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Bilateral Monopoly, Non-durable Goods and Dynamic Trading Relationships*

by

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Abstract

A dynamic signalling game is examined in which a monopolist seller of a nondurable good faces a buyer who is privately informed about the position of his demand curve. The seller names a price in each period and is committed to supply whatever quantity is demanded at that price. The buyer chooses the quantity in the awareness that the seller will use the quantity sold to infer information about the seller. The paper exploits the literature on the refinement of equilibria in signalling games to focus on a unique equilibrium which is characterized by complete separation in each period. Nevertheless, buyer types are forced to continue separating behaviour throughout the game.

Keywords: Repeated signalling games, refinements of equilibria, bilateral monopoly, bargaining.

Bilateral Monopoly

Playing hard to get is as time-honoured in markets as it is in love. The coy and clever buyer knows that betraying too much eagerness to a seller can often place him at a disadvantage as their relationship develops. Nevertheless, feigned indifference comes at a cost. Delayed consumption destroys irrevocably some opportunities for satisfaction and, at worst, can discourage a seller from pursuing the trading relationship any further. A careful buyer must always balance his wish for immediate gratification with a caution against betraying his true desires; an interested seller must balance her desire to benefit from the current transaction with the need to extract information about the future of their relationship.

In trading situations, the natural environment in which to analyze this behaviour is bilateral monopoly. Much is known about static monopoly behaviour -- much less is known about an environment where a demander has an incentive to alter his quantity demanded in order to alter his supplier's future pricing behaviour. The phenomenon is examined in a dynamic trading game in which two agents desire to trade, period by period, a non-durable good or a service. One agent, the buyer, has private information about his demand curve. The monopolistic seller fixes a price in each period and supplies any quantity the buyer chooses at that price. Since the buyer can choose any quantity, the potential screening device can be very fine. Furthermore, since the buyer has a downward sloping demand curve, the cost to him of revealing information is lower than standard models in which the buyer has an inelastic demand for a single unit. The opposing desires of the seller to obtain and the buyer to conceal private knowledge typically generate inefficiencies in models of asymmetric information. It is important, therefore, to see how quickly (if at all) information is revealed in these environments.

Section I describes the formal model of the paper. The next section discusses the relationship with other research and the third section describes equilibrium behaviour in the dynamic bilateral monopoly trading game. An argument is provided there for concentrating on a unique equilibrium outcome which satisfies a relatively weak refinement of sequential equilibrium in each subform of the game. The equilibrium consists of an extremely simple profile of strategies and is characterized by complete separation by the informed buyers in every period. However, the revelation of information fails to prevent the loss of resources in the subsequent play of the game. In a twist on the theme of love, it is the less interested buyer who must prove his lack of interest again and again throughout the relationship.

Section I: The Model

A monopolistic supplier with a constant marginal cost of production faces a market with linear demand. She sets a price for her product in a given period and supplies whatever quantity is demanded at that price. At the end of the period, she observes the quantity sold and fixes a new price for the next period. The seller has a finite time horizon, T, and a discount factor $\delta \in [0,1]$.[1]

Demand for the product is uncertain from the perspective of the supplier. The market is made up of a 'substantial portion', λ, of passive or

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[1] To ease notation, throughout the paper, the analysis proceeds assuming $\delta = 1$. It will be clear that this is a harmless restriction.

non-strategic buyers and a relatively large strategic, monopsonistic buyer responsible for $1-\lambda$ of the total market. Given a price p_t, the non-strategic portion demands according to a fixed relationship, $Q_t = A - p_t$. The large strategic buyer (henceforth, just the 'buyer') seeks to maximize the sum of per period expected utility of consumption q_t,

$$U = E \{\Sigma V(q_t;p_t)\}$$

Per period payoffs to a buyer who purchases quantity q at a price p is given by

$$V(q;p) = (a - p - .5q)q = aq - .5q^2 - pq$$

The buyer is privately informed about the value of a which may take one of two values in any period, a_H and a_L. Throughout the paper, it will be assumed that $a_L = ka_H > .5(\lambda A+(1-\lambda)a_H)$ so that naive low types would demand even at the highest possible price.

The variable a is stochastic and follows a simple Markov process. Fix $\mu_H \in (0,1)$ and $\mu_L \in (0,1)$, $\mu_H \geq \mu_L$. If a_t is the value of a in period t then $\mu_H = \text{Prob}(a_{t+1} = a_H \mid a_t = a_H)$ and $\mu_L = \text{Prob}(a_{t+1} = a_H \mid a_t = a_L)$. The game is, therefore, parametrized in part by the transition probabilities, μ_H and μ_L and by k and λ. Note that the restrictions on μ_i imply that it is weakly more likely to obtain a value of a_i in the next period given a_i is realized this period. Note, also, that the limiting case $\mu_H = 1 = 1 - \mu_L$ is the special case in which a realization of a in the first period fixes a for the rest of the game. The case $\mu_H = \mu_L = \alpha_0$ generates a simple repeated static game in which information about a in the current period conveys no information about a's value in the next period.

The model can be interpreted as one in which the seller faces a monopsonistic market with a competitive fringe. The strategic buyer might be a wholesaler (for example, an exporter) who can sell the product to a separate market at a privately known price, a_i, but who incurs a cost $.5q^2$ in addition to the per unit cost, $p_t q_t$, paid to the monopolist supplier. The variation in a may be explained by uncertainty as to whether the buyer is subject to some per unit cost or not or uncertainty about which of two markets he will ultimately sell to. The presence of a substantial non-atomic market justifies, in part, the restriction on the seller's strategy space -- that is, her inability to choose non-linear pricing schemes to extract more surplus. In general, she prefers to charge the strategic buyer a lump sum fee as well, however, the presence of the non-strategic fringe makes it difficult for the seller to prevent the buyer from avoiding such a charge by purchasing the object from the passive buyers. It also provides some convenient constraints on the size of the seller's price space which will be exploited in the equilibrium analysis.

The seller knows only a (posterior) distribution represented by α_t, the probability that $a = a_H$ in period t. Her initial prior is $\alpha_0 \in (0,1)$. The per period 'demand curve' is, of course, $q(p_t;a) = \lambda A + (1-\lambda)a - p_t$ were all agents to act naively.[2] It is in this sense that the seller knows the shape of the 'demand curve' (that it is linear with a known slope) but knows only that the intercept a may be high or low.

A restriction on the intercept of the non-strategic buyers and their weight in the market is embodied in Assumption A1. Let $A = ma_H$, $\mu = .5(\mu_H+\mu_L)$, and $\Delta\mu = \mu_H-\mu_L$, $n = (\mu_H+(1-\mu_H)k)$,

Assumption A1:

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[2] The demand curve actually has a kink in it of course. Restrictions on the values that a can take will ensure that this kink will be too high on the demand curve to matter.

$$.5(\lambda A+(1-\lambda)a_H) \leq x^* = \frac{1}{1} \frac{-}{-} \frac{(n-\lambda m/2-(1-\lambda)/2\mu)(1-\lambda)\Delta\mu}{(1-\lambda)(1-\lambda)(1-\mu)\Delta\mu/2}$$

The need for Assumption A1 comes out of the equilibrium analysis and will imply that in no stage of the game will the seller offer a price for which the low-type's equilibrium demand is zero. A1 rules out a fairly small set of parameters. However, the case where there are no passive buyers and first period information is perfect ($\mu_H = 1-\mu_L = 1$) violates the assumption.

