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INFORMATION REVELATION IN A MARKET WITH PAIRWISE MEETINGS

by

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Abstract

The paper presents a simple pairwise meetings model of trade. The new feature is that agents have asymmetric information about the true state of the world. The focus is on the transmission of the information through the process of trade. The qualitative question: to what extent is the information revealed to uninformed agents through the trading process, when the market is in some sense frictionless? In particular: does the decentralized process give rise to full revelation results as derived by the literature on rational expectations for centralized and competitive environments? In the context of the model of this paper, it turns out that the information is not fully revealed to uninformed agents, even when the market is in some sense approximately frictionless.

1. Introduction

Much of the received economic analysis is concerned with markets characterized by centralized trading. In such markets the trades are carried out at publicly announced prices and all traders have access to the same trading opportunities. In many important markets, however, the trading process is decentralized--prices are quoted and transactions are concluded in private meetings among agents. The present paper considers a market in which the trading process is both decentralized and takes place under conditions of asymmetric information. The main objective is to study the transmission of information via the trade process in such a market.

Under conditions of asymmetric information the process of trade itself may transmit some of the relevant information, and agents may attempt to extract it before they trade. This insight motivates, of course, the literature on rational expectations, which observes that in a market characterized by centralized trading the price may aggregate and transmit information. The concept of Rational Expectations Equilibrium (REE) refers to a centralized market which is also competitive and requires that agents indeed utilize whatever information that can be extracted from the equilibrium price.

In markets in which the trading process is decentralized, prices may not be publicly observed and it is of interest to investigate other forms of transmission of information via the trade process. The present paper investigates the transmission of information through the process of trade in a market in which transactions are concluded in pairwise meetings of agents. The agents have asymmetric information on some underlying parameter which affects the value of the goods throughout the market, and they are aware of the relationship between the value of this parameter and the distribution of agreements reached in the market. The counterpart of extracting information from price in a competitive market is sampling alternative trading partners in an attempt to learn from the distribution of their offers about the value of the parameter of interest. The main question concerns the extent to which information is revealed to the participants through the trade process. In particular, we shall inquire about the extent of information revelation when such market is approximately frictionless in the sense that the cost of sampling is negligible.

To address these questions we use a simple model with the following features. There are two populations of agents: sellers who have one unit of an indivisible good for sale, and buyers who seek to buy one unit. The market operates over time. In each period all agents are matched with agents of the opposite type at random. If two matched agents agree on the terms of the transaction, they exchange the good and leave the market and if they disagree, they stay in the market to be rematched. The asymmetric information is about some parameter which we shall think of as the value of all units of the good traded in this market and which can be either high throughout or low throughout. Some of the agents who enter the market each period know the true value of the good and others do not. The range of possible "bargaining positions" that agents can adopt is also restricted to two: a seller can either insist that the true value is high and demand the high price or be willing to concede to the low price, and similarly a buyer can insist that the true value is low and demand the low price or be willing to agree to the high price.

We characterize the steady state equilibria of this market. The distribution of the agreements depends on the true value. The uninformed agents are aware of this relationship and their equilibrium behavior incorporates the optimal (given their information) amount of search to learn about the true value. The force that limits the agents' learning at equilibrium is their impatience which is captured by a constant discount factor δ.

To address the question of whether or not information is fully revealed to the uninformed agents when the market becomes approximately frictionless, we consider the equilibria when δ approaches 1. We show that the information is not fully revealed in the sense that a non-negligible fraction of those who are uninformed as they come into the market end up transacting at the wrong price--a price at which they would not want to transact if they knew the true value. This is because when the market is made frictionless there are two opposing effects. On the one hand, it becomes less costly for an uninformed agent to insist on the more favorable price and collect more observations before he concedes to the less favorable price. On the other hand, when all uninformed agents prolong their search in an attempt to acquire more information, there is less to be learned from each meeting. Therefore, the overall effect need not be full revelation of the information.

The related literature includes three lines of work. Firstly, there is the literature on information transmission in REE (e.g., Grossman [1981] and Grossman and Stiglitz [1980]) and, secondly, there is the literature that looks at the microstructure of the REE (Hellwig [1982]; Dubey, Geanakoplos and Shubik [1984]; Blume and Easley [1984]; Kyle [1986]; and Laffont and Maskin [1986]). These two literatures are related to the present paper because they deal with similar qualitative questions in the context of a different type of market. An important difference is that here there are no commonly observed prices, and hence information is transmitted by other means than the price. Finally, in terms of its basic model the paper is related to the literature on matching and bargaining markets. It differs from most other contributions to this literature in that the agents are asymmetrically informed about some payoff relevant marketwide parameters. In this respect the most closely related models are Rosenthal and Landau [1981] and Samuelson [1987], which include some form of imperfect information. They differ from the present model in that they consider the case of imperfect information of the independent values variety, while here the asymmetric information is of the common values variety--it concerns marketwide parameters which affect all traders.

2. The Model

The market is envisioned as an ocean of agents. There are two populations of equal size: sellers who are interested in selling a unit of some indivisible good, and buyers who are interested in buying a unit of the good.

The market operates over time which is divided into discrete periods. We do not consider an initial or final period so that the time index can be thought of as going from $-\infty$ to ∞. In each time period each agent meets at random exactly one agent of the other type. In the end of the period a meeting terminates either with an agreement, in which case the two agents transact and leave the market, or in disagreement, in which case the two agents stay in the market to be rematched. There are constant streams of new arrivals: in the beginning of each period, before the matching stage, a measure M of new buyers and a measure M of new sellers join in.

The payoffs from an agreement depend on the underlying state of the market, which affects both the value of the good to the buyer and the cost to the seller. There are two possible states: the high value-high cost state, H, and the low value-low cost state, L. The true state is the same for all units of the good traded throughout the market and it does *not* change over time. It is either H for all units throughout, or L for all units throughout. (The reason that we introduce two states though only one state actually prevails is that, as we shall see later, some agents will be uncertain as to whether the true state is H or L.) Let c_i and u_i be the seller's cost and the buyer's utility in state $i = H,L$. That is, if in state i a good is sold for price p, the resulting seller's and buyer's surpluses will be $p - c_i$ and $u_i - p$, respectively. It is assumed that $u_H > c_H > u_L > c_L$. Note that $u_i > c_i$, $i = H,L$, means that in each state there is room for a mutually beneficial transaction, and $u_L < c_H$ means that, if a seller thinks that the state is H and a buyer thinks it is L, there is no room for agreement.

All agents discount expected future benefits using a constant discount factor $\delta < 1$. Thus, the payoff to an agent who ends up with surplus y after having spent t periods in the market is $\delta^t y$. The payoff to an agent who never transacts is zero.

The prices are determined in <u>bargaining</u>. The bargaining takes place within one period and can be described by a simultaneous announcements game in which each agent names a state. Let the lower case letters h and ℓ stand for announcements of states H and L, respectively. The following matrix describes how the two announcements determine whether and at what price the exchange takes place.

(1)

		buyer: h	buyer: ℓ
seller	h	p^{hh}	disagree
	ℓ	$p^{\ell h}$	$p^{\ell\ell}$

where $u_H > p^{hh} > c_H > u_L > p^{\ell\ell} > c_L$, and $p^{hh} > p^{\ell h} > p^{\ell\ell}$.

Thus, when both agents agree on a state, they trade at the corresponding price p^{hh} or $p^{\ell\ell}$; when both are "soft," i.e., the seller conceding to ℓ and the buyer to h, they trade at $p^{\ell h}$; and when both are "tough," i.e., each insisting on the state which is more favorable to him, they do not transact. The assumptions that relate the prices to the parameters u_i and c_i assure that, when the agents agree on the true state, both enjoy positive surplus. But if they agree on the wrong state, one of them will suffer a loss. The latter property follows from $u_L < c_H$ and its purpose is to rule out a class of pooling equilibria in which there is one position which all agents adopt in both states, since these equilibria are uninteresting for the analysis of the present paper.

As mentioned above, the true state is the same for all units and all agents and the reason we consider two states is that some agents will be uncertain as to whether the true state is H or L. Regarding the <u>information</u>, it is assumed that a fraction x_S of the sellers and a fraction x_B of the buyers who enter each period know the true state. The other entrants do not know the true state and upon entry they just have (the same) prior beliefs that the true state is H with probability α_H and L with probability $\alpha_L = 1 - \alpha_H$. In subsequent periods an agent's information includes the information with which he entered the market and his personal history of meetings since entry. It is assumed that what an agent learns in a meeting is just the position of the other agent.

Having specified the agents' information, we can define a strategy. A <u>strategy</u> for an agent is a sequence of decision rules prescribing the agent's position, h or ℓ, in each meeting, as a function of his information.

