Discussion Paper No. 805

Veto Threats: Rhetoric in a Bargaining Model*

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November 1988

^{*} This paper is a revision of a working paper by the same name, CARESS WP #87-06 at the University of Pennsylvania. It has new comparative statics results, but has fewer technical discussions about the applicability of equilibrium refinements.

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ABSTRACT:

A specific bargaining game is studied, motivated by the speech-making, bill-proposing, and bill-vetoing observed in legislative processes. The game has two players, a chooser and a proposer, with the preferences of the chooser not known to the proposer. The chooser starts the game by talking. Then the proposer proposes an outcome, which the chooser accepts or vetoes. Only two kinds of perfect equilibria exist. In the more interesting kind, the chooser tells the proposer which of two sets contains his type. Two proposals are possibly elicited, a compromise proposal and the proposer's favorite proposal. Ironically, only the compromise proposal is ever vetoed.

^{*} Much of this work was done while I visited the Department of Economics at the University of Pennsylvania. I thank generally the members of that department for their hospitality and congeniality. For their comments, I thank Robert Gibbons, Daniel Ingeberman, George Mailath, Andrew Postlewaite, Joel Sobel, Dennis Yao, and two referees. I received support from NSF Grants SES-8410157-01 and SES-8810615.

I. INTRODUCTION

One motivation for this paper is the following, perhaps apocryphal story: A President of the United States eloquently threatens to veto any Congressional legislation that increases taxes. But this President is an avowed deficit-cutter, so that many do not believe he would veto <u>any</u> tax increase. Pundits think he bluffs, hoping that Congress will give his rhetoric enough credence that it legislates only a small tax increase (and a large budget cut). The interesting questions for us are, first, does the President's rhetoric influence the ultimate nature of proposed bills? Second, is his rhetoric only a bluff? If so, will his bluff be called and will he renege on his promises to veto?

An initial model takes the form of a two-stage proposal-veto game with the following equilibrium: the Congress proposes a bill it prefers to any bill that the President prefers as much as the status quo, and the President accepts it, thereby achieving only his reservation utility level. This model is inadequate on two counts. First, it is counterfactual in its prediction than no vetoes occur. Second, rhetoric in it is meaningless, despite the common belief that Presidential veto threats are at least partially credible. Thus, Ingberman and Yao [1986] observe in their model of Congress-President bargaining that a President may bear political costs, presumably transmitted via a fall in his reputation for "effectiveness," for not carrying out a public threat to veto. Such costs could convert empty veto threats into credible commitments.

This paper presents a different rationale for meaningful rhetoric. The premise is that only the President knows which bills he would prefer to the status quo. Consequently, the Congress is uncertain about which bills would be vetoed. The President's rhetoric serves to dispel some of this uncertainty.

Although still simple, the model exhibits two features that previous models of Congress-President bargaining do not: a well-specified mechanism whereby presidential rhetoric has influence, and a non-counterfactual prediction that proposed bills must sometimes be vetoed. Furthermore, the model can give predictions about which bills are likely to be vetoed.

At another level, this paper contributes to a growing literature on cheap talk in bargaining games.² The phrase "cheap talk", due to Farrell [1988], refers to what is called "rhetoric" here: messages sent from one party to another that are payoff-irrelevant to both. For such messages to convey information, the sorting condition made exogenously in the usual signalling models must be an endogenous consequence of the receiver adopting a nonconstant message-response function. For this to happen in equilibrium, the preferences of the sender and receiver must not be too dissimilar.

A seminal paper on such games is by Crawford and Sobel [1982].³ They consider the sparest possible game, one with two stages and two players. A player who has private information sends a message to another who does not, whereupon the latter responds with a decision affecting them both. The game in this paper differs by having a third stage in which the first player can veto the other's decision. The "chooser," who privately knows his own preference "type," first sends a message to the proposer. Next, the "proposer" responds with a proposal. Then the chooser either accepts or vetoes this proposal. The final outcome is the proposal if it is accepted, and an exogenously given status quo if it is vetoed. The chooser's rhetoric in the first stage serves to convey information about his preferences to the proposer, which is to the benefit of both parties if it helps the proposer find an acceptable proposal.

