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IMPLEMENTATION IN UNDOMINATED STRATEGIES A LOOK AT BOUNDED MECHANISMS*

by

Matthew O. Jackson**

Preliminary Draft: May 1989

*I wish to thank Leo Hurwicz, Ehud Kalai, Alejandro Manelli, Roger Myerson, Andrew Postlewaite, and Ket Richter for helpful conversations.

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Abstract: This paper addresses the issue of full implementation when we place a natural restriction on the class of mechanisms we consider. In particular, we rule out a feature common in the mechanisms used in constructive proofs in the literature: an integer construction. That is, a part of the message space in which the agent who announces highest integer is rewarded. These constructions are used to assure that undesired message combinations do not form an equilibrium.

In our setting, agents know their own preferences. It is shown that if preferences satisfy a basic condition, then *any* social choice function can be fully implemented in undominated strategies. In contrast, if we place a restriction (which rules out integer games) on the class of mechanisms we admit, then a social choice function which can be fully implemented in undominated strategies is strategy–proof. It is shown that under strong assumptions the Walrasian social choice function for large replications of an exchange economy can be fully implemented by a mechanism satisfying the restrictions.

In general, given a solution concept for implementation, we should limit our consideration to mechanisms which are appropriate for that solution concept. In the last portion of the paper we present examples which point out aspects of this issue as it relates to implementation under Nash equilibria and undominated Nash equilibria.

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1. Introduction

A goal of implementation theory is to characterize the results of decentralized decision making processes, given various information structures. The theory helps us to answer two questions. First, suppose that for each possible state of the environment, we have an allocation which we wish to see achieved. Can we structure the interactions of individuals so that, given their information, they choose actions which result in the desired allocation? Second, if individuals possess information about the state of the environment and interact through a given mechanism, what are the properties of the resulting allocation as a function of the state? In answering these questions, the outcomes of a given information structure and mechanism are predicted through the application of game theoretic solution concepts. The relevance of implementation theory depends on the qualities of the mechanisms and the solution concepts we consider.

Although the theory of implementation has been quite successful in identifying the social choice functions which can be implemented in different informational settings, a nagging criticism of the theory is that the mechanisms used in the general constructive proofs have 'unnatural' features. A natural response to this criticism is that the mechanisms in the constructive proofs are designed to apply to a broad range of environments and social choice functions . Given this versatility, it is not surprising that the mechanisms possess questionable features. With this in mind, we would hope that for particular settings and social choice functions we could find 'natural' mechanisms with desirable properties. To the extent that there are social choice functions which we can only implement using questionable mechanisms, the existing theory of implementation is inadequate.

At this point let us begin to be a bit more specific about what a questionable feature of a mechanism is. Various solution concepts seem more or less compelling depending on the the particular mechanism they are applied to. Some of the constructive proofs in the implementation literature employ mechanisms which push *any* solution concept to its limit.

In particular, many of the constructive proofs employ some sort of 'integer game' or 'modulo game'.[1] These are employed in situations in which there is some sort of 'disagreement' in the messages of agents. In an integer game the agent announcing the highest integer gets to name the allocation. [For those not familiar with these sorts of mechanisms, a mechanism with an integer construction is presented in Appendix 1. Example 1 also shows the implicit use of such a construction.] If there is no unanimous best outcome, then there is no solution to such a game since there is always someone who can benefit from increasing their announced integer. In the modulo games the agents announce numbers from some set and the modulo of the sum of the numbers is taken and the agent who's identification (say i) matches the modulo gets to name the allocation. If this allocation is not best for each agent, then there is some agent who wishes to change their announcement. It is clear that there are no pure strategy Nash equilibria to a modulo game (in the absence of unanimity), but there may exist mixed strategy equilibria. Integer and modulo constructions are used to assure that certain action combinations do not form equilibria.

We give a brief argument for why such constructions are inappropriate for various solution concepts. [We refer you to Examples 1 and 5 for more concrete illustrations and arguments.] Let us look at an isolated integer game in a situation where agents are not in agreement about what the best allocation is. As discussed above, there will be no solution to the integer game regardless of what solution concept we apply. Agents playing this game have to choose actions in any case, so all that we can say is that we have no prediction concerning those actions. Now suppose we take this integer game, and append it to a given game so that whenever agents choose actions which we wish to rule out as equilibria, they end up playing the integer game. It does not make sense to say that since there are no stable

[1] For instance see Dutta and Sen (1988a) (1988b), Jackson (1988a), McKelvey (1985), Moore and Repullo (1986) (1988), Palfrey and Srivastava (1986) (1987) (1988) (1989), Postlewaite and Schmeidler (1986), Saijo (1988). Of course integer games can be replaced with 'half-open interval' games where agents announce a number from a half-open interval. Maskin (1977) does not completely describe an implementing mechanism, but the proof of his theorem which does, Saijo (1988), involves a modulo game.

points in the integer game, agents will not choose actions which place them in the integer game. In a nutshell, it is unreasonable to say that a given solution concept 'solves' the integer game, and likewise, it is unreasonable to say that a given solution concept 'solves' a game which is augmented with the addition of an integer game.

A parallel argument can be made against modulo constructions, but only for solution concepts which look only at pure strategy Nash equilibria or a refinement of pure strategy Nash equilibria.[2] [The literature on Nash and undominated Nash implementation considers only pure strategies. For example, see Maskin (1977), Moore and Repullo (1988), Saijo (1988), Williams (1984) and Palfrey and Srivastava (1986).] Furthermore, as we shall see, the class of mechanisms which are problematic for pure strategy Nash solution concepts goes beyond those with modulo constructions.

The goal of this paper is to begin to understand the impact of restricting the class of mechanisms considered for implementation, and to start the process of identification of classes of mechanisms which are appropriate for different solution concepts.

We begin by using a single elimination of dominated strategies as a solution concept. This sheds light on the problem in several ways. First, it allows us to take the current theory to its logical extreme: We show that if we place no restrictions on mechanisms, then we can fully implement any social choice function with a weak solution concept and a minor assumption concerning preferences. This implies that either implementation theory can achieve anything, or that we must have been applying the solution concept to too broad a class of mechanisms. Example 1 suggests the latter. Second, in using undominated strategies as a solution concept, the appropriate class of mechanisms for consideration is easy to identify. We can then examine the implications of restricting our attention to mechanisms in an appropriate class. Third, performing this exercise for undominated strategies reveals a very stark contrast between what we can implement when we consider

[2] For other solutions, such as undominated strategies, modulo constructions are not a problem.

the appropriate mechanisms and what we can implement when we consider all mechanisms. Under a minor assumption on preferences all social choice functions are implementable by some mechanism, but only strategy–proof social choice functions are implementable by a bounded mechanism. Thus we have a partial answer to the question we posed: some of the power of implementation theory is derived from the absence of appropriate restrictions on mechanisms.

It turns out to be more difficult to identify appropriate classes of mechanisms for other solution concepts, and we leave these issues largely unresolved. We provide examples which raise some of the issues which need to be considered for implementation in undominated Nash or Nash equilibria. One example shows that there exist social choice functions which are fully implementable in undominated Nash equilibria by unbounded mechanisms, but which are not fully implementable in undominated Nash equilibria by a bounded mechanism. Another example raises issues associated with mixed strategy equilibria and their omission from definitions of implementation. In particular, it demonstrates a mechanism which has mixed strategy equilibria which always result in outcomes which all agents strictly prefer to the outcome associated with the pure strategy equilibria considered in the implementation definitions. These equilibria pose a difficulty for the current definitions of implementation involving the Nash property, when there are no addtional restrictions on mechanisms.

2. Definitions and Notation

In general, for a given set of scalars or functions $\{v^1, \ldots, v^n\}$, define the vectors v, v^{-i}, $v/\tilde{v}^i$, and (v^{-i}, v^i), by $v = (v^1, \ldots, v^n)$, $v^{-i} = (v^1, \ldots, v^{i-1}, v^{i+1}, \ldots, v^n)$, and $v/\tilde{v}^i = (v^{-i}, \hat{\tilde{v}}^i) = (v^1, \ldots, v^{i-1}, \tilde{v}^i, v^{i+1}, \ldots, v^n)$.

There are a finite number, N, of agents in an environment. The letters i, j, and k are used to represent generic individual agents. The set of allocations to be considered is denoted A. We assume that $\#A \geq 2$, since the implmentation issue is trivial otherwise. Agent i has preferences over the allocations in A which are denoted R^i. We assume that R^i is a binary relation which is complete, transitive, and reflexive. The set of all such relations is denoted $\Re(A)$. The set of possible preferences for i is $\mathcal{R}^i$, $\mathcal{R}^i \subseteq \Re(A)$. Let $R = (R^1, \ldots, R^n)$ be the vector of agents' preference orderings and $\mathcal{R} = \mathcal{R}^1 \times \cdots \times \mathcal{R}^N$ be set of possible preference vectors. The strict preference relation associated with R^i is denoted P^i. That is, P^i is the irreflexive relation defined by $aP^ib \Leftrightarrow$ *not* bR^ia.[3] The indifference relation associated with R^i is denoted I^i and defined by $aI^ib \Leftrightarrow (aR^ib$ and $bR^ia)$.

Agents are assumed to know their own preferences. Agents are also assumed to know the structure of any mechanism they participate in. However, our analysis is independent of agents' knowledge about the preferences of the other agents. One of the appeals of a single elimination of weakly dominated actions as a solution concept is that it does not require specification of priors or any common knowledge.

An *environment* is a collection $(N, A, \mathcal{R})$.

A *social choice correspondence* is a correspondence which associates a subset of A with each $R \in \mathcal{R}$. If a social choice correspondence is single valued then it is called a *social choice function* .

