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DYNAMIC HOUSING MARKET EQUILIBRIUM WITH TASTE HETEROGENEITY, IDIOSYNCRATIC PERFECT FORESIGHT AND STOCK CONVERSIONS

by

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ABSTRACT

A discrete-time, nonstationary dynamic equilibrium model of the housing market is developed in which consumers exhibit taste heterogeneity and investors act with perfect foresight subject to idiosyncratic uncertainty in costs. Housing is treated as a discrete, durable and differentiated good which is indivisible in consumption and a convertible asset in investment. The dynamic market equilibrium determines the allocation of each consumer type among the housing types, the rent and vacancies of each housing type, the asset prices of housing and land, and the conversions between land and housing and among the stocks of housing of each type. The model relies on probabilistic discrete choice theory, whereby the investor's and consumer's choice probabilities are given the multinomial logit specification. An algorithm is developed which solves for dynamic equilibrium by assuming a relationship between rents and asset prices in the terminal period. The computable model is used to examine the "filtering hypothesis": that technological progress or government subsidies targeted to the construction of high-quality housing eventually benefit the poor. While this hypothesis is correct when subsidies do not induce stock conversions, it is false if investors demolish sufficient numbers of low-quality housing units in order to construct high-quality housing.

1. INTRODUCTION

Economists are well aware of the difficulties in modeling housing markets. These stem from the characteristics of housing, the most important of which are : durability (housing is the most durable of major commodities), complexity and multidimensional heterogeneity (a housing unit has a large number of features from which utility is derived), indivisibility in consumption (households typically do not mix fractions of housing units), adaptability (housing characteristics change over time due to depreciation and costly conversion and maintenance), nonconvexity in production (rehabilitation, demolition, construction, merger and subdivision of units all entail discontinuous changes), spatial fixity (with minor exceptions housing units cannot be transported), the dual role of housing as a consumption good and a major investment asset, the importance of transactions costs (search costs, moving costs and transaction fees), and the presence of asymmetric information (sellers and buyers or landlords and tenants have different information and are not motivated to fully reveal what they know). Most goods contain some or all of these characteristics to some degree. But only in housing are they all so pronounced.

The now-classical model of Muth (1969) remains the most widely employed. In this model all housing characteristics are collapsed into a single index, "housing services". The housing market is treated as a linked pair of perfectly competitive markets, a market for housing services, the flow consumption good, and a market for housing stock. The stock generates the flow of housing services and the value (or price) of a unit of stock equals the present value of net revenue from this flow. The Muthian model can be extended to treat structure type, spatial fixity, and tenure choice by assuming a set of linked, differentiated submarkets. The main weakness of the Muthian model is that it is too crude for many purposes. It fails to distinguish among alternative technologies for expanding the housing stock (construction, rehabilitation, conversion, improved maintenance, downgrading etc.); it fails to identify whether more housing stock entails more housing units, larger housing units, or higher-quality housing units; and its treatment of housing durability and of the relationship between the flow consumption demand for housing services and the asset demand for the housing stock is imprecise.

Sweeney (1974a,b,c) extended the Muthian model by treating housing submarkets as differentiated in quality ; a housing unit is constructed at a certain quality and gradually deteriorates at a speed depending on the level of maintenance expenditures. Sweeney's model was a conceptual breakthrough, but solved only for the stationary state, and failed to treat space and housing quantity : each household consumed one unit of housing of variable quality, but fixed size.

More recently, Arnott, Braid, Davidson and Pines, in a series of papers [Arnott, Davidson and Pines (1983, 1986), Braid (1981, 1984) and Arnott, Braid, Davidson and Pines (1988) - ABDP hereafter] extended the Sweeney model to allow for differentiated space, housing quantity (i.e. on the demand side a household decides on the size and quality of housing it consumes) and upgrading, rehabilitation, demolition and reconstruction. However, like Sweeney, they treated only the stationary state and assumed that apartment sizes within the shell of a building adjust costlessly. Anas (1980, 1982) departed from the Muthian framework, by developing a theory of housing market equilibrium founded on the microeconomics of discrete choice [McFadden (1978)]. This approach addresses the multidimensional heterogeneity of the housing commodity and the dispersion of tastes among housing consumers together with the indivisibility of housing units in consumption. The demand for each housing unit is measured as the expected number of households that prefer it to any other, and households adjust their housing consumption by switching from one indivisible dwelling to another as prices, incomes and preferences change. This contrasts with the Muthian models in which households adjust their housing consumption by costlessly modifying the quantity of housing services consumed. Anas (1982) and Anas and Duann (1985) extended the discrete choice approach further to model the behavior of landlords and builders. These papers derive the housing offer function as the expected number of existing dwellings of a particular type which are offered for rent (as opposed to kept vacant) at a given time, and the housing supply function as the expected number of newly-created dwellings. Temporary equilibrium rents are treated as deterministic and found by equating the expected demands for each housing type with the expected offers of each housing type in each time period (stochastic equilibrium). The approach lends itself to direct econometric estimation [Anas (1982)].

The current paper develops a housing market theory and resulting housing market policy simulation model that synthesizes the approach of ABDP with the discrete choice and stochastic equilibrium approach of Anas. In particular, the probabilistic discrete choice approach is extended to model the offer and supply decisions of investors, including stock conversions, under conditions of perfect foresight subject to idiosyncratic uncertainty in investors' costs, which leads to choice dispersion among investors.

The model treats nonstationary states in perfect foresight equilibrium, which ABDP could not because of analytical intractability. As in ABDP and in Anas, all agents are price takers and all households rent; thus no attempt has been made to treat aspects of imperfect competition in housing markets, and the tenure choice decision has been suppressed. As in Anas, several household groups are allowed and each group exhibits internal taste heterogeneity and choice dispersion, and rents and vacancies in each year are found by temporary stochastic equilibrium.

The general structure of the model is as follows : In period t there are { S_k^t } housing units of type k where k is a compound index for the set of units of a certain floor area, quality, structural density and location. Competition between households for these dwellings generates temporary equilibrium rents

$\{ R_k^t \}$. With perfect foresight, investors know the future rents and asset prices (or values) $\{ V_k^t \}$, including land prices. Based on this information, as well as knowledge of the costs of converting a housing unit of type k in period t to a unit of type k' in period t+1, $\{ C_{kk'}^t \}$, investors decide how to convert their current units which may entail demolition, reconstruction, quality upgrading or downgrading, size conversions inside a building, etc. Prices are computed as asset prices which equilibrate the asset market for residential buildings and land, making investors indifferent between investing their capital in the housing market or in a financial asset yielding the interest rate.

Thus far, we have placed our model in the context of only the housing theory literature. But, since the model was constructed with policy application in mind, it is also of interest to compare it with other dynamic housing market simulation models. These are the National Bureau of Economic Research (NBER) Model [Ingram, Kain and Ginn (1972), Kain, Apgar and Ginn (1976)], the Urban Institute Model (UIM) [de Leeuw and Struyk (1975), Struyk and Turner (1986), Vanski and Ozanne (1978)] and the Chicago Area Transportation-Land Use Model (CATLUM) [Anas and Duann (1985)].

The NBER model is a multiperiod dynamic model which does not assume perfect foresight; instead, it generates rents and prices by means of a disequilibrium adjustment procedure. Disequilibrium prices are not determined as true asset prices. Stock conversions are not distinguished from quality transitions. The Urban Institute model is Muthian in character and essentially static, though it allows treatment of multiple periods as a sequence of static equilibria. Housing quality is treated as a continuous "quantity of services" which flows from a dwelling of a particular type. Stock conversions between distinct dwelling types are not treated and there is no distinction between rents and asset prices. Both of these models are, in some cases, computationally intractable. The Chicago model was developed to examine the effects of transportation investments on the spatial aspects of the housing market. It is a multiperiod model in which asset prices are determined under adaptive foresight. While construction and demolition are included in the model, stock conversions and quality maintenance are not. The model computes a temporary equilibrium in each year (similar to the one employed in this paper) from which rents, vacancies and the utilization of the housing stock by households are determined.

