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BIDDING OFF THE WALL: WHY RESERVE PRICES ARE KEPT SECRET

by

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Abstract

This note shows by means of an example that in a common value auction a seller with a random reservation value can increase her ex ante expected profits by following a policy of conducting an auction in which her reserve price is kept secret compared to one in which the reserve price is announced. By keeping the reserve price secret, the seller is able to encourage greater participation from the bidders and can, therefore, increase the linkage of the price paid to the value of the purchased object.

Bidding Off the Wall: Why reserve prices are kept secret

It is well-known that if a seller of a good at a common-value auction has private information about an object to be sold, she can increase her expected profits by following a policy of credibly revealing the information. By making relevant information public, the seller is able to alleviate some of the costs due to the winner's curse and therefore increase the average bid. In view of this wisdom, it has seemed to be a puzzle that in many auctions, sellers will typically not announce the reserve price in advance. At auctions of fine wines and art, auctioneers will generate phantom bids 'off the wall' or 'from the chandelier' in order to keep the object in-house when the bidding from the floor is not high enough to warrant selling the good.[1]

Does such behaviour violate the principle of the optimality of information revelation? This paper shows that a policy of keeping private reserve prices can be revenue-enhancing for a seller in a common-value auction. The conclusion, of course, does not overturn the standard wisdom. Instead it follows from it. The announcement of a reserve price may have an inhibiting effect on the participation of bidders in a given auction -- for some potential bidders, the only possibility of winning is to win at the reserve price and such an event may occur only when the object is not worth purchasing. This possibility will discourage some bidders from participating. As a result, their information does not play a role in the

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[1] For an interesting discussion of this phenomenon, see Ashenfelter (1989). According to Ashenfelter, "If you sit through an auction you will find that every item is hammered down and treated as if it were sold. ... In short, the auctioneers do not reveal the reserve price and they make it as difficult as they can for bidders to infer it."

process of the auction even though it may be relevant for the valuation of other bidders. The consequence is to prevent some sales from being made even though the aggregate information would imply that a transaction should occur. This note illustrates that in a Bayesian game in which the reservation price is kept secret, the seller can induce more participation by bidders. In general, bidders submit lower bids since it is not known whether the rival price is that of another bidder or the seller but this cost may be worth incurring if the policy encourages more bidders to participate and therefore induces a greater aggregation of information.

Section One characterizes an equilibrium of a common value auction game and shows that, in general, the announcement of a reserve price discourages the participation of bidders. The next section focusses on a particular class of common value games and characterizes as well the Bayesian Nash equilibrium of a game in which the seller communicates to the auctioneer her desired reserve price but instructs the auctioneer not to divulge this information to the bidders. It is shown that this policy will yield a significantly higher expected revenue to the seller when the common value element is high.

Section One: Equilibrium With An Announced Reserve Price

For ease of analysis, this paper will concentrate on a second-price auction. While second-price auction and English auctions generally are not equivalent, the effect highlighted in this paper -- that is, the inhibiting effect that the announcement of a reserve price will have on participation rates -- will only be stronger in an English auction where the participation of bidders conveys even more information. Furthermore, the speed at which wine auctions, for example, proceed suggests that most participants are not actually aware of more than the fact that some other bidders are active. They rarely can tell if and at what price bidders drop out. Therefore, the practical consequences of focussing on a second-price auction seem relatively harmless.

This section analyzes equilibrium behaviour of bidders in a second-price auction with a reserve price. A seller has a reservation value, s, (thus her payoff from a sale at price p is p-s) which is distributed randomly with a distribution function H(s) over $[s_l, s_h]$. There are n+1 risk neutral buyers whose value, v, for the object is affiliated but who differ in their information about what the realized value of v is. A buyer of type i observes information X_i. The variables X_i are identically distributed over $[0, x_h]$ with a strictly increasing distribution function F(x). For a full treatment of the implications of affiliated random variables see Milgrom and Weber (1982). Here, it is sufficient to note that an implication of affiliation is the fact that the function

$$h(a_1, b_1; \ldots a_{n+1}, b_{n+1}) = E(v_i | a_1 \le X_1 \le b_1, \ldots, a_{n+1} \le X_{n+1} \le b_{n+1})$$

is increasing in all its arguments. It is assumed here that the monotonicity is strict. A high signal x_i (should it be known) generates a higher estimate by all buyers of the value of the good. Observe that it is assumed that the reservation value of the seller does not affect the value of the object of the buyers.[2]

