

Discussion Paper No. 831

COORDINATION FAILURE AND LONG RUN GROWTH

by

Hiroshi Osano*
Shiga University

April 1989

Abstract

This paper explores the long run implications of the coordination problem between the firm and workers which arises from a shared accumulation process of the stock of knowledge in the stage of production. Coordination failure emerging from the accumulation process of the stock of knowledge can produce multiple, Pareto-ranked equilibrium balanced growth paths in which identical economies can converge to steady states with different rates of growth. A reform of the internal organization of firms can increase the long run growth rate of an economy not only by improving the efficiency of production but also by changing the expectations of one agent about the other agents' input decisions in the accumulation process of the stock of knowledge.

*The author is currently visiting the Department of Economics,
Northwestern University, Evanston, IL 60208, U.S.A.

1. Introduction

The standard neoclassical growth model such as Solow's (1956) predicts that the growth rates in per capita income across otherwise identical closed economies will eventually converge even if these economies start off with different per capita stocks of physical and human capital. However, as noted by Kaldor (1961), Romer (1988) and De Long (1988), there are substantial cross country variations in average growth rates in per capita income. The only way to account for this empirical fact in the standard neoclassical growth model is to introduce cross sectional differences in exogenous technological progress rates; but a successful theory of economic development needs a model in which cross country differences in growth patterns are endogenously determined.

Recent work by Romer (1986), Lucas (1988), Rebelo (1987), Azariadis and Drazen (1988), Barro (1988), and King and Rebelo (1988) has explored the implications of the class of endogenous growth models pioneered by Uzawa (1965) in which the long run growth rate is an endogenous outcome of time-invariant technology and preferences. The endogenous growth model of Romer (1986) stresses three elements: externalities of new knowledge, decreasing returns in the production of new knowledge, and increasing returns in the production of output. His model predicts that growth rates accelerate over time, rather than constant as in any steady state model, and that large countries may always grow faster than small countries. On the other hand, the endogenous growth models developed by Lucas (1988), Rebelo (1987), Azariadis and Drazen (1988), Barro (1988), and Rebelo and King (1988) do not possess any form of increasing returns to scale. Instead, their models investigate a case of constant returns to scale with linear production of human capital or the

"primary capital good". In these models, the possibility of wide and sustained differences in growth rates in per capita income across countries can be explained by production patterns with learning-by-doing technology (Lucas), influences of public policy (Rebelo, Barro, Rebelo and King) or technological externalities with threshold features (Azariadis and Drazen).

The purpose of this paper is to give an alternative, complementary explanation for the possibility of wide and sustained differences in growth rates in per capita income across countries, and to discuss which forces affect long run growth rates in per capita income. To achieve this purpose, we consider the problem of coordination among input suppliers to a shared accumulation process of the stock of knowledge in the stage of production.

The accumulation process of the stock of knowledge inside production firms includes a broad range of activities such as on-the-job training, basic scientific research, product development, and innovation in systems and management. The stock of knowledge generated by the accumulation process can contribute to the production of goods in several ways. First, the accumulation of some part of the stock of knowledge increases the stock of labor in efficiency units because this part of the stock of knowledge is embodied in human capital of workers. Second, another part of the stock of knowledge directly increases the production efficiency of firms, for this part of the stock of knowledge is embodied in some kind of tangible capital such as conventional physical capital.

To capture the basic idea of the problem of coordination in the accumulation process of the stock of knowledge inside firms, we assume that the accumulation process of the stock of knowledge needs labor inputs, both from management (capitalists) and from workers. Then "coordination failure"

arises from the inability of capitalists and workers to coordinate their actions successfully in a many-person, decentralized economy.¹ Because of technological complementarities, capitalists and workers could benefit from a collusive agreement to allocate more labor inputs in the accumulation process of the stock of knowledge. Although this agreement would be Pareto-improving, each agent would have an incentive to shirk, not allocating its stipulated labor input in the accumulation process of the stock of knowledge. These features in the accumulation process of the stock of knowledge can generate multiple, Pareto-ranked balanced growth paths so that identical economies may converge to steady states with different rates of growth.²

The paper is organized as follows. Section 2 describes the basic environment under study and presents dynamic competitive market equilibria for the aggregate economy. Section 3 derives equilibrium balanced growth paths from the dynamic competitive market equilibria given in section 2. Section 4 discusses the problem of coordination failure implied by the equilibrium balanced growth paths. Section 5 explores the implications of economic structure on equilibrium balanced growth rates. The final section is devoted to conclusions.

2. The Model

We begin with specifying the environment under study. There are two classes of agents, who are distinguished by their ownership of different factors. The first consists of infinitely-lived capitalists, who own one unit of physical capital in each period; since physical capital accumulation is ignored here, we may think of physical capital as land or entrepreneurial ability. Their preferences over consumption and leisure are represented by

$$V = \int_0^{\infty} \exp(-\rho t) \frac{(C_{ct})^{1-a}}{1-a} \cdot \frac{(1-E_t)^{1-e}}{1-e} dt. \quad (1)$$

Here, ρ is the positive discount rate; C_{ct} is their commodity consumption in period t ; and E_t is their time devoted to the accumulation of the stock of knowledge in period t and is restricted by $0 \leq E_t \leq 1$. The parameters a and e must satisfy $0 < a < 1$ and $0 < e < 1$ because V is strictly increasing and concave in C_{ct} and $1-E_t$.³ For simplicity, capitalists are assumed to devote no time to commodity production. The other class is made up of infinitely-lived workers, who are only endowed with their one unit of time in each period to divide between leisure and labor. Their preferences are written by

$$U = \int_0^{\infty} \exp(-\rho t) \frac{(C_{wt})^{1-b}}{1-b} \cdot \frac{(1-H_{Yt}-H_{Kt})^{1-h}}{1-h} dt. \quad (2)$$

Here, C_{wt} is their commodity consumption in period t ; H_{Yt} is their labor supply to commodity production in period t ; H_{Kt} is their labor supply to the accumulation of the stock of knowledge in period t ; and H_{Yt} and H_{Kt} are restricted by $0 \leq H_{Yt} + H_{Kt} \leq 1$. The parameters b and h are restricted to be $0 < b < 1$ and $0 < h < 1$.

Each capitalist produces commodity in period t using labor and physical capital. This process follows a constant returns to scale neoclassical production process given by

$$Y_t = A(L_t)^n(Z_t)^{1-n}, \quad (3)$$

where L_t is the capitalist's demand for labor in period t measured in

efficiency units, Z_t is her stock of physical capital in period t measured in efficiency units, and the parameters A and n must satisfy $0 < A$ and $0 < n < 1$. Output is used only for consumption and is not storable.

To simplify the analysis, we assume that the capitalist-worker ratio is equal to unity at the initial period and that there is no population growth. Thus, each firm is made up of at most one capitalist and one worker. We also assume that all firms produce the same level of output and accumulate the same level of the stock of knowledge, so that we will focus our attention on the representative firm here.

The stock of knowledge generated inside each firm can contribute to the commodity production of the firm in several ways even though this knowledge cannot directly help the commodity production of the other firms.⁴ Some part of the stock of knowledge accumulated inside each firm is embodied in human capital of its worker. Let us assume that an increase of one unit in the stock of knowledge will yield an increase of α unit in the worker's skill of commodity production. We also assume that this augmentation in the worker's skill of commodity production arises from an increase in his general human capital but not in his specific human capital; therefore, even if the worker moves to another firm at period t , he can maintain the level of human capital attained inside the original firm until period t . Given that all firms operate at the same level of the stock of knowledge, these assumptions imply that the capitalist's demand for the effective work force is expressed by

$$L_t = \alpha K_t N_t, \tag{4}$$

where K_t is the stock of knowledge of the firm in period t , N_t the capitalist's demand for hours worked in period t , and α is a positive

parameter. Another part of the stock of knowledge is embodied in physical capital owned by the capitalist. The accumulation of the stock of knowledge thus increases the stock of physical capital in efficiency units owned by the capitalist. Since the capitalist is endowed with one unit of physical capital, the process is represented in the form:

$$Z_t = \gamma K_t, \quad (5)$$

where $\gamma > 0$. For simplicity, we assume $\alpha = \gamma = 1$. Substituting (4) and (5) into (3) with $\alpha = \gamma = 1$, we obtain

$$Y_t = A(N_t)^\alpha K_t. \quad (6)$$

The accumulation process of the stock of knowledge can summarize a broad range of activities in the stage of production. These activities include on-the-job training, basic scientific research, product development, innovation in systems and management, and so on. In the subsequent discussion, we will suppose that the accumulation process of the stock of knowledge inside the firm needs labor inputs supplied both by the capitalist and by the worker.⁵ To incorporate endogenous growth in the present model, we must also eliminate the diminishing returns to the accumulation process of the stock of knowledge in some manner. In this paper, we adopt the approach pioneered by Uzawa (1965) and Lucas (1988), assuming that changes in the stock of knowledge are governed by the following linear function in K_t :⁶

$$\dot{K}_t = K_t B(E_t)^k (H_{K_t})^{1-k}. \quad (7)$$

Here, $\dot{K}_t = dK/dt$, and the parameters B and k satisfy $0 < B$ and $0 < k < 1$. The striking feature of (7) is that it exhibits sustained per-capita income

growth from endogenous accumulation of the stock of knowledge. This feature stems from the linearity assumption in the production of the stock of knowledge. Equation (7) also shows that the stock of knowledge does not increase if one agent does not devote his or her labor input to the accumulation process of the stock of knowledge although the other agent does. This set-up strengthens the possibility of the occurrence of coordination failure in the production of the stock of knowledge.⁷