Consider now a snapshot of the entire market in period t. There are K(t) sellers and (by assumption) an equal number of buyers. The distributions of agents between positions are given by

	Fraction adopting position	
	h	ℓ
sellers	$S^h(t)$	$S^\ell(t) = 1 - S^h(t)$
buyer	$B^h(t)$	$B^\ell(t) = 1 - B^h(t)$

That is, S(t) with the superscript h means the fraction of all sellers who at that period adopt position h, etc.

The description of the matching process can be made now more precise: at date t the probability that a seller (resp. buyer) has of meeting a buyer (resp. seller) who adopts position j is $B^j(t)$ (resp. $S^j(t)$). In the aggregate, it is assumed that in date t the number of meetings between buyers who adopt position $j, j = h, \ell$, and sellers who adopt position $i, i = h, \ell$, is $K(t)S^i(t)B^j(t)$. This assumption can be motivated by assuming continuum of agents and taking proper care to avoid the complications that arise when one considers a continuum of random variables, but here we take it as a primitive of the model.

The strategies and histories of the agents who are present in the market in period t together with the strategies of the agents who enter at period t+1 determine the evolution of K(t+1), $S^j(t+1)$ and $B^j(t+1)$ from their predecessors in period t. For example, recall from table (1) that the meetings that result in disagreement are between sellers who adopt position h and buyers who adopt position ℓ. Therefore, only the $K(t)S^h(t)B^\ell(t)$ sellers (buyers) who participated in such meetings at t continue to t+1 and hence $K(t+1) = K(t)S^h(t)B^\ell(t) + M$.

We shall say that the market is in a <u>steady state</u> if K(t), S(t), and B(t) are the same for all t. In what follows we shall focus on steady states and therefore omit the argument t.

An <u>equilibrium</u> consists of two numbers K_H and K_L; two distributions of the seller population $S_H = (S_H^h, S_H^\ell)$ and $S_L = (S_L^h, S_L^\ell)$; two distributions of the buyer population $B_H = (B_H^h, B_H^\ell)$ and $B_L = (B_L^h, B_L^\ell)$; and assignment of strategies to agents such that:

(i) Each agent's strategy maximizes his payoff, given the distributions S_i and B_i, $i = H,L$.

(ii) If the true state is $i = H,L$, the steady state characterized by K_i, S_i, and B_i is consistent with the strategies.

Thus, the equilibrium is based on two steady state configurations: one corresponding to state H and one corresponding to state L. All agents know that the market is in one of the two. Condition (i) requires that each agent's strategy is optimal, given this knowledge and his private information regarding the true state. Condition (ii) requires that, if the true state is i, the equilibrium behavior of all agents combined together indeed sustains the appropriate steady state. That is, if $K(t) = K_i$, $S(t) = S_i$ and $B(t) = B_i$ and all agents follow the equilibrium strategies, then these magnitudes evolve so that $K(t+1) = K_i$, $S(t+1) = S_i$, and $B(t+1) = B_i$.

3. The Equilibrium

Agents know the distribution of the positions h and ℓ prevailing among agents of the opposite type, conditional on the true state. Thus, an optimal strategy maximizes an agent's expected payoff, given these distributions and given what the agent knows about the true state. An informed agent knows the true distribution and hence the optimal strategy is one of optimal search from a known two-point distribution. An uninformed agent does not know the true state and hence the optimal strategy is one of optimal search from an unknown distribution which belongs to a family of two such distributions.

Notice that once a seller adopts the position ℓ or a buyer adopts the position h, they reach agreement immediately. Therefore, the only relevant seller's strategies are of the form: insist on h for n meetings in a row and then switch to ℓ (where n can also be 0 or infinity). Similarly, the relevant buyer's strategies are of the form: insist on ℓ for n meetings in a row and then switch to h. Let us refer to a strategy by the integer that characterizes it.

Suppose that the market is in a steady state. All agents know that, if the true state is H, the steady state distributions of buyers' and sellers' positions will be $B_H = (B_H^h, B_H^\ell)$ and $S_H = (S_H^h, S_H^\ell)$; if the true state is L, the steady state distributions will be $B_L = (B_L^h, B_L^\ell)$ and $S_L = (S_L^h, S_L^\ell)$. Given B_H and B_L, let $V_S(n,Q)$ denote the expected value of strategy n to a seller who believes with probability Q that the true state is H. Define the set $N_S(Q)$ by

(2) $$N_S(Q) = \operatorname{Arg}\,\underset{n}{\operatorname{Max}}\; V_S(n,Q) \quad ,$$

where n can assume the value ∞ as well. Now, an optimal strategy for the seller is an integer in the appropriate set N_S: for an informed seller this is an integer in $N_S(1)$ or in $N_S(0)$ according to whether the true state is H or L; for an uninformed seller this is an integer in $N_S(\alpha_H)$.

Notice that $V_S(n,Q)$ and $N_S(Q)$ depend on B_H and B_L. However, although B_H and B_L are endogenous to the model, we chose to supress them as arguments of V_S and N_S, since for much of the following discussion the distributions B_H and B_L are fixed and the focus is on how the optimal strategies depend on Q. It will be useful, however, to remember this dependency when we consider later the equilibrium determination of B_H and B_L.

Claim 1:

(i) For any B_H and B_L, $N_S(1) = \{\infty\}$.

(ii) $N_S(0) = \{0\}$ or $\{\infty\}$ or $\{0,\ldots,\infty\}$.

(iii) If $B_H^\ell < B_L^\ell$, then $N_S(\alpha_H) = \{n_S\}$ or $\{n_S, n_S+1\}$, where $0 \le n_S \le \infty$.

(iv) If $B_H^\ell \ge B_L^\ell$, then $N_S(\alpha_H) = \{0\}$ or $\{\infty\}$ or $\{0,\infty\}$ or $\{0,\ldots,\infty\}$.

(v) For any B_H and B_L, Max $N_S(0) \le$ min $N_S(\alpha_H)$.

The claim is proved in appendix A, but it is explained informally below. Note that (i) and (ii) deal with the best responses of the informed sellers. Part (i) follows immediately from the facts: $p^{\ell\ell} < c_H$ and $p^{hh} > p^{\ell h}$, since they imply that, when the true state is H, it is a dominant strategy for the informed seller to insist on h perpetually. Part (ii) follows from the fact that the seller always prefers to sell at a higher price, but in state L he is also willing to sell at $p^{\ell\ell}$. Thus, if B_L^h is sufficiently small, say $B_L^h = 0$, it will be optimal for the informed seller to adopt position ℓ and sell for $p^{\ell\ell}$, while if B_L^h is high, say B_L^h = 1, it is optimal to adopt position h and sell for p^{hh}, and for some intermediate value of B_L^h the seller will be just indifferent between the two positions.

Parts (iii), (iv), and (v) deal with the best responses of the uninformed sellers. These best responses, $N_S(\alpha_H)$, are simply sandwiched between $N_S(0)$ and $N_S(1)$. If $\infty \in N_S(0)$ so that in state L it is optimal for the informed seller to insist on h, then a fortiori it is optimal for the uninformed, who assigns some positive probability to the possibility that the true state is H. If $N_S(0) = \{0\}$, then there exists a level $Q_S, 0 < Q_S < 1$, such that it is optimal for the uninformed seller to adopt position h or ℓ according to whether the probability of the true state being H is above or below Q_S. Now, after sampling n buyers who took the position ℓ, the updated belief in the true state being H is (using Bayes' formula)

$$\text{(3)} \qquad \text{Prob}(H|n) = \alpha_H (B_H^\ell)^n / [\alpha_H (B_H^\ell)^n + \alpha_L (B_L^\ell)^n] \quad .$$

Part (iii) of the claim follows from the fact that if $B_H^\ell < B_L^\ell$, i.e., in state L the fraction of buyers who adopt position ℓ is greater than it is in state H, then Prob(H|n) is strictly decreasing in n. Therefore, there is a finite n such that $\text{Prob}(H|n) \le Q_S$. Let n_S denote the minimal such n. If $\text{Prob}(H|n_S) < Q_S$, then $N_S(\alpha_H) = n_S$. If $\text{Prob}(H|n_S) = Q_S$, then after n_S unsuccessful draws the uninformed is just indifferent between positions h and ℓ and $N_S(\alpha_H) = \{n_S, n_S+1\}$.

Part (iv) follows from the fact that if $B_H^\ell \geq B_L^\ell$, then Prob(H|n) is increasing with n and hence $N_S(\alpha_H) = \{0\}$ or $\{\infty\}$ or $\{0,\ldots,\infty\}$ according to whether $\alpha_H < Q_S$ or $\alpha_H > Q_S$ or $\alpha_H = Q_S$.

