

Discussion Paper No. 806

CALCULATING THE COST OF CAPITAL FOR  
THE PURPOSES OF INPUT SUBSTITUTION \*

by

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November 1988

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\*I would like to thank Kathleen Hagerty and Dan Siegel for extremely helpful discussions.



## Introduction

An important function of management is capital budgeting -- deciding what sorts of new investment projects to undertake. Not surprisingly, therefore, this problem has been extensively studied in the finance literature.<sup>1</sup> A major stumbling block has been that problems involving valuation of risky returns are very difficult to analyze. In contrast the problem facing a risk neutral decision maker with a known cost of borrowing money is extremely simple. Thus an important achievement of modern financial theory has been to show that it is often the case that management's complicated decision making problem of whether to accept or reject a risky project is equivalent to a problem where management is risk neutral and can borrow money at some interest rate  $R^*$  where  $R^*$  is greater than the risk free rate  $R_f$ . The value  $R^*$  is often called the "hurdle rate." Furthermore elegant derivations of the appropriate hurdle rate can be obtained through the capital asset pricing model (CAPM). In particular, the hurdle rate can be calculated by observing the Beta coefficient of firms involving in similarly risky activities to the project being considered. See Rubinstein [1973] for a very nice treatment using this approach.

An important aspect of the capital budgeting problem which has not been explicitly considered in the finance literature is that of input substitution. Often management must decide not only whether to undertake a project or not but also what mix of inputs to use. In particular, the use of greater amounts of capital will probably reduce the need for other inputs such as labor. As for the accept/reject decision, determining an optimal level of capital investment is complicated by the issue of valuing uncertain returns.

Therefore an important step in analyzing this problem would be to derive an analogous result to that obtained for the accept/reject decision.

This is the purpose of this paper. It is shown that Rubinstein's techniques for deriving a hurdle rate,  $R^*$ , in the CAPM model for the accept/reject decision can also be used to derive a cost of capital function,  $B^*(x)$ , for the more general case where management decides not only to accept or reject a project but also chooses an input mix. Management will make an optimal decision by pretending it is risk neutral and it can borrow  $x$  dollars at a certain cost of  $B^*(x)$ . (i.e. -- If  $x$  dollars are borrowed then  $x + B^*(x)$  must be repaid next period.) As in Rubinstein's analysis,  $B^*(x)$  can in principle be calculated by observing Beta coefficients of appropriate firms.

The cost of capital function  $B^*(x)$  has a number of interesting properties. First, it has the following relationship to the hurdle rate. Let  $R^*(x)$  be the hurdle rate calculated for an accept/reject decision on the project assuming a fixed capital level of  $x$ . Then

$$(1.1) \quad R^*(x) = \frac{B^*(x)}{x} .$$

That is, the standard hurdle rate calculation yields an average cost of capital.

Second,  $B^*(x)$  is in general not linear -- i.e. it is not of the form

$$(1.2) \quad B^*(x) = bx$$

for some constant  $b$ . In fact it is difficult to even create natural examples where (1.2) is true. The reason this is important is that (1.2) is a necessary and sufficient condition for the marginal cost of capital,  $B^{*'}(x)$ ,

to equal the hurdle rate  $R^*(x)$ . Thus, the marginal cost of capital for a firm is very unlikely to equal its hurdle rate,  $R^*(x)$ .

This has important practical significance. Suppose management provisionally chooses a capital level of  $\hat{x}$  for a contemplated project and derives a hurdle rate of 15%. Suppose the question now arises as to whether more capital and less of other inputs should be used. The marginal cost of this additional capital in general is not 15% even though 15% is the correctly calculated hurdle rate. Therefore correct capital budgeting requires separate calculation of both the average cost of capital for an accept/reject decision given the input mix and a marginal cost of capital for the input mix decision.

Third, when more specific assumptions are made about the nature of uncertainty, the techniques of this paper can often be used to derive rather sharp characterizations of the nature of  $B^*(x)$ . For example, suppose that the firm must choose its non-capital inputs each period before observing the resolution of demand uncertainties. Then it is shown that  $B^*(x)$  has the form

$$(1.3) \quad B^*(x) = \alpha + R_F x$$

where  $\alpha$  is a positive constant and  $R_F$  is the risk free interest rate. That is, in this case the marginal cost of capital is the risk-free rate even though the project is risky and the hurdle rate for any given level of capital is greater than  $R_F$ .

