Discussion Paper No. 811

EQUILIBRIUM WAGE DISTRIBUTIONS: A SYNTHESIS*

by

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November 1988

Abstract

The problem of wage determination is formulated as a non-cooperative game in which employers post wage offers and workers sequentially search among them. Establishing the existence of an equilibrium distribution of wage offers and associated reservation wage rates for the general case of any finite number of worker and employer types given that workers receive offers both when employed and when not at frequencies that generally depend on employment status is the principal task accomplished in the paper. The constructed representation of the equilibrium wage offer distribution is used to determine the data required to identify the model's structural parameters. Finally, an extended version of the model is outlined that incorporates the equilibrium provision of job attributes other than the wage. The model suggests that OLS estimates of the marginal willingness to pay for a desirable attribute may be biased down.

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* Prepared for the Symposium on Panel Data and Labour Market Studies, Amsterdam, the Netherlands, December 15-17, 1988. The author has benefitted from comments given in workshops presented at the University of Pittsburgh, Texas A&M University, and the University of Chicago. The implications of the approach studied in the paper for the theory of compensating differentials was suggested by W. Robert Reed.

1. Introduction

What should equilibrium wage rates be when individual employers post offers and workers search among them randomly and sequentially? When interpreted in a labor market context, Diamond [1971] formulated the problem as a non-cooperative wage-setting and wage-search game played by profit maximizing employers and wealth maximizing workers. In equilibrium, each employer maximizes profit given the search strategies of the workers and the wages offered by the other employers while each worker searches sequentially from the wage offer distribution using an expected wealth maximizing stopping strategy. When employers and workers are respectively identical and workers search only while unemployed, Diamond found only a single equilibrium offer equal to the workers' common reservation wage. Surprisingly, the equilibrium offer and reservation wage is the monopsony wage, the wage that would be offered were there only a single employer.

More recently, Albrecht and Axell [1984] extended the analysis by considering the case of workers and employers of different types. They show that if there are two groups of workers who have different values of non-market time and if labor productivity varies across employers and is sufficiently dispersed, then a two wage equilibrium offer distribution exists. Although the two equilibrium offers equal the endogenously determined reservation wage rates of the two worker types, the lower offer generally exceeds the monopsony wage. Under restrictions that imply that the support of the equilibrium offer distribution is identical to the set of reservation wage rates, Eckstein and Wolpin [1987] provide an algorithm for computing an equilibrium wage offer distribution for the case of any finite number of worker types. This effort represents only part of their original attempt to estimate the structural parameters underlying the model.

In these papers, a worker does not receive job offers while employed. That every equilibrium wage offer must be the reservation wage of some worker type is a consequence of this counter-factual assumption. In a labor market version of a product price and advertising model introduced by Mortensen [1986] and extended by Wernerfelt [1988], Burdett and Mortensen [1988] show that an equilibrium is a non-degenerate continuous distribution of wage offers with lower support equal to the common reservation wage when workers are identical and offers arrival frequencies are independent of employment status.

The creation and study of a synthesis of the Albrect-Axell and the Burdett-Mortensen models is the focus of the paper. Establishing the existence of an equilibrium dispersed distribution of wage offers and an associated equilibrium reservation wage rates for the general case of any finite number of worker and employer types given that workers receive offers both when employed and when not at frequencies that generally depend on employment status is the principal accomplishment reported in the paper. As a second theoretical task, the framework is extended in a natural way that permits study of the equilibrium provision of job attributes as well as the wage.

The fact that the existence proof provides a method of constructing the equilibrium wage offer distribution facilitates the problem of estimating the model's structural parameters. Indeed, data on the length of an unemployment spell, the post spell wage received, and the duration of a subsequent job spell for a sample of workers are sufficient to identify maximum likelihood estimates of all the structural parameters of the model, at least in the case of homogenous workers and employers.

The extended model implies that one can obtain estimates of compensating wage differentials from observations relating the lengths of job spells experienced by individual workers to their associated wage-attribute vectors. Indeed, this estimation method is implemented by Gronberg and Reed [1988]. Finally, the extended model also suggests that OLS estimates of the willingness to pay for a valued job attribute likely to be inconsistent and biased down.

The remainder of the paper is organized as follows: Section 2 contains a formal statement of the model's structure and the definition of an equilibrium solution to it. The salient necessary properties of any candidate for an equilibrium distribution of wage offers are summarized by the four propositions stated and proved in Section 3. Given these properties, the functional form of the unique equilibrium distribution is derived in Section 4 for the case of homogeneous workers and employers. The comparative static properties of the equilibrium for that special case are also studied in the section.

Section 5 contains the existence proof for the general case. The argument has two parts. First, an algorithm is derived for computing the wage offer distribution reflecting the unique non-cooperative solution to the game of posting wage offers played by the employers given reservation wage rates and measures of worker availability. Second, the existence of an equilibrium reservation wage rate and measure of availability for each worker type given the offer distribution generated by the algorithm is established as an application of Brouwer's fixed point theorem. In the special case of offer arrival rates that are the same whether employed or not, uniqueness of equilibrium is corollary of the first part of the argument because reservation wage rates and measures of worker availability are independent of the offer distribution.

The extension of the existence of equilibrium results to a model in which worker's value job attributes as well as the wage is presented in Section 6. In the extended model, equilibrium dispersion exists over both the wage and job attributes offered if employers differ with respect to the cost of providing attributes. The equilibrium offer distribution in the extended model is a distribution of job values where the value of a job is a given function of wage and attributes offered. Finally, the paper concludes with a discussion of empirical issues in Section 7. The problems of estimating the model's structural parameters, including willingnesses to pay for desirable attributes, is explicitly considered.

2. The General Model

Workers differ with respect to the value of non-market time. Let b_i, $i = 1,\ldots,n$, represent the value of non-market time for a worker of type i. Without loss of generality, $b_{i+1} > b_i$ in the sequel. Workers of every type receive offers at given Poisson rates, λ_0 when unemployed and λ_1 when employed, and job-worker matches separate at the exogenous rate δ. Assuming that the wage offer distribution, $F(w)$, is known to each worker and stationary over time, an expected wealth maximizing worker pursues a reservation wage acceptance strategy when not employed and moves from a current employer to another when an opportunity arises if the alternative offer exceeds the current wage.

One can show (See Mortensen and Neumann [1988]) that the wealth maximizing reservation wage required by a worker of type i when unemployed, denoted as r_i, is the unique solution to

$$(1) \quad r_i = b_i + (\kappa_0 - \kappa_1)\int_{r_i}^{\infty} \left[\frac{1-F(x)}{1+\kappa_1[1-F(x)]}\right] dx, \; i = 1,\ldots,n,$$

where

$$(2) \quad \kappa_0 \equiv \lambda_0/\delta \text{ and } \kappa_1 \equiv \lambda_1/\delta$$

represent the two offer arrival rate to job separation rate ratios. Of course, (1) and $b_{i+i} > b_i$ imply $r_{i+1} > r_i$.\1

Let m_i denote the given number of type i worker and let u_i represent the endogenous number who are unemployed. Define

$$(3.a) \quad m \equiv \sum_{i=1}^{n} m_i$$

and

$$(3.b) \quad u \equiv \sum_{i=1}^{n} u_i$$

as the total number of workers and the total number unemployed respectively. Finally, let

$$(3.c) \quad F(w) = F(w^-) + \nu(w)$$

where $\nu(w)$ is the fraction of employers that offer exactly w.