The paper is organized as follows. Section 2 develops the behavior of investors and derives the asset bid price equations for housing and land under perfect foresight with idiosyncratic uncertainty in costs, showing how the realization of such uncertainty leads to choice dispersion among investors' supply activities. Section 3 develops the behavior of the housing consumer treating idiosyncratic taste heterogeneity in preferences for housing. Section 4 defines the dynamic equilibrium problem which incorporates within it two equilibria, the matching of households to dwellings in each year and the competitive bidding equilibrium among investors in each year. A computational algorithm which solves the model is described in the Appendix. Section 5 employs a simple version of the model to illustrate its application to policy by examining the validity of the "filtering hypothesis" : that the most efficient way to improve the housing quality of the poor is to subsidize the construction of high-quality housing, thereby allowing older, but still good-quality housing to filter down to the poor.

2. SUPPLY

Investors, buy, let (or keep vacant), convert, and sell property (buildings or vacant land). Transaction costs in buying and selling are ignored and thus, without loss of generality, investors' holding periods are assumed to be single years. Investors face uncertainty concerning their profits from conversions. In our model, this uncertainty is resolved after their decision to buy property but before their conversion decisions. To simplify the presentation, we shall treat an investor as owning only one property, though this is not necessary.

We assume that the idiosyncratic random variables affecting an investor's costs for each conversion activity on each property are independently distributed and serially uncorrelated. We also assume that these random variables are independently distributed across investors with variances which are specific to property type (location, structural density and apartment size). One implication of these assumptions is that an investor's decisions are unaffected by his investment history. Another is that the aggregate behavior of asset prices in such a housing economy with large numbers of investors is completely deterministic even though the decisions of individual investors are made under uncertainty. For these reasons, we describe our model as entailing <u>perfect foresight with idiosyncratic uncertainty</u>, or <u>idiosyncratic perfect foresight</u>.

An investor purchasing a building (or unit amount of land) faces a three-stage nested decision. In stage one (the beginning of the year), the investor decides under uncertainty how much to bid for the property. In stage two (the early part of the year), a housing investor's uncertainty about the occupancy-vacancy costs he is facing is resolved and he decides whether to rent the building or to keep it vacant. In stage three (the end of the year), the investor's uncertainty about the conversion activities is resolved and he decides which conversion to perform on the property. Suppose further, that the occupancy decision for a property (prior to its conversion) is independent of the conversion decision for it. Then, utilizing Bellman's (1957) principle, it is natural to examine the stage-two occupancy and the stage-three conversion decisions as parallel decisions nested within the outer level (or stage-one) bidding decision.

All investors are <u>ex ante</u> identical. The representative investor who has purchased a k-property in the beginning of the year t, is concerned with the sum of the expected maximized utility from the decision to let k or keep it vacant during t and the expected maximized utility from the decision to convert k to k' at the end of t. There are k = 1,..., K, K+1 property assets (combinations of structural type and quality), of which the first K are housing assets and $K+1$ is vacant land.[1] A subscript $k = 0$ denotes the financial asset which yields the interest rate. The double positive subscripts kk' represent any possible conversion of property asset k to k', including $k' = k$ (keeping the property unchanged); $k' = K+1$, $k \leq K$ represents demolition, and $k = K+1$ and $k' \leq K$ represents construction of a new building on vacant land. Because some transitions are impossible or prohibitively costly (locations of buildings cannot be changed and certain structural conversions require demolition first), we define A_k to be the subset of states $k' = 1 \ldots K+1$ that can be reached from k in one year, i.e. the <u>activity choice set</u> of investors buying k in the beginning of the year.

We must now make our specification of utilities precise. We will use capital letters to denote utilities related to the conversion activity and lower case letters to denote utilities related to the occupancy activity. Both of these utilities are defined as the sum of three components : (i) the deterministic <u>common net revenue</u> or <u>profit</u> received from the activity (denoted by H and h); (ii) the deterministic <u>common nonfinancial cost</u> associated with the activity (denoted by D and d); and (iii) the random <u>idiosyncratic nonfinancial cost</u> associated with the activity

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[1] To simplify, we have treated the situation with only one location; the extension to treat multiple locations is straightforward.

(denoted by F and f).[2] Precisely, we express such an investor's <u>expected</u> <u>maximized</u> <u>after-tax</u> <u>investment</u> <u>utility</u> in period t (discounted to the beginning of the period) subsequent to the purchase of k as,

$$E\left[\underset{k' \in A_k;\ x \in \{0,1\}}{\text{Maximize}} \left(U^t_{kk'} + u^t_{kx}\right)\right], \tag{1}$$

where,

$$U^t_{kk'} = H^t_{kk'} - D^t_{kk'} - F^t_{kk'}, \tag{1'}$$

and,

$$u^t_{kx} = h^t_{kx} - d^t_{kx} - f^t_{kx}. \tag{1''}$$

(1') is the investor's utility from the k to k' <u>conversion</u> at the end of year t, and (1'') is the investor's utility from the <u>occupancy</u> of k during year t, where x = 0 denotes that property k remains vacant during t and x = 1 denotes that property k is occupied during t. H, h, D and d, are invariant across investors for the same investment activity, whereas F and f have the same distribution across investors owning the same type of property.

We make the following assumptions about the probability distributions of the idiosyncratic components. They are <u>i.i.d.</u> (<u>independently</u> <u>and</u> <u>identically</u> <u>distributed</u>) : (1) for any conversion k' or occupancy x activities that can be performed on the same

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[2] Nonfinancial costs (common and idiosyncratic) reflect the income equivalents of nuisance, know-how value, value of time and other nonfinancial aspects of engaging in property investment. These include such things as dealing with bad tenants, monitoring the work of an inefficient builder, searching for information about conversion technology etc. The larger these nonfinancial costs, the lower the investor's sensitivity to changes in net revenue.

property k in year t; (2) for different properties of type k and conversion k' or occupancy x activities in the same year t; (3) for different years and property type k and activity k' or x. We assume also that the c.d.f. (cumulative density function) of the idiosyncratic costs is the <u>type I extreme value</u> distribution,

$$G(s < z) = \exp \{-\exp[-a_k (z + (g/a_k))]\} \tag{2}$$

where s stands for any random realization of an idiosyncratic cost. In (2), $E[s] = 0$, $Var[s] = \pi^2/6a_k^2$, $g=0.5772$ is Euler's constant and a_k, which is inversely proportional to the variance, is the "dispersion parameter" or "heterogeneity coefficient". As a dispersion parameter approaches infinity, investment uncertainty vanishes and all investors who purchased k choose the same k' (or x) activity with the highest level of non-idiosyncratic utility; thus, heterogeneity in investors' choices also vanishes. And as a dispersion parameter approaches zero, investment outcomes are perceived as extremely uncertain, all activities become equiprobable and choices are extremely heterogeneous.

Since idiosyncratic conversion and occupancy costs are independent, (1) can be written as the sum of the expected maximum conversion and occupancy utilities,

$$E [\underset{k'\in A_k}{Max} (U^t_{kk'})] + E [\underset{x\in\{0,1\}}{Max} (u^t_{kx})]. \tag{3}$$

It follows that the investor's conversion and occupancy choice probabilities are independent and determined from stochastic utility maximization,

$$Q^t_{kk'} = \text{Prob}[U^t_{kk'} > U^t_{ks}; \text{all } s \in A_k, \ s \neq k']; \ k=1...K+1 \tag{4}$$

and,

$$q^t_{k1} = \text{Prob}[u^t_{k1} > u^t_{k0}]; \ k=1...K+1, \tag{4'}$$

where $\Sigma_{k' \in A_k} Q^t_{kk'} = 1$ and $q^t_{k0} + q^t_{k1} = 1$. Imposing the distributional assumption (2) on the idiosyncratic costs, we obtain the conversion choice probabilities as multinomial logit and the occupancy choice probabilities as binary logit with dispersion parameters Φ_k and ϕ_k respectively:

$$Q^t_{kk'} = \exp \Phi_k(H^t_{kk'} - D^t_{kk'}) / \sum_{s \in A_k} \exp \Phi_k(H^t_{ks} - D^t_{ks}), \tag{5}$$

and,

$$q^t_{k1} = \exp \phi_k(h^t_{k1} - d^t_{k1}) / \sum_{s \in \{0,1\}} \exp \phi_k(h^t_{ks} - d^t_{ks}). \tag{5'}$$

At the beginning of year t, just before the purchase decision (stage-one), an investor can predict the outcome of his own conversion and occupancy decision only up to the probabilities given by (5) and (5'), since his idiosyncratic costs associated with the decision alternatives have not yet been realized. After the purchase of k, the idiosyncratic costs are realized in stages two and three. Each investor then makes a deterministic choice and, with a large number of investors purchasing k-type properties, the probabilities (5) and (5') give the expected proportion of investors realizing each activity.