In an announced reserve price auction (ARP) the seller announces a reserve price, r_A and commits herself to sell the object only at a price r_A

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[2] This is not a very strong restriction since it may be presumed that, as in wine auctions, information that the seller has which is relevant to the buyers' valuations and can be made public has been made public already.

or higher. If the second highest bid exceeds r_A then the highest bidder buys the object at a price equal to the second highest bid but if the second highest bid is less than r_A and the highest bid exceeds r_A the high bidder purchases the object at the price r_A.

Let $(x_1, x_2, \ldots x_{n+1})$ be the realization of the bidders' information and let $Y_1 = \max\{X_i\}$ be the first-order statistic of X, Y_2 is the second order statistic. Define $v(x,y) = E(v|X_1 = x, Y_{-1} = y)$[3]. In Milgrom and Weber (1982) it is shown that if the seller's reserve price r is known to be zero, the profile of bids, $b(x) = v(x,x)$ constitutes a Nash equilibrium of the second price auction. Let $r_A \geq 0$ and define $d(r)$ such that $r_A = E(v \mid X_1 = d(r), Y_{-1} \leq d(r))$.

Theorem One: The profile of strategies

$$b(x) = E(\ v \mid X_1 = x,\ Y_{-1} = x) \text{ for } x \geq d(r),$$

$$b(x) = E(v \mid X_1 = x,\ Y_{-1} \leq d) \text{ for } x < d(r)$$

constitutes a Nash Equilibrium of the second price auction game when the seller's reserve price is known to be r_A.

Proof: Consider buyer 1 and suppose all other buyers follow the strategy listed above. Let $\beta(b)$ be the inverse of the strategy $b(.)$. Note that $b(.)$ is discontinuous at $d(r)$ since affiliation implies that $E(\ v \mid X_1 = x,\ Y_{-1} = x) \geq E(\ v \mid X_1 = x,\ Y_{-1} \leq x)$.

Case 1: $x_1 \geq d(r)$. Suppose that bidder 1 submits a bid, $b > r_A$. His payoff is given by

$$U(b;x) = \text{Prob}(Y_{-1} < d|\ x\ )(E(\ v \mid X_1 = x,\ Y_{-1} \leq x) - r_A) +$$
$$\text{Prob}(Y_{-1} \geq d) E\{[E(\ v \mid X_1 = x, Y_{-1}) - v(Y_{-1}, Y_{-1})] 1_{\{b(Y-1)<b\}} \mid X_1 = x, Y_{-1} \geq d\}$$

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[3] Here, Y_{-i} should be read as the first order statistic excluding bidder i -- that is, the highest of the other bidders.

Since the first term is independent of b, we can rewrite the expression as

$$U(b;x) = K + \int_{d}^{\beta(b)} v(x,y) - v(y,y) \; f_{Y_{-1}}(y)dy$$

Since $v(.,y)$ is increasing in x, the second term is maximized at $\beta(b) = x$ or $b = b(x)$.

Case 2: $x < d(r)$. From the argument above, if a bid higher than $v(d,d)$ is made and the object is won, then the expected utility of the object to the buyer is less than b. If a bid in the interval $(r_A, v(d,d))$ is made and the object is won, the object is worth $E(v \mid X_1 = x, Y_{-1} \leq d)$ which is less than r by definition of d and by affiliation. Therefore any 'serious' bid always generates negative profits in the event that the bidder buys the object -- the bid specified above is only one of many possible best responses for the bidder with $x < d(r)$. ||

Corollary : For a fixed $r_A > 0$, there is a set of positive measure of realizations of X for which $E(v \mid Y_1 = y_1, Y_2 = y_2) \geq r$ and yet no trade occurs.