Summary of Results

The model is described in Section II. The chief assumptions are that the outcome space is unidimensional and the preferences of the different types of chooser are nicely ordered. Attention is confined to perfect equilibria, which are defined in Section III. A central result -- proved in Section V -- is that only two kinds of such equilibria exist.

The first kind are referred to as size one equilibria and characterized in Section IV.

Rhetoric has no function in these equilibria. The proposer always makes the same proposal,
a compromise proposal located between the proposer's ideal outcome and the status quo.

This proposal has positive probability of being accepted and, generally, of being vetoed.

The second kind of equilibria are referred to as size two equilibria and characterized in Section VI. In a canonical size two equilibrium, the chooser sends either an "accomodating message' or a "threat message." The proposer responds to an accomodating message by proposing his own ideal outcome, and this proposal is surely accepted by the types of chooser who send accomodating messages. The proposer responds to a threat message by proposing a compromise. Some of the chooser types who send a threat message accept this compromise proposal, but generally others of them veto it. Consequently, the resulting prediction is that only compromise proposals will ever be vetoed.

Section VII considers two comparative statics questions. First, is communication beneficial in the sense that a size two equilibrium is preferred to a size one equilibrium? The answer is unambiguous for the proposer: he always prefers a size two equilibrium because it gives him better information upon which to base his decision. Although the answer is ambiguous for the chooser, he does tend to also ex ante prefer a size two equilibrium, since it gives him some influence over which outcome is proposed. The second question concerns the status quo. Surprisingly, increasing the separation between the status quo and the proposer's ideal outcome can either increase or decrease the player's ex ante expected utilities. The analysis of this question highlights the role of the status quo.

Section VIII contains a brief re-assessment of the model's application to politics, and the Appendix contains proofs.

II. THE MODEL

A <u>proposer</u> and a <u>chooser</u> play a three-stage game to determine an outcome in a set X, assumed to be the real line. Think of the chooser as the President, the proposer as the Congress, and, prosaically, the outcome as a level of military expenditure. The chooser sends a message in the rhetoric stage, the proposer responds to it by proposing an outcome in the proposal stage, and the chooser either accepts or vetoes in the veto stage. If he vetoes, the final outcome is the status quo, s.

The von Neumann-Morgenstern utility function of the proposer over outcomes is u. It is continuously differentiable, and it strictly declines to either side of an ideal outcome, r: u is unimodal at r. To avoid triviality, assume $r \neq s$; further, assume r < s. (Congress wants to cut military spending.)

The proposer does not know the chooser's utility function. It depends on a <u>type</u> variable t which the proposer cannot observe, but believes to be a realization of a real random variable with a distribution function F. These beliefs of the proposer are common knowledge. We return to this distribution function after making assumptions that will allow an interpretation of the type variable.

Denote the utility function of a type t chooser over outcomes as $v(\cdot,t)$, and assume the following:

- (A1) $v(\cdot,\cdot)$ is continuous;
- (A2) $v(\cdot,t)$ is unimodal at z(t) for each t; and
- (A3) if type t is indifferent between outcomes x_1 and $x_2>x_1$, then higher types prefer x_2 and lower types prefer x_1 : for all x_1 , x_2 , t, and t', if $x_1<x_2$ and $v(x_1,t)=v(x_2,t)$, then $(t'-t)[v(x_2,t')-v(x_1,t')]>0$.

These assumptions imply that a continuous, strictly increasing function $e(\cdot)$ exists such that a chooser of type t is indifferent between outcome e(t) and the status quo. The type variable can thus be transformed so that e(t)=t. So, append the assumption that type t is indifferent between t and s:

(A4) v(t,t)=v(s,t) for all t.