Since workers of type i receive offers at rate λ_0 and accept an offer when greater than or equal to r_i when unemployed but are separated at rate δ

when employed, the time rate of change in the number of type i workers who are unemployed is given by

$$du_i/dt = \delta[m_i - u_i] - \lambda_0[1-F(r_i^-)]u_i .$$

Hence, the steady-state number of unemployed workers of type i is

$$\text{(4.a)} \quad u_i = \frac{m_i}{1 + \kappa_0[1-F(r_i^-)]}.$$

Let G(w) represent the fraction of all employed workers who earn wage w or less. The steady state earnings distribution associated with any given offer distribution can be derived using an analogous argument. Specifically, a wealth maximizing worker currently employed at wage w or less will transit to a job offering a higher wage when available. Because offers larger than w are received when employed at a rate equal to the product of the offer arrival rate λ_1 and the probability that an offer exceeds w, 1-F(w), because exogenous separations occurs at rate δ, because an unemployed worker receives acceptable offers less than or equal to w at a rate equal to the product of the offer arrival rate when unemployed, λ_0, and the probability that such a wage is offered and is acceptable, $F(w)-F(r_i^-)$, the time rate of change in the total number of workers employed at wage w or less is

$$d\{G(w)[m-u]\}/dt = \lambda_0 \sum_{i=1}^{n} \max[F(w)-F(r_i^-),0]u_i - \left[\delta+\lambda_1[1-F(w)]\right]G(w)[m-u].$$

Therefore, the steady-state distribution of wages earned by employed workers is proportional to a mixture of the n conditional distributions of acceptable offers with weights equal to the fraction of total employment represented by each worker type:

$$(4.b)\quad G(w) = \kappa_0 \sum_{i=1}^{n} u_i \max[F(w)-F(r_i^-),0]/[1 + \kappa_1[1-F(w)]][m-u]$$

$$= \left[\frac{1}{1+\kappa_1[1-F(w)]}\right] \sum_{i=1}^{n} \left[\frac{\max[F(w)-F(r_i^-),0]}{1 - F(r_i^-)}\right]\left[\frac{m_i-u_i}{m-u}\right].$$

The factor of proportionality decreases with κ_1 reflecting the effect of job-to-job movements from lower to hiring paying employers on the steady state earnings distribution.

Consider the wage interval $(w-\varepsilon,w]$: The equations of (4) imply that the steady number of workers employed at wage rates in this interval is

$$[G(w)-G(w-\varepsilon)](m-u) = \kappa_0[F(w)-F(w-\varepsilon)] \sum_{r_i \le w} h_i / \left[1+\kappa_1[1-F(w)]\right]\left[1+\kappa_1[1-F(w-\varepsilon)]\right]$$

and

$$(5)\quad h_i = [1+\kappa_1[1-F(r_i^-)]]u_i = m_i\left[1+\kappa_1[1-F(r_i^-)]\right]/\left[1+\kappa_0[1-F(r_i^-)]\right].$$ \2

Since $F(w)-F(w-\varepsilon)$ represents the number of employers who offer wage rates in the interval $(w-\varepsilon,w]$, the steady-state number of workers per firm offering a specific wage w can be defined as

$$\ell(w) = \lim_{\varepsilon\to 0}\left[\frac{[G(w)-G(w-\varepsilon)](m-u)}{F(w)-F(w-\varepsilon)}\right]$$

given $F(w)-F(w-\varepsilon) > 0$ for all sufficiently small $\varepsilon > 0$. Hence,

$$(6.a)\quad \ell(w) = \left[\kappa_0 \sum_{r_i \le w} h_i\right]/\left[1+\kappa_1[1-F(w)]\right]^2$$

if F(w) is left-continuous at w and

$$(6.b)\quad \ell(w) = \left[\kappa_0 \sum_{r_i \le w} h_i\right] / \left[1+\kappa_1[1-F(w)]\right]\left[1+\kappa_1[1-F(w^-)]\right]$$

if w is a mass point. Because only the relative wage offer determines whether an employed worker prefers an alternative, equation (6.a) is also the appropriate definition of a steady state labor force associated with a wage not in the support of the offer distribution.

The equations of (6) define the typical employer's steady state labor force for every possible wage offer given the wages of other employers described by the wage offer distribution. The associated steady-state profit flow per period earned by an employer who is of productivity p and offers w is

$$(7)\quad \pi(p,w) = (p-w)\ell(w).$$

The productivity of any job-worker match depends only on the employer's type by assumption. Let p_j, $j = 1,\ldots,s$, represent the value of productivity in the case of type j employers. Define

$$(8)\quad W_j = \operatorname*{argmax}_w\{\pi(p_j,w)\},\ j = 1,\ldots,s,$$

as the set of wage rates that maximize steady state profit for type j employers. Without loss of generality, $p_{j+1} > p_j$ in the sequel.

Let $\gamma(p)$ represent the distribution of employers over productivity, the proportion with productivity p or less. Obviously, if $F_j(w)$ is the wage offer distribution of type j employers, its support is contained in W_j, and the market's wage offer distribution is the mixture

$$(9)\quad F(w) = \sum_{j=1}^{s} F_j(w)[\gamma(p_j)-\gamma(p_{j-1})].$$

A steady-state market equilibrium is a collection of reservation wages, one for each worker type, and a collection of wage offer distributions, one for each employer type, that represent mutually consistent non-cooperative best responses for all the individual workers and employers.

Definition: A **steady-state market equilibrium** is a reservation wage for each worker type r_i, $i = 1,..,n$, a wage offer distribution for each employer type F_j with an associated support W_j, $j = 1,...,s$, and a market wage offer distribution F such that the reservation wage rate for each worker type maximizes the expected wealth of a worker of that type, i.e. r_i satisfies (1) given F, every wage offer in the support of the offer distribution of each employer type maximizes the steady-state profit of an employer of that type, i.e., W_j satisfies (8) given F and all the reservation wage rates, and the market offer distribution is the mixture defined by (9).

3. Properties of an Equilibrium Wage Offer Distribution

The character of an equilibrium distribution of wage offers depends critically on whether or not employed workers are able to receive and respond to alternative wage offers. The principal result implies that only continuous offer distributions are candidates for equilibrium if they can and only discrete distributions are candidates if they can not. The reason for this rather dramatic difference reflects the different natures of the function relating an individual employer's steady state labor force to its offer in the two cases.

If the offer arrival rate when employed is zero, an employer losses no workers by lowering its offer to the next reservation wage. Hence, an offer between two adjacent reservation wage rates can never be profit maximizing. The fact that the support of the offer distribution must be contained in the set of reservation wage rates provides the structure exploited by Eckstein and Wolpin [1987] in their algorithm for computing an equilibrium.

However, if the offer arrival rate when employed is strictly positive, then employed workers move whenever a job offering a higher wage becomes available. Although no employer of many offering the same wage loses workers to the others, any one that offers a higher wage, no matter how small, gains workers from the others. Because doing so guarantees a strictly larger steady-state profit, there can be no mass of employers offering the same wage in equilibrium. This fact complicates the construction of an equilibrium to the wage posting game played by the employers given the reservation wage rates.

Proposition 1: (a) If employed workers do not receive alternative offers, then a wage offer, w, that attracts workers is profit maximizing only if w is also the reservation wage of some worker type. (b) However, if employed workers do receive alternative offers, then a wage offer, w, that attracts workers is profit maximizing for employers of type $p > w$ only if no mass of other employers offer w.

Proof. (a) If $\kappa_1 = \lambda_1/\delta = 0$, then $\ell(w)$ is constant on any interval defined by two adjacent reservation wage rates which is closed on the left and open on the right, i.e. $[r_i, r_{i+1})$. Hence, $\pi(p,r_i) > \pi(p,w)\ \forall\ w \in (r_i, r_{i+1})$ for all p given $\ell(w) > 0$.

Proof. (b) If $\kappa_1 > 0$, then equations (6) and (7), $\ell(w) > 0$, $p > w$, and the fact that the c.d.f. F(w) is right continuous imply

$$\lim_{\varepsilon\to 0}\{\pi(p,w+\varepsilon)\} = (p-w)\left[k_0 \sum_{r_i \le w} h_i\right]/\left[1+k_1[1-F(w)]\right]^2 >$$

$$(p-w)\left[k_0 \sum_{r_i \le w} h_i\right]/\left[1+k_1[1-F(w)]\right]\left[1+k_1[1-F(w^-)]\right] = \pi(p,w)$$

since $F(w) - F(w^-) = \nu(w) > 0$ when w is a mass point. \\\\

The discontinuous jump up in the steady state labor force function at any reservation wage implies that there will always be a "gap" in the support of the equilibrium offer distribution to the left of a reservation wage. However, the fact that only an employer's relative wage offer affects the size of its steady state labor force between reservation wage rates implies no "gaps" occur elsewhere.