The activities of agents in the labor market and the production of the stock of knowledge inside the firm are competitive in the following sense. The capitalist selects the time paths of N_t (labor demand for the worker) and E_t (labor input devoted by the capitalist to the accumulation process of the stock of knowledge), taking as given a time path of H_{Kt} (labor input devoted by the worker to the accumulation process of the stock of knowledge) and recognizing that an auctioneer will set wages to clear markets. The worker also selects the time paths of H_{Yt} (labor input devoted by the worker to commodity production) and H_{Kt} , taking as given a time path of E_t chosen by the capitalist and acting as wage takers. Because of technological complementarities, the capitalist and the worker could benefit from a collusive agreement to devote more labor supply to the accumulation process of the stock of knowledge. Although this agreement could be Pareto-improving in this model, it cannot be supported for the reasons that collusive agreements fail in asymmetric information models. To make this point clear, we assume that it would be prohibitively costly to describe E_t and H_{Kt} in such a way that an outside court could determine whether or not they have been made.⁸ As a result of this imperfect information, each capitalist or worker would have an incentive to shirk under the collusive agreement, not devoting her or his

stipulated labor input in the accumulation process of the stock of knowledge.

We now consider the optimal decision of each agent. Taking H_{Kt} and W_t as given, the capitalist maximizes her lifetime utility (1) with respect to C_{ct} , N_t and E_t subject to (i) the accumulation equation of the stock of knowledge, (7), (ii) the non-negativity constraints associated with C_{ct} , N_t and E_t , (iii) the time constraint of E_t ($E_t \leq 1$), and (iv) the following budget constraint:

$$\begin{aligned} C_{ct} &= Y_t - W_t N_t K_t \\ &= A(N_t)^{\alpha} K_t - W_t N_t K_t, \end{aligned} \tag{8}$$

where W_t is the real wage rate in period t and is paid to the effective work force for commodity production;⁹ and the final equality is obtained from (6). Note that the capitalist cannot save or borrow, because commodity Y_t is not storable and there is no money in this economy. Taking E_t and W_t as given, the worker also maximizes his lifetime utility (2) with respect to C_{wt} , H_{Yt} and H_{Kt} subject to (i) the accumulation equation of the stock of knowledge, (7), (ii) the non-negativity constraints associated with C_{wt} , H_{Yt} and H_{Kt} , (iii) the time constraint of H_{Yt} and H_{Kt} ($H_{Yt} + H_{Kt} \leq 1$), and (iv) the following budget constraint:

$$C_{wt} = W_t H_{Yt} K_t. \tag{9}$$

The worker cannot save or borrow for the same reasons as the capitalist.

We next proceed to specify the market-clearing conditions for labor and commodity markets. The market clearing condition for labor in each period implies that

$$N_t = H_{Ft}. \tag{10}$$

The market clearing condition for commodity in each period is written in the form

$$C_{ct} + C_{wt} = Y_t = A(N_t)^{\alpha} K_t, \quad (11)$$

which is automatically satisfied by budget constraints (8) and (9) and labor market clearing condition (10).

We now define a competitive market equilibrium for the aggregate economy. A competitive market equilibrium is a set of market price and quantity sequences $[W_t, C_{ct}, N_t, E_t, C_{wt}, H_{yt}, H_{kt}, K_t; 0 \leq t \leq \infty]$ which satisfies the following conditions: (a) Taking $[W_t, H_{kt}; 0 \leq t \leq \infty]$ as given, $[C_{ct}, N_t, E_t; 0 \leq t \leq \infty]$ maximizes (1) subject to (7), (8), the non-negativity constraints associated with C_{ct} , N_t and E_t , and the time constraint of E_t ; (b) Taking $[W_t, E_t; 0 \leq t \leq \infty]$ as given, $[C_{wt}, H_{yt}, H_{kt}; 0 \leq t \leq \infty]$ maximizes (2) subject to (7), (9), the non-negativity constraints associated with C_{wt} , H_{yt} and H_{kt} , and the time constraint of H_{yt} and H_{kt} ; (c) The accumulation equation of the stock of knowledge, (7), and the labor market clearing condition, (10), hold in all $t \in [0, \infty]$.

3. Equilibrium Balanced Growth Paths

In this section, we derive equilibrium balanced growth paths from the dynamic competitive market equilibria defined in the previous section. We begin by solving the optimal decision problem for the capitalist in the competitive market equilibria. For this purpose, using (1), (7) and (8), let us introduce the current-value Hamiltonian for the capitalist's maximization problem:

$$\theta_c = \frac{[A(N_t)^a K_t - W_t N_t K_t]^{1-a}}{1-a} \cdot \frac{(1-E_t)^{1-e}}{1-e} + p_t K_t B(E_t)^k (H_{Kt})^{1-k}, \quad (12)$$

where p_t is the *capitalist's imputed price* used to value increments to the stock of knowledge at period t . We focus on the analysis of an interior solution case in the text because Appendix A discusses a corner solution case. The first-order conditions for this problem are obtained as follows:

$$[A(N_t)^a K_t - W_t N_t K_t]^{-a} [A n(N_t)^{a-1} K_t - W_t K_t] \frac{(1-E_t)^{1-e}}{1-e} = 0, \quad (13)$$

$$-\frac{[A(N_t)^a K_t - W_t N_t K_t]^{1-a}}{1-a} (1-E_t)^{-e} + p_t K_t B(E_t)^{k-1} (H_{Kt})^{1-k} = 0, \quad (14)$$

$$\dot{p}_t = \rho p_t - [A(N_t)^a - W_t N_t]^{1-a} (K_t)^{-a} \frac{(1-E_t)^{1-e}}{1-e} - p_t B(E_t)^k (H_{Kt})^{1-k}, \quad (15)$$

$$\lim_{t \rightarrow \infty} p_t K_t \cdot \exp(-\rho t) = 0, \quad (16)$$

where $\dot{p}_t = dp_t/dt$.

We next solve the optimal decision problem for the worker. Using (2), (7) and (9), let us define the current-value Hamiltonian for the worker's maximization problem in the following form:

$$\theta_w = \frac{(W_t H_{Yt} K_t)^{1-b}}{1-b} \cdot \frac{(1-H_{Yt} - H_{Kt})^{1-h}}{1-h} + q_t K_t B(E_t)^k (H_{Kt})^{1-k}, \quad (17)$$

where q_t is the *worker's imputed price* used to value increments to the stock of knowledge at period t . Since Appendix A deals with a corner solution case,

we restrict our attention to the analysis of an interior solution case, and have the following first-order conditions:

$$(W_t K_t)^{1-b} (H_{Yt})^{-b} \frac{(1 - H_{Yt} - H_{Kt})^{1-h}}{1-h} - \frac{(W_t H_{Yt} K_t)^{1-b}}{1-b} (1 - H_{Yt} - H_{Kt})^{-h} = 0, \quad (18)$$

$$- \frac{(W_t H_{Yt} K_t)^{1-b}}{1-b} (1 - H_{Yt} - H_{Kt})^{-h} + q_t K_t B (1-k) (E_t)^k (H_{Kt})^{-k} = 0, \quad (19)$$

$$\dot{q}_t = \rho q_t - (W_t H_{Yt})^{1-b} (K_t)^{-b} \frac{(1 - H_{Yt} - H_{Kt})^{1-h}}{1-h} - q_t B (E_t)^k (H_{Kt})^{1-k}, \quad (20)$$

$$\lim_{t \rightarrow \infty} q_t K_t \cdot \exp(-\rho t) = 0, \quad (21)$$

where $\dot{q}_t = dq_t/dt$.

