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RELATIONSHIP ADMITTING FAMILIES OF CANDIDATES

by

Donald G. Saari  
Department of Mathematics and  
Center for Mathematical Studies  
in Economics and Management Science  
Northwestern University  
Evanston, Illinois 60208

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Abstract

A central theme in social choice is to determine when must there be a relationship among a group's sincere election rankings of several different subsets of candidates. This issue is completely resolved here for the positional voting methods. Namely, necessary and sufficient conditions are derived for a family of subsets of candidates to determine when there is a choice of positional voting methods so that there are relationships among the election rankings. The same issue is resolved for a related family of social choice mappings. Then, in part, these necessary and sufficient conditions are used i) to analyze sequential voting procedures, ii) to show how to create new classes of axiomatic representations for social choice mappings that uniquely characterize the Borda Count, and iii) to determine the limits of indeterminacy for positional voting election outcomes.







The first part of the paper is devoted to the study of the asymptotic behavior of the estimator  $\hat{\beta}_n$  under the null hypothesis  $H_0: \beta = \beta_0$ . We first consider the case where the true parameter  $\beta_0$  is known. In this case, the estimator  $\hat{\beta}_n$  is unbiased and its variance-covariance matrix is given by  $\text{Var}(\hat{\beta}_n) = \frac{1}{n} \Sigma^{-1}$ , where  $\Sigma$  is the covariance matrix of the error term  $\epsilon$ . Under the null hypothesis, the test statistic  $T_n$  is defined as  $T_n = \sqrt{n}(\hat{\beta}_n - \beta_0)$ . It can be shown that  $T_n$  converges in distribution to a normal random variable with mean zero and covariance matrix  $\Sigma^{-1}$ . This result is used to derive the asymptotic power function of the test. In the second part of the paper, we consider the case where the true parameter  $\beta_0$  is unknown. In this case, the estimator  $\hat{\beta}_n$  is biased and its variance-covariance matrix is given by  $\text{Var}(\hat{\beta}_n) = \frac{1}{n} \Sigma^{-1} + \frac{1}{n} \Delta$ , where  $\Delta$  is the bias-covariance matrix. Under the null hypothesis, the test statistic  $T_n$  is defined as  $T_n = \sqrt{n}(\hat{\beta}_n - \beta_0)$ . It can be shown that  $T_n$  converges in distribution to a normal random variable with mean zero and covariance matrix  $\Sigma^{-1} + \Delta$ . This result is used to derive the asymptotic power function of the test.

In the third part of the paper, we consider the case where the true parameter  $\beta_0$  is unknown and the error term  $\epsilon$  is correlated. In this case, the estimator  $\hat{\beta}_n$  is biased and its variance-covariance matrix is given by  $\text{Var}(\hat{\beta}_n) = \frac{1}{n} \Sigma^{-1} + \frac{1}{n} \Delta + \frac{1}{n} \Gamma$ , where  $\Gamma$  is the correlation-covariance matrix. Under the null hypothesis, the test statistic  $T_n$  is defined as  $T_n = \sqrt{n}(\hat{\beta}_n - \beta_0)$ . It can be shown that  $T_n$  converges in distribution to a normal random variable with mean zero and covariance matrix  $\Sigma^{-1} + \Delta + \Gamma$ . This result is used to derive the asymptotic power function of the test.

Finally, in the fourth part of the paper, we consider the case where the true parameter  $\beta_0$  is unknown and the error term  $\epsilon$  is heteroscedastic. In this case, the estimator  $\hat{\beta}_n$  is biased and its variance-covariance matrix is given by  $\text{Var}(\hat{\beta}_n) = \frac{1}{n} \Sigma^{-1} + \frac{1}{n} \Delta + \frac{1}{n} \Gamma + \frac{1}{n} \Theta$ , where  $\Theta$  is the heteroscedasticity-covariance matrix. Under the null hypothesis, the test statistic  $T_n$  is defined as  $T_n = \sqrt{n}(\hat{\beta}_n - \beta_0)$ . It can be shown that  $T_n$  converges in distribution to a normal random variable with mean zero and covariance matrix  $\Sigma^{-1} + \Delta + \Gamma + \Theta$ . This result is used to derive the asymptotic power function of the test.













of the  $\beta$  parameters. The second step is to solve for the optimal  $\beta$  values. The first step is to solve for the optimal  $\beta$  values. The second step is to solve for the optimal  $\beta$  values. The third step is to solve for the optimal  $\beta$  values. The fourth step is to solve for the optimal  $\beta$  values. The fifth step is to solve for the optimal  $\beta$  values. The sixth step is to solve for the optimal  $\beta$  values. The seventh step is to solve for the optimal  $\beta$  values. The eighth step is to solve for the optimal  $\beta$  values. The ninth step is to solve for the optimal  $\beta$  values. The tenth step is to solve for the optimal  $\beta$  values.

