theta)+T_S] = C(\theta)$, and therefore Assumption 6 is satisfied.[11]

Note that, given ℓ_S is very large, there exists strong incentives for the supervisor to falsify the signal. The assumption that the signal is publicly verifiable is therefore crucial. There also exist incentives for the supervisor to bribe the agent to shirk so that the supervisor obtains the prize ℓ_S. We need to assume that contracts involving covert payments among the two agents are not enforceable.[12]

A couple of real world examples suggest how large the prizes from the principal (owner, in our terminology) to the delegate (supervisor) can be:

1) Suppose the owner is society, the supervisor is a bounty hunter, and the agent is a potential outlaw. The existence of large reward signs discourages theft: It induces larger search intensities as the best reply to small probabilities of committing crimes. Probabilities of crime are small because search is intense. However, crime occurs in equilibrium.[13]

2) Suppose the owner is society, the supervisor is a malpractice lawyer, and the worker is the medicine doctor. Doctors exert extreme care since doctors are aware of the huge benefit lawyers derive form catching a careless doctor. Examples in this line include product liability and worker safety.

5. CONCLUSIONS AND EXTENSIONS

One of the most general results in this paper is that the probability that the worker shirks is positive in equilibrium when commitment is not feasible. Moreover, in the false positives-no commitment game, the probability that the worker shirks is bounded away from 0 for any cost function. From a theoretical point of view, this result highlights the difference between zero monitoring costs and public observability. If p is publicly observable, contracts on p can be signed, so the incentive problem disappears. That monitoring costs are zero does not preclude the possibility that <u>both</u> the worker and the owner shirk in equilibrium. In the NFP-NC game, the owner's incentive to not monitor vanishes when monitoring is costless, and therefore $\theta = 1$ is credible, so the moral hazard problem disappears.

We have also shown that the delegation of the monitoring task may be advantageous for the owner when false positives can occur (in general), and also when false positives are impossible if the supervisor is risk neutral.

Also important is the empirical implication that the supervisor should be rewarded if the signal is high when false positives are possible, whereas he should be rewarded if the signal is low if false positives are impossible. For instance, suppose that in a public school (firm) a teacher's (worker's) effort cannot be observed directly. The teacher's effort is evaluated by interviewing his students (monitoring technology). If only one interview is performed, a diligent teacher could be accused of shirking (false positive). Society (owner) delegates this evaluation process to the school principal (supervisor).[14] The principal should be rewarded when the student's interviews indicate that the teacher was diligent because, supposing the teacher is diligent, such a reward scheme would motivate the school principal to conduct enough interviews to reveal the teacher's diligence. The teacher would be diligent because he would know that so many interviews were going to be conducted.

On the other hand, in the case of a secretary who is fully responsible for his typos and who cannot be accused of shirking if no typos are detected (no false positives), a supervisor should be rewarded for finding typos.

Extensions

The author has explored (see Olivella (1989), Appendix C) the case in which the monitoring technology is a two-parameter information structure: let θ be the probability that Y=L when the worker shirks, and let ψ be the probability that Y=H when the worker is diligent. If the owner can choose θ and ψ independently without any other restriction than to satisfy the Monotone Likelihood Ratio Condition, the equilibrium probability of shirking in the NC game is one even if monitoring costs are 0 for all (θ,ψ). The intuition is the following: the MLRC is binding in equilibrium since the owner tries to maximize the probability of a false positive $(1-\psi)$ independently of his choice of θ. If the MLRC is binding then the signal is not informative about the worker's action, and the worker's best reply to this is to shirk.

The results in our analysis of the delegation game hinge on two assumptions:

1) Any private contract between any two of the three players is not enforceable.

2) The signal generated by monitoring is publicly observable.

An interesting extension would be to relax (2). In that case a reporting problem would arise between supervisor and owner. Contracts would have the additional constraint that the supervisor be induced to report the signal truthfully. Note, however, that if the signal was not publicly observable, explicit contracts could not be written on the signal even if the owner did not delegate monitoring: the owner would always report to the court that the signal was low so as to collect the fine from the worker.

Our model could also be applied to a situation where shareholders (owner) hire an auditor (supervisor) to evaluate the decisions of a manager (worker). Since auditing is a form of ex-post supervision, we argue that it is adequate to treat auditing as a no-commitment game. Also, it is unlikely that evidence of malfeasance on the part of the manager can be produced if the manager was diligent. Therefore, our key prediction is that auditors should be rewarded when producing such evidence. This is not the case in the real world, where auditors usually receive a flat rate. The static character of our model explains this shortcoming. The relations between firms and auditors are built in a dynamic setting, where reputation (of the auditor) plays a crucial role as an incentive device.[15] Studying the repeated version of the DG game would therefore be a natural extension of our model.