The expected maximized income-equivalent returns from the conversion (stage-two) and occupancy (stage-three) decisions implied by the distributional assumption (2) are :

$$W_k^t = E[\underset{k' \in A_k}{Max} (U_{kk'}^t)] = (1/\Phi_k) \ln [\sum_{k' \in A_k} \exp \Phi_k (H_{kk'}^t - D_{kk'}^t)], \qquad (6)$$

and,

$$w_k^t = E[\underset{x \in \{0,1\}}{Max} (u_{kx}^t)] = (1/\phi_k) \ln [\sum_{x \in \{0,1\}} \exp \phi_k (h_{kx}^t - d_{kx}^t)]. \qquad (6')$$

In specifying the investor's discounted after-tax conversion net revenue, we need to define several variables and parameters. First, n_k for $k < K+1$ is the number of identical "apartments" in one type k building with $n_{K+1} = 1$, a unit amount of vacant land. V_k^t is the asset price at the beginning of year t of an apartment in a type k building (or of a unit amount of land for $k=K+1$), so that $n_k V_k^t$ for $k < K+1$ is the asset price of a type k building. $m_{kk'}$ is the number of units (apartments or land) of type k needed to create one unit of type k' through the conversion activity kk' at a total financial cost $C_{kk'}^t$ incurred at the end of year t. μ^t is the investor's income tax rate, θ_k^t is the ad-valorem property tax rate, p is the downpayment rate required of investors, r is the interest rate and $\rho^t = (1-\mu^t)r$ is the after-tax interest rate, and d^t is the investor's rate of discount. Then, the investor's discounted net revenue over the year is :

$$H_{kk'}^t = n_k \frac{1}{(1+d^t)} \{ [\frac{V_{k'}^{t+1} - C_{kk'}^t}{m_{kk'}} - V_k^t - r(1-p) V_k^t - (1+d^t) p V_k^t]$$

$$- \theta_k^t V_k^t - \mu^t [\frac{V_{k'}^{t+1} - C_{kk'}^t}{m_{kk'}} - V_k^t] + \mu^t [\theta_k^t V_k^t + (1-p) r V_k^t] \}. \qquad (7)$$

The right side of (7) is obtained by combining terms in the investor's one-year discounted cash flow of direct cash disbursements and receipts : at the beginning of the year, the investor makes the downpayment; at the end of the year, he incurs the conversion cost and pays the property tax and the capital gains tax; also at the end of the year he pays interest on his mortgage, he collects the sales price and the income tax rebates (on mortgage interest, the property tax and the conversion cost), and pays off the mortgage. The first term in (7) is the discounted gross-of-tax return (discounted capital gain net of conversion cost, less discounted mortgage interest payments, less the downpayment), the second term is the discounted ad-valorem property tax on the building, the third term is the discounted capital gains tax on the building assumed payable on an accrual basis every year at the same rate as on other income (and with full loss offset) and the fourth term is the investor's discounted income tax rebate.[3]

(7) can also be written,

$$H^t_{kk'} = n_k \frac{1 - \mu^t}{1 + \rho^t} (V^{t+1}_{k'} - C^t_{kk'})/m_{kk'} + (\text{constant})^t_k \quad , \qquad (7')$$

where the constant depends on k and t only. From (5), this

[3] This is a somewhat crude representation of the U.S. tax law in two respects. First, with a one-year holding period, deducted depreciation is fully offset by the increase in the capital gains tax on the depreciated property; and second, the rules on the taxation of capital gains and losses have been simplified. Also, the investor is depicted as carrying a fully adjustable rate mortgage and as refinancing (or selling) every year at zero cost.

constant part cancels out of the multinomial logit choice probabilities.[4]

The after-tax annual net rental revenue from the type k property is,

$$h_{k0}^{t} = -(1-\mu^{t})\, c_{k0}^{t}\, n_k \tag{8}$$

if it is vacant and,

$$h_{k1}^{t} = (1-\mu^{t})(R_k^{t} - c_{k1}^{t})\, n_k \tag{8'}$$

if it is occupied, where R_k^t denotes the rent for a k-apartment (for k <K+1) or the rent on land (k=K+1) in a nonresidential use and the c's are the investor's annual operating costs depending on occupancy status.[5]

The investor's stage-one problem is a bidding problem. In competitive asset market equilibrium, the price of each asset is bid up until the internal rate of return generated by that asset becomes equal to the internal rate of return of the financial investment. Since the internal rate of return of the financial investment is the after-tax interest rate, the competitive bid for a type k property is determined by setting the investor's discount rate equal to the after-tax interest rate and solving for the asset

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[4] This constant is the after-tax <u>sunk</u> <u>net</u> <u>cost</u> associated with an already purchased property of type k and thus does not influence the conversion decision on that property.

[5] The income tax system is assumed to work on a settle-as-you-go basis. Thus, in the beginning of the year, the investor collects rents, pays the income tax on this rental income, incurs the operating expenses and collects an income tax rebate on these expenses, while the conversion-related tax settlements occur at year-end as explained earlier.

price such that the expected net income-equivalent present worth of each investment (given by (3)) is zero. Making the necessary substitutions, the year-beginning asset bid prices are :

$$V_k^t = \frac{\ln \sum_{k' \in A_k} \exp\{ \Phi_k [w_k^t - D_{kk'}^t + n_k (1-\mu^t)(V_{k'}^{t+1} - C_{kk'}^t)/(m_{kk'}(1+\rho^t))] \}}{\Phi_k \, n_k (1 + r + \theta_k^t)(1 - \mu^t)/(1 + \rho^t)} . \tag{9}$$

Partial derivatives give unambiguous signs and show that asset bid prices increase with increasing year-end asset prices, decreasing conversion costs, decreasing ad-valorem property tax rates, increasing leverage (decreasing p), decreasing common nonfinancial costs, increasing rents, and higher idiosyncratic uncertainty (decreasing Φ_k or ϕ_k). The effect of raising the income tax rate is more complex because the marginal increase in the capital gains tax can offset the marginal benefit from the tax deductibility of mortgage payments, but also because the income tax rate affects the investor's rate of discount. Higher interest rates make financial assets more attractive, increasing the opportunity cost of the investor's capital and thus lowering asset bid prices on property investments.[6]

3. DEMAND

Consumers of housing (households) are grouped into h = 1...H types by income and demographics. The number in group h is N^{ht}.

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[6] This holds as long as $V_{k'}^{t+1} - C_{kk'}^t \geq 0$ for each k', but if $V_{k'}^{t+1} - C_{kk'}^t < 0$ and sufficiently negative for at least one $k' \in A_k$ then the interest rate increase discounts such potential losses and can result in higher asset prices for k in year t.

Each consumer faces choice in a market of k = 1...K housing types (submarkets or bundles of dwellings) with S_k^t apartments in k.Each year, each consumer must choose and rent <u>one</u> apartment, so utility maximization is a problem of discrete choice. Households are subject to a contemporaneous budget : each year's income is fully spent on that year's consumption. Since in our model housing is altered only by investors, households do not have the option of adjusting to market changes by modifying the quality or size of their apartment. In the absence of transactions costs (which we ignore) households adjust to market changes, such as a rise in rent, by moving to a new dwelling each year if necessary. We assume that all apartments are available for choice each year, regardless of whether they were occupied or vacant in the previous year.

We treat the housing market in question as a part, such as a city, within a larger economy. Each of the N^{ht} consumers (in each group h) in this larger economy decides whether to enter the housing market, which submarket to select if it enters, and which apartment to select in that submarket. The utility function is composed of additively separable subutilities common to all households of the same group and idiosyncratic utilities which vary among households to reflect taste heterogeneity. Letting the subscript 0 denote choice of an apartment outside the housing market, and letting 1 denote the choice of the housing market,

$$\hat{U}_0^{ht} = U_0^{ht} + \beta_0^{ht} , \qquad (10)$$

$$\hat{U}^{ht}_{1kz} = U^{ht}_{1k} + \beta^{ht}_{1k} + \beta^{ht}_{1kz} \quad , \tag{10'}$$

for each submarket k = 1...K and each dwelling z = 1... S^t_k, $z \in k$. The U's are the common utilities and the β's are the idiosyncratic income-equivalent premia which households attach to submarkets and to specific apartments. The subutility, U^{ht}_0, of remaining outside of the market is exogenous for each t = 1,...,T and the submarket subutilities U^{ht}_{1k} have the linear form,

$$U^{ht}_{1k} = (y^{ht} - R^t_k - E^{ht}_k) + Y^{ht}_k , \tag{11}$$

where Y^{ht}_k measures the income-equivalent premium attached to submarket k by a type h household, y^{ht} is the household's income, R^t_k is the rent on an apartment in submarket k, and E^{ht}_k is other residential expenditure related to submarket k.