Since the role of buyers and sellers is completely symmetric, we can repeat the above discussion to obtain the analogous magnitudes for the buyers. Letting $V_B(n,Q)$ denote the expected value of strategy n for a buyer who believes with probability Q that the true state is L, and $N_B(Q) = \text{ArgMax}_n V_B(n,Q)$, we can restate claim 1 for the buyer's strategies: for any S_H and S_L, $N_B(1) = \{\infty\}$; $N_B(0) = \{0\}$ or $\{\infty\}$ or $\{0\ldots\infty\}$; if $S_L^h < S_H^h$, $N_B(\alpha_L) = n_B$ or $\{n_B, n_B+1\}$, $0 \leq n_B \leq \infty$; and if $S_L^h \geq S_H^h$, $N_B(\alpha_L) = \{0\}$ or $\{\infty\}$ or $\{0,\infty\}$ or $\{0,\ldots,\infty\}$. Note that V_B and N_B depend, of course, on S_H and S_L although they do not appear as arguments. Note also that to keep the symmetry, a strategy n for the buyer means adopting position ℓ (rather than h in the seller's case) for n times, and that the argument of N_B is the probability that the true state is L (rather than H).

The agents' strategies determine jointly the market magnitudes: the distributions $S_i = (S_i^h, S_i^\ell)$ and $B_i = (B_i^h, B_i^\ell)$, i = H,L, and the total numbers K_H and K_L. At equilibrium, these magnitudes satisfy the steady-state conditions which require that they are constant over time. The stationarity of K_i, i = H,L, is equivalent to the requirement that, in each of the states, the flow of arrivals M is equal to the endogenously determined flow of departure. That is, in state H,

(4) $$M = K_H S_H^\ell + K_H S_H^h B_H^h \quad .$$

Since each seller meets a buyer, the group of sellers who transact and depart consists of those $K_H S_H^\ell$ who adopted position ℓ and those $K_H S_H^h B_H^h$ who adopted the position h and met a buyer who agreed to it. Similarly, in state L,

$$(5) \qquad M = K_L B_L^h + K_L B_L^\ell S_L^\ell \quad .$$

The stationarity of S_i and B_i, i = H,L, amounts to similar requirements on the equality of inflows and outflows into and from the populations of agents of each particular type who adopt a particular position. Suppose, for example, that all informed sellers use strategy ∞ in state H and strategy 0 in state L and all the uninformed sellers use strategy $n_S, 0 \le n_S < \infty$. Then the stationarity of S_H^ℓ requires,

$$(6) \qquad S_H^\ell = (1 - x_S)M(B_H^\ell)^{n_S}/K_H \quad .$$

In state H, the LHS captures the fraction of the seller population who adopted position ℓ last period (and hence transacted and left), and the RHS captures the number of sellers who switched to position ℓ only this period divided by K_H. The latter group consists of those out of the $(1-x_S)M$ uninformed sellers, who entered the market n_S periods ago and who have not met a buyer who would agree to h. Since an uninformed seller experiences such a history with probability $(B_H^\ell)^{n_S}$, the RHS indeed captures the size of that group. Similarly, the stationarity of S_L^ℓ requires.

$$(7) \qquad S_L^\ell = \left[x_S M + \left(1 - x_S\right)M\left(B_L^\ell\right)^{n_S}\right]/K_L \quad .$$

That is, in state L the fraction of all sellers who adopted position ℓ last period and departed (on LHS) is equal to the number of those, who have just adopted position ℓ, divided by K_L (on RHS). This number consists of those uninformed who were disappointed for n_S periods and the $x_S M$ informed who have just entered.

Analogously, if all informed buyers use strategy ∞ in state L, and strategy 0 in state H, and if all the uninformed buyers use strategy $n_B, 0 \leq n_B < 1$, then the stationarity of B_L^{ℓ} and B_H^h amounts to

$$(8) \qquad B_L^h = (1 - x_B) M (S_L^h)^{n_B} / K_L$$

$$(9) \qquad B_H^h = [x_B M + (1 - x_B) M (S_H^h)^{n_B}] / K_H \quad .$$

Now, suppose there exist values $K_i, S_i^h, S_i^{\ell}, B_i^h, B_i^{\ell}, i = H, L$ and numbers n_S and n_B such that $N_S(0) = \{0\}$, $N_S(1) = \{\infty\}$, $N_S(\alpha_H) = \{n_S\}, N_B(0) = \{0\}$, $N_B(1) = \{\infty\}$, $N_B(\alpha_L) = \{n_B\}$, and such that equations (4)-(9) hold. Then this configuration is an equilibrium. Notice, however, that an arbitrary equilibrium does not have to satisfy (6)-(9), since these equations are derived for a particular configuration of optimal strategies and, as claim 1 asserts, there are several possible configurations of the optimal strategies. In the end of this section we shall use the same principles as above to derive the other relevant versions of conditions (6)-(9).

The first proposition shows that, at least, for some interpretable range of the parameters there exists an equilibrium.

Proposition 1: If $p^{\ell\ell} - [\alpha_H c_H + \alpha_L c_L] > 0$ and or $\alpha_H u_H + \alpha_L u_L - P^{hh} > 0$, then there exists an equilibrium.

The sufficient condition states that, at least for one type of agent, the expected benefit from trading at the less favorable price, evaluated at the prior probabilities is positive. This condition rules out a situation in which the uninformed agents of both sides keep accumulating in the market without ever taking the risk of agreeing to trade at the less favorable price.

The following proposition provides two observations that help in the characterization of equilibrium.

Proposition 2: In equilibrium:

(i) If $\infty \in N_S(\alpha_H)$, then $\infty \notin N_B(\alpha_L)$

(ii) $B_H^{\ell} < B_L^{\ell}$ and $S_L^h < S_H^h$.

The proof is deferred to appendix A. The implication of part (ii) of the proposition together with claim 1 is that, in equilibrium, the sellers' best responses are either

(10) $N_S(1) = \{\infty\}$; $N_S(0) = \{0\}$; $N_S(\alpha_H) = \{n_S\}$ or $\{n_S, n_S+1\}$, $n_S < \{\infty\}$.

or

(11) $N_S(1) = \{\infty\}$; $N_S(0) = \{0,\ldots,\infty\}$; $N_S(\alpha_H) = \{\infty\}$.

Similarly, the buyers' best responses are either

(12) $N_B(1) = \{\infty\}$; $N_B(0) = \{0\}$; $N_B(\alpha_L) = \{n_B\}$ or $\{n_B, n_B+1\}$, $n_B < \{\infty\}$.

or

(13) $N_B(1) = \{\infty\}$; $N_B(0) = \{0,\ldots,\infty\}$; $N_B(\alpha_L) = \{\infty\}$.

Furthermore, it follows from part (i) that the only possible configurations in equilibrium are (10) and (12); (10) and (13); (11) and (12). That is, the configuration consisting of (11) and (13) is impossible in equilibrium.

Recall that the steady-state conditions (6)-(9) were derived for the case in which the sellers' and buyers' strategies are given by (10) and (12), respectively, and where both $N_S(\alpha_H)$ and $N_B(\alpha_L)$ are singletons. Now, to each possible configuration of the strategies corresponds a different version of the steady-state conditions (6)-(9) (conditions (4) and (5) are always the same).

Thus, if (10) holds and $N_S(\alpha_H) = \{n_S, n_S+1\}$, then (6) and (7) are replaced by

$$(6') \qquad S_H^{\ell} = (1 - x_S)M(B_H^{\ell})^{n_S}\left[g_S + (1 - g_S)(B_H^{\ell})\right]/K_H$$

$$(7') \qquad S_L^{\ell} = \left\{x_S M + (1 - x_S)M(B_L^{\ell})^{n_S}\left[g_S + (1 - g_S)(B_L^{\ell})\right]\right\}/K_L \quad ,$$

where $g_S \in [0,1]$ captures the fraction of the uninformed who adopt strategy n_S, while the remaining fraction $1-g_S$ adopt n_S+1 (note that when $g_S = 1$, (6') and (7') coincide with (6) and (7)).

If the sellers' strategies are given by (11), then (6) and (7) are replaced by

$$(6'') \qquad S_H^{\ell} = 0$$

$$(7'') \qquad S_L^{\ell} = r_S x_S M/K_L \quad ,$$

where $r_S \in [0,1]$ is the fraction of the entering informed sellers who adopt strategy 0, while the remaining adopt strategy[1] ∞.

Analogously, if the buyers' strategies are given by (12) and if $N_B(\alpha_L) = \{n_B, n_B+1\}$, then (8) and (9) are replaced by

$$(8') \qquad B_L^h = (1-x_B)M(S_L^h)^{n_B}\left[g_B + (1 - g_B)S_L^h\right]/K_L$$

$$(9') \qquad B_H^h = \left\{x_B M + (1 - x_B)M(S_H^h)^{n_B}\left[g_B + (1-g_B)S_H^h\right]\right\}/K_H \quad ,$$

where $g_B \in [0,1]$ is the fraction of the uninformed buyers who adopt strategy n_B.