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FIGURES

Figure 1: The Full Commitment Game

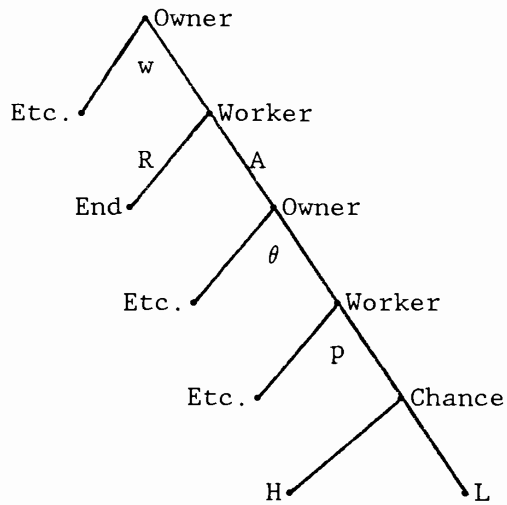


Figure 2: The No Commitment Game. The dotted line represents a single information set for the worker.

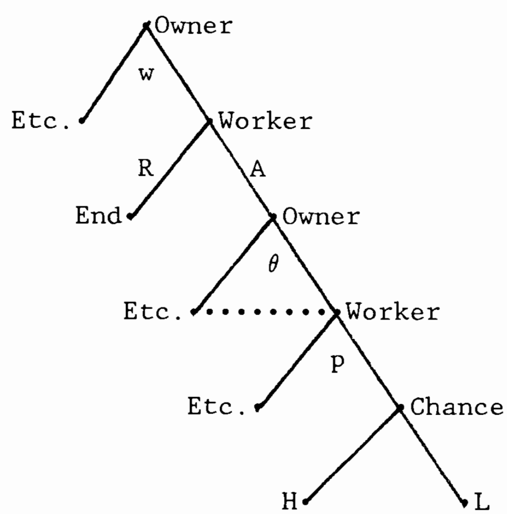
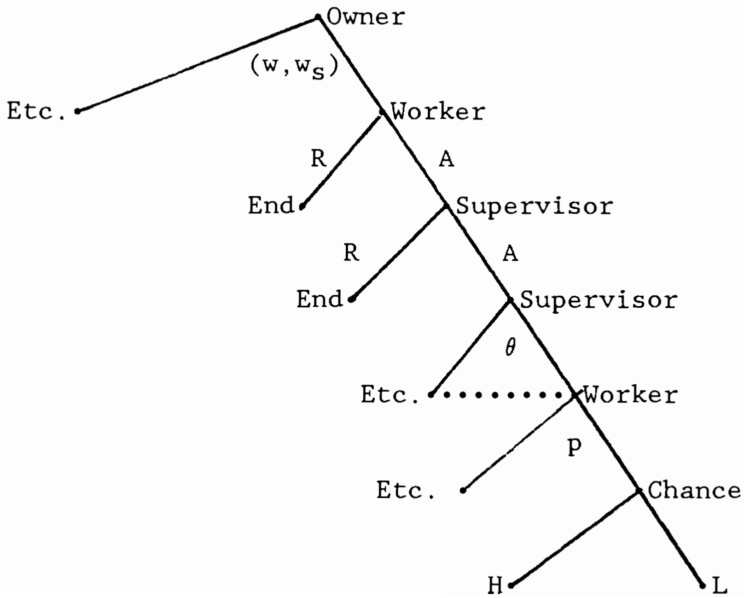


Figure 3: The Delegation Game. The dotted line represents a single information set for the worker.



FOOTNOTES

1.There are many reasons why such an assumption may hold. Output may not be verifiable to a third party because, for example, some aspects of output are intangible (good will, reputation, high quality service). Second, it might be the case that the owner can manipulate the observable output by, for instance, consuming part of the output or garbling the accounts. (This is a moral hazard problem on the part of the owner). Third, the worker might be able to misrepresent the output to a third party by, for instance, hiding the fact that some of the units produced are defective.

2.Otherwise, the following scheme would implement the full information first best if output is not random: at the beginning of the game, the owner sells the property rights to output to the worker, at a price that ensures the worker a residual that exactly covers his disutility of effort and his opportunity utility. If output is random, this scheme would also be used if the worker is not too risk averse and the distribution of output is not too risky (even though the first best would not be implemented).

3.See also Fershtman and Judd (1986a) and (1986b).

4.See Milgrom (1981).