Section 2 describes Rubinstein's [1973] method for calculating a hurdle rate in the context of the CAPM. Section 3 describes the model and Section 4 uses Rubinstein's basic results to derive a cost of capital function. Then Section 5 and 6 explore the properties of two different cases

in more detail. Finally Section 7 contains a brief conclusion and suggests avenues for future research.

## 2. Rubinstein's Results

Prior to presenting this paper's model, it will be useful to summarize Rubinstein's major results from applying the CAPM to capital budgeting. A firm is assumed to face a mutually exclusive investment decision. That is there is a set of possible investment projects indexed by  $i \in I$ . The firm can choose to undertake at most one of the projects or to undertake none of them. Project  $i$  involves an initial investment expense of  $x_i \geq 0$  in period 1 and yields a (random) dollar payoff of  $\tilde{y}_i$  in period 2.<sup>2</sup>

The CAPM allows one to calculate the market price of the firm's shares. Rubinstein calculates the market price of the firm's shares for the case when it adopts none of the projects and for the cases when it adopts each of the projects. He thus is able to calculate the change in current shareholders' wealth from undertaking each project. Under the assumption that the firm acts to maximize current shareholders' wealth, the optimal decision for the firm is to select the project which causes the largest increase in existing shareholders' wealth. If all projects cause a decrease in current shareholders wealth then it is optimal to adopt none of the projects.

Since the CAPM is so well known, its assumptions and general structure will not be fully described or discussed. Rather, Rubinstein's key results and the notation necessary to state them will be presented. See Rubinstein [1973] for proofs and fuller discussions.

$R_F$  , the risk free interest rate -- i.e. -- a dollar invested in period 1 in a risk free security will yield  $1 + R_F$  dollars in period 2.

$\tilde{R}_M$  , the (random) rate of return on the market portfolio -- i.e. -- if  $p_1$  is the price of the market portfolio in period 1 and  $\tilde{p}_2$  is the (random) price of the market portfolio in period 2 then  
 $\tilde{R}_M = (\tilde{p}_2 - p_1) / p_1$ .

$E(\tilde{Z})$  , the expected value of the random variable  $\tilde{Z}$

$\text{Cov}(\tilde{Z}, \tilde{U})$ , the covariance of the random variables  $\tilde{Z}$  and  $\tilde{U}$

$\text{Var}(\tilde{Z})$  , the variance of the random variable  $\tilde{Z}$

$\lambda$  , a constant equal to  $(E(\tilde{R}_M) - R_F) / \text{Var}(\tilde{R}_M)$

Rubinstein's major result can now be stated.

Theorem 1 [Rubinstein, 1973]

Suppose the firm adopts project  $i$ . Then the increase in the wealth of existing shareholders is given by  $\text{NPV}_i$ . This is defined by

$$(2.1) \quad \text{NPV}_i = -x_i + \frac{E(\tilde{y}_i) - \lambda \text{Cov}(\tilde{y}_i, \tilde{R}_M)}{1 + R_F}.$$

If  $x_i > 0$  this can be equivalently written as

$$(2.2) \quad \text{NPV}_i = \frac{1}{1 + R_F} \left\{ E(\tilde{y}_i) - x_i (1 + R_i^*) \right\}$$

where

$$(2.3) \quad R_i^* = R_F + \frac{\lambda}{x_i} \text{Cov}(\tilde{y}_i, \tilde{R}_M).$$

Therefore the firm's optimal investment decision is to choose the project with largest NPV if this is non-negative. Otherwise no project should be selected.

proof:

See Rubinstein [1973]

QED

When the firm is considering only project  $i$  (or equivalently, the projects are not mutually exclusive) expression (2.2) provides the standard hurdle rate formulation for the accept/reject decision. Suppose management was risk neutral and could borrow money for one period at an interest rate of  $R$  -- i.e. -- if it borrows  $x$  dollars in period 1 it must repay  $(1+R)x$  dollars in period 2. Then management would accept project  $i$  if and only if

$$(2.4) \quad E(\tilde{y}_i) - (1+R)x_i \geq 0 .$$

By comparing (2.2) and (2.4) it is clear that management will make an optimal decision if it acts as though it were risk neutral and its borrowing cost is  $R_i^*$ .



### 3. The Model

A simple two-period investment model of the sort analyzed by Rubinstein [1973] will be considered.<sup>3</sup> The only difference will be that the firm can now exert some control over its input mix as well as deciding whether or not to undertake the project.

The basic structure of the problem is that the firm invests capital in period 1. It then chooses a level of labor usage (and thus output) in period 2. It sells this output and earns profit. Uncertainty enters the problem because the demand curve for the firm's output is only probabilistically known in period 1.<sup>4</sup>

The following notation will be used to formalize this problem.