We are now in a position to summarize equilibrium growth paths using (7), (10), (13)-(16), and (18)-(21). Since C_{ct} given by (8) is assumed to be positive, equation (13) reduces to

$$A n (N_t)^{n-1} = W_t. \quad (22)$$

Rearranging (14) implies

$$\frac{[A(N_t)^n - W_t N_t]^{1-a}}{1-a} (K_t)^{-a} (1-E_t)^{-e} = p_t B k (E_t)^{k-1} (H_{Kt})^{1-k}. \quad (23)$$

It is found from (15) with (7) and (23) that

$$\frac{\dot{p}_t}{p_t} = \rho - \frac{\dot{K}_t}{K_t} \left[\frac{(1-a)k}{(1-e)E_t} + 1 \right]. \quad (24)$$

Substituting (10) into (18) and (19) and rearranging them, we find

$$(W_t N_t)^{1-b} \frac{(1 - N_t - H_{Kt})^{1-h}}{1-h} = N_t \frac{(W_t N_t)^{1-b}}{1-b} (1 - N_t - H_{Kt})^{-h}, \quad (25)$$

$$\frac{(W_t N_t)^{1-b}}{1-b} (K_t)^{-b} (1 - N_t - H_{Kt})^{-h} = q_t B(1-k)(E_t)^k (H_{Kt})^{-k}. \quad (26)$$

Substituting (10) into (20) and rearranging it with (7), (25) and (26) yield

$$\frac{\dot{q}_t}{q_t} = \rho - \frac{\dot{K}_t}{K_t} \left[\frac{(1-k)N_t}{H_{Kt}} + 1 \right]. \quad (27)$$

It is immediate from (7) that

$$\frac{\dot{K}_t}{K_t} = B(E_t)^k (H_{Kt})^{1-k}. \quad (28)$$

Equilibrium growth paths for this economy, $[K_t, p_t, q_t, W_t, N_t, E_t, H_{Kt}; 0 \leq t \leq \infty]$, are solved simultaneously from equations (22)-(28) for a given initial value K_0 and transversality conditions (16) and (21).

In the subsequent analysis, we will confine our attention to balanced growth paths on which (i) the stock of knowledge K_t and the imputed prices of the stock of knowledge $[p_t, q_t]$ are growing at constant rates, and (ii) the real wage rate W_t and the input variables $[N_t, E_t, H_{Kt}]$ are constant. The restriction of the balanced growth path class is rationalized partly by causal empiricism (see Lucas (1988) and King and Rebelo (1988)). The preferences and production technology assumed in this paper ensure that balanced growth is feasible.

We now derive equilibrium balanced growth paths from the system of equations consisting of (22)-(28). Let g_K , g_P , and g_Q denote \dot{K}_t/K_t , \dot{p}_t/p_t , and \dot{q}_t/q_t . Since we restrict our attention to balanced growth paths, we can drop the time subscript from W_t , N_t , E_t and H_{Kt} , denoting them by W , N , E and H_K . Taking logarithms of both sides of (23) and (26) and differentiating them with respect to time, we obtain

$$g_P = -ag_K, \quad (29)$$

$$g_Q = -bg_K. \quad (30)$$

Rewriting (22) and rearranging (25) yield

$$W = An(N)^{\alpha-1}, \quad (31)$$

$$N = (1 - b)(1 - H_K)/(2 - b - h). \quad (32)$$

Substituting g_K , g_P , and g_Q into (24), (27) and (28) leads to

$$g_P = \rho - g_K \left[\frac{(1 - a)k}{(1 - e)E} + 1 \right], \quad (33)$$

$$g_Q = \rho - g_K \left[\frac{(1 - k)N}{H_K} + 1 \right], \quad (34)$$

$$g_K = B(E)^k (H_K)^{1-k}. \quad (35)$$

Equilibrium balanced growth paths [g_K , g_P , g_Q , W , N , E , H_K] are determined as the solutions of equations (29)-(35). The balanced growth paths satisfy transversality conditions (16) and (21) because the asymptotic rates of p_t and q_t are given by (33) and (34).

To characterize the properties of the equilibrium balanced growth paths, we must transform the system of equations consisting of (29)-(35) to a reduced system of two equations with respect to g_k and H_k . For this purpose, we substitute (29) into (33), and rearrange it as follows:

$$E = \frac{(1 - a)kg_k}{(1 - e)[\rho - (1 - a)g_k]} . \quad (36)$$

Substituting (36) into (35) and rearranging it give us

$$H_k = \frac{g_k}{(B)^{1/(1-k)}} \left\{ \frac{(1 - e)[\rho - (1 - a)g_k]}{(1 - a)k} \right\}^{k/(1-k)} . \quad (37)$$

Combining (30) and (34) yields

$$g_k = \frac{\rho H_k}{(1 - b)H_k + (1 - k)N} . \quad (38)$$

Substituting (32) into (38) and rearranging it, we see

$$H_k = \frac{(1 - b)(1 - k)g_k}{\rho(2 - b - h) - (1 - b)(1 + k - b - h)g_k} . \quad (39)$$

The values of g_k and H_k in the equilibrium balanced growth paths are now determined as the solutions of equations (37) and (39). From these values, the paths for the other variables can be easily derived.

In Appendix A, we show that a balanced growth path associated with $(g_k, H_k) = (0, 0)$ becomes an equilibrium balanced growth path generated by the competitive market equilibria defined in the previous section.¹⁰ This balanced growth path satisfies (37) and (39) even though it corresponds to the

corner solution case of $(E, H_K) = (0, 0)$. Therefore, the system of equations (37) and (39) provides a complete description of the values of g_K and H_K in equilibrium balanced growth paths generated by the competitive market equilibria in the previous section.

4. Coordination Failure in the Equilibrium Balanced Growth Paths

We now consider the features of the equilibrium balanced growth paths by exploiting the system of equations consisting of (37) and (39). First, we restrict the range of g_K using (32), (36), (37) and (39) with the time restrictions $0 < N < 1$, $0 \leq E < 1$, $0 \leq H_K < 1$, and $N + H_K < 1$. It follows from $0 < b < 1$, $0 < h < 1$ and $0 \leq H_K < 1$ that labor demand for the worker, N , is always determined from (32) in the range $(0, 1)$. The value of N also satisfies $N + H_K < 1$ as long as $H_K < 1$. Thus, we confine our attention to the restrictions imposed on g_K by (36) with $0 \leq E < 1$ and by (37) and (39) with $0 \leq H_K < 1$. To simplify the analysis, we assume from now on that B is great enough to ensure $H_K < 1$ in (37). Using $0 < a < 1$ and $0 < e < 1$, g_K is restricted from (36) with $0 \leq E < 1$ in the range $[0, \rho(1-e)/\{(1-a)(1-e+k)\}]$. Given $0 < b < 1$, $0 < h < 1$ and $0 < k < 1$, g_K is also restricted from (39) with $0 \leq H_K < 1$ in the range $[0, \rho/(1-b))$. Furthermore, it is seen from (37) with $0 \leq H_K$ that g_K must lie in the range $[0, \rho/(1-a))$. Combining these restrictions on g_K , we now have

$$0 \leq g_K < \text{Min} \left[\frac{\rho(1-e)}{(1-a)(1-e+k)}, \frac{\rho}{1-b}, \frac{\rho}{1-a} \right]. \quad (40)$$

We proceed to scrutinize the characteristics of the loci of (37) and (39) to depict these two curves in the (g_K, H_K) diagram. Let $H_K = \phi(g_K)$ and $H_K =$

$\psi(g_K)$ denote the locus of (37) and the locus of (39). Totally differentiating (37) with respect to g_K and H_K , we obtain

$$\phi'(g_K) = \frac{1-e}{k \cdot (B)^{1/(1-k)}} \left(\frac{\rho}{1-a} - \frac{g_K}{1-k} \right) \left\{ \frac{(1-e)[\rho - (1-a)g_K]}{(1-a)k} \right\}^{(2k-1)/(1-k)}. \quad (41)$$

