Discussion Paper No. 829

OLIGOPOLY LIMIT PRICING

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April 1989*

*This paper has benefited greatly from comments by Steve Matthews.

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1. Introduction

The basic notion of limit pricing involves an incumbent firm choosing a low price and thereby convincing a potential entrant that entry would be unprofitable. This informal idea becomes a complete theory of entry deterrence when two further issues are addressed. First, the linkage between the preentry price and the postentry profits of the entrant must be made explicit. Second, true monopolies are certainly the exception, and any useful model of limit pricing must therefore be consistent with the existence of multiple, uncoordinated incumbents.

Past literature has studied two kinds of linkages. The early work of Bain [1956], Modigliani [1958] and especially Sylos-Labini [1962] proposed a commitment linkage, whereby the incumbent is able to commit to sustain its output level if entry occurs. This idea has been extended to commitment of a wider class of strategic variables, such as capacity, and to commitments by multiple incumbents, made either simultaneously or sequentially.[1] Milgrom and Roberts' [1982] introduced the idea of an informational linkage, in which the incumbent reduces its price in order to signal to the entrant that entry prospects are unfavorable. Despite the distortion to preentry pricing, entry decisions are exactly the same as under complete information.

This information-based approach to limit pricing has been extended in several directions.[2] Of particular importance are two papers by Harrington [1986; 1987]. In his early paper, Harrington argues that an entrant will often expect its costs to be positively correlated with those of the incumbent. If the correlation is sufficiently strong, this can lead the incumbent to distort its price upward, in order to signal high costs. Harrington extends this framework in his second paper to allow for multiple incumbents. Assuming that the entrant observes only the market price, he shows that high prices again arise when incumbents with common, privately known costs simultaneously choose preentry output.

In our view, however, the extension of the informational theory of limit pricing to the possibility of multiple incumbents remains incomplete. It seems most plausible that an entrant would be able to observe individual behavior of each incumbent separately, as opposed to a single, summary statistic of all incumbents' behavior. We show below that this difference is of fundamental importance in understanding limit pricing behavior.

Our model takes the following form. Industry costs are the common costs incurred by each active firm. These costs can be either high or low. For simplicity, we assume there are two incumbent firms, of whom each knows the industry cost. The firms choose prices in a differentiated product market. Both prices are observed by the entrant, who then tries to infer industry costs. Higher costs correspond to lower profits, so incumbents would like to signal high costs.

One might expect that prices would tend to be distorted upward. We argue, however, that a very robust equilibrium exists in which the incumbents simply play as if there were complete information, or equivalently, no entry threat! Prices are not distorted in either direction, and entry takes place exactly when it is profitable. While the latter conclusion is consistent with previous single incumbent models, the former is not and points to a significant difference between multiple and single incumbent models.

The intuition is actually straightforward and serves to illustrate the importance of the assumption that each incumbent's choice is observable. Hypothesize a sequential equilibrium (Kreps and Wilson [1982]) in which complete-information choices are made. Of course, such an equilibrium would typically violate incentive constraints in a single incumbent setting, as well as in Harrington's multiple incumbent model. But suppose in our model that a low-cost incumbent attempts to feign high costs by raising its price. Because preentry pricing is noncooperative, the deviating low-cost incumbent expects that its rival will continue to choose its equilibrium price. Thus, any deviation is guaranteed to produce a pair of prices lying "off the equilibrium path," to which the entrant may respond with an inference of low costs. The key point is that when incumbents select observable signals simultaneously, noncooperative behavior implies that they are <u>unable to coordinate deception</u>. This in turn means that it is possible to credibly signal high costs with no distortions at all. We refer to such an outcome as a <u>no distortion equilibrium</u>.

As is usually the case in signaling models, there may exist many other signaling equilibria, involving a wide variety of possible distortions. We show that for a large class of entry situations, the no distortion equilibrium <u>uniquely</u> satisfies a pair of refinements which eliminate unreasonable inferences by the entrant.

The first variety of equilibria are <u>two-sided separating equilibria</u>, in which each incumbent plays a separating pricing strategy (the no distortion equilibrium fits into this category). For these equilibria we introduce the refinement of <u>unprejudiced beliefs</u>, which requires that the entrant never places infinitely less posterior likelihood on a non-equilibrium dominated deviation relative to any other deviation. In other words, an entrant should not be "prejudiced" against deviations unless they are equilibrium dominated.

Suppose that a high-cost incumbent must in a two-sided separating equilibrium choose a price off its complete-information reaction function. If this incumbent deviates to a preferred price closer to the reaction function, it can be sure that separating behavior by the rival incumbent will reveal high costs, as long as the entrant is not prejudiced against the deviation. That is, with unprejudiced beliefs the high-cost incumbent will informationaly free-ride on the separation of its rival, unless there are no gains available from deviating. Only the no distortion equilibrium eliminates all such gains, making it the only two-sided separating equilibrium which is supportable by unprejudiced beliefs.[3]

We also consider one-sided separating equilibria, in which one of the incumbents plays a pooling strategy, and pooling equlibria in which both play pooling strategies. Requiring unprejudiced beliefs need not have force in such equilibria, since informational free-riding is impossible when the rival incumbent pools. We show that the intuitive criterion of Cho and Kreps [1987] eliminates all equilibria with pooling, under a set of assumptions which guarantees that (1) high-cost incumbents differentially prefer price increases which reduce the probability of entry; and (2) there exist price increases which are equilibrium dominated for the low-cost incumbents. For entry situations satisfying these assumptions, the no distortion equilibrium is the only equilibrium which can be supported by unprejudiced and intuitive beliefs.

We show by parameterized example that the assumptions will fail only when entry deterrence is sufficiently important relative to preentry profits. This will be the case, for example, in a rapidly growing market. The interesting point is that the equilibria which now satisfy the intuitive criterion will typically involve downward distortions to preentry prices, in contrast to Harrington's results. This is caused by the fact that an incumbent's preentry profits are reduced when its rival chooses a low price. When both incumbents choose low pooling prices, preentry profits may be so low that no price increase will be equilibrium dominated under either cost level, and the equilibrium will then satisfy the intuitive criterion. Thus, the distortions first demonstrated by Milgrom and Roberts emerge here in the form of intuitive pooling equilibria, which exist when market growth is sufficiently rapid.

The plan of the paper is as follows. Section 2 describes the basic model, and section 3 develops an adaptation of Kreps and Wilson's sequential equilibrium to our setting. Sections 4, 5, and 6 consider two-sided separating equilibria, one-sided separating equilibria and pooling equilibria, respectively. Section 7 presents our parameterized example. Section 8 gives brief comments concerning implications of our model for incentives to pursue collusive strategies, and section 9 concludes.

2. Model

Consider the following situation. There are two incumbent firms, Firm 1 and Firm 2, and one potential entrant firm, Firm 3, who compete in a two-period market. In the first period the incumbent firms alone produce the product. At the outset of the second period, the potential entrant may choose to enter the market, and in the second period the market may have either two or three sellers. The key feature of this market is that the incumbent firms possess information about production costs which the entrant cannot observe prior to making its entry decision.

This market will be modeled by means of the following three-stage game:

<u>Stage 1</u>: Firms 1 and 2 observe a cost parameter ω and choose prices P_1 and P_2. The set of possible ω is given simply by $\{L,H\}$. P_1 and P_2 are chosen noncooperatively from nondegenerate intervals $[0,\bar{P}_1]$ and $[0,\bar{P}_2]$, respectively.

<u>Stage 2</u>: Firm 3 observes P_1 and P_2, but not ω, and makes an entry decision $E \in \{0,1\}$, where 0 denotes no entry, and 1 denotes entry.

<u>Stage 3</u>: The firms play a second period oligopoly game whose structure depends on the entry decision.

We abstract from the details of second-period interaction and simply specify payoffs conditional on the entry decision. Let $\Pi_i^N(\omega)$ give the second-period profit of Firm i when $E = 0$ and the cost parameter is ω, and $\Pi_i^E(\omega)$ the profit when $E = 1$. Assume $\Pi_i^N(\omega) > \Pi_i^E(\omega)$ for $i \neq 3$ and all ω, and $\Pi_3^E(\omega) > \Pi_3^N(\omega) = 0$. Incumbents always prefer no entry, while an entrant makes positive second-period profits. If $E = 1$, Firm 3 incurs a sunk entry cost of $K \geq 0$; we will suppose that the value of K is the private information of Firm 3. Finally, $\omega = L$ is associated with lower production costs than is $\omega = H$, so $\Pi_i^N(L) > \Pi_i^N(H)$ for $i \neq 3$, and $\Pi_i^E(L) > \Pi_i^E(H)$ for all i.

For the incumbents, first-period profits are given by $\Pi_i(P_i,P_j,\omega)$, $i,j = 1,2$, $i \neq j$, which are assumed to be continuous functions of the prices. For each P_j and ω, Π_i is uniquely maximized by $P_i^R(P_j,\omega)$, which is a reaction function that is continuous, strictly increasing in P_j and such that $P_i^R(P_j,H)$

$> P_i^R(P_j,L)$. We assume that there is a unique static Nash equilibrium in prices for each ω, given by $(P_1^*(\omega),P_2^*(\omega))$. Figure 1 illustrates these assumptions.

3. Sequential Equilibria

As our solution concept we adopt a straightforward adaptation of Kreps and Wilson's sequential equilibrium, which gives restrictions on strategies as well as beliefs of players. The firms' strategies are given by $\hat{P}_i(\omega)$ for Firms 1 and 2, and $\hat{E}(P_1,P_2,K)$ for Firm 3. ω and K are chosen by "Nature" via randomization. Let $\rho \in (0,1)$ give the probability that $\omega = L$, and suppose that K is drawn from nondegenerate $[0,\bar{K}]$ according to the strictly positive density $f(K)$. Firm 3's beliefs when it makes its entry decision are given by $\hat{\rho}(P_1,P_2,K)$, the posterior probability of $\omega = L$ when (P_1,P_2,K) has been observed. To ensure that there is positive probability of entry being unattractive, assume $\bar{K} > \Pi^E(H)$. The beliefs of the incumbents are represented by densities $\hat{f}_i(K|\omega)$.

The definition of sequential equilibrium is comprised of two components. First, strategies must be sequentially rational: Each time a player makes a decision, the choice must maximize his expected payoff given his beliefs at the decision point, and assuming that players will use their equilibrium strategies in all future moves. Second, beliefs must be consistent: There must exist a sequence of fully-mixed strategies, converging to the equilibrium strategies, such that the sequence of beliefs formed by applying Bayes' rule to the fully-mixed strategies converges to the equilibrium beliefs.