Given the joint probability density function of the idiosyncratic tastes across the population of households, choices are determined from stochastic utility maximization where the P's denote the probability that a representative (randomly selected) household chooses a particular alternative :

$$P^{ht}_{1kz} = \text{Prob}\ [\hat{U}^{ht}_{1kz} > \max\ (\hat{U}^{ht}_0 , \hat{U}^{ht}_{1nx}\ ; \text{all}\ (n,x) \neq (k,z) ; n \in J^h)] , \tag{12}$$

for each (k,z) where $k \in J^h$, J^h being the choice set of the h<u>th</u> household type, and $z \in k$, $x \in n$.[7] Choices are conserved by :

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[7] The formulation assumes that each household is able to obtain its preferred unit in its preferred submarket. In fact, however, more than one household may prefer a particular unit. In the event of such a tie, one of the households would then have to settle in a less-preferred unit. This could be treated, but would considerably complicate the analysis.

Note that if an additional unit is added to some submarket, it expands the choice set of households in this submarket. This

$$P_0^{ht} + \sum_{k \in J^h} \sum_{z=1}^{S_k^t} P_{1kz}^{ht} = 1 . \tag{12'}$$

It is convenient to specialize to choice probabilities that are sequentially decomposable into a marginal probability of market choice and successive conditional probabilities of submarket and apartment choice. Thus,

$$P_{1kz}^{ht} = P_1^{ht} \; P_{k|1}^{ht} \; P_{z|k1}^{ht}, \text{ for each } k \in J^h, z = 1 \ldots S_k^t . \tag{13}$$

Under stylized assumptions about the joint distribution of the β's, these choice probabilities can be given the <u>nested</u> <u>logit</u> specification [see McFadden (1978) or Anas (1982 ; ch.2)]. In this case, the nested logit model takes the form :

$$P_{z|1k}^{ht} = 1/S_k^t \quad \text{for each } z = 1 \ldots S_k^t \text{ and each } k, \tag{14'}$$

$$P_{k|1}^{ht} = \exp[\alpha^h \frac{U_{1k}^{ht}}{1-\sigma^h} + \ln S_k^t] \;/\exp(I^{ht}) ; \; k \in J^h , \tag{14''}$$

for $k = 1 \ldots J^h$ and $\sum_{k \in J^h} P_{k|1}^{ht} = 1$ and where,

$$I^{ht} = \ln \sum_{m \in J^h} \exp \; [\alpha^h \frac{U_{1m}^{ht}}{1-\sigma^h} + \ln S_m^t] . \tag{14'''}$$

$$P_0^{ht} = \exp(\alpha^h \; U_0^{ht} \;) / \{ \exp(\alpha^h \; U_0^{ht} \;) + \exp[\; (1-\sigma^h) \; I^{ht} \;] \} , \tag{14''''}$$

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stimulates demand for the submarket, benefiting all investors who own housing in the submarket. Since the investor of the additional unit cannot capture these benefits, there is an uninternalized externality.

with $P_1^{ht} = 1 - P_0^{ht}$.

This nested logit model embodies the following assumptions. First, within the same submarket k, β_{1kz}^h and β_{1ks}^h are independent across different households of type h, for any two apartments z and s $\in$ k. This means that households evaluate apartments in the same submarket as being extremely distinct in idiosyncratic premia. Second, the coefficients σ^h measure the similarity among different submarkets across households of type h. More precisely, σ^h is the correlation coefficient between the idiosyncratic premia β_{1m}^h and β_{1n}^h for <u>any</u> <u>two</u> submarkets n and m across type h households. Consistency with utility maximization requires that $1 > \sigma^h \geq 0$. As σ^h approaches one, households evaluate submarkets to be identical in their idiosyncratic premia and as σ^h approaches zero submarkets are evaluated to be distinct in their idiosyncratic premia. Third, the coefficients α^h are measures of the variance in the idiosyncratic premia among households of the same type h. Precisely, α^h is inversely proportional to the variance of the idiosyncratic premia β_0^h and β_{1k}^h for each k across the households in group h. As α^h approaches infinity idiosyncratic heterogeneity in tastes vanishes and all households are identical, and as α^h approaches zero idiosyncratic heterogeneity becomes infinite and households are distributed randomly among the choices they face. The submarket choice probabilities, (14''), exhibit <u>strict</u> <u>gross</u> <u>substitution</u> : when one submarket becomes more desirable, say because of a drop in rent, its probability of being preferred rises and the probabilities of each of the other submarkets being preferred fall. Similarly, the probabilitiy of remaining outside the market, (14''''), rises as the utility from remaining outside increases. Setting the outside utility level at negative infinity insures a closed housing market, in that each household must locate within the market.

4. DYNAMIC EQUILIBRIUM

Dynamic equilibrium of the housing market must satisfy three sets of conditions. First, given the number of households and the stock of dwellings available in each submarket, the expected demands of households and the expected offers of landlords match each year to clear the market by establishing a temporary stochastic equilibrium [Anas (1982)]. Second, given these market-clearing rents for each year, house and land asset prices determined by competitive bidding under perfect foresight should equalize in each year the after-tax expected rate of return from each housing type or land with the after-tax exogenous rate of interest. Third, the housing and land asset prices determined under perfect foresight should give rise to stock conversion activities that generate (from an arbitrary housing stock configuration in year 1) the housing stock sequence supported by the market clearing rents of each year.

Stochastic Equilibrium

Let $\bar{R}^t = \{ R_1^t, \ldots, R_k^t, \ldots, R_K^t \}$ be the vector of market rents in year t. Then, stochastic equilibrium in year t requires the equality of expected demands by households with the expected offers by investors [see Anas (1982)]. This is expressed by the following excess demand system,

$$X_k^t(\bar{R}^t) \equiv \Sigma_{h=1}^{H} N^{ht} P_1^{ht}(\bar{R}^t) P_{k|1}^{ht}(\bar{R}^t) - S_k^t q_{k1}^t(R_k^t) = 0, \qquad (15)$$

for k= 1...K, where the summation on the right is the expected demand (expected number of households) choosing submarket k and the last term is the expected offers (expected number of apartments offered by the landlords) in submarket k.

The choice probabilities $P_1^{ht}(.)$, $P_{k|1}^{ht}(.)$ and $q_{k1}^t(.)$ are given by (14''), (14'''') and (5').[8]

<u>Asset Market Equilibrium</u>

In a competitive asset market, equilibrium asset prices equal the asset bid prices given by (9). Thus, the asset price vector for each year can be determined from (9) recursively backward in time, given an appropriate choice of terminal asset prices for some terminal time period T. These terminal prices must be chosen in such a way that the finite horizon problem with T periods serves as an adequate approximation to the underlying infinite horizon problem.

To determine such asset prices we proceed as follows. First,

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[8] The excess demand equations (17) can be shown to satisfy <u>strict gross substitution</u> in the submarket rents. Also, the Jacobian of the excess demand system has a negative dominant diagonal which is sufficient to prove that if an equilibrium exists it is unique [see McKenzie (1960)]. Existence of stochastic temporary equilibrium follows from an application of Brouwer's theorem. Negative rents are possible since investors may subsidize tenancy in the short run if the annual operating cost of a vacancy is sufficiently higher than the annual cost of an occupied unit.

we assume that investors' choice sets in the terminal period do not admit conversions from one property asset to another property asset $(A_k^T = \{k\})$. Second, we let $V_k^{T+1} = (1 + b_k)V_k^T$, where b_k is an assumed rate of change in the k<u>th</u> asset price in the terminal period. Imposing these conditions on (9) and expressing the expected terminal year occupancy utility (6') as the function $w_k^T(.)$, the terminal relationship between asset prices and rents is obtained :[9]

$$V_k^T = \frac{(1+\rho^T)[\, w_k^T(R_k^T) - D_{kk}^T \,] - (1-\mu^T)C_{kk}^T\, n_k}{(1-\mu^T)n_k(r + \theta_k^T - b_k)} . \qquad (16)$$

<u>Stock</u> <u>Adjustment</u>

Given vectors of asset prices and rents, $\bar{V} = \{\bar{V}^1, \bar{V}^2, \ldots, \bar{V}^T\}$, $\bar{R} = \{\bar{R}^1, \bar{R}^2, \ldots, \bar{R}^T\}$, the stock conversion probabilities are single-valued as defined by (5). These are used to update the stock of housing and land for t=1...T-1 recursively from any <u>given</u> initial stock vector S_k^t , as follows :

$$S_k^{t+1} = \sum_{k' \in B_k} (n_k/n_{k'})\, S_{k'}^t\, Q_{k'k}^t , \qquad (17)$$

for each k=1...K, where B_k is the set of housing types that can be converted to k in one period.