If the buyers' strategies are given by (13), then (8) and (9) are replaced by

$$(8'') \qquad B_H^h = 0$$

$$(9'') \qquad B_H^h = r_B x_B M/K_H \quad ,$$

where $r_B \epsilon [0,1]$ is the fraction of entering informed buyers who in state H adopt strategy 0.

Finally, notice that the proposition does not allow all possible strategy configurations. It may not happen at equilibrium that an informed agent strictly prefers to "misrepresent" and insist on the other state. Thus, at equilibrium, in state L(resp. H) an informed seller (resp. buyer) is at most indifferent between h and ℓ, but may not strictly prefer h (resp. ℓ). It also may not happen at equilibrium that both (11) and (13) hold simultaneously. Thus, if (11) holds so that uninformed sellers do not modify their behavior due to learning and some of the informed sellers may "misrepresent" their information in state L, then (12) must hold so that the informed buyers strictly prefer position h in state H.

4. Revelation of Information

The information in the market is originated in the informed agents who enter the market knowing the true state. This information is expressed in the market by the different distributions of positions which prevail in the different states. Thus, if an uninformed agent could observe the distribution of positions throughout the market, he could learn the true state. However, uninformed agents cannot make such an observation, but can only learn about the distribution by sampling agents of the opposite type and observing their positions. Obviously, since due to the discounting, search is costly, uninformed agents will usually draw only a limited sample before they transact, and hence will not learn the true state with certainty. Therefore, as long as there are frictions which make the search costly, it is not surprising that the information is not fully revealed through the trading process. That is, a non-negligible percentage of the uninformed agents end up transacting at the "wrong price"--a price at which they would not want to trade if they knew the true state. The interesting question then is <u>to what extent is the information revealed through the trading process when the frictions are made negligible</u>?

We shall say that the market is approximately frictionless if the common discount factor δ is sufficiently near 1. Of course, if everything remains the same and only the discount factor of one uninformed individual is made arbitrarily close to 1, then this individual will be inclined to search more and hence will be unlikely to end up transacting at a price that he would reject if he were informed. When the frictions are lessened for all participants in the market, then all the uninformed will be prepared to search more. However, this does not guarantee that they will end up being better informed, since the equilibrium will be such that, at each state and each period, more uninformed agents will be searching the market, and hence the informative content of each meeting will be poorer.

Thus, it is not immediately obvious whether or not the information is revealed through the trading process when it is made approximately frictionless. To examine this issue in detail, let f_S denote the fraction of the uninformed sellers who in state H end up transacting at $p^{\ell h}$ or $p^{\ell\ell}$. To compute f_S notice that, when the true state is H, the uninformed sellers who depart in each period after transacting at $p^{\ell h}$ or $p^{\ell\ell}$ are exactly the $K_H S_H^{\ell}$ sellers who adopt position ℓ in that period. Therefore, $f_S = K_H S_H^{\ell}/(1- x_S)M$, which is the above number divided by the number

$(1 - x_S)M$ of the uninformed sellers who enter each period. It follows from (6) and (6") that

$$\text{(14)} \qquad f_S = \frac{K_H S_H^{\ell}}{(1-x_S)M} = \begin{cases} \left(B_H^{\ell}\right)^{n_S} & \text{if } N_S(\alpha_H) = n_S \in (0,\infty) \\ 1 & \text{if } N_S(\alpha_H) = 0 \\ 0 & \text{if } N_S(\alpha_H, B_H) = \infty \end{cases}$$

Analogously, let f_B denote the fraction of the uninformed buyers who at state L end up transacting at $p^{\ell h}$ or p^{hh}. Observe from (8) and (8") that

$$\text{(15)} \qquad f_B = \frac{K_L B_L^{h}}{(1-x_B)M} = \begin{cases} \left(S_L^{\ell}\right)^{n_B} & \text{if } N_B(\alpha_L) = n_B \in (0,\infty) \\ 1 & \text{if } N_B(\alpha_L) = 0 \\ 0 & \text{if } N_B(\alpha_L) = \infty \end{cases}$$

Now, the question of whether the information is fully revealed at equilibrium, when the market is approximately frictionless, reduces to the question of whether the limiting values of f_S and f_B are positive, when the limit is taken over a sequence of equilibria corresponding to a sequence of δ converging to 1. The meaning of positive limits is that, even when the market is approximately frictionless, a non-negligible fraction of the uninformed traders do not learn the information and transact at the wrong price.

Proposition 3: Consider a sequence of δ converging to 1 and a corresponding sequence of equilibria such that $\lim f_S$ and $\lim f_B$ exist. Then at least one of these limits is positive.

Since the complete proof is rather long, it will be useful to provide first an informal explanation of the intuition. Suppose that the true state is H and that $N_S(\alpha_H) = n_S < \infty$ and $N_B(\alpha_L) = n_B < \infty$. If f_S is close to zero so that almost no sellers trade at $p^{\ell h}$ or $p^{\ell\ell}$, it must be that buyers rarely trade as long as they adopt position ℓ, so that almost all the uninformed buyers stay in the market for the full n_B periods until they switch to h, and almost all the M buyers who depart each period are those who adopted h. Thus, in a given period the number of buyers who adopt ℓ is about $n_B(1-x_B)M$, while the number who adopt h is about M, so that

$$B_H^{\ell} \approx \frac{n_B(1 - x_B)M}{n_B(1 - x_B)M + M} = \frac{n_B(1 - x_B)}{n_B(1 - x_B) + 1} \quad .$$

Since $f_S = (B_H^{\ell})^{n_S}$ for it to be small, say smaller than $(1 - x_B)/(2 - x_B)$, it must be that either $n_B = 0$ or n_S is larger than n_B because

$$f_S = (B_H^{\ell})^{n_S} \approx \left[\frac{n_B(1 - x_B)}{n_B(1 - x_B) + 1}\right]^{n_B \frac{n_S}{n_B}} \geq \left(\frac{1 - x_B}{2 - x_B}\right)^{\frac{n_S}{n_B}} \quad .$$

But by analogous arguments, $S_L^h \approx n_S(1 - x_S/[n_S(1 - x_S) + 1]$ and if $n_B = 0$ or $n_S > n_B$ then $f_B = 1$ or $f_B = (S_L^h)^{n_B} \geq (1 - x_S)/(2 - x_S)$. Therefore, it may not be that both f_S and f_B are arbitrarily small.

<u>Proof</u>: Throughout the proof we shall ignore the case of $N_S(\alpha_H) = \{n_S, n_S+1\}$ or $N_B(\alpha_L) = \{n_B, n_B+1\}$. That is, we shall assume that in any of the equilibria in the sequence considered, each agent has a unique best response: either a finite integer or ∞. We make this assumption just to reduce the complexity of the following expressions. The reader can verify that all the arguments go through, essentially without any modification, for cases with more than one best response by replacing expressions such as $(B_H^\ell)^{n_S}$ by $(B_H^\ell)^{n_S}[g_S + (1 - g_S)B_H^\ell]$.

In each period the number of sellers who sell at a particular price is equal to the number of buyers who buy at that same price. Thus, in equilibrium, if the true state is H, the equality between the number of sellers (on the LHS) and the buyers (on the RHS) who transact at the price $p^{\ell h}$ or $p^{\ell\ell}$ is expressed in terms of the equilibrium parameters as

$$(16) \qquad (1 - x_S)Mf_S = (1 - x_B)M\left[1 - (S_H^h)^{n_B+1}\right] \quad ,$$

in the case $n_B < \infty$; and it is

$$(17) \qquad (1-x_S)Mf_S = (1 - r_B x_B S_H^\ell)M \quad , \qquad \text{where } 0 \le r_B \le 1 \quad ,$$

in the case $n_B = \infty$. To verify the RHS of (16), recall from (8) that the uninformed buyers who end up not transacting at $p^{\ell h}$ or $p^{\ell\ell}$ are those who switched to position h after n_B meetings with sellers who adopted position h and in the (n_B+1) time met once again a seller who adopted h. They make up fraction $(S_H^h)^{n_B+1}$ of the uninformed buyers and hence the RHS of (16) captures the number of the remaining buyers. The RHS of (17) follows from (9").