A *mechanism* is an action space $M = M^1 \times \cdots \times M^N$ and a function $g : M \to A$.

An action $m^i \in M^i$ is *weakly dominated* at R^i if there exists $\tilde{m}^i \in M^i$ such that $g(m/\tilde{m}^i)R^ig(m/m^i)$, for all $m \in M$ and $g(m/\tilde{m}^i)P^ig(m/m^i)$ for some $m \in M$. In this case, we say that $\tilde{m}^i$ *weakly dominates* m^i at R^i. An action $m^i \in M^i$ is *undominated* at R^i if there is no action in M^i which weakly dominates it.

A social choice correspondence F is *fully implemented in undominated strategies* if there exists a mechanism (g, M) such that for all $R \in \mathcal{R}$,[4]

$$F(R) = \{a \in A \mid a = g(m) \text{ for some } m \in M \text{ s.t. } \forall i \; m^i \text{ is undominated at } R^i\}.$$

[3] We use the notation aR^ib to say "a weakly preferred by i to b" (rather than the notation $(a,b) \in R^i$).

[4] We remark that this is quite different from implementation in dominant strategies (see Dasgupta, Hammond, and Maskin (1979)). There it is required that for each i $g(m^{-i}, m^i)R^ig(m^{-i}, \overline{m}^i)$, for all $\overline{m}^i$ and *for all* m^{-i}.

3. Unbounded Mechanisms

We first examine the implementation question with no restrictions on the class of mechanisms we consider.

We say that agents' preferences are *strictly value distinguished* if given any R^i and $\overline{R}^i$ in $\mathcal{R}^i$ ($R^i \neq \overline{R}^i$), there exist x and y in A such that xP^iy and $y\overline{P}^ix$. [See Palfrey and Srivastava (1988).] Many interesting preference classes satisfy strict value distinction. For instance, it is satisfied if preferences are strict or if A is a convex subset of $\mathbb{R}^l$ and preferences are continuous and locally not completely indifferent.[5]

THEOREM 1. *If agents' preferences are strictly value distinguished, then any social choice function can be fully implemented in undominated strategies.*

The proof of Theorem 1 appears in Appendix 1.

Theorem 1 can be extended to cover the case where F is not a function and where preferences are not strictly value distinguished, with some qualifications. The statement of the theorem above allows us to work with a relatively simple mechanism.

Theorem 1 provides what appears to be a very strong result, especially given the weakness of the solution concept. That is, if we agree that a basic requirement of a solution concept is that agents not choose weakly dominated actions, then the solutions we consider should be a subset of the undominated strategies.[6] If the set of undominated actions provide us with our unique desired outcome, then any (nonempty) subset of the undominated actions will also coincide with that outcome.

However, there is a problem with considering only the undominated actions of some mechanisms, as is illustrated in the following example. Example 1 shows that the proof

[5] By locally not completely indifferent, we mean that for any $x \in A$, $R^i \in \mathcal{R}^i$, and neighborhood U of x, there exists some $z \in U$ such that either zP^ix or xP^iz. We do not, in fact, need that A is a convex subset of $\mathbb{R}^l$. If A is a perfect metric space (for any $x \in A$ and $\epsilon > 0$ there exists $z \neq x$ such that $d(x, z) < \epsilon$) and preferences are continuous and not completely indifferent, then preferences are strictly value distinguished.

[6] For an alterative view see Bernheim (1984) and Pearce (1984).

of Theorem 1 relies on the construction of mechanisms which put to question solving for undominated actions. In particular, the mechanisms have infinite strings of dominated strategies with no undominated strategy 'on top'. It turns out that it is this feature which allows us to fully implement any social choice function . The example below gives an idea of how this is achieved for a particularly nasty social choice function .

EXAMPLE 1

Consider the environment $(N, A, \mathcal{R})$ where $N = 2$, $A = \{a, b\}$, and $\mathcal{R}$ is described as follows: $\mathcal{R}^1 = \{R^1\}$ where aP^1b, and $\mathcal{R}^2 = \{R^2, \overline{R}^2\}$ where aP^2b and $a\overline{P}^2b$.

The social choice function F is defined by $F(R^1, R^2) = b$ and $F(R^1, \overline{R}^2) = a$. This function violates any sort of incentive compatibility condition we may wish to consider. Furthermore, is fails to satisfy efficiency, monotonicity (in Maskin's sense (1977) (1983)), or even unanimity. However, it can be fully implemented in undominated strategies by the mechanism pictured below.

$$
\begin{array}{cccccccccc}
 & & & & & M^1 & & & & \\
 & & m^1 & & & & & & & \\
 & m^2 & b & a & a & a\ldots & a & a & a & a\ldots \\
 & & b & a & a & a\ldots & b & b & b & b\ldots \\
 & & b & b & a & a\ldots & b & b & b & b\ldots \\
 & & b & b & b & a\ldots & b & b & b & b\ldots \\
 & & \vdots & & & & \vdots & & & \\
M^2 & & & & & & & & & \\
 & & & & & & & & & \\
 & \overline{m}^2 & a & b & b & b\ldots & b & b & b & b\ldots \\
 & & a & a & a & a\ldots & a & b & b & b\ldots \\
 & & a & a & a & a\ldots & a & a & b & b\ldots \\
 & & a & a & a & a\ldots & a & a & a & b\ldots \\
 & & \vdots & & & & \vdots & & &
\end{array}
$$

The table above describes the outcome as a function of the actions of the agents. The only undominated action for agent 1 is m^1. At R^2, the only undominated action for agent 2 is m^2, while at $\overline{R}^2$ the only undominated action for agent 2 is $\overline{m}^2$. It follows that this mechanism fully implements F.

This ends Example 1.

We make two remarks about this example.

First, the number of allocations and preferences in this example are finite, and therefore so is the range of any social choice correspondence defined on this environment. It seems reasonable to expect that if we can implement a given social choice correspondence on this environment, that we should be able to do so with a fairly simple mechanism (at least a finite one). The fact that we need such a large mechanism in Example 1 gives us a clue that something is wrong.

Second, as mentioned previously, the way in which the undesired actions are ruled out in the above mechanism is through an infinite string of actions, each one dominating the previous one (essentially an integer game). In this context it is no longer compelling to argue that agents will not play weakly dominated actions. If we were to look simply at an integer game, eliminating dominated actions tells us that there are no actions which agents can play. [Alternatively, eliminating dominated actions from the above mechansim when m^2 and $\overline{m}^2$ are not in the action space M^2, leaves us with no actions.] We should not take this to mean that agents would not play the game anyway, simply that eliminating dominated actions cannot predict how agents would play such a game. Given this, it seems unreasonable to look at a more complicated game which includes an integer construction, and then say that agents would not choose any actions in the integer construction, since it has no undominated actions.[7]

[7] Notice that there are no Nash equilibria to the mechanism (in Example 1) at $(R^1, \overline{R}^2)$, and all the Nash equilibria at (R^1, R^2) give outcome a. However, if we add a third agent (with any preferences) then we can extend this mechanism so that F is fully implemented

The example and discussion above indicate that it does not make sense to apply the elimination of dominated strategies to the sort of mechanisms used in the proof of Theorem 1, at least for some environments. This raises two questions. First, for what class of mechanisms is it reasonable to eliminate dominated strategies. And second, which social choice functions can we implement in undominated strategies by some mechanism in that class? With these questions in mind, we turn to the next section.

4. Bounded Mechanisms

There is a natural notion of 'bounded' which identifies mechanisms which do not have the property of the mechanism in Example 1. A mechanism (g, M) defined on an environment $(N, A, \mathcal{R})$ is said to be *bounded* if, whenever an action $m^i \in M^i$ is dominated at some $R^i \in \mathcal{R}^i$, there exists an action $\tilde{m}^i \in M^i$ which dominates m^i at R^i and which is undominated at R^i.

The definition of a bounded mechanism is made with reference to the environment it is defined on. This is quite natural, since the preferences of the agents determine what the 'payoffs' to the mechanism are. Thus the properties of the mechanism are linked directly to the set of preferences which are possible in an environment. There are mechanisms which are bounded regardless of the possible preferences: It follows from the transitivity of weak domination that any finite mechanism ($\#M < \infty$) is bounded.

Clearly, the mechanisms in Example 1 and the proof of Theorem 1 are not bounded.[8]

The notion of bounded mechanisms has very strong implications for the implementation issue. If we consider the outcome for an agent's action against a vector of possible actions

in undominated Nash equilibria. This is done in Appendix 5. It is not surprising to find that identifying undominated Nash equilibria is not compelling when done for the extended mechanism.

[8] We notice that modulo sorts of constructions are possible in a bounded mechanism. Modulo constructions are not inappropriate with undominated strategies. If we consider a simple modulo game there exists a solution since all actions are undominated (in contrast to the non–existence of pure strategy Nash equilibria). Furthermore, mixed strategy equilibria are not a relevant issue when we use undominated strategies as a solution.

of the other agents, there has to exist an undominated action which gives an outcome which is at least preferred to this outcome. The implication of this for implementation is captured in the following property.

