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REAL BUSINESS CYCLES IN A DYNAMIC LABOR CONTRACT EQUILIBRIUM

by

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Abstract

This paper show that the dynamically Pareto optimal allocation generated by a real business cycle model is equivalent to the equilibrium allocation generated by long-term labor contracts. The result provides and justifies the econometric models of long-term labor contracts for testing the real business cycle model. This finding can be exploited to explain why an economy strongly characterized by long-term labor contracts (i.e., the Japanese economy) have more tendencies towards equilibria than an economy with more competitive labor markets (i.e., the US economy).

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1. Introduction

The real business cycle theory developed by Kydland and Prescott (1982), Long and Plosser (1983) and King and Plosser (1984) explains business cycle phenomena by stressing the importance of real shocks to production technology. The models of these researchers consider an economy populated by a single infinite-lived individual with given initial resources, production possibilities, and tastes. The representative individual chooses a preferred consumption-employment plan and the resulting allocation is Pareto optimal. The implications of the real business cycle theory are thus derived from the Pareto optimal equilibrium allocation as calculated from the planning problem for a "social planner" or representative agent. The policy implications of this theory is that there is no significant effect of monetary policies on output. In this view, observed correlations between money and output are the consequences of responses of the money shocks to output fluctuations brought about by random real shocks to production technology.

Since the real business cycle theory characterizes equilibrium prices and quantities by studying the social planning problem, the resulting Pareto optimal equilibrium must be decentralized. The most usual way of decentralizing Pareto optimal equilibrium is to use spot commodity markets. Then, the real business cycle theory typically exploits intertemporal substitution of leisure to account for large fluctuations in employment with small fluctuations in real wages. This idea, often called the intertemporal substitution theory, stems from the fundamental contribution of Lucas and Rapping (1968).

The recent studies of Sargent (1979, chapter 15), Homen (1985) and Wright (1988) have suggested another way for decentralizing Pareto optimal equilibrium although they have not explicitly dealt with this problem within the real
business cycle model. They show that long-term contracts support precisely the same dynamic quantity allocation as that which obtains with spot markets in labor.

The real business cycle model with long-term contracts has several advantages. First, this model can account for the failure of the econometric tests of the intertemporal substitution model which the real business cycle model with spot competitive labor markets in constructed on. Almost all empirical studies of intertemporal substitution suggest that the test results provide substantial evidence against the intertemporal substitution model or that the elasticity of intertemporal substitution in labor supply is substantially low. The main reason arises from their assumption that measured wage rates reflect either the true marginal product of labor or the marginal rate of substitution between consumption and leisure. However, the long-term contract model reveals that measured wage rates do not index either of them if long-term contracts prevail in the economy. In fact, the long-term contract model does not share the same Euler equation in labor supply with the intertemporal substitution model (see Bern and Franke (1988)).

Second, the real business cycle models developed by Hansen (1985), Greenwood and Shleifer (1987) and Rogerson (1988) can account for layoff unemployment as the outcome of models with indivisible labor, where (a) employment is determined by lotteries and (b) the incomes for employed and unemployed individuals are chosen to allocate risk efficiently. However, in their models, employed workers are worse off than unemployed workers. In the long-term contract model, we may avoid this problem by assuming that the firm pays no severance payments to workers under the imperfect capital market.

Third, the real business cycle model with long-term contracts can be
interacted within a bilateral bargaining framework between a representative capital owner and a representative worker (see McDonald and Solow (1981) for the discussion of the bilateral bargaining model). Thus, we can exploit this model to examine the effect of a change in the distribution parameter between profits and wages on the dynamic equilibrium allocation. The investigation helps us discuss the effect of income policies on the dynamic equilibrium allocation.

The purpose of this paper is to explore the theoretical properties of the real business cycle model with long-term contracts. The analysis establishes the fundamental equivalence theorem that the dynamically Pareto optimal allocation generated by the real business cycle model is equivalent to the equilibrium allocation generated by long-term contracts. Although Sargent (1979), Rosen (1985), and Wright (1988) have suggested that the dynamically Pareto optimal allocation is attained by long-term contracts, they have not verified this theorem within the real business cycle model. This missing link poses a problem of which model should be formulated and estimated to test the real business cycle hypothesis with long-term contracts. The contract equilibrium model considered in this paper can provide one possible answer to this question.