It is straightforward to analyze the model when the buyer acts non-strategically. What happens when the buyer recognizes that his quantity choice in any period affects the seller's future prices? The buyer now must weigh the benefits of consuming his 'optimal' per period quantity against the costs of revealing information to the seller. The seller, in turn, must determine the information content of a given quantity purchased and choose prices in the awareness that the buyer is behaving with a twofold purpose. The seller's strategy consists of a sequence of functions σ_i from the history of past prices and quantities purchased to a current period price. The buyer's strategy is a sequence of functions from the past history of price offers, quantities demanded and current price to a current quantity demanded. Since the model is a dynamic game of incomplete information, the natural solution concept to examine is perfect Bayesian equilibrium. It should be clear, though, that the richness of the strategy space of the informed agents will generate a large set of pBe outcomes.[3] An aim of this paper is to examine the implications for this model of exploiting the belief-based refinements of sequential equilibrium. It will be seen that the game can be recursively segmented into what may be thought of as two-period

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[3] The large scope of pBe outcomes will be evident in the analysis of Section III.

signalling games. Results on the use of belief restrictions in such games are exploited to characterize the unique equilibrium which satisfies the restrictions of the intuitive criterion of Cho and Kreps(1987) in any subform of the T-period game.

Section II: Connected Research

The bilateral monopoly model is related to three separate strands of literature. There are obvious connections with research on dynamic trading games for durable goods in bargaining theory and the theory of dynamic monopoly. Such games can be thought of as true dynamic games. A seller attempts to obtain a high price for a single indivisible good. Once trade occurs, the game ends. The game in this paper formally corresponds to a repeated game with incomplete information. Trade in one period is generally followed by trade in subsequent periods. In durable goods bargaining models with one-sided uncertainty, as negotiations proceed, an uninformed player gains successively better information about the sort of player he is facing.[4] In the model of this paper, however, trade is followed by more trade. A reasonable conjecture might be that since information is more harmful to the buyer, the revelation of information will occur more slowly, if at all. In fact, this paper shows that in general, information is revealed immediately -- in each period, buyer types separate completely to reveal their information.

The model is also an extension of research on the behaviour of a monopolist who faces an unknown demand curve. (See McLennan (1984), Aghion, Bolton,Julien (1986), and Grossman, Kihlstrom and Mirman (1977)). The

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[4] See Fudenberg, Levine,Tirole (1986), Gul Sonnenschein, Wilson (1986), Vincent (1988) for examples of this characteristic in these models.

typical approach in this area has been to assume that the monopolist faces a non-atomic or non-strategic market. Buyers choose quantities in any given period without taking into consideration the effects this will have on future prices. In a market where large buyers are also present, this assumption is not very palatable. The strategic choice of quantities in order to alter the monopolist's beliefs should be anticipated.

Finally, the model is closely related to analyses of the ratchet effect in repeated adverse-selection problems. (See Aron (1987), Freixas, Guesnerie and Tirole (1985) and Hart and Tirole (1988)). Freixas, Guesnerie and Tirole examine the problem of a central planner choosing a linear incentive scheme to induce firm which is privately informed about its cost structure to produce the socially optimal level of output. The interaction occurs for two periods and they find that, in general, the principal is forced to offer a more generous first period contract in order to extract the information from the low cost firm. However, they also find that often information is only imperfectly revealed or not revealed at all until the last period when privately informed agents no longer need worry about the effects of revealing their information to the principal. The strategic power of the principal in their model suggests why information is often hidden. Once the principal determines the agent's private information, he is able to extract all the surplus from the remainder of the game. While the payoff structure is different in the model of this paper, it should be clear that the likelihood of information revelation may be higher in the dynamic bilateral monopoly case. Even when the monopolist supplier knows the buyer's information for sure, she is forced to share some surplus due to the limits on her pricing scheme. On the other hand, the model here also examines trading relationships which may endure for more than two periods -- the payoffs to a buyer from pretending to be other than he truly is may make hiding information more attractive.

Hart and Tirole (1988) study a many (more than two) period non-durable goods trading relationship. Their motivation is to examine the effects of renegotiation on both durable and non-durable goods relationships. However, in the process they provide an analysis of a very simple version of the strategic buyer versus a supplier of a rental good. In their model, the buyer has an inelastic demand for a single unit of the good each period but has a reservation price which, ex ante, may take one of two values. The seller would like to discover this value. They find that there is essentially no learning in this game. For large, finite period games, high types pool with low types for almost every period. Only in the final two periods does any separation occur. Again, this result is partly intuitive because if a seller learns the buyer's type at any point, the buyer gains zero surplus from then on as the seller simply charges the reservation price each period. The severe punishment for revelation in this model might be driving the result. Furthermore, the high-type buyer incurs no cost by pretending to be a low-type. On the contrary, he gains the object for a lower price. In the model examined here, both of these features are absent. Even if the buyer's type is known exactly, the buyer will gain some surplus through the rest of the game. The buyer <u>can</u> act as if he were a lower type, but he must do so by foregoing some current trading opportunities. Furthermore, since the buyer may choose any level of quantity to purchase, the signalling space of the informed agent is larger.

In light of the previous research, a number of questions can be addressed. Is information revealed more slowly in the strategic model and is there ever a point at which the monopolist learns the buyer's type exactly? Is the seller forced to shade her price even further relative to the non-strategic model in order to induce the buyer to reveal his information? What are the efficiency effects of the buyer's behaviour? The equilibrium analysis suggests that for a large family of such games, information is fully revealed in each period. Nevertheless, the revelation of such information, fails to eliminate inefficiencies even as the game progresses.

Section III: Equilibrium Analysis

The equilibrium analysis of the model is simplified greatly by the assumption that the game is of finite periods. Each period is made up of two stages -- the first stage when a price offer is made and the last when a quantity is chosen. In the final stage of period T, the behaviour of the buyer is fully determined by sequential rationality. A buyer of type a chooses $q_T = a - p_T$. Given this behaviour and given any seller belief α_T, the seller's last period price offer is also fully determined, $p_T = .5((1-\lambda)(\alpha_T a_H+(1-\alpha_T)a_L)+\lambda A)$. The trick, now, is to observe that the overlapping stages consisting of the last stage of period T-1 and the first stage of period T constitute, formally, a two-stage signalling game parametrized by a prior seller belief, α_{T-1} and an outstanding seller price offer, p_{T-1}. The informed agent chooses a signal, q_{T-1}, the uninformed agent forms a belief, α_T, and then takes a response, p_T, conditional on this belief. Games of this structure have been analyzed extensively in the literature on the refinements of sequential equilibrium and the paper exploits these results to narrow the set of equilibrium outcomes.