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[9] Positivity of terminal asset prices is guaranteed by having both the numerator and denominator of (16) positive. The numerator is positive with sufficiently high terminal rents. To insure that the denominator is positive, the assumed rate of growth in each terminal asset price must be less than the interest rate plus the ad-valorem tax rate.

5. POLICY ANALYSIS

Government housing policy has the potential to improve efficiency by offsetting the effects of three distortions : i) distortionary taxation (see Rosen (1985) for a discussion of the efficiency effects associated with the tax treatment of housing); ii) the absence of an insurance market against idiosyncratic risk (but since we treat investors as risk neutral, this distortion causes no efficiency loss); iii) the externalities associated with the thinness of the market (recall footnote 7).[10] Government housing policy may also seek to improve equity.

Conventional wisdom about the workings of the housing market has led to the well-known filtering hypothesis, that across-the-board construction subsidies are a more effective means of improving the housing consumed by the poor, than the construction of low-income housing by the public sector. The hypothesis rests on the assumptions that : in the absence of distortions or government policy the technology of housing production is such that construction occurs only at high-quality levels; and maintenance costs are such that housing deteriorates as it ages, and finally becomes unoccupied and is removed from the market.[11] Under the

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[10] There are other important distortions which operate on the housing market in the U.S. The most important of these are land use controls and neighborhood externalities. These are also not included in our model.

[11] These assumptions are implicit or explicit in Sweeney's model (1974 a,b), in Ohls' (1975) simulation model, in de Leeuw and Struyk's (1976) Urban Institute Model and in the NBER model [Kain and Apgar (1976)].

conditions described, it is widely held that the poor will benefit from construction subsidies by obtaining hand-me-down housing more cheaply. This occurs since the construction subsidies expand the supply of high-quality housing, thereby lowering the demand and hence the rents on middle- and low-quality housing. Hence, it is argued a construction subsidy which benefits the well-to-do in fact benefits all households. Expositions of this scenario, expressed in varying degrees of detail, can be found in Muth's seminal book (1969; p.98); a decade later in Quigley's survey (1979; pp.418-419) of conventional knowledge about housing markets, which also explains how filtering would work in Sweeney's model; or, most recently, in the textbook by Mills and Hamilton (1989; p.228).

Simulations with applied models have confirmed the hypothesis. Notably, Ohls (1975) employed a model akin to Sweeney's, and in a numerical example found that, in response to a subsidy on high-quality housing construction, rents fall throughout the housing market due to filtering. de Leeuw and Struyk (1977) undertook the same policy experiment using the Urban Institute Model and showed that filtering would benefit the poor in the six metropolitan areas for which their model was calibrated.

To our knowledge, Lowry (1964) is the only one to have critiqued the filtering hypothesis by arguing that filtering might lead to disinvestment in lower-quality housing to the detriment of the occupants. Our simulation results with the current model show that the conventional wisdom is, indeed, deficient because of conceptual limitations in the previous filtering models. In particular, none of these models included land. As a result, buildings are demolished only when their rent falls to zero and, because there is no land, demolitions do not augment the land supply. In a model with land such as ours, however, low-quality buildings are demolished while they still command positive rents to make space available for the construction of high-quality housing. In this case, construction subsidies may stimulate the demolition of low-quality housing, thereby reducing the supply of such housing and raising the rent on housing consumed by the poor.

We now apply our theoretical model to gain insight into the workings of "filtering" within a nonstationary economic environment. To do so, a simplified version of the model is utilized, in which there are two household groups (rich and poor), three dwelling types (shacks, houses and mansions) and twenty time periods. We select specific parameter values to solve this simplified version for a dynamic equilibrium. Then, the effects of government policies are evaluated in several base market environments.

The algorithm discussed in the Appendix is extended to determine the level of a construction subsidy (determined as a percentage of construction cost) which exhausts a given policy budget. This is accomplished by solving the dynamic market equilibrium repeatedly with different values of the subsidy level until the policy budget balances. From such a solution, the annual change in the consumer surplus (ΔCS) of rich and poor is calculated as,

$$\Delta CS^{ht} = (B^{ht})^{policy} - (B^{ht})^{base} , \tag{18}$$

where B^{ht} is the gross year-t utility benefit to type h households computed as the expected maximized utility level of the representative household times the number of households :[12]

$$B^{ht}=N^{ht}\left(\frac{1}{\alpha^h}\right)\ln\left\{\left(\sum_{m\in J^h}\exp\left[\alpha^h\frac{U_{1m}^{ht}}{1-\sigma^h}+\ln S_m^t\right]\right)^{(1-\sigma^h)}+\exp\left(\alpha^h\, U_0^{ht}\right)\right\}. \quad (18')$$

Gross present value benefits (GPVB) are then obtained as the discounted sum of the annual consumer surplus changes.

The government's annual tax revenue is given by [13]

$$G^t = \Sigma_{k=1}^{K+1} S_k^t \,\{\, q_{k1}^t\, \mu^t\, (R_k^t - c_{k1}^t) - q_{k0}^t\, \mu^t\, c_{k0}^t \}$$

$$+ \Sigma_{k=1}^{K+1} S_k^t \sum_{k'\in A_k} \frac{Q_{kk'}^t}{1+r} \left\{\left[\frac{V_{k'}^{t+1} - C_{kk'}^t}{m_{kk'}} - (1+(1-p)r+\theta_k^t)V_k^t\right]\mu^t + \theta_k^t V_k^t\right\} \quad (18'')$$

These gross consumer surplus benefits for rich and poor and the tax revenue changes induced by the policy are added in each year, the exogenous policy budget (PB) for that year is subtracted from this sum and these annual net benefits are discounted by the interest rate, r,[14] to obtain a net present value benefit (NPVB). In doing so it is assumed that the gross benefits and the tax revenue

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[12] Small and Rosen (1981) have shown that the expected maximum utility level in probabilistic discrete choice models with a constant marginal utility of income can also be calculated as the consumer surplus measure associated with the expected demand function.

[13] This tax revenue ignores taxes from nonhousing investment income and taxes from outside the housing market.

[14] Since the model is not fully general equilibrium, our assumption that r is the social rate of discount is arbitrary.

changes of the terminal year (t=T) remain constant for the future (t>T) :[15]

$$NPVB= \left[\Sigma_{t=1}^{T} \frac{\Delta CS^{1t}+\Delta CS^{2t}+\Delta G^{t}-PB^{t}}{(1 + r)^{t-1}} \right]+ \frac{\Delta CS^{1T} +\Delta CS^{2T} +\Delta G^{T}}{r\ (1 + r)^{T}} \qquad (19)$$

<u>A Simple Housing Market : Shacks, Houses and Mansions</u>

Consider a simple housing market with three discrete single family (n_k= 1) housing types : shacks (k=1), houses (k=2) and mansions (k=3). There is also vacant land (k=4). For simplicity, assume that one dwelling of each type requires only one unit of land (or dwellings are identical in lot sizes). Suppose that each type can be constructed on vacant land at a fixed cost per dwelling, and only shacks can be demolished to create land at a fixed cost per shack. It also takes a fixed cost to convert one shack to one house, and a different fixed cost to convert one house to one mansion. Thus, all $m_{kk'}$ = 1. Assume also that housing units do not deteriorate to lower quality levels and that the investors' conversion costs for maintaining units at the same quality are identically zero (C_{kk}^{t}= 0 ,k=1,2,3).