Further differentiation of $\phi'(g_K)$ with respect to g_K yields

$$\phi''(g_K) = \frac{(1-e)^2}{k(1-k) \cdot (B)^{1/(1-k)}} \left(\frac{g_K}{1-k} - \frac{2\rho}{1-a} \right) \left\{ \frac{(1-e)[\rho - (1-a)g_K]}{(1-a)k} \right\}^{(3k-2)/(1-k)}. \quad (42)$$

Given $0 < a < 1$, $0 < e < 1$, $0 < k < 1$, $0 < B$, and (40), it is found from (41) and (42) that

$$\phi'(g_K) \gtrless 0 \quad \text{iff} \quad g_K \gtrless \frac{\rho(1-k)}{1-a}, \quad (43)$$

and

$$\phi''(g_K) \gtrless 0 \quad \text{iff} \quad g_K \gtrless \frac{2\rho(1-k)}{1-a}. \quad (44)$$

Similarly, totally differentiating (39) with respect to g_K and H_K leads to

$$\psi'(g_K) = \frac{\rho(1-b)(1-k)(2-b-h)}{[\rho(2-b-h) - (1-b)(1+k-b-h)g_K]^2} > 0, \quad (45)$$

where the final inequality is derived from $0 < b < 1$, $0 < h < 1$, $0 < k < 1$, $1 + k - b - h < 2 - b - h$, and (40). Further differentiation of (45) with respect to g_K generates

$$\psi''(g_K) = \frac{2\rho(1-b)^2(1-k)(2-b-h)(1+k-b-h)}{[\rho(2-b-h) - (1-b)(1+k-b-h)g_K]^3}, \quad (46)$$

which implies that

$$\psi''(g_k) \geq 0 \quad \text{iff} \quad 1 + k \geq b + h. \quad (47)$$

We now discuss the features of the equilibrium balanced growth paths in the (g_k, H_k) diagram using the loci of $H_k = \phi(g_k)$ and $H_k = \psi(g_k)$ characterized by (40) and (43), (44), (45) and (47). For simplicity, it is assumed from now on that there exists no difference between the intertemporal elasticities of substitution in consumption of the capitalist and the worker, that is, $a = b$. From (40), this assumption ensures that g_k is restricted only from (36) with $0 \leq E < 1$ in the following range:

$$0 \leq g_k < \frac{\rho(1-e)}{(1-a)(1-e+k)}. \quad (48)$$

Given the parametric conditions in (43), (44), (47) and (48), the discussion is divided into the following four cases:

(I) $1 + k \geq a + h$ and $e \leq k$.

In this case, it is immediate from (47) that $\psi''(g_k) \geq 0$. Thus, in the light of (45) and $\psi(0) = 0$, the $H_k = \psi(g_k)$ locus is depicted by $\psi(g_k)$ in Fig. 1.¹¹ It is also found from (43), (44), (48), $e \leq k$ and $a = b$ that¹²

$$\phi'(g_k) > 0 \text{ and } \phi''(g_k) < 0 \text{ for } g_k < \rho(1-k)/(1-a),$$

$$\phi'(g_k) \leq 0 \text{ and } \phi''(g_k) \leq 0$$

$$\text{for } \frac{\rho(1-k)}{1-a} \leq g_k \leq \text{Min} \left[\frac{2\rho(1-k)}{1-a}, \frac{\rho(1-e)}{(1-a)(1-e+k)} \right],$$

$$\phi'(g_k) < 0 \text{ and } \phi''(g_k) > 0 \text{ for } \text{Min} \left[\frac{2\rho(1-k)}{1-a}, \frac{\rho(1-e)}{(1-a)(1-e+k)} \right] < g_k.$$

Thus, in view of $\phi(0) = 0$, the $H_k = \phi(g_k)$ locus can be illustrated by $\phi(g_k)$ in Fig. 1.¹³ The equilibrium balanced growth paths are now indicated by the intersections of the $H_k = \phi(g_k)$ locus and the $H_k = \psi(g_k)$ locus. As shown in Fig. 1, two equilibrium balanced growth paths can exist.¹⁴ The point 0, associated with the no growth rate of per capita income, corresponds to the pessimistic state of the capitalist's and the worker's expectations about the accumulation of the stock of knowledge. On the other hand, the point E, associated with a high growth rate of per capita income, corresponds to the optimistic state of the capitalist's and the worker's expectations about the accumulation of the stock of knowledge. The difference between the expectations of these two equilibrium balanced growth paths arises from technological complementarities of the accumulation process of the stock of knowledge.

(II) $1 + k \geq a + h$ and $e > k$.

Fig. 2 depicts the loci of $H_k = \phi(g_k)$ and $H_k = \psi(g_k)$ in this case. As shown in Fig. 2, we may have two equilibrium balanced growth paths, 0 and E.

(III) $1 + k < a + h$ and $e \leq k$.

Fig. 3 illustrates a location of the loci of $H_k = \phi(g_k)$ and $H_k = \psi(g_k)$. In this case, the locus of $H_k = \phi(g_k)$ is similar to that in Fig. 1, whereas the locus of $H_k = \psi(g_k)$ is a concave function of g_k . It is still possible that multiple equilibrium balanced growth paths exist. Fig. 3 indicates an example that there are three equilibrium balanced growth paths, 0, E_1 , and E_2 .¹⁵

(IV) $1 + k < a + h$ and $e > k$.

Fig. 4 describes a location of the loci of $H_K = \phi(g_K)$ and $H_K = \psi(g_K)$ in this case, and suggests that there exist multiple equilibrium balanced growth paths.

These four figures show that coordination failure in the accumulation process of the stock of knowledge can yield multiple, Pareto-ranked equilibrium balanced growth paths. In the present model, the coordination failure emerges from technological complementarities within the accumulation process of the stock of knowledge inside firms. There may exist some mechanisms that can overcome the coordination failure. The introduction of these mechanisms, which is probably related with a reform of the internal organization of firms, can explain some aspects of takeoffs in economic development.

5. Determinants of Equilibrium Balanced Growth Rates

Since the present model generates endogenous growth rates, we can explore how economic structure affects these growth rates. In this section, we discuss the effects on long run growth rates of the parameters of commodity production (A and n), the parameters of the accumulation process of the stock of knowledge (B and k), and the parameters of the preferences of agents (a , e , h and ρ).

We first examine the effects on long run growth rates of a change in the parameters of commodity production. In fact, a shift in A or n causes no effects on long run growth rates because neither (37) nor (39) includes these parameters. Thus, we obtain the following proposition.

Proposition 1: Equilibrium balanced growth rates are invariant to a change in any of the parameters of commodity production (A and n).

Some remarks about this proposition should be mentioned. First, the standard endogenous growth model such as Lucas (1988) and Rebelo (1987) predicts that the equilibrium balanced growth rate is invariant to linear transformation of the production function of consumption commodities. Proposition 1 shows that this prediction also holds for the endogenous growth model with coordination failure. Second, Proposition 1 implies that a change in n does not affect equilibrium balanced growth rates in the present model. The reason stems from the fact that commodity production function (3) is transformed to be linear in the stock of knowledge in this model (see equation (6)).

We next consider the implications of changing the other parameters. Suppose that the effectiveness of labor inputs in the accumulation process of the stock of knowledge changes so that B increases. Then, it is seen from Table 1 that the $H_K = \phi(g_K)$ locus shifts down whereas the $H_K = \psi(g_K)$ locus does not shift. Given the discussion in the previous section, the analysis of the effects of the shift in B on long run growth rates is divided into the following two cases:

(I) If $1 + k \geq a + h$, we can have two equilibrium balanced growth paths depicted by points O and E in Figs. 1 and 2, as represented in the previous section. If B increases, then the new equilibrium balanced growth solution with a high growth rate of per capita income lies to the left of E . This finding implies that the new equilibrium balanced growth path with a high growth rate of per capita income is characterized by a slower balanced growth

rate. On the other hand, the shift in B causes no effect on the equilibrium balanced growth path with the no growth rate of per capita income at 0.

(II) If $1 + k < a + h$, we may suppose that equilibrium balanced growth paths are illustrated by points O , E_1 and E_2 , as shown in Figs. 3 and 4. Then, the effects of the shift in B on the equilibrium balanced growth paths, both with the maximum growth rate and with the no growth rate, are similar to those obtained when $1 + k \geq a + h$. However, the new equilibrium balanced growth solution with the medium growth rate lies to the right of E_1 , so that this solution is characterized by a higher growth rate. In general, we can show that a rise in B decreases (increases) the equilibrium balanced growth rate of an equilibrium balanced growth path with $\psi' > \phi'$ ($\psi' < \phi'$).