Consider the last stage of period T-1 when a price, p_{T-1}, is outstanding. The informed buyer must decide on a 'message', q_{T-1} to send to the seller which will, among other things, help to determine a price offer in the next period. The appendix shows that with the payoffs for this model, the preferences of a high-type buyer in period T-1 over the space (q_{T-1},p_T) vary in a systematic way from those of a low-type. In particular, in the region of possible equilibrium outcomes, the slope of the high-type's indifference curves are steeper than those of the low-type (See Figure One). The highest indifference curve I_H represents the worst that the high-type can get in any equilibrium of the continuation game. Clearly, a pBe may be pooling, semi-pooling or separating. A pooling equilibrium is represented by an outcome with p_T strictly in the interval (p_L,p_H) -- p_L is the last period price when the seller believes that the type was low in period T-1 for sure and p_H the price when she believes the type was high for sure. To support such an equilibrium as a pBe the beliefs of the seller when a deviant quantity is offered must be specified. It should be clear that, in general, beliefs do exist to support a vast range of such outcomes.[5] However, a simple and plausible restriction on the seller's updating rules suffices to rule out all non-separating equilibria.

Fix a candidate pooling outcome, B in Figure One and consider the seller's beliefs and subsequent response when a quantity such as q' is demanded. To support B as an equilibrium, the seller must believe that the high-type was sufficiently likely to have deviated as to induce the seller to offer a price above I_H'. The application of the intuitive criterion of

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[5] Simply set beliefs conditional on a deviation to put a high or full weight on the type having been high.

Cho and Kreps seems natural here. Notice that there is no belief that the seller can have which would generate a best response that yields the high-type a higher payoff following the deviation than does the proposed equilibrium B. However, if the seller believed that the low-type deviated, her subsequent best response does, in fact, justify the deviation. The intuitive criterion requires the updating rule following the deviation q′ to put zero weight on the high-type. Such a rule, though, breaks the pooling outcome since the low-type is strictly better off demanding q′. The proof of Theorem One in the appendix shows that the outcome in which the high-type chooses his static optimum and the low-type chooses either his static optimum or a quantity such as q_L in Figure One (depending on the parameters of the game) is the only pBe which survives the restriction imposed by the intuitive criterion. The actual value of q_L, it turns out conveniently, is linear in the current price, p_{T-1}. Furthermore, it is independent of the seller's prior, α_{T-1}.

The equilibrium for the whole game is characterized by adopting the analogy of subgame perfection. The T-period equilibrium is required to satisfy the belief restriction in any subform of the game when the continuation path following the seller's next period price offer is replaced by the expected buyer payoffs when they follow the strategies determined by the (in our case, unique) continuation equilibrium.[6] In the Appendix it is shown that for any period t, if the buyers expect to separate in period t+1, their preferences over (q_{t+1}, p_{t+1}) again correspond to those of Figure One. Similar arguments are applied to this subform to isolate separation there as

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[6] In what follows, by 'equilibrium' is meant a perfect Bayesian equilibrium which satisfies this restriction on beliefs.

well. Furthermore, the demand of the low-type is again linear in the current period price and independent of α_t.[7]

The use of this technique imposes strong restrictions on the seller's beliefs. In effect, it requires the seller, once a deviation is observed, to ignore all events earlier in the game, and to consider only the consequences that a current deviant signal has for her future behaviour. The stochastic behaviour of the variable a provides a foundation for this form of updating. The positive probability that the buyer's type may have changed between the past and the present grants the seller the freedom to attribute a deviation to either type. Updating then proceeds by introspecting on which type now has the greater incentive to signal. The equilibrium behaviour generated by this type of belief formation yields an extremely simple form of strategies as shown in Theorem One.[8]

Theorem One: Fix μ_H, μ_L, λ, k such that A1 is satisfied.There exists a monotonic, bounded sequence, $\{a_{Lt}\}$, for t from 0 to T such that the unique equilibrium outcome is characterized by buyers following the strategy: for any p_t, if $a_t = a_H$, demand $a_H - p_t$, if $a_t = a_L$, demand $a_{Lt} - p_t$.

Proof: See Appendix.

If the low-type buyer's value of a is low, $k \epsilon (p_H, x^*)$, then the high and low types separate at no cost in each period. However, if the low-type buyer's value of a is fairly close to a_H, $k > x^*$, the low-type must

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[7] In finite two-period signalling games, if the equilibrium satisfying the intuitive criterion is unique then it is also the unique outcome corresponding to the stable component of the game. It is not known whether in multi-period games, the satisfaction of this restriction subform by subform generates a similar correspondence.

[8] Similar refinement techniques are used in Noldeke and van Damme (1987) and (as one of a number of possible outcomes) in Bagwell and Riordan (1988).

underdemand in each period $t < T$ to ensure that the seller does not believe that he is a high-type pretending to be a low type. Furthermore, the longer the game (the higher is T) the more the low-type is forced to underdemand earlier on in the game. Theorem Two illustrates the effects of this behaviour:

Theorem Two: Let $k > x^*$. As T becomes large, a_{L0} converges monotonically to x^*.

Proof: Fixing $k_T = k$, k_t is determined recursively by $k_t = f(k_{t+1})$ and $a_{Lt} = k_t a_H$. The function f(x) is given by

$f(x) = 1 - [((2n-\lambda m-(1-\lambda)\mu)-(1-\mu)(1-\lambda)x)(.5)(1-\lambda)(1-x)\Delta\mu]^{.5}$.

Note that $f(1) = 1$ and $f(x^*) = x^*$. For x between x^* and one, $f(x) < x$ and $f'(x) > 0$. These facts give us the limit in the Theorem (see Figure Two). ||

If $k > x^*$, the equilibrium of a T+1-period game can be characterized by drawing a hypothetical sequence of T demand curves with unit slope and with intercept a_{Lt} below a_L and above x^* (see Figure Three). In any period t, if the type is a_L in period t, the buyer acts as if his demand curve was $a_{Lt} - p_t$. As the game approaches the final period, the low type is required to underdemand less since the advantages to the high type of mimicking the lower type are correspondingly lower. The low-type buyer is constrained from demanding off his true demand curve because he must consider the effects such a demand will have on the future behaviour of the seller. If the seller observes an out-of-equilibrium higher quantity demanded following behaviour corresponding to a low-type earlier in the game she must form a conjecture about the deviant. The shadow demand curves are constructed in such a way that the set of possible best responses the seller might offer which make the high type made better off is strictly larger by set inclusion than those which make the low type better off. In this sense, the temptation to deviate is greater for the high type.