Sequential rationality allows the game to be solved via backward induction, so we first consider the entry decision. For given P_1, P_2, K, entry gives a best response if:

$$\hat{\rho}(P_1,P_2,K)\Pi^E(L) + (1-\hat{\rho}(P_1,P_2,K))\Pi^E(H) - K \geq 0 \quad . \tag{1}$$

Thus, the entry strategy is sequentially rational if $\hat{E}(P_1,P_2,K) = 1$ when (1) holds, and $\hat{E}(P_1,P_2,K) = 0$ otherwise. Consider next the pricing decisions of the incumbent firms. Pricing is sequentially rational if, for $i,j = 1,2$, $i \neq j$, $\omega = L,H$:

$$\hat{P}_i(\omega) \in \underset{P_i}{\operatorname{argmax}} \{\Pi_i(P_i,\hat{P}_j(\omega),\omega) + \Pi^N(\omega)$$

$$+ (\Pi^E(\omega) - \Pi^N(\omega)) \int_0^{\overline{K}} \hat{E}(P_i,\hat{P}_j(\omega),K)\hat{f}_i(K|\omega)dK\} \quad . \tag{2}$$

In defining consistent beliefs, we must depart somewhat from the Kreps-Wilson formulation, since our strategy spaces have uncountably many elements. First we must indicate what we mean by "fully-mixed strategies converging to the equilibrium strategies." For Firm i of type ω, these strategies are given by a sequence of probability distribution functions $\{\Psi_i^n(P_i|\omega)\}_{n=1}^{\infty}$, each element of which has full support on $[0,\overline{P}_i]$. The random variables associated with these distributions must converge in probability to the equilibrium strategy $\hat{P}_i(\omega)$. That is, for every $\varepsilon > 0$:

$$(3) \qquad \lim_{n\to\infty}\left[\int_{0}^{\hat{P}_i(\omega)-\varepsilon} d\Psi_i^n(P_i|\omega) + \int_{\hat{P}_i(\omega)+\varepsilon}^{\bar{P}_i} d\Psi_i^n(P_i|\omega)\right] = 0 \quad .$$

A set of strategy sequences $\{\{\Psi_i^n(P_i|\omega)\}_{n=1}^{\infty}\}_{i=1,2;\omega=L,H}$ satisfying these properties will be called a test sequence.

Next, we must define the Bayesian posteriors formed from the distributions $\Psi_i^n(P_i|\omega)$ by conditioning on observed price pairs (P_1,P_2). This is made difficult by the fact that the events on which we must condition typically have probability zero under the prior joint distribution. If we extend the notion of conditional probability to events having strictly positive density, placing suitable restrictions on the distributions, we are left with sequences of posteriors whose limits may exhibit very odd behavior at particular price pairs. For example, the limiting beliefs need not be Bayes-consistent with the equilibrium strategies for equilibrium-path prices. To see this, suppose we have a pooling equilibrium, with $\hat{P}_i(L) = \hat{P}_i(H) = P_i^P$, $i = 1,2$. Fix a sequence $\{\varepsilon^n\}_{n=1}^{\infty}$ of small strictly-positive real numbers converging to zero. Let the test sequence be defined by the following densities, for $i=1,2$:

$$\psi_i^n(P_i|L) = \begin{cases} \varepsilon^n \ , & |P_i - P_i^P| \geq \varepsilon^n \\ [1-\varepsilon^n(\bar{P}_i - 2\varepsilon^n)]/2\varepsilon^n \ , & \text{otherwise} \end{cases}$$

$$\psi_i^n(P_i|H) = \begin{cases} \varepsilon^n \ , & |P_i - P_i^P| \geq (\varepsilon^n)^2 \\ [1-\varepsilon^n(\bar{P}_i - 2(\varepsilon^n)^2)]/2(\varepsilon^n)^2 \ , & \text{otherwise} \end{cases}$$

which clearly implies convergence in probability to the equilibrium strategies. Convergence under $w = H$ is much more rapid than under $w = L$, so much so that the probability of L conditional on (P_1^P, P_2^P) approaches zero. Thus, the limit of the Bayesian posteriors differs from the posterior implied by the equilibrium strategies, which is ρ.

We will instead employ a better-behaved notion of conditional probability. Rather than conditioning directly on events having prior probability zero, we will condition on positive-probability events which approximate the zero-probability events, and define consistent beliefs in terms of conditional probabilities obtained using the positive-probability events. Let Z_i denote an open subinterval of $[0,\overline{P}_i]$, and let the length of the subinterval be given by $l(Z_i)$. Events of the form $(P_1,P_2) \in Z_1 x Z_2$ will be called <u>simple events</u>. Put:

$$R_i^n(Z_i|\omega) = \int_{Z_i} d\Psi_i^n(P_i|\omega)$$

For simple events, the conditional probability of $\omega = L$ is given by:

$$\rho^n(Z_1,Z_2) = \frac{\rho R_1^n(Z_1|L)R_2^n(Z_2|L)}{\rho R_1^n(Z_1|L)R_2^n(Z_2|L) + (1-\rho)R_1^n(Z_1|H)R_2^n(Z_2|H)} \tag{4}$$

For a given set of price strategies, beliefs $\hat{\rho}(P_1,P_2,K)$ are said to be <u>consistent</u> if there exists a test sequence, converging to the price strategies, such that for all (P_1,P_2) and $\delta,\varepsilon > 0$ the following is true: There exists a simple event $(P_1,P_2) \in Z_1 x Z_2$ with $l(Z_1), l(Z_2) < \delta$, such that:

$$|\hat{\rho}(P_1,P_2,K) - \lim_{n\to\infty} \rho^n(Z_1,Z_2)| < \varepsilon \quad .$$

Thus, beliefs are consistent if at each price pair they can be approximated by the limiting posteriors, under a given test sequence, obtained from arbitrarily small simple events. It is easy to see that this notion of consistency ensures Bayes-consistency with the equilibrium strategies, since $\hat{P}_i(\omega) \in Z_i$ implies:

$$\lim_{n\to\infty} R_i^n(Z_i|\omega) = 1 \quad .$$

Note that using the test sequence to derive inferences means that K has no effect on the posteriors, and $\hat{\rho}$ will be independent of K if it is consistent; we will henceforth write the entrant's beliefs $\hat{\rho}(P_1,P_2)$. Finally, since the incumbents obtain no information about K prior to choosing prices, consistency of the incumbents' beliefs means simply that beliefs agree with the prior f(K); thus, consistency requires $\hat{f}_i(K|\omega) = f(K)$ for all i and ω.

4. No Distortion Equilibria

In the limit pricing theory of Milgrom and Roberts, the fact that price is a signal of cost forces the incumbent to depart from its complete-information optimal pricing. With multiple incumbents, however, it turns out that signaling need not induce distortions to preentry pricing. In this section we consider a class of sequential equilibria called <u>no distortion equilibrium</u> (NDE), in which $\hat{P}_i(\omega) = P_i^*(\omega)$ for all i and ω. Here, complete-information Nash prices are played for each cost type. Note that under our assumptions multiple NDE arise only to the extent that the entrant's reactions may differ for off-equilibrium-path prices. We now show:

Proposition 1: There exists an NDE.

Proof: Choose a scalar $\beta > 0$ and a sequence $(\varepsilon^n)_{n=1}^{\infty} \subset \mathbb{R}_{++}$ with $\varepsilon^n \to 0$, and let the test sequence be given by the following densities:

$$\psi_i^n(P_i|L) = \begin{cases} \varepsilon^n , & |P_i - P_i^*(L)| \geq \varepsilon^n \\ [1 - \varepsilon^n(\bar{P}_i - 2\varepsilon^n)]/2\varepsilon^n , & \text{otherwise} \end{cases} \tag{5}$$

$$\psi_i^n(P_i|H) = \begin{cases} \beta\varepsilon^n , & |P_i - P_i^*(H)| \geq \varepsilon^n \\ [1 - \beta\varepsilon^n(\bar{P}_i - 2\varepsilon^n)]/2\varepsilon^n , & \text{otherwise} \end{cases} \tag{6}$$

These densities may be used to define a set of consistent beliefs $\hat{\rho}(P_1,P_2)$, by taking limits of the posteriors (4) formed using sufficiently small Z_1,Z_2, with $l(Z_1) = l(Z_2)$. Let $\hat{E}(P_1,P_2,K)$ be specified in accordance with these beliefs and (1).

Consider the equilibrium condition (2) for $\omega = H$ Any deviation from $P_i^*(H)$ will only make Firm i worse off, since $P_i^*(H)$ uniquely maximizes preentry profits given the pricing strategy of Firm j, while consistency implies $\hat{\rho}(P_i^*(H),P_j^*(H)) = 0$, which means that the probability of entry is as low as possible under the entrant's equilibrium strategy. Now consider $\omega = L$. For a deviation to $P_i \neq P_i^*(H)$, consider the simple event $(P_i, P_j^*(L)) \in Z_i x Z_j$, with the intervals chosen small enough to ensure $P_i^*(L), P_i^*(H) \notin Z_i$ and $P_j^*(H) \notin Z_j$. For sufficiently large n we have:

$$\rho^n(Z_i, Z_j) = \frac{\rho\varepsilon^n l(Z_i) R_j^n(Z_j|L)}{\rho\varepsilon^n l(Z_j) R_j^n(Z_j|L) + (1-\rho)\beta^2(\varepsilon^n)^2 l(Z_i) l(Z_j)} ,$$

which converges to unity. Such a deviation cannot benefit Firm i. Finally, consider the deviation $P_i = P_i^*(H)$. Choosing Z_i and Z_j to satisfy $P_i^*(L) \notin Z_i$, $P_j^*(H) \notin Z_j$ and $l(Z_i) = l(Z_j)$, we have, for sufficiently large n:

$$\rho^n(Z_i, Z_j) = \frac{\rho\varepsilon^n l(Z_i) R_j^n(Z_j|L)}{\rho\varepsilon^n l(Z_i) R_j^n(Z_j|L) + (1-\rho) R_i^n(Z_i|H)\beta\varepsilon^n l(Z_j)}$$

$$= \frac{\rho R_j^n(Z_j|L)}{\rho R_j^n(Z_j|L) + (1-\rho) R_i^n(Z_i|H)\beta} ,$$

which converges to $\rho/[\rho + (1-\rho)\beta]$. Thus, the consistent beliefs satisfy $\hat{\rho}(P_i^*(H), P_j^*(L)) = \rho/[\rho + (1-\rho)\beta]$, which may be made arbitrarily close to unity by taking β sufficiently small. In particular, we may take β small enough to ensure that Firm i would not gain by deviating to $P_i^*(H)$.

Q.E.D.

The key distinction between this result and that of Milgrom and Roberts is that, with multiple incumbents, noncooperative pricing prevents the firms from coordinating their defections from the equilibrium prices. A low-cost incumbent cannot "fool" the entrant by imitating the equilibrium strategy of a high-cost incumbent, since the rival incumbent will continue to play its low-cost equilibrium strategy. In the NDE, every unilateral defection leads to off-equilibrium-path prices, and the consistent beliefs constructed in the proof place sufficient weight on $\omega = L$ to make deviation unattractive.