Applying the similar analysis to all the remaining parameters, we can establish the following propositions:

Proposition 2: Suppose that $1 + k \geq a + h$. Then:

- (1) If an equilibrium balanced growth path is characterized by the no growth rate, the long run growth rate is independent of a change in any parameter.
- (2) If an equilibrium balanced growth path is characterized by a positive growth rate, the long run growth rate *decreases* with the effectiveness of labor inputs in the accumulation process of the stock of knowledge, B , and *decreases* with the intertemporal elasticity of substitution in consumption of each agent, $1/a$. The long run growth rate also *increases* with the intertemporal elasticity of substitution in labor supply of each agent, $1/e$ or $1/h$, and *increases* with the discount rate, ρ . An increase in the substitution elasticity of labor inputs in the accumulation process of the stock of knowledge, k , causes ambiguous effects on the long run growth rate.

Proposition 3: Suppose that $1 + k < a + h$. Then:

(1) If an equilibrium balanced growth path is characterized by the no growth rate, the result of Proposition 2(1) holds.

(2) If an equilibrium balanced growth path with a positive growth rate is characterized by $\psi' > \phi'$, the long run growth rate satisfies the result of Proposition 2(2).

(3) If an equilibrium balanced growth path with a positive growth rate is characterized by $\psi' < \phi'$, the long run growth rate *increases* with the effectiveness of labor inputs in the accumulation process of the stock of knowledge, B , and *increases* with the intertemporal elasticity of substitution in consumption of each agent, $1/a$. The long run growth rate also *decreases* with the intertemporal elasticity of substitution in labor supply of each agent, $1/e$ or $1/h$, and *decreases* with the discount rate, ρ . An increase in the substitution elasticity of labor inputs in the accumulation process of the stock of knowledge, k , causes ambiguous effects on the long run growth rate.

Some comments about these two propositions are in order. First, if $1 + k \geq a + h$, we always obtain $\psi' > \phi'$ on the equilibrium balanced growth path with a positive growth rate, as depicted in Figs. 1 and 2. Thus, we may state that Proposition 3 holds true irrespective of whether $a + h$ is greater than $1 + k$. Furthermore, if the worker's intertemporal elasticities of substitution in consumption and labor supply are small enough, we can exclude the case of Proposition 2. Second, the standard endogenous growth model developed by Lucas (1988) and Rebelo (1987) predicts that the equilibrium balanced growth rate *increases* with the effectiveness of investment in human capital or the stock of knowledge, *increases* with the intertemporal elasticity of

substitution in consumption, and decreases with the discount rate. However, their results are confirmed in the endogenous growth model with coordination failure only if an equilibrium balanced growth path with a positive growth rate is characterized by $\psi' < \phi'$. Furthermore, the opposite results are obtained in this model if an equilibrium balanced growth path with a positive growth rate is characterized by $\psi' > \phi'$. The reason is attributed to the fact that, in this model, parameter shifts affect long run growth rates by changing the expectation of each agent about the level of labor input devoted by the other agent to the accumulation process of the stock of knowledge. Thus, if an increase in the effectiveness of the accumulation of the stock of knowledge leads one agent to expect that the other agent devotes less labor input to the accumulation of the stock of knowledge, then the long run growth rate declines. Third, if the equilibrium balanced growth paths with $\psi' > \phi'$ are unstable, we can state that the results of the standard endogenous growth model still hold in the present model. Since the features of the equilibrium balanced growth paths with $\psi' > \phi'$ seem to be counterintuitive, we may conjecture that these equilibrium balanced growth paths are unstable.

6. Conclusions

This paper has considered the long run implications of the coordination problem between the capitalist and the worker which arises from technological complementarities within a shared accumulation process of the stock of knowledge in the stage of production. Coordination failure emerging from the accumulation process of the stock of knowledge can produce multiple, Pareto-ranked equilibrium balanced growth paths in which identical economies can converge to steady states with different rates of growth. This finding

suggests that a reform of the internal organization of firms is beneficial not only by improving the efficiency of production but also by changing the expectations of one agent about the other agents' input decisions in the accumulation process of the stock of knowledge inside the firms. This paper has also explored the implications of economic structure on equilibrium balanced growth rates. The obtained results have shown that the conclusions derived from the standard endogenous growth model are not necessarily confirmed in the endogenous growth model with coordination failure.

Appendix A

This appendix shows that a balanced growth path associated with $(E, H_K) = (0, 0)$ satisfies the equilibrium conditions generated by the competitive market equilibria defined in section 2.

We begin with deriving the equilibrium conditions under the assumption that all inequality constraints except the non-negativity constraints of E_t and H_{Kt} are satisfied with strict inequality. To this end, let us notice that the first-order conditions from the capitalist's and the worker's problems with the inequality assumption mentioned above are represented by (13), (15), (16), (18), (20), (21), and the following equations:

$$-\frac{[A(N_t)^a K_t - W_t N_t K_t]^{1-a}}{1-a} (1-E_t)^{-e} + p_t K_t B k (E_t)^{k-1} (H_{Kt})^{1-k} + \lambda_t = 0, \quad (A1)$$

$$-\frac{(W_t H_{Yt} K_t)^{1-b}}{1-b} (1 - H_{Yt} - H_{Kt})^{-h} + q_t K_t B (1-k) (E_t)^k (H_{Kt})^{-k} + \mu_t = 0, \quad (A2)$$

where λ_t and μ_t are the nonnegative multipliers associated with $E_t \geq 0$ and $H_{Kt} \geq 0$. Rearranging these first-order conditions with accumulation equation (7) and labor market clearing condition (10) produces the equilibrium conditions, which are expressed by (15), (16), (21), (22), (25), (28), and the following equations:

$$\dot{q}_t = \rho q_t - (W_t N_t)^{1-b} (K_t)^{-b} \frac{(1 - N_t - H_{Kt})^{1-h}}{1-h} - q_t B (E_t)^k (H_{Kt})^{1-k}, \quad (A3)$$

$$\frac{[A(N_t)^a - W_t N_t]^{1-a}}{1-a} (K_t)^{1-a} (1-E_t)^{-e} = p_t K_t B k (E_t)^{k-1} (H_{Kt})^{1-k} + \lambda_t, \quad (A4)$$

$$\frac{(W_t N_t K_t)^{1-b}}{1-b} (1 - N_t - H_{Kt})^{-h} = q_t K_t B(1-k)(E_t)^k (H_{Kt})^{-k} + \mu_t. \quad (A5)$$

We next show that growth paths with $(E, H_K) = (0, 0)$ can be constructed so as to satisfy the equilibrium conditions specified by (15), (16), (21), (22), (25), (28), (A3), (A4) and (A5). Let us define the growth paths by

$$E_t = 0, \quad (A6)$$

$$H_{Kt} = 0, \quad (A7)$$

$$N_t = (1 - b)/(2 - b - h), \quad (A8)$$

$$W_t = A n (N_t)^{n-1}, \quad (A9)$$

$$\lambda_t = \frac{[A(N_t)^n - W_t N_t]^{1-a} (K_t)^{1-a}}{1-a}, \quad (A10)$$

$$\mu_t = \frac{(W_t N_t K_t)^{1-b} (1 - N_t)^{-h}}{1-b}, \quad (A11)$$

$$\dot{K}_t = 0, \quad (A12)$$

$$\dot{p}_t = \rho p_t - \frac{[A(N_t)^n - W_t N_t]^{1-a} (K_t)^{-a}}{1-e}, \quad (A13)$$

$$\dot{q}_t = \rho q_t - \frac{(W_t N_t)^{1-b} (K_t)^{-b} (1 - N_t)^{1-h}}{1-h}. \quad (A14)$$

It is immediate that these growth paths satisfy (15), (22), (25), (28), (A3), (A4) and (A5). Given that the capitalist's consumption and the worker's

consumption are positive, it is also found from (A13) and (A14) that the transversality conditions of (16) and (21) are satisfied. Thus, the growth paths defined by (A6)-(A14) are proved to be equilibrium growth paths.