Proposition 2: (a) If w is the reservation wage of some employer type, then $w-\varepsilon$ is not profit maximizing for all sufficiently small values of $\varepsilon > 0$.
(b) Conversely if $w-\varepsilon$ attract worker and if $F(w-\varepsilon) = F(w)$ for some $\varepsilon > 0$, then w is profit maximizing only if it is the reservation wage of some worker type.

Proof. Because $\ell(w)$ jumps up discontinuously at any reservation wage r_i and is right continuous at such a wage,

$$\pi(p,r_i) = (p-r_i)\ell(r_i) > \lim_{\varepsilon\to 0}\{[p-(r_i-\varepsilon)]\ell(r_i-\varepsilon)\} = \lim\{\pi(p,r-\varepsilon)\}\ \forall\ p.$$

Hence, the claim (a) follows.

Under the hypothesis to (b), (6.a) implies $\ell(w) = \ell(w-\varepsilon) > 0$. Consequently, $\pi(p,w) = (p-w)\ell(w) < [p-(w-\varepsilon)]\ell(w-\varepsilon) = \pi(w-\varepsilon,p)$ given $\varepsilon > 0$.

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Not surprisingly, equilibrium offers are non-decreasing with productivity across firms in the sense that more productive employers do not offer lower wage rates.

Proposition 3: If w" and w' are profit maximizing offers for two employers of types p" and p' respectively and both offers attract workers, then $p'' > p'$ implies $w'' \geq w'$.

Proof. Because (7), the hypothesis, and $p'' > p'$ all imply

$$(p''-w'')\ell(w'') = \pi(p'',w'') \geq \pi(p'',w') = (p''-w')\ell(w') >$$
$$(p'-w')\ell(w') = \pi(p',w') \geq \pi(p',w'') = (p'-w'')\ell(w'') > 0,$$

$(p''-p')\ell(w'') \geq (p''-p')\ell(w')$. Because $\ell(w)$ is non-decreasing by virtue of (6) and is positive at both w' and w" by hypothesis, the claim follows. \\\\

An obvious implication of Proposition 3 and the convention $p_{j+1} > p_j$ is

(10.a) $w_{1j} \leq w_{0j+1}$

where

(10.b) $w_{0j} \equiv \inf W_j$ and $w_{1j} \equiv \sup W_j$

respectively denote the smallest and the largest profit maximizing wage offered by employers of type j given (8). Consequently, in the case of a strictly positive offer arrival rate when employed, $\kappa_1 > 0$,

(11.a) $F(w_{01}) = 0$ and $F(w_{0j}) = F(w_{1j-1}) \ \forall\ j > 1$

and

(11.b) $F(w_{1j}) = \gamma(p_j) \quad \forall\ j \geq 1$

where $\gamma(p)$ is the fraction of employers of productivity p or less.

Because h_i, as defined by equation (5), is equal to the steady state number of workers of type i who are unemployed, u_i, when employed worker do not receive alternative offers, the following is a generalization of Diamond's [1971] conclusion that the equilibrium offer is the "monopsony" wage rate.

Proposition 4: The smallest profit maximizing wage offer for employers of type j, w_{0j}, is a solution to the problem

$$(12)\quad \max_{w \geq w_{1j-1}} \left\{ (p_j - w) \sum_{r_i \leq w} h_i \right\} \text{ where } w_{10} = -\infty .$$

Proof. That $w_{0j} \geq w_{1j-1}$ for all j>1 is implied by Proposition 2. For j=1, there is no restriction, so one can define $w_{10} = -\infty$ without loss of generality. In either case, if the claim is not true and $\kappa_1 = 0$, then an offer $w \geq w_{1j-1}$ exists such that

$$\pi(p_j, w_{0j}) = (p_j - w_{0j}) \kappa_0 \sum_{r_i \leq w_{0j}} h_i < (p_j - w) \kappa_0 \sum_{r_i \leq w} h_i = \pi(p_j, w)$$

which contradicts the fact that w_{0j} must be profit maximizing. A similar contradiction obtains in the case of $\kappa_1 > 0$. In particular, if w_{0j} does not solve the problem defined by (12), then an offer $w \geq w_{1j-1}$ exists such that

$$\pi(p_j, w_{0j}) = (p_j - w_{0j}) \left[\kappa_0 \sum_{r_i \leq w_{0j}} h_i \right] / \left[1 + \kappa_1 [1 - F(w_{0j})] \right]^2$$

$$< (p_j - w) \left[\kappa_0 \sum_{r_i \leq w} h_i \right] / \left[1 + \kappa_1 [1 - F(w_{0j})] \right]^2$$

$$\leq (p_j - w) \left[\kappa_0 \sum_{r_i \leq w} h_i \right] / \left[1 + \kappa_1 [1 - F(w)] \right]^2 = \pi(p_j, w)$$

by virtue of equations (6) and (7) because $F(w) = F(w^-)$ for all w and because (11.a) holds when $\kappa_1 > 0$, and because $F(w) \geq F(w_{1j-1})$ for all $w \geq w_{1j-1}$. \\\\

As a corollary, the lowest wage offer of each employer type j is either the highest offer of employer type j-1 or the larger reservation wage of some worker type.

4. The Case of Homogeneous Agents

In the special case of workers and employers who are respectively identical, one can construct the unique equilibrium as follows: By virtue of Proposition 4, the lowest wage offered is the common reservation wage, i.e., $w_0 = r$. Obviously, this is the only wage offered if no employed worker receives information about alternatives from other employers by virtue of Proposition 1.a. Because there are no gains from continued search when the only offer is r in the sense that the last term on the right of equation (1) is zero, the offer and the reservation wage both equal the common value of non-market time in equilibrium, i.e., $w_0 = r = b$. Of course, this conclusion in Diamond's [1971].

A different conclusion obtains when employed workers receive alternative offers, even when the offer arrival rate when employed is arbitrarily small. In this case, an equilibrium distribution has no mass points by virtue of Proposition 1.b. Furthermore, Proposition 2 implies that the support of an equilibrium offer distribution is a connected interval, i.e., $[w_0, w_1]$ where of course the lowest wage is the common reservation wage,

(13.a) $w_0 = r$,

as already noted. Finally, because all employers of the same type must earn the same profit in equilibrium, this interval must equal the set of profit maximizing wage rates, i.e., $W = [w_0, w_1]$. Equations (6) and (7) imply that the only continuous distribution function, $F(w)$, consistent with the equal profit requirement, that $\pi(p,w) = \pi(p,w_0)$, is

$$\text{(13.b)} \quad F(w) = \left[\frac{1+\kappa_1}{\kappa_1}\right]\left[1 - \left[\frac{p-w}{p-w_0}\right]^{1/2}\right] \quad \forall\ w \in W = [w_0, w_1].$$

Finally, because the largest offer, w_1, is the smallest solution to $F(w_1) = 1$,

$$\text{(13.c)} \quad w_1 = p - [1/(1+\kappa_1)]^2 (p-w_0).$$

The shape and location of the distribution function implied by (13) is illustrated in Figure 1.

The equations of (13) characterize the only wage offer distribution consistent with profit maximization for any given reservation wage. Hence, a market equilibrium is a reservation wage and distribution function pair that simultaneously satisfy (1) and (13). Obviously, if the offer arrival rate is independent of employment status so that $\kappa_0 = \kappa_1$, then $r = b$ and the associated solution to (13) is the unique market equilibrium. This observation is a corollary of the existence and uniqueness proof offered by Burdett and Mortensen [1988].