The paper is organized as follows. Section 2 presents a social planning problem with heterogeneous agents. Section 3 describes an equilibrium model with long-term contracts under the same environment as in section 2. Section 4 proves the equivalence theorem between the allocations generated by the social planning problem and the contract equilibrium model. Section 5 discusses the econometric implications of the contract equilibrium model. The final section provides a brief summary.
2. Social Planning Problem

We begin with considering the preferences, production technology and endowments of the environment under study. Let us first give the preferences of each individual. In our economy, there are two classes of individuals. The first consists of infinitely lived "workers" who are endowed with one unit of time in every period of his life to divide between leisure and labor. Each worker has preferences given by $V(C_t, L_t)$ in each period, where $C_t$ is his real consumption in period $t$ and $L_t$ his working time in period $t$. The utility function $V(C_t, L_t)$ is increasing and concave in $C_t$ and $L_t$. The other class consists of infinitely lived "capitalists" who have the management skills of producing only one final good. Each capitalist has preferences represented by $V(C_t)$ in each period, where $C_t$ is her per worker real consumption in period $t$. The utility function $V(C_t)$ is increasing and concave in $C_t$.

We next describe the per worker production technology in this economy in period $t$: 

$$y_t = f(N_t, L_t), \quad t = 0, \ldots, \infty, \tag{1}$$

where $N_t$ is the per worker labor input in period $t$, $L_t$ the per worker real stock of capital in period $t$, and $f$ a technology shock in period $t$. It is assumed that $f$ is increasing and concave in $N_t$ and $K_t$.

The final good can be either consumed or invested. The per worker capital stock evolves according to 

$$K_{t+1} = (1 - \delta) K_t + I_t, \quad t = 0, \ldots, \infty, \tag{2}$$

where $I_t$ is per worker real gross investment and $\delta$ the rate of depreciation of capital.
In each period, this economy must face three resource constraints. First, the per worker labor input must satisfy

\[ \lambda_t \leq \lambda_t^{\max}, \quad t = 0, \ldots, \infty, \]  

(3)

where \( \lambda_t \) is the employment probability of each worker in period \( t \). Second, the employment probability of each worker in period \( t \) must be restricted such that

\[ 0 \leq \lambda_t \leq 1, \quad t = 0, \ldots, \infty. \]  

(4)

Third, total use of the commodity must not exceed output. Since worker’s consumption \( (C_{t+1}) \) depends on whether he is employed \( (C_{t+1}) \) or not \( (C_{t+1}) \), we have

\[ C_{t+1} = \lambda_t C_{t+1} - (1-\lambda_t) U_{t+1} + \lambda_t \geq Y_t, \quad t = 0, \ldots, \infty. \]  

(5)

Given the initial per worker real stock of capital \( k_0 \), the social planning problem is to choose a contingency plan for \( (C_t, C_t, C_t, 1, 1, 1, 1) \), \( 0 \leq t \leq \infty \), to maximize

\[ \bar{V}_0 = E^0 \left[ \sum_{t=0}^{\infty} \beta^t V(C_{t+1}) \right], \]  

(6)

subject to (1)-(5) and

\[ E^0 \left[ \sum_{t=0}^{\infty} \beta^t \left| 1U(C_{t+1}, 1-L_t) + (1-L_t)U(C_{t+1} - 1) \right| \right] \geq U^*, \]  

(7)

Here, \( E^0 \) is an expectation operator conditional on information available at the initial period; \( \beta \) is a constant discount factor; and \( U^* \) is a worker’s expected discounted lifetime utility level which is ensured by social planners. We can support the solution of this social planning problem as a competitive spot market equilibrium by requiring workers to choose employment lotteries with a probability of working, \( \lambda_t \) (see Rogerson [1988]).
3. Contract Equilibrium

In this section, we describe an equilibrium model with long-term contracts under the same environment as in section 2. As a manager of the firm, a representative capitalist can make a contract offer to a representative worker. This offer, drawn up on the initial period when the states of nature in the present and future periods are unknown, specifies a compensation-employment sequence contingent on the set of available information history in each period. Let $w_t$ and $s_t$ denote worker's contract real wages and severance payments in period $t$. Then $(w_t, s_t, h_t, l_t)$ represents the contingent contract arrangements to be implemented in period $t$.

In this economy, the worker is only endowed with one unit of time to divide between leisure and labor in each period. Furthermore, he does not have any access to the capital market. On the other hand, the capitalist is endowed capital assets at the beginning of each period to consume commodities or to buy capital assets. It is also assumed that, as a manager of the firm, the capitalist receives all the profit of the firm after the payment to capital owners.