We now confine our attention to balanced growth paths on which (i) the stock of knowledge K_t , the capitalist's and the worker's imputed prices of the stock of knowledge $[p_t, q_t]$, and the nonnegative multipliers $[\lambda_t, \mu_t]$ are growing at constant rates, and (ii) the input variables $[N_t, E_t, H_{Kt}]$ and the real wage rate W_t are constant. To derive a balanced growth path from the equilibrium growth paths defined by (A6)-(A14), we drop the time subscript from N_t, E_t, H_{Kt} and W_t , and denote $\dot{\lambda}_t/\lambda_t$ and $\dot{\mu}_t/\mu_t$ by g_M and g_N . Taking logarithms of both sides of (A10) and (A11) and differentiating them with respect to time, we see

$$g_M = g_N = g_K = 0, \tag{A15}$$

where the final inequality follows from (A12). Thus, we obtain a balanced growth path from (A6)-(A14) by dropping the time subscript from all variables except p_t and q_t . This balanced growth path is verified to be an equilibrium balanced growth path because it is generated by the equilibrium growth paths given by (A6)-(A14).

Appendix B

The purpose of this appendix is to explore the implications of economic structure on equilibrium balanced growth rates. To this end, we must examine the effects of a change in each structural parameter on the loci of $H_K = \phi(g_K)$ and $H_K = \psi(g_K)$. These results are represented as follows (note that b is assumed to be equal to a):

(1) Partial differentiation of ϕ and ψ with respect to B ;

$$\frac{\partial \phi}{\partial B} = - \frac{(B)^{(k-2)/(1-k)} \cdot g_k}{1-k} \left\{ \frac{(1-e)[\rho - (1-a)g_k]}{(1-a)k} \right\}^{k/(1-k)}, \quad (B1)$$

$$\frac{\partial \psi}{\partial B} = 0. \quad (B2)$$

(2) Partial differentiation of ϕ and ψ with respect to a ;

$$\frac{\partial \phi}{\partial a} = \frac{(1-e)\rho g_k}{(1-a)^2(1-k)(B)^{1/(1-k)}} \left\{ \frac{(1-e)[\rho - (1-a)g_k]}{(1-a)k} \right\}^{(2k-1)/(1-k)}, \quad (B3)$$

$$\frac{\partial \psi}{\partial a} = - \frac{(1-k)[\rho(1-h) + (1-a)^2 g_k] g_k}{[\rho(2-a-h) - (1-a)(1+k-a-h)g_k]^2}. \quad (B4)$$

(3) Partial differentiation of ϕ and ψ with respect to e ;

$$\frac{\partial \phi}{\partial e} = - \frac{[\rho - (1-a)g_k] g_k}{(1-a)(1-k)(B)^{1/(1-k)}} \left\{ \frac{(1-e)[\rho - (1-a)g_k]}{(1-a)k} \right\}^{(2k-1)/(1-k)}, \quad (B5)$$

$$\frac{\partial \psi}{\partial e} = 0. \quad (B6)$$

(4) Partial differentiation of ϕ and ψ with respect to h ;

$$\frac{\partial \phi}{\partial h} = 0, \quad (B7)$$

$$\frac{\partial \psi}{\partial h} = \frac{(1-a)(1-k)[\rho - (1-a)g_k] g_k}{[\rho(2-a-h) - (1-a)(1+k-a-h)g_k]^2}. \quad (B8)$$

(5) Partial differentiation of ϕ and ψ with respect to ρ ;

$$\frac{\partial \phi}{\partial \rho} = \frac{(1-e)g_k}{(1-a)(1-k)(B)^{1/(1-k)}} \left\{ \frac{(1-e)[\rho - (1-a)g_k]^{(2k-1)/(1-k)}}{(1-a)k} \right\}, \quad (\text{B9})$$

$$\frac{\partial \psi}{\partial \rho} = - \frac{(1-a)(1-k)(2-a-h)g_k}{[\rho(2-a-h) - (1-a)(1+k-a-h)g_k]^2}. \quad (\text{B10})$$

Given $0 < a < 1$, $0 < e < 1$, $0 < h < 1$, $0 < k < 1$, $0 < B$, $0 < \rho$ and (48), we obtain the results summarized in Table 1.

Notes

1. For various models of coordination failure, see Cooper and John (1988).
2. As indicated above, Azariadis and Drazen (1988) showed that technological externalities with threshold features can produce multiple balanced growth paths in their endogenous growth model. Their threshold features imply that the effectiveness of investment in human capital rises very rapidly if the stock of human capital achieves some critical values. On the other hand, our model need not assume such threshold externalities in the accumulation process of the stock of knowledge to verify the possibility of multiple balanced growth paths. Prescott and Boyd (1987) have developed an overlapping generation model of endogenous economic growth in which workers' productivity depends not only on their own human capital but also on that of their coworkers. Although the spirit of their study is close to ours, they exhibit that there exists the unique equilibrium balanced growth path. The difference stems from their assumption that young workers can make with old workers contract arrangements such that strategic complementarity effectively disappears.
3. Note that $(a)^{-1}$ is the intertemporal elasticity of substitution in consumption and $(e)^{-1}$ the intertemporal elasticity of substitution in labor supply. The parameters b and h defined below are also interpreted in a similar way.
4. This qualification implies that the stock of knowledge accumulated inside each firm displays no spillovers among firms. Several studies, however, discuss the implications of spillovers of knowledge in the economy. See Romer (1986), Lucas (1988), Stokey (1988), and Grossman and Helpman (1988).

5. It is possible to construct a model in which the capitalist invests her foregone consumption instead of supplying her labor effort to accumulate the stock of knowledge. However, our main conclusions are not modified by this alternative specification.
6. For the justification of the linearity assumption, see Lucas (1988). Note that an increase in the stock of knowledge raises the efficiency of the capitalist's or the worker's labor input for the accumulation of the stock of knowledge.
7. Our main conclusions do not rely on the specific functional form of (7).
8. It is important to emphasize that E_t and H_{Kt} are observable to the capitalist and the worker. The difficulty that the agents face is conveying this common information to the court.
9. We assume that the stock of knowledge K_t is publicly observable, so that wages are paid to labor supply measured in efficiency units. Note that the assumption of public observability on K_t does not imply that E_t and H_{Kt} are publicly observable.
10. Note that this result depends on the assumption of the multiplicative functional form of (7).
11. Note that $\psi(\rho(1-e)/[(1-a)(1-e+k)]) < 1$ because $E = 1$ and $H_K < 1$ at $g_K = \rho(1-e)/[(1-a)(1-e+k)]$ with the assumption of $a = b$. This remark also holds in Figs. 2-4.
12. More precisely, if $2\rho(1-k)/(1-a) \geq \rho(1-e)/[(1-a)(1-e+k)]$, the $H_K = \phi(g_K)$ locus does not have any portion on which $\phi'(g_K) < 0$ and $\phi''(g_K) > 0$.
13. Note that the $H_K = \phi(g_K)$ locus lies everywhere below the $H_K = 1$ locus in the range of (48) because B is assumed to be great enough to ensure the relation between these two curves. This remark also holds in Figs. 2-4.

14. Although Fig. 1 suggests the existence of two equilibrium balanced growth paths, it is possible that there exists the unique equilibrium balanced growth path characterized by the no growth rate of per capita income. This remark also holds in Fig. 2.

15. Although Fig. 3 exhibits an example of three equilibrium balanced growth paths, we may have less than or more than three equilibrium balanced growth paths. This remark also holds in Fig. 4.

REFERENCES

- Azariadis, C. and A. Drazen, "Threshold Externalities in Economic Development," mimeo, 1988.
- Barro, R.J., "Government Spending in a Simple Model of Endogenous Growth," mimeo, 1988.
- Cooper, R., and A. John, "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics*, 103(1988), 441-63.
- De Long, B., "Productivity Growth, Convergence, and Welfare: Comment," *American Economic Review*, 78(1988), 1138-54.
- Grossman, G.M., and E. Helpman, "Comparative Advantage and Long-run Growth," mimeo, 1988.
- Kaldor, N., "Capital Accumulation and Economic Growth," in *The Theory of Capital*, Lutz and Hague, eds. (New York: St. Martin's Press, 1961).
- King, R.G., and S. Rebelo, "Public Policy and Economic Growth: Developing Neoclassical Implications," mimeo, 1988.
- Lucas, R.E.Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22 (1988), 3-42.
- Prescott, E.C., and J.H. Boyd, "Dynamic Coalitions: Engines of Growth," *American Economic Review, AEA Papers and Proceedings*, 77(1987), 63-67.
- Rebelo, S., "Long Run Policy Analysis and Long Run Growth," mimeo, 1987.
- Romer, P.M., "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94(1986), 1002-37.
- , "Capital Accumulation in the Theory of Long Run Growth," in *Modern Business Cycle Theory*, R.J.Barro, ed. (New York: Wiley & Sons, 1988).
- Solow, R.M., "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70(1956), 65-94.
- Stokey, N.L., "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy*, 96(1988), 701-17.
- Uzawa, H., "Optimum Technical Change in an Aggregative Model of Economic Growth", *International Economic Review*, 6(1965), 18-31.