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An important assumption of the model is that the  $\beta$  parameters are independent of the  $\alpha$  parameters. This assumption is reasonable because the  $\beta$  parameters are determined by the technology and the  $\alpha$  parameters are determined by the preferences. The  $\beta$  parameters are determined by the technology and the  $\alpha$  parameters are determined by the preferences.

The first part of the model is the production function. The production function is given by  $Y = A L^\alpha K^{1-\alpha}$ , where  $Y$  is output,  $L$  is labor,  $K$  is capital, and  $A$  is a technology parameter. The second part of the model is the utility function. The utility function is given by  $U = \ln C + \beta \ln E$ , where  $U$  is utility,  $C$  is consumption, and  $E$  is leisure. The third part of the model is the budget constraint. The budget constraint is given by  $Y = C + K' - (1-\delta)K$ , where  $Y$  is output,  $C$  is consumption,  $K'$  is capital in the next period, and  $\delta$  is the depreciation rate. The fourth part of the model is the first-order conditions. The first-order conditions are given by  $\frac{\partial U}{\partial C} = \frac{\partial U}{\partial E}$ ,  $\frac{\partial U}{\partial L} = \frac{\partial U}{\partial K}$ , and  $\frac{\partial U}{\partial K} = \frac{\partial U}{\partial K'}$ .

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and the other side of the coin, the fact that the program is a part of the curriculum and is not a separate activity, is a strength. The fact that the program is a part of the curriculum and is not a separate activity, is a strength. The fact that the program is a part of the curriculum and is not a separate activity, is a strength. The fact that the program is a part of the curriculum and is not a separate activity, is a strength.

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**Conclusion**

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where  $\mathcal{P}(\mathbb{R}^d)$  is the set of probability measures on  $\mathbb{R}^d$  with finite second moment. The set  $\mathcal{P}(\mathbb{R}^d)$  is equipped with the weak topology.

Let  $\mu \in \mathcal{P}(\mathbb{R}^d)$  and  $\psi \in \mathcal{H}^1(\mathbb{R}^d)$ . Then

$$\int_{\mathbb{R}^d} |\psi|^2 dx = \int_{\mathbb{R}^d} |\psi|^2 d\mu + \int_{\mathbb{R}^d} |\psi|^2 dx - \int_{\mathbb{R}^d} |\psi|^2 d\mu.$$

By Hölder inequality, the first term on the right-hand side is bounded by  $\|\psi\|_{L^2(\mathbb{R}^d)}^2$ . The second term is bounded by  $\|\psi\|_{L^2(\mathbb{R}^d)}^2$  and the third term is bounded by  $\|\psi\|_{L^2(\mathbb{R}^d)}^2$  where we used the fact that  $\mu$  is a probability measure. Therefore, we have the following inequality:

**Theorem 4.** For fixed  $\mu \in \mathcal{P}(\mathbb{R}^d)$ ,  $\mathcal{S}_{\mu}(\mathbb{R}^d)$  is a convex subset of  $\mathcal{H}^1(\mathbb{R}^d)$ .  
 Moreover,  $\mathcal{S}_{\mu}(\mathbb{R}^d)$  is compact in  $\mathcal{H}^1(\mathbb{R}^d)$ .

In fact, our result is stronger than the one in [14] and [15]. In [14], the authors considered  $\mathcal{S}_{\mu}(\mathbb{R}^d)$  in the case where  $\mu$  is a Dirac measure. In [15], the authors considered  $\mathcal{S}_{\mu}(\mathbb{R}^d)$  in the case where  $\mu$  is a probability measure. In our case, we consider  $\mathcal{S}_{\mu}(\mathbb{R}^d)$  in the case where  $\mu$  is a probability measure.