To display a dynamic contract equilibrium, we consider the optimal decision of the capitalist by distinguishing between the consumption and saving decision as a capital owner and the factor input and contract arrangement decision as a manager of the firm.

We first discuss the optimal consumption-saving decision of the capitalist as a capital owner. To this end, we must introduce the budget constraint of the capitalist represented by

$$f_t + S_t \leq x_{t1} + (R_t + \delta_s)h_t, \quad t = 0, \ldots, \infty. \quad (8)$$
where $S_t$ is per worker real gross saving in period $t$; $\pi_t$ is the per worker real profit of the firm after the payment to capital owners in period $t$; $\delta_t$ is the real net rental rate of capital between $t$ and $t+1$; and $A_t$ is the per worker real stock of capital possessed by the capitalist at the beginning of period $t$. Note that the real gross rental rate of capital is $\delta_t S_t$ in period $t$. He must also account for the accumulation process of the per worker real stock of capital, $A_t$. This process is expressed by

$$A_{t+1} = (1-\delta_t)A_t + S_t, \quad t = 0, \ldots, n, \quad (9)$$

where $\delta_t$ is the rate of depreciation of capital stock. Given the initial real stock of capital $A_0$ and $(R_t, \pi_t: 0 \leq t \leq \infty)$, the capitalist chooses a contingency plan for $(\tau, S_t, A_{\tau+1}: 0 \leq \tau \leq \infty)$ to maximize (6) subject to (8) and (9).

We next explore the factor input and contract arrangement decision of the capitalist as a member of the firm. When offering the long-term contract to the worker, the capitalist must at least ensure the worker a minimum expected discounted lifetime utility level $U^*$:

$$E^\tau \sum_{t=1}^\tau (1+i)^{t-\tau}[[U(h_t, 1-i_t) + (1-i_t)U(w_t, 1)] \geq U^*, \quad (10)$$

In general, $U^*$ is determined by dynamic bilateral bargaining between the firm and the worker or from alternative wages available to the worker. However, the specification of the determination process of $U^*$ does not modify any results in this paper, so that $U^*$ is regarded as exogenously given.

Now, the capitalist desires to maximize the expected value of the discounted sum of real profits with respect to the contingency plan for the factor input and long-term contract arrangements subject to the layoff...
probability constraints (4) and the minimum expected utility constraint of the worker (10). Since the per worker real profit of the firm after the payment to capital owners in period $t$ is described by

$$w_t = a(t, k_t) - l k_t - (1 - l) c_{t-1} = \alpha k_t - (1 - l) c_{t-1}, \quad t \geq 0, \ldots, \infty,$$  \hspace{1cm} (11)$$

the dynamic maximization problem of the capitalist with respect to $[k_t, w_t, a_t, l_t, \lambda_t; 0 \leq t \leq \infty]$ is formally represented by

$$\begin{align*}
\max_{\{k_t, w_t, a_t, l_t, \lambda_t\}} & \left[ \sum_{t=0}^{\infty} \left( \frac{\lambda_t}{\rho} \right)^t \left( \alpha k_t - (1 - l) c_{t-1} - \beta r_{t+1} k_t \right) \right] \\
\text{subject to} & \quad (4) \text{ and } (10) \text{ given } \lambda_t \text{ and } \beta r_{t+1} \geq 0 \leq \infty. 
\end{align*} \hspace{1cm} (12)$$

We are in a position to close a contract equilibrium by specifying the market clearing conditions of capital and product markets. The market clearing conditions of the capital markets are

$$k_t = \lambda_t, \quad t \geq 0, \ldots, \infty,$$  \hspace{1cm} (13)$$

because workers do not have access to the capital markets. The imperfect capital market assumption also implies that the contract wages (severance payments) are equal to the consumption of employed workers (unemployed workers) in each period. Thus, the market clearing conditions of the product markets are given by