5.Throughout this paper, the possibility that the worker randomizes between the options of accepting or rejecting the contract is ignored. Indeed, a strategy that would involve the worker accepting the contract with probability q, $0 < q < 1$, could only be part of an equilibrium if the owner, as well as the worker, was indifferent between the worker accepting or rejecting the contract, since otherwise the owner could earn a discrete gain via an arbitrarily small increase in wages. If the owner was indifferent between the worker accepting or rejecting the contract, his equilibrium payoff would have to be zero, an uninteresting case, since the owner can always obtain zero by offering the worker an unacceptable contract.

6.The issue of existence of a solution to that first stage problem is addressed in Olivella (1989). It is shown that a sufficient condition for existence is that u(x) be bounded above. This, of course, rules out the risk neutral case $u(x) = x$.

7.See footnote 5, replacing "supervisor" for "worker".

8.The term <u>renegotiation proof</u> is taken from the papers by Bernheim and Ray (1987) and Farrell and Maskin (1987), which discuss renegotiation in repeated games. Although our definition differs from theirs, it is similar in spirit.

9.See Tirole (1984) for similar considerations.

10.Using a proof analogous to the one for Proposition 3.2.1, one can show that, in the DG-NFP game, a unique subgame equilibrium exists for any $\underline{w}$. Therefore, one need not worry about the "renegotiation proofness" issue.

11. The assumption that the supervisor is risk neutral may be too strong: In some cases it may be enough to assume that the supervisor is asymptotically risk neutral, that is $u_S'(x) \rightarrow \alpha > 0$ as $x \rightarrow$ infinity.

12.Again, see Tirole (1984) for similar considerations.

13.One might argue that the probability of theft is large and that search is not intense in today's American society. Our model suggests that things might change if large prizes (for catching a thief) were introduced. Some individuals would be accepting the risk involved in bounty hunting, and would become the "society's supervisors". Such policy recommendation hinges on the assumptions that 1) false positives are impossible and 2) bounty hunters would find it impossible to write contracts with (potential) thieves.

14.The word "principal" has the opposite meaning to the usual meaning in agency theory: the school principal is an agent of society.

15.See, for example, Watts and Zimmermann (1983) for a historical perspective on auditing and the role of reputation in the issue of auditor's independence.

APPENDIX

Proof of Proposition 3.3.1

Break problem (3.3.5) into two maximization problems, M1 and M2. In M2, $\underline{w}$ is restricted to be in $A \equiv \{\underline{w} \in \mathbb{R}^4 | h_s \geq \ell_s\}$. In M1, $\underline{w}$ is restricted to be in the complement of A in $\mathbb{R}^4$, A^c. We show that for π large enough the solution to M2 must also be the solution to (3.3.5).

Claim 1: $E(w_{si}|\theta,p) \geq 0$ for all $(p,\theta,\underline{w})$ that are feasible.

By (iv), $E(u_s(w_{si})|\theta.p) \geq T_s + C(\theta)$. By the concavity of $u_s(\cdot)$ and Assumption 6 we have $E(w_{si}|\theta,p) \geq u_s^{-1}[T_s + C(\theta)] \geq C(\theta) \geq 0$.

Claim 2: $E(w_i|\theta,p) \geq 0$ for all $(p,\theta,\underline{w})$ that are feasible.

By (iii), $E(u(w_i)|\theta,p) \geq pv + T$. By the concavity of $u(\cdot)$ and $u(0) = 0$, we have $E(w_i|\theta,p) \geq u^{-1}(pv+T) \geq 0$.

Claim 3: $U^{o\prime} = U^o(p',\theta',(A,A),w') \leq \frac{1}{2}\pi$ for any (p',θ',w') feasible in M1.

Note that any feasible tuple $(p',\theta',\underline{w}')$ in M1 has $p' \leq 1/2$ (by constraint (ii')). Therefore, $U^{o\prime} \leq \frac{1}{2}\pi$, since $E(w_i+w_{si}|\theta',p') \geq 0$ by Claims 1 and 2, done.

Claim 4: There exists a tuple $L = (p,\theta,\underline{w})$ that is feasible in M2, and yields $U^o = U^o(p,\theta,(A,A),\underline{w}) = \pi - E(w_i+w_{si}|\theta,p)$.

Pick $1/2 < \theta < 1$, and let $p = 1$, $h = u^{-1}[T + \theta v/(2\theta-1)]$, $\ell = u^{-1}[T - (1-\theta)v/(2\theta-1)]$, $h_s = u_s^{-1}[C(\theta)+(1-\theta)C'(\theta)+T_s]$, $\ell_s = u_s^{-1}[C(\theta)-\theta C'(\theta)+T_s]$. By condition (4), wages h and ℓ are well defined since $\theta > 1/2$, and wages h_s and ℓ_s are well defined since $\theta < 1$. It is easy to check that all the constraints in (3.3.5) are satisfied. The owner's payoff is:

$U^o = U^o(p,\theta,(A,A),\underline{w}) = \pi - [\theta(h+h_s)+(1-\theta)(\ell+\ell_s)]$, done.