Proof: Define d' such that $E(v \mid X_1 = d', Y_1 = d') = r$. By affiliation, $d' < d$. The set of events $D = \{Y_1 \in (d',d) \text{ and } Y_2 \in (d',d)\}$ occur with positive measure, the value of v conditional on learning $(Y_1,Y_2) \in D$ exceeds r but no acceptable bid is made. ||

The Corollary indicates that with positive probability a transaction fails to occur which would have occurred if the bidders knew the realization of the highest and second highest valuations.

In general, the seller will wish to set her reserve price strictly higher than her use value, s, in order to extract a higher surplus from the bidders. In a second price auction, for a seller of type s the expected return from a reserve price of r is given by

$$\pi_A(r;s) = r\text{Prob}(Y_1 \geq d(r))\text{Prob}(Y_2 \leq d(r) \mid Y_1 > d(r))$$
$$+ E(v(Y_2,Y_2) \mid Y_2 \geq d(r))\text{Prob}(Y_2 \geq d(r)) - \text{Prob}(Y_1 \geq d(r))s.$$

She obtains r when only Y_1 exceeds the cut-off point d(r) and obtains the second highest type's bid when they both exceed d.

Analytic solutions for the optimal choice of r are in general rare in common value auctions. In what follows, therefore, we concentrate on a specific form of distributions of bidders types and common value. Suppose that all bidder types are drawn independently from the uniform [0,1] distribution. Suppose further that the valuation of bidder i is given by $V_i = X_i + TY_2$, $T \geq 0$. That is, it depends on his private information X_i and a common value component determined by the second order statistic. From Theorem One, it is straightforward to compute the bidding strategies for any announced r, that is, $b(x) = (T+1)x$ for $x \geq d(r)$, where $d(r) = dr = (n+1)/(n+1+Tn)r$.[4] Using this behaviour, the choice of r which maximizes the seller's expected revenue is given by

$$dr(s) = 0, \text{ for } s \leq -1,$$
$$= d(s+1)/(d+1), \text{ for } s \in [-1,1/d]$$
$$= 1 \text{ otherwise.}$$

Observe that when s exceeds 1/d, no bidder type would submit a bid which the seller would ever accept.

Section Two: A Better Way

If the seller's use value s were common knowledge, then whether or not r was announced would make no difference in an auction -- bidders would simply compute the seller's optimal r and behave as if it were announced.

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[4] This can be found by noting that $E(v \mid X_1 = d(r), Y_{-1} \leq d(r)) = d(r) + Tn/(n+1)d(r) = r$ implies that $d(r) = (n+1)/(n+1+Tn)\ r = dr$.

Suppose, though, that s is random. The seller observes s and could, if she desires, announce the reserve price $r_A(s)$ and conduct the auction as in Section One. Alternatively, she could tell the reserve price to the auctioneer and instruct him not to reveal it to the bidders. If the bidding does not go high enough, the auctioneer will have to create bids "off the wall". Again, sales occur whenever the highest bid exceeds the reserve price but now that the reserve price is not known, it will not be common knowledge whether or not the winning price is that of a bidder or that of the seller.

Call this game a secret reserve price auction (SRP). The SRP is a Bayesian game between the seller and the bidders. In a Bayesian Nash equilibrium of this game, given the behaviour of the bidders, each seller type must choose her best reserve price, r and given this choice rule of seller types (and the prior distribution of seller types) bidders must choose their optimal bidding strategy. In general, the characterization of an equilibrium of this game is not tractable, however, for the specification described in Section One and for a particular seller distribution, we can arrive at a closed form solution of the game.

Suppose s is distributed over $[-m,m]$ according to the distribution function $H(s) = ((s+m)/(2m))^m$. Suppose further that all bidders follow the linear bidding strategy $b(x) = mx$. Lemma Two characterizes the best response of a seller of type s.

Lemma Two: If n+1 bidders each with private information x drawn independently from the uniform [0,1] distribution submit bids mx in the SRP auction, then the optimal reserve price for a seller of type $s \in [-m,m]$ is $r(s) = (m+s)/2$.