Establishing the existence and uniqueness of equilibrium in the case of employment dependent offer arrival rates is only slightly more complicated. By substituting from (13.b) into (1), by integrating the resulting expression, and then by substituting from (13.a) and (13.c), one obtains

$$r = b + \frac{\kappa_0 - \kappa_1}{\kappa_1}\int_r^{w_1}\left[1 - \frac{1}{1+\kappa_1}\left[\frac{p-x}{p-w_0}\right]^{-1/2}\right]dx$$

$$= b + \frac{\kappa_0-\kappa_1}{\kappa_1}\left[w_1-w_0 - \frac{2(p-w_0)}{1+\kappa_1}\left[\left[\frac{p-w_1}{p-w_0}\right]^{1/2} - \left[\frac{p-r}{p-w_0}\right]^{1/2}\right]\right.$$

$$= b + \frac{\kappa_0-\kappa_1}{1+\kappa_1}\left[1 - \frac{1}{(1+\kappa_1)^2} + \frac{2}{1+\kappa_1}\left[\frac{1}{1+\kappa_1} - 1\right]\right][p-r]$$

$$= b + \frac{(\kappa_0-\kappa_1)\kappa_1}{(1+\kappa_1)^2}[p-r].$$

Consequently, the equilibrium offer distribution is given by (13) where the equilibrium reservation wage is the following weighted average of the worker's value of non-market and market time:

$$r = \frac{(1+\kappa_1)^2 b + (\kappa_0-\kappa_1)\kappa_1 p}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1}. \tag{14}$$

Because (14) implies that p-r is proportional to p-b, (13) represents a meaningful equilibrium if and only if the value of market time, p, is at least as large as the value of non-market time, b. Note that the market equilibrium offer distribution is non-degenerate distribution when this required inequality is strict. The comparative static properties of equilibrium, reviewed below, are easily derived from (13) and (14).

Any increase in the value of non-market time, b, increases both the lowest and the highest wage offer without otherwise affecting the functional form of the equilibrium wage offer distribution. Hence, the offer distribution increases with the value of non-market time in the sense of first order stochastic dominance. In other words, higher offers are more probable in an equilibrium associated with a higher value of non-market time. For the same reasons, the equilibrium offer distribution is also stochastically increasing in the offer arrival rate when unemployed given $p > b$. These two results are equilibrium consequences of the fact that the optimal reservation wage increases with both the opportunity cost of accepting employment and the frequency with which offers arrive when not employed.

Counter-intuitively, the equilibrium wage offer distribution need not be stochastically increasing in the common value of labor productivity, p. Formally, the equations of (13) and (14) imply that the equilibrium wage offer distribution is stochastically increasing in p if and only if the offer arrival rate when not employed exceeds the offer arrival rate when employed. Higher offers are not always more probable because it is individually rational for each worker to reduce the minimum acceptable wage in response to an improvement in the offer distribution in the sense of first order stochastic dominance when this condition is violated, on the one hand, while the reservation wage is always the smallest profit maximizing offer, on the other.

As noted above, the reservation wage is equal to the value of non-market time, b, when the offer arrival rate is the same whether employed or not. Because the worker is indifferent between searching while employed and while unemployed, any job that compensates for the forgone value of non-market time is acceptable in this case. Finally, because an increase in the offer arrival rate when employed makes employment more attractive, the reservation wage decreases with κ_1 given κ_0 and, consequently, the reservation wage is less than b if and only if the offer when employed exceeds the offer arrival rate when unemployed.

An increase in the offer arrival rate when employed has two different effects on the equilibrium wage offer distribution. First, the lowest wage offer decreases with κ_1 because the reservation wage does and because the reservation wage is the lowest wage. Second, the probability that an offer is less than or equal to any wage w, F(w), decreases with κ_1 for every w in the interior of the support of the equilibrium offer distribution by virtue of (13.b). Because these two effects on the equilibrium offer distribution are offsetting, the offer distributions associated with two different employed offer arrival frequencies cannot generally be ordered with respect to the criterion of stochastic dominance. However, the highest and the lowest offers do both converge to b, the value of non-market time, as κ_1 tends to zero, i.e., Diamond's [1971] equilibrium is the limiting solution obtained by letting the offer arrival rate when employed tends to zero.

Because we have abstracted from time preference in the model, the only cost of "friction", the lags in the arrival of offers, arise as a consequence of job separation risk. Hence, less friction is appropriately associated with larger values of the two arrival rates relative to the job separation rate, i.e., larger values of κ_0 and κ_1. In the limit as both tend to infinity, there is no friction in the sense that all the offers arrive instantaneously, whether employed or not, relative to the duration of any job-worker match.

Interestingly, the equations of (13) and (14) imply that the limiting wage offer distribution is dispersed as friction vanishes provided that the limiting ratio of the two offer arrival rates is strictly positive and finite. Recalling that $\kappa_0 = \lambda_0/\delta$ and $\kappa_1 = \lambda_1/\delta$, equations (13) and (14) imply that the equilibrium offer distribution function and its support limits to

(15.a) $w_0 = (\lambda_1/\lambda_0)b + (1-\lambda_1/\lambda_0)p,$

(15.b) $F(w) = 1 - \left[\frac{p-w}{p-w_0}\right]^{1/2} \quad \forall\ w \in [w_0, w_1],$

and

(15.c) $w_1 = p$

as λ_1 and λ_0 both tend to infinity holding their ratio constant. In particular, equilibrium wage offers do not all converge to the competitive wage, which is p in this formulation, as both offer arrivals increase without bound relative to the job separation rate in both market states.

However, note that (15.c) does imply that the largest wage tends to p as κ_1 tends to infinity whatever the limiting behavior of κ_0. Since the employer offering this wage makes no profit, every employer makes zero profit in the limiting equilibrium. For this condition to hold, employers offering less than p attract no workers. To establish this fact, we simply note that the equations of (4) imply that the steady state distribution of wages earned over employed workers is

(16) $G(w) = \frac{F(w)}{1 + \kappa_1[1-F(w)]}$

in the case of a single worker type. Since (16) implies that the fraction of workers earning w or less, G(w), converges to zero for all $w < w_1$ as κ_1 tends to infinity, it follows that all workers earn the highest wage offer in the limiting equilibrium, which is the competitive equilibrium wage p.

5. The General Case

Propositions 1-4 can be used to derive the form of an equilibrium offer distribution in the general case of any number of worker and employer types as follows: Profit maximization requires that employers of the same type earn the same profit in equilibrium. By virtue of (6) and (7) the common equilibrium profit of type j employers is

$$(16)\quad \pi_j = (p_j - w_{0j}) \sum_{r_i \le w_{0j}} h_i \kappa_0 / \Big[1+\kappa_1[1-F(w_{0j})]\Big]^2 .$$

where w_{0j} is the lowest offer of type j employers. The equal profit condition requires that $\pi_j = \pi(p_j, w)\ \forall\ w \in W_j$ where W_j, defined by (8), is the set of offers that are profit maximizing for type j employers. Hence, the equations of (6) and (7) imply that the value of the equilibrium offer distribution function employers, F(w), must equal

$$(17)\quad \phi_j(w) = \frac{1+\kappa_1}{\kappa_1}\left[1 - \frac{1+\kappa_1[1-F(w_{0j})]}{1+\kappa_1}\left[\frac{(p_j - w) \sum_{r_i \le w} h_i}{(p_j - w_{0j}) \sum_{r_i \le w_{0j}} h_i}\right]^{1/2}\right]$$

for all w in W_j.

The right side of (17) provides all the possible values of F(w) for each wage w that yields profit equal to that associated with the lowest profit maximizing wage offered by an employer of type j. Its graph is represented by the "saw-toothed" shaped curves drawn in the panels of Figure 2. As (17) implies, $\phi_j(w)$ is continuous, upward sloping, and convex between adjacent reservation wage rates and jumps down at each reservation wage rate. This pattern is simply implied by the nature of the steady state labor force function, defined in (6), and the equal profit condition. That the value of

$\phi_j(w)$ for all $w \geq w_{0j}$ at any reservation wage rate is no less than $\phi_j(w_{0j})$ is implied by the fact that w_{0j} solves the maximization problem defined by (12) for each j. Hence, $\phi_j(w) = \phi_j(w_{0j})$ if and only if w is also a solution to problem (12). However, $\phi_j(r_{i+1}) < \phi_j(r_i)$ for some i is a possibility. This case is illustrated in panel b of Figure 2.