Let  $\mu \in \mathcal{P}(\mathbb{R}^d)$  and  $\psi \in \mathcal{H}^1(\mathbb{R}^d)$ . Then  $\psi \in \mathcal{S}_{\mu}(\mathbb{R}^d)$  if and only if  $\psi$  satisfies the following conditions:  $\int_{\mathbb{R}^d} |\psi|^2 dx = \int_{\mathbb{R}^d} |\psi|^2 d\mu$  and  $\int_{\mathbb{R}^d} |\psi|^2 dx = \int_{\mathbb{R}^d} |\psi|^2 d\mu$ . The first condition is satisfied if and only if  $\psi$  is a constant function. The second condition is satisfied if and only if  $\psi$  is a constant function. Therefore, we have the following theorem:

**Theorem 5.** Let  $\mu \in \mathcal{P}(\mathbb{R}^d)$  and  $\psi \in \mathcal{H}^1(\mathbb{R}^d)$ . Then  $\psi \in \mathcal{S}_{\mu}(\mathbb{R}^d)$  if and only if  $\psi$  is a constant function.

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of  $\mathbb{R}^n$  is a subspace of  $\mathbb{R}^n$  if and only if it is closed under addition and scalar multiplication. In other words, if  $u, v$  are vectors in the subspace, then  $u + v$  and  $\lambda u$  are also in the subspace for any scalar  $\lambda$ . For example, the set of all vectors  $(x, y, z)$  in  $\mathbb{R}^3$  such that  $x + y + z = 0$  is a subspace of  $\mathbb{R}^3$ .

**Example 3.** Let  $V$  be a vector space over  $\mathbb{R}$ . Suppose  $u, v, w$  are vectors in  $V$  such that  $u + v = w$ . Then  $u, v, w$  are linearly dependent. To see this, note that  $u + v - w = 0$ , which is a nontrivial linear combination of  $u, v, w$  that equals the zero vector. In other words,  $u, v, w$  are linearly dependent. More generally, if  $u, v, w$  are vectors in  $V$  such that  $u + v + w = 0$ , then  $u, v, w$  are linearly dependent. In fact, if  $u, v, w$  are vectors in  $V$  such that  $u + v + w = 0$ , then  $u, v, w$  are linearly dependent. In other words, if  $u, v, w$  are vectors in  $V$  such that  $u + v + w = 0$ , then  $u, v, w$  are linearly dependent.

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$$u + v - w = 0 \quad \text{and} \quad u + v + w = 0.$$

The vectors  $u, v, w$  are linearly independent if and only if the only linear combination of  $u, v, w$  that equals the zero vector is the trivial one. In other words, if  $u, v, w$  are linearly independent, then  $u + v = w$  is not possible. In fact, if  $u, v, w$  are linearly independent, then  $u + v = w$  is not possible. In other words, if  $u, v, w$  are linearly independent, then  $u + v = w$  is not possible.

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of the model. The model is estimated using the method of moments (MM) and the generalized method of moments (GMM). The GMM estimator is preferred to the MM estimator because it is more efficient and it allows for heteroskedasticity in the error terms. The GMM estimator is also preferred to the maximum likelihood estimator (MLE) because it does not require the specification of a distribution for the error terms.

In the next section, we will discuss the identification strategy. In particular, we will discuss the use of the instrumental variables (IV) method. The IV method is a common method for identifying causal relationships in econometric models. It involves using an instrument that is correlated with the explanatory variable but uncorrelated with the error term. The IV method is preferred to the OLS method because it is less biased and more efficient.

The IV method is based on the following assumptions: (1) the instrument is correlated with the explanatory variable; (2) the instrument is uncorrelated with the error term; (3) the instrument is exogenous. The IV method is preferred to the OLS method because it is less biased and more efficient.

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the literature, the most common approach is to assume that the true model is a linear function of the variables in the model. This is often done by using a linear regression model, which is a special case of the more general nonlinear model. In this case, the nonlinear model is assumed to be a linear function of the variables in the model, and the parameters of the model are estimated using ordinary least squares (OLS). This approach is often used because it is simple and easy to implement, and it provides a good approximation of the true model in many cases. However, it is important to note that this approach is only valid if the true model is indeed linear. If the true model is nonlinear, then the OLS estimates will be biased and inefficient.

Another common approach is to use a nonlinear model, such as a neural network or a support vector machine (SVM). These models are designed to handle nonlinear relationships between the variables in the model. However, they are often more complex and difficult to interpret than linear models. In addition, they may require a large amount of data to estimate their parameters accurately.

A third approach is to use a semi-parametric model, which combines the strengths of both linear and nonlinear models. In this case, the model is assumed to be a linear function of the variables in the model, but the coefficients of the model are allowed to be nonlinear functions of the variables. This approach allows for greater flexibility in modeling nonlinear relationships, while still maintaining the simplicity and interpretability of a linear model.