In Claim 4, we have found a tuple L feasible in M2 that yields $U^o = \pi - [\theta(h+h_s)+(1-\theta)(\ell+\ell_s)]$. Note that feasibility is independent of π. Moreover, by Claim 3, $U^{o\prime} \leq \frac{1}{2}\pi$ for any (p',θ',w') feasible in M1. Therefore, for π large enough the solution to M2 is also the solution to (3.3.5). This proves part (1) of the Proposition.

Construct now the Lagrangian $L(\cdot)$ associating multipliers λ_1, λ_2, λ_3 and λ_4 to the constraints (i'), (ii'),(iii) and (iv), respectively. By Kuhn-Tucker, $\lambda_1 \geq 0$, $\lambda_3 \geq 0$, and $\lambda_4 \geq 0$.

Step 1: Show part (3) of the Proposition. It is enough to show that $\lambda_1 > 0$, since then $(2\theta-1)[u(h)-u(\ell)] = v$ and one obtains $\theta = \theta^c(w)$ by solving for θ. Let $Q = p\theta + (1-p)(1-\theta) = pr(Y{=}H)$. We now show that $0 < Q < 1$. Show first that a $0 < \varepsilon < 1/2$ exists such that a tuple $(p,\theta,\underline{w})$ is feasible only if $\theta \leq 1-\varepsilon$. From constraint (iv), $E(u(w_{si}|\theta,p) \geq C(\theta) + T_s$. By condition (2) in Lemma 3.3.1, this implies $\bar{u}_s \geq C(\theta) + T_s$. By condition (3) in Lemma 3.3.1, there exists $1/2 < \theta' < 1$ such that $\bar{u}_s = C(\theta') + T_s$. Hence $\theta \leq \theta'$ (otherwise the voluntary participation constraint for the supervisor would be violated), and we can let $\varepsilon = 1 - \theta'$.

Since $1/2 \leq \theta \leq 1-\varepsilon$, we have $Q \leq p(1-\varepsilon) + (1-p)\frac{1}{2} = p(\frac{1}{2}-\varepsilon) + \frac{1}{2} \leq 1 - \varepsilon$.

On the other hand, since $\theta \geq 1/2$ (by definition) and $p \geq 1/2$ by constraint (ii) and $h_s > \ell_s$, we have $Q = p\theta + (1-p)(1-\theta) \geq p\theta \geq 1/4$. Hence, $Q > 0$. That is, $0 < Q < 1$.

Two of the Kuhn-Tucker conditions are:

$$\partial L/\partial h = -Q + \lambda_1(2\theta-1)u'(h) + \lambda_3 Q u'(h) = 0$$

$$\partial L/\partial \ell = -(1-Q) - \lambda_1(2\theta-1)u'(\ell) + \lambda_3(1-Q)u'(\ell) = 0.$$

Suppose $\lambda_1 = 0$. Then these expressions imply $\ell = h$ since $0 < Q < 1$, which violates constraint (i'). This proves part (3) of the Proposition.

Step 2: Show part (2) of the Proposition, i.e., show $p = 1$. Since $\lambda_1 > 0$, we can reduce problem (3.3.5): solve for θ in (i'), which holds with equality, and substitute into (iii) to obtain:

(iii') $\frac{1}{2}[u(h) + u(\ell) - v] \geq T$.

We can also solve for p in (ii') and substitute into (iv) to obtain:

(iv') $\frac{1}{2}[u_s(h_s)+u_s(\ell_s)] + (\theta - \frac{1}{2})C'(\theta) - C(\theta) \geq T_s$.

Then (3.3.5) becomes:

$$\max_{p\in[0,1],\theta\in[\frac{1}{2},1],\underline{w}\in\mathbb{R}^4} \quad p\pi - p[\theta(h+h_s)+(1-\theta)(\ell+\ell_s)] - (1-p)[(1-\theta)(h+h_s)+\theta(\ell+\ell_s)]$$

subject to:

(i') $(2\theta-1)[u(h)-u(\ell)] = v$

(ii') $(2p-1)[u_s(h_s)-u_s(\ell_s)] = C'(\theta)$

(iii') $\frac{1}{2}[u(h) + u(\ell) - v] \geq T$

(iv') $\frac{1}{2}[u_s(h_s)+u_s(\ell_s)] + (\theta - \frac{1}{2})C'(\theta) - C(\theta) \geq T_s$.

