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THE PERSISTENCE AND INDETERMINACY OF UNEMPLOYMENT IN SEARCH EQUILIBRIUM*

by

Dale T. Mortensen
Northwestern University

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Abstract

Existing theories that explain persistent and indeterminant unemployment are brought together within the unifying framework of search equilbrium. External economies that exhibit increasing returns to production and exchange are identified as potential causes of both the indeterminacy and the persistence of unemployment for a wide range of assumptions about wage determination. Those considered include a 'market clearing' wage, an 'efficiency' wage, and an 'insider-outsider' wage model. Although either of the non-market clearing specifications can induce greater persistence, multiple equilibria require increasing returns in the technologies of either production or exchange.

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I. Introduction

Current concern about European unemployment is focussed on both its high level and its persistence. A recent fifteen year period of high measured unemployment rates justifies this emphasis. As in the Thirties, a long period of high unemployment also challenges our understanding of how economies with voluntary labor markets work.

Keynes rejected the classical paradigm of production coordinated by competitive market clearing prices as inconsistent with the persistence of depressed employment experienced in the Thirties. According to Leijonhufvud [1968], Keynes argues in the General Theory that the system is subject to significant coordination failures in the absence of Walras's auctioneer. Once the economy is "stuck" at a low employment level, government can and must provide a substitute for the dysfunctional "invisible hand."

Subsequently, the Walrasian approach to macro economic analysis was rehabilitated in the form of the "new-classical" theory of the business cycle. Positive co-movement between employment and other macro aggregates as well as other stylized features of business fluctuations are viewed as by-products of inter-temporal substitution in a stochastic world with signal extraction problems but rational expectations in the most familiar version of this theory. (See Lucas and Sargent [1981].) Because fiscal and monetary policy has very limited positive influence on employment and wage and incomes policies can only cause distortions in any Walrasian general equilibrium model, the establishment of this approach as the principal framework for macroeconomic analysis, at least in the United States and in some degree in Great Britain, has been a major factor in the policy debates of the past several years.

The recent European experience has called the "new-classical" reinterpretation into question and has led some to "new-Keynsian" ideas. (See Gordon [1987] for summary of these developments.) However, Blanchard and Summers [1986, 1988] suggest that the continued persistence of high unemployment in the face of relatively low inflation rates is inconsistent with both of these broad approaches to unemployment theory.

Specifically, Blanchard and Summers argue that the evidence contradicts the idea that the unemployment rate tends to return to some unique stationary 'natural rate' which is independent of initial conditions and is relatively insensitive to shocks. To explain unemployment time series outcomes that are highly sensitive to past shocks, they suggest alternative theories of 'fragile' equilibria. A dynamic system is 'fragile' when multiplier processes exist that create instability and multiplicity of equilibria. For such systems, history can determine the particular steady state toward which the system tends and shocks abruptly change the system's tendency by creating discontinuities in its history.

Blanchard and Summers [1988] argue that multiple unemployment equilibria occur for two general reasons: either labor costs decrease or the value of labor productivity increases with employment. In their language, multiple intersections can occur if either the 'supply of labor' is downward-slowing or the 'demand for labor' is upward-sloping in some range. In Blanchard and Summers [1986], they suggest that the 'insider-outside' wage determination model proposed by Lindbeck and Snower [1986] provides an example of the former.

Labor productivity that increases with employment can be modeled directly by assuming increasing returns in production and imperfect competition in the goods market as in Weitzman [1982], Hart [1982], and Roberts [1986]. A perversely sloped 'labor demand' curve obtains as well when the exchange process in the labor market exhibits increasing returns as suggested by Diamond [1982a] and developed by Howitt and McAfee [1984,1987] and Howitt [1987]. Finally, positive feed back from the goods market to the labor market for the reasons suggested by Diamond [1982b, 1984] and by Drazen [1988a,1988b] can also induce instability and multiplicity of equilibrium.

The purpose of the paper is to review and to critique the existing collection of 'fragile equilibria' theories within the unifying framework of the 'search equilibrium' model developed from the work of Diamond [1981, 1982b], Mortensen [1982a, 1982b] and Pissarides [1984, 1985]. The model provides a parsimonious analytic framework for such a study for two principal reasons. First, the exchange process is simply but explicitly modelled within this framework. Consequently, its role as a potential destablizing influence in the aggregate can be made explicit. Second, the existence of transactions lags and costs in the labor market implies that the wage is indeterminant in the sense that individual rationality requires only that it divide match rents between employee and employer. Hence, the effects of different stories about relative work-employer bargaining power on aggregate employment outcomes can be explicitly modelled within the same framework.

The general conclusions drawn from the exercise follow: Increasing returns in production, even if only in some range, is a powerful potential cause of both the indeterminacy and the persistence of unemployment for all the wage determination models considered, provided that the positive relationship between aggregate employment and the productivity of an individual worker is an external effect attributable to agglomeration or specialization. Multiple equilibria arising in this case pose a coordination problem of the kind discussed by Cooper and John [1986]. Specifically, which equilibria obtains depends on whether expectations are optimistic and pessimistic since either are self-fulfilling.

These same conclusions hold for a wage that 'clears' the market in the sense that workers are indifferent between unemployment and employment, a wage that is 'efficient' in the sense that it induces the optimal tradeoff between monitoring and recruiting costs, and an 'insider-outsider' wage set solely for the benefit of employed workers. In the 'market clearing' case, unemployment is 'voluntary' by definition. However, some component of unemployment is 'involuntary' given either alternative wage determination model in the sense that unemployed workers strictly prefer employment at the going wage and employers are either unwilling or unable to offer them a lower wage. As a consequence, steady state employment is lower and the speed of adjustment to the steady state is slower given either alternative to the 'market clearing' wage model.

Neither the 'efficiency wage' nor the 'insider-outsider' wage model alone generates indeterminacy. However, either can yield multiple equilibria not present in the 'market clearing' wage case when the job-worker matching process is characterized by increasing returns to scale in the sense that matching rates per unemployed worker and per vacancy both increase with proportionate increases in the number of vacant jobs and searching workers. In both cases, a multiplier process is induced by the fact that the profit are pro-cyclic -- vary directly with aggregate employment. Again, multiple equilibria occur because the effect is external and the multiplier process created by it can support different expectations.