Observe that the equilibrium isolated in the component two-stage signalling game is also the perfect sequential equilibrium (see Grossman and Perry (1986)) when it exists. However, an argument reminiscent of that showing non-existence of equilibrium in the Rothschild-Stiglitz competitive insurance market shows that a pse may not exist in many realizations of the signalling game. Refer to Figure Four. The points (A,B) represent the intuitive criterion equilibrium; the price p* represents the next period optimal price if the seller obtains no information from this period's behaviour. For (A,B) to be a pse, it must be the case that there does not exist an out of equilibrium message such that if the seller takes a best response conditional on the belief that a set K deviated, exactly that set is made better off by having deviated. Obviously, a message such as $a_L - p_t$ violates this requirement since if the seller believes that each type had an equal likelihood of deviating, the response p* is rational on her part and it yields both types of buyers a higher payoff. However, a similar argument breaks $(a_L - p_t, p^*)$ as a potential pse. Notice that the intuitive criterion, since it does not put such a stringent condition on how the updating rule is formed is not broken by such an argument -- there exists a best response on the seller's part which would justify a belief which puts high weight on a high type, and, so, the belief passes the test of the criterion.[9]

[9] Notice that if we allow the transition parameters to be zero or one so that the information of the buyer is fixed throughout the game, we must confront the question of increasing support. That is, off the equilibrium path, beliefs must be such that the seller goes from believing with probability one that the buyer is of a low-type to believing with some probability that he is a high-type buyer. Given the characteristic of the equilibrium path, I feel that such a construction does not gravely violate

Section IV: Observations

The equilibrium singled out by Theorem One exhibits some striking characteristics. First, of course, is the feature that, in each period, all private information is fully revealed -- buyer types who observe a_L in the t'th period demand a quantity strictly less than those who observe a_H. At the end of any given period, the seller and the buyer know all the available information in the game. Only when new information is generated at the beginning of the next period, does asymmetric information re-emerge.

Second, in the multi-period game, the so-called ratchet effect is still present although in a different sense than in Freixas, Guesnerie and Tirole. In their model, the principal is forced to offer a more generous scheme in order to induce information revelation. In the case of the bilateral monopoly, the informed agents separate for <u>any</u> price offer of the seller. However, it is this separating behaviour which forces the seller's price offer to be lower than in a static price setting problem. For any given belief of the seller, her optimal price is higher with the same belief as the game nears the final period.

Third, note that even though information is fully revealed, screening costs are incurred throughout the game. In particular, consider a game in which a realization of a_i today leads to (almost) perfect information that a_i will be realized tomorrow. In the extreme, where $\mu_H = 1-\mu_L = 1$, after the first period, both the buyer and the seller know all the relevant information for the rest of the game. Nevertheless, the equilibrium strategy of the low type is to underdemand for the remainder of the game (except the

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our intuition about belief formation but the matter is clearly a controversial one and not one that lies within the scope of the paper.

final period). He acts as if he had a demand curve with a strictly lower intercept. Given this behaviour, the seller can do no better than to post a lower price. The buyer who is informed that he is of a low type signals this to the seller in the first period but is 'forced' to continue to convince the seller throughout the game.

The welfare effects of this model are, in general, difficult to analyze since the framework is inherently monopolistic and because of the presence of the non-strategic buyers. The stationary character of the equilibrium behaviour, however, suggests one natural comparison. Since, in equilibrium, information is completely revealed in each period, it is interesting to compare the results here with those of a similar model where the buyer has the same preferences which follow the same stochastic process but acts non-strategically. Notice first, that if $k < x^*$, then high- and low-type buyers simply choose their naive optimal quantity $a - p_t$ so the two environments yield identical results. For the case in which $k > x^*$, though, the results are ambiguous. To see this, consider any given period, t. For any prior of the seller, α_t, the price offer of the seller is lower in the strategic game. If the true state is high, the lower price is a straight gain to the buyer. When the true state is low, the lower price is a benefit but the buyer is also forced to underdemand relative to his true demand curve. Which effect dominates depends on how low a price the seller is forced to offer. If the non-strategic portion (λ) is relatively low and if the seller's prior places relatively high weight on the state being low (α small), then the net effect on the low type is positive as well. Both types benefit from the strategic behaviour. For these parametrizations, the buyers is made unambiguously better off.

This result is interesting because it represents a situation in which informed types are in a position in which they are forced to separate and the separation benefits both types. In a typical screening model, some types of the informed players are usually forced to incur screening costs to separate themselves from other types. Here the screening can bestow an advantage. In a static or non-strategic game, a low type would also prefer it if he could convince the seller to offer a lower price by committing to demand a lower quantity. In general, the technology to enforce such a commitment is lacking. Here, though, the low type has a credible concern that the seller will mistake him for a high type in the remainder of the game. The concern serves as a credible commitment device and allows him to induce a lower price from the seller to shift some of the surplus from the trade in his direction.

The direct characterization of a_{Lt} make comparative statics analysis straightforward. We can examine how the behaviour of the types change as time preferences change (δ), as their information becomes better ($\Delta\mu$), and as the role of the strategic agents in the market changes (λ). The results are encapsulated in Propositions One to Three:

Proposition One: Let $k > x^*$. For any T, for any period t, a_{Lt} is lower the higher is δ.

Proposition Two: Let $k > x^*$. For any T, for any period t, holding $\mu = \mu_H + \mu_L$ fixed, a_{Lt} is lower the higher is $\Delta\mu$.

Proposition Three: Let $k > x^*$, $n > m$. For any T, for any period t, a_{Lt} is higher the higher is λ.

Proof: The proofs follow directly by observing that $a_{Lt} = f(k_{t+1})a_H = k_t a_H$ and differentiating f(.) with respect to the parameters of interest. ||

The results are not surprising. First, observe that in equilibrium, Theorem One shows that the behaviour of the high type is unaffected by variations in these parameters. The high types always demands $q_t = a_H - p_t$. Also, if $k < x*$, the same is true for the low type, $q_t = a_L - p_t$. However, the behaviour of the low-type depends on the various parameters via the function f if $k > x*$. In any period, t, as δ rises, a_{Lt} falls -- the low type is forced to underdemand more, the more important is the future. This is because it is then more attractive for high types to pretend to be low types.

As the share of the non-strategic market rises, a rise in λ, the amount demanded by the low types also rises. The presence of a large non-strategic market reduces the beneficial effects to the high type of a seller belief that the type was actually low and the costs to the low type of separating are correspondingly lower. Finally, holding μ fixed but allowing $\Delta\mu$ to increase represents an increase in the precision of current period private information. As $\Delta\mu$ rises, an agent who learns that he is of type i today has a higher probability of being of type i tomorrow. This also increases the cost of screening since a seller belief that the type is low this period induces a higher seller belief in a low type in subsequent periods.

The presence of the non-strategic buyers in the market provides a partial explanation for the complete revelation of information. Technically, the presence of these buyers allows the signalling space to be large enough for the two types of buyers to separate in each period. In the absence of such buyers, if the two buyer types are different enough (a_L low) and if the game lasts long enough (T large) the incentives for the high types to pretend to be low types are too great to screen out with positive demand quantities. To see this, put $\lambda = 0$ in the equation of Theorem Two to get the lower bound on a_{Lt} to be $a_H/3$. A high seller belief α_t close to one would generate a price offer close to $a_H/2$ and the proposed equilibrium strategy for a low type would be to demand zero. Such a demand is not enough to deter high types from mimicking the low types. The characterization of behaviour under these conditions has not proved to be tractable since it appears that some mixing on the part of high types throughout the game will be required. Furthermore, since the low-types demand curve is very low, the kink in the market demand curve becomes relevant -- the sellers optimization problem is no longer concave, her best response correspondence may not be convex and she may also be forced to mix in equilibrium. In such cases, it seems reasonable to expect that the private information will be leaked out more gradually in the game. Furthermore, it is possible that, as was the case for Freixas, Guesnerie, Tirole, the seller may have some latitude in determining the rate at which information is revealed by the appropriate choice of prices.