$p$	,	the market price of output in period 2
$q$	,	units of output produced in period 2
$K$	,	dollars of capital invested by the firm in period 1
$L$	,	units of labor used by the firm in period 2
$w$	,	the wage rate in period 2
$\tilde{s}$	,	the random variable affecting demand
$s$	,	the realization of $\tilde{s}$
$\phi(q,s)$	,	the period 2 inverse demand curve
$F(K,L)$	,	the firm's production function.

Two separate cases will be considered, depending upon whether the firm can observe  $s$  before choosing  $L$ . If it can, the firm's labor choice will be said to be flexible because it can adjust to demand conditions. If the firm must choose  $L$  before it observes  $s$ , the labor choice will be said to be

inflexible. In reality, the flexibility of firms' variable input choices to short run fluctuations in demand is probably intermediate between these two extremes and varies among industries depending on a variety of factors. Thus ideally one would want to build a model which incorporated a "flexibility parameter" which could be varied smoothly between 0 and 1. However it was possible to create a significantly simpler and more elegant analysis by simply considering the two extreme cases. Therefore this has been done. For purposes of applying this paper's results to real situations, it seems reasonable to assume that industries with less (more) flexible labor inputs will behave more like the case of inflexible (flexible) labor choice.

The case of inflexible labor choice exhibits the following sequence of events.

Period 1:       K is chosen  
 Period 2:       L is chosen  
                   s is observed.

The case of flexible labor choice reverses the last two events.

Period 1:       K is chosen  
 Period 2:       s is observed  
                   L is chosen.

An input mix decision has the firm solve for an optimal amount of capital usage given it holds output constant. To formally define this, let  $\psi(q,K)$  be the amount of labor required to produce  $q$  units of output given  $K$  dollars of capital are purchased. First consider the case of inflexible labor. The firm can be viewed as choosing a capital level,  $K$ , and an output level  $q$ . Thus the input mix decision problem given  $q$  has the firm choose an investment level  $K$  in period 1 in order to receive profits in period 2 of

$$(3.1) \quad \phi(q, \bar{s})q - w\psi(q, K).$$

Now consider the case of flexible labor. The firm can now be viewed as choosing a capital level,  $K$ , and an output function,  $q(s)$ . Once again, the input mix decision has the firm hold its output decision fixed, and calculate an optimal investment level. In this case the firm holds the function  $q(s)$  fixed. Thus the input mix decision problem given  $q(s)$  has the firm choose an investment level  $K$  in period 1 in order to receive profits in period 2 of

$$(3.2) \quad \phi(q(\bar{s}), \bar{s}) q(\bar{s}) - w \psi(q(\bar{s}), K).$$

Notice that the firm's decision-making problem in both cases has the same general form. Namely the firm must choose an investment level of  $K$  in period 1 in order to receive a profit of  $\pi(K, \bar{s})$  in period 2. For the case of inflexible labor,  $\pi(K, \bar{s})$  is defined by (3.1). When labor is flexible,  $\pi(K, \bar{s})$  is defined by (3.2). Section 4 will derive the cost of capital function for this general investment problem. Then Sections 5 and 6 will develop

qualitative predictions about the cost of capital function for the two cases of flexible and inflexible labor.

#### 4. Analysis of the General Problem

The key point to realize about this problem is that the firm can be viewed as facing a continuum of mutually exclusive projects indexed by  $K \in [0, \infty)$  -- i.e. -- project  $K$  involves investing in a level of capital  $K$ . The firm can choose to invest in at most one of the projects from the set  $[0, \infty)$  or reject them all. Therefore Rubinstein's analysis immediately yields a solution to this problem. This will be stated as Proposition 2

##### Proposition 2

An investment of  $K$  will increase shareholder's wealth by  $NPV(K)$  where  $NPV(K)$  is defined by

$$(4.1) \quad NPV(K) = -K + \frac{E\left[\pi(K, \tilde{s})\right] - \lambda \text{Cov}\left[\pi(K, \tilde{s}), \tilde{R}_M\right]}{1 + R_F} .$$

An equivalent expression for (4.1) when  $K > 0$  is given by

$$(4.2) \quad NPV(K) = \frac{1}{1 + R_F} \left\{ E\left[\pi(K, \tilde{s})\right] - K\left[1 + R^*(K)\right] \right\}$$

where

$$(4.3) \quad R^*(K) = R_F + \frac{\lambda}{K} \text{Cov}\left[\pi(K, \tilde{s}), \tilde{R}_M\right] .$$

Therefore the optimal decision for the firm is to choose  $K$  to maximize  $NPV(K)$  so long as the maximized value is positive. If  $NPV(K) < 0$  for every  $K$  then the optimal decision is to reject the project.

proof:

This follows immediately from Proposition 1.