II. The Transaction Approach to Labor Market Analysis

In this section a simple model of the process by which unemployed workers and employers with vacant job meet in the labor market is set down. Although the model considered is in the tradition of Diamond [1982b], Pissarides [1985, 1987], and Drazen [1988a, 1988b] in the sense that only employers actively engage in search, the external effects present are analogous to those in two sided search models considered by Mortensen [1982a, 1982b], Pissarides [1984] and Howitt and McAfee [1984, 1987].

Given current aggregate employment, denoted as e, marginal revenue product of labor productivity, y, is identical across potential job-worker pairs and the value of leisure forgone when employed, b, is positive and identical across workers. Of course, there is a gain from trade only when y exceeds b. Because no matching problem exists and the value of marginal productivity compensates for the opportunity cost of forming a match by assumption, pairs form at the meeting rate represented by a function of the number of actively searching unemployed workers, u, and the number of vacant jobs that employers are attempting to fill, v. Given that matches dissolve at the exponential rate δ for exogenous reasons, the number of employed workers and filled jobs solves the differential equation

$$\dot{e} = m(u,v) - \delta e. \qquad (1)$$

The relationship between the aggregate meeting rate and the numbers of unmatched participants, the function m(u,v), represents the "exchange technology". The number of searching unemployed workers, u, and the number of jobs that employers are seeking to fill, v, are "inputs" in the meeting process that the function characterizes. Following Diamond [1982b], we assume that both inputs are essential, that the marginal contribution of each is positive, that the average return to each input diminishes, and, for simplicity of exposition, that the exchange technology is homogeneous of degree k. Formally, all the following hold:

$$m(0,v) = m(u,0) = 0. \tag{2.a}$$

$$m(u,v) \text{ is increasing in } u \text{ and } v. \tag{2.b}$$

$$m(u,v)/u \text{ and } m(u,v)/v \text{ are decreasing in } u \text{ and } v \text{ respectively.} \tag{2.c}$$

$$m(u,v) = u^k m(1,v/u). \tag{2.d}$$

Note that (2.c) restricts but does not rule out increasing returns to scale in the transaction technology. Indeed, $m(u,v) = uv$ is on the boundary and any scale parameter satisfying $0 < k < 2$ is permitted.

Because job-worker matches dissolve for exogenous reasons at the given frequency δ, the expected present value an employer's future profit stream per worker, J, solves the asset pricing equation

$$rJ = y - w - \delta(J-V) + \dot{J} \tag{3}$$

where V represents the capital value of a vacant job. In other words, the opportunity interest on holding the asset, a filled job, is equal to the profit earned per period less the expected loss per period attributable to exogenous separation plus any capital gain that can be expected in the immediate future. Suppose that the recruiting cost per period required in the attempt to fill a vacancy is fixed at c. Then, because the rate at which vacancies are filled per vacancy per period is $m(u,v)/v$ and the capital gain associated with filling a vacancy is J-V, the expected present value of future profit attributable to holding a vacancy, V, must solve

$$rV = [m(u,v)/v][J-V] - c + \dot{V}. \tag{4}$$

For simplicity, we restrict attention to the special case of trivial out-of-pocket search costs. Given this assumption, analogous arguments imply that the expected present value of the typical worker's future income when employed, W, and the expected present value of future worker income when not, U, solve the following equations obtained using analogous arguments:

$$rW = w - \delta(W-U) + \dot{W} \tag{5}$$

$$rU = b + [m(u,v)/u][W-U] + \dot{U}. \tag{6}$$

All workers participate if the value of employment, W, is no less than the value of unemployment, U. Hence,

$$u = \ell - e \tag{7}$$

where ℓ is the given fixed total number of workers. Employer participation requires that the expected present value of the future returns to recruiting cover recruiting cost; equivalently that the value of holding a vacancy V is non-negative. Following Pissarides [1985], we assume that entry will eliminate this rent. In short, the number of vacant jobs instantaneously adjusts to equate the return and cost of recruiting per period in the sense that

$$Jm(u,v)/v = c, \text{ or equivalently } V(t) = 0 \ \forall \ t. \tag{8}$$

III. Alternative Models of Wage Determination

The specification of a wage is required to close the model. In the search equilibrium literature, Diamond [1981,1982b], Mortensen [1982a,1982b], and Pissarides [1984,1985] all view wage determination as a bilateral bargaining problem over match specific quasi-rents that are induced by the time lags and the transactions costs that characterize the meeting and exchange process. Although this solution begs the question, it illustrates one fundamental fact: Without further specification of the institutional detail required to determine the relative bargaining powers of the two parties, the wage is fundamentally indeterminant, even under conditions that would otherwise be regarded as competitive. The procedure followed in the paper is to consider several plausible alternative specifications that have been suggested in the literature.

The oldest solution to the "pricing problem" in the search literature is due to Diamond [1971]. When applied to the labor market, the model supposes that employer post wage offers and that workers are assumed to search among them -- sequentially, at random, and with no recall -- knowing only their distribution over employers. It is well known that a reservation wage policy is optimal under these circumstances. The optimal reservation wage is the same for all worker when the workers are identical in production, have the same preferences, and have the same information about offers. Given this reservation wage and the wages offered by other employers, each employer sets his wage to maximize expected wealth.

There is only one non-cooperative equilibrium to this "wage setting game." It is the lowest wage offer that any worker will accept given that all offers are equal, the common value of leisure, b. The logic used to establish Diamond's result is straight forward. Under the assumption that each worker receives offers sequentially without recall, an assumption implicit in our specification of the search technology, any employer offering a wage greater than the common reservation wage forsakes no beneficial hiring opportunity by lowering that offer to the common reservation wage. Hence, there can be no wage dispersion in equilibrium and the common offer, w, is equal to the common reservation wage. But, the optimal reservation wage is the wage offer that equates the capital value of search unemployment, U, with the capital value of employment, W. Given equation (5) and (6),

$$W = U \tag{9.a}$$

is equivalent to

$$w = b. \tag{9.b}$$

In sum, the assumptions that employers post wage offers and that workers can only accept or reject offers sequentially are sufficient to insure that the employers possess all the bargaining power in this simple world. As a consequence of (9.b), the extent of recruiting effort is maximal by virtue of (4) and (5) and unemployment is "voluntary" by virtue of (9.a) in the sense that workers are indifferent between employment and unemployment in this equilibrium. In other words, $w = b$ is the 'market clearing' wage. It is this property of the solution rather than Diamond's story about how it might be attained that are of primary interest in the sequel.