The entrant's beliefs reflect the fact that deviations by the incumbent firms are uncorrelated. We might think of (5) and (6) as giving the entrant's actual conjectures as to the incumbents' strategies; for large n, the entrant's conjectures become arbitrarily close to the equilibrium strategies, but the entrant does not rule out the possibility that any price might appear. The entrant does rule out correlated pricing, since the conjectures presume independent price choices by the incumbents.

There are two other key properties of (5) and (6) which make it possible for the NDE to satisfy the conditions for sequential equilibrium. These properties involve the relative likelihoods which the entrant places on possible deviations which support a given observed price pair. We will henceforth use the notation (i,ω) to mean, "Firm i of type ω." Suppose the entrant anticipates that the NDE obtains, but observes $(P_i, P_j^*(L))$ with $P_i \neq P_i^*(L), P_i^*(H)$. There are two ways which the unexpected observation can be rationalized. First, the entrant can hypothesize that (i,L) deviated to P_i while (j,L) played its equilibrium strategy, and second, the entrant can posit that both (i,H) and (j,H) deviated.

The entrant will place greater weight on the prospect of $\omega = L$ following the observation, if it believes that one deviation is more likely than two. The strategy conjectures (5) and (6) capture this intuition by the fact that they do not place infinitely greater liklihood, as n approaches infinity, on any one deviation relative to another. This property alone ensures that consistent beliefs satisfy $\hat{\rho}(P_i, P_j^*(L)) = 0$ for any such price pair.

Observation of $(P_i^*(H), P_j^*(L))$ requires a stronger restriction, however, since it can be rationalized by one deviation under either cost-state. The second property is that (5) and (6) assign sufficiently greater liklihood to the prospect of deviation by (i,L), relative to deviation by (j,H).[4] It is not necessary that the strategy conjectures place infinitely greater weight in the limit on the former prospect.[5]

5. <u>Two-Sided Separating Equilibria and Unprejudiced Conjectures</u>

In this model, a separating equilibrium arises whenever $(\hat{P}_i(L), \hat{P}_j(L)) \neq (\hat{P}_i(H), \hat{P}_j(H))$. Since there are two incumbents in possession of the cost information, there are two sorts of separating equilibria. A <u>two-sided separating equilibrium</u> (TSE) is one in which both incumbents play separating strategies, i.e., $\hat{P}_i(L) \neq \hat{P}_i(H)$, $i = 1,2$. In a TSE, the entrant can learn the cost parameter by observing the price of either incumbent alone. When $\hat{P}_i(L) \neq \hat{P}_i(H)$ but $\hat{P}_j(L) = \hat{P}_j(H)$, we have a <u>one-sided separating equilibrium</u> (OSE). Here, the entrant can learn the cost parameter only by observing the price of Firm i. In this section we will develop a refinement of sequential equilibrium which greatly reduces the set of possible TSE. We consider OSE in the next section.

First, it should be noted that in any separating equilibrium, $\hat{P}_i(L) = P_i^*(L)$ for $i = 1,2$, i.e., the pricing of the low-cost incumbents is never distorted. This is because (i,L)'s equilibrium pricing in a separating equilibrium leads to the largest possible entry, so that there is no punishment which can deter (i,L) from deviating to $P_i^R(\hat{P}_j(L),L)$. But the threat of increased entry can induce a wide range of pricing behavior by the high-cost incumbents. We will argue that threats which lead to pricing distortions in a TSE are based on unreasonable inferences by the entrant.

Consider a TSE with $\hat{P}_i(H) \neq P_i^R(\hat{P}_j(H),H)$. Then there exists some $P_i \neq \hat{P}_i(L)$ which would give (i,H) greater profits, if it did not increase the probability of entry. The entrant can rationalize observation of $(P_i,\hat{P}_j(H))$ by positing a single deviation under ω = H, but a joint deviation must be entertained if ω = L. Thus, for the probability of entry to rise, the entrant must place greater weight on the prospect of a defection by one of the incumbents of type L, versus the defection to P_i by (i,H). In fact, the only way that $\hat{\rho}(P_i,\hat{P}_j(H)) > 0$ can satisfy consistency is for the strategy conjectures to place <u>infinitely</u> greater liklihood, as n approaches infinity, on one of the former deviations relative to the latter.

It seems unreasonable that the entrant should assign such large differences to the conjectured probabilities of defection, especially given that the defection by (i,H) could potentially increase its profits. This indicates the need for further restrictions on the entrant's conjectures, which we develop as follows.

Let us abuse notation and write the equilibrium probability of entry as a function of the entrant's beliefs:

$$\hat{E}(\hat{\rho}) = \int_0^{\hat{\rho}\Pi^E(L)+(1-\hat{\rho})\Pi^E(H)} f(K)dK$$

We will refer to the statement "P_i was chosen by (i,ω)" as a <u>hypothesis</u>, and denote it by (P_i,ω). (P_i,ω) is called a <u>hypothesized deviation</u> if $P_i \neq \hat{P}_i(\omega)$. A hypothesized deviation is <u>equilibrium admissible</u> if:

$$\Pi_i(P_i,\hat{P}_j(\omega),\omega) + \hat{E}(0)(\Pi^E(\omega) - \Pi^N(\omega))$$

$$\geq \Pi_i(\hat{P}_i(\omega),\hat{P}_j(\omega),\omega) + \hat{E}(\hat{\rho}(\hat{P}_i(\omega),\hat{P}_j(\omega)))(\Pi^E(\omega) - \Pi^N(\omega))$$

That is, by deviating to P_i, (i,ω) could improve on its equilibrium payoff, or at least do no worse, if the entry response were sufficiently low. Any hypothesized deviations which are not equilibrium admissible are called <u>equilibrium dominated</u>.

Fix a sequential equilibrium and a hypothesized deviation (P_i,ω) which is equilibrium admissible. We say that a test sequence is <u>prejudiced</u> against (P_i,ω) if these exists a hypothesized deviation $(P_{i'},\omega')$ ($P_{i'} = P_i$ when $i' = i$) such that the following is true: For all $\delta > 0$, there exists a simple event $(P_i,P_{i'}) \in Z_i x Z_{i'}, (Z_i = Z_{i'}$ when $i = i'$) with $l(Z_i),l(Z_{i'}) < \delta$, such that:

$$\lim_{n\to\infty} \frac{R_i^n(Z_i|\omega)}{R_{i'}^n(Z_{i'}|\omega')} = 0$$

This means that the test sequence places infinitely greater liklihood, as n approaches infinity, on small simple events associated with deviations by (i',ω') to $P_{i'}$, relative to those associated with deviations by (i,ω) to P_i. Such conjectures seem unreasonable, as the latter hypothesis is not entirely implausible due to its equilibrium admissibility. The test sequence is <u>unprejudiced</u> if it is not prejudiced against any equilibrium admissible hypothesized deviation.

With this, we have:

<u>Proposition 2</u>: The NDE is the only TSE which can be supported by beliefs which are consistent with respect to an unprejudiced test sequence.

<u>Proof</u>: We know $\hat{P}_i(L) = P_i^*(L)$, i=1,2, in every TSE. Suppose $\hat{P}_i(H) \neq P_i^R(\hat{P}_j(H),H)$. Then there exists $P_i \neq P_i^*(L)$ such that:

$$\Pi_i(P_i, \hat{P}_j(H), H) > \Pi_i(\hat{P}_i(H), \hat{P}_j(H), H)$$

so that P_i increases preentry profit for (i,H); it follows that (P_i,H) is equilibrium admissible. Consider the simple event $(P_i, \hat{P}_j(H)) \in Z_i x Z_j$ with $\hat{P}_i(L), \hat{P}_i(H) \notin Z_i$ and $\hat{P}_j(L) \notin Z_j$. Posterior beliefs (4) may be written:

$$(7) \qquad \rho^n(Z_i, Z_j) = \frac{\rho R_i^n(Z_i|L)}{\rho R_i^n(Z_i|L) + (1-\rho) R_j^n(Z_j|H)[R_i^n(Z_i|H)/R_j^n(Z_j|L)]} \quad ,$$

and we have:

$$(8) \qquad \lim_{n\to\infty} R_i^n(Z_i|L) = 0, \ \lim_{n\to\infty} R_j^n(Z_j|H) = 1 \quad .$$

Suppose $\hat{\rho}(P_i,\hat{P}_j(H)) > 0$ under the consisent beliefs which support the TSE. For all $\delta > 0$ we can find a simple event of the sort specified above, satisfying $l(Z_i), l(Z_j) < \delta$. Further, for δ sufficiently small, all simple events must be of this sort. Thus, the supposition implies:

$$\lim_{n\to\infty} \rho^n(Z_i,Z_j) > 0 \quad . \tag{9}$$

Combining (7), (8) and (9) gives:

$$\lim_{n\to\infty} \frac{R_i^n(Z_i|H)}{R_j^n(Z_j|L)} = 0 \quad .$$

But this is impossible if the test sequence which supports the consistent beliefs is unprejudiced. It follows that any consistent beliefs supported by an unprejudiced test sequence satisfy $\hat{\rho}(P_i,\hat{P}_j(H)) = 0$, and clearly (i,H) will deviate to P_i. Thus, the only TSE that can be supported by unprejudiced conjectures is the NDE. That the latter is true can be seen by noting that (5) and (6) define an unprejudiced test sequence. Q.E.D.

Thus, there is at most one TSE supported by an unprejudiced test sequence and in this equilibrium the signaling of cost information has no effect whatsoever on the preentry pricing of incumbents. It is important that attention is restricted to TSE: An incumbent of type H can deviate without risking increased entry, since the pricing of the other incumbent will ensure separation. This <u>informational free riding</u> is explained as follows. When (i,H) deviates to $P_i \neq \hat{P}_i(L)$, it expects (j,H) to choose $\hat{P}_j(H)$. As long as $\hat{P}_j(L) \neq \hat{P}_j(H)$ and the test sequence which defines beliefs is unprejudiced, observing $(P_i, \hat{P}_j(H))$ will lead the entrant to place infinitely greater likelihood, in the limit, on $\omega = H$ versus $\omega = L$, as $\omega = H$ can be rationalized by a single equilibrium admissible deviation. Thus, the cost level will still be revealed even if (i,H) deviates. In a TSE, each high cost incumbent knows that the other's strategy guarantees separation, and deviations will occur unless both firms' prices are best responses.