$$c_{t+1} = w_t, \quad t \geq 0, \ldots, \infty,$$  \hspace{1cm} (14)$$

$$c_{t+1} = w_t, \quad t \geq 0, \ldots, \infty,$$  \hspace{1cm} (15)$$

$$c_{t+1} + (1 - l) c_t + (1 - l) c_{t-1} + \beta r_{t+1} k_t = \alpha k_t, \quad t \geq 0, \ldots, \infty.$$  \hspace{1cm} (16)$$
We can now define a contract equilibrium for this economy. A contract equilibrium is a set of interest rate, quantity, and contract sequences \( (R_t, \pi_t, e_t, c_t, x_t, y_t, z_t, \lambda_t, \lambda^*_t, \omega_t, \omega^*_t, h_t, h^*_t, l_t, l^*_t) \) which satisfy the following conditions: (a) \( (c_t, e_t, x_t, y_t, z_t, \lambda_t, \lambda^*_t, \omega_t, \omega^*_t, h_t, h^*_t, l_t, l^*_t) \) maximizes (6) subject to (8) and (9) given \( \lambda_t \) and (8), \( \pi_t ; 0 \leq t \leq \infty \); (b) \( (x_t, y_t, z_t, \omega_t, \omega^*_t, h_t, h^*_t, l_t, l^*_t) \) maximizes (12) subject to (11) and (16) given \( U^* \) and (10); (c) market clearing conditions (13)-(16); and (d) the definition of \( \pi_t, \omega_t \). 6

4. Equivalence Theorem

Comparing the efficiency conditions of the social planning problem with those of the contract equilibrium, we can show the following proposition (see Appendix):

**Proposition 4.** The allocation generated by the social planning problem described in section 2 can be supported as the contract equilibrium defined in section 3 if \( U = U^* \).

Since the allocation of the social planning problem can also be supported as a competitive spot market equilibrium with employment lotteries, we additionally obtain the following corollary of Proposition 4:

**Corollary.** The allocation that can be supported as a spot competitive market equilibrium with employment lotteries can also be supported as the contract equilibrium for a suitably adjusted \( U^* \).

In the contract economy considered in section 3, we permit the firm to pay severance payments to workers. However, in the actual economy, severance
ments are infrequently observed or are incomplete even if they are observed. 
There are several possible explanations of this empirical evidence although we do not discuss the reasons here (see Oswald (1989)). If the firm cannot offer severance payments to laid off workers and if laid off workers receive only unemployment insurance benefits, we must modify the definition of the contract equilibrium in the way such that laid off workers can consume only the amount of unemployment insurance benefits. The corresponding social planning problem is specified as follows: Given, h, a contingency plan for \( h(t, t, h, h, L, \ldots, \lambda(t)) \) maximizes \( (\lambda) \) subject to (11)-(4) and

\[
\begin{align*}
F_{N} \leq h \leq F_{W}, & \quad (1 \\
0 \leq h \leq H, & \quad (2)
\end{align*}
\]

Some remarks about this social planning problem are in order. First, in this social planning problem, we need not specify how unemployment insurance benefits are financed. Second, if the amount of unemployment insurance benefits and the utility level of leisure are small enough, this social planning problem above yields involuntary unemployment: employed workers are better off than laid off workers. Third, the allocation generated by this social security planning problem is not Pareto optimal, so that the corresponding contract equilibrium is nonoptimal. This finding also implies that the allocation of this social planning problem cannot be supported as a competitive spot market equilibrium.

5. Economic Implications

There are no econometric models like the contract equilibrium presented in
section 3. The first is a bilateral bargaining model between the capitalist and the worker in which a continuity plan for \( k_t, \pi_t, \alpha_t, \beta_t, \mu_t \), \( \forall \leq \infty \) maximizes \( (12) \) subject to \( (11) \) and \( (10) \) given \( (8) \) and \( (9) \). The Euler equations implied by the bilateral bargaining model have been tested in Osako and Inoue (1998) using aggregate Japanese data. The empirical results give some evidence for the bilateral bargaining model.

The second is a capital owner’s lifecycle consumption model where a continuity plan for \( (k_t, \pi_t, \beta_t, \mu_t) \), \( \forall \leq \infty \) maximizes \( (6) \) subject to \( (6) \) and \( (9) \) given \( \alpha_t \) and \( \beta_t \). This lifecycle consumption model is similar to the standard lifecycle consumption model considered by Hall (1978) and Hayashi (1982) and the asset pricing model developed by Hansen and Singleton (1982, 1983). However, there remains a question of which consumption or asset data are to be used for the empirical test. It may be desirable to estimate the Euler equation in asset investment instead of consumption to test this lifecycle consumption model.