This parameterization allows a nice classification of chooser types. As each type has a unimodal utility function, (A4) implies that type t prefers to the status quo precisely the outcomes between t and s. Types t<r prefer the proposer's ideal outcome r to the status quo; refer to these types as accommodating. Types t>s prefer the status quo to any outcome the proposer prefers to the status quo; refer to these types as recalcitrant. Finally, types t \in (r,s)

agree with the proposer that outcomes in the interval (t,s) are preferred to the status quo; refer to these types as <u>compromising</u>. Figure I illustrates.

[Figure I about here]

The often used "Euclidean" ("Type 1") utility functions can be expressed so as to satisfy (A1)-(A4). Simply define the utility function of a type t chooser by $v(x,t)=-(x-z(t))^2$, where the ideal point z(t) is the midpoint (t+s)/2.

Meaningful assumptions about the type distribution F can now be made. Let an interval $T=(\underline{t},\overline{t})$ be the set of possible types, and assume F is continuous with $F(\underline{t})=0$ and $F(\overline{t})=1$. As only one chooser type can be indifferent between the status quo and any other outcome, the continuity of F implies that such indifference occurs with zero probability. Assume also that F has a positive and continuously differentiable density function, f. To prevent triviality, assume \underline{t} <s and \overline{t} >r, so that the chooser is a compromising type with positive probability. The chooser is recalcitrant with positive probability if \overline{t} >s, in which case the proposal in any equilibrium must have positive probability of being vetoed. But even if \overline{t} <s, so that it is common knowledge that both the chooser and the proposer prefer the outcomes in the interval $(\overline{t}$,s) over the status quo, we shall see that vetoes may occur.

The set of messages is M. Messages have no literal meanings -- their contextual (equilibrium) meanings are discussed later. Assume M contains more than one message, but no more than a countable number. This rules out the possibility of separating equilibria; however, it shall be seen that separating equilibria are not possible anyway. Sequential rhetoric is not ruled out: any sequence of speeches made over the course, say, of a long campaign can be viewed as one, very long message. The chooser is required to send some message in M: "silence" is a message.

A strategy for the chooser specifies a collection of rhetoric rules in the first stage and acceptance rules in the third stage. Each type $t \in T$ of chooser has a <u>rhetoric rule</u> $\rho(\cdot \mid t)$: $M \rightarrow [0,1]$ that specifies for each message m its probability $\rho(m \mid t)$ of being sent by type t. The function ρ satisfies

(1) $\sum_{m \in M} \rho(m \mid t) = 1 \text{ for all } t \in T, \text{ and } \rho(m \mid t) \ge 0 \text{ for all } m \in M \text{ and } t \in T.$

An <u>acceptance rule</u> for a chooser of type t is a function $\alpha(\cdot,t)$: $X \rightarrow \{0,1\}$. He accepts proposal x if $\alpha(x,t)=1$, and he vetoes x if $\alpha(x,t)=0$.

The chooser's strategy set consists of the measurable pairs of functions (ρ,α) such that ρ satisfies (1). The proposer's strategy set consists of all <u>proposal rules</u> π : $M \rightarrow X$ mapping messages into proposed outcomes. (Conditions under which it is not restrictive to disallow mixed proposal rules are discussed later.)

Given a pair of strategies $\langle (\rho, \alpha), \pi \rangle$, say that type t <u>sends</u> message m if $\rho(m \mid t) > 0$. Message m <u>induces</u> proposal x if $\pi(m)=x$, and x is <u>elicitable</u> if some message induces it. Proposal x is <u>elicited</u> by t if it is induced by a message sent by t. Proposal x is <u>accepted</u> by t if $\alpha(x,t)=1$. The <u>final outcomes</u> are the elicited proposals accepted by some of the types who elicit them; the status quo is also a final outcome if it is elicited or if any elicited proposal is vetoed by some of the types who elicit it.

III. PERFECT EQUILIBRIA

The proposer-chooser game has many equilibria, some not very plausible. For example, the following is an equilibrium in which most chooser types get their ideal outcomes: the chooser truthfully tells the proposer his type and plans to veto all proposals not equal to his ideal outcome, and the proposer proposes the ideal outcome of the chooser type he learns he is facing if he prefers it to the status quo, and otherwise he proposes the status quo. Of course, this equilibrium is not subgame perfect — it would not be optimal for the chooser to actually veto the proposals that he prefers to the status quo.