Efficiency wage theory provides a simple rational for modifying (9). In the shirking model version of the theory proposed by Stiglitz and Shapiro [1984], effort is not costlessly observable. Hence, the worker can collect both the wage, w, and the value of leisure, b, if successful at "taking a vacation on the job." Assuming that the employer monitors effort with frequency λ and fires the worker when he or she is caught shirking, the equilibrium wage must exceed b to insure that the expected worker cost of shirking per period, which is $\lambda[W-U]$, is no less than the benefit, b. Because the two are equal in equilibrium,

$$W - U = b/\lambda. \tag{10.a}$$

In this case, equations (5) and (6) imply

$$w = b + [r+\delta+m(u,v)/u]b/\lambda. \tag{10.b}$$

There are two important differences between (9) and (10), both of which were emphasized by the originating authors. First, this particular equilibrium 'efficiency wage' exceeds that required to compensate the worker for leisure, hence some unemployment is 'involuntary' in the sense that the value of employment, W, strictly exceeds the value of unemployment, U. Second, the bonus required to prevent shirking is increasing in the ease with which an unemployed worker finds employment, the unemployment duration hazard $m(u,v)/u$.

So called 'insider-outsider' story provides another rational for a wage in excess of the value of leisure. Because existing employees have bargaining power when ever replacing them is costly, one can expect them to demand a share of the rents generated by these costs and to be less concerned about the effect of their demands on unemployment than otherwise identical unemployed workers would be.

Although it is typical in this literature to assume that the insiders, the employed workers, have 'complete bargaining power' in the sense that the wage is set as high as possible consistent their continued employment, Lindbeck and Snower [1987, footnote 3] clearly state that this extreme assumption is not a strict requirement of the theory:

> This strong assumption ('complete bargaining power') is a convenient simplification but is not necessary for the subsequent analysis. It would be sufficient to assume that the insiders receive some part of the rent generated by the turnover costs and that the greater these costs, the greater are their wages.

In other words, current employee and employer share their match rents just as the search equilibrium literature assumes. Since the match rent is $J - V + W - U$, the worker's share, θ, is

$$W - U = \theta[W - U + J - V] \qquad (11.a)$$

by definition. Hence, the equations (3) - (7) imply

$$w = b + \theta[y - b + cv/u]. \qquad (11.b)$$

IV. Dynamic Labor Market Equilibrium: A Definition

Except for the shirking model, the marginal revenue product of labor, y, is the marginal product, denoted as

$$y = f(e) \qquad (12.a)$$

where f(e) is productivity per worker expressed as a function of the aggregate level of employment. We assume that there are "many" employer, that productivity per employee is independent of the size of the employer's own labor force and is identical across employers, but that the common value depends on the aggregate level of employment.

In the case of the shirking model, the net value of the marginal worker's productivity is

$$y = f(e) - a\lambda \qquad (12.b)$$

where λ is the average number of times that the effort of each worker is checked per unit time period, the monitoring frequency, and a is the fixed cost required to check a worker's effort.

By virtue of assumption (2.d), that the exchange technology is homogeneous of degree k, the equilibrium condition for vacancies, equation (8), can be written as

$$c(v/u)/m(1,v/u) = J(\ell-e)^{k-1}. \qquad (13.a)$$

Because the left side of (13.a) is increasing in v/u by virtue of assumption (2.c), the equilibrium vacancy/unemployment ratio is a strictly increasing function of the capital value that an employer places on a new hire, J. The effect of the level of unemployment on the vacancies per unemployed worker depends on the returns to scale in the exchange technology. Namely, given J, an increase in unemployment, ℓ-e, increases the vacancy/unemployment ratio if and only if the meeting rate function is homogeneous of degree greater than one. This fact and the assumption that the meeting rate increases with the number of unemployed, equation (2.b), imply that the propensity that an unemployed worker is hired, the unemployment duration hazard m(u,v)/u, also increases with the value of a new hire and increases (decreases) with the number of unemployed workers if the exchange technology exhibits increasing (decreasing) returns to scale.

Formally, the equilibrium hiring frequency per unemployed worker, denoted as $\eta(J,e)$, is defined by

$$m(u,v)/u = (\ell - e)^{k-1} m(1,v/u) = cv/uJ = \eta(J,e) \qquad (13.b)$$

where v/u is the solution to (13.a). Because m(1,0) = 0 from (2.a), it follows that

$$\eta(0,e) = 0 \text{ and } \lim_{e \to \ell} \{\eta(J,e)\} = 0\ (\infty) \text{ if } k > (<)\ 1 \qquad (14.a)$$

$$\partial\eta(\cdot)/\partial J > 0 \text{ everywhere} \qquad (14.b)$$

$$\partial\eta(\cdot)/\partial e <(>)\ 0 \text{ as } k >(<)\ 1, \qquad (14.c)$$

Given the hiring frequency per unemployed worker function, $\eta(J,e)$, one can express the wage and profit per worker as related reduced form functions of the value of a new hire, J, and the level of employment, e, for each wage determination model. Denote these as $\omega(J,e)$ and $\pi(J,e)$ respectively. Of course, equations (9.b) and (12.a) trivially imply that the wage and profit functions are

$$w = \omega(J,e) = b \qquad (15.a)$$

and

$$y - w = \pi(J,e) = f(e) - b \qquad (15.b)$$

in the case of a 'market clearing' wage.