A social choice correspondence is *strategy–resistant* if for all i, $R \in \mathcal{R}$, $\overline{R}^i \in \mathcal{R}^i$, and $a \in F(R/\overline{R}^i)$, there exists $b \in F(R)$ such that $bR^i a$. If F is a function which is strategy–resistant, then it is said to be *strategy–proof.*

LEMMA 1. *If a social choice correspondence can be fully implemented in undominated strategies by a bounded mechanism then it is strategy–resistant.*

PROOF: Pick any i, $R \in \mathcal{R}$, $\overline{R}^i \in \mathcal{R}^i$, and $a \in F(R/\overline{R}^i)$. By the definition of implementation, there exists (g, M) and $m \in M$ such that m^j is undominated for all j at the j^{th} component of $R/\overline{R}^i$ and $g(m) = a$. If m^i is undominated at R^i then $a \in F(R)$ and strategy–resistance is satisfied. If m^i is dominated at R^i then, since (g, M) is bounded, there exists some undominated action $\tilde{m}^i$ which dominates it. It follows that $g(m/\tilde{m}^i)R^i g(m)$ and that $g(m/\tilde{m}^i) \in F(R)$. Therefore strategy–resistance is satisfied. ∎

COROLLARY 1. *If a social choice function can be fully implemented in undominated strategies by a bounded mechanism then it is strategy–proof.*

It would seem that strategy–proofness should also be sufficient for full implementation in undominated strategies by a bounded mechanism. However, this is not true since we require *full* implementation. Strategyproofness makes it easy to assure that there exists a vector of undominated actions with outcome $f(R)$, but it does not rule out undominated actions which lead to some other outcome.

The following example shows that, in general, strategyproofness is not sufficient for full implementation in undominated strategies by a bounded mechanism.

EXAMPLE 2.

Consider the environment $(N, A, \mathcal{R})$ where $N = 2$, $A = \{a, b, c, d\}$, and $\mathcal{R}$ is described as follows: $\mathcal{R}^1 = \{R^1\}$ where $dP^1aP^1bP^1c$, and $\mathcal{R}^2 = \{R^2, \overline{R}^2\}$ where $aI^2bP^2cP^2d$ and

$a\overline{I}^2 b\overline{P}^2 d\overline{P}^2 c$. These preferences are pictured below.

R^1	R^2	$\overline{R}^2$
d	a-b	a-b
a	c	d
b	d	c
c		

The social choice function F is defined by $F(R^1, R^2) = a$ and $F(R^1, \overline{R}^2) = b$.

Suppose that a bounded mechanism (g, M) fully implements F in undominated strategies. It follows that there exists $m \in M$ such that $g(m) = a$ and m is undominated at R^1, R^2. Therefore, since $g(m) \neq F(R^1, \overline{R}^2)$, m^2 is dominated at $\overline{R}^2$ by an undominated action $\overline{m}^2$. Given the fact that $\overline{m}^2$ does not dominate m^2 at R^2, this implies that there exists $\tilde{m}^1$ such that $g(\tilde{m}^1, \overline{m}^2) = d$ and $g(\tilde{m}^1, m^2) = c$. Since these outcomes do not coincide with F, $\tilde{m}^1$ must be dominated (at R^1) by an undominated action $\hat{m}^1$; which implies that $g(\hat{m}^1, \overline{m}^2) = d$. Since $\hat{m}^1$ and $\overline{m}^2$ are both undominated at $R^1, \overline{R}^2$, this implies that $d \in F(R^1, \overline{R}^2)$ which contradicts the fact that (g, M) fully implements F in undominated strategies. We have shown that F cannot be fully implemented in undominated strategies by a bounded mechanism, even though it is strategy–proof.

This ends Example 2.

Although Example 2 shows that strategy–proofness is not a sufficient condition for full implementation by a bounded mechanism on some environments, strategy–proofness is sufficient when we consider exchange economies, the topic of the next section.

5. Exchange Economies

In this section, we restrict our attention to environments satisfying the special properties described below. We call such an environment an *exchange economy*.

Each agent i has an endowment of l different goods, $e^i \in \mathbb{R}^l_+$. The aggregate endowment $\sum_i e^i$ is assumed to be strictly positive. It is assumed that the endowments of all the agents are known and fixed.[9] An allocation is an Nl–dimensional vector x describing each agent's allocation x^i, where x^i is a vector in $\mathbb{R}^l_+$. The allocation set can be represented as

$$A = \{x \in \mathbb{R}^{lN}_+ \mid \sum_i x^i \leq \sum_i e^i\}.$$

Agent i's preferences are assumed to depend only on the allocation x^i. That is, for a, b, x, y in A, if xR^iy and $a^i = x^i$ and $b^i = y^i$, then aR^ib. We also require preferences to the be strictly increasing: if $x^i \geq y^i$ then xP^iy. [Given l–dimensional vectors u and v, $u \geq v$ indicates that $u_k \geq v_k$ for all $k \in \{1, 2, \ldots, l\}$ and $u_k > v_k$ for some $k \in \{1, 2, \ldots, l\}$.]

THEOREM 2. *Consider an exchange economy $(N, A, \mathcal{R})$ such that preferences are strictly value distinguished and $N \geq 3$. A social choice function defined on $(N, A, \mathcal{R})$ can be fully implemented in undominated strategies by a bounded mechanism if and only if it is strategy–proof.*

The proof of Theorem 2 appears in Appendix 2.

A nice feature of the mechanism used in the proof of Theorem 2 is that the unique actions which are undominated at R, also form a Nash or Bayesian Nash equilibrium at R (depending on the information structure). This is assured by the strategy–proofness of F.

Corollary 1 (and Theorem 2) show that a very restrictive condition is associated with implementation in undominated strategies by a bounded mechanism: strategy–proofness. It is well known (Gibbard (1973) and Satterthwaite (1975)) that if $\#A \geq 3$ and the domain $\mathcal{R}$ is unrestricted, then a social choice function which satisfies citizen sovereignty (the range

9 Our notion of implementation assumes a fixed allocation space. This derives from the fact that the mechanism is to be independent of the uncertainty in the environment. [In fact it is designed to resolve the uncertainty.] Allowing the allocation space to vary is certaintly an interesting question, but beyond the scope of this paper. For a discussion see Hurwicz, Maskin, and Postlewaite (1984).

of F is A) is strategy–proof if and only if it is dictatorial. This result does not carry over to situations in which preferences are restricted to satisfy certain properties, and in particular it does not hold for exchange economies. Although strategy–proofness is restrictive, it still admits social choice functions of interest. We show below that under strong assumptions, the Walrasian allocation for large replications of an exchange economy is strategy–proof.

First, we develop the notion of replicating an exchange economy.

The k^{th} *replication* of an exchange economy $E = (N, A, \mathcal{R})$ is the exchange economy $(kN, A(k), \mathcal{R}^k)$, where $A(k) = \{x \in I\!R_+^{lkN} \mid \sum_{i=1}^{kN} x^i \leq \sum_{i=1}^{kN} e^i\}$, and $e^{ni} = e^i$ for all $i \in \{1, \ldots, N\}$ and $n \in \{1, \ldots, k\}$. Notice that although a 'replication' agent has the same set of possible preferences as his or her initial counterpart, in any particular realization of the replicated economy they do not necessarily have the same preferences. For example, if $N = k = 2$, then $\mathcal{R}^1 = \mathcal{R}^3$ while it is possible that in a realization of the economy $R^1 \neq R^3$. We use the notation E^k to represent the k^{th} replication of E.

Let $T(E) = \cup_{i, R^i \in \mathcal{R}^i} (R^i, e^i)$ be the set of possible types of agents in the economy E. [Notice that $T(E) = T(E^k)$ for all k.] We can represent a realization of E by a probability measure on $T(E)$. That is, if μ is a probability measure on $T(E)$, the interpretation of $\mu(t)$, $t \in T(E)$, is that a proportion $\mu(t)$ of the agents in the economy are of type t. We consider the metric $d(\mu, \nu) \doteq sup_{t \in T} |\mu(t) - \nu(t)|$ (and the topology it induces) on the space of all probability measures defined on $T(E)$.

For an exchange economy, E, only some probability measures (defined on the type space $T(E)$) correspond to a possible realization of the economy. Let $\mathcal{M}(E)$ be that set of probability measures. We define $\overline{\mathcal{M}}$ to be the closure of $\cup_{k=1}^{\infty} \mathcal{M}(E^k)$.

We remark that if preferences are continuous, strictly increasing, strictly convex, then associated with each $t \in T$ is a continuous competitive demand function $h^t : S_{++}^l \rightarrow I\!R_+^l$, where $S_{++}^l = \{p \in I\!R_{++}^l \mid \sum_{i=1}^l p_i = 1\}$. [So h^t is the function such that $h^t(p) \in \{a^i \in I\!R_+^l \mid p \cdot a^i \leq p \cdot e^i\}$ and $h^t(p) R^i a^i$ for all $a^i \in \{a^i \in I\!R_+^l \mid p \cdot a^i \leq p \cdot e^i\}$, where $t =$

(R^i, e^i).] This follows from Richter (1989).[10][11] Under these conditions, the Walrasian price correspondence is well defined. The *Walrasian price correspondence* of an exchange economy is the correspondence Π from $\overline{\mathcal{M}}$ to S^l_{++} such that for any $\mu \in \overline{\mathcal{M}}$

$$\Pi(\mu) = \{p \in S^l_{++} \mid \int_T h^t(p) d\mu(t) = \sum_{i=1}^N e^i / N\}.$$

The *Walrasian social choice correspondence* W defined on an exchange economy E is the correspondence from $\mathcal{R}$ to A such that for all $R \in \mathcal{R}$

$$W(R) = \{a \mid \exists p \in \Pi(\mu^R) \text{ such that } a^i = h^{(R^i, e^i)}(p)\ \forall i\},$$

where μ^R is the probability measure in $\mathcal{M}(E)$ which corresponds to the realization R.

An exchange economy is *heterogeneous* if its Walrasian price correspondence Π is a continuous *function*. This coupled with single valued demand implies a unique Walrasian equilibrium, and so the Walrasian social choice correspondence is a social choice function . Roughly, this says no good has an aggregate demand which is becoming arbitrarily large or small relative to the demands for the other goods.