QED.

It will now be useful to identify the hurdle rate that the standard analysis would calculate for the project if it was making an accept/reject decision holding the level of capital fixed at  $K$ . From (4.2) it is clear that the hurdle rate is  $R^*(K)$ .

The cost of capital function,  $B^*(x)$ , will now be calculated. Suppose management was risk neutral and could borrow  $x$  dollars at a cost of  $B(x)$  -- i.e. -- if it borrows  $x$  dollars in period 1 it must repay  $x + B(x)$  dollars in period 2. Then management would use the following decision rule for choosing a level of capital and deciding whether to accept or reject. The firm would choose the level of  $K$  which maximizes

$$(4.4) \quad E\left[\pi(K, \tilde{s})\right] - K - B(K)$$

so long as the maximized value of (4.4) was positive and reject otherwise. By comparing (4.2) and (4.4) it is clear that the optimal strategy for management is obtained when  $B(K)$  equals  $R^*(K)K$ . This is stated as Proposition 3.

Proposition 3:

The cost of capital function,  $B^*(K)$ , is given by

$$(4.5) \quad B^*(K) = R^*(K)K.$$

proof:

as above.

QED.

Thus the standard hurdle rate calculation yields an average cost of capital for the level of capital being considered --i.e. -- from (4.5),

$$(4.6) \quad R^*(K) = \frac{B^*(K)}{K}.$$

This is intuitively reasonable. For an accept/reject decision it is sufficient to know the average input cost. However, when one is additionally considering the marginal choice of altering the input mix, the marginal cost of capital,  $B^{*'}(x)$ , is also important. The standard relationship between marginal and average quantities yields

$$(4.7) \quad B^{*'}(K) \begin{matrix} < \\ \equiv \\ > \end{matrix} R^*(K) \Leftrightarrow R^{*'}(K) \begin{matrix} < \\ \equiv \\ > \end{matrix} 0.$$

That is, if the hurdle rate is declining (increasing) then the marginal cost of capital is less than (greater than) the hurdle rate. Only when the hurdle rate is constant, will the marginal cost of capital be equal to the hurdle rate. In particular if the hurdle rate is constant and equals  $R^*$  then the cost of capital is defined by

$$(4.8) \quad B^*(K) = R^*K$$

and of course the marginal and average cost of capital are the same. For all other cases the hurdle rate is not the marginal cost of capital.

This has important practical significance. It means that when input mix is a decision variable management cannot necessarily calculate a hurdle rate for a "representative" capital/labor mix and then use this hurdle rate as its marginal cost of capital. In regulatory situations it means that regulators must recognize that regulated firms may view their marginal cost of capital as being different than their average cost.

It is difficult to develop any general propositions about  $B^*(K)$ . However the following intuition is useful. The shape of  $B^*(K)$  is essentially determined by the shape of  $R^*(K)$ , and in particular by whether  $R^*(K)$  increases or decreases. Equation (4.3) breaks the required average return to capital into two components. First, every investment must return the risk-free rate  $R_F$ . Second, to the extent it exhibits non-diversifiable risks, a project must promise a premium to compensate for this risk. For a project using  $K$  units of capital the size of the risk premium is

$$(4.9) \quad \delta(K) = \lambda \text{Cov} \left[ \pi(K, \bar{s}), \bar{R}_M \right].$$

Therefore the premium can be expressed as a return per unit of capital as

$$(4.10) \quad \frac{\delta(K)}{K} = \frac{\lambda \text{Cov} \left[ \pi(K, \bar{s}), \bar{R}_M \right]}{K}.$$

This is the second term of (4.3). Thus (4.3) can be written as

$$(4.11) \quad R^*(K) = R_F + \frac{\delta(K)}{K}$$

where  $\delta(K)$  is the required risk premium for a project using  $K$  units of capital.

Given this interpretation, it can be seen that whether  $R^*$  increases or decreases when capital increases depends on whether the risk premium grows proportionately with capital or not. This is stated as Corollary 1.

Corollary 1:

$$(4.12) \quad R^{*'}(K) \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \frac{\delta'(K)}{\delta(K)} \begin{matrix} > \\ < \end{matrix} \frac{1}{K}$$

proof:

This follows immediately from (4.11)

QED.