The rich consumers (h=2) attach a higher premium on mansions than do the poor (h=1). Rich and poor attach the same premium to houses and the rich hate shacks. Then, (J^1={k=1,2,3}, J^2={k=2,3}) and $Y_1^{1t} < Y_2^{1t} = Y_2^{2t} < Y_3^{1t} < Y_3^{2t}$.

As shown in Figure 1a, investors in shacks have three choices

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[15] Because the housing market is open to inmigration and the model is not fully closed, the NPVB is a <u>parochial</u> or city-specific net benefit measure, it is not a measure of total net economic benefit.

every year : demolish, do nothing or upgrade to a house. Thus, $A_1=\{1,2,4\}$. Investors in houses can either do nothing or upgrade, ($A_2=\{2,3\}$), and investors in mansions cannot make any conversions, ($A_3=\{3\}$). Investors in land can either build one of the three types of housing or keep the land vacant for future use, ($A_4=\{1,2,3,4\}$).

A Base Market Simulation

Parameter values, shown in Tables 1 and 2, are chosen to set up a plausible base market simulation. Inflation is zero.[16] Table 1 lists the demand side parameters. The initial incomes of the rich are twice as high and growing three times faster, while the poor are more numerous and growing seven times faster in exogenous population. Since our city is open, not all of the given population can actually move into it, but *ceteris paribus* higher exogenous income or population growth increases the demand for entering the city because the utility levels of rich and poor obtainable outside are assumed fixed over time as are the premia which households attach to each submarket. [17] The coefficient of correlation among submarket premia, σ^h, is set to 0.1 for h=1,2.

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[16] Our model applies as well to an economy with a non-zero inflation rate where either : i) all variables, including nonfinancial costs, are expressed in nominal terms and the tax system is not inflation-indexed; or ii) all variables are expressed in real terms and the tax system is fully inflation-indexed.

[17] Assuming an exogenous growth rate for the outside utility level would simply slow down the endogenous population growth of the city.

This means that households evaluate the three submarkets as being highly dissimilar. The logit demand models are calibrated in such a way that the elasticity of demand at equilibrium (in the first simulated period) is about minus one for rich and poor. In the base simulation, the endogenous population of the poor falls by 21 % and that of the rich rises by 64.4 % over the twenty-year span.

Table 2 lists the supply side parameters. The year-one housing stock (which is exogenous) consists of 50 % houses, 25 % shacks and 25 % mansions, with the amount of vacant land equal to 12.5 % of the total land occupied by the housing stock. It is assumed that the investor incurs higher operating costs on vacant than on occupied units, since when a unit is occupied the bulk of the operating cost passes on to the tenant. Conversion costs are high relative to construction costs (it is assumed that conversions require upgrading which may include gutting a building and replacing wiring, plumbing etc.). The cost of demolishing a shack is set at about 40 % of the cost of building it. All costs are assumed to increase at the annual rate of 0.2 % (lower than income growth). Land rent is $ 2,000 and grows exogenously at 1% annually. We assume that investors do not have any common nonfinancial costs from investing in housing.

With these parameters, the housing market's household population in year one is 74,062 which gives an aggregate vacancy rate of 7.42 %. About 53 % of the market's occupants are poor. Over the twenty year simulation span the total stock grows by 10,000 units or 12.5 %, as all of the available land is developed.

Internally, the initial stock of shacks declines by 74.6 %, and the stock of houses by 61.3 %, with the stock of mansions growing by 247 % due to a combination of new construction and conversion of houses as the market responds to the rapid income growth of the rich, relative to the cost of housing. Land and shack values approximately double, house values grow by 59 %, and mansions appreciate by 42 %. Shack rents rise by 191 %, house rents by 54 % and mansion rents by 2 %. Over time, the rich get better off as their expected utility level climbs by 4.4 % whereas that of the poor declines by 2.07 %. This indicates that housing is becoming less affordable to the poor. As a result, the proportion of poor in the market falls from 53 % to 35 %. Vacancies, meanwhile fall from the initial rate of 7.42 % to a mere 1.9 %. Table 3 samples three periods from the twenty year growth path, showing vacancy rates, rents, asset prices, and the stock of units as well as the distribution of rich and poor across the three submarkets. We note from this table that as the rich get richer, rich and poor tend to become segregated with a growing percentage of rich occupying mansions and a growing number of houses becoming occupied by the poor. Table 4 shows the stock conversion matrix for the sampled years.

In comparative dynamic simulations we found significant deviations from the base case described above. For example, if the costs of upgrading shacks and houses and demolishing shacks are sufficiently low and the income growth rate of the rich is sufficiently high, the market (over a sufficiently long time span) will become a market of mansions only as house investors will upgrade to mansions and shack investors will demolish to create land which can then be put into mansion building. If the income growth rate of the rich is sufficiently less rapid, houses and mansions can coexist in the long run while shacks may disappear. A polarized long run outcome in which there are only shacks and mansions is also possible. This can occur, for example, if the cost of building a shack is sufficiently low relative to the cost of building a house and the conversion of houses to mansions is rapid.

To allow an examination of policies under alternative conversion assumptions, the above base simulation was modified in two different ways. First, it was assumed that the building of mansions is the only allowable conversion. This may correspond to a situation where land unavailability, zoning or other land development restrictions do not allow the construction or upgrading of low-income housing. Second, this base simulation was further modified by assuming that vacant land can be created by demolishing shacks. Figure 1 shows these conversion assumptions in the three base market cases relative to which the filtering hypothesis will be examined.

<u>The Filtering Hypothesis Revisited</u>

Recall that, according to the filtering hypothesis, it is efficient for the poor to live in hand-me-down housing and for the quality of housing occupied by the poor to be improved through subsidies to the construction of <u>high-quality</u> housing. The basic idea is that the rent reduction at the top end of the market induced by the policy, cascades down to the lower-quality levels benefitting all households. Furthermore, the addition of stock at the top end of the market, combined with the rent reductions at all quality levels, results in all household groups moving up the quality hierarchy.

The first part of Table 5 contains the results of simulations which show some of the pitfalls associated with the filtering hypothesis. We consider an ad-valorem subsidy on the construction of mansions where the subsidy rate (as a percentage of the construction cost) is set endogenously to exhaust a policy budget of $ 5 million annually. The economy is otherwise the same as in the base case. Six cases are considered : three policy durations and two real interest rates.

In all cases, the consumer surplus of the rich rises as mansion-building brings down the rent of mansions. However, the benefits to the poor depend on the <u>induced demolition effect</u> : that investors who own shacks have an incentive to tear them down to free up the land for subsidized mansion construction. A large quantity of demolitions creates a relative scarcity of shacks, raising the rents the poor must pay. While some poor filter up to houses and benefit, those who are left behind in the shacks have losses which offset the gains from filtering. The effect on house rents is ambiguous in general. Rents can rise if the pace of demolitions is rapid relative to the pace of new construction. In most cases, present value net benefits are positive. With lower interest rates, net benefits are lower and can become negative, since a lower rate raises values relative to construction costs, thus making the marginal impact of the policy smaller.

The second set of simulations in Table 5 is designed to examine how the effects of the construction subsidy are altered when, in the spirit of previous filtering models discussed earlier, the induced demolition effect is absent. This is achieved by effectively assuming that demolition costs are prohibitively high (see Figure 1b).[18] As one would expect on the basis of the reasoning underlying the filtering hypothesis, both rich and poor benefit from the susbsidy program. When the interest rate is 8 %, net benefits are positive, and when the rate is 5 % net benefits are negative as the impact of the policy is reduced <u>and</u> the present value of the policy budget outweighs the gross benefits to rich and poor.

The third set of simulations highlights the induced demolition effect. Relative to the first set of simulations, the cost of demolishing a shack is halved while upgrading costs are assumed to be prohibitively high (see Figure 1c). Furthermore, to demonstrate the anticipatory effects of the policy which arise because of perfect foresight, the construction subsidy is applied for only a single year, year 5. Investors postpone demolitions until the end of year 4, and mansion construction until the end of year 5, in order to take advantage of the construction subsidy in year 5.

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[18] In the second set of simulations, the initial stock of land is increased to 50,000, because land cannot be created by demolition.