LEMMA 2. *Consider an exchange economy E such that (i) $\#\mathcal{R}$ is finite, (ii) preferences are continuous, strictly increasing, and strictly convex,*[12] *(iii) E is heterogeneous, and (iv) for all i, if $t = (R^i, e^i)$ and $\bar{t} = (\overline{R}^i, e^i)$ then $h^t(p) \neq h^{\bar{t}}(p)$ for all p in the range of Π. There exists a number K such that for all $k > K$, the Walrasian social choice function associated with E^k can be fully implemented in undominated strategies by a finite mechanism.*

The proof of Lemma 2 appears in Appendix 3.

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[10] In fact, it is not necessary that preferences be strictly convex. We need the weaker condition Richter defines as $>$–convexity, which roughly requires that strict convexity hold only with respect to points which differ in all coodinates. This allows indifference curves to be flat in sections which are not supported by strictly positive prices (which is ruled out by strict convexity). This is sufficient to obtain a unique demand point for any strictly positive price and hence assure that the demand correspondence is a function.

[11] Demand is often defined over prices and income. Here we are in fact considering the Slutsky compensated demand, so that $h^t(p)$ represents $h^t(p, p \cdot e^i)$.

[12] Again, we need only Richter's $>$–convexity.

To better understand condition (iv), notice that it is equivalent to requiring that indifference curves corresponding to different preferences are never tangent at a point where they can be supported by a strictly positive price.

The intuition behind the lemma is straightforward; it is really just the standard competitive story. Let us think of the Walrasian setting in which agents submit demand functions to an auctioneer, who in turn chooses a price to clear markets. In choosing a demand to submit there are two opposing motivations. The first is to influence the price in your favor, which may involve submitting a demand which differs from your price taking demand. The second is to assure that at the resulting price to have your best quantity choice, which involves submitting the price taking demand. There is clearly a tension between these two motivations. As the economy is replicated, the tension decreases as each agent's influence on the price decreases. Eventually, any two demand functions an agent might consider result in almost the same price. Then an agent is simply concerned with receiving the best allocation at that price.

Postlewaite and Roberts (1976) show that, for replicated economies, submitting the competitive demand is limiting individually incentive compatible. [Given any ϵ, there is a large enough replication of the economy so that the incentive compatibility inequality is satisfied if ϵ is added to the utility of submitting the competitive demand.] Their result shows that for large economies, there is little incentive to deviate from purely competitive behavior. In a sense, Lemma 2 is an extension of their results to show that the outcomes of a sequence of economies with a finite number of agents converge to a competitive equilibrium.

The lemma is of the most interest when information is incomplete (recall that we have not specified the information structure beyond each agent knowing his or her preferences). If there is complete information then the Walrasian correspondence can be fully implemented in Nash, strong Nash, or undominated Nash equilibria (see Hurwicz (1979), Schmeidler (1980), Palfrey and Srivastava (1986), and Nakamura (1988)). Palfrey and Srivastava (1986b) have a result similar to Lemma 2 showing that there are outcomes which converge to the Walrasian allocation, when information is incomplete. In their setting, however, replicated agents have the same realized preferences as their base counterpart agent. Here we examine the situation in which as the economy is replicated the state space expands (replica agents may be different from base agents). Mas–Colell and Vives (1989) also examine this setting. They use a Bayesian approach and look at games with a continuum of agents in which the distribution of characteristics is known. The intuition behind their approach is that uncertainty is negligeable with large numbers of agents. Here, we have not specified priors or how the distribution of agents characteristics is resolved; instead we have made use of the fact that the gains from misrepresentation are negligeable in large economies, regardless of the realization.

We have confined our analysis to situations in which agents know their own preferences. Situations which involve a common value with incomplete information are not covered. There it seems plausible that an agent may have private information which permits manipulation of the price, despite the size of the economy. [See Jackson (1988b) for an example along these lines.]

6. Other Solution Concepts

So far, we have restricted our attention to implementation in undominated strategies. Our success in implementing desired social choice functions by bounded mechanisms is limited, as is evidenced by Lemma 1 and Corollary 2. Yet we should not be discouraged since our look at undominated strategies was partly an exercise to help us understand the importance of identifying an appropriate class of mechanisms for a given solution concept. We may, in general, be willing to look at stronger solution concepts.

Since our analysis has demonstrated a stark contrast between what we can fully implement in undominated strategies with bounded mechanisms and without them, we should be interested in restricting attention to appropriate mechanisms when we use other solution concepts. However, identifying the right class of mechanisms is not as easy for some solution concepts. Our notion of a bounded mechanism corresponds nicely with undominated strategies as a solution. Although this notion of bounded mechanism rules out integer games, it fails to rule out the modulo games and other mechanisms which seem objectionable for instance when Nash or undominated Nash equilibria are considered.

Let us proceed first things first. We begin by looking at the restriction to bounded mechanisms for implementation in undominated Nash equilibria. Given an environment $(N, A, \mathcal{R})$ and a mechanism (g, M), an action m is a *Nash equilibrium* at R if for all i $g(m)R^i g(m/\overline{m}^i)$ for all $\overline{m}^i \in M^i$. An action m is an *undominated Nash equilibrium* at R if, for each agent i, m^i is undominated at R^i, and m is a Nash equilibrium at R. [See Palfrey and Srivastava (1986).] A social choice correspondence F is *fully implementable in undominated Nash equilibria* if there exists a mechanism (g, M) such that for each $R \in \mathcal{R}$

$$F(R) = \{a \mid \exists m \in M \text{ s.t. } m \text{ is an undominated Nash equilibrium at } R \text{ and } g(m) = a\}.$$

The following example shows that considering only bounded mechanisms narrows the set of social choice functions which can be implemented in undominated Nash equilibria.

EXAMPLE 3.

Consider the environent $(N, A, \mathcal{R})$, where $N = 5$, $A = \{a, b\}$, and $\mathcal{R}$ is defined as follows: $\mathcal{R}^1 = \mathcal{R}^2 = \{R^i\}$ where $aP^i b$, $\mathcal{R}^3 = \mathcal{R}^4 = \{R^i\}$ where $bP^i a$, and $\mathcal{R}^5 = \{R^5, \overline{R}^5\}$ where $aP^5 b$ and $b\overline{P}^5 a$.

Consider F defined by $F(R) = b$ and $F(R/\overline{R}^5) = a$. Preferences are strictly value distinguished and F satisfies no–veto power, so it follows from Palfrey and Srivastava (1986) that F can be fully implemented in undominated Nash equilibria. However, F cannot be fully implemented in undominated Nash equilibria by a bounded mechanism.

To see this suppose the contrary: There exists a bounded mechanism (g, M) which fully implements F in undominated Nash equilibria. Thus $\exists m \in M$ such that $g(m) = b$ and m is an undominated Nash equilibrium at R. It is clear that m is also a Nash equilibrium at $R/\overline{R}^5$. Therefore, since $g(m) \neq F(R/\overline{R}^5)$, m^i is dominated for some i at

$R/\overline{R}^5$. Since only agent 5's preferences have changed it must be that m^5 is dominated at $\overline{R}^5$. Since (g, M) is bounded, it follows that there exists an undominated $\overline{m}^5$ which dominates m^5 at $\overline{R}^5$. This implies that $g(m/\overline{m}^5) = b$. Since $m/\overline{m}^5$ is undominated at $R/\overline{R}^5$ and $g(m/\overline{m}^5) \neq F(R/\overline{R}^5)$, $m/\overline{m}^5$ cannot be a Nash equilibrium at $R/\overline{R}^5$. It follows that there exists $j \neq 5$ (since 5 is getting the best outcome under $\overline{R}^5$) such that $g(m/\overline{m}^5/\tilde{m}^j)P^j b$, and so $g(m/\overline{m}^5/\tilde{m}^j) = a$. Since m is Nash at R it follows that $g(m/\tilde{m}^j) = b$. This implies that $g(m/\tilde{m}^j)\overline{R}^5 g(m/\overline{m}^5/\tilde{m}^j)$, which contradicts the fact that $\overline{m}^5$ dominates m^5 at $\overline{R}^5$.

We remark that it is not critical to this example that $\#A = 2$. Similar examples can be constructed with $\#A \geq 3$. [Consider the addition of a third allocation c. All preferences are unchanged between a and b, and c is always the worst outcome, except for $\overline{R}^5$ which is such that $bP^i cP^i a$.]

This ends Example 3.

Example 3 shows that considering bounded mechanisms for full implementation in undominated Nash equilibria is a non–trivial restriction. The class of social choice functions which we can implement in undominated Nash equilibria by bounded mechanisms is strictly smaller than the class we can implement when we admit unbounded mechanisms.

At this point, we might ask what are the additional conditions necessary for implementation in undominated Nash equilibria by bounded mechanisms. Given our previous analysis and knowledge of implementation in Nash equilibria, we check strategy–proofness and monotonicity.

The following example demonstrates a social choice function which is fully implemented in undominated Nash equilibria by a bounded mechanism, but is neither strategy–proof nor monotonic.

EXAMPLE 4.

Consider the environent $(N, A, \mathcal{R})$, where $N = 2$, $A = \{a, b, c, d\}$, and $\mathcal{R}$ is defined as follows: $\mathcal{R}^1 = \{R^1\}$ where $cP^1 aP^1 dP^1 b$, and $\mathcal{R}^2 = \{R^2, \overline{R}^2\}$ where $bP^2 aP^1 cI^2 d$ and

$a\overline{P}^2 b\overline{P}^2 c\overline{I}^2 d$. These preferences are represented below.