That is, when the amount of capital is doubled, if the risk premium doubles the hurdle rate is unchanged. If it is less than (more than) doubles the hurdle rate falls (rises).

Thus in order to make specific qualitative predictions about the cost of capital, the behavior of the risk premium  $\delta(K)$  is crucial. From (4.9) this depends on the nature of the  $\pi(K, \bar{s})$  function. The next two sections will consider the nature of  $\pi(K, \bar{s})$  under the two cases of inflexible and flexible labor.



5. The Input Mix Decision with Inflexible Labor

From (4.11), if the risk premium  $\delta(K)$  does not depend on  $K$  then

$$(5.1) \quad R^*(K) = R_F + \frac{\delta}{K} .$$

Therefore

$$(5.2) \quad B^*(K) = \delta + R_F K$$

That is, the marginal cost of capital is constant and equals the risk free rate  $R_F$ . The average cost of capital declines and approaches  $R_F$  as  $K$  approaches infinity. However,  $R^*(K)$  always exceeds  $R_F$ . This is intuitively reasonable. When  $\delta(K)$  does not depend on  $K$ , the risk of the project is unaffected by  $K$ . In order to undertake the project a firm must be compensated for the risk,  $\delta$ , independent of its capital choice. However, since no more risk is borne as capital usage increases, the firm only requires a return of  $R_F$  on incremental units of capital.

Thus the model of this paper yields a very clear and rather startling result for cases where  $\delta$  does not depend on  $K$ . Firms should view their marginal cost of capital to be the risk free rate (which is always lower than their hurdle rate.)

It will now be shown that  $\delta$  does not depend on  $K$  when labor choice is inflexible and the firm is making an input mix decision. Substitution of (3.1) into (4.9) yields

$$(5.3) \quad \delta(K) = \lambda q \text{Cov}\left[\phi(q, \tilde{s}), \tilde{R}_M\right]$$

which does not depend on  $K$ .

This intuition for this result is straightforward. The firm is conducting a thought experiment of varying  $K$  and  $L$  while holding  $q$  fixed. The period 2 income of the firm is given by

$$(5.4) \quad \phi(q, \tilde{s})q - wL.$$

Given that  $K$  and  $L$  are chosen to keep  $q$  constant, the risk of the project does not vary as  $K$  and  $L$  vary. Therefore, for purposes of substituting capital for labor, its marginal cost should be viewed as the risk free rate.

#### 6. The Input Mix Decision with Flexible Labor

The firm is assumed to have chosen an output function  $q(s)$ . For this section assume that  $\psi(q, K)$  can be written as

$$(6.1) \quad \psi(q, K) = \alpha(K)\beta(q)$$

where  $\beta$  is strictly increasing. Assume that  $\alpha$  is strictly decreasing, strictly convex, and satisfies

$$(6.2) \quad \lim_{K \rightarrow 0} \alpha(K) = \infty.$$

Let  $\bar{\alpha}$  denote the limit value of  $\alpha(K)$  as  $K$  converges to  $\infty$ .

$$(6.3) \quad \bar{\alpha} = \lim_{K \rightarrow \infty} \alpha(K)$$

The Cobb-Douglas production satisfies the above assumptions. In this case  $\bar{\alpha}$  equals 0. The following shorthand notation for some random variables is useful.

$$(6.4) \quad \bar{q} \quad , \quad q(\bar{s})$$

$$(6.5) \quad \bar{p} \quad , \quad \phi(q(\bar{s}), \bar{s})$$

Then the risk premium is defined by

$$(6.6) \quad \delta(K) = \lambda \left\{ \text{Cov} \left[ \bar{p}\bar{q}, \bar{R}_M \right] - w\alpha(K) \text{Cov} \left[ \beta(\bar{q}), \bar{R}_M \right] \right\} .$$

Assume that

$$(6.7) \quad \text{Cov} \left[ \bar{p}\bar{q}, \bar{R}_M \right] > 0$$

$$(6.8) \quad \text{Cov} \left[ \beta(\bar{q}), \bar{R}_M \right] > 0 .$$

$$(6.9) \quad \text{Cov} \left[ \bar{p}\bar{q}, \bar{R}_M \right] - w\bar{\alpha} \text{Cov} \left[ \beta(\bar{q}), \bar{R}_M \right] > 0 .$$

also covary positively with the market at least for high levels of capital usage. The output choice of a firm will generally display these properties if, for example, higher values of  $s$  cause demand to shift out and  $\tilde{s}$  covaries positively with the market. When demand is larger, revenues, labor usage, and profits will all be higher.