Section V: Conclusions

The bilateral monopoly game provides a simple first step towards understanding dynamic incentives in adverse selection problems. Although, the model is restrictive, some justification can be given in addition to the overriding one of tractability.

Although, assumption A1 seems natural in many situations, it is obviously restrictive. It rules out, for example, many environments in which a single seller confronts a single buyer. An analysis of the consequences of relaxing this assumption is of some interest, since it can provide insights into alternative processes of information transmission in dynamic trading games and the role for pooling.

The restriction of the support to only two points can be explained in various ways. The buyer may or may not be subject to certain costs, he may resell on market H or market L, his preferences may just be one of two types. However, the restriction is by no means innocuous. The analysis of the equilibrium suggests that with two types, if k is in the interval $(x^*, 1)$, the low-types behaviour converges to a demand curve with intercept $x^* a_H$. If there are three types, the equilibrium analysis suggest that the behaviour of the lowest type will converge on $x^{*2} a_H$, if there are four, on $x^{*3} a_H$ and so on. The more the number of types, the more costly the screening behaviour, more or less independent of how different the types are (if the number of periods in the game is large). The impact of an assumption such as A1 will become more stringent and the examination of behaviour when it is violated more urgent. The problem is clearly relevant for more general classes of models. Pure separating equilibria are more difficult to support with many types in games which last for many periods since the advantages of cheating are usually greater and the size of the signalling space in any given period no larger. The resolution of this question is certainly an important one for future research on dynamic signalling games.

It should be clear that the equilibrium outcome isolated in the paper will remain an equilibrium as T goes to infinity. It is also clear from previous work on repeated games that many other outcomes may also be supported as equilibria in the infinite horizon case. Until more progress is made on narrowing the scope of equilibrium outcomes in infinitely repeated games, the problem of predicting outcomes in this case remains open.

The results suggest, though, that for a large class of bilateral monopoly games, the signalling space is large enough for informed buyers to separate in each period. From the standpoint of the strategic players even if this separation conveys a great deal of information, it does not relieve the players of the burden of separation in subsequent play of the game. Some transactions are not consummated even if the players are virtually certain that they should be. The disinterested partner shows his true colours in the first period and proves it over and over for the rest of the relationship by demanding less of his partner than he truly desires. Playing hard to get can result in the persistence of an under-requited love.

Appendix One

The characterization of the equilibrium is preceded by a few useful facts.

Fix $p_t \le a_H/2$, $\alpha_t \epsilon (0,1)$, $a_{Lt+1} \le a_L$, $a_{Lt+1} \ge p_H$, $K_{Lt} \ge 0$, $K_{Ht} \ge 0$, and consider preferences over the space (q,p) represented by the functions

$$V(q,p;a_H,p_t) = (a_H - p_t - .5q)q + \mu_H(.5(a_H - p)^2 + K_{Ht}) + (1-\mu_H)((a_L - p - .5(a_{Lt+1}-p))(a_{Lt+1}-p) + K_{Lt}). \quad (H')$$

$$V(q,p;a_L,p_t) = (a_L - p_t - .5q)q + (1-\mu_L).5(a_L - p - .5(a_{Lt+1}-p))(a_{Lt+1}-p) + K_{Lt}) + \mu_L(.5(a_H - p)^2 + K_{Ht}). \quad (L')$$

Observe that these may be rewritten as

$$V(q,p;a_H,p_t) = (a_H - p_t - .5q)q + .5(\mu_H(a_H - p)^2 + (1-\mu_H)(a_L - p)^2) + M_{Ht} \quad (H)$$

$$V(q,p;a_L,p_t) = (a_L - p_t - .5q)q + .5(\mu_L(a_H - p)^2 + (1-\mu_L)(a_L - p)^2) + M_{Lt} \quad (L)$$

The slopes of the indifference curves generated by these payoff functions are given by

$$dp/dq(a_i) = (a_i-p_t-q)/(a^*_i-p), \text{ where } a^*_H = \mu_H a_H+(1-\mu_H)a_L \text{ and}$$
$$a^*_L = \mu_L a_H+(1-\mu_L)a_L.$$

The following two facts are straighforward implications of these equations:

Fact One: If $p \le p_t+q+a^*_i-a_i$, the indifference curves are convex and the slope of the curves is less than one.

Fact Two: If $(a_H a^*_L - a_L a^*_H - p(a_H-a_L) + (p_t+q)(a^*_H-a^*_L)) > 0$, then the slope of the H indifference curve is greater than the slope of the L curve.

Fix $q_H = a_H-p_t$ and $p_H = .5((1-\lambda)(\mu_H a_H+(1-\mu_H)a_{Lt+1})+\lambda A)$ and define the set $QP_H = \{(q,p) | V(q,p;a_H,p_t) \ge V(q_H,p_H;a_H,p_t)\}$, that is all the points which are preferred by a high-type buyer to (q_H,p_H). Facts One and Two can be combined to yield

Fact Three: If $(q,p) \epsilon QP_H$, indifference curves are convex and the slope of the H curve is greater than that of the L curve.

The next Lemmas concern a two-stage signalling game with $p_t \le a_H/2$, $\alpha_t \in (0,1)$, $a_{Lt+1} \le a_L$, $a_{Lt+1} > p_H$, $K_{Lt} \ge 0$, $K_{Ht} \ge 0$ all fixed and where an informed player chooses the signal q and the uninformed player responds with action p. Expected payoffs for the informed player are given by (H) and (L) while the payoff of the uninformed player is given by $U = p((1-\lambda)(\mu_H(a_H-p)+(1-\mu_H)(a_{Lt+1}-p))+\lambda(A-p)) + N_H$ if the current state is H and $U = p((1-\lambda)(\mu_L(a_H-p)+(1-\mu_L)(a_{Lt+1}-p))+\lambda(A-p)) + N_L$ if the current state is L. If, after observing the quantity demanded in the first stage, the seller's beliefs are that the state was H with probability α_{t+1}, the seller's best response is given uniquely by $p(\alpha_{t+1}) = .5[\lambda A + (1-\lambda)(\alpha_{t+1}(a^*_H)+(1-\alpha_{t+1})a^*_L)]$.

Lemma One: In any sequential equilibrium of the signalling game, for any value of the fixed parameters, if (q,p) is an outcome which occurs with positive probability for a high type, then $(q,p) \in QP_H$.

Proof: Sequential rationality imposed on the seller's response implies that the worst that a high type can do is bounded below by (q_H,p_H) which implies the Lemma. ||

Let $p_L = .5((1-\lambda)(\mu_L a_H+(1-\mu_L)a_{Lt+1})+\lambda A)$ and q_L satisfy $V(q_L,p_L;a_H,p_t) = V(q_H,p_H;a_H,p_t)$. Note that since a_{Lt+1} is time dependent, so too is p_L. The t subscript is suppressed.