R^1	R^2	$\overline{R}^2$
c	b	a
a	a	b
d	c-d	c-d
b		

Consider that social choice function F defined by $F(R^1, R^2) = a$ and $F(R^1, \overline{R}^2) = c$. It is easily checked that F is fully implemented in undominated Nash equilibria by the following mechanism.

	m^1	$\tilde{m}^1$	$\overline{m}^1$
m^2	b	d	a
$\overline{m}^2$	a	c	a

We remark that F is not strategy–proof and is not monotonic.

This example is easily extended to situations in which $N \geq 3$, simply by adding agents who have singleton action spaces.

This ends Example 4.

Although monotonicity is not necessary for implementation in undominated Nash equilibria by a bounded mechanism, it is sufficient when we confine our analysis to exchange economies. Furthermore, exchange economies have features which permit implementation by bounded mechanisms which have no modulo constructions. In addition, we can find such a mechanism for which all the Nash equilibria are in undominated actions. This is all captured in the following lemma.

LEMMA 3. *Consider any monotonic social choice correspondence F defined on an exchange economy with $N \geq 3$, such that $aP^i 0$ for all $R \in \mathcal{R}$, $a \in F(R)$, and i. F can be fully* *implemented in Nash or undominated Nash equilibria by a bounded mechanism which has no modulo constructions.*

The proof of Lemma 3 appears in Appendix 4.

The mechanism used in the proof of Lemma 3 takes advantage of the properties of an exchange economy, in place of an integer or a modulo construction. For situations in which at least two agents choose actions incongruent with the actions of the others, the largest group of agents with congruent actions evenly split the aggregate endowment, while the other agents get nothing. [In the mechanism all agents have the same action space $\mathcal{R} \times A$, so two actions are congruent if they announce the same (R, a).] This assures that there cannot exist an equilibrium with 'too much' disagreement.

However, by eliminating the modulo games, we have not eliminated the possiblility that there are mixed strategy equilibria which we have not considered. These could be a problem for our notion of implementation, especially if the mixed strategy equilibria result in an allocation not in the social choice correspondence . The following example provides a mechanism which has a mixed strategy (undominated) Nash equilibrium which *always* results in a better allocation, for both agents, than the pure strategy (undominated) Nash equilibrium identified for full implementation.

EXAMPLE 5.

Consider the environent $(N, A, \mathcal{R})$, where $N = 2$, $A = \{a, b, c, d\}$, and $\mathcal{R}$ is defined as follows: $\mathcal{R}^1 = \{R^1\}$ where $cP^1aP^1dP^1b$, and $\mathcal{R}^2 = \{R^2, \overline{R}^2\}$ where $aP^2cP^1bP^2d$ and $c\overline{P}^2a\overline{P}^2d\overline{P}^2b$. These preferences are represented below.

R^1	R^2	$\overline{R}^2$
a	a	b
b	b	a
c	d	c
d	c	d

Consider that social choice function F defined by $F(R^1, R^2) = a$ and $F(R^1, \overline{R}^2) = c$. It is easily checked that F is fully implemented both in Nash and in undominated Nash equilibria by the following mechanism.

	m^1	$\tilde{m}^1$	$\overline{m}^1$
m^2	a	b	d
$\tilde{m}^2$	b	a	d
$\overline{m}^2$	d	d	c

The only (pure strategy) Nash equilibria at R are m and $\tilde{m}$, which both result in a. At $(R^1, \overline{R}^2)$, the unique (pure strategy) Nash equilibrium is $\overline{m}$, which results in b. The Nash equilibria of this mechanism coincide with the undominated Nash equilibria.

Remark that F is monotonic, and that we can easily extend the example to situations in which $N \geq 3$ (by adding agents with singleton action spaces and having F be independent of their preferences).

This ends Example 5.

There is a difficulty with identifying the pure strategy Nash equilibria of the above mechanism as the outcomes. At $(R^1, \overline{R}^2)$ there is a mixed strategy Nash equilibrium (where each agent plays m^i and $\tilde{m}^i$ each with probability .5) which *always* results in an outcome (either a or b) which both agents strictly prefer to c (the result of the pure strategy Nash equilibrium $\overline{m}$).

This example points out the basic difficulty we face in assuring that we have a convincing notion of full implementation when the solution concept has the Nash property. We should be sure that we properly take into account mixed strategy equilibria. There are alternative approaches to this problem, for example (i) defining a notion of implementation which accounts for mixed strategy equilibria, (ii) looking only at mechanisms for which only the pure strategy equilibria are reasonable, or (iii) looking only at mechanisms for which any existing mixed strategy equilibria result in the same outcomes as the pure strategy equilibria. What we view as the 'proper' approach to setting up the implementation problem for solution concepts with the Nash property, depends on how we view mixed strategy equilibria. We leave this issue for future consideration.

7. Summary and Concluding Remarks

Essentially much of what we have said is captured by a simple rule. Given a solution concept for an implementation problem, we should consider only those mechanisms for which the solution concept is appropriate. The difficulty, and the focus of this paper, is identifying the mechanisms which are appropriate for a given solution concept. We argue that for undominated strategies the appropriate mechanisms are bounded, in the sense we defined. We saw that this was a very restrictive property: it narrowed the class of social choice functions which can be fully implemented in undominated strategies from any social choice function to ones which are strategy–proof.

We should say that this is does not go against our intuition. A weak solution concept corresponds to a strong requirement of full implementation. Full implementation requires that all the solutions of the mechanism be in the social choice correspondence . This is a great deal to ask of a weak solution. A weak solution concept, in general, will provide us with a large set of solutions. If we view a weak solution as providing a necessary condition for solutions to satisfy, then it may be that some of these solutions can be ruled out on other grounds. With this in mind, we turned in the last part of the paper to outlining the problem for other solution concepts. In particular we looked at Nash equilibria (which may be viewed as another weak solution concept), and combining the two, undominated Nash equilibria.

We showed that the problem of identifying appropriate mechanisms also had non–trivial consequences for solutions with the Nash property. Example 3 identifies a social choice function which can be fully implemented in undominated Nash equilibria, but not by any bounded mechanism. Example 4, however, indicates the restriction to bounded mechanisms will not produce as drastic a reduction in the class of implementable social choice functions using undominated Nash equilibria as using undominated strategies. Example 4 provides a social choice function which is not monotonic and not strategyproof, but which is fully implementable in undominated Nash equilibria by a bounded mechanism.

Finally, we discussed a problem which needs to be addressed for solution concepts with the Nash property, which is not captured by our notion of bounded. Namely, the existence of mixed strategy equilibria and their consideration in the definition of implementation and identification of an appropriate class of mechanisms.

There are many issues which we did not touch on. These include looking at restricting the class of mechanism when we consider subgame perfect implementation (Moore and Repullo (1986)), implementation in strong equilibria (Dutta and Sen (1988b)), or implementation in an approximate sense such as virtual implementation (Abreu and Sen (1987)) or approximate implementation (Sen (1988)). We also did not discuss the issues associated with Bayesian implementation (Jackson (1988a), Palfrey and Srivastava (1987) (1988) (1989), and Postlewaite and Schmeidler (1986)). The issues we discussed in this paper should have parallels for these other notions of implementation.

Eventually, the sort of analysis suggested in this paper should be synthesized with some of the other analyses concerning the properties of mechanisms used for implementation. These include looking for mechanisms with action (message) spaces of minimal dimension (McKelvey (1985), Reichelstein and Reiter (1988), Saijo (1988), Williams (1986)), or mechanisms which are continuous, balanced, etc. (Hurwicz (1979), Nakamura (1987), Tian (1985)). Ultimately, our goal would be, given a solution concept, a class of appropriate mechanisms with desired properties, and a class of environments, to be able to identify the class of social choice functions which we can implement.

APPENDIX 1

PROOF OF THEOREM 1.

We prove the theorem by constructing a mechanism which fully implements an arbitrary social choice function F. The structure of the mechanism presented in this proof is partly derived from the mechanisms designed by Palfrey and Srivastava (1986) and (1987) to prove implementation theorems concerning undominated Nash equilibria and undominated Bayesian Nash equilibria. Similar notation is used when convenient.

In the following definitions $R^i, \widetilde{R}^i$, and $\overline{R}^i$ represent distinct preferences in $\mathcal{R}^i$, unless otherwise noted.

Given R^i and $\overline{R}^i$ in $\mathcal{R}^i$, by strict value distinction we can find x and y in A such that xP^iy and $y\overline{P}^ix$. We denote x and y by $a(R^i, \overline{R}^i, i)$ and $a(\overline{R}^i, R^i, i)$, respectively.

$$M^i = [\cup_{i=1}^{N}(\mathcal{R}^i \times \mathcal{R}^i \times i)] \times \{0,1,2,\ldots\} \times A \times \Re(A)$$

We partition the set M into sets. The function g is defined according to which set an action belongs. The sets are defined on the left below with the corresponding choice of g given on the right below.

$$D_1 = \{m \mid \forall i \; m^i = (R^i, R^i, i, 0, \cdot, \cdot) \text{ or } m^i = (R^i, \widetilde{R}^i, i, \cdot, \cdot, \cdot)\} \qquad g(m) = F(R)$$

$$D_2^i = \{m \mid \exists i \text{ s.t. } \forall j \neq i \; m^j = (R^i, \overline{R}^i, i, \cdot, \cdot, \cdot) \text{ and either } m^i = (R^i, R^i, i, 0, \cdot, \cdot) \text{ or } m^i = (R^i, \widetilde{R}^i, i, \cdot, \cdot, \cdot)\} \qquad g(m) = a(R^i, \overline{R}^i, i)$$

$$D_3^i = \{m \mid \exists i \text{ s.t. } \forall j \neq i \; m^j = (R^i, \overline{R}^i, i, \cdot, \cdot, \cdot) \text{ and either } m^i = (R^i, \overline{R}^i, i, \cdot, \cdot, \cdot) \text{ or } m^i = (\overline{R}^i, \cdot, i, \cdot, \cdot, \cdot)\} \qquad g(m) = a(\overline{R}^i, R^i, i)$$

$$D_4 = \{m \mid m \notin D_1, \; m \notin \cup_i (D_2^i \cup D_3^i) \text{ and } \forall i \; m_4^i = 0\} \qquad g(m) = \overline{a}$$

$$D_5 = \{m \mid m \notin D_1 \cup D_4 \text{ and } m \notin \cup_i (D_2^i \cup D_3^i)\} \qquad g(m) = a(i^*)$$

where $\overline{a}$ is an arbitrary (fixed) allocation in A, i^* is the unique agent such that $m_4^{i^*} + \frac{i^*}{N} > m_4^j + \frac{j}{N} \; \forall j \neq i^*$, and $a(i^*)$ is the most preferred m_5^i according to $m_6^{i^*}$ (ties are broken by choosing the m_5^i with the lowest index i).