Given the above assumptions,  $\delta(K)$  has the form drawn in Figure 6.1. Namely, it is strictly increasing, strictly concave, converges to  $-\infty$  as  $K$  approaches 0, and converges to  $\text{Cov}(\tilde{p}\tilde{q}, \tilde{R}_M) - \bar{w}\alpha\text{Cov}(\beta(\tilde{q}), \tilde{R}_M)$  as  $K$  approaches  $\infty$ . From the figure it is also clear that  $\delta(K)/K$  has the shape drawn. Namely, it is single peaked and converges to 0 as  $K$  grows large. Note that the risk premium may become negative for small values of  $K$ . This is because extremely small values of  $K$  can cause labor usage to grow extremely large and for profits to grow negative. At this point profits covary negatively with the market and the risk premium is negative.

Given the definitions of  $B^*(K)$  and  $R^*(K)$ , it is clear from Figure 6.1 then the total, average and marginal cost of capital have shapes as drawn in Figure 6.2. In particular the marginal cost of capital,  $B^{*'}(K)$ , is decreasing and approaches  $R_F$  in the limit. The average cost of capital,  $R^*(K)$ , is single peaked and also approaches  $R_F$  in the limit.

Note that the marginal cost of capital may be greater than or less than the hurdle rate. Thus it seems very few qualitative predictions are possible in general. The hurdle rate is in general not a good estimate of the marginal cost of capital. However the true marginal cost may be higher or lower than the hurdle rate.

The results of Section 5 may provide some guidance. Even when labor is flexible, variation in the firm's output function may be relatively small --

i.e. -- the firm may plan to respond to increases (decreases) in demand primarily by raising (lowering) price. In this case, the results of Section 5 suggest that the marginal cost of capital will be close to the risk free rate (and be less than the hurdle rate).