Because an equilibrium cumulative offer distribution function, F(w), must be non-decreasing by definition and continuous by virtue of Proposition 1.b, $F(w) = \phi_j(w)$ on W_j implies that F(w) is composed of continuous upward sloping segments of $\phi_j(w)$ and of "flats" connecting these segments. Of course, the "flats" are supported by open intervals that are not in W_j. Because "gaps" in the support of an equilibrium c.d.f. occur only to the immediate left of the reservation wage of each worker type by virtue of Proposition 2, each connecting "flat" is supported on the right by the reservation wage rate of some worker type. The only function consistent with these requirements is

$$(18) \quad F(w) = \min_{x \geq w}\{\phi_j(x)\} \;\; \forall \; w \in [w_{0j}, w_{1j}].$$

Its graphs is illustrated in the panels of Figure 2 by the smooth curves obtained by eliminating the "teeth" of the "saw" with the appropriate horizontal "flats."

Note that if $\phi_j(r_{i+1}) \leq \phi_j(r_i)$, then the reservation wage r_i is offered by no employer by inspection of Figure 2.b.\3 Consequently, if some $w > w_{0j}$ is a solution to (12), then $F(w) = F(w_{0j})$. Hence, the lowest wage is largest solution to the problem defined by (12) without loss of generality, i.e.,

$$(19) \quad w_{0j} = \max\left\{ \operatorname*{argmax}_{w \geq w_{1j-1}} \left\{ (p_j - w) \sum_{r_i \leq w} h_i \right\} \right\} \;\; \forall \; j \geq 1 \text{ where } w_{10} = -\infty.$$

Because the equilibrium wage offer distribution has no mass point, (20.a) $F(w_{01}) = 0$ and $F(w_{0j}) = F(w_{1j-1})\ \forall\ j > 1$

where w_{1j} is the unique solution to

(20.b) $F(w_{1j}) = \gamma(p_j)\ \ \forall\ j \geq 1$

as already noted in (11) above.

Finally, if the solution to (19) is $w_{0j+1} = w_{1j}$ for some j, the equilibrium distribution must have a "kink" at the common wage offered by employer types j and j+1, as illustrated by the solid curve in Figure 3. The claim holds because $p_{j+1} > p_j$ and $F(w_{0j+1}) > F(w_{0j})$ imply $\phi'_{j+1} < \phi'_j(w)$ by virtue of (17) and because $\phi_j(w_{1j}) = F(w_{1j}) = F(w_{0j+1}) = \phi_{j+1}(w_{0j+1})$ by virtue of (18) and (20).

In sum, equations (17)-(20) provide a closed form solution for the wage offer distribution function consistent with non-cooperative profit maximization by all employers given any vector of reservation wage rates $(r_1,\ldots,r_n)$ and any modified distribution of worker types, defined by the vector $(h_1,\ldots,h_n)$. An algorithm for constructing the function given these vectors, the ratios of the offer arrival rate to the job separation rate, and the distribution of employers over productivity follows: (1) Solve for the smallest wage offered, w_{0j}, by employer type j=1 using (19). (2) Substitute $\gamma(p_{j-1})$ for $F(w_{0j})$ in equation (17), as required by the equations of (20), to obtain a closed form for $\phi_j(w)$ for j=1. (3) Use the fact that $\gamma(p_j) = F(w_{1j}) = \phi_j(w_{1j})$, implied by (20.b) and (18), to find the largest wage offered, w_{1j}, for employer type j=1. Having obtained w_{1j-1} for j = 2 in this manner, iterate back to step (1) until j = n. Having computed the triple $(w_{0j},\phi_j(w),w_{1j})$ for all j = 1,...,n, F(w) can be constructed using (18).

Because equations (1) and (5) imply that neither r nor h depend on F when $\kappa_0 = \kappa_1$, a unique equilibrium exists if offer arrival rates are independent of employment status.

Theorem 1: If the offer arrival rates when employed and unemployed are equal, then equations (17)-(20) and $r_i = b_i$ and $h_i = m_i$ for all $i = 1,\ldots,n$ characterize the unique equilibrium.

Obviously, this result constitutes a generalization of the principal conclusion found in Burdett and Mortensen [1988]. The following general existence proof includes the models of both Albrecht and Axell [1984] and Eckstein and Wolpin [1987], at least as limiting cases.

Let $r = (r_1,\ldots,r_n)$ denote the vector of worker reservation wage rates and let $h = (h_1,\ldots,h_n)$ represent the modified distribution of workers over types defined by (5). Of course, equations (17)-(20) define the unique profit maximizing distribution of offers associated with any given pair of such vectors, denoted as $F(w;r,h)$ in the sequel. Because the function maximized in (19) is continuous in r and h, the lowest wage offered by type j employers, w_{0j}, is a continuous function of r and h by virtue of (19) and the maximum theorem.\4 Because $F(w_{0j}) = \gamma(p_{j-1})$ is invariant with respect to r and h by virtue the equation of (20), equation (17) implies that $\phi_j(w)$ is continuous in r and h for every j and w. Finally, these two facts together with (18) imply that $F(w;r,h)$ is continuous in both r and h for every w.

Let

$$(21)\quad \rho_i(r,h) = b_i + (\kappa_0-\kappa_1)\int_{r_i}^{\infty}\left[\frac{1-F(x;r,h)}{1+\kappa_1[1-F(x;r,h)]}\right]dx,\ i = 1,\ldots,n.$$

and

$$(22)\quad \eta_i(r,h) = \frac{m_i[1+\kappa_1[1-F(r_i;r,h)]]}{1+\kappa_0[1-F(r_i;r,h)]},\quad i = 1,\ldots,n.$$

represent the right sides of (1) and (5) respectively with the c.d.f. $F(w;r,h)$ defined by equation (17)-(20) substituted for F. Because $F(w;r,h)$ is continuous is w, by virtue of Proposition 1.b, as well as in r an h, both $\rho_i(r,h)$ and $\eta_i(r,h)$ are continuous in r and h for all i by virtue of (21) and (22) respectively.

As corollary of (17)-(20), the largest wage offered by each employer type is no larger than that type's productivity, i.e., $w_{1j} \le p_j$. Because w_{1s} is the upper support of $F(x;r,h)$ where s represents the most productive employer type, i.e., $p_s \ge p_j$ for all j, equation (21) implies that $\rho_i(r,h)$ is bounded below by b_i and above by $b_i + (\kappa_0-\kappa_1)p_s/\kappa_1$ when $\kappa_0 \ge \kappa_1$ and is bounded above by b_i and below by $b_i + (\kappa_0-\kappa_1)p_s/\kappa_1$ when $\kappa_0 < \kappa_1$. Of course, $\eta_i(r,h)$ is non-negative and bounded above by $m_i(1+\kappa_1)$. Hence, if we let R represent the cartesian product of the n compact intervals that are the ranges of $\rho_i(r,h)$, $i = 1,\ldots,n$ and if we let H denote the n compact intervals that are the ranges of $\eta_i(r,h)$, $i = 1,\ldots,n$, then the pair of real vector function (ρ,η) defined by

$$(23.a)\quad \rho(r,h) = (\rho_1(r,h),\ldots,\rho_n(r,h))$$

$$(23.b)\quad \eta(r,h) = (\eta_1(r,h),\ldots,\eta_n(r,h))$$

map R×H into itself. Since we have just shown that this map is continuous and R×H is compact and convex, it has at least one fixed point,

$$(24.a)\quad r^* = (r_1^*,\ldots,r_n^*) = \rho(r^*,h^*)$$

$$(24.b)\quad h^* = (h_1^*,\ldots,h_n^*) = \eta(r^*,h^*),$$

by virtue of the Brouwer fixed point theorem. Of course, r^* and $F(x;r^*,h^*)$ are an equilibrium reservation wage vector and wage offer distribution pair given any fixed point of (ρ,η) by construction.

Theorem 2: A market equilibrium exists.