Analogously, if the true state is L, the equality between the number of buyers (on the LHS) and the sellers (on the RHS) who transact at the price $p^{\ell h}$ or p^{hh} is expressed in terms of the equilibrium parameters as

$$(18) \qquad (1 - x_B)Mf_B = (1 - x_S)M\left[1 - (B_H^{\ell})^{n_S+1}\right] \quad ,$$

in the case $n_S < \infty$; and it is

$$(19) \qquad (1-x_B)Mf_B = (1 - r_S x_S B_L^h)M \quad , \qquad \text{where } 0 \le r_S \le 1 \quad ,$$

in the case $n_S = \infty$.

Suppose that there exists a sequence of δ approaching 1 and a corresponding sequence of equilibria such that over this sequence $\lim f_S = 0$ and $\lim f_B = 0$. In what follows we show that the implications of this assumption lead to a contradiction.

<u>Implication 1</u>: In all equilibria in the sequence (except perhaps a finite number $n_S < \infty$ and $n_B < \infty$.

To see this observe that, if there is a subsequence over which $n_B = \infty$, then equation (17) holds. But, since $x_B < 0$, the RHS of (17) is bounded away from zero, while by the supposition f_S approaches zero as δ goes to 1. Therefore, in all equilibria far enough in the sequence $n_B < \infty$, and using (19) in an analogous manner, we have $n_S < \infty$ as well.

<u>Implication 2</u>: $n_S \to \infty$ and $n_B \to \infty$ over the sequence.

Suppose to the contrary that there is a subsequence over which $n_S < \bar{n} < \infty$. Recall from (14) that $f_S = (B_H^\ell)^{n_S}$. Since $\lim f_S = 0$ and since n_S is bounded, it must be that B_H^ℓ approaches zero. But, observe that B_H^ℓ is either bounded away from zero (when $n_B \geq 1$), or equal to zero (when $n_B = 0$), and hence in all the equilibria far enough in this subsequence $n_B = 0$. Therefore, it follows from (15) that $f_B = 1$--contradiction.

<u>Implication 3</u>: $\lim(S_H^h)^{n_B} = 1$ and $\lim(B_L^\ell)^{n_S} = 1$.

Since both $n_S < \infty$ and $n_B < \infty$, the relevant equations are (16) and (18). Equation (18) together with $\lim f_S = 0$ imply $\lim(S_H^h)^{n_B} = 1$, and equation (16) together with $\lim f_B = 0$ imply $\lim(B_L^\ell)^{n_S} = 1$.

Observe from (14), (15), and the fact that $n_S < \infty$ and $n_B < \infty$ (see implication 1) that $f_S = (B_H^\ell)^{n_S}$ and $f_B = (S_L^h)^{n_B}$. As well, recall that when both $n_S < \infty$ and $n_B < \infty$, equations (6)-(9) hold. Solving (5) for M/K_H, substituting the result into (7), using the fact $B_L^h + B_L^\ell = 1$ and rearranging, we have

$$(20) \qquad S_L^h = 1 - \frac{(1 - B_L^\ell)[x_S + (1 - x_S)(B_L^\ell)^{n_S}]}{(1 - B_L^\ell) + (1 - x_S)[1 - (B_L^\ell)^{n_S}]B_L^\ell}$$

Dividing both numerator and denominator by $(1-B_L^\ell)$ and by $(1 - x_S)$ we have

$$(21) \qquad S_L^h = 1 - \frac{\frac{x_S}{1 - x_S} + (B_L^\ell)^{n_S}}{\frac{x_S}{1 - x_S} + 1 + B_L^\ell + \ldots (B_L^\ell)^{n_S}}$$

Analogously, we get from (4) and (9)

$$(22) \qquad B_L^{\ell} = 1 - \frac{\frac{x_B}{1 - x_B} + (B_L^{\ell})^{n_B}}{\frac{x_B}{1 - x_B} + 1 + S_L^h + \ldots (S_L^h)^{n_B}} .$$

We may now use (21) and (22) to evaluate $\lim f_S = \lim(B_H^{\ell})^{n_S}$ and $\lim f_B = \lim(S_L^h)^{n_B}$.

Claim 2:

(i) If both $\lim(B_L^{\ell})^{n_S} = 1$ and $\lim(S_L^h)^{n_B} = 0$, then the sequence $\frac{n_B}{n_S}$ approaches ∞.

(ii) If both $\lim(S_H^h)^{n_B} = 1$ and $\lim(B_H^{\ell})^{n_S} = 0$, then the sequence $\frac{n_S}{n_B}$ approaches ∞.

The details of the proof are deferred to appendix B. The idea, however, is quite simple. When $\lim(B_L^{\ell})^{n_S} = 1$, the RHS of (21) is of the order of magnitude of $1 - \frac{1}{n_S}$. Therefore, $(S_L^h)^{n_B}$ is on the order of

$$(1 - \frac{1}{n_S})^{n_B} = \left((1 - \frac{1}{n_S})^{n_S}\right)^{\frac{n_B}{n_S}} .$$

Recall that $n_S \to \infty$ and $n_B \to \infty$ and note that if $\lim \frac{n_B}{n_S} < \infty$, then $\lim(S_L^h)^{n_B}$ is on the order of $e^{-\lim(n_B/n_S)} > 0$. Thus, $\lim(S_L^h)^{n_B} = 0$ implies that $\frac{n_B}{n_S}$ approaches infinity.

The assumption that $\lim f_S = 0$ and $\lim f_B = 0$ means $\lim(S_L^h)^{n_B} = 0$ and $\lim(B_H^\ell)^{n_S} = 0$ and implies that $\lim(B_L^\ell)^{n_S} = 1$ and $\lim(S_H^h)^{n_B} = 1$ (see implication 3). Hence, the claim contradicts the assumption, since it is impossible to have both $\frac{n_S}{n_B}$ and $\frac{n_B}{n_S}$ approaching infinity. Therefore, at least one of $\lim f_S$ or $\lim f_B$ is positive. Q.E.D.

Recall that $f_S = (B_H^\ell)^{n_S}$ is the fraction of the uninformed sellers who end up transacting at $p^{\ell h}$ or $p^{\ell\ell}$. Note that the fraction who transact just at $p^{\ell\ell}$ is given by $(B_H^\ell)^{n_S+1}$ but since $n_S \to \infty$ that limit is, of course, the same as $\lim f_S$.

Proposition 3 has established that the revelation of information is incomplete. That is, even when the market is approximately frictionless in the sense that δ is close to 1, at least in one of the states, a non-negligible fraction of the uninformed traders transact at the "wrong" price--a price at which they would not trade if they knew the true state. Roughly speaking, the result owes to the fact that the cost of acquiring a given amount of information is not negligible even when δ is close to 1. This is because this cost depends both on the cost of making an additional observation and on how informative each observation is. When δ is close to 1, the cost per observation is small, but at the same time the number of uninformed agents in the market is relatively high, so that each observation is less informative.

Notice that the proposition does not require that both $\lim f_S$ and $\lim f_B$ are positive. If, for example, α_H is very small, it is conceivable that all uninformed agents assume that the true state is L and adopt always position

ℓ. In this case $S_L^\ell = 1$ and the uninformed buyers never err in buying for p^{hh} or $p^{\ell h}$ when the true state is L. But then, of course, uninformed sellers will err, if the true state is H.

Finally, recall that we have not claimed uniqueness of the equilibrium in the model. Indeed, proposition 3 does not rely on any uniqueness result. It speaks about <u>any</u> sequence of equilibria.

5. Discussion

Comparison with Centralized Trading

Since the actual interaction in markets with decentralized trade seems very different from the interaction in markets characterized by centralized trading, it is not obvious that the Walrasian paradigm is appropriate for the analysis of decentralized trading processes. Gale [1987] explained the sense in which, under conditions of perfect information, the Walrasian paradigm provides an approximation of the outcome of decentralized trading processes when the frictions are small in the specific sense that δ is close to 1 (i.e., when the pace of meetings is rapid).

The natural extension of this line of the thought is to inquire whether such a result also holds for models of decentralized trade with asymmetric information. The relevant question in this case is whether the outcomes are approximated by a Rational Expectations Equilibrium (REE) of a corresponding centralized trading model, when the frictions are small. Obviously, if the sense in which we think of the frictions as being small is that δ is close to 1, then the answer is negative. This is because, as we know from proposition 3, in the equilibrium of the present model, non-negligible volumes of transactions can take place at each of two different prices, while in a market with centrally announced prices, by definition, all transactions take place at one price. Thus, our analysis suggests that, under conditions of asymmetric information, it might be impossible to approximate what goes on in a nearly frictionless decentralized market by using the REE of a corresponding centralized market.