Figure 6.1

The Shape of  $\delta(K)$  and  $\delta(K)/K$

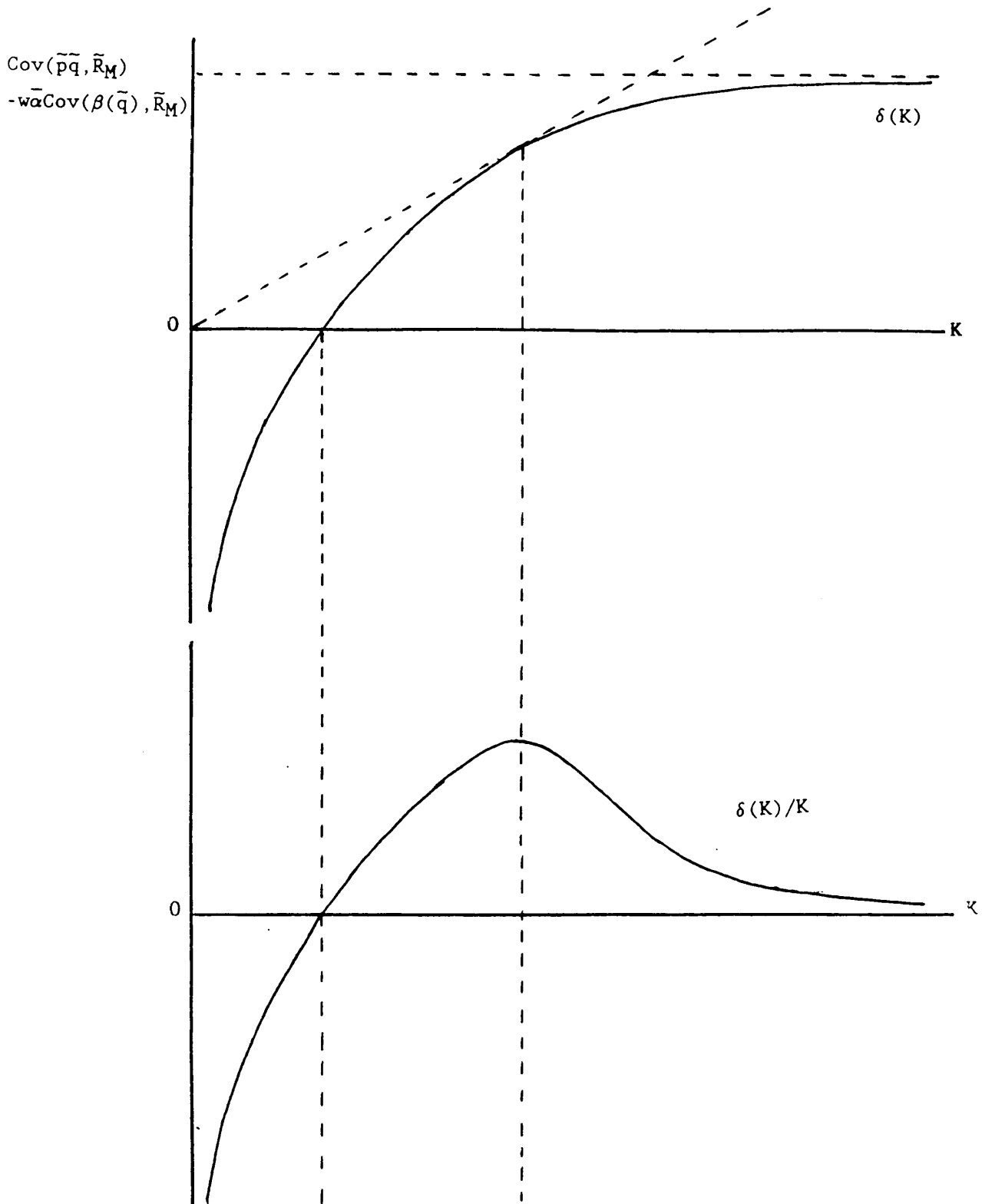
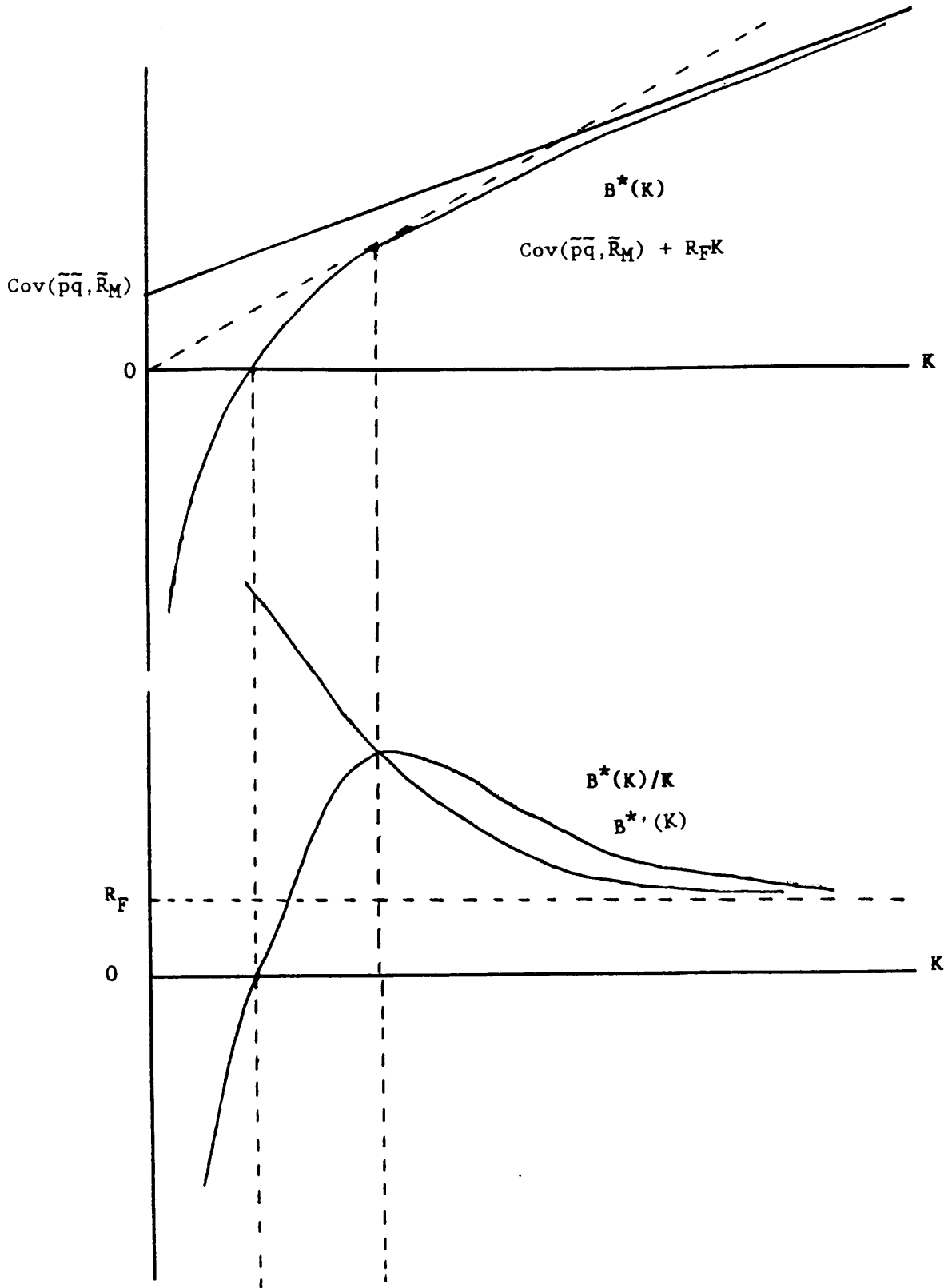