Subgame perfection is not enough. Significant multiplicities are created by the fact that a chooser type who would veto every elicitable proposal is not restricted by the best-reply maxim as to which proposal to elicit.⁸ However, it seems implausible that the chooser would elicit a proposal that is not his most preferred, even if he were planning to veto. If there were <u>any</u> probability that he might not carry out a planned veto, the best

strategy for the chooser would be to safely elicit only his most preferred elicitable proposal. This argument is based on a possible "tremble" of the chooser's hand in the veto stage, which seems especially compelling in a political context. In any case, we adopt an equilibrium refinement that corresponds to Selten's [1975] "trembling-hand perfection."

To avoid the technical problem of defining trembles in a game with a continuum of information sets (one for each type of chooser), the definition is put directly in terms of the derived properties of perfect equilibria. A key property to recall from Selten [1975] is that an equilibrium of an extensive form game is perfect if and only if it is a perfect equilibrium of the corresponding agent-normal form.

A <u>perfect equilibrium</u>, henceforth to be called an <u>equilibrium</u>, is a strategy pair $<(\rho,\alpha),\pi>$ satisfying (E1)-(E3):

$$(E1) \qquad \alpha(x,t) \ = \ \begin{cases} 1 & \text{if} \quad v(x,t) > v(s,t) \\ 0 & \text{if} \quad v(x,t) < v(s,t) \end{cases} \qquad \text{for all } x \in X \text{ and } t \in T;$$

$$(E2) \qquad \rho(m \mid t) > 0 \quad \Rightarrow \quad m \in \underset{\widehat{m} \in M}{argmax} \ v(\pi(\widehat{m}), t) \quad \text{ for all } m \in M \text{ and } t \in T;$$

(E3) for all
$$m \in M$$
, $u(\pi(m)) \ge u(s)$ and
$$\int_T \rho(m \mid t) f(t) dt > 0 \implies \pi(m) \in \underset{x \in X}{\operatorname{argmax}} \beta(x \mid m) u(x) + (1 - \beta(x \mid m)) u(s),$$

where $\beta(x|m)$ is the conditional probability that x is accepted:

(2)
$$\beta(x \mid m) = \frac{\int_{T} \alpha(x,t) \rho(m \mid t) f(t) dt}{\int_{T} \rho(m \mid t) f(t) dt}.$$

Condition (E1) -- subgame perfection -- requires the third-stage chooser to accept proposals he prefers to the status quo. In view of assumptions (A2) and (A4), it requires that type t accept proposals in the interval (t,s):

(3)
$$\alpha(x,t) = \begin{cases} 1 & \text{if } x \in (t,s) \\ 0 & \text{if } x \notin [t,s] \end{cases} \text{ for all } x \in X \text{ and } t \in T.$$

Condition (E2) requires the chooser to elicit only his most preferred elicitable proposals. This strengthens the best-reply maxim, as discussed above.

The first part of (E3) requires the proposer to propose only outcomes that he weakly prefers to the status quo, so that he does not play a dominated strategy. The second part of (E3) requires that he play a best-reply. A proposal rule π is a best reply to (ρ,α) if and only if for every message m sent with positive probability, $x=\pi(m)$ maximizes the proposer's expected utility calculated using the correct conditional probability that x will be accepted, $\beta(x|m)$. Refer to $\beta(\cdot|\cdot)$ as the proposer's belief function.

Equilibria are classified conveniently in terms of their elicited proposals. Define the <u>size</u> of an equilibrium $<(\rho,\alpha),\pi>$ to be its number of elicited proposals:

$\{x \in X \mid m \in M \text{ and } t \in T \text{ exist such that } x = \pi(m) \text{ and } \rho(m \mid t) > 0\}.$

It is shown in Section V that the size of any equilibrium is one or two, so that reference can be made to <u>size one</u> and to <u>size two</u> equilibria.