Table 1
The effects of parameter changes on ϕ and ψ

	ϕ	ψ
B	-	0
k	?	?
a	+	-
e	-	0
h	0	+
ρ	+	-

Notes: +: upward shift; -: downward shift; 0: no shift; ?: ambiguous effect.
The derivation procedure of the results is shown in Appendix B.

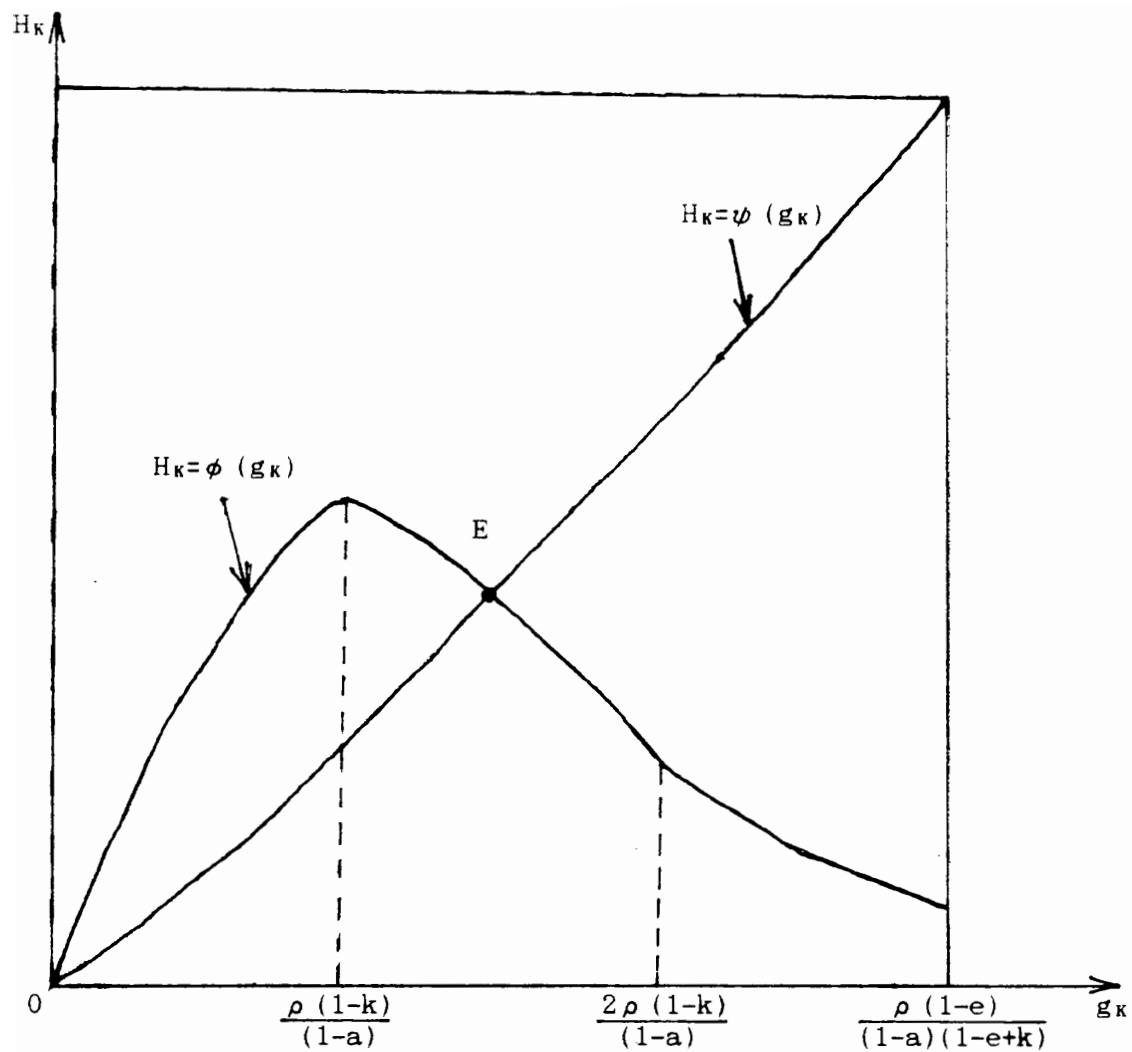


Fig. 1. Equilibrium Balanced Growth Paths for $1 + k \geq a + h$ and $e \leq k$.

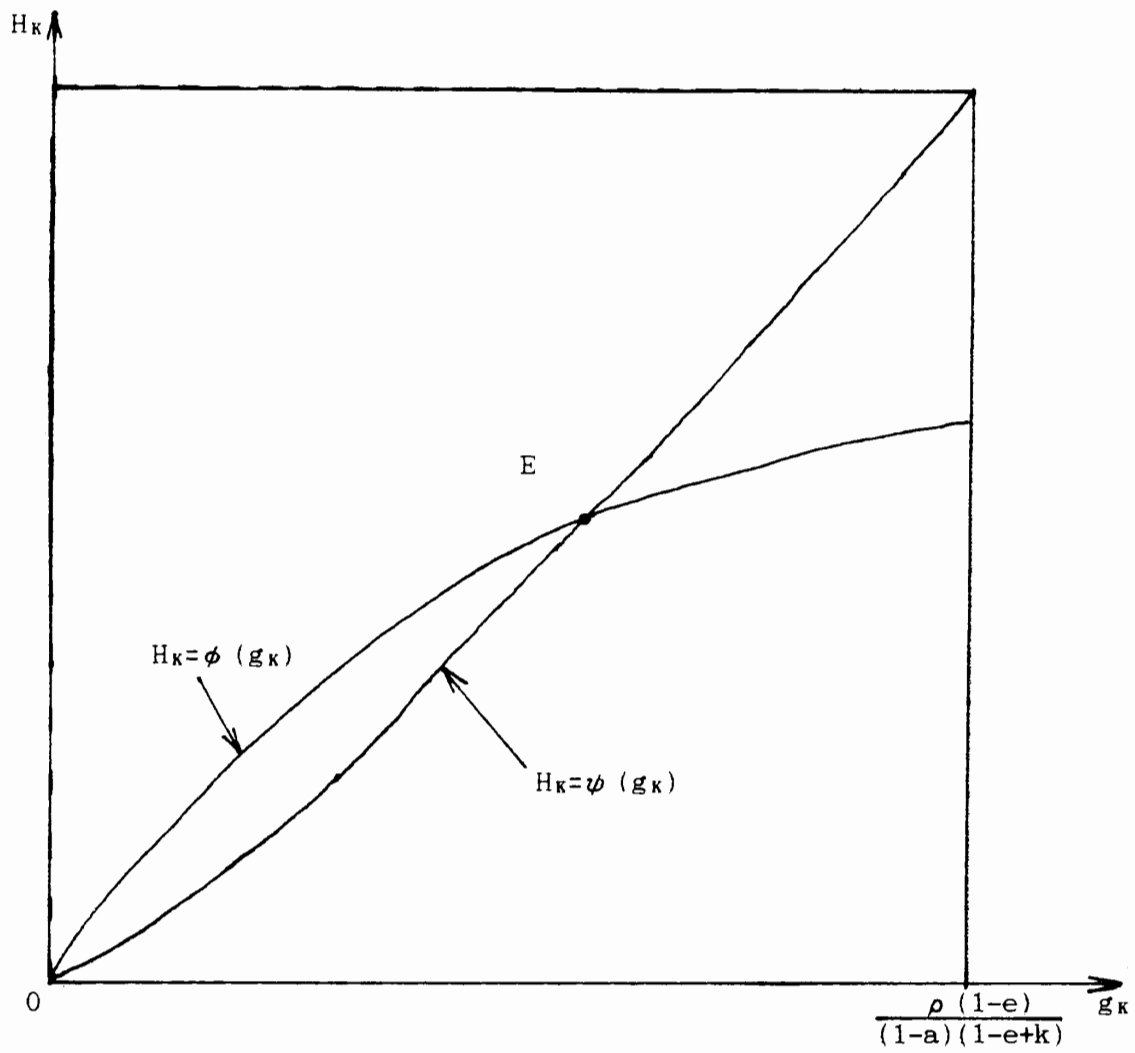


Fig. 2. Equilibrium Balanced Growth Paths for $1 + k \geq a + h$ and $e > k$.