Lemma Two: Suppose $q_L < a_L-p_t$. In any equilibrium of the signalling game, if (q,p) is an outcome which occurs with positive probability, then $(q,p) \in QP_H$.

Proof: Suppose not. Then it must be the case that in equilibrium only L types choose q. Therefore, $p = p_L$. Now suppose that a quantity q' is chosen such that $V(q',p;a_H,p_t) < V(q_H,p_H;a_H,p_t)$ and $q' > q$. Since there does not exist a best response of the seller conditional on any posterior belief which would yield the high type a better payoff than the equilibrium payoff and since (q',p) is strictly better for the low type than (q,p), the seller must believe that the low type deviated and sent q'. But his subsequent response of p breaks the proposed (q,p) equilibrium. ||

Lemma Three: Let $q_L < a_L - p_t$. Then $\{(q_L,p_L),(q_H,p_H)\}$ is the unique equilibrium outcome of the two-stage signalling game.

Proof: Refer to Figure One and note that points A and C represent $(q_L,p_L),(q_H,p_H)$ respectively. Suppose some other point (q,p) inside the set QP_H occurs with positive probability in some D1 equilibrium. Without loss of generality suppose the price p strictly exceeds p_L so that some pooling occurs at (q,p). Define q_L' and q_H' by $V(q_L',p_L;a_L,p_t) = V(q,p;a_L,p_t)$ and $V(q_H',p_L;a_H,p_t) = V(q,p;a_H,p_t)$. Facts One through Three imply that $q_L' < q_H'$. Now suppose some out of equilibrium message $q' \epsilon (q_L',q_H')$ is offered. Facts One through Three again imply that there is no best response of the seller following q' which is preferred by the high type to the proposed equilibrium outcome. If the deviation q' is followed by p_L, the deviation is strictly preferred by the low type. The equilibrium outcome (q,p) must be robust to an updating rule of the seller which puts zero weight on state H after observing q'. This implies a response of p_L to a message q' but since this outcome is strictly preferred by the low types to (q,p) it destroys its candidacy as an equilibrium outcome. It is straighforward to show that $\{(q_L,p_L),(q_H,p_H)\}$ can be supported as a equilibrium -- all messages q greater than q_L result in a seller belief with probability one that state H has occurred and therefore generates a subsequent price offer of p_H. Any message $q \le q_H$ can generate a belief that the state was L and the outcome will be supportable. ||

Lemma Four: Let $q_L > a_L - p_t$. The unique equilibrium is given by $\{(a_L - p_t, p_L), (a_H - p_t, p_H)\}$.

Proof: The proof follows the same lines as Lemma Three. ||

Observe that the definition of q_L implies that q_L is linear in p_t and independent of α_t. In particular,

$$q_L = \min\{a^*_H - [(2n - p_L - p_H)(p_H - p_L)]^{.5}, a_L\} - p_t.$$

$$= \min\{ f(k_t)a_H, a_L\} - p_t \text{ where } \mu = .5(\mu_H + \mu_L),\ \Delta\mu = \mu_H - \mu_L,\ n = \mu_H + (1-\mu_L)k,$$

$k_t = a_{Lt+1}/a_H$ and

$$f(k_t) = 1 - [((2n - \lambda m - (1-\lambda)\mu) - (1-\mu)(1-\lambda)k_t)(.5)(1-\lambda)(1-k_t)\Delta\mu]^{.5}.$$

Set $k_T = k$. Define k_i recursively by

$$k_i = \min\{f(k_{i+1}), k_{i+1}\}.$$

Note that $f'(x) > 0$, and that $f(x) = x$ at $x = 1$ and at

$$x^* = \frac{1 - (1-\lambda m/2 - (1-\lambda)\mu/2)(1-\lambda)\Delta\mu}{1 - (1-\lambda)(1-\lambda)(1-\mu)\Delta\mu/2}$$

Finally observe that for $x \in (x^*, 1)$, $f(x) < x$ and for $x \in (0, x^*)$, $f(x) > x$. Therefore, if $k_T > x^*$, the sequence $\{k_t\}$ converges monotonically to x^* and for $k_T < x^*$, $k_t = k_T$ for all t.

Set $a_{Lt} = k_t a_H$. In any two-stage signalling game, then, the strategy of the L buyer is given by $q_L = a_{Lt} - p_t$ and the expected utility of the buyers is given by

$$V_L = .5(a_L - p_t)^2 - (a_L - a_{Lt})^2 + K_{Lt} \text{ for the low type and}$$

$$V_H = .5(a_H - p_t)^2 + K_{Ht} \text{ for the high type.}$$

The simple linear best response functions of the two types of buyers in any two-stage signalling game parametrized by p_t, leads naturally into Theorem

One.

Theorem One: Let $a_L \geq .5a_H$ and $\delta = 1$. The unique equilibrium path of the T-period game is characterized by a monotonic sequence of numbers $\{a_{Lt}\}$, $t = 0,1,..T$, such that in any period, t, after any history h_t, a buyer of type L in period T facing a price p_t demands $q_t = a_{Lt} - p_t$. A buyer of type H in period t demands $q_t = a_H - p_t$. The seller in any period, t, with posterior beliefs α_t offers a price $p(\alpha_t) = .5(\alpha_t a_H + (1-\alpha_t)a_{Lt})$.

Proof: The response of the buyers in period T clearly satisfy the theorem as does the price offer of the seller. Consider the second stage of period T-1 when a price p_{T-1} is offered. For any seller belief, α_{T-1}, Lemmas One and Two show that the unique outcome is $q_L = a_{LT-1} - p_{T-1}$, $q_H = a_H - p_{T-1}$ and p_T equal to either p_H or p_L (calculated for $a_{LT} = a_L$). Therefore, for any belief of the seller in the first stage of period T-1, α_{T-1}, sequential rationality determines her best response price offer to be $p_{T-1} = .5((1-\lambda)(\alpha_{T-1}a_H + (1-\alpha_{T-1})a_{LT-1}) + \lambda A)$.