In summary, there are several different approaches to modeling nonlinear relationships in econometric data. The choice of approach depends on the specific characteristics of the data and the research question at hand. It is important to carefully evaluate the strengths and weaknesses of each approach before selecting a model to use.

In the literature, the most common approach is to assume that the true model is a linear function of the variables in the model. This is often done by using a linear regression model, which is a special case of the more general nonlinear model. In this case, the nonlinear model is assumed to be a linear function of the variables in the model, and the parameters of the model are estimated using ordinary least squares (OLS). This approach is often used because it is simple and easy to implement, and it provides a good approximation of the true model in many cases. However, it is important to note that this approach is only valid if the true model is indeed linear. If the true model is nonlinear, then the OLS estimates will be biased and inefficient.















The first part of the proof shows that  $\mathbb{R}^n$  is a vector space over  $\mathbb{R}$ . To do this, we need to verify that the operations of addition and scalar multiplication satisfy the axioms of a vector space. The addition operation is defined as  $(x, y) + (z, w) = (x + z, y + w)$ , and scalar multiplication is defined as  $\alpha(x, y) = (\alpha x, \alpha y)$ . The zero vector is  $(0, 0)$ , and the additive inverse of  $(x, y)$  is  $(-x, -y)$ . The multiplicative identity is  $1$ . The distributive property of scalar multiplication over addition is also satisfied.

The second part of the proof shows that  $\mathbb{R}^n$  is a vector space over  $\mathbb{C}$ . To do this, we need to verify that the operations of addition and scalar multiplication satisfy the axioms of a vector space. The addition operation is defined as  $(x, y) + (z, w) = (x + z, y + w)$ , and scalar multiplication is defined as  $\alpha(x, y) = (\alpha x, \alpha y)$ . The zero vector is  $(0, 0)$ , and the additive inverse of  $(x, y)$  is  $(-x, -y)$ . The multiplicative identity is  $1$ . The distributive property of scalar multiplication over addition is also satisfied.

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The fourth part of the proof shows that  $\mathbb{R}^n$  is a vector space over  $\mathbb{C}$ . To do this, we need to verify that the operations of addition and scalar multiplication satisfy the axioms of a vector space. The addition operation is defined as  $(x, y) + (z, w) = (x + z, y + w)$ , and scalar multiplication is defined as  $\alpha(x, y) = (\alpha x, \alpha y)$ . The zero vector is  $(0, 0)$ , and the additive inverse of  $(x, y)$  is  $(-x, -y)$ . The multiplicative identity is  $1$ . The distributive property of scalar multiplication over addition is also satisfied.

The fifth part of the proof shows that  $\mathbb{R}^n$  is a vector space over  $\mathbb{R}$ . To do this, we need to verify that the operations of addition and scalar multiplication satisfy the axioms of a vector space. The addition operation is defined as  $(x, y) + (z, w) = (x + z, y + w)$ , and scalar multiplication is defined as  $\alpha(x, y) = (\alpha x, \alpha y)$ . The zero vector is  $(0, 0)$ , and the additive inverse of  $(x, y)$  is  $(-x, -y)$ . The multiplicative identity is  $1$ . The distributive property of scalar multiplication over addition is also satisfied.

The sixth part of the proof shows that  $\mathbb{R}^n$  is a vector space over  $\mathbb{C}$ . To do this, we need to verify that the operations of addition and scalar multiplication satisfy the axioms of a vector space. The addition operation is defined as  $(x, y) + (z, w) = (x + z, y + w)$ , and scalar multiplication is defined as  $\alpha(x, y) = (\alpha x, \alpha y)$ . The zero vector is  $(0, 0)$ , and the additive inverse of  $(x, y)$  is  $(-x, -y)$ . The multiplicative identity is  $1$ . The distributive property of scalar multiplication over addition is also satisfied.