The fact that non-negligible volumes of trade occur at two different prices means that the information is not fully revealed through the trading mechanism. This aspect in itself is not special to the case of decentralized trade and may appear in models of centralized trade as well (see the discussion of Grossman and Stiglitz [1980] below). However, in the centralized version of the example analyzed here, the information is fully revealed by the REE price. In this version a single price in announced in each period and sellers and buyers decide whether to sell or buy, respectively. The steady-state REE is a price function specifying a price as a function of the state such that the uninformed know the price function and base their decision on it; demand is equal to supply in each period; and the market is in steady state. It is easily verified that a steady-state REE here is fully revealing: the equilibrium price will be higher in state H, say p^{hh}, and lower in state L, say $p^{\ell\ell}$. This result does not depend on the magnitude of the fractions x_S and x_B of sellers and buyers who are informed, but only on the fact that they are positive.

The Relationship Between External Information and Revelation

Proposition 3 does not capture explicitly, in terms of the parameters of the model, the extent to which the information is not revealed. To get some idea of the magnitude of this phenomenon and on how it depends on other parameters, consider the completely symmetric model in which $x_S = x_B = x, \alpha_H = \frac{1}{2} = \alpha_L$ and the payoffs are symmetric in the sense that $u_H - p^{hh} = p^{hh} - c_H$, $u_L - p^{\ell\ell} = p^{\ell\ell} - c_L$, and $p^{\ell h} = \frac{1}{2}(p^{hh} + p^{\ell\ell})$. In this case there exists a symmetric equilibrium in which $S_H^h = B_L^\ell$, $S_L^\ell = B_H^h$, and $n_S = n_B = n$. As $\delta \to 1$, let us look at a sequence of symmetric equilibria and let $k = \lim(S_L^h)^n$ over such a sequence.

<u>Proposition 4</u>: The limit k exists and satisfies $k = (1 - k)^{\frac{1}{k(1 - x)} - 1}$.

The proposition is proved in appendix B. The formula is interesting because it captures a qualitative relationship between k and the percentage of informed agents x. It can be verified that k is a decreasing function of x: the more external information there is, the lower the percentage of transactions carried out at the wrong prices. Further, as x varies between 0 and 1, k attains any value between 1/2 and 0. That is, the magnitude of the phenomenon is non-negligible: when the percentage of informed is close to zero, almost half of all transactions (the half owes to the symmetry assumed here) are made at the wrong price.

The relationship between x and k points to another qualitative difference between the equilibrium of the present model and the REE of the centralized trading version of the model. In the REE the information is fully revealed regardless of the magnitude of the external information as captured by the x_i's and this is in some sense an artifact of the too perfect transmission of information through prices. In contrast, in the present model, the extent of information revelation is closely related to the amount of external information. When x is close to 1, so that most agents are informed, the fraction of uninformed agents who end up transacting at the "wrong" price is close to zero.

This property of the model is in some sense reminiscent of the work of Grossman and Stiglitz [1980], where we can also find an analogous result to the nonrevelation of information result of proposition 3. They considered a centralized market for an asset about the (expected) value of which agents can get information at a cost. Uninformed agents can extract some information from observing the price, but the supply noise prevents them from learning all the information from the price. The extent of information acquisition is determined endogenously so as to equate the net benefit of acquiring the information with the benefit of using whatever information that can be extracted from the price. What corresponds in their model to the reduction of frictions in our model is making the supply noise smaller. Other things equal, this reduction enhances the revelation of information through the price, but in equilibrium this effect is counteracted by a resulting decrease in the purchase of information and hence an increase in the fraction of the uninformed. This is reminiscent of the irrevelation of information in our model in which the effect of a reduction in sampling cost is counteracted by the diminished informativeness of the search due to the increase in the fraction of the uninformed.

6. Concluding Remarks

The framework of a random pairwise-meetings model already incorporates some special assumptions. In addition, the foregoing analysis has imposed a number of extra assumptions: the populations of buyers and sellers are equal; an agent is matched in every period; the menu of bargaining positions is limited to two. Although I did not analyze the model in the absence of these assumptions, the intuition that I developed leads me to believe that the first two assumptions simplify the analysis significantly but are probably not essential for the qualitative results. If, for example, the assumption on equal populations and/or the choice of the particular matching technology were relaxed, agents may be able to extract information from the frequency of their meetings. This would imply that an agent's strategy will not be characterized by a single integer, but rather will also depend on the information that can be learned from the frequency of past meetings as well. Nevertheless, this extra complexity does not seem likely to affect the basic forces that prevent full revelation in the present model. However, I do not know how essential the assumption that limits the range of bargaining positions is. It is not intuitively obvious from our analysis whether or not a richer set of prices will facilitate full revelation. Therefore, this feature of the model presents probably the most pressing need for further investigation.

Appendix A

Claim 1:

(i) For any B_H and B_L, $N_S(1) = \{\infty\}$

(ii) $N_S(0) = \{0\}$ or $\{\infty\}$ or $\{0,\ldots,\infty\}$

(iii) If $B_H^{\ell} < B_L^{\ell}$, then $N_S(\alpha_H) = \{n_S\}$ or $\{n_S, n_S + 1\}$ where $0 \le n_S \le \infty$

(iv) If $B_H^{\ell} \ge B_L^{\ell}$, then $N_S(\alpha_H) = \{0\}$ or $\{\infty\}$ or $\{0,\infty\}$ or $\{0,\ldots,\infty\}$

(v) For any B_H and B_L, If $\infty \in N_S(0)$, then $N_S(\alpha_H) = \{\infty\}$

Proof: Recall that $V_S(n,Q)$ and $V_B(n,Q)$ denote the expected benefit of strategy n to a seller (buyer) who believes with probability Q that the true state is H(L). Observe that

$$(A.1) \qquad V_S(n,1) = \sum_{i=0}^{n-1} \delta^i \left(B_H^{\ell}\right)^i B_H^h \left(p^{hh} - c_H\right) + \delta^n \left(B_H^{\ell}\right)^n \left[B_H^h \left(p^{\ell h} - c_H\right) + B_H^{\ell}\left(p^{\ell\ell} - c_H\right)\right] ,$$

where the first term on the RHS captures the seller's expected discounted profit in the event that, during his first n periods, he meets a buyer who agrees to h (the probability of such meeting in the i-th period is $(B_H^{\ell})^{i-1} B_H^h$); the second term captures the expected discounted profit in the event that the seller does not meet such a buyer in the first n periods and switches to ℓ. Similarly, observe that, (A.2) $$V_S(n,0) = \sum_{i=0}^{n-1} \delta^i \left(B_L^{\ell}\right)^i B_L^h \left(p^{hh} - c_L\right) + \delta^n \left(B_L^{\ell}\right)^n \left[B_L^h p^{\ell h} + B_L^{\ell} p^{\ell\ell} - c_L\right] .$$

Notice that (A.1) can be written as

(A.3) $$V_S(n,1) = V_S(0,1) + \sum_{i=0}^{n-1} \delta^i \left(B_H^{\ell}\right)^i \left[V_S(1,1) - V_S(0,1)\right] .$$

That is, the value of adopting position h for n periods in a row and then switching to ℓ, $V_S(n,1)$ is equal to the value of switching immediately, $V_S(0,1)$, plus the incremental value of postponing the switch one period at a time for n periods. Similarly,

(A.4) $$V_S(n,0) = V_S(0,0) + \sum_{i=0}^{n-1} \delta^i \left(B_L^{\ell}\right)^i \left[V_S(1,0) - V_S(0,0)\right] .$$

Finally, since $V_S(n,Q) = QV_S(n,1) + (1-Q)V_S(n,0)$, we have from (A.3) and (A.4)

(A.5) $$V_S(n,Q) = V_S(0,Q) + \sum_{i=0}^{n-1} \delta^i \left\{ \left(B_H^{\ell}\right)^i \left[V_S(1,1) - V_S(0,1)\right] Q + \left(B_L^{\ell}\right)^i \left[V_S(1,0) - V_S(0,0)\right] (1-Q) \right\} .$$

Since $p^{hh} - c_H > 0$, $p^{\ell\ell} - c_H < 0$, and $p^{hh} \geq p^{\ell h}$, we have $V_S(1,1) - V_S(0,1) > 0$. But $V_S(1,0) - V_S(0,0)$ may be positive or negative.

(i) Since $V_S(1,1) - V_S(0,1) > 0$, expression (A.3) is clearly maximized for $n = \infty$.

(ii) Observe that (A.4) is maximized at $n = \infty$, $n = 0$, or is constant for all n, according to whether $V_S(1,0) - V_S(0,0)$ is positive, negative, or zero.