Prior to year 5, the poor benefit and the rich are worse off because the slowdown in demolitions dominates the slowdown in filtering, but after year 5 the welfare effects are reversed due to the demolitions induced by the subsidy. In discounted terms, the policy hurts the poor and by more the larger the policy budget and the larger, therefore, the induced demolition effect. Meanwhile, whether the policy hurts or helps the rich in discounted terms depends on the interest rate.

6. CONCLUSIONS

This paper presented a dynamic housing market simulation model with taste heterogeneity, idiosyncratic perfect foresight, and stock conversions. The model is firmly rooted in economic theory, and appropriately elaborated should prove to be a valuable tool in the analysis of housing policy at both the metropolitan and national levels.

The model incorporates probabilistic discrete choice theory into a dynamic perfect foresight context. The advantages of doing so, at least for housing markets, are two-fold. First, the model is parametrized so as to capture observed heterogeneity in the market on both the demand and supply sides; all investors do not make the same conversion decisions and all households within a particular income-demographic group do not choose the same type of housing. Second, and relatedly, introducing heterogeneity via discrete probabilistic choice theory smooths the response of the market to exogenous changes.

The realism of the model can be improved by extending its theoretical structure to incorporate two aspects of the housing market which we have thus far neglected. Moving costs are quantitatively important [Hanushek and Quigley (1978)], and may significantly influence the dynamic response of the housing market to policy changes and exogenous shocks. For example, contrary to expectations on the basis of models which ignored mobility costs, the temporary housing allowances granted in the U.S. housing demand experiments induced little upward housing mobility [Friedman and Weinberg (1981)]. But considering the magnitude of moving costs, this result is quite understandable. Tenure choice is important for a number of reasons. First, almost all housing policies are directed at either owner-occupied or rental housing, but not both. Relatedly, tax policy for the two tenure modes is markedly different in most countries. Second, owner-occupiers and investors have different production functions for modifying housing units; scale economies favor investors while owner-occupiers can more easily input their own time and effort in rehabilitating housing they own. Third, in an extended model which incorporates tenure choice it will be important to include liquidity constraints together with an explicit treatment of the home as a component of the household's investment portfolio.

Finally, an important task concerns the empirical implementation of the model for one or more metropolitan areas. This is necessary in order to compare our model's predictions with those of its antecedents : the NBER, UIM and CATLUM models discussed in the introduction. Some work in this direction has already occurred and an expanded version of the current model, named the N.C.I. Dynamic Housing Market Model, has been tentatively operationalized for the Chicago metropolitan area. On the demand side, this version of the model contains thirty household types classified by five income groups, two races and three life-cycle intervals. On the supply side, there are two building types (single and multiple family), each classified into five quality categories, and three geographic areas each with its own stock of vacant land.

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APPENDIX

This Appendix describes the algorithm devised for finding the dynamic housing market equilibrium. The algorithm guesses an arbitrary vector of rents for each year, and generates a vector of asset prices and a vector of stocks for each year. Using the stocks generated, the market is equilibrated to find a new vector of rents which replace the original guess. This process continues until the next vector of rents generated is arbitrarily close to the previous vector of rents. The algorithm is efficiently convergent regardless of the initial rent vector guess.

Letting $\bar{R} = [\bar{R}^1,\ldots,\bar{R}^T]$; $\bar{V} = [\bar{V}^1,\ldots,\bar{V}^T]$; and $\bar{S} = [\bar{S}^1, \ldots,\bar{S}^T]$ denote the rent, asset price and stock vectors respectively, the algorithm works as follows and is generally rapidly convergent.

<u>Step 0</u> : Set the iteration counter i=1, set $\Omega_i=1$ and set $D=10^{10}$.

<u>Step 1</u> : Guess the initial rent vector sequence, $\bar{R}_i$.

<u>Step 2</u> : Calculate the initial terminal values, $\bar{V}_i^T$, from the terminal condition (16) and calculate the rest of the asset price vector from the asset bid price equation (9), recursively backward in time (t=T, t=T-1, t=T-2,..., t=1), for each k = 1...K.

<u>Step 3</u> : Calculate the vector of stocks, $\bar{S}_i$, recursively forward in time for t=1,2,...T, using the stock adjustment equations (17).

<u>Step 4</u> : Solve (15) for stochastic market equilibrium using

$\bar{S}_i$, and obtain $\bar{R}_i^*$, a tentative vector of rents. In this step an efficient nonlinear simultaneous equation solver is utilized.

Step 5 : For each k=1...K, and t=1...T define the deviation in rent as $s_{ki}^t = | R_{ki}^{t*} - R_{ki}^t |$. Also define $s_i = \max_{k,t} (s_{ki}^t ; k=1...K, t=1...T)$. If $s_i < \epsilon$, where ϵ is a small number such as \$ 0.50, then stop and finalize solution. Otherwise, continue.

Step 6 : (6a) If $s_i \leq D$, set $D = s_i$. For each k=1...K and t=1...T set $R_k^{tU} = \text{Max} (R_{ki}^t , R_{ki}^{t*})$ and $R_k^{tL} = \text{Min} (R_{ki}^t , R_{ki}^{t*})$.

(6b) If $s_i > D$ and $\Omega_i \geq 0.01$, set $\Omega_{i+1} = (1/2) \Omega_i$.

Step 7 : (7a) If $\Omega_i \geq 0.01$ then $R_{k,i+1} = (1-\Omega_i) R_{ki}^t + \Omega_i R_{ki}^{t*}$, for each k=1...K and t=1...T. Set i = i+1 and go to step 2.

(7b) If $\Omega_i < 0.01$, then for each k=1...K and t=1...T :

(7b1) if $R_k^{tL} < R_{ki}^{t*} < R_k^{tU}$ and $R_{ki}^{t*} < R_{ki}^t$ then set $R_k^{tL} = R_{ki}^{t*}$ and $R_k^{tU} = R_{ki}^t$.

(7b2) if $R_k^{tL} < R_{ki}^{t*} < R_k^{tU}$ and $R_{ki}^{t*} > R_{ki}^t$, then set $R_k^{tL} = R_{ki}^t$ and $R_k^{tU} = R_{ki}^{t*}$.

(7b3) if $R_{ki}^{t*} < R_k^{tL}$ and $R_k^{tU} - R_k^{tL} < \epsilon$, then $R_k^{tL} = R_{ki}^{t*}$.

(7b4) if $R_{ki}^{t*} > R_k^{tU}$ and $R_k^{tU} - R_k^{tL} < \epsilon$, then

$R_k^{tU} = R_{ki}^{t*}$.

Step 8 : For each k=1...K and t=1...T, set $R_{k,i+1}^{t} = (R_k^{tU} + R_k^{tL})/2$.

Then, set i = i+1 and go to step 2.

	Poor (h=1)	Rich (h=2)
Populations (N^{h1})	120,000	75,000
Annual growth rate	3.5 %	0.5 %
Incomes (y^{h1})	\$ 20,000	\$ 40,000
Annual growth rate	0.25 %	0.75 %
Submarket premia (Y^{h})		
Shacks	-\$ 5,000	---
Houses	\$ 0	\$ 0
Mansions	\$ 2,125	\$ 24,545
Maintenance costs for housing (E^{h})		
Shacks	\$ 2,000	---
Houses	\$ 4,000	\$ 6,000
Mansions	\$ 7,000	\$ 9,000
Heterogeneities among households*	\$ 8,900	\$ 18,494
Outside utility (U_0^h)	\$ 81,770	\$ 173,766

TABLE 1 : Demand side parameter values

*Calculated as the standard deviation of the idiosyncratic premium β_{1k}^{h} attached to submarkets by households = $\pi/(2.45\alpha^{h})$.

	Shacks	Houses	Mansions	Land
Initial stock (S_k^1)	20,000	40,000	20,000	10,000
Operating costs				
occupied units (c_{1k}^1)	$ 360	$ 400	$ 800	$10
vacant units (c_{0k}^1)	$ 1300	$ 2500	$ 5000	
Conversion costs ($\{C_{kk'}^1\}$)				
shacks	$ 0	$ 98,000	---	$20,000
houses	---	$ 0	$ 138,000	---
mansions	---	----	$ 0	---
land	$ 49,000	$ 118,000	$ 174,000	$ 200
Heterogeneity*				
occupancy costs	$ 2,465	$ 4,286	$ 6,572	---
conversion costs	$ 5,477	$14,084	$16,431	$ 9,858

TABLE 2 : Supply side parameter values for base market simulation

(Other parameters, which remain constant over time : investor's income tax rate, μ=0.22; ad-valorem property tax rate for each type of property, θ = 0.01; downpayment rate, p=0.20; interest rate, r=0.08; terminal period growth rate in values, b_k =0.03 for k=1,...,4.)