IV. SIZE ONE EQUILIBRIA

A particularly simple kind of size one equilibrium that always exists is a <u>babbling</u> equilibrium. All types of chooser pool together in such an equilibrium by using the same rhetoric rule, and all messages are sent:

(4)
$$\rho(m \mid t) = \rho(m \mid t') > 0 \quad \text{for all } m \in M \text{ and } t, t' \in T.$$

Babbling rhetoric is neither informative nor persuasive. The probability that a proposal x is accepted after any message is sent is simply the probability that x is between the chooser's type and the status quo. It is given by the <u>babbling belief function</u>, which, from (2)-(4), is

$$\beta^b(x\mid m) \ \equiv \ \int_T \alpha(x,t)f(t)dt \ = \ \begin{cases} F(x) & \text{if} \quad x < s \\ 1 - F(x) & \text{if} \quad x > s \end{cases} \qquad \text{for all } m \in M.$$

Any elicited proposal is to the left of s, by (E3). So, from (5) and (E3), a proposal x_0 that is elicitable in a babbling equilibrium must satisfy

(6)
$$x_0 \in \underset{x \in X}{\operatorname{argmax}} [u(x) - u(s)] F(x).$$

The same solution to (6) must be proposed regardless of the message sent: if there were multiple elicitable proposals, almost no type of chooser would babble.

It can be shown from (6) that $r < x_0 < s$ and $x_0 \le \overline{t}$. Thus, the elicited proposal in a babbling equilibrium is strictly between the proposer's ideal outcome and the status quo. It has positive probability of being accepted. It also generally has positive probability of being vetoed, since the corner solution $x_0 = \overline{t}$ can be regarded as special. 11

Not all size one equilibria are babbling equilibria. Different messages may induce different proposals, and some messages may never be sent. All that is required is that every message actually sent induce the same proposal, one preferred by all types of chooser to any other elicitable proposal. Unelicited elicitable proposals are vacuously optimal for the proposer because they are proposed only at information sets never reached.

Rhetoric can even be informative in a size one equilibrium. For example, in case $s<\overline{t}$, choose $\varepsilon>0$ and alter a babbling equilibrium with elicited proposal x_0 by arbitrarily perturbing the rhetoric rules $\rho(\cdot|t)$ for types $t>s-\varepsilon$. The proposer then updates his beliefs differently for different messages, so that the chooser does convey information about his type. But if ε is small, the proposer should still propose x_0 in response to all sent messages.

Condition (6) was derived only for a babbling equilibrium. The next proposition shows that it holds for all size one equilibria. It also gives a sufficient condition for all size one equilibria to have the same elicited proposal. (The Appendix contains proofs.)

PROPOSITION 1. The elicited proposal in any size one equilibrium satisfies (6), which has a unique solution if

(7)
$$\frac{u'(x)}{u(x)-u(s)} + \frac{f(x)}{F(x)} \quad \text{decreases in } x \text{ on } (\max(\underline{t},r), \min(s,\overline{t})).$$

REMARK 1: Condition (7) holds if u is concave and f/F is decreasing. Also, it insures that a no-communication equilibrium in which the proposer responds to every sent message by choosing a proposal according to the same probability distribution does not exist.

V. ONLY SIZE ONE AND SIZE TWO EQUILIBRIA EXIST

The chooser never always reveals his type. In particular, a compromising chooser who revealed his type would receive only his reservation utility, since he would be indifferent between the forthcoming proposal and the status quo. Yet, the chooser has an incentive to say something about which proposals he would veto in order to persuade the proposer to make an acceptable proposal. It turns out that the chooser can credibly communicate only a limited amount of information, as is indicated by the fact that the number of proposals he ever elicits is at most two.

The intuition for comes from understanding why the "compromise interval," (r,s), can in any equilibrium contain only one elicited proposal. Suppose to the contrary that it holds two, say x and y>x, in some equilibrium. Then some types of chooser prefer x to y, and others prefer y to x. Some type t is indifferent. This type is to the left of x, since $v(\cdot|t)$ is unimodal, as shown in Figure II.