6. Equilibrium Compensating Wage Differentials

The framework can easily be extended to generate an equilibrium theory of the relationship between the wage and other endogenous job attributes offered, a so called hedonic wage function. For the sake of illustration, let x represent a finite row vector of desirable job characteristics and let $\beta > 0$ denote an associated column vector of values that all workers place on the components of the attribute vector. In other words, the total value flow associated with a job characterized by the wage-attribute pair (w,x) follows:

$$(25)\quad v = w + x\beta.$$

Workers care only about the total value of a job offer, v, and employers choose the wage-attribute vector once and for all with this fact in mind. A worker accepts the first job arrival that offers a value greater than or equal to an appropriately chosen reservation value when unemployed and moves to alternative jobs offering larger values as they arrive when unemployed. By reinterpreting F(v) as the fraction of employers offering jobs of value v or less, a distribution induced by (25) and the joint distribution of wage and characteristics offered by employers, it follows that the optimal reservation value is the solution to equation (1) for each worker type.

Similarly, a particular employer offering a wage-attribute pair of value v obtains a steady-state labor force of size $\ell(v)$, where $\ell(\cdot)$ is the function defined by the equations of (6) when $F(\cdot)$ is interpreted as the distribution of job values offered by all the employers. Hence, if employers differ only with respect to the cost of providing the vector of attributes x, denoted as $c_j(x)$, then steady state profit of type j employers associated with any wage-characteristic pair (w,x) is

$$\pi_j = (p - w - c_j(x))\ell(w+x\beta) = (p + x\beta - c_j(x) - v)\ell(v)$$

by virtue of (25). Obviously, if a profit maximizing employer of type j offers jobs of value v, then the optimal characteristic choice is

$$(26.a) \quad x_j = \underset{x\geq 0}{\text{argmax}}\{x\beta - c_j(x)\}$$

and the employer's net labor productivity is

$$(26.b) \quad p_j = p + \max_{x\geq 0}\{x\beta - c_j(x)\}.$$

The assumption that $c_j(x)$ is strictly convex, $c_j'(x) > 0 \; \forall \; x > 0$, and $c_j(0) = 0$ guarantees that both x_j and p_j are well defined and unique.

Finally, the steady state profit of an employer of type j offering jobs of value v is

$$(27) \quad \pi(p_j,v) = (p_j-v)\ell(v).$$

Given this representation of the profit function, all the existence arguments apply as well to the extended version of the model. Specifically, the equilibrium wage offer distribution constructed above also can be viewed as the equilibrium distribution of job value offers. In other words, the theory implies a equilibrium stochastic relationship between wage and attributes offered of the form $w = v - x\beta$ where v is a random variable distributed according to the equilibrium $F(\cdot)$ derived above.

7. Structural Parameter Estimation

Models of optimal job search have provided a framework for interpreting empirical finding on both the duration of unemployment spells experienced by individuals and on the post spell wage rates earned by the same individuals in many papers in the existing literature. Some authors even attempt to estimate structural parameters. In the most closely related examples, Mortensen and Neumann [1988] consider the theoretical possibility of estimating the offer arrival rates and an arbitrary wage offer distribution under the maintained hypothesis that the reservation wage rate for each worker type satisfies equation (1). Eckstein and Wolpin [1987] make an original attempt to estimate the structure of the equilibrium market model of wage determination suggested by Albrecht and Axell [1984].

This section contains a brief discussion of the problem of identifying the structural parameters of the general equilibrium market model formulated in the paper. For this purpose, we suppose that observations on at least the duration of one unemployment spell and the post spell wage earned are available for a sample of identical individuals. The hypothetical plan is to use these observations and the model to obtain maximum likelihood estimates of the structural parameters, at least those that can be identified. As is clear from a reading of Eckstein and Wolpin, the identification problem differs from that previously considered, say by Flinn and Heckman [1982], because the wage offer distribution, as well as the distribution of unemployment spell lengths, is an endogenous consequence of the model's structure.

In the simplest case of identical workers and employers, the structural parameters of interest are the workers' common value of non-market time, b, the employers' common value of productivity, p, the two offer arrival rate to job separation rate ratios, λ_0 and λ_1, and the job separation rate δ. According to the model, the sample of realized unemployment spell durations are independently and identically distributed as exponential random variables with hazard equal to $\lambda_0[1-F(r)]$ and the sample of post spell wages earned are independent and identical random draws from the distribution of acceptable offers, which is $\max[F(w)-F(r),0]/[1-F(r)]$. Because in equilibrium all offers are acceptable when workers are identical, i.e., $F(r) = 0$, the observations on unemployment spell durations identify the maximum likelihood estimate of λ_0 while the observations on the post spell wage offers together with its functional form, given in equation (13), identify maximum likelihood estimates of the parameters of F(w). The latter include the ratio of the offer arrival rate when employed to the job separation rate $\kappa_1 = \lambda_1/\delta$, productivity p, and the common reservation rate, r.

Without either more structure or more data, estimates of the value of non-market time, b, the job separation rate, δ, and the offer arrival rate when employed, λ_1, are not separately identified. Because an estimate of b requires an inference using equation (14), the estimates of r, λ_0, and the parameters of F(w) are not sufficient. One needs to know the ratio of the offer arrival rate when unemployed to the job separation rate, $\kappa_0 = \lambda_0/\delta$, as well. Given that an estimate of the ratio $\kappa_1 = \lambda_1/\delta$ is available, the assumption that the offer arrival rates are independent of employment status, i.e., $\lambda_0 = \lambda_1$, would permit complete identification of the structure.

Alternatively, observations on the length of each individual's subsequent employment spell as well as the wage earned during that spell would be sufficient to identify maximum likelihood estimates of these parameters since the model implies that the duration of any such spell is exponential with hazard $\delta + \lambda_1[1-F(w)]$. Furthermore, over identifying restrictions exist and can be tested using these data.

The marginal willingnesses to pay for job attributes, the vector β, represents additional structural parameters in the extended model. As Gronberg and Reed [1988] observe, the value of this vector is reflected in job duration data when jobs differ in value and workers obtain information about alternatives while employed. Because the job separation hazard associated with job to job movement given that a worker is current employed at value $v = w + \beta x$ and information about alternatives arrives at Poisson rate λ_1 is $h(w,x) = \lambda_1[1-F(w+x\beta)]$, the marginal values of job attributes equal the ratio of the partial derivatives of the hazard function with respect to x divided by the partial derivative with respect to w, i.e., $\beta = h_x(\cdot)/h_w(\cdot)$. Gronberg and Reed use this fact, appropriate job duration data, and observations on wage and job attributes to obtain consistent estimates of β.

In the received literature, the standard estimate of the attribute values is the coefficient vector obtained by regressing job characteristics on the wage received. Because our market model generates a stochastic equilibrium relationship between wage offered and job attributes, one can ask and answer the following questions: Is the OLS estimate of β consistent under the hypothesis that the data are generated by a market equilibrium in the sense of this paper? The argument below suggest a negative answer. Indeed, the OLS estimate is inconsistent and biased in the opposite direction of the true sign when more productive employer types are so because of a cost advantage in providing desirable job attributes.

By virtue of equations (26), an employer with a cost advantage in providing desired attributes supplies them in greater quantity and enjoys a larger net productivity per worker as a consequence. Formally, if $c_j(0) = 0$, $c_j(x)$ is strictly convex, and $c_j'(x) > c_{j+1}'(x) > 0$ for all $x > 0$ and j, then $x_{j+1} > x_j$ by virtue of (26.a) and $p_{j+1} > p_j$ by virtue of (26.b). Of course, higher net productivity also implies that jobs of no less value are offered by virtue of Proposition 3, i.e., $v_{1j} \leq v_{0j+1}$ if $p_j < p_{j+1}$.