Yes, suppose that the capital owner’s lifecycle consumption model is rejected by data even though the bilateral bargaining model is supported by data. Then, the discussion of the policy implications of the real business cycle model is qualified in the way that systematic expansionary policies may affect total consumption and employment if consumption from profit income is sensitive to systematic expansionary policies.

6. Conclusion

This paper has shown that the dynamically Pareto optimal allocation generated by a real business cycle model is equivalent to the equilibrium allocation generated by long-term labor contracts. The result provides and
justifies the economic models of long-term labor contracts for testing the real business cycle model. This finding also serves to explain why an economy strongly characterized by long-term labor contracts (i.e., the Japanese economy) has more tendencies towards equilibria than an economy with more competitive labor markets (i.e., the US economy).

However, the policy implications of the real business cycle model considered here depend on several restrictive assumptions. First, if informational asymmetries between contracting agents are serious, systematic expansionary policies may increase consumption and employment. Second, the real business cycle model with long-term contracts consists of two parts: the bilateral bargaining model between the capitalist and the worker, and the capitalist woman's life-cycle consumption model. In the bilateral bargaining model, consumption can be viewed as consumption from wages and not as consumption from profit income. Thus, systematic expansionary policies may affect total consumption and employment even if the empirical analysis finds evidence in support of the bilateral bargaining model. This prediction arises from the possibility that the empirical analysis may give some evidence against the capitalist woman's life-cycle consumption model.
The efficiency conditions for the social planning problem are summarized as follows:

\[(31)\]

\[\beta^* \cdot V^*(C_{it}) = \lambda \beta^* \cdot U_{it}(C_{it}, 1-L_{it})\]

\[(32)\]

\[\beta^* \cdot U^*(C_{it}) = \lambda \beta^* \cdot U_{it}(C_{it}, 1)\]

\[(33)\]

\[\beta^* \cdot V^*(C_{it}, D_{it}, K_{it}) = \lambda \beta^* \cdot U_{it}(C_{it}, 1-L_{it})\]

\[(34)\]

\[\varepsilon_{it} \beta^* \cdot U_{it}(C_{it}, 1-L_{it}) - C_{it} + \varepsilon_{it} \cdot [U_{it}(C_{it}, 1-L_{it})]^{1/2} \cdot [U_{it}(1-L_{it})]^{1/2} \cdot \varepsilon_{it} + \xi_{it} \leq 0\]

\[(35)\]

\[\{1-\delta_{it}\} \cdot \varepsilon_{it} \beta^* \cdot V^*(C_{it}) = \beta^* \cdot V^*(C_{it-1})\]

\[(36)\]

\[C_{it} + \varepsilon_{it} \cdot (1-\delta_{it}) \cdot C_{it} + \xi_{it} - \{1-\delta_{it}\} \cdot \varepsilon_{it} \beta^* \cdot U_{it}(C_{it}, K_{it})\]

\[(37)\]

\[\varepsilon_{it} \beta^* \cdot [U_{it}(C_{it}, 1-L_{it}) + \{1-\delta_{it}\} \cdot U_{it}(1-L_{it})] \geq \xi_{it}\]

\[\text{where } \varepsilon_{it}, C_{it}, U_{it} = \partial \phi / \partial C, L_{it} = \partial \phi / \partial L, K_{it} = \partial \phi / \partial K, \lambda, \text{ and } \xi_{it}, \zeta_{it}, \text{ are the non-negative multipliers associated with constraint } (i)\; \text{and } \xi_{it}, \zeta_{it}, \text{ are the non-negative multipliers associated with } l_{it} \geq 0 \text{ and } 1 - l_{it}. \text{ We can also describe the efficiency conditions for the contract equilibrium in the same form as (31)-(37). The equivalence of the efficiency conditions verify Proposition 1.}\]

2. This problem is particularly interesting in the Japanese economy because it has been often stated that, after the oil crises, the Japanese government has succeeded in reducing an increase in wages by announcing the guidelines on the wage income.

3. It is assumed that the ratio of capitalists to workers in this economy is constant.

4. In fact, if it is divisible, the solution of the social planning problem always yields full employment of workers because we consider a convex environment where workers and hours are perfect substitutes. See Mortensen (1978).

5. \[ \frac{1}{1 + \epsilon} = \frac{1}{1 + \epsilon} \]

6. Note that the product price in each period is normalized to unity because of the Walras law.

7. Cooper (1988a) and Farmer (1988a, 1988b) show the non-neutrality of monetary policies using the contractual, overlapping generation model under the asymmetric information structure.
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