In the case of the 'efficiency wage', the monitoring frequency, λ, is chosen continuously to maximize the expected present value of future profits attributable to the marginal worker. Since J is the resulting maximal capital value of the hiring another worker, the optimal monitoring frequency solves the problem

$$rJ = \max_{\lambda}\{y - w - \delta J + \dot{J}\}$$

$$= Pf(e) - b - \delta J + \dot{J} - \min_{\lambda}\{a\lambda + [r+\delta+\eta]b/\lambda\}$$

by virtue of (10.b), (12.b), and (13.b). In other words, the monitoring frequency is set to minimize the sum of the monitoring cost and the wage premium required to prevent shirking. Because the solution is

$$\lambda = [(b/a)(r+\delta+\eta(J,e))]^{1/2}, \tag{16}$$

it follows that the "reduced form" wage and profit functions are

$$w = b + b[r+\delta+\eta]/\lambda = \tag{17.a}$$

$$\omega(J,e) = b + [(ab)(r+\delta+\eta(J,e))]^{1/2}$$

and

$$y - w = y - b - a\lambda - b[r+\delta+\eta]/\lambda = \tag{17.b}$$

$$\pi(J,e) = f(e) - b - 2[(ab)(r+\delta+\eta(J,e))]^{1/2}$$

in the 'efficiency wage' case. The optimal wage paid exceeds the 'market clearing' wage and is an increasing function of the unemployment duration hazard, $\eta(J,e)$, if and only if monitoring is costly, i.e., $a > 0$.

The distinction between the 'insider-outsider' model and other theories

The distinction between the 'insider-outsider' model and other theories of worker-employer rent sharing is based on the observation that it is employed rather than unemployed workers who have bargaining power when turnover costs are important. Still, given this distinction, the question of whether the currently employed are viewed as bargaining collectively with all the employers through a comprehensive industrial or trade union or whether each employed labor force bargains separately with its own employer only as a company union is an important issue. In the "comprehensive" union case, 'insiders' with 'complete bargaining power' would take account of the effect of their collective wage demand on any future earnings that might be received as an employee of another firm as well as of the effect on current earning. In the "company" union case, the employees of a single firm don't control the future earning that might be received from some other employer.

The significance of this distinction can be illustrated using the formal rent sharing model introduced earlier. First, an employed worker's current income is monotone increasing in the current worker share of match surplus, θ, provided that the value of marginal productivity, y, exceeds the capital value of leisure, b, by virtue of equation (11.b). However, currently employed workers become unemployed at the rate δ for exogenous reasons and the loss in wealth associated with that event is W - U, the difference between the value of being employed and not employed. A "comprehensive union" can be expected to take into account the fact that the loss incurred in the event of unemployment is increasing in the worker share, specifically $W-U = \theta J/(1-\theta)$ by virtue of (11.a), given employer expectations about the future profitability of hiring another worker, J.

Indeed, the optimal collective wealth maximizing choice solves the problem

$$rW = \max_{\theta\epsilon[0,1]} \{w - \delta(W-U) + \dot{W}\}$$

$$= \max_{\theta\epsilon[0,1]} \{b + \theta[f(e)-b] + \theta J\eta(J,e) - \delta\theta J/(1-\theta) + \dot{W}\}$$

by virtue of the equations of (11) and equations (12.a) and (13.b). The unique interior solution is

$$1 - \theta = [\delta J]^{1/2}[f(e)-b+J\eta(J,e)]^{-1/2}. \tag{18}$$

Consequently, the associated wage and profit functions are

$$w = b + \theta[f(e)-b+J\eta(J,e)] = \tag{19.a}$$

$$\omega(J,e) = f(e) + J\eta(J,e) - [\delta J]^{1/2}[f(e)-b+J\eta(J,e)]^{1/2}$$

and

$$y - w = (1-\theta)[f'(e)-b] - \theta J\eta(J,e) = \tag{19.b}$$

$$\pi(J,e) = [\delta J]^{1/2}[f(e)-b+J\eta(J,e)]^{1/2} - J\eta(J,e).$$

It is quite clear that Lindbeck and Snower [1987, p.412] have the "company" rather than "comprehensive" union model in mind:

> To fix ideas, we suppose that entrants receive the reservation wage ... and that the insider wage ... is determined by a bargaining process between the firm and its insiders. For simplicity let insiders bargain 'individualistically' (i.e., each insider assumes the wage and employment of all other insiders to be exogenously given) and let them have 'complete market power' (i.e., each insider sets his wage as high as possible consistent with his continued employment).

In short, because the wage demand made of a current employer does not affect the future income received after a subsequent unemployment spell, 'insiders' in "company" unions demand all the match rent, i.e., $\theta = 1$. The equilibrium outcome implied by this solution is no hiring and eventual autarky as employment erodes at the exogenous rate δ because employers can expect no quasi-rent to compensate recruiting effort. It is precisely this version of the model that Blanchard and Summers [1986] apply.

But wait, "entrants receive the reservation wage" rather than the "insider wage" according to the quote. Since an entrant must eventually become an insider, the "reservation wage" would equal b, the value of leisure, **less** the expected present value of the insider's share of the match rent amortized over the time period required for the entrant to become an insider. Hence, if the employer has 'complete bargaining power' over entrants, as the quote above suggests, then the employer can extract all the rent up front before the entrant becomes an insider. In the case of instantaneous insidership, the employer collects the rent as a lump sum entrance fee exactly equal to the expected present value of the future stream of differences between the value of marginal product, y, and the value of leisure, b, for the expected duration of the match. Because this fee is exactly equal to the value of the marginal worker to the employer in the 'market clearing' case by virtue of equation (6), the equilibria are the same. The fact that insiders receive the full value of their marginal product after paying this entrance fee is of no consequence.

The point of this story is well known to every union organizer: 'Insiders' have no effective power unless they can determine the wage that 'entrants' receive. "Company" unions simply do not allow insiders to control entrant wage rate, at least not if employer entry is also relatively costless. Hence, 'insiders' can exploit their collective bargaining power only through a "comprehensive" union.

The equations (18) and (19) represent the outcome of rational bargaining by a "comprehensive" union controlled by all employed workers, the 'insiders.' As already noted, account is taken of the effects of the wage choice on both current and future income of the **current** insiders. Notice that both the optimal worker share of the rent, given by (18), and the wage, given by (19.a), increase with the difference between the value of margin product and the value of leisure, f(e) - b, as Lindbeck and Snower [1986, 1987] and Blanchard and Summers [1986] suggest. However, the elasticity of the share with respect to this difference is less than unity so that profit earned on the marginal worker is also increasing in this difference by virtue of (19.b).