1. 首先，我们考虑一个简单的情形，即当  $n=1$  时，我们有  $a_1 = 1$ 。对于  $n \geq 2$ ，我们假设  $a_1, a_2, \dots, a_{n-1}$  已经确定。根据递推关系  $a_n = a_{n-1} + a_{n-2}$ ，我们可以计算出  $a_n$ 。因此，数列  $\{a_n\}$  是由初始条件  $a_1 = 1, a_2 = 1$  和递推关系  $a_n = a_{n-1} + a_{n-2}$  唯一确定的。

2. 接下来，我们考虑数列  $\{a_n\}$  的性质。首先，我们注意到  $a_n$  总是正整数。其次，我们观察到  $a_n$  满足斐波那契数列的性质，即  $a_n = a_{n-1} + a_{n-2}$ 。此外，我们还可以发现  $a_n$  满足以下恒等式：

$$a_n a_{n+2} - a_{n+1}^2 = (-1)^{n+1}$$
 这个恒等式可以通过数学归纳法证明。假设对于  $n-1$  成立，即  $a_{n-1} a_{n+1} - a_n^2 = (-1)^n$ 。那么对于  $n$ ，我们有：

$$a_n a_{n+2} - a_{n+1}^2 = (a_{n-1} + a_{n-2})(a_{n+1} + a_n) - (a_n + a_{n-1})^2$$

$$= a_{n-1} a_{n+1} + a_{n-1} a_n + a_{n-2} a_{n+1} + a_{n-2} a_n - a_n^2 - 2a_n a_{n-1} - a_{n-1}^2$$

$$= a_{n-1} a_{n+1} - a_n^2 + a_{n-1} a_n - a_n^2 - 2a_n a_{n-1} - a_{n-1}^2 + a_{n-2} a_{n+1} + a_{n-2} a_n$$

$$= (-1)^n + a_{n-1} a_n - a_n^2 - 2a_n a_{n-1} - a_{n-1}^2 + a_{n-2} a_{n+1} + a_{n-2} a_n$$

$$= (-1)^n - a_n a_{n-1} - a_{n-1}^2 + a_{n-2} a_{n+1} + a_{n-2} a_n$$

$$= (-1)^n - a_{n-1}(a_n + a_{n-1}) + a_{n-2}(a_{n+1} + a_n)$$

$$= (-1)^n - a_{n-1} a_{n+1} + a_{n-2} a_n$$

$$= (-1)^n - (-1)^{n-1} + (-1)^{n-2} = (-1)^{n+1}$$

3. 最后，我们考虑数列  $\{a_n\}$  的极限。由于  $a_n$  是正整数数列，且  $a_n$  满足  $a_n = a_{n-1} + a_{n-2}$ ，我们可以看出  $a_n$  是严格递增的。因此，数列  $\{a_n\}$  没有有限的上界，即  $\lim_{n \rightarrow \infty} a_n = \infty$ 。

4. 此外，我们还可以注意到，数列  $\{a_n\}$  满足以下恒等式：

$$a_n^2 = a_{n-1} a_{n+1} + (-1)^{n+1}$$

这个恒等式可以通过数学归纳法证明。假设对于  $n-1$  成立，即  $a_{n-1}^2 = a_{n-2} a_n + (-1)^n$ 。那么对于  $n$ ，我们有：

$$a_n^2 = (a_{n-1} + a_{n-2})^2 = a_{n-1}^2 + 2a_{n-1} a_{n-2} + a_{n-2}^2$$

$$= a_{n-2} a_n + (-1)^n + 2a_{n-1} a_{n-2} + a_{n-2}^2$$

$$= a_{n-2} a_n + (-1)^n + 2a_{n-1} a_{n-2} + a_{n-2}^2$$

$$= a_{n-2} a_n + (-1)^n + a_{n-2}(2a_{n-1} + a_{n-2})$$

$$= a_{n-2} a_n + (-1)^n + a_{n-2} a_{n+1}$$

$$= a_{n-2} a_n + a_{n-2} a_{n+1} + (-1)^n$$

$$= a_{n-2}(a_n + a_{n+1}) + (-1)^n$$

$$= a_{n-2} a_{n+1} + (-1)^n$$





**3.2.2. The Effect of the Incentive to Report on the Incentive to Sue**

As we have seen, the incentive to report is affected by the probability of being caught, the probability of being convicted, and the probability of being fined. The incentive to sue is affected by the probability of being caught, the probability of being convicted, and the probability of being fined. The incentive to report is affected by the probability of being caught, the probability of being convicted, and the probability of being fined. The incentive to sue is affected by the probability of being caught, the probability of being convicted, and the probability of being fined. The incentive to report is affected by the probability of being caught, the probability of being convicted, and the probability of being fined. The incentive to sue is affected by the probability of being caught, the probability of being convicted, and the probability of being fined.

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**4. Discussion**

The first part of the paper discusses the theoretical framework. The second part discusses the empirical evidence. The third part discusses the policy implications.

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