Proof: A seller of type s who sets a reserve price of r receives an expected profit of

$$\pi_S(r;s) = r\mathrm{Prob}(Y_1 \geq r/m)\mathrm{Prob}(Y_2 \leq r/m) + E(mY_2 | Y_2 \geq r/m)\mathrm{Prob}(Y_2 \geq r/m) - \mathrm{Prob}(Y_1 \geq r/m)s$$

$$= (n+1)r(1-r/m)(r/m)^n + (n+1)n\, m \int_{r/m}^{1} z(1-z)z^{n-1}\, dz - (1 - (r/m)^{n+1})s.$$

Maximizing this expression with respect to r yields the desired result. ||

If the seller types follow this strategy but do not anounce s or r, then from the point of view of the bidders, the reserve price is random and is distributed over [0,m] according to $H(r) = (r/m)^m$. Now consider the viewpoint of bidder one. Suppose that the seller types follow the strategy in Lemma Two and that all other n bidders follow the bidding strategy, b(x) = mx where $m = (1+Tn(n+2)/(n+1)^2)$. Suppose that bidder one wins at a price, p. Let P(p) be the probability that the highest other bidder observation (excluding bidder one) is p/m given that the winning price is p. 1 - P(p) is the probability that $Y_{-1} < p/m$ or in other words the probability that the price to be paid is, in fact, the seller's reserve price. The density of the seller's reserve price conditional on p being the price to be paid and on $Y_{-1} < p/m$ is

$$\hat{h}(p) = F^n(p/m)h(p).$$

The density of Y_{-1} conditional on p and $Y_{-1} = p/m$ is

$$\hat{f}(p/m) = nH(p)F^{n-1}(p/m)f(p/m).$$ Thus

$$P(p) = (nH(p)F^{n-1}(p/m)f(p/m)/(nH(p)F^{n-1}(p/m)f(p/m) + F^n(p/m)h(p))$$

$$= nH(p)/[nH(p)+F(p/m)h(p)]$$

$$= np^m/[np^m+p^m] = n/(n+1).$$

That is, this choice of distribution yields a particularly simple updating rule. For any price p, with probability 1/(n+1), the price is the seller's reserve price.

Define the function

$$v(x,p) = E(v \mid X_1 = x,\ Y_{-1} \le p/m)/(n+1) + nE(v \mid X_1 = x,\ Y_{-1} = p/m)/(n+1).$$

$$= \{x + T(n/(n+1))(p/m)\}/(n+1) + n\{x + T\ p/m\}/(n+1)$$

Observe that $v(.,.)$ is increasing in both its arguments and that $v(x,mx) = (1+Tn(n+2)/(n+1)^2)x = mx$.

Theorem Three: The profile of strategies, $b(x) = mx$ for all x, is a Bayesian Nash Equilibrium of the second-price auction game when the seller's reserve price is private.

Proof: Let p be the (random) price paid. With $b(.)$ fixed, the distribution of $p = \max(r,b(x_2),\ldots b(x_{n+1}))$ is determined by the parameters of the model, F, H, b. Call it $T(p)$. Given that all other bidders use the strategy $b(.)$, the expected payoff from a bid, b, is

$U(x,b) = E\left((v(x,p) - p)1_{\{p \le b\}} \mid X_1 = x\right)$ where the expectation is taken with respect to $T(p)$. Thus,

$$U(x,b) = \int_0^b v(x,p) - p\ \ dT(p).$$

Since $v(.,p)$ is increasing in x and, by definition of m, $v(p/m,p) = p$, this expression is maximized by choosing $b = mx$. Therefore, given the presumed behaviour of the seller and the n other bidders, $b(x) = mx$ is a best response for any bidder of type x. Given that bidders choose mx, Lemma Two shows that a reserve price strategy $r(s) = (s+m)/(2m)$ is a best response on the part of the seller which completes the proof. ||

Observe that all bidders submit bids in this auction but that for a given r, high bidders typically shade their bid down in order to account for the possibility that the object is won at a reserve price bid instead of a buyer's offer. If r was known to be zero, that is, in the standard second price auction form, a Nash Equilibrium profile of bids is $b(x) = (T+1)x$. On the other hand, if r is greater than zero and is announced, then Theorem One shows that a significant and ex post inefficient measure of bidders do not participate in the auction. These are the two effects that a seller wishes to trade off in the choice of an auction policy.