6. <u>One-Sided Separating Equilibria and Intuitive Conjectures</u>

In this section we consider OSE, in which $\hat{P}_i(L) \neq \hat{P}_i(H)$ but $\hat{P}_j(L) = \hat{P}_j(H)$. In such equilibria (i,H) can no longer informationally free ride on Firm j, since Firm j's strategy no longer signals the cost level. Correspondingly there may exist OSE supported by unprejudiced test sequences, which are not NDE. An example is given in Figure 2, in which $\hat{P}_1(L) = \hat{P}_1(H)$, but separation occurs since $\hat{P}_2(L) \neq \hat{P}_2(H)$. (1,H) could free ride on separation by (2,H), but since $\hat{P}_1(H) = P_1^R(\hat{P}_2(H),H)$ there is no need to deviate. (2,H) could increase preentry profits by deviating, but Firm 1's pricing does not ensure separation; observing $(\hat{P}_1(H), P_2)$ can be rationalized by one deviation under either state, and as in Proposition 1 we may support the OSE beliefs with an unprejudiced test sequence which puts sufficiently large weight on $\omega = L$. The key point is that when Firm 1 is not separating, the entrant need not place infinitely less weight on deviations by (2,H) in order to support an inference of low costs. This seems especially plausible when the deviation is equilibrium admissible for (2,L) as well as (2,H).

Suppose, however, that the incumbents differentially prefer to increase their price under high costs. This is plausible since the decrease in sales would lead to a larger reduction in production costs when costs are high. Then (2,H) might be able to find some price $P_2 > \hat{P}_2(L)$ which (2,L) would never choose even if it generated the most favorable entry response. It seems reasonable that the entrant would place little weight on the prospect that (2,L) would choose such a price. Choosing P_2 would then allow (2,H) to separate without having to free ride on Firm j.

This is the idea which underlies the <u>intuitive criterion</u> of Cho and Kreps [1987]. In our setting we formalize it as follows. A test sequence is <u>unintuitive</u> if there exists a pair of disequilibrium hypotheses $(P_i,\omega),(P_{i'},\omega')$ $(P_i=P_{i'}$ when $i=i')$, with (P_i,ω) equilibrium dominated and $(P_{i'},\omega')$ equilibrium admissible, such that the following is true: For all $\delta > 0$, there exists a simple event $(P_i,P_{i'}) \in Z_i x Z_{i'} (Z_i=Z_{i'}$ when $i=i')$ with $l(Z_i), l(Z_{i'}) < \delta$, such that:

$$\lim_{n\to\infty} \frac{R_i^n(Z_i|\omega)}{R_{i'}^n(Z_{i'}|\omega')} > 0 \quad .$$

Thus, a test sequence is unintuitive when an equilibrium dominated hypothesis is given positive likelihood in the limit relative to an equilibrium admissible hypothesis. A test sequence is <u>intuitive</u> when it is not unintuitive relative to any pair of disequilibrium hypotheses.

For the intuitive criterion to have force, we must strengthen our assumptions. First, price increases must be differentially preferred under high costs:

<u>Assumption A</u>: Whenever $P_i' > P_i$, we have:

$$\Pi_i(P_i',P_j,L) - \Pi_i(P_i,P_j,L) < \Pi_i(P_i',P_j,H) - \Pi_i(P_i,P_j,H) \quad .$$

Next, there must exist some price increase which gives the low-cost firm lower profits than in the OSE, even for the most favorable entry response. To ensure this, we need:

<u>Assumption B</u>: For all P_i, P_j with $P_i < \overline{P}_i$, there exists $P_i' > P_i$ such that:

$$\Pi_i(P_i',P_j,L) < \Pi_i(P_i,P_j,L)$$

We now have the following sharpening of our previous result:

<u>Proposition 3</u>: Under Assumptions A and B, the NDE is the only separating equilibrium which can be supported by beliefs which are consistent with respect to an unprejudiced and intuitive test sequence.

<u>Proof</u>: In view of Proposition 2, we need only consider OSE, where $\hat{P}_i(L) \neq \hat{P}_i(H)$ for Firm i and $\hat{P}_j(L) = \hat{P}_j(H)$ for Firm j. Since the equilibrium is separating, $\hat{P}_i(L) = P_i^*(L)$ and $\hat{P}_j(L) = P_j^*(L)$ are necessary. Thus, if $P_j^*(L) \neq P_j^R(\hat{P}_i(H),H)$, the fact that $\hat{P}_i(L) \neq \hat{P}_i(H)$ allows us to use the argument of Proposition 2 to generate a deviation by (j,H) when beliefs are formed from an unprejudiced test sequence. Now suppose $P_j^*(L) = P_j^R(\hat{P}_i(H),H)$. The fact that $P_j^R(P_j^*(L),L) = P_j^*(L) = P_j^R(\hat{P}_i(H),H)$ guarantees $\hat{P}_i(H) < P_i^*(L) < \overline{P}_i$

(this is clear from Figure 2 with $i = 2$). We may use Assumptions A and B to find $P_i' > \hat{P}_i(H)$ such that:

$$\text{(10)} \qquad \Pi_i(P_i', P_j^*(L), L) < \Pi_i(\hat{P}_i(H), P_j^*(L), L)$$

$$\text{(11)} \qquad \Pi_i(P_i', P_j^*(L), H) > \Pi_i(\hat{P}_i(H), P_j^*(L), H) \quad .$$

Combining (10) with equilibrium condition (2) gives:

$$\begin{aligned} & \Pi_i(P_i', P_j^*(L), L) + \hat{E}(0)(\Pi^E(L) - \Pi^N(L)) \\ & < \Pi_i(\hat{P}_i(H), P_j^*(L), L) + \hat{E}(0)(\Pi^E(L) - \Pi^N(L)) \\ & \leq \Pi_i(\hat{P}_i(L), P_j^*(L), L) + \hat{E}(1)(\Pi^E(L) - \Pi^N(L)) \quad , \end{aligned}$$

so that (P_i', L) is equilibrium dominated. From (11) we have:

$$\text{(12)} \qquad \begin{aligned} & \Pi_i(P_i', P_j^*(L), H) + \hat{E}(0)(\Pi^E(H) - \Pi^N(H)) \\ & > \Pi_i(\hat{P}_i(H), P_j^*(L), L) + \hat{E}(0)(\Pi^E(H) - \Pi^N(H)) \quad , \end{aligned}$$

and (P_i', H) is equilibrium admissible. Thus, for an intuitive test sequence we have:

$$\lim_{n\to\infty} \frac{R_i^n(Z_i|L)}{R_i^n(Z_i|H)} = 0 \quad ,$$

for all sufficiently small simple events $(P_i', P_j^*(L)) \in Z_i x Z_j$. Posteriors (4) may be written:

$$\rho^n(Z_i, Z_j) = \frac{\rho R_j^n(Z_j|L)}{\rho R_j^n(Z_j|L) + (1-\rho) R_j^n(Z_j|H)[R_i^n(Z_i|H)/R_i^n(Z_i|L)]} .$$

Thus for any sufficiently small simple events $(P_i', P_j^*(L)) \in Z_i x Z_j$, we have:

$$\lim_{n\to\infty} \rho^n(Z_i, Z_j) = 0 \quad ,$$

and the corresponding consistent beliefs satisfy $\hat{\rho}(P_i', P_j^*(L)) = 0$. From (12) it follows that (i,H) deviates to P_i'.

Finally, the NDE can be supported by an intuitive test sequence since any $P_i \neq P_i^*(H)$ is an equilibrium dominated deviation for (i,H); thus, the corresponding beliefs will only reduce the weight which the entrant puts on $\omega = H$. To be more specific, define:

$$D_i = \{P_i | \Pi_i(P_i, P_j^*(L), L) + \hat{E}(0)(\Pi^E(L) - \Pi^N(L))$$

$$< \Pi_i(P_i^*(L), P_j^*(L), L) + \hat{E}(1)(\Pi^E(L) - \Pi^N(L))\} \quad .$$

That is, D_i gives prices such that (P_i, L) is equilibrium dominated. Note that for sufficiently small $\varepsilon > 0$, D_i does not intersect $(P_i^*(L) - \varepsilon, P_i^*(L) + \varepsilon)$. Let L_i be the Lebesgue measure of D_i. For $\{\varepsilon^n\}_{n=1}^{\infty} \subset \mathbb{R}_{++}$, $\varepsilon^n \to 0$, define the test sequence by:

$$\psi_i^n(P_i|L) = \begin{cases} (\varepsilon^n)^2 , & P_i \in D_i \\ [1 - (\varepsilon^n)^2 L_i - \varepsilon^n(\bar{P}_i - L_i - 2\varepsilon^n)]/2\varepsilon^n , & |P_i - P_i^*(L)| < \varepsilon^n \\ \varepsilon^n , & \text{otherwise} \end{cases}$$

$$\psi_i^n(P_i|H) = \begin{cases} (\varepsilon^n)^2 , & |P_i - P_i^*(H)| \geq \varepsilon^n \\ [1 - (\varepsilon^n)^2(\bar{P}_i - 2\varepsilon^n)]/2\varepsilon^n , & \text{otherwise} \end{cases}$$

This test sequence is clearly unprejudiced and intuitive. Moreover, it generates consistent beliefs which support the NDE, since $\hat{\rho}(P_i, P_j^*(L)) = 1$ for any P_i which could possibly be a profitable deviation for (i,L).

Q.E.D.

Thus, should separating equilibria of the form depicted in Figure 2 exist, the intuitive criterion gives a quite plausible argument for ruling them out. We conclude that the NDE is the focal separating equilibrium.

7. Pooling Equilibria

Thus far we have considered separating equilibria, but there may also exist pooling equilibria, in which $(\hat{P}_i(L), \hat{P}_j(L)) = (\hat{P}_i(H), \hat{P}_j(H))$. In pooling equilibria the entrant learns nothing from observing the preentry price, so the equilibrium probability of entry is given by $\hat{E}(\rho)$. Requiring unprejudiced conjectures has no force in pooling equilibria because neither firm can free ride on separation by the other. Further, the intuitive criterion is of limited use under the assumptions given up to now, since we cannot be sure that equilibrium dominated deviations under low costs are not also equilibrium dominated under high costs.

The key difference between OSE and pooling equilibria is that, in the former, the low-cost separating incumbent does not profit from choosing the high-cost equilibrium price, even though this would give the lowest probability of entry. This fact is used to construct a price increase which is equilibrium dominated under low costs, but not under high. With pooling, the equilibrium conditions permit no inferences as to which cost-type benefits more from a given entry-reducing price increase.