* Calculated as the standard deviation of the uncertain and idiosyncratic cost : std.dev. = $\pi/(2.45\ a_k)$, where for occupancy profits $a_k = \phi_k$, and for conversion profits $a_k = \Phi_k$.

	Shacks	Houses	Mansions	Land
Year_1				
Rent	$ 3,359	$ 9,451	$ 20,928	$ 2,000
Asset price	$ 86,349	$ 158,818	$257,896	$91,709
Vacancy rate (%)	14.88	6.33	2.14	
poor	43.40	52.93	3.67	
rich	0.0	47.95	52.05	
Year_9				
Rent	$ 5,910	$ 11,243	$ 19,943	$ 2,166
Asset price	$119,109	$192,510	$285,945	$118,524
Vacancy rate (%)	5.82	4.23	2.45	
poor	34.54	48.43	17.03	
rich	0.0	18.54	81.46	
Year_17				
Rent	$ 8,739	$ 13,762	$ 21,294	$2,345
Asset price	$ 154,731	$ 235,547	$ 338,018	$162,911
Vacancy rate (%)	1.91	2.38	1.99	
poor	23.68	43.06	33.26	
rich	0.0	10.21	89.79	

TABLE 3 : Rents, asset prices, vacancy rates and the household distribution in the three years sampled from the base market simulation.

	Shacks	Houses	Mansions	Land	Total
Year 1					
Shacks	18,608	219	---	1,173	20,000
Houses	---	37,521	2,479	---	40,000
Mansions	---	---	20,000	---	20,000
Land	44	60	3,099	6,797	10,000
Year 9					
Shacks	13,207	161	---	372	13,740
Houses	---	26,919	1,204	---	28,123
Mansions	---	---	46,119	---	46,119
Land	13	19	611	1,376	1,748
Year 17					
Shacks	6,530	269	---	1,072	7,871
Houses	---	18,458	1,539	---	19,997
Mansions	---	---	60,216	---	60,216
Land	3	9	877	1,027	1,916

TABLE 4 : The conversion matrix sampled for three years

Annual Budget ($M)	Years Subsidy in Place	GPVB Poor ($M)	GPVB Rich ($M)	GPVB Government ($M)	Present Value Policy Budget ($M)	NPVB
Part I: Base market simulation (Figure 1a)						
(interest rate, r = 0.08)						
5	1	+2.67	+10.94	+1.67	-5.00	+10.28
5	1-5	-10.25	+48.12	+4.83	-21.56	+21.14
5	1-10	-34.53	+72.79	+5.23	-36.23	+ 7.26
(interest rate, r = 0.06)						
5	1	+1.89	+ 7.59	+1.38	-5.00	+5.86
5	1-5	-11.30	+44.89	+6.50	-22.33	+17.76
5	1-10	-26.05	+48.98	+7.41	-39.00	-8.67
Part II: Modified base market simulation (Figure 1b)						
(interest rate,r = 0.08)						
1	1-5	+2.08	+7.81	+0.27	-4.31	+5.85
2	1-5	+4.16	+15.56	+0.57	-8.62	+11.66
5	1-5	+10.29	+38.53	+1.48	-21.56	+28.74
(interest rate,r = 0.05)						
1	1-5	+0.62	+1.36	-0.07	-4.55	-2.64
2	1-5	+1.24	+2.74	+0.04	-9.09	-5.08
5	1-5	+3.10	+6.98	+0.40	-22.73	-12.24
Part III: Modified base market simulation (Figure 1c)						
(interest rate,r = 0.08)						
2	5	-1.31	+2.91	+0.20	-1.47	+0.34
5	5	-3.11	+6.95	+0.49	-3.68	+0.66
10	5	-5.81	+13.08	+0.99	-7.35	+0.91
(interest rate,r = 0.04)						
2	5	-0.04	-3.61	-0.50	-1.71	-5.78
5	5	-0.01	-8.56	-1.17	-4.27	-13.89
10	5	-0.27	-16.03	-2.09	-8.55	-26.39

TABLE 5: The Welfare Effects of Filtering Under Different Base Market Conditions.
(r: interest rate; $M: million dollars; GPVB: gross present value benefits; NPVB: net present value benefits)

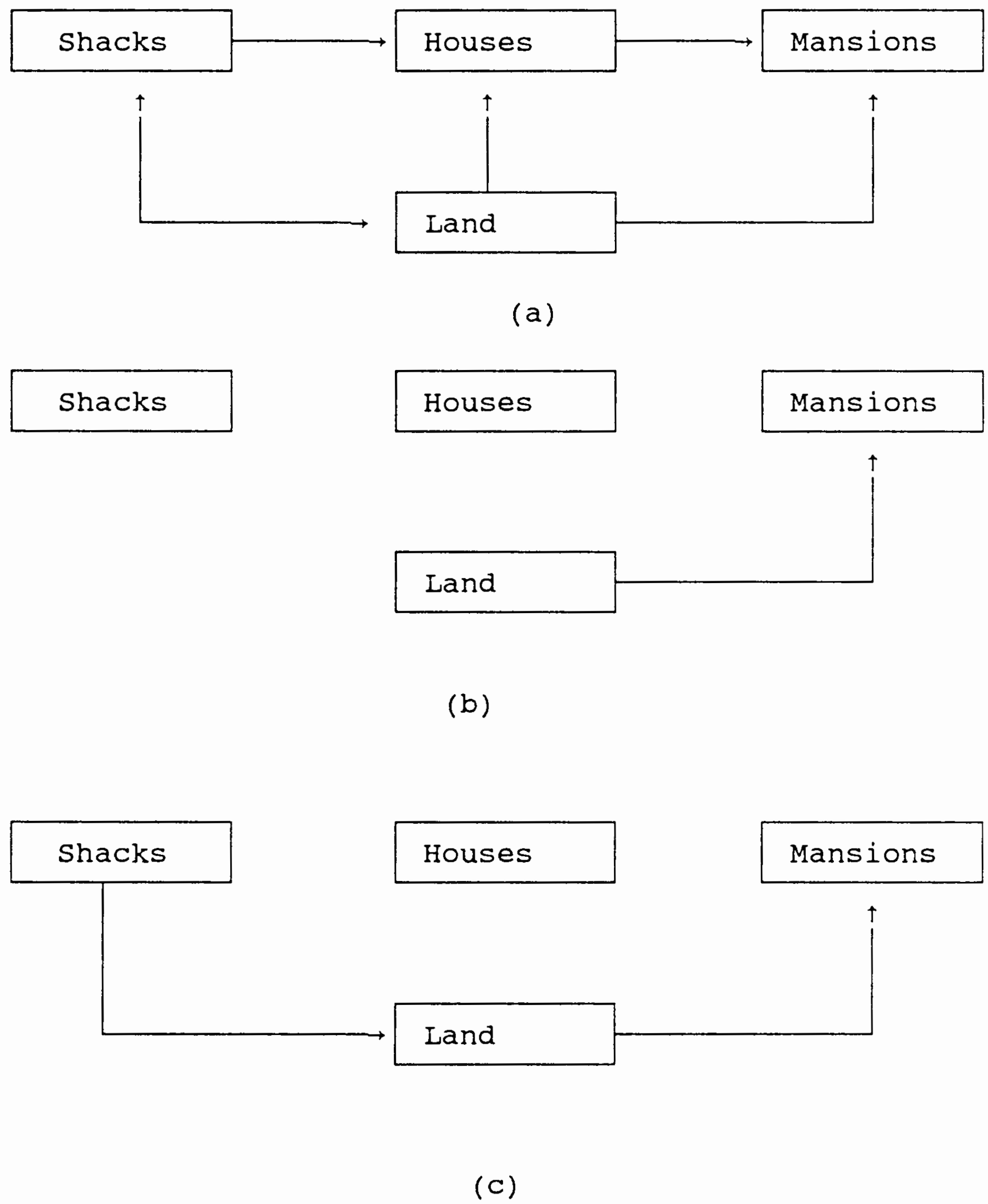

FIGURE 1 : Allowed conversion paths in the simulations.