[Figure II about here]

No type higher than t elicits x, since higher types prefer larger outcomes and hence prefer y to x. So when the proposer receives a message to which he supposedly responds by proposing x, he knows that the chooser's type is not in the interval (t,x). All proposals in (t,x] thus have the same conditional probability of acceptance. Contradiction: proposing x cannot be the proposer's best reply because he prefers the outcomes just to the left of x that have the same probability of acceptance. Proposition 2 gives the formal statement and begins the characterization of size two equilibria.

PROPOSITION 2. Only size one and size two equilibria exist.¹² Furthermore, if x_1 and $x_2 > x_1$ are elicited in an equilibrium, then $x_1 = r$ and $x_2 \in (r,s)$.

VI. SIZE TWO EQUILIBRIA

Proposition 3 below characterizes size two equilibria, and gives a necessary and sufficient condition for their existence. The proposition is put in terms of a correspondence $g: T \rightarrow X$, where g(t) is defined to be the set of proposals that are optimal for the proposer when he knows that the chooser's type exceeds t:

(8)
$$g(t) = \underset{x>t}{\operatorname{argmax}} [u(x)-u(s)] [F(x)-F(t)].$$

PROPOSITION 3. If x_1 and $x_2 > x_1$ are the elicited proposals in a size two equilibrium, then $x_1 = r$, $x_2 \in (r,s)$, $x_2 \le \overline{t}$, and there exists $t \in (\underline{t},r)$ such that $x_2 \in g(t)$ and

(9)
$$v(r,\tau) = v(x_2,\tau).$$

Types less than τ elicit only r, and types greater than τ elicit only x_2 . A size two equilibrium exists if and only if

(10)
$$v(r,t) \ge v(x,t)$$
 for some $t \in T$ and $x \in g(t)$.

REMARK 2: Condition (10) holds if there is a relatively large probability mass of chooser types in the upper part of the interval (r,s) -- so that the proposer's optimal proposals in g(t) will be large -- and if some chooser types prefer the proposer's ideal outcome r to such large proposals. A sufficient condition for (10) is $v(r,\underline{t})>v(x_0,\underline{t})$, so that the most accomodating types prefer the proposer's ideal outcome to the proposal made in a size one equilibrium. A stronger sufficient condition is $z(\underline{t})< r$. A necessary consequence of (10) is $\underline{t}< r$: the chooser must be accomodating with some probability.

REMARK 3. All size two equilibria can be shown to have the same elicited proposals $\{r,x_2\}$ if the following regularity conditions hold:

(11)
$$u(\cdot)$$
 and $ln(F(\cdot))$ are concave, one strictly, and

(12)
$$\frac{\mathbf{v}_2(\mathbf{x},t) - \mathbf{v}_2(\mathbf{r},t)}{-\mathbf{v}_1(\mathbf{r},t)} > \frac{\mathbf{B}_2(\mathbf{x},t)}{-\mathbf{A}'(\mathbf{x}) - \mathbf{B}_1(\mathbf{x},t)} \quad \text{for all } t \in [\underline{t},r] \text{ and } \mathbf{x} \in (r,s),$$

where
$$A(x) = \frac{u'(x)}{u(x)-u(s)}$$
 and $B(x,t) = \frac{f(x)}{F(x)-F(t)}$.

Condition (11) insures that $g(\cdot)$ is single-valued and hence continuous, and (12) insures that all size two equilibria have the same τ . Although seemingly not restrictive (it is satisfied in examples), (12) has no ready interpretation.

To facilitate our discussion of size two equilibria, consider first a maximal-pooling size two equilibrium, where a <u>maximal-pooling equilibrium</u> is an equilibrium in which all chooser types who elicit a particular proposal use the same rhetoric rule. (The babbling equilibria are the maximal-pooling size one equilibria.)