Which policy, the ARP or SRP, yields the seller the highest expected revenue on average can be found by substituting in $r_A(s)$ and $r_S(s)$ into the profit function, yielding $\pi_A(r_A(s))$ and $\pi_S(r_S(s))$ as expected profits for any given s and integrating over the range of s. From the results above computation yields

$\pi_A(dr_A(s)) = n(T+1)/(n+2) - s$, for $s < -1$,

$\quad = \{n/(n+2)(T+1)+1\} - dr/b + (dr)^{n+2}/(b(n+2))$ for $s \in [-1,1/d]$

where $dr_A(s) = d(v+1)/(d+1)$ and $b = d/(1+d)$, and

$\quad = 0$ otherwise.

$\pi_S(r_S(s)) = 2[m(n+1)/(n+2) - r + r(r/m)^{n+1}/(n+1)$ where $r_S(s) = (m+s)/2$ for all $s \in [-m,m]$.

Ex ante expected profits from the two different policies are given by

$\pi_A = E(\pi_A(s))$

$\quad = ((m-1)/(2m))^m(2m/(m+1)-1)+((1+b(m-1))/(2bm))^m\{n(T+1)/(n+2)\}$

$+ (Prob(s\in[-1,1/d])E\{(dr(s))^{n+2}/(n+2) - dr(s) \mid s \in [-1,1/d]\}/b$

and $\pi_S = E(\pi_S(s))$

$\quad = 2[m(n+1)/(n+2) + m^2/[(m+n+2)(n+2)] - m^2/(m+1)]$. [5]

Table One shows the values of these profits for various values of T and n. Notice that for T = 0, the two policies generate the same revenue since the common value element is absent in that case. However, as T becomes

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[5] The computations are rather tedious and are available from the author on request.

large, the common value element becomes more critical and the SRP is preferred. The analysis suggests, then, that in auctions with a high common value element and with relatively few active bidders, an auction policy of bidding off the wall will be more profitable.

Table One

$n+1 = 2$

T	0	1	2	3	9
π_S	.500	.491	.427	.365	.143
π_A	.500	.536	.519	.489	.330

$n+1 = 3$

π_S	.600	.615	.554	.490	.212
π_A	.600	.666	.651	.615	.415

Section Three: Discussion

The welfare analysis for the seller is done in an ex ante context. Observe that it is important, therefore, that the seller be able to commit to a policy of always revealing or never revealing before she learns her private information. This is because whenever the seller's type is in fact very low, she has an incentive to announce the fact, ex post, since the resulting low reserve price will not discourage many bidders in any case. The role of the auctioneer may be seen, in part, as serving the function of such a commitment device.

Other forms of common valuations -- for example, letting the buyers' ex post value be the sum (or average) of all buyers' observations -- can be put into this framework and yield similar results. What is important is the exploitation of what Milgrom (1987) calls the 'linkage principle'. The announcement of a reserve price which may be above the bids of some buyers breaks part of the linkage between the price paid and the value of the object. Keeping the reserve price secret is a way of restoring this linkage by inducing greater participation and thereby increasing the seller's profits. The robustness of this example is difficult to assess since the choice of the distribution of the seller's type (which is important in order to yield a tractable solution to the Bayesian game) also plays a role in the determination of ex ante profits of course. The particular distribution used here has the property of putting relatively high weight on high seller types and it is in the event of a high seller type when the greatest gains from secrecy arise. It seems reasonable to conjecture that other distributions which put greater weight on lower seller types could reverse the conclusion of this example but such a formulation does not seem to be a tractable one. Nevertheless, the example in this note highlights an important factor to consider in naming reserve prices. The announcement of a reserve price may inhibit the ability of the auction mechanism to aggregate the information of other bidders and, as a result, lower expected revenues.

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