The claims below show that this mechanism implements the arbitrary social choice function F. Claim 1 identifies conditions which are necessary for an action to be undominated for the given mechanism. The outcomes corresponding to action profiles satisfying the necessary conditions of Claim 1 coincide with F. Claim 2 shows that there exists actions which are undominated, thus concluding the proof.

CLAIM 1. *If m^i is undominated at R^i then $m^i = (R^i, R^i, i, 0, \cdot, \cdot)$.*

PROOF: First we show that if m^i is undominated at R^i then $m^i = (R^i, R^i, i, 0, \cdot, \cdot)$ or $m^i = (\overline{R}^i, \overline{R}^i, i, 0, \cdot, \cdot)$. Let m^i be any action which is not of the specified form. Consider the action $\tilde{m}^i$, which is the same as m^i except that $\tilde{m}^i_4 = m^i_4 + 2$, $\tilde{m}^i_6 = R^i$, and $\tilde{m}^i_5 R^i \overline{a}$, $\tilde{m}^i_5 R^i m^i_5$ (and $\tilde{m}^i_5 P^i m^i_5$ if possible). We show that m^i is dominated at R^i by $\tilde{m}^i$. Against any m^{-i} such that $m^{-i}, m^i \notin D_5$, $m^{-i}, \tilde{m}^i$ results in the same allocation. From the construction of $\tilde{m}^i$, it follows that $m^{-i}, \tilde{m}^i \in D_5$ whenever $m^{-i}, m^i \in D_5$, and that in this case $g(m^{-i}, \tilde{m}^i) R^i g(m^{-i}, m^i)$. It remains to be shown that there exists $m^{-i} \in M^{-i}$ such that $(m^{-i}, m^i) \in D_5$ and $g(m^{-i}, \tilde{m}^i) P^i g(m^{-i}, m^i)$. First we note that we can find m^{-i} such that $i^* \neq i$ and $\tilde{m}^i_4 + \frac{i}{N} > m^{i^*}_4 + \frac{i^*}{N} > m^i_4 + \frac{i}{N}$. Next we note that by the definition of $\tilde{m}^i_5$ there exists $a \in A$ such that $\tilde{m}^i_5 P^i a$. Now we adjust m^{-i} so that $m^{i^*}_5 = a$ and $m^{i^*}_6 \in \Re(A)$ such that $a(i^*) = a$. For this m^{-i} the outcome for (m^{-i}, m) is $a(i^*)$ while the outcome for $(m^{-i}, \tilde{m}^i)$ is $a(i)$. Since $a(i) R^i \tilde{m}^i_5 P^i a(i^*)$, the outcome for $\tilde{m}^i$ is strictly preferred to the outcome for m^i.

Next we show that $m^i = (\overline{R}^i, \overline{R}^i, i, 0, \cdot, \cdot)$ is dominated at R^i by $\tilde{m}^i = (\overline{R}^i, R^i, i, 0, \cdot, \cdot)$. The only time these actions produce a different outcome is against $m^{-i} \in M^{-i}$ such that $(m^{-i}, m^i) \in D^i_2$ and $(m^j_1, m^j_2) = (\overline{R}^i, R^i)$ for all $j \neq i$. In that case, the outcome for $\tilde{m}^i$ is $a(R^i, \overline{R}^i, i)$ which is strictly preferred to the outcome for m^i, which is $a(\overline{R}^i, R^i, i)$. ∎

CLAIM 2. *Actions of the form $m^i = (R^i, R^i, i, 0, a, \cdot)$, such that $\exists b \in A$ with $a P^i b$, are undominated at R^i.*

PROOF: First we remark that there always exists such an action. [It follows from strict value distinction that there exist a and b with $a P^i b$.]

We partition the set of actions $\tilde{m}^i$, which differ from m^i, into the six cases listed below. We show that in the first five cases there exists some m^{-i} where $g(m^{-i}, m^i) P^i g(m^{-i}, \tilde{m}^i)$, which implies that $\tilde{m}^i$ cannot dominate m^i at R^i. Since this is true of all the $\tilde{m}^i$ which differ from m^i, m^i is undominated at R^i. In the sixth case, $g(m^{-i}, m^i) = g(m^{-i}, \tilde{m}^i)$ for all m^{-i}.

CASE 1: $\tilde{m}^i_3 \neq i$.

In this case $\tilde{m}^i = (R^j, \tilde{R}^j, j, n, \cdot, \cdot)$ (with the possiblility that $R^j = \tilde{R}^j$). Consider m^{-i} defined by $m^j = (R^i, \overline{R}^i, i, n+1, a(\overline{R}^i, R^i, i), \tilde{R}^j)$ for all $j \neq i$, where $\tilde{R}^j$ has $a(\overline{R}^i, R^i, i)$ as the unique most preferred in A. The action m/m^i is in D_2^i with outcome $a(R^i, \overline{R}^i, i)$, while $m/\tilde{m}^i$ is in D_5 with outcome $a(i^*) = a(\overline{R}^i, R^i, i)$. Note that $a(R^i, \overline{R}^i, i) P^i a(\overline{R}^i, R^i, i)$.

CASE 2: $\tilde{m}_1^i \neq R^i$, $\tilde{m}_3^i = i$.

In this case $\tilde{m}^i = (\overline{R}^i, \cdot, i, \cdot, , \cdot, \cdot)$. Consider m^{-i} defined by $m^j = (R^i, \overline{R}^i, i, \cdot, \cdot, \cdot)$ for all $j \neq i$. The action m/m^i is in D_2^i with outcome $a(R^i, \overline{R}^i, i)$, while $m/\tilde{m}^i$ is in D_3^i with outcome $a(\overline{R}^i, R^i, i)$. Again, note that $a(R^i, \overline{R}^i, i) P^i a(\overline{R}^i, R^i, i)$.

CASE 3: $\tilde{m}_1^i = R^i$, $\tilde{m}_2^i \neq R^i$, $\tilde{m}_3^i = i$.

In this case $\tilde{m}^i = (R^i, \overline{R}^i, i, \cdot, \cdot, \cdot)$. The argument for this case is the same as the argument for Case 2.

CASE 4: $\tilde{m}_1^i = R^i$, $\tilde{m}_2^i = R^i$, $\tilde{m}_3^i = i$, $\tilde{m}_4^i \neq 0$.

In this case $\tilde{m}^i = (R^i, R^i, i, n > 0, \cdot, \cdot)$. The argument for this case is the same as the argument for Case 1.

CASE 5: $\tilde{m}_1^i = R^i$, $\tilde{m}_2^i = R^i$, $\tilde{m}_3^i = i$, $\tilde{m}_4^i = 0$, $\tilde{m}_5^i = c$.

In this case $\tilde{m}^i = (R^i, R^i, i, 0, c, \cdot)$. Consider m^{-i} such that $m^{-i}, m^i \in D_5$. In this case it must be that $i^* \neq i$, since $m_4^j > 0$ for some $j \neq i$ (otherwise we would be in D_4). There exists such a m^{-i} such that for all $j \neq i$ $m_5^j = b$ where b is as defined in the claim, and $m_6^j = R^j$, where $aP^j bP^j c$ (or just $aP^j b$ if $b = c$). Then $g(m^{-i}, m^i) = aP^i b = g(m^{-i}, \tilde{m}^i)$.

CASE 6: $\tilde{m}_1^i = R^i$, $\tilde{m}_2^i = R^i$, $\tilde{m}_3^i = i$, $\tilde{m}_4^i = 0$, $\tilde{m}_5^i = a$, $\tilde{m}_6^i \neq m_6^i$.

In this case m^i and $\tilde{m}^i$ lead to the same allocation, regardless of m^{-i}. This follows since the sixth part of the action space only matters in D_5 when $i = i^*$, which is not possible given that $m_4^i = \tilde{m}_4^i = 0$. ▮

Claim 1 shows that any undominated set of actions produces an action in D_1 with outcome $F(R)$, where R is the true preference profile. Claim 2 shows that there exists an undominated set of actions for each preference profile. It follows that the mechanism presented above fully implements a given social choice function.

This ends the proof of Theorem 1.

APPENDIX 2

PROOF OF THEOREM 2.