Thus, to invoke the intuitive criterion in the case of pooling equilibria, we must further strengthen the assumptions. First, we require that a version of the "single-crossing property" holds.[6] In particular, the marginal rate of substitution of price increases for reductions in the probability of entry must be greater under high costs:

<u>Assumption A'</u>: For all P_i, P_j:

$$- \frac{\frac{\partial}{\partial P_i}\Pi_i(P_i, P_j, H)}{\Pi^E(H) - \Pi^N(H)} > - \frac{\frac{\partial}{\partial P_i}\Pi_i(P_i, P_j, L)}{\Pi^E(L) - \Pi^N(L)} \quad .$$

This assumption is related to Assumption A, in that both presume that high-cost incumbents differentially prefer price increases. If we have:

$$\Pi^E(L) - \Pi^N(L) \leq \Pi^E(H) - \Pi^N(H) \quad ,$$

then Assumption A' implies Assumption A. Second, it must be the case that a price increase can be found which is equilibrium dominated under low costs. This means the low-cost incumbents must place sufficiently high value on preentry profits relative to entry deterrence:

<u>Assumption B'</u>: For all P_i, P_j with $P_i < \overline{P}_i$, there exists $P_i' > P_i$ such that:

$$\Pi_i(P_i', P_j, L) + \hat{E}(0)(\Pi^E(L) - \Pi^N(L)) < \Pi_i(P_i, P_j, L) + \hat{E}(\rho)(\Pi^E(L) - \Pi^N(L))$$

.

This strengthens Assumption B. Finally, we must rule out the possibility of pooling at the upper bound of possible prices, which would preclude upward price deviations by the high-cost incumbent:

<u>Assumption C</u>: For all P_j:

$$\Pi_i(\overline{P}_i, P_j, L) + \hat{E}(\rho)(\Pi^E(L) - \Pi^N(L)) < \Pi_i(P_i^R(P_j, L), P_j, L) + \hat{E}(1)(\Pi^E(L) - \Pi^N(L)) \quad .$$

<u>Proposition 4</u>: Under Assumptions A', B', and C, pooling equilibria cannot be supported by beliefs which are consistent with respect to an intuitive test sequence.

<u>Proof</u>: Let $(\hat{P}_i(L), \hat{P}_j(L)) = (\hat{P}_i(H), \hat{P}_j(H)) = (P_i^P, P_j^P)$. Under Assumption C we know $P_i^P < \bar{P}_i$, and using Assumptions A′ and B′ we can find $P_i' > P_i^P$ such that:

$$\Pi_i(P_i', P_j^P, L) + \hat{E}(0)(\Pi^E(L) - \Pi^N(L)) < \Pi_i(P_i^P, P_j^P, L) + \hat{E}(\hat{\rho}(P_i^P, P_j^P))(\Pi^E(L) - \Pi^N(L))$$

$$\Pi_i(P_i', P_j^P, H) + \hat{E}(0)(\Pi^E(H) - \Pi^N(H)) > \Pi_i(P_i^P, P_j^P, H) + \hat{E}(\hat{\rho}(P_i^P, P_j^P))(\Pi^E(H) - \Pi^N(H)) \tag{13}$$

(P_i', L) is equilibrium dominated, while (P_i', H) is equilibrium admissible so that observing P_i leads the entrant to place infinitely more weight on (i,L) when the test sequence is intuitive. It follows that $\hat{\rho}(P_i', P_j^P) = 0$ when beliefs are consistent with an intuitive test sequence, and (13) then guarantees deviation by (i,H).

Q.E.D.

In Proposition 4, the intuitive criterion is used in the standard way: High-cost incumbents differentially prefer price increases which lead to entry reductions, and this allows them to benefit from entry-reducing price increases which would not be contemplated under low costs.

8. <u>Example</u>

In this section we analyze the plausibility of our added assumptions by means of a simple differentiated-product oligopoly model proposed by Shubik and Levitan [1980]. The example demonstrates that when the assumptions break down, the distortions associated with OSE and pooling equilibria will tend to involve price <u>reductions</u> by the incumbent firms.

Suppose there is a representative consumer, whose utility in a given period is:

$$U = \frac{\alpha \sum_{i=1}^{n} q_i}{\beta} - \frac{\left(\sum_{i=1}^{n} q_i\right)^2}{2\beta} - \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left(q_i - q_j\right)^2}{4\beta(1 + \gamma)} - \sum_{i=1}^{n} P_i q_i \quad ,$$

where n is the number of firms, P_i is Firm i's price, q_i gives the number of units of Firm i's product purchased by the consumer in the period, and α, β, γ are positive parameters. Profits for each firm are given by:

$$\Pi_i = (P_i - \omega) q_i \quad .$$

For our purposes, there are two periods to consider. In period one we have $n = 2$ and $\alpha = \alpha_1$. For period two, $\alpha = \alpha_2 > \alpha_1$, with the growth in demand leading to the prospect of entry by a third firm. The value of n in period two is either two or three, depending on the entry decision. The parameters β, γ, and ω are constant across the two periods. ω may assume the values L,H, with:

$$0 < L < H < \frac{\alpha_1}{\beta} \quad .$$

Finally, $\overline{P}_i = \alpha/\beta$, i.e., the upper bound of possible prices is taken to be the lowest price at which Firm i is guaranteed to sell no units, no matter what prices are chosen by rival firms. It is easy to check that the assumptions of section two are satisfied by this example.

Let us first consider Assumptions A and B. For Firm i, the lowest price at which period one sales are zero, i.e., the period one "choke price," is given by:

$$P_i^C(P_j) = \frac{\alpha_1/\beta + \gamma P_j/2}{1 + \gamma/2} \quad .$$

Assumption A is satisfied as long as $P_i \le P_i^C(P_j)$. Once $P_i > P_i^C(P_j)$, profits are zero under both P_i and P_i', and the assumption fails to hold. This does not affect Proposition 3, however, since in the OSE we have $\hat{P}_i(H) < P_i^*(L) \le P_i^C(P_j^*(L))$, where the latter inequality follows from $L < \alpha_1/\beta$. Thus, starting at $\hat{P}_i(H)$, any price increase is differentially preferred by (i,H), and the intuitive criterion can be applied as long as there is some price increase which is equilibrium dominated for (i,L).

Assumption B holds if and only if $L < P_i < P_i^C(P_j)$; if $P_i \ge P_i^C(P_j)$, preentry profits are zero whether or not prices are increased, while for $P_i \le L$ the profits of (i,L) are nonpositive, and no $P_i' > P_i$ can give lower profits. The latter case creates difficulties for Proposition 3 and in fact the OSE may satisfy the intuitive criterion when $\hat{P}_i(H) \le L$.

Two conditions are necessary and sufficient for an OSE to satisfy the intuitive criterion. First, $\hat{P}_i(H) \le L$ must be consistent with $P_j^R(\hat{P}_i(H),H) = P_j^*(L)$. This means L and H must not be too close together to give $\hat{P}_i(H) > L$. Second, $(P_i^C(P_j^*(L)),L)$ must be equilibrium admissible. To see why this condition is necessary, note that $\hat{P}_i(H) \le L < H$ implies equilibrium admissibility of $(P_i^C(P_j^*(L)),H)$. Thus, the OSE would not be intuitive if $(P_i^C(P_j^*(L)),L)$ were equilibrium dominated. To rule this out, it is sufficient that α_2 be large enough, and the probability of entry be sensitive enough to the entrant's cost inference, so that reducing the probability from $\hat{E}(1)$ to $\hat{E}(0)$ increases (i,L)'s expected postentry profits by more than $\Pi_i(P_i^*(L),P_j^*(L),L)$. In particular, the density f(K) must put sufficient weight on the subinterval $(\Pi_3^E(H),\Pi_3^E(L))$ of entry costs under which entry occurs when costs are low, and not when costs are high.

It is interesting to note that intuitive OSE lead to pricing distortions in the form of reduced prices by both incumbents, when costs are high. The pooling incumbent imitates complete-information pricing which would arise under low costs, while the separating incumbent chooses price strictly below its unit costs. The latter pricing policy does not, however, represent predatory behavior by the separating incumbent against the pooling incumbent; in fact, the separating incumbent bears a disproportionate share of the burden of signaling.

The possibility of price reductions in intuitive separating equilibria stands in contrast to the price increases which arise when there is a single incumbent signaling a common cost parameter (e.g., Harrington [1986]). The key point is that if informational free riding is ruled out by rival pooling behavior, credible transfer of the cost information must involve price reduction by the high-cost incumbent, and reductions must be large if the equilibrium is to be intuitive. Of course, existence of an OSE is itself problematic (L and H cannot be too far apart, and profits from entry deterrence must be sufficiently great), and it seems reasonable that the most plausible separating outcome will involve no distortions at all.

Next, consider Assumptions A', B', and C. Because in this example the profit functions are strictly concave in own price, Assumption A' clearly holds for $P_i^R(P_j,L) \leq P_i \leq P_i^R(P_j,H)$. For $P_i^R(P_j,H) < P_i < P_i^C(P_j)$, the assumption is equivalent to:

$$\text{(14)} \qquad \frac{\alpha_1/\beta - 2(1 + \gamma/2)P_i + \gamma P_j/2 + (1 + \gamma/2)H}{\alpha_1/\beta - 2(1 + \gamma/2)P_i + \gamma P_j/2 + (1 + \gamma/2)L} < \left(\frac{\alpha_2 - \beta H}{\alpha_2 - \beta L}\right)^2 .$$

The right-hand side of (14) is less than one, reflecting the fact that reducing the probability of entry is differentially beneficial to the low-cost incumbents. At $P_i = P_i^R(P_j,H)$ the left-hand side of (14) is zero, but it increases as P_i rises. It is possible that (14) fails to hold for large P_i. Similarly, Assumption A' may fail for small $P_i < P_i^R(P_j,L)$. The assumption cannot hold when $P_i > P_i^C(P_j)$.

Assumption B' breaks down when the gains from entry deterrence are large relative to preentry profits, since the low-cost incumbent would be willing to accept any price increase in order to reduce the probability of entry from $\hat{E}(\rho)$ to $\hat{E}(0)$; similarly for Assumption C. Thus, the existence of intuitive pooling equilibria hinges on the comparison between preentry and postentry profits. If the former are relatively important, pooling equilibria will lie close to the incumbents' reaction functions, and small price increases will be equilibrium dominated for the low-cost incumbent; Assumption A' can then be invoked to find price increases which overturn the equilibria. If the latter are relatively important, then all price increases will be equilibrium admissible for the low-cost incumbent, or else price increases which are large enough to be equilibrium dominated under low costs will be equilibrium dominated under high costs, due to failure of Assumption A'.

Whether preentry or postentry profits are more important depends on the amount by which demand grows between the two periods, which in this setting is determined exogeneously. But the comparison also depends endogeneously on the preentry pricing policies of the incumbent firms. Choosing low prices serves to reduce the preentry profits, and pooling equilibria may therefore be intuitive precisely because pooling occurs at very low prices.