Now consider the period T-2. In period T-1, if the buyer is a high type, he chooses $q = a_H - p_{T-1}$, if he is a low type he will choose $q = a_{LT-1} - p_{T-1}$. Thus for buyers of high type in period T-2, facing a price p_{T-2}, preferences over q_{T-2} and p_{T-1} are given by

$$V(q,p;a_H,p_{T-2}) = .5[(a_H-p_{T-2})^2-(a_H-p_{T-2}-q)^2 + \mu_H(a_H-p_{T-1})^2 + (1-\mu_H)[(a_L-p_{T-1})^2-(a_L-a_{LT-1})^2] + K$$

and for low types in period T-2,

$$V(q,p;a_L,p_{T-2}) = .5[(a_L-p_{T-2})^2-(a_L-p_{T-2}-q)^2 + \mu_L(a_H-p_{T-1})^2 + (1-\mu_L)[(a_L-p_{T-1})^2-(a_L-a_{LT-1})^2] + K$$

(Notice that since $a_L > p_H$, individual rationality constraints will not bind in the space of equilibrium candidate price- quantity pairs.) Since these preferences are precisely those used in Lemmas One and Two and behaviour in period T-1 onward is independent of the history of period T-2, the same analysis applies, high and low type buyers completely separate in the third to last period. If the seller observes a quantity greater than $a_{LT-2} - p_{T-2}$, the equilibrium requires her to put relatively high weight on the probability that the buyer's type was high in period T-2 and her updated belief in period T-1 is (close to) $\alpha_{T-1} = \mu_H$. For any lower quantities, her belief is that the type was low in period T-2 and her subsequent belief is therefore $\alpha_{T-1} = \mu_H$. These beliefs uniquely determine her best response price offer in period T-1. The same arguments obviously apply to periods before T-2. The seller's initial price offer is determined by the initial prior α. That is, $p_0 = .5(\alpha a_H + (1-\alpha)a_{L0})$. ||