Appropriate substitution from (4) and (13) into (1) yields he following equilibrium employment adjustment equation

$$\dot{e} = \eta(J,e)[\ell-e] - \delta e. \qquad (20.a)$$

By virtue of (5), the value of a new hire solves

$$\dot{J} = (r+\delta)J - \pi(J,e) \qquad (20.b)$$

where $\pi(J,e)$ is the profit function specified above in the case of each wage determination model. **Solutions to these equations that are consistent with rational expectations about the future constitute the equilibria of this dynamics model of the labor market.**

V. 'Natural' Employment Dynamics: The Case of Decreasing Returns

The purpose of this section and the next is to characterize equilibrium solutions to the differential equation system (20) for each wage model. In this section, we restrict attention to the case of diminishing returns in both production, $f'(e) \leq 0$, and exchange, $k \leq 1$. In this case, a unique steady state solution exists for each wage model and associated with each is a 'natural rate' of unemployment.

In both panels of Figure 1, the upward sloping curve represents the locus of point along which the time rate of change in employment is zero; hereafter referred to as the **employment singular curve**. By virtue of (20.a), the curve is implicitly defined by the requirement that the hire flow must equal the turnover flow in steady state, i.e.,

$$(\ell\text{-}e)\eta(J,e) = \delta e \text{ along } \dot{e} = 0. \qquad (21.a)$$

Its positive slope reflects the fact that higher expectations about future profitability per worker, a higher J, are needed to induce the additional recruiting effort required to sustain a higher level of steady state employment. A formal proof follows: Because the left side is equal to m(u,v) and because the equilibrium number of vacancies, v, is always increasing in the number unemployed, u, by virtue of condition (4) and the assumptions of (2), the left side of (21.a) decreases with e in general. Hence, (21.a) has a unique solution for every positive value of J. Because the left side is also strictly increasing in J by virtue of (14.b), the relationship between e and J defined by (21.a), represented as 0E in Figure 1a, has a strictly positive slope.

Of course, (14.a) and (21.a) imply steady state employment is zero given no demand for another worker, i.e., $e = 0$ when $J = 0$. Finally, employment is bounded above by the given size of the available labor force, i.e., $e \leq \ell$ by virtue of (21.a), for all J. Hence, the curve asymptotes to ℓ as J increases without bound.

Each of the three negatively sloped curves in Figure 1a represents the locus of points consistent with a zero rate of change per period in J, the value of a new hire, associated with one of the three wage determination models. Each **value singular curve** is simply the present value of the future profit stream attributable to the marginal worker where the discount rate is the sum of the interest rate and the turnover rate by virtue of (20.b), i.e.,

$$(r+\delta)J = \pi(J,e) \text{ along } \dot{J} = 0. \qquad (21.b)$$

The curve labeled AA′ in Figure 1a represents the value singular curve for the case of the 'market clearing' wage, b. Because b is a constant, the negative slope of AA′ simply reflects the assumption that labor productivity falls with aggregate employment. At the point A′ on the horizontal axis, the value of marginal product and b are equal. The unique steady state values of a new hire and of employment are represented by the intersection of AA′ and 0E at the point (J^*,e^*). Steady state employment is less than the available labor force in the model only because turnover occurs, $\delta > 0$, and recruiting replacements is costly, $c > 0$. As either tend to zero, one can show that employment tends to the minimum of line segments 0ℓ and 0A′ on the horizontal axis because the curve 0E converges to the vertical at $e = \ell$ for all $J > 0$. Of course, steady state unemployment, $u^* = \ell - e^*$, represents only "voluntary frictional" unemployment and e^* is "full employment."

The curves BB′ and CC′ represent the steady state condition (21.b) in the case of the 'efficiency wage' and 'insider-outsider' models respectively. Both are downward sloping given diminishing returns in production and exchange. To establish the claim, it is sufficient to show that the profit attributable to the marginal worker, $\pi(J,e)$, is decreasing in both e and J for each wage determination model.

Formally, the assumptions of diminishing returns in production and exchange imply that f(e) is decreasing in e by definition and that $\eta(J,e)$ is increasing in e by virtue of (14.c) respectively. Since it is obvious that profit per marginal worker, π, is increasing in the value of marginal product, f(e), in both cases by virtue of (17.b) and (19.b), $\pi(J,e)$ is decreasing in e if profit per marginal worker is also decreasing in η. As one can verify by differentiating (17.b) with respect to η in the 'efficiency wage' case and by differentiating (19.b) and then applying (18) in the 'insider-outsider' case, both wage models have this implication. In the 'efficiency wage' case, a higher wage must be offered in response to an increase in the unemployment duration hazard, η, in order to maintain the capital cost of losing ones job, W-U, at the level required to discourage shirking. The 'insider-outsider' model implies that the worker share of the match rent increase with η because employed workers expropriate an increasing share as the cost of becoming unemployment declines and because the capital cost of becoming unemployment, W-U, decreases with the unemployment duration hazard, η. Consequently, the wage responds positively and profits negatively to the unemployment duration hazard.

Finally, $\pi(J,e)$ is also decreasing in J in the 'efficiency wage' case because π depends on J only through its dependence on η and because $\eta(J,e)$ is always increasing in J by virtue of (14.b). To obtain the same qualitative result for the 'insider-outsider' model, simply note that partial derivatives of the last term on the right side of (19.b) with respect to J and η are both negative given (18) and the fact that $0 \leq \theta < 1$. In sum,

$$\partial\pi(J,e)/\partial J \leq 0, \qquad (22.a)$$

and

$$\partial\pi(J,e)/\partial e \leq 0 \text{ if } f'(e) \leq 0 \text{ and } 0 \leq k \leq 1 \qquad (22.b)$$

given any one of the wage determination models.