(iii) If $V_S(1,0) - V_S(0,0) \geq 0$, then observe that (A.5) is maximized at $n = \infty$. If $V_S(1,0) - V_S(0,0) < 0$, and $Q < 1$, then since $B_H^\ell < B_L^\ell$ there must be some finite n such that

$$\text{(A.6)} \qquad (B_H^\ell)^n[V_S(1,1) - V_S(0,1)]Q + (B_L^\ell)^n[V_S(1,0) - V_S(0,0)](1-Q) \leq 0 \quad .$$

Let n_S denote the minimal such n. Observe that for $n > n_S$ inequality (A.6) holds strictly. Thus, (A.5) is maximized at $n = n_S$ and if n_S satisfies (A.6) with equality, then (A.5) is maximized at $n = n_{S+1}$ as well.

(iv) If for some n inequality (A.6) holds, then since $B_H^\ell \geq B_L^\ell$ it must hold for any integer smaller than n and hence $V_S(0,Q) \geq V_S(n,Q)$. If the reverse of (A.6) holds for some n, then it must hold for any larger integer and hence $V_S(\infty,Q) \geq V_S(n,Q)$. Therefore, (A.5) is maximized at $n = 0$, $n = \infty$, or at both depending on whether $V_S(0,Q)$ is greater, smaller, or equal to $V_S(\infty,Q)$. If $B_H^\ell = B_L^\ell$ and (A.6) holds with equality, then (A.5) is constant for all n.

(v) If $\infty \in N_S(0)$, then $V_S(1,0) \geq V_S(0,0)$. Therefore, (A.5) is maximized at $n = \infty$.

Q.E.D.

Recall that the different versions of the steady-state conditions (6)-(7), (6')-(7'), and (6")-(7") correspond to the different strategy configurations described in claim 1 except in the case of $N_S(\alpha_H) = \{0,\infty\}$ or $\{0,\ldots,\infty\}$, described in part (iv) of the claim. In this case part (v) implies that $N_S(0) = \{0\}$, and the appropriate version of the steady-state conditions is

$$(6''') \qquad S_H^\ell = \left(1 - x_S\right)Mt_S/K_H$$

$$(7''') \qquad S_L^\ell = \left[x_S M + \left(1-x_S\right)Mt_S\right]/K_L \quad ,$$

where $t_S \in [0,1]$ captures the fraction of the entering uninformed sellers who adopt strategy 0 while the rest choose strategy ∞.

Analogously, the appropriate steady-state conditions for the case $N_B(\alpha_L) = \{0,\infty\}$ or $\{0,\ldots,\infty\}$ are

$$(8''') \qquad B_L^h = \left(1 - x_B\right)Mt_B/K_L$$

$$(9''') \qquad B_H^h = \left[x_B M + \left(1 - x_B\right)Mt_B\right]/K_H \quad .$$

<u>Proposition 1</u>: If $p^{\ell\ell} - [\alpha_H c_H + \alpha_L c_L] > 0$ and/or $\alpha_H u_H + \alpha_L u_L - p^{hh} > 0$, then there exists an equilibrium.

<u>Proof</u>: Consider the following point-to-set mapping from $[0,1]^4$ into the set of its subsets. For a given 4-tuple $(S_H^\ell, S_L^\ell, B_L^h, B_H^h)$, let $S_H^h = 1-S_H^\ell$, $S_L^h = 1-S_L^\ell$, $B_L^\ell = 1-B_L^h$, and $B_H^\ell = 1-B_H^h$. If $(S_H^\ell, B_H^h) \neq (0,0)$ solve (4) to obtain

(A.7) $$K_H = M/[S_H^h B_H^h + S_H^\ell] \quad .$$

If $(B_L^h, S_L^\ell) \neq (0,0)$, rearrange (5) to obtain

(A.8) $$K_L = M/[B_L^\ell S_L^\ell + B_L^h] \quad .$$

Use (A.5) and (2) to compute $N_S(\alpha_H)$ and $N_S(0)$ and analogously compute $N_B(\alpha_L)$ and $N_B(0)$. On the basis of this, choose the appropriate version of the steady-state conditions (6)-(9). Substitute (A.7), (A.8), and the chosen values of B_i^j, S_i^j $i=H,L$, $j=h,\ell$ into the RHS of the appropriate version of (6)-(9). Let $\hat{S}_H^\ell, \hat{S}_L^\ell, \hat{B}_L^h, \hat{B}_H^h$ denote the sets obtained from the LHS of the described four equations when: r_i and t_i, $i=S,B$, are varied over [0,1]; g_S varies over [0,1] if $N_S(\alpha_H) = \{n_S, n_S + 1\}$ and otherwise $g_S = 1$; and g_B is varied over [0,1] if $N_B(\alpha_L) = \{n_B, n_B + 1\}$ and otherwise $g_B = 1$.

The above defines a point-to-set mapping that maps $(S_H^\ell, S_L^\ell, B_L^h, B_H^h)$ such that $(S_H^\ell, B_H^h) \neq (0,0)$ and $(B_L^h, S_L^\ell) \neq (0,0)$ to the set $(\hat{S}_H^\ell, \hat{S}_L^\ell, \hat{B}_L^h, \hat{B}_H^h)$. This mapping will be extended continuously by defining $\hat{S}_H^\ell = 0$ and $\hat{B}^H = \{1/[(1-x_B)(n_B + 1 - g_B) + 1] | g_B \in [0,1]\}$ when $(S_H^\ell, B_H^h) = (0,0)$, and by defining $\hat{B}_L^h = 0$ and $\hat{S}_L^\ell = \{1/[(1-x_S)(n_S + 1 - g_S) + 1] | g_S \in [0,1]\}$ when $(B_L^h, S_L^\ell) = (0,0)$. The correspondence is convex valued since $\hat{S}_i^j$ and B_i^j, $i=H,L$, $j=h,\ell$, are either singletons or closed intervals. The correspondence is also upper semicontinuous, since over the ranges in which the strategy sets N_S and N_B are singletons, the correspondence is, in fact, a continuous function. The discontinuity points of this function are where one of the sets N_S or N_B (or both) is not a singleton, but these gaps are filled by letting g_i, r_i, and t_i, i=S,B, range over [0,1]. Thus, the correspondence satisfies the conditions of Kakutani's fixed point theorem (see, e.g., Todd [1976]) and hence has a fixed point.

If the fixed point is such that $(S_H^\ell, B_H^h) \neq (0,0)$ and $(B_L^h, S_L^\ell) \neq (0,0)$, then there is an equilibrium. This is because, by the construction of the correspondence, the fixed point is consistent with optimal strategies for all agents and the appropriate version of the steady-state conditions holds.

Thus, to complete the proof it remains to show that $(S_H^\ell, B_H^h) \neq (0,0)$ and $(B_L^h, S_L^\ell) \neq (0,0)$. Suppose, to the contrary, that the fixed point has $(S_H^\ell, B_H^h) = (0,0)$. This implies that $\infty \in N_S(\alpha_H)$ and $\infty \in N_B(\alpha_L)$ and also $(B_L^h, S_L^\ell) = (0,0)$. It follows immediately from $B_H^h = B_L^h = 0$ that $V_S(\infty, \alpha_H) = 0$ and analogously $V_B(\infty, \alpha_L) = 0$. Observe from (A.1)-(A.5) that $V_S(0, \alpha_H) = p^{\ell\ell} - [\alpha_H c_H + \alpha_L c_L]$ and analogously that $V_B(0, \alpha_H) = \alpha_H u_H + \alpha_L u_L - p^{hh}$. Thus, it follows from the assumption of the proposition that either $V_S(0, \alpha_H) > 0 = V_S(\infty, \alpha_H)$ and/or $V_B(0, \alpha_L) > 0 = V_B(\infty, \alpha_L)$, so that either $\infty \notin N_S(\alpha_H)$ and/or $\infty \notin N_B(\alpha_L)$. This contradicts the earlier conclusion and hence both $(S_H^\ell, B_H^h) \neq (0,0)$ and $(B_L^h, S_L^\ell) \neq (0,0)$.

Q.E.D.

<u>Proposition 2</u>:

(i) In equilibrium if $\infty \in N_S(\alpha_H)$, then $\infty \notin N_B(\alpha_L)$.

(ii) In equilibrium if $B_H^\ell < B_L^\ell$ and $S_L^h < S_H^h$.

Proof: Part (i) was proved in the course of the proof of proposition 1. To prove (ii) suppose, to the contrary, that in equilibrium $S_L^h \geq S_H^h$, and let us check the following possibilities.

[1] If $N_S(\alpha_H) = n_S < \infty$ and $B_H^\ell < B_L^\ell$, then

(A.9)
$$S_L^\ell = \left[x_S M + (1-x_S)M\left(B_L^\ell\right)^{n_S}\right]/K_L > (1-x_S)M\left(B_L^\ell\right)^{n_S}/K_L$$
$$\geq (1-x_S)M\left(B_H^\ell\right)^{n_S}/K_L > (1-x_S)M\left(B_H^\ell\right)^{n_S}/K_H = S_H^\ell \quad ,$$

where the first equality follows from (7); the first inequality is obvious; the second inequality follows from $B_L^\ell > B_H^\ell$; the third inequality is obtained from (A.7) and (A.8) by observing that $S_L^h \geq S_H^h$ and $B_H^\ell < B_L^\ell$ imply $K_H > K_L$; and the last equality follows from (6). But the conclusion from (A.9) that $S_L^\ell > S_H^\ell$ contradicts the supposition.