Associate multipliers λ_1, λ_2, λ_3 and λ_4 to the constraints (i'), (ii'),(iii') and (iv'), respectively. Three of the Kuhn-Tucker conditions for this problem are:

$$\frac{\partial L}{\partial p} = \pi - (2\theta-1)[(h-\ell)+(h_s-\ell_s)] + 2\lambda_2[u_s(h_s)-u_s(\ell_s)] \begin{cases} > 0 \rightarrow p = 1 \\ = 0 \rightarrow p \in [0,1] \\ < 0 \rightarrow p = 0 \end{cases}$$

$$\frac{\partial L}{\partial h_s} = -Q + u_s'(h_s)[\lambda_2(2p-1) + \tfrac{1}{2}\lambda_4] = 0$$

$$\frac{\partial L}{\partial \ell_s} = -(1-Q) + u_s'(\ell_s)[\tfrac{1}{2}\lambda_4 - \lambda_2(2p-1)] = 0.$$

Using the last two expressions, solve for λ_2 and λ_4 to obtain:

$$\lambda_2 = \frac{\tfrac{1}{2}}{2p-1}\left[\frac{Q}{u_s'(h_s)} - \frac{1-Q}{u_s'(\ell_s)}\right].$$

Therefore,

$$2\lambda_2[u_s(h_s)-u_s(\ell_s)] = \frac{1}{2p-1}\left[\frac{Q}{u_s'(h_s)} - \frac{1-Q}{u_s'(\ell_s)}\right][u_s(h_s)-u_s(\ell_s)].$$

By concavity of $u_s(\cdot)$, we have

$$\frac{u_s(h_s)-u_s(\ell_s)}{u_s'(h_s)} \geq h_s - \ell_s \geq \frac{u_s(h_s)-u_s(\ell_s)}{u_s'(\ell_s)}.$$

Since $h_s > \ell_s$, (ii') implies $2p - 1 > 0$. Hence,

$$2\lambda_2[u_s(h_s)-u_s(\ell_s)] \geq \frac{1}{2p-1}[Q - (1 - Q)](h_s - \ell_s).$$

Note that $Q/(2p-1) = [p\theta + (1-p)(1-\theta)]/(2p-1) = \theta + (1-p)/(2p-1)$, and

$(1-Q)/(2p-1) = p/(2p-1) - \theta$. Substitute these into the previous expression to get:

$2\lambda_2[u_s(h_s)-u_s(\ell_s)] \geq (2\theta-1)(h_s-\ell_s)$.

Substitute this into the first Kuhn-Tucker condition listed:

$\partial L/\partial p = \pi - (2\theta-1)[(h-\ell)+(h_s-\ell_s)]$

$+ 2\lambda_2[u_s(h_s)-u_s(\ell_s)] \geq \pi - (2\theta-1)(h-\ell)$.

By contradiction, assume that $\pi - (2\theta-1)(h-\ell) \leq 0$. Then

$U^o = p\pi - E(w_i|\theta,p) - E(w_s i|\theta,p) =$

$p[\pi - (2\theta-1)(h-\ell)] - (1-\theta)h - \theta\ell - E(w_{si}|\theta,p) \leq$

$-(1-\theta)h - \theta\ell - E(w_{si}|\theta,p)$.

Note that $-(1-\theta)h - \theta\ell \equiv E(w_i|\theta,0)$, i.e., the expected worker's wage outlay when the worker shirks. However, since the worker is left indifferent between shirking and being diligent (by Step 2, in which we showed that $\lambda_1 > 0$), we have $T \leq E(u(w_i)|\theta,p) - pv = E(u(w_i)|\theta,0)$. By concavity of $u(\cdot)$, we have

$E(w_i|\theta,0) \geq u^{-1}(T) \geq 0$. Also, $E(w_{si}|\theta,p) > 0$ since $E(u_s(w_{si})|\theta.p) \geq T_s + C(\theta)$ (otherwise the supervisor would reject the contract), and using the concavity of $u_s(\cdot)$ and Assumption 6 we have: $E(w_{si}|\theta,p) \geq u_s{}^{-1}[T_s + C(\theta)] \geq C(\theta) > 0$ since $\theta > 1/2$ by constraint (i'). We have shown that $\pi - (2\theta-1)(h-\ell) \leq 0$ implies $U^o < 0$, contradiction. Therefore $\pi - (2\theta-1)(h-\ell) > 0$ and $\partial L/\partial p > 0$, which implies $p = 1$, done. This proves part (2) of the Proposition. Q.E.D.