The curves BB′ and CC′ both lie below AA′ simply because the wage paid exceeds the 'market clearing' wage given either alternative wage determination model, i.e., $\omega(J,e) > b$ for all (J,e). As a consequence, steady state employment, uniquely defined by the intersection of the two singular curves in each case, is smaller under either alternative wage determination model. Furthermore, steady state unemployment is "involuntary" in the sense that the value of employment exceeds that of unemployment in both cases. One measure of the extent of "involuntary" unemployment is the difference between steady state unemployment under the alternative model and ℓ-e^*, that attained under the 'market clearing' wage. Finally, the steady state levels of unemployment under the two alternative wage models are not ordered. Which is larger depends on the parameters of both models.

Figure 1b represents the phase diagram implied by the system (20) given any one of the three wage determination models. Of course, the upward and downward sloping relations are respectively the singular curves for employment and the value of a new hire. The vector arrows in the diagram reflect the directions of motion for each variable implied by the two equations of (20) in the case of diminishing returns in both production and exchange. Obviously, the unique steady state is a saddle point for every wage determination model. The only solution trajectories that converge to the steady state are represented in Figure 1b by the curve labeled SS.

Given the current level of employment, say e_0, the trajectory converging to the steady state along this curve from the point (J_0,e_0) represents the only dynamic rational expectations labor market equilibrium of interest for the following reasons: First, such a trajectory is a rational expectations equilibrium because the current expected present value of hiring another worker is J_0 when all employers expect the economy to follow the path defined by the trajectory initiating at (J_0, e_0). Second, it is the only one of interest because all the other solutions to the system (20) either lead eventually to autarky (paths below SS') or generate an unsustainable speculative bubble (paths above SS).

Contrary to the suggestion of Blanchard and Summers [1988], our representation of the 'insider-outsider' model does not alone imply multiple equilibria in the downward sloping labor demand case under consideration. However, the theory does suggest greater persistence in the sense that the time required for the labor market to return to the steady state after a disturbance is longer. Because the workers' share of match rent increases with the value of labor productivity by virtue of (18), profits are less responsive to productivity than in the 'market clearing' wage case, as one can easily verify by comparing (15.b) with (19.b). As a consequence, the speed of adjustment to the steady state solution, which is the absolute value of

$$\partial \dot{e}/\partial e = - \delta - \eta(J,e)$$

given constant returns in both production and exchange, is smaller because the smaller value of a new hire, J, implies that the unemployment duration hazard, $\eta(J,e)$, is smaller at every employment level.

For the sake of subsequent comparison, it is of interest to note one general dynamic property of the equilibrium established for this version of the model. After the initial impact effect of any shock to the system, the value of expected future labor productivity, J, and the level of employment, e, are negatively related. Since labor is capital in this model, J reflects the asset value of a producing firm. These implied negative co-movements between employment and the value of the typical firm are not consistent with the positive co-movements between asset prices and employment typically observed over the business cycle in most market economies. A multiplier process generated by feedback from employment to profitability does not get started in this model because labor productivity falls and wages rise to cut it short in the upswing while increasing productivity and decreasing wages temper downturns independent of the wage determination rule.

VI. 'Fragile' Employment Dynamics: The Case of Increasing Returns

In describing the recent behavior of European unemployment, Blanchard and Summers [1988] use the analogy of a ball on a relatively flat but dimpled surface. Instead of always returning to one spot, shocks move it to and fro from one local resting place to another. The ball is extra sensitive to past shocks and eventually comes to rest at locations that depend on its origin and on the direction, nature, and strength of the shocks. Blanchard and Summers coin the term 'fragile equilibria' to describe the solutions to dynamics system with these properties. As they point out, systems that exhibit this kind of behavior are characterized by strong feedback mechanisms, local instability, and/or multiple equilibria.

The unique stable dynamic rational expectations labor market equilibrium derived in the previous section has none of these characteristics. As the analysis of the dynamic behavior illustrated in Figure 1b demonstrates, any stimulative or depressive shock is quickly countered by offsetting wage and/or productivity changes. The purpose of this section is to show that this implication can easily be reversed if either (1) the production process exhibits increasing returns or (2) the exchange process exhibits increasing returns **and** the wage is determined by either alternative to the 'market clear' wage model.

As noted in the introduction, the demonstrations that follow bring together the results of several authors. Increasing returns in the production process is an important ingredient of the unemployment theories of Hart [1982], Weitzman [1982], and more recently of Roberts [1986]. The models of Diamond [1982a], Howitt and McAfee [1984,1987], and Howitt [1987] all derive multiple equilibria in the case of increasing returns in the exchange process. Finally, Blanchard and Summers [1986,1988] suggest that 'hysteresis' and 'fragile' equilibria occur when employed worker's demand a larger share of the rent attributable to turnover and recruiting cost in response to an increase in the difference between the values of marginal productivity and leisure, as Lindbeck and Snower [1986] argue. All of these arguments are reviewed in the section.

In the analysis that follows, increasing returns in production is simply interpreted as the consequence of some external economy. Figure 2 represents the phase diagram for the system of differential equations, (20), in this case. The singular curve for employment is drawn as before, upward sloping, reflecting the positive steady state effect of increases in the capital value of a new hire on each employer's recruiting effort. Under increasing returns, the value singular curve is also upward sloping, at least in the case of a 'market clearing' wage.

The particular value singular curve drawn can be derived using (21.b) given $w = b$ and a value of marginal product, $f(e)$, that increases initially at an increasing rate and then at a decreasing rate. Assuming that productivity is bounded, there must be at least one intersection of the two steady state conditions. However, any odd integer number as well as a continuum of steady state solutions are possible given that employment is bounded by the fixed size of the available worker force, ℓ. The picture is qualitatively identical for the other two wage determination models in the case of a constant returns exchange technology because the hiring frequency per worker, η, and, consequently, the wage are independent of the employment level conditional on J when k = 1 by virtue of (14.c) and the reduced form wage equations of (17.b) and (19.b).

Obviously, there are three steady state equilibria in the case illustrated in Figure 2, at a low (L), medium (M), and high (E) value-employment pair. Both the high and the low equilibria are saddle points. Consequently, for some set of initial values of employment, those in a neighborhood of e_0 in the figure, there can be two different values of J consistent with rational expectations about the future evolution of the economy and each of these leads to a different steady state. Because both L and H have unique stable trajectories associated with them, labeled S_lS_l and S_hS_h respectively, the common expectation that the economy will follow either one to L or the other to H from the given current employment level are both self-fulfilling.