[2] If $N_S(\alpha_H) = \{n_S, n_S+1\}$, $n_S < \infty$ and $B_H^\ell < B_L^\ell$, then the argument is exactly as above except that we replace $(B_L^\ell)^{n_S}$ by $[g_S(B_L^\ell)^{n_S} + (1 - g_S)(B_L^\ell)^{n_S+1}]$.

[3] If $N_S(\alpha_H) = \{\infty\}$ then $S_H^h = 1$ and hence $S_H^h \leq S_L^h$ implies $S_L^h = 1$. Part (i) implies that it may not be that $\infty \in N_B(\alpha_L)$. It also may not be that $N_B(\alpha_L) = \{0,\infty\}$ or $\{0,\ldots,\infty\}$, since then (8''') and (9''') hold and therefore $K_H B_H^h > K_L B_L^h$, but $S_H^h = 1$ and $S_L^h = 1$ together with (4) and (5) imply that $K_H B_H^h = K_L B_L^h$.

Steps [1]-[3] eliminate all the possible configurations of $N_S(\alpha_H)$ which may arise if $B_H^{\ell} < B_L^{\ell}$ (see claim 1). Furthermore, step [3] eliminates the possibility of $N_S(\alpha_H) = \{\infty\}$. Therefore, $B_H^{\ell} \geq B_L^{\ell}$ and $N_S(\alpha_H) = \{0\}$ or $\{0,\infty\}$ or $\{0,\ldots,\infty\}$. Observe from (6), (7) and (6'''), (7''') that in all these three cases $S_L^h \geq S_H^h$ implies $K_L > K_H$.

Thus, the supposition $S_L^h \geq S_H^h$ implies $B_H^{\ell} \geq B_L^{\ell}$ and $K_L > K_H$. But by going through the analogous arguments we have that $B_H^{\ell} \geq B_L^{\ell}$ implies $S_L^h \geq S_H^h$ and $K_H > K_L$, contradiction. Therefore, in equilibrium $S_L^h < S_H^h$ and $B_H^{\ell} < B_L^{\ell}$.

Q.E.D.

Appendix B

Proofs of claim 2 (in proposition 3) and proposition 4:

The proofs use the result of the following lemma.

Lemma: Let $\{z_n\}$ be a sequence such that for all n, $0 \le z_n \le 1$. Suppose that $\lim z_n^n = z > 0$, and let R be a constant. Then,

(i) $$\lim_{n\to\infty} \left(1 + R(1-z_n)\right)^n = z^{-R}$$

(ii) $$\lim \left(1 + \frac{z_n^n + R}{1 + z_n + \ldots + z_n^{n-1}}\right)^n = \begin{cases} z^{-(z + R)/(1 - z)} & \text{if } z < 1 \\ e^{1 + R} & \text{if } z = 1 \end{cases} .$$

Proof of the lemma:

(i) Consider the sequence $\{n(1-z_n)\}$. It must be bounded from above. Otherwise, for any F there is an n such that $n(1-z_n) > F$ and hence there is a subsequence such that $z_n < 1 - \frac{F}{n}$ implying that $z = \lim z_n^n \le \lim(1 - \frac{F}{n})^n = e^{-F}$ for any arbitrarily large F, in contradiction to $z > 0$. Let v be a cluster point of the sequence $n(1-z_n)$, and consider a subsequence converging to v. For any ε there is $n(\varepsilon)$ such that for $n > n(\varepsilon)$

$$1 - \frac{v + \varepsilon}{n} \le z_n \le 1 - \frac{v - \varepsilon}{n} .$$

Therefore, for any cluster point v,

$$z = \lim z_n^n = \lim (1 - \frac{v}{n})^n = e^{-v} \quad ,$$

which means that $v = -\ell nz$ and that $\lim n(1-z_n)$ exists and is equal to v. Therefore,

$$\lim\left(1 + R(1-z_n)\right)^n = \lim[1 + Rn(1-z_n)/n]^n = e^{-R\ell nz} = z^{-R} \quad .$$

(ii) If $z = 1$, then for any $\varepsilon > 0$ and sufficiently large n,

$$1 + \frac{z_n^n + R}{n} \le 1 + \frac{z_n^n + R}{1+z_n + \ldots + z_n^{n-1}} \le 1 + \frac{z_n^n + R}{n(1 - \varepsilon)} \quad .$$

By raising to the power of n and taking limits we get that the desired limit is sandwiched between e^{1+R} and $e^{(1+R)/(1-\varepsilon)}$ and hence it is e^{1+R}.

If $z < 1$, rewrite

$$\lim\left[1 + \frac{z_n^n + R}{1+z_n + \ldots + z_n^{n-1}}\right]^n = \lim\left[1 + \frac{z_n^n + R}{1-z_n^n}\left(1-z_n\right)\right]^n \quad .$$

Now, it follows from (i) that the last expression is equal to $z^{-(z+R)/(1-z)}$.

Q.E.D.

<u>Corollary</u>: Let $\{z_n\}$ and R be as above,

$$\lim_{n\to\infty}\left[1 - \frac{R + z_n^n}{R + 1 + z_n + \ldots + z_n^n}\right]^n = \begin{cases} z^{(z + R)/(1 - z)} & \text{if } z < 1 \\ e^{-(1 + R)} & \text{if } z = 1 \end{cases} .$$

<u>Proof</u>: This follows from the lemma and the fact that

$$1 - \frac{R + z_n^n}{R + 1 + z_n^n + \ldots + z_n^n} = 1/\left[1 + \frac{R + z_n^n}{1 + z_n^n + \ldots + z_n^n}\right] .$$

Q.E.D.

<u>Claim 2</u>:

(i) If both $\lim (B_L^{\ell})^{n_S} = 1$ and $\lim (S_L^h)^{n_B} = 0$, then the sequence $\frac{n_B}{n_S}$ approaches ∞.

(ii) If both $\lim (S_L^h)^{n_B} = 1$ and $\lim (B_L^{\ell})^{n_S} = 0$, then the sequence $\frac{n_S}{n_B}$ approaches ∞.

<u>Proof</u>: Raise both sides of (21) to the power of n_B to get

$$\text{(B.1)} \qquad (S_L^h)^{n_B} = \left\{\left[1 - \frac{\frac{x_S}{1-x_S} + (B_L^\ell)^{n_S}}{\frac{x_S}{1-x_S} + 1 + B_L^\ell + \ldots + (B_L^\ell)^{n_S}}\right]^{n_S}\right\}^{\frac{n_B}{n_S}}$$

Recall from implication 2 that, as $\delta \to 1$, both $n_S \to \infty$ and $n_B \to \infty$. Suppose that over some subsequence $\lim \frac{n_B}{n_S} = F < \infty$. Then, it follows from the corollary above that over this subsequence the RHS of (B.1) approaches $e^{-[1+x_S/(1-x_S)]F} > 0$, in contradiction to $\lim(S_L^h)^{n_B} = 0$. Therefore, $\frac{n_B}{n_S}$ is unbounded over any subsequence.

The proof of part (ii) is completely analogous. It just uses (22) instead. Q.E.D.

<u>Proposition 4</u>:

The limit $k = \lim(S_L^h)^n$ exists and satisfies $k = (1-k)^{\frac{1}{k(1-x)} - 1}$.

<u>Proof</u>: In the symmetric equilibrium $n_S = n_B = n$ and $S_L^h = B_H^\ell$. Consider a subsequence such that $\lim(S_L^h)^n$ exists and call it k. By proposition 3, $k > 0$ and from (18) $\lim(B_L^\ell)^n = 1 - \lim(B_H^\ell)^n = 1 - k$. Raising both sides of (21) to the power of n and using the corollary for the case $z < 1$, we have

$$k = (1-k)^{\frac{1}{k(1-x)} - 1} .$$

Since this equation has a unique solution, it must be that $\lim(S_L^h)^n$ exists and equal to this solution. Q.E.D.

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Notes

1. Note that in this case the informed sellers are indifferent among all strategies $0,\ldots,\infty$, so that there might be an equilibrium in which informed sellers adopt strategies other than 0 and ∞. However, for any such equilibrium there exists $r_S \in [0,1]$ such that (7") holds, where r_S here is some weighted average of the fractions of informed sellers who choose the different finite strategies. Thus, for our purposes we need not treat this case separately.