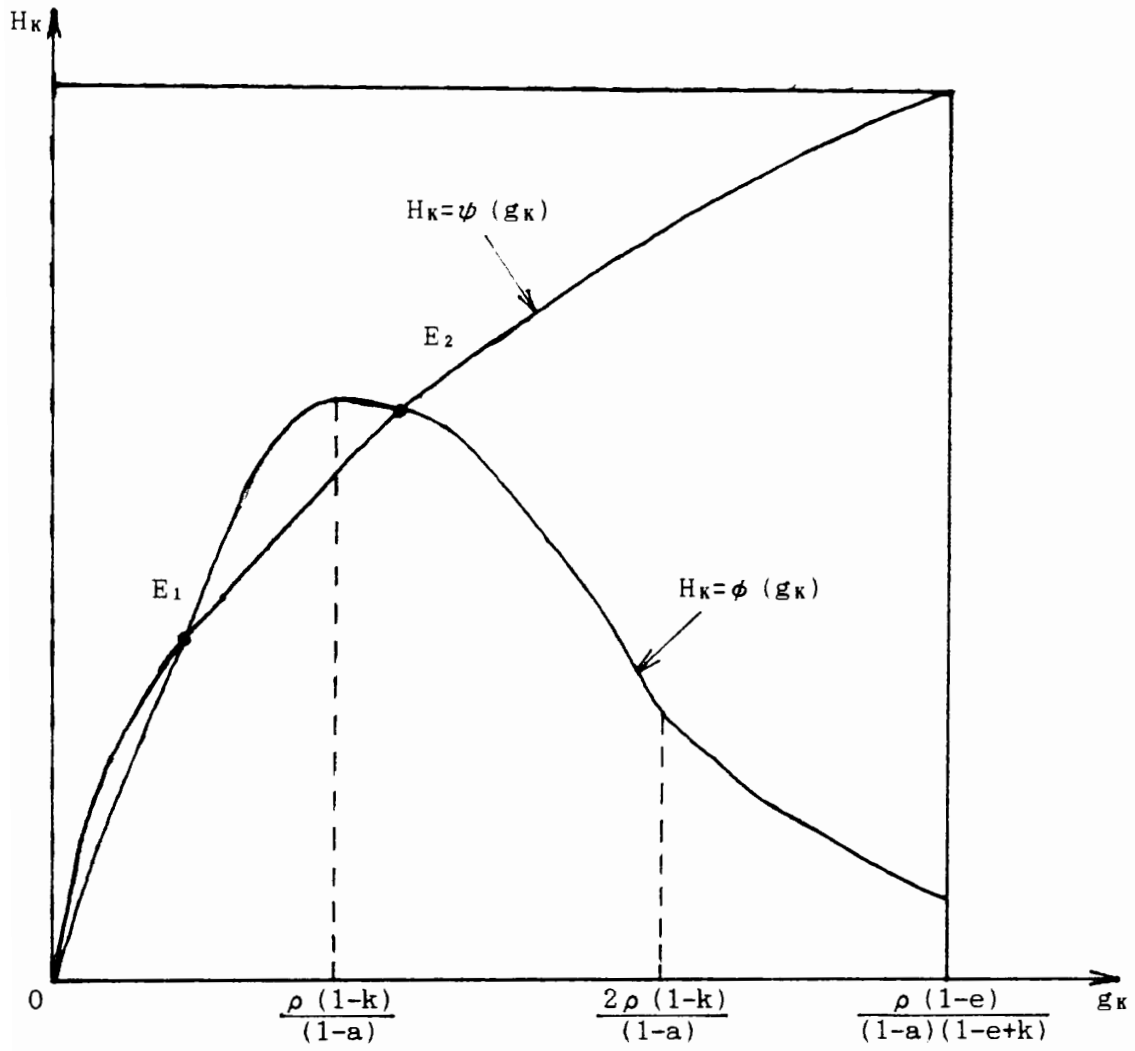


Fig. 3. Equilibrium Balanced Growth Paths for $1 + k < a + h$ and $e \leq k$.

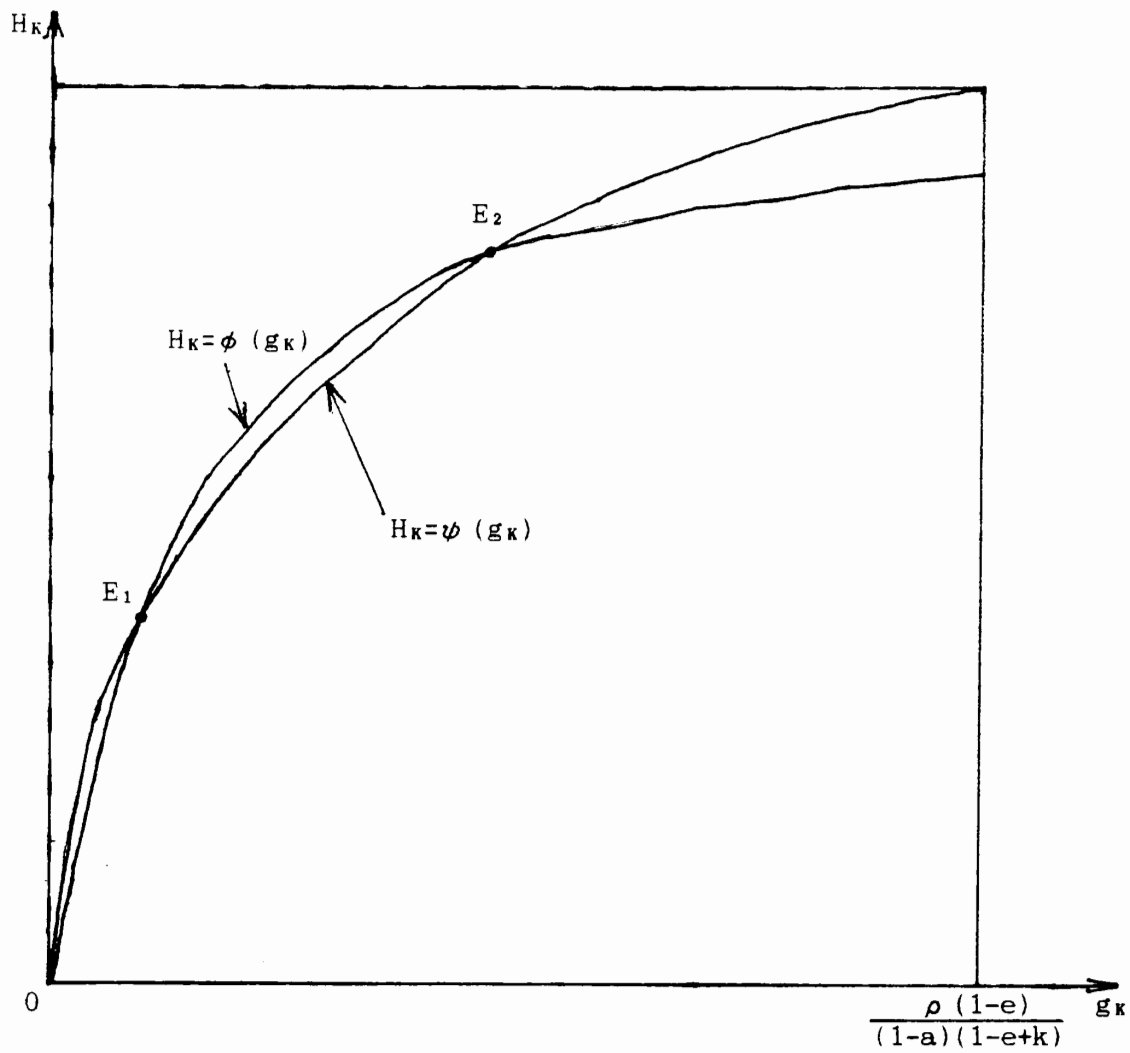


Fig. 4. Equilibrium Balanced Growth Paths for $1 + k < a + h$ and $e > k$.