Let F be a social choice function defined on an exchange economy satisfying the conditions of the theorem. Lemma 1 shows that if F can be fully implemented in undominated strategies by a bounded mechanism then it is strategyproof. To complete the proof, it is shown that if F is strategyproof then the mechanism given below is bounded and fully implements F in undominated strategies.

$$M^i = \mathcal{R}^i \times \left[0 \cup (\cup_{j \neq i}[\mathcal{R}^j \times \mathcal{R}^j \times j])\right]$$

We partition the set M into sets. The function g is defined according to which set an action belongs. The sets are defined on the left below with the corresponding choice of g given on the right below.

$$D_1 = \{m \mid \forall i\ m^i = (R^i, 0)\} \qquad g(m) = F(R)$$

$$D_2^i = \{m \mid \exists i \text{ s.t. } \forall j \neq i\ m^j = (R^j, [R^i, \overline{R}^i, i]) \text{ and } m^i = (R^i, 0)\} \qquad g(m) = a(R^i, \overline{R}^i, i)$$

$$D_3^i = \{m \mid \exists i \text{ s.t. } \forall j \neq i\ m^j = (R^j, [R^i, \overline{R}^i, i]) \text{ and } m^i = (\overline{R}^i, 0)\} \qquad g(m) = a(\overline{R}^i, R^i, i)$$

$$D_4^i = \{m \mid \exists i \text{ s.t. } \forall j \neq i\ m^j = (R^j, [R^i, \overline{R}^i, i]) \text{ and } m^i = (\widetilde{R}^i, 0)\} \qquad g(m) = a(\widetilde{R}^i, R^i, \overline{R}^i, i)$$

$$D_5^i = \{m \mid \exists i \text{ and } J \subset \{1, 2, \ldots, N\} \text{ s.t. } N - 2 \geq \#J \geq 1,\ m^j = (R^j, [R^i, \overline{R}^i, i])\ \forall j \in J, \text{ and } m^k = (R^k, 0)\ \forall k \notin J\} \qquad g(m) = a(i, J)$$

$$D_6 = \{m \mid m \notin D_1 \cup (\cup_i [D_2^i \cup D_3^i \cup D_4^i \cup D_5^i])\} \qquad g(m) = 0$$

where $a(\widetilde{R}^i, R^i, \overline{R}^i, i)$ is defined by

$$a(\widetilde{R}^i, R^i, \overline{R}^i, i) = \begin{cases} a(R^i, \overline{R}^i, i) & \text{whenever } a(R^i, \overline{R}^i, i)\ \widetilde{R}^i\ a(\overline{R}^i, R^i, i) \\ a(\overline{R}^i, R^i, i) & \text{otherwise,} \end{cases}$$

and where $a(i,J)^k = \sum_i e^i/(N - \#J - 1)$, for any agent $k \notin J$, $k \neq i$, and $a(i,J)^k = 0$ for the other agents.

We show that at R^i the only undominated action is $(R^i, 0)$. We do this by showing that this action dominates all other actions. This establishes that the mechanism is bounded and that it fully implements F.

Consider the action $m^i = (\overline{R}^i, 0)$. By strategyproofness, we know that for any m^{-i} such that $m \in D_1$, $m/(R^i,0) \in D_1$ and $g(m/(R^i,0))R^i g(m)$. For any m^{-i} such that $m \in D_6$ or $m \in \cup_i D_5^i$ the outcomes for m^i and $(R^i, 0)$ are the same. For any m^{-i} such that $m \in D_2^i \cup D_3^i \cup D_4^i$ the outcome for $(R^i,0)$ is weakly preferred to the outcome for m^i. In particular, when $m^j = (R^j, [R^i, \overline{R}^i, i])$ for all $j \neq i$, then $m \in D_2^i$ with outcome $a(\overline{R}^i, R^i, i)$ while $m/(R^i, 0) \in D_3^i$ with outcome $a(R^i, \overline{R}^i, i)$. This is strictly preferred by i. Hence, m^i is dominated by $(R^i, 0)$.

Consider any action of the form $m^i = (\cdot, [R^j, \widetilde{R}^j, j])$. We verify that $(R^i, 0)$ dominates such an action. For any m^{-i} such that $m \in D_5^j \cup D_6$, the allocation for i is 0 and so $m/(R^i, 0)$ will do at least as well. For any m^{-i} such that $m \in D_2^j \cup D_3^j \cup D_4^j$, the outcome is $a(R^j, \widetilde{R}^j, j)$ or $a(\widetilde{R}^j, R^j, j)$, while $m/(R^i, 0) \in D_5^j$ with resulting allocation for i $a(j,J)^i = \sum_i e^i$. From the definition of $a(\cdot,\cdot,j)$ it follows that $\sum_i e^i \geq a(\cdot,\cdot,j)$. Since preferences are strictly increasing, agent i strictly prefers the outcome associated with the action $m/(R^i,0)$ to the outcome associated with m. We have shown that m^i is dominated by $(R^i, 0)$.

This ends the proof of Theorem 2.

APPENDIX 3

PROOF OF LEMMA 2.

We examine a k^{th} replication of the economy E. We prove the lemma by verifying that there exists K such that if $k > K$, then agent i prefers the Walrasian outcome associated with i's true type to the outcome associated with any demand associated with some other type in $\mathcal{R}^i$. We show that this is true independent of the (announced) types of the other agents. We need only verify this for the base types in the original economy E. Since there are a finite number N of base agents and a finite number of possible types of each base agent, this establishes that for any large enough replication, the Walrasian social choice function is strategyproof and thus by Theorem 2, is fully implementable in undominated strategies by a bounded mechanism. [Notice that the assumptions of Lemma 2 assure that preferences are strictly value distinguished.] Furthermore, it is fully implementable by a finite mechanism since for a finite $\#\mathcal{R}$ the mechanism used in the proof of Theorem 2 is finite.

First we remark that, since $\overline{\mathcal{M}}$ is compact and Π is continuous on $\overline{\mathcal{M}}$, there exists a compact set $\mathcal{P} \subset S^l_{++}$ such that $\Pi(\mu) \in \mathcal{P}$ for all $\mu \in \overline{\mathcal{M}}$.

Second we remark that for any R^i and i there exists a $\delta > 0$ such that $h^{R^i,e^i}(p) R^i h^{\overline{R}^i,e^i}(p) + \delta \mathbf{1}$ for all $p \in \mathcal{P}$, where $\mathbf{1}$ is the unit vector in $\mathbb{R}^l$. This follows from the strict convexity of R^i, the continuity of the demand functions, the compactness of $\mathcal{P}$, and the fact that $h^{R^i,e^i}(p) \neq h^{\overline{R}^i,e^i}(p)$ for all $p \in \mathcal{P}$. Then, for the Walrasian social choice function to be strategyproof, it is sufficient to show that $h^{\overline{R}^i,e^i}(\Pi(\overline{\mu})) - h^{\overline{R}^i,e^i}(\Pi(\mu)) < \delta \mathbf{1}$, for any $\overline{R}^i \neq R^i$ and realizations of the economy $\overline{\mu}, \mu$ in $\mathcal{M}^k$ which differ only by the change of i from $\overline{R}^i$ to R^i.

Since $\mathcal{P}$ is compact, $h^{\overline{R}^i,e^i}$ is uniformly continuous on $\mathcal{P}$. Likewise, since $\overline{\mathcal{M}}$ is compact, Π is uniformly continous on $\overline{\mathcal{M}}$. Therefore, there exists some $\gamma > 0$ such that $h^{\overline{R}^i,e^i}(\Pi(\overline{\mu})) - h^{\overline{R}^i,e^i}(\Pi(\mu)) < \delta \mathbf{1}$, whenever $d(\overline{\mu}, \mu) < \gamma$. By the definition of $d(\cdot,\cdot)$ it follows that $d(\overline{\mu}, \mu) = \frac{1}{Nk}$. Hence, there is a K large enough so that if $k > K$ then $h^{\overline{R}^i,e^i}(\Pi(\overline{\mu})) - h^{\overline{R}^i,e^i}(\Pi(\mu)) < \delta \mathbf{1}$. There exists such a K for each $h^{\overline{R}^i,e^i}$. Since $\#\mathcal{R}^i < \infty$, we can find a K which works for all $h^{\overline{R}^i,e^i}$.

APPENDIX 4

PROOF OF LEMMA 3.

We prove the lemma by constructing the following mechanism which fully implements any monotonic social choice correspondence F defined on the exchange economy $(N, A, \mathcal{R})$, which satisfies $a \in F(R) \Rightarrow aP^i 0$ for all i and R.

$$M^i = \mathcal{R} \times A.$$

We partition the set M into sets. The function g is defined according to which set an action belongs. The sets are defined on the left below with the corresponding choice of g given on the right below.

In the following definitions R^i and $\widetilde{R}^i$ represent distinct preferences in $\mathcal{R}^i$, and a and b are distinct allocations in A.

$D_1 = \{m \mid \ m^i = (R, a) \forall i, \ a \in F(R) \ \}$ $\qquad g(m) = a$

$D_2^i = \{m \mid m^j = (R, a) \ \forall j \neq i,$ $\qquad g(m) = b$ if $aR^i b$ and $b\widetilde{P}^i a$

$\qquad m^i = (\widetilde{R}, b), \ a \in F(R)\}$ $\qquad g(m) = a$ otherwise

$D_3 = \{m \mid \exists$ disjoint nonempty i, J, K s.t.

$\qquad i \cup J \cup K = \{1, \ldots, N\},$

$\qquad m^j = (R, a) \forall j \in J, \ m^k = (R, b) \forall k \in K,$

$\qquad$ and $m^i = (\widetilde{R}, b)\}$ $\qquad g(m) = a(K)$

$D_4 = \{m \mid m \notin D_1, \ m \notin \cup_i D_2^i, \ m \notin D_3\}$ $\qquad g(m) = a(K^*)$

where $a(K)$ is the allocation which gives $\sum_i e^i / \# K$ to all $k \in K$ and 0 to $j \notin K$ if $K \neq \emptyset$ and $a(\emptyset) = 0$; and where K^* is the largest set containing at least 2 agents such that $m^k = m^j$ for all $k, j \in K$ (and ties are broken by choosing the set containing the lowest indexed agent).