Finally, because $w = v - x\beta$ and because the values of x offered differ only over employer type, conditioning on x is equivalent to conditioning on employer type. Indeed,

$$\text{(29.a)} \quad E\{w|x=x_j\} = \alpha_j - x_j\beta + \varepsilon_j,$$

$$\text{(29.b)} \quad \alpha_j = E\{v|x=x_j\} = \int_{v_{0j}}^{v_{1j}} v dF_j(v)$$

and

$$\text{(29.c)} \quad E\{\varepsilon_j|x=x_j\} = 0$$

where $F_j(v) = [F(v)-F(v_{0j})]/[F(v_{1j})-F(v_{0j})]$ is the conditional distribution of job values offered by employers of type j over the range $[v_{0j},v_{1j}]$. Obviously, $v_{0j+1} \geq v_{1j}$ and dispersion in the distribution of values offered by each employer type imply $\alpha_{j+1} > \alpha_j$. Because $x_{j+1} > x_j$ as well, α_j and x_j are positively correlated across employers which implies that the coefficient of x obtained from an OLS wage equation is inconsistent and biased upward as an estimate of $-\beta$.[5]

The inconsistency in the OLS estimator is graphically illustrated in Figure 4. Workers are identical but there are two types of employers. Given that the second type can provide the attribute at a lower cost, the level of the attribute supplied and the values of the jobs offered by type 2 employers are larger that those of type 1 employers. In Figure 4, the negative sloped lines are iso-value curves. Since $w = v - \beta x$ on any one of them, all have slope equal to $-\beta$, the parameter of interest. In the case under consideration, the lowest wage offered by type 2 employers is equal to the highest offer of type 1 employers, as illustrated in the Figure. The observed sample of wage-attribute pairs offered by the employers of a given type will lie on the set of iso-value curve associated with the range of value offered at the value of the attribute supplied. These points are illustrated by the heavy vertical line segments in Figure 4.[6] Obviously, the OLS line through these points, indicated by the dashed line in the Figure, necessarily has slope greater the $-\beta$. Indeed, it can even be positive as illustrated.

ENDNOTES

\1 In this papers, neither workers nor employers are assumed to discount the future for simplicity of exposition. In Mortensen and Neumann [1988], an optimal reservation wage maximizes the expected discounted stream of future income. Formally, the solution to (1) is the limit of the wealth maximizing reservation wage obtained as the rate of discount is allowed to converge to zero. Wernerfelt's [1988] analysis of the case of positive discounting suggests that the results presented in this paper can be regarded as approximations to those that obtain when the time rate of discount is small.

\2 Here and in the sequel, the inequality $r_i \leq w$ below the summation sign is short-hand for the sum over the set of worker types that find the specified offer w acceptable.

\3 Eckstein and Wolpin [1987] impose a condition on the distribution of workers over values of non-market time that implies $\phi_j(r_{i+1}) > \phi_j(r_i)$. Given this condition, they then establish that the support of the wage offer distribution is the set of reservation wage rates given no search while employed. Existence does not require the inequality, which is a fact established below.

\4 See Green and Heller [1981,p. 49].

\5 The difference between the estimates of β obtained by Gronberg and Reed using the two methods seem to be consistent with this implication of the model.

\6 Note that an overlap of the supports of the wage rates offered by the two types is permitted in the extended model. Proposition 3 requires only that the job values offered by type 2 employer exceed those of type 1.

REFERENCES

Albrecht, J.W. and B. Axell (1984): "An Equilibrium Model of Search Employment," **Journal of Political Economy**, 92.

Burdett, K. and D.T. Mortensen (1988): "An Equilibrium Wage Distribution," mimeo, Cornell University.

Burdett, K. and K. Judd (1983): "Equilibrium Price Distributions," **Econometrica**, 51, 955-970.

Diamond, P. (1971): "A Model of Price Adjustment." **Journal of Economic Theory**, 3, 156-168.

Eckstein, Z. and K.I. Wolpin (1987): "Estimating a Market Equilibrium Search Model From Panel Data on Individuals," mimeo, Tel-Aviv University and Ohio State University.

Flinn, C. and J. Heckman (1982): "New Methods for Analyzing Structural Models of Labor Force Dynamics." **Journal of Econometrics**, 18.

Green, J. and W.P. Heller (1981): "Mathematical Analysis and Convexity with Applications to Economics," in K.J. Arrow and M.D. Intriligator eds., **Handbook of Mathematical Economics**, North-Holland.

Gronberg, T.J., and W.R. Reed (1988): "Estimating Workers' Marginal Willingness to Pay For Job Attributes Using Duration Data", mimeo, Texas A&M University.

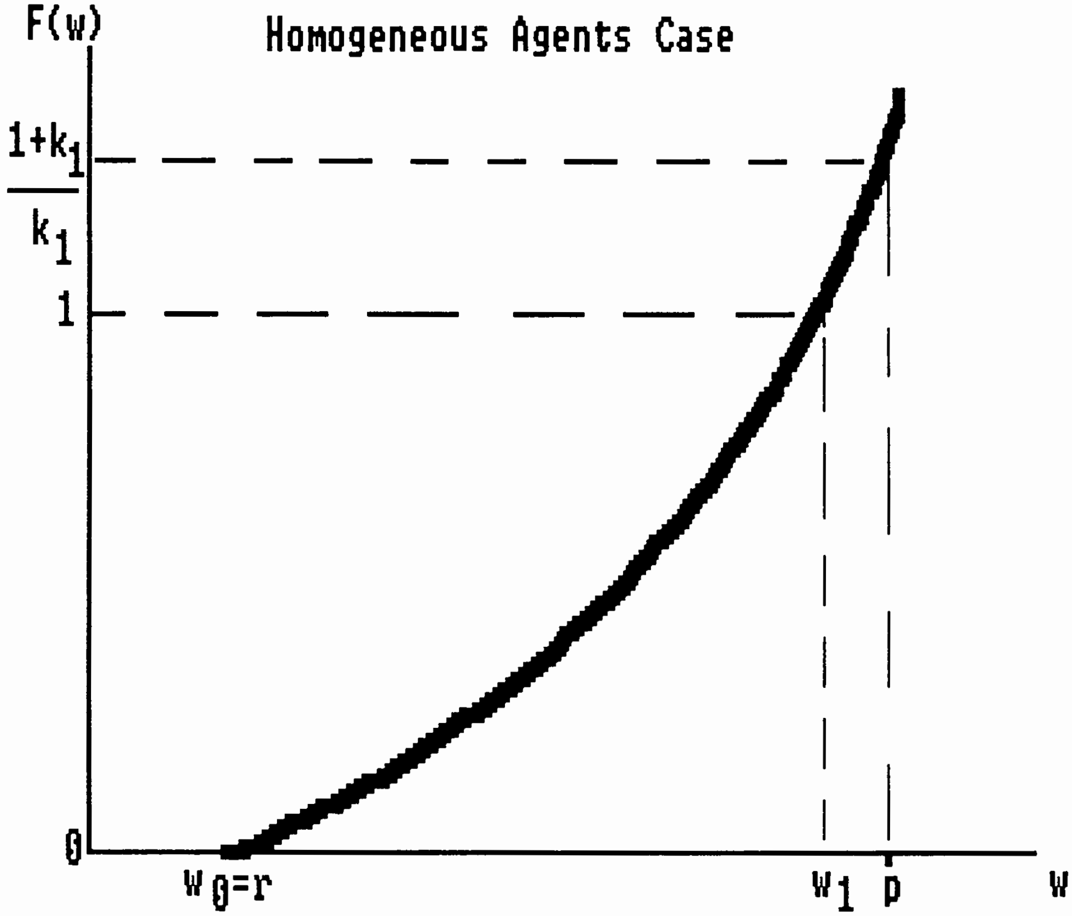
Mortensen, D.T. (1986): "Closed Form Equilibrium Price Distributions, mimeo, Northwestern University.

Mortensen, D.T., and G.R. Neumann (1988): "Estimating Structural Models of Unemployment and Job Duration" in **Dynamic Econometric Modeling**, Proceedings of the Third International Symposium in Economic Theory and Econometrics, Edited by W.A. Barnett, E.R. Berndt, and H. White, Cambridge University Press.

Wernerfelt, B. (1988): "General Equilibrium with Real Time Search in Labor and Products Markets," **Journal of Political Economy**, 96, 4, 821-831.