These points are illustrated in the parameterized examples in Figure 3.[7] In the examples, the densities f(K) are chosen to generate large gains from entry deterrence. As α_2 rises, the range of intuitive pooling equilibria rises; for sufficiently large α_2 all price pairs give intuitive pooling equilibria. Short of the latter case, intuitive pooling equilibria are associated with low prices, as illustrated in the examples. The key factor is that when one incumbent chooses a low price, the variability of the other incumbent's preentry profits is reduced: any price increase by the latter causes only a small change in preentry profits relative to the large possible change in postentry profits. There is then no price increase which decreases preentry profits sufficiently to be equilibrium dominated for the low-cost incumbent. This gives a limit pricing theory for growing markets, in which pooling behavior leads to price reductions, and equilibria are intuitive due to the rent dissipation associated with low prices.

9. Incentives to Collude

This section contains brief remarks on the implications of our model for the incentives of oligopolists to pursue collusive strategies. In the preceding analysis, we have assumed that preentry prices are determined noncooperatively. Let us consider the possibility that the incumbents collude in setting prices, and to compare profitability under the two pricing regimes. To this end, suppose that collusion means that the incumbents choose prices to maximize industry profits. Let $(P_1^M(\omega), P_2^M(\omega))$ be the unique maximizer of $\Pi_1(P_1, P_2, \omega) + \Pi_2(P_2, P_1, \omega)$ for each ω. The key difference between noncooperative and collusive pricing is that the incumbents can coordinate their preentry pricing under the latter, and in particular the low-cost incumbents can use joint defections to imitate high-cost equilibrium pricing. Thus, the following becomes a necessary condition for separating equilibrium with preentry collusion:

$$(15) \qquad \Pi_1(P_1^M(L), P_2^M(L), L) + \Pi_2(P_2^M(L), P_1^M(L), L) + 2\hat{E}(1)(\Pi^E(L) - \Pi^N(L))$$

$$\geq \Pi_1(\hat{P}_1(H), \hat{P}_2(H), L) + \Pi_2(\hat{P}_2(H), \hat{P}_1(H), L) + 2\hat{E}(0)(\Pi^E(L) - \Pi^N(L)) \quad .$$

Because of the need to satisfy (15), potential entry affects the incentives to adopt collusive pricing. This is illustrated in Figure 4, in which the collusive prices are graphed along with the noncooperative prices. Consider first the low-cost incumbents. Under noncooperative pricing, the NDE prices are $(P_1^*(L), P_2^*(L))$, while separation under collusion implies $(P_1^M(L), P_2^M(L))$. Thus, preentry profits are greater under collusion for the low-cost incumbents. This is not surprising, since low-cost incumbents do not distort their monopoly pricing under collusion.

Now consider the high-cost incumbents. If entry is quite important to the low-cost incumbents, then (15) will rule out many prices which the high-cost incumbents may wish to adopt; in Figure 4, prices inside the region enclosed by the dashed line fail (15). The shaded region indicates the prices which give the high-cost incumbents greater preentry profits than under the noncooperative solution. In the figure, the NDE under noncooperative pricing gives the high-cost incumbents greater preentry profits than does any separating equilibrium which could arise under collusion. This means that high-cost firms facing potential entry may have little incentive to collude, because of the large pricing distortion away from monopoly levels that collusion would require.

Collusion will also affect post-entry profits. These will be greater under collusion if there is no entry, but the prospect of post-entry collusion may lead to an increased probability of entry. We may conclude that potential entry reduces the returns from collusion, particularly for industries with above-average costs, and that *ex ante* profitability actually may be higher when pricing is noncooperative.

10. Conclusion

We have examined the extension of the information-based approach to limit pricing to the important possibility of multiple, uncoordinated incumbents. Assuming that the incumbents share private information, we find that the inability of incumbents to coordinate deception results in separating equilibria in which no distortion occurs. Moreover, the incentive that incumbents have to free ride on the signaling of others precludes the existence of other signaling equilibria in which both incumbants separate. Thus, the lack of coordination among incumbents results in a very focal equilibrium in which incumbents simply ignore the threat of entry and choose their complete-information, Nash prices.

This work breaks new ground in the context of the signaling literature. To our knowledge, no other work has explored the possibility of several signal senders who possess common, private information and who simultaneously choose observable signals.[8] The no distortion equilibrium appears to be a robust equilibrium for such models. Its existence does not require a "single crossing property" to hold. Further, in the limit pricing context, the no distortion equilibrium exists whether incumbents choose observable quantities or prices, have costs which are or are not independent of entrant costs, or have common, private information about costs or demand.

The assumption that incumbents have common, private information seems quite plausible. One can imagine that an entrant is often interested in rather coarse information, such as whether input prices are high or low or whether demand will profitably support entry or not, and incumbents will generally agree about this information. For such settings, the informational theory of limit pricing is correct in predicting that entry will occur exactly when it is profitable, but incorrect in claiming that this process involves a distortion in preentry pricing.

In other settings, however, it may be plausible to assume that each incumbent has idiosyncratic private information as well. Pricing distortions clearly will re-appear in this environment. Useful future work might search for a general relation between the degree of idiosyncratic information and the size of the pricing distortion.

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Notes

1. See Gilbert and Vives [1986] and Waldman [1987] for multiple incumbent models in which output is committed before entry. Vives [1982] explores this model with sequential output choices. Dixit [1980], Eaton and Lipsey [1980], Fudenberg and Tirole [1983], and Spence [1977] construct models with strategic capacity choices. Related models with multiple incumbents and a dynamic structure are studied by Bernheim [1984], Eaton and Ware [1987], and McLean and Riordan [1989]. Finally, the entry-deterring role of product proliferation is analyzed by Bonanno [1987], Eaton and Lipsey [1979], Judd [1985], Prescott and Visscher [1977], and Schmalensee [1978].

2. See, for example, Bagwell and Ramey [1988; forthcoming] for the possibility of signaling costs and demand with price and advertising expenditures, Cho [1987] for a careful analysis of the single crossing property, Matthews and Mirman [1982] for a model in which price is observed with noise, Ramey [1987] for an analysis that includes a capacity choice, and Roberts [1987] for a survey of an extended literature.

3. A public good problem is thus associated with entry deterrence. Incumbents would like to deter entry by signaling high costs with very high prices, but free rider effects result in signaling at lower prices. This "underinvestment" in entry deterrence is similar to results found by Harrington, but opposite of those found by Gilbert and Vives.

4. This belief is actually very plausible, since (j,H) could never improve upon equilibrium profits with a deviation. We discuss standards of plausibility more carefully below.

5. Note that existence of the no distortion equilibrium does not require a single crossing property to hold.

6. See Cho and Ramey for more on the importance of the single crossing property in the limit pricing context.

7. The shape of the set of pooling equilibria is easily understood. For each (i,ω), one can imagine a band about $P_i^R(P_j,\omega)$ capturing the prices at which (i,ω) would be willing to pool. The intersection of all such bands then gives the set of pooling equilibria, which has a diamond shape as shown in Figure 3A. The diamond is distorted partially in Figures 3B and 3C, because under these parameterizations the band about (i,H)'s reaction curve hits the choke price, meaning that (i,H) will pool at the choke price and hence any higher price.

8. Fertig and Matthews [1989] examine a model in which an entrant's quality of product is known by the entrant and the incumbent, who successively choose advertising levels. This sequential structure results in separating equilibria in which a very small amount of entrant advertising is sufficient to signal high quality. Mimicry by the low-quality entrant fails to occur, because of the low-quality incumbent's threat to "counteract" any false entrant signal with a sufficiently high level of incumbent advertising.