When the middle equilibrium is locally stable, a continuum of cyclic dynamic rational expectations equilibria also exist that converge to M for any initial employment level in a sufficiently small neighborhood of the steady state level of e at M. Because the Jacobian of the system defined by (20) (the 2×2 matrix of partials of the right sides with respect to J and e) has a strictly positive determinant at M, the steady state M is locally stable if its trace is negative. In the case of w = b, the trace is simply $r - \eta(J,e) + (\ell - e)\partial\eta(J,e)/\partial e$, which is always negative for all sufficiently small values of r by virtue of (13) and (14). As the phase diagram in Figure 2 suggests, it is possible that cases exist in which every point between the two saddle trajectory curves, S_lS_l and S_hS_h, initiate a rational expectations equilibrium path that converges to M. Finally, even when unstable, M can be surrounded by stable limit cycles. Indeed, dynamic rational expectations labor market equilibria in this environment are fragile!

The coordination problem posed by Figure 2 is apparent. The assumptions that each agent knows the structure of this economy and has rational expectations are not enough to determine aggregate outcomes. As a theoretical matter, all paths that converge to some steady state are equally likely in the sense that if all individuals expects any one to represent the future, so it will be. As agent welfare is quite different on the different solution paths, the need for a policy of coordination is evident.

Finally, note that there is a multiplier process operating in this version of the model and that the process tends to induce positive co-movements between the level of employment and the asset value of hiring another worker, at least for those solutions to the system that converge to saddle points. Again, equation (21.b) implies that the value singular curve will shift up everywhere in response to an increase in output price. The dynamic effect will be to shift up both equilibrium saddle point trajectories reflecting the anticipated increase in the future values of labor productivity. However, as employment increases from its initial level in response to the greater recruiting effort induced by the jump in J, J continues to rise as a consequence of the anticipated future increase in the value of productivity induced by increasing employment, causing even more recruiting effort in the future. J continues to rise because increased employment now and anticipated increases in future employment increase the value of hiring a worker now under the assumption of increasing returns. Eventually the employment boom is choked off even though J continues to rise because recruiting effort as measured by the number of active vacancies falls with the unemployment rate by virtue of (13.a). Movement down and to the left along either of the stable trajectory path to L and R represent the same process in reverse.

The upswing in any one of the cyclic trajectories surrounding the middle steady state represents the same interaction between the stimulative effect of a rising valuation on employment followed by positive feedback of higher employment on the value of the marginal worker that occurs in the increasing returns case. However, in any one of these cases, the value of the marginal worker overshoots and begins to fall before the employment boom is reversed by the scarcity of unemployed workers. When this happens, the multiplier process quickly reverses direction causing a recession as falling employment reduces labor productivity still further.

The equilibrium model can produce all the same phenomenon given constant returns in production and an exchange technology that exhibits increasing returns to scale **if and only if the wage responds positively and profit responds negatively to the frequency with which unemployed worker are hired, the unemployment duration hazard**. As noted earlier, both the 'efficiency wage' and the 'insider-outsider' models have this implication but the 'market clearing' wage does not. Consequently,

$\partial\pi(J,e)/\partial e > 0$ if and only if $f'(e) > 0$ given 'market clearing' (23.a)

and

$\partial\pi(J,e)/\partial e > 0$ if $f'(e) \geq 0$ and $k > 1$ given either alternative. (23.b)

Although the qualitative properties of the model are identical, the nature of the multiplier process is different given increasing returns in the exchange technology and constant returns in production. Higher levels of employment are associated with lower wage payments, given the value of hiring another worker J, because the wage increases with the hire frequency per unemployed worker, $\eta(J,e)$, and because this escape rate from unemployment decreases with the level of employment given increasing returns in the exchange technology. **The resulting positive association between future profitability and employment is the source of the multiplier processes in this case.**

Multiple equilibria reflect the fact that different expectations can be self-fulfilling when multiplier processes operate as a consequence of some external effect. Specifically, if all employers expect high employment in the future, then the current value of hiring another worker is high by virtue of equation (6) given increasing returns in production or exchange. Recruiting is vigorous as a result, and future employment and profits will be high as originally expected. Conversely, pessimism about future profitability depresses current recruiting efforts which results in lower employment and profits in the future. Either is a consistent future course for the economy.

VIII. Conclusions

As stated in the introduction, the primary purpose of this paper is expository: To view a number of related and recent contributions to unemployment theory -- those that suggest problems with the notion of a stable relatively low 'natural rate' of unemployment -- within a unified framework. The structure chosen for this purpose is a variant of the transactions approach to search equilibrium, in part, because several of the contributions stem from that literature and because alternative models of wage and price determination can parsimoniously be added to the structure. Because the paper is also motivated by a desire to provide some insight into the dismal recent European experience with unemployment, the following question arises: What practical measures, if any, do these theories suggest?

Certainly, the view that the European unemployment problem is caused by real wage rates that are set too high by organized employed workers who are insulated from unemployment risk by hiring and training costs as well as politically imposed separation costs is popular. Its appeal is enhanced by comparing the European experience with the recent job creating ability of the U.S. economy, where worker bargaining and political power have clearly deteriorated. There is no doubt that differential bargaining power and artificial costs of separation can account for some part of the differences in the 'natural rates' across economies in the model studied in this paper. However, to establish monopoly unions are the culprit, one must explain why worker bargaining power in Europe has increased so significantly since the "full employment" days of the 60's and early 70's.

It is the inability of the unemployment to bounce back after the negative supply shocks of the 70's that motivate the interest of Blanchard and Summers [1986] in the 'insider-outsider' model of rent sharing. Essentially, they argue that wage setting by the insiders is a dynamic process, one in which the worker's change their wage demands in response to changes is labor productivity. Specifically, employment falls initially in response to temporary negative shocks to derived demand as almost any theory will predict. However, when normalcy returns, employed 'insiders' have the incentive and the power to respond by increasing their wage demand preventing the creation of jobs required to induce a return of unemployment to the 'natural rate.'