Since it is not difficult to verify that this mechanism implements F (in either Nash or undominated Nash equilibria), we only sketch the proof.

First, any (R,a) where $a \in F(R)$ is undominated. Consider some other action $\tilde{m}^i$ which might dominate (R,a). There is a combination of other agents actions so that the $\tilde{m}^i$ places the action in D_4 with i getting 0, while (R,a) results in an allocation for i which is not 0. It is critical to note that since this is an exchange economy $\#A > 2$.

Second, all Nash equilibrium are in D_1. In any other set, some agent has a deviation which results in an allocation which that agent prefers.

Third, there is a Nash equilibrium in which all agents choose (R,a), where R is the true profile and a is any allocation in $F(R)$. A deviation by some agent can only move the action to D_2 and result in a or else some b such that aR^ib.

Finally, if (R,a) is a Nash equilibrium at $\overline{R} \neq R$, then $a \in F(\overline{R})$. Suppose that $a \notin F(R)$. By monotonicity there exists an agent i and allocation b such that $b\overline{P}^i a$ and aR^ib. This agent can then announce $(\overline{R},b)$, shifting the action to D_2 and resulting in allocation b. This contradicts the fact that (R,a) is a Nash equilibrium.

APPENDIX 5

EXTENSION OF EXAMPLE 1.

In this appendix we extend the mechanism described in Example 1 to include the actions of a third agent. The extension fully implements F in undominated Nash equilibria. We extend the mechanism in such a way so that we do not need to specify the preferences of the third agent since all three choices are undominated for that agent and the Nash equilibria exist independent of the preferences. Likewise, we extend F to be independent of the preferences of agent 3.

The additional third agent is given the choice of which of the following three tables should apply.

$\tilde{m}^3$					M^1				
	$\overline{m}^1$	m^1							
m^2	b	b	a	a	a ...	a	a	a	a ...
	a	b	a	a	a ...	b	b	b	b ...
	b	b	b	a	a ...	b	b	b	b ...
	b	b	b	b	a ...	b	b	b	b ...
M^2	⋮					⋮			
$\overline{m}^2$	a	a	b	b	b ...	b	b	b	b ...
	b	a	a	a	a ...	a	b	b	b ...
	a	a	a	a	a ...	a	a	b	b ...
	a	a	a	a	a ...	a	a	a	b ...
	⋮					⋮			

m^3 M^1

	$\overline{m}^1$	m^1							
m^2	b	b	b	b	b ...	b	b	b	b ...
	b	b	a	a	a ...	a	a	a	a ...
	b	b	a	a	a ...	a	a	a	a ...
	b	b	a	a	a ...	a	a	a	a ...
M^2	$\vdots$					$\vdots$			
$\overline{m}^2$	a	b	a	a	a ...	a	a	a	a ...
	a	b	a	a	a ...	a	a	a	a ...
	a	b	a	a	a ...	a	a	a	a ...
	a	b	a	a	a ...	a	a	a	a ...
	$\vdots$					$\vdots$			

$\overline{m}^3$ M^1

	$\overline{m}^1$	m^1							
m^2	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
M^2	$\vdots$					$\vdots$			
$\overline{m}^2$	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
	a	b	b	b	b ...	b	b	b	b ...
	$\vdots$					$\vdots$			

For agent 3, m^3, $\overline{m}^3$ and $\tilde{m}^3$ are all undominated, regardless of the preferences.

For agent 1, $\tilde{m}^1$ and m^1 are undominated actions. For any other action, we can progress steadily to the right of m^1 and eventually find an action which dominates it.

For agent 2, the only undominated action at R^2 is m^2, and at $\overline{R}^2$ is $\overline{m}^2$.

At R^2, the only Nash equilibrium which uses only undominated actions (regardless of R^3) is m^1, m^2, m^3. At $\overline{R}^2$, the only Nash equilibrium which uses only undominated actions (regardless of R^3) is $\overline{m}^1, \overline{m}^2, \overline{m}^3$.

References

ABREU, D. AND A. SEN [1987], "Virtual Implementation in Nash Equilibria," Department of Economics, Harvard.

ARROW, K. [1963], *Social Choice and Individual Values,* Cowles Foundation Monograph 12, Yale University Press: New Haven.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN [1979], "The Implementation of Social Choice Rules: some General Results on Incentive Compatibility," *Review of Economic Studies*, Vol. 46, pp. 185–216.

DUTTA, B. AND A. SEN [1988], "Necessary and Sufficient Conditions for 2–Person Nash Implementation," mimeo: Indian Statistical Institute.

——————[1988b], "Implementation Under Strong Equilibria: A Complete Characterization," mimeo: Indian Statistical Institute.

GIBBARD, A. [1973], "Manipulation of Voting Schemes: A General Result," *Econometrica*, Vol. 41, pp. 587–601.

HARRIS, M. AND R. TOWNSEND [1981], "Resource Allocation with Assymetric Information," *Econometrica*, Vol. 49, pp. 33–64.

HILDENBRAND, W. [1974], *Core and Equilibria of a Large Economy*, Princeton University Press.

HURWICZ, L. [1973], "The Design of Mechanisms for Resource Allocation," *American Economic Review*, Vol. 61, pp. 1–30.

——————[1979], "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies*, Vol. 46, pp. 217–225.

HURWICZ, L., E. MASKIN, AND A. POSTLEWAITE [1984], "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," Mimeo.

JACKSON M. [1988a], "Full Bayesian Implementation," Working Paper 791, CMSEMS Northwestern University.

——————[1988b], "Equilibrium, Price Formation, and the Value of Private Information," Working Paper 790, CMSEMS Northwestern University.

MASKIN, E. [1977], "Nash Equilibrium and Welfare Optimality," Department of Economics, MIT.

MASKIN, E. [1985], "The Theory of Implementation in Nash Equilibrium." In Ed.: L. Hurwicz, D. Schmeidler, and H. Sonnenschein, *Social Goals and Social Organization: Essays in Honor of Elisha A. Pazner*, Cambridge: Cambridge University Press.

MCKELVEY, R. [1985], "Game Forms For Nash Implementation of General Social Choice Correspondences," mimeo: Cal–Tech.

MOORE, J. AND R. REPULLO [1986], "Subgame Perfect Implementation," Working Paper, London School of Economics.

——————[1988], "Nash Implementation: A Full Characterization," Working Paper, London School of Economics.

MOOKHERJEE, D. AND S. REICHELSTEIN [1987], "Implementation via Augmented Revelation Mechanisms," Working Paper , Stanford Graduate School of Business.

MYERSON, R. [1979], "Incentive Compatibility and the Bargaining Problem," *Econometrica*, Vol. 47, pp. 61–74.

NAKAMURA, S. [1988], "Feasible Nash Implementation of Competitive Equilibria in an Economy with Externalities," Working Paper 248, Center for Economic Research, University of Minnesota.

PALFREY, T., AND S. SRIVASTAVA [1986], "Nash Implementation Using Undominated Strategies," Carnegie Mellon University.

————[1986b], "Private Information in Large Economies," *Journal of Economic Theory*, Vol. 39, pp. 34–58.

————[1987], "On Bayesian Implementable Allocations," *The Review of Economic Studies*, Vol. 54, pp. 193–208.

————[1988], "Mechanism Design with Incomplete Information: A Solution to the Implementation Problem," Forthcoming *Journal of Political Economy*.

————[1989], "Implementation with Incomplete Information in Exchange Economies," *Econometrica*, Vol. 57, pp. 115–134.

POSTLEWAITE, A. AND D. J. ROBERTS [1976], "The Incentives for Price Taking Behavior in Large Exchange Economies," *Econometrica*, Vol. 44, pp. 115–127.

POSTLEWAITE, A. AND D. SCHMEIDLER [1986], "Implementation in Differential Information Economies," *Journal of Economic Theory*, Vol. 39, pp. 14–33.

REICHELSTEIN S. AND S. REITER [1988], "Game Forms with Minimal Strategy Spaces," *Econometrica*, Vol. 56, pp. 661–692.

RICHTER, M.K. [1989], "Continuous Demand Functions," mimeo: University of Minnesota.

SAIJO, T. [1988], "Strategy Space Reduction in Maskin's Theorem: Sufficient Conditions for Nash Implementation," *Econometrica*, Vol. 56, pp. 693–700.

SATTERTHWAITE, M. [1975], "Strategy–proofness and Arrow' Conditions: Existence and Correspondecne Theorems for Voting Procedures and Social Welfare Theorems," *Journal of Economic Theory*, Vol. 10, pp. 187–217.

SCHMEIDLER D. [1980], "Walwrasian Analysis via Strategic Outcome Functions," *Econometrica*, Vol. 48, pp. 1585–1593.

SEN, A. [1988], "The Approximate Implementation of Social Choice Functions by Social Choice Correspondences," mimeo: Indian Statistical Institute.

TIAN, G. [1985], " Completely Feasible Continuous Outcome Functions Nash–Implementing Lindahl Allocations with a Message Space of Minimal Dimensions," mimeo: University of Minnesota.

WILLIAMS, S. [1984], "Sufficient Conditions for Nash Implementation," mimeo.

————[1986], "Realization and Nash Implementation: Two Aspects of Mechanism Design," *Econometrica*, Vol. 54, pp. 139–151.