Figure 1

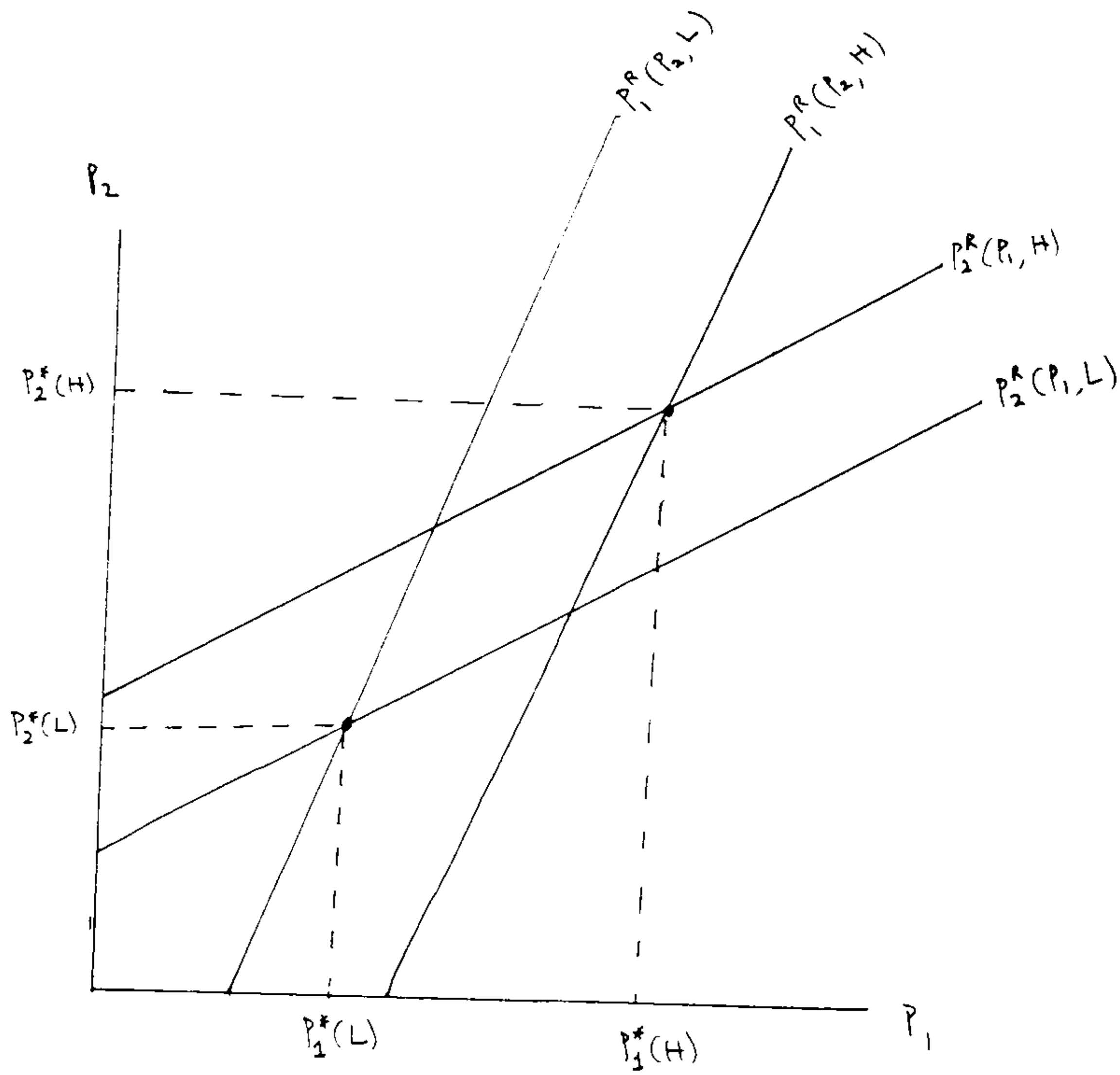
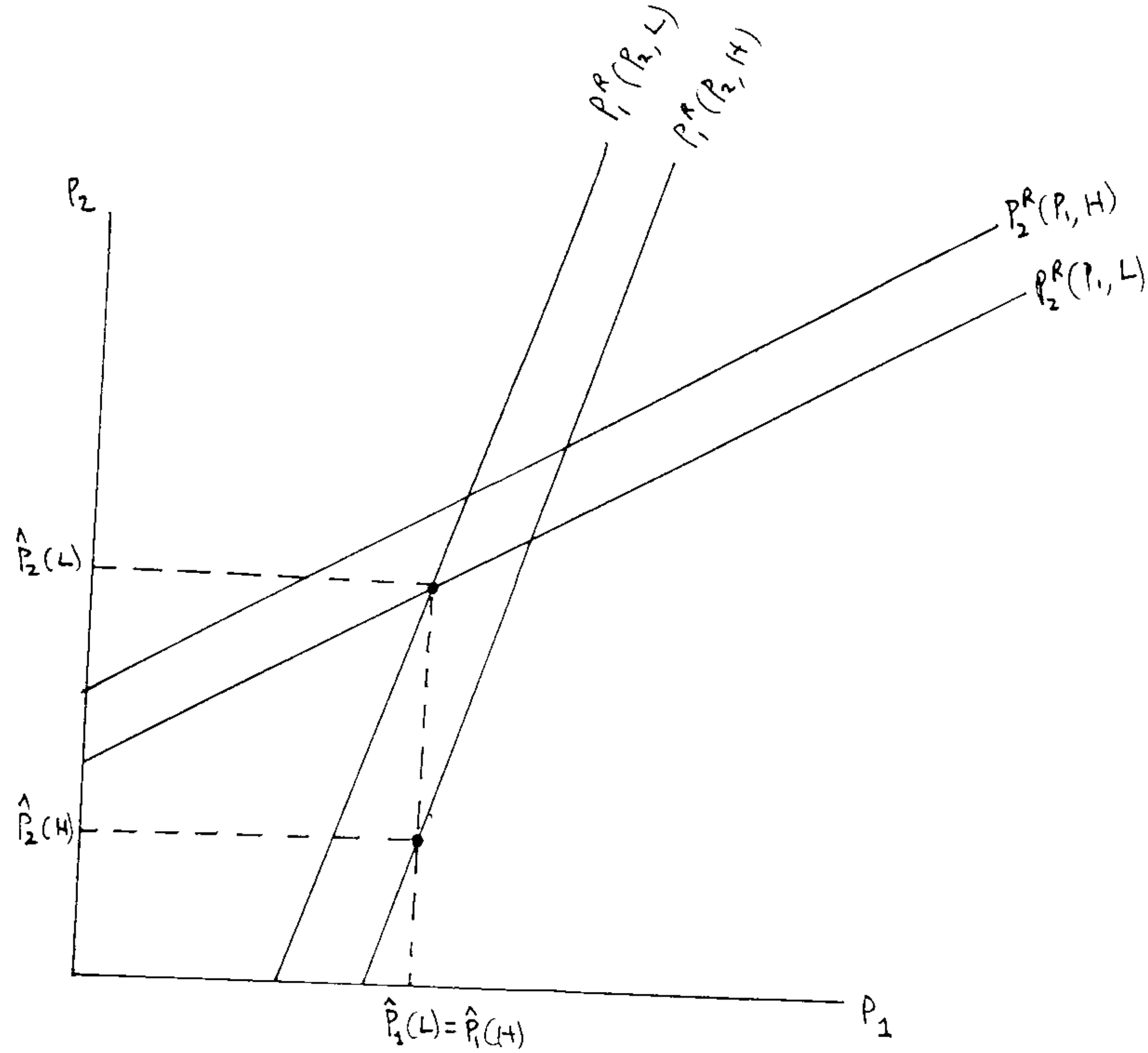
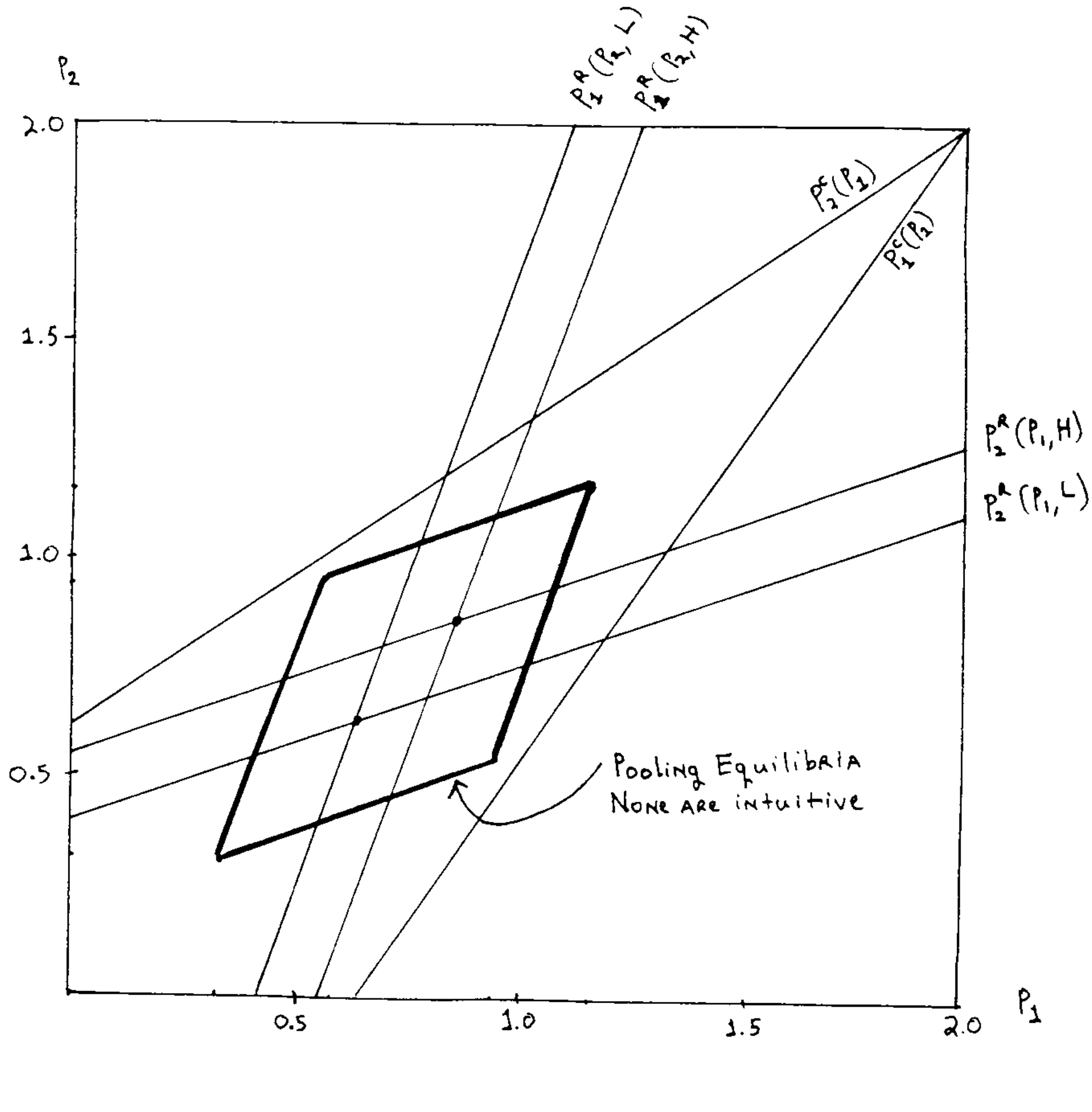


Figure 2



$p_1^R(p_2, L)$
$p_1^R(p_2, H)$
$p_2^R(p_1, H)$
$p_2^R(p_1, L)$
p_2
$\hat{p}_2(L)$
$\hat{p}_2(H)$
$\hat{p}_1(L) = \hat{p}_1(H)$
p_1

FIGURE 3A



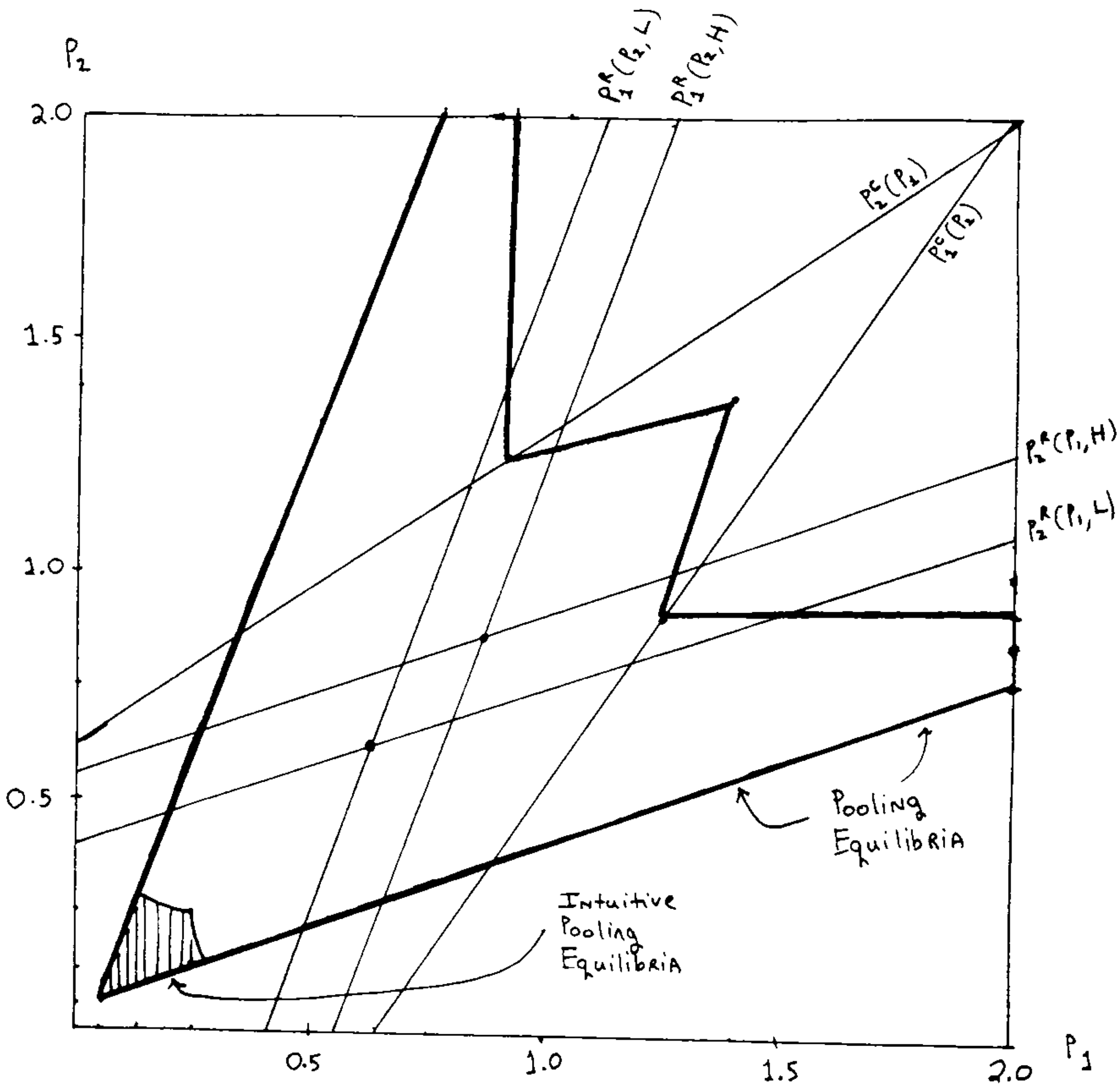
PARAMETERIZATION:

$\alpha_1 = 10, \ \alpha_2 = 20, \ \beta = 5, \ \gamma = 5$

$H = 0.5, \ L = 0.2, \ \rho = 0.5$

$$f(K) = \begin{cases} .15, & 0 \le K \le 3.4 \\ 2.15, & 3.4 < K < 3.6 \\ .15, & 3.6 \le K \le 4 \end{cases}$$

FIGURE 3B



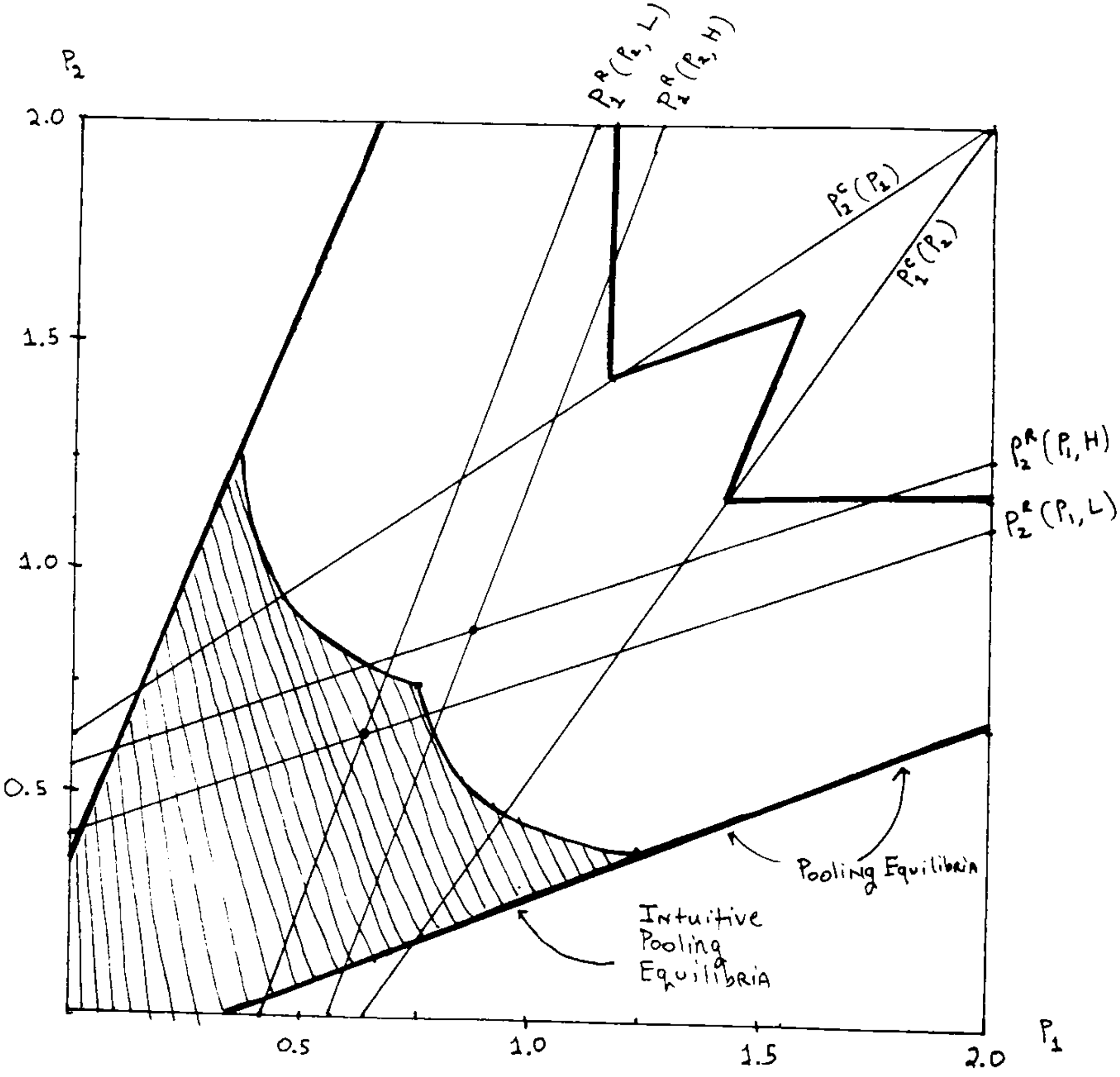
PARAMETERIZATION:

$\alpha_1 = 10$, $\alpha_2 = 30$, $\beta = 5$, $\gamma = 5$

$H = 0.5$, $L = 0.2$, $\rho = 0.5$

$$f(K) = \begin{cases} 0.07, & 0 \le K \le 8.2 \\ 1.3033, & 8.2 < K < 8.5 \\ 0.07, & 8.5 \le K \le 9 \end{cases}$$

FIGURE 3C



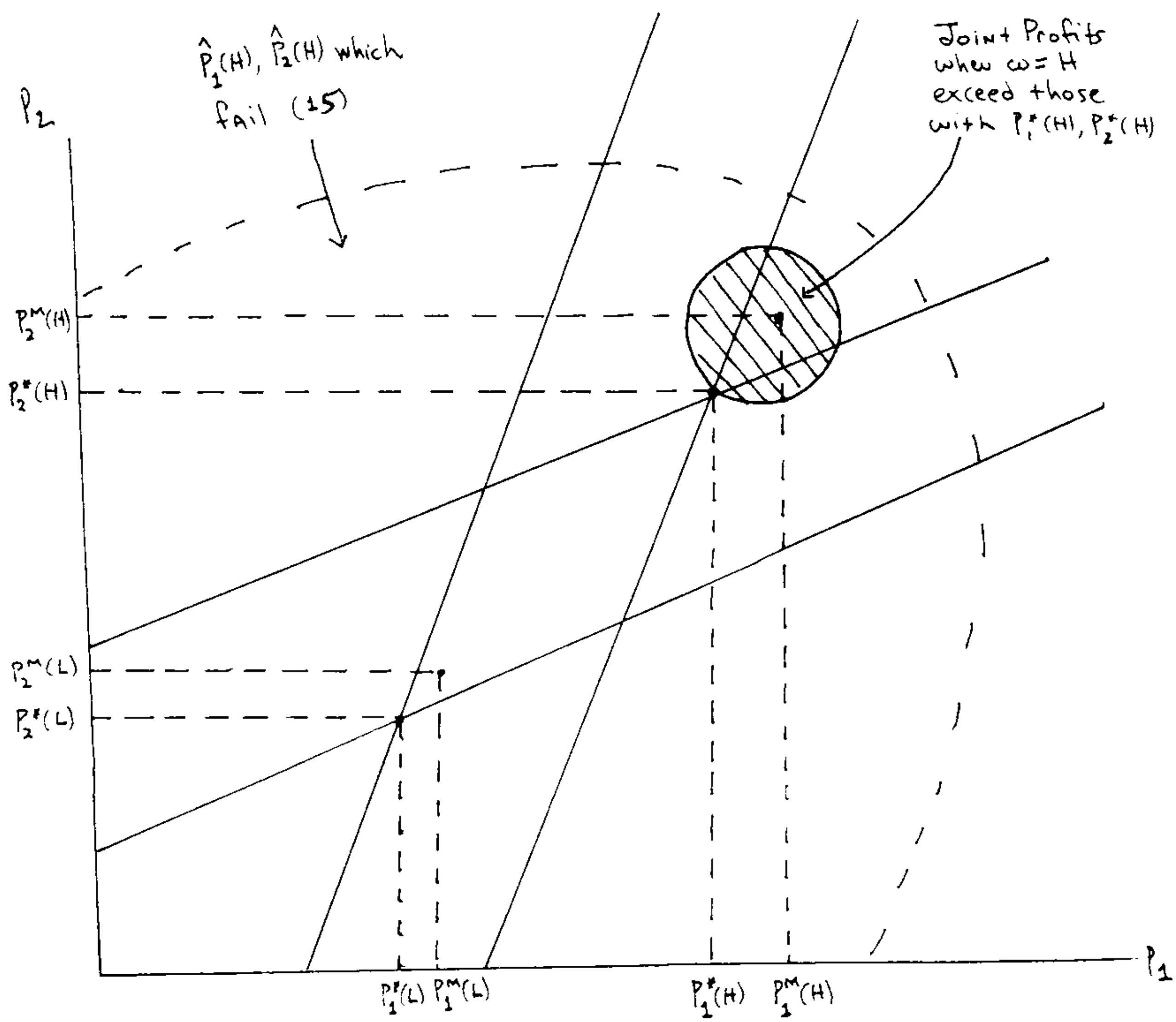
PARAMETERIZATION:

$\alpha_1 = 10$, $\alpha_2 = 50$, $\beta = 5$, $\gamma = 5$

$H = 0.5$, $L = 0.2$, $\rho = 0.5$

$$f(K) = \begin{cases} 0.03, & 0 \le K \le 23.5 \\ 0.53, & 23.5 < K < 24 \\ 0.03, & 24 \le K \le 25 \end{cases}$$

Figure 4


$\hat{P}_1(H)$, $\hat{P}_2(H)$ which fail (15)
Joint Profits when $\omega = H$ exceed those with $P_1^*(H)$, $P_2^*(H)$
P_2
$P_2^m(H)$
$P_2^*(H)$
$P_2^m(L)$
$P_2^*(L)$
P_1
$P_1^*(L)$
$P_1^m(L)$
$P_1^*(H)$
$P_1^m(H)$