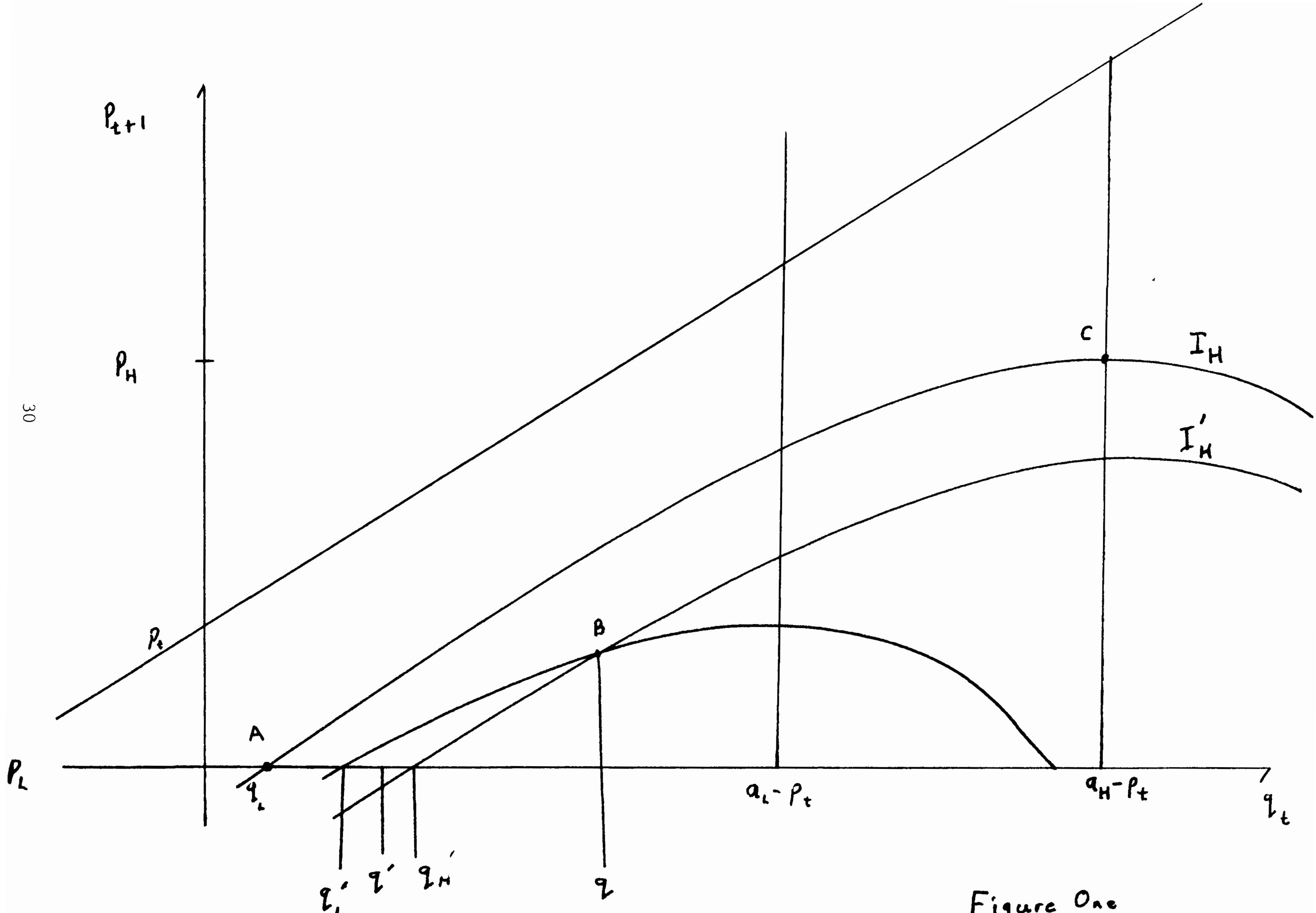

Figure One

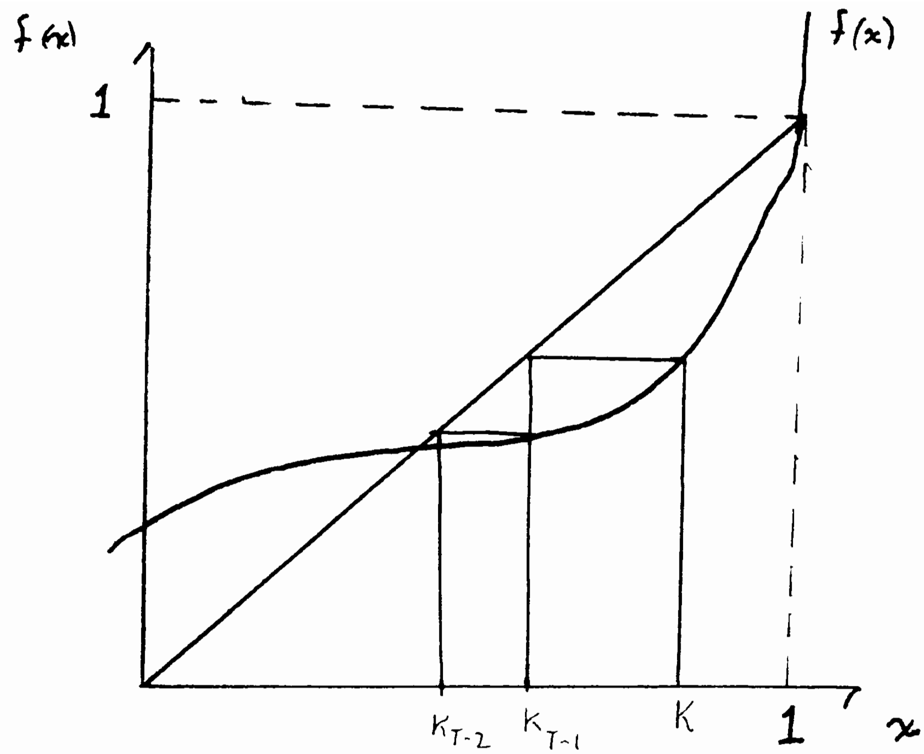

Figure Two

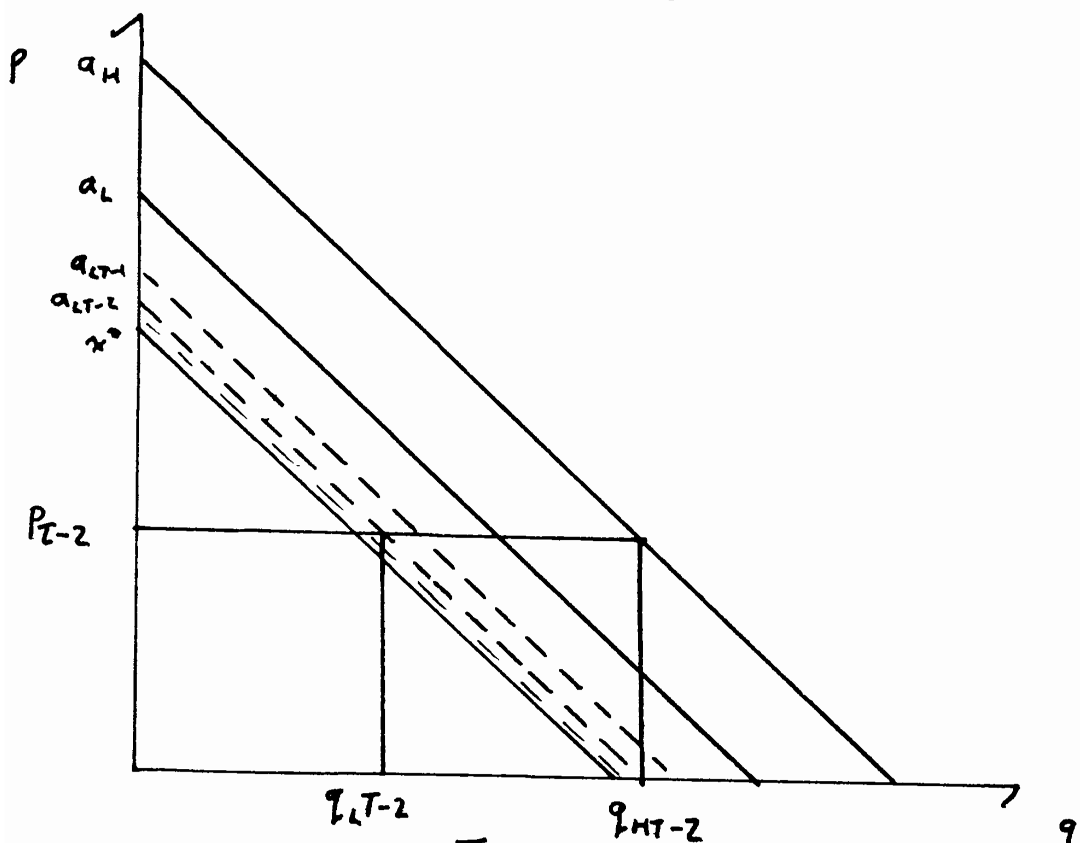

Figure Three

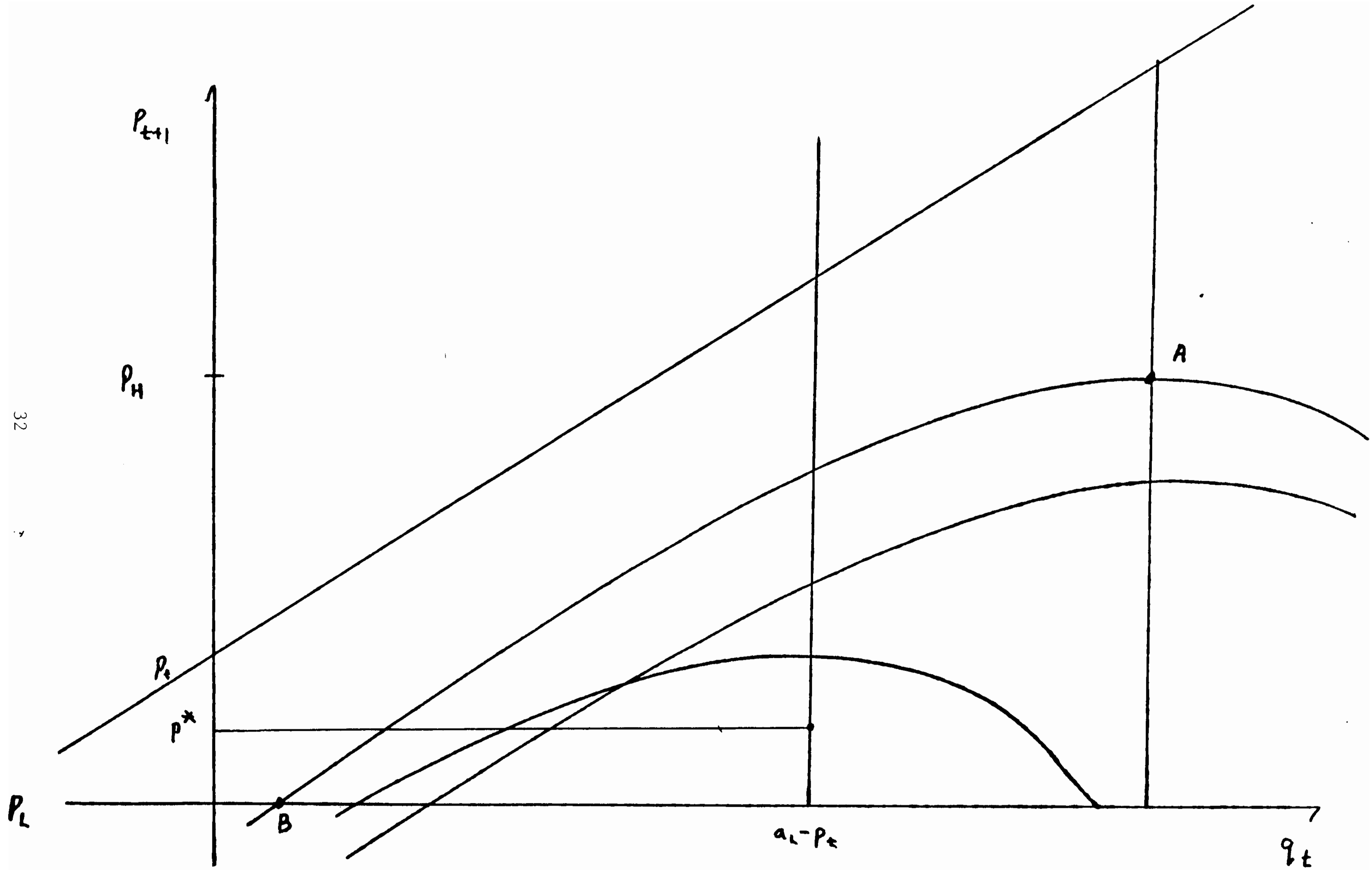

Figure Four.

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