Figure 1: Wage Offer Distribution

Homogeneous Agents Case



Heterogeneous Workers

Figure 2.a: Wage Offer Distribution

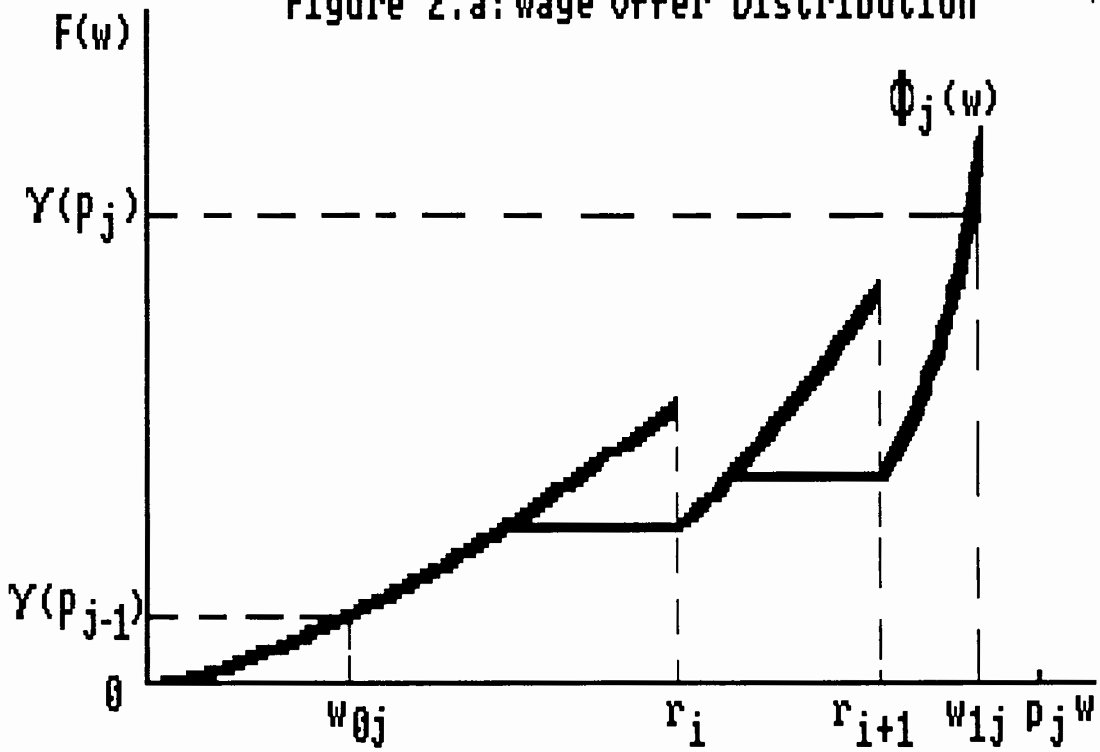


Figure 2.b: Wage Offer Distribution

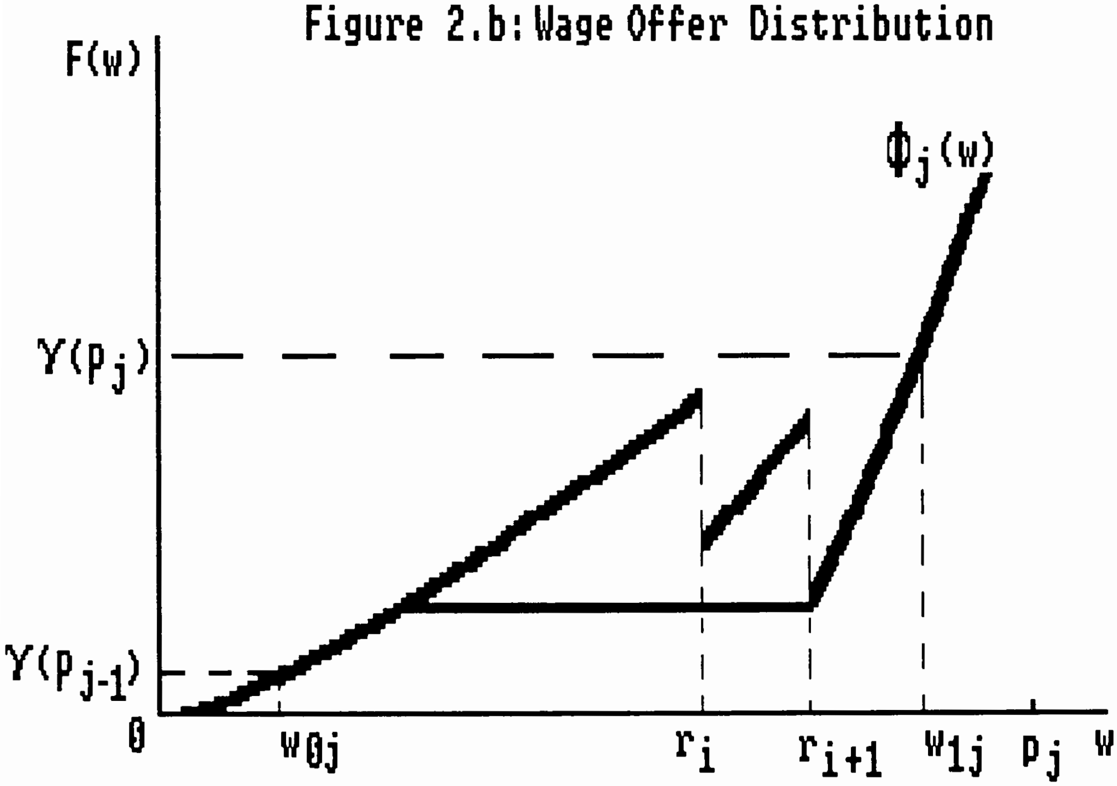


Figure 3: Heterogeneous Employers

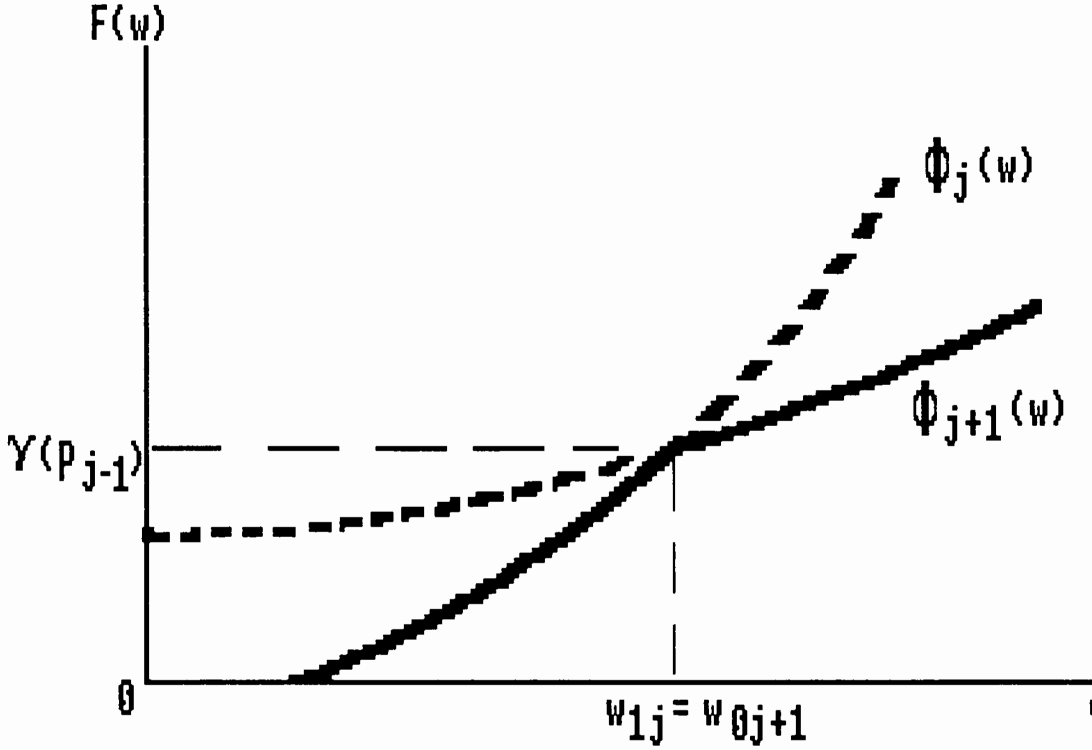


Figure 4: Compensating Differentials