I argue that this phenomena alone cannot cause unemployment indeterminacy, although it can certainly contribute to persistence in the sense of the time required for adjustment to a 'natural rate'. Specifically, it is true that 'insiders' with 'complete bargaining power' rationally demand a larger share of the rent attributable to recruiting and firing costs as the value of labor productivity increases. Consequently, an increase in productivity attributable to a decrease in employment increases the wage demand as they suggest. However, the elasticity of the optimal wage with respect to productivity is less than unity if there is any exogenous chance that the typical insider will become unemployed in the future. Consequently, profitability per worker increases when employment decreases under diminishing returns, but not by as much as it would if the worker share of rent were independent or decreased with labor productivity.

If excessive 'insider' bargaining power were the problem, then politics is the answer and economists have little else to contribute. If instead the European experience is indicative of a low employment equilibrium in an economy with many equilibria as suggested by the theories reviewed in this paper, then one must consider other possibilities. Specifically, some form of collective coordination is needed to achieve higher employment equilibria.

In the paper, I show that multiple employment equilibria are possible given any one of the following circumstances: (1) External economies in the production process that cause labor productivity to increase with aggregate employment. (2) Scale economies in the process by which unemployed workers and vacant jobs are matched in the sense that a doubling of both more than doubles the matching rate. If either (1) obtains given any of the wage determination models considered or if (2) obtains and workers and employers share the rents associated with an existing job-worker match in sense of either the 'efficiency wage' or the 'insider-outsider' model, then anticipated increases (decreases) in aggregate employment increase (decrease) profitability per worker which in turn stimulates (depresses) recruiting effort. When multiple equilibria exist for these reasons, there is a coordination problem in the sense that a multiplicity of conjectures about the future health of the economy are self-fulfilling. Consequently, the economy can settle in a low employment trap simply because everyone expects it to do so.

How one actually identifies whether an economy is in a low employment trap and, once identified, what policies might generate movement out are open questions. Obviously, testing for the plausibility of the conditions identified above is at least a start on the research program needed. Assuming that one can answer that question, the self-fulfilling nature of equilibria suggests that the policies required must somehow generate collective confidence in the future. Whether these instruments are within the existing scope of government powers or require some new form of collective action remains to be seen.

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Figure 1: Labor Market Equilibria

(a) Alternative Wage Models

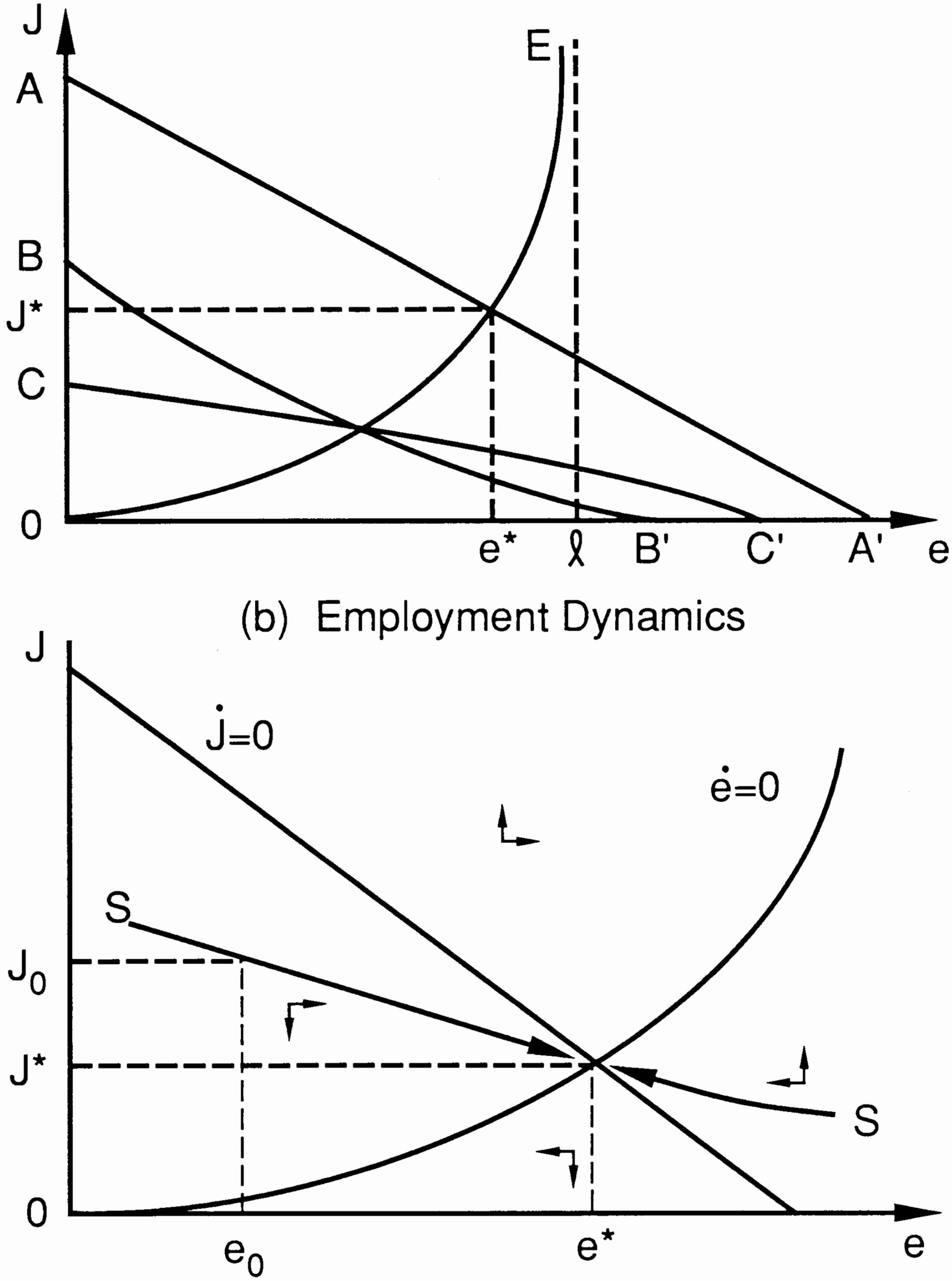

Figure 2: The Increasing Returns Case