Figure 6.2  
The Shape of  $B^*(K)$ ,  $B^{*'}(K)$  and  $R^*(K)$



## 7. Conclusion

This paper has shown that the standard techniques for calculating a hurdle rate in the context of the CAPM model for a firm making an accept/reject decision can be used to calculate an entire cost of capital function for a firm when it decides on a level of capital usage as well as whether to accept or reject the project. The standard hurdle rate is shown to be the average cost of capital. Incremental decisions on the level of capital investment are of course determined by the marginal cost of capital, not the average cost. In general the marginal cost of capital will be unequal to the average cost. Unfortunately in general one cannot predict whether it will be higher or lower. However, in some special cases one can predict not only that it will be lower, but that it will equal the risk free rate.

This paper suggests a number of interesting avenues for future research. First it would be interesting to empirically investigate whether hurdle rates do vary with capital labor ratios and whether some practical method exists for incorporating this factor into firms' capital budgeting decisions.

Second, the general investment problem analyzed in Section 4 can also be interpreted as applying to scale decisions as well as to input mix decisions. It may be that doubling the scale of a project by doubling all input usage does not simply double the risk involved. In this case the marginal cost of capital for purposes of scale expansion will not be equal to the average cost. It would be interesting to use the results of Section 4 to develop a cost of capital function for scale decisions and to characterize its properties.

Third, the concept that the marginal and average cost of capital may diverge and that both can be explicitly calculated in the context of a CAPM model given a structural description of the firm's environment may prove to be



an extremely useful idea for the study of regulatory economics. A key concern of regulators is that incentives not be created for the firm to choose non-cost-minimizing input mixes. Beginning with the pioneering work of Averch and Johnson [1962], a voluminous literature has now established the idea that if regulators over-reimburse a regulated utility for its capital expenditures, then the utility will have an incentive to over-capitalize.<sup>5</sup> In the standard economic models of this issue the cost of capital is simply assumed to be linear. Thus the average and marginal cost of capital are identical and distinctions between the two concepts are not made. However, it is clear that correct incentives require the marginal reimbursement of the firm to equal the marginal cost of capital. Standard regulatory practice is to calculate a current average cost of capital and use this to determine a marginal reimbursement rate. In fact there is an increasing trend to use financial analysis to determine utilities' hurdle rates, by for example, using their Beta coefficients.<sup>6</sup> However all of these exercises are devoted towards calculating an average cost of capital for the firm. This average rate is then used to determine marginal reimbursements for marginal capital expenditures until the next hearing. However if the marginal cost of capital differs significantly from the average cost, firms will then have an incentive to choose a non-cost-minimizing capital labor ratio. The techniques of this paper could be used to investigate this issue by calculating the cost of capital function for a firm operating under regulatory constraint.

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Footnotes

1. See Hirshleifer [1958], Meyers [1968,1974], Robichek and Meyers [1966], Rubinstein [1973] and Treynor and Black [1976].
2. The analysis also applies in almost unchanged form to the case where the firm makes an investment which yields an infinite sequence of iid returns. See Rubinstein [1973].
3. As in Rubinstein's [1973] case, the analysis applies in almost unchanged form to the case where the firm makes an investment which generates an infinite sequence of i.i.d. returns.
4. It is possible to perform exactly the same type of analysis when the period 2 wage rate and/or the marginal productivity of capital and/or the marginal productivity of labor are also uncertain. The simpler case is considered solely for purposes of expositional clarity.
5. See Bailey [1973] for a very elegant and complete analysis of this topic.
6. See Breen [1973], Hayes and Siegel [1986], Leland [1974], Meyers [1972], and Robichek [1966], for example.