Chooser types in a maximal-pooling size two equilibrium pool into two groups, those in the intervals (\underline{t},τ) and (τ,\overline{t}) . The chooser types in (τ,\overline{t}) send messages that convey the meaning, "I want you to propose a big outcome, and I will veto a small one." Refer to these as "threat messages." The proposer gives credence to a threat message, responding with a compromise proposal x_2 to the right of his ideal outcome. The compromise proposal has a positive probability of being accepted, since (τ,\overline{t}) contains all compromising types and even some accomodating types. The compromise proposal also has positive probability of being vetoed, except in the corner case $x_2=\overline{t}$ (which occurs only if \overline{t} is quite close to r). On the other hand, the chooser types in (\underline{t},τ) send "accomodating messages" that convey the meaning, "I will accept your ideal outcome." The proposer believes accomodating messages wholeheartedly, responding to them by proposing his ideal outcome. His ideal outcome is then surely accepted, as all chooser types in (\underline{t},τ) are accomodating.

Size two equilibria exist that are not maximal-pooling equilibria. As a simple example, assume recalcitrant chooser types exist (i.e. $s<\overline{t}$) and a maximal-pooling size two equilibrium exists. Alter this equilibrium by having the recalcitrant types send a new message, say m_V , and by making the proposer respond to m_V by proposing x_2 . This yields another size two equilibrium (any outcome that the proposer prefers to s will be vetoed by the chooser types who send m_V , so he might as well respond to m_V by proposing x_2). This

equilibrium is not a maximal-pooling equilibrium because the recalcitrant types are not pooling with the compromising types. However, this equilibrium is <u>equivalent</u> to the initial maximal-pooling equilibrium in the precise sense of having the same mapping of chooser types into elicited proposals and final outcomes. This is a general property implied by Propositions 1 and 3: corresponding to any size one or size two equilibrium is an equivalent maximal-pooling size one or size two equilibrium, respectively.

Returning to a maximal-pooling size two equilibrium, note that the chooser types in (τ,r) are bluffing when they send a threat message -- these types would in fact accept the proposer's ideal outcome if it were proposed. Thus, if a "credible veto threat" is defined to be a message sent only by chooser types who would <u>surely</u> veto the proposer's ideal outcome [Ingberman and Yao, 1986], the threats sent in a maximal-pooling size two equilibrium are not credible veto threats. Some size two equilibria do have veto threats that are credible in this strong sense. For example, the messages sent by the recalcitrant types in the equilibrium constructed in the previous paragraph are credible veto threats. But credible veto threats do not play a fundamental role, since every equilibrium is equivalent to a maximal-pooling equilibrium that has no credible veto threats.¹³

VII. COMPARATIVE STATICS

Two comparisons seem especially interesting. The first is between equilibria of different size, asking whether communication is beneficial in the sense that a size two equilibrium is preferred to a size one equilibrium. The second comparison is between equilibria with different status quo points.

Comparing Equilibria of Different Size

The proposer always prefers a size two over a size one equilibrium, since his decisions are based on better information in a size two equilibrium. The proof is by revealed preference: even if the chooser plays a size two equilibrium strategy, the proposer could still, if he wanted to, respond to all messages by making the same proposal that he does in a size one equilibrium, which would give him his size one equilibrium expected utility.

On the other hand, the preference of the chooser before he knows his type, his <u>ex ante</u> preference, is not so clear. Consider first the preferences of his various types. By Lemma 1(iv) in the Appendix, the proposal in a size one equilibrium, x_0 , is always between the two proposals in a size two equilibrium, r and x_2 . Moderate chooser types, those whose ideal outcomes are near x_0 , consequently prefer the size one equilibrium. These types include the type τ who is indifferent between r and x_2 in the size two equilibrium, since this type prefers x_0 to both r and x_2 . More extreme leftist and rightest types prefer the size two equilibrium; for example, the size two equilibrium is preferred by the rightmost non-vetoing types, the types between x_0 and x_2 . $\frac{14}{2}$

The <u>ex ante</u> preference of the chooser can go either way. Consider a three-type example in which the status quo is s=10, the chooser's ideal outcome is r=3, and the chooser types are $t_1=0$, $t_2=4$, and $t_3=8$. The utility functions are u(x)=-|x-r| and v(x,t)=5-|x-z(t)|, where z(t)=(t+s)/2. Let $p_i=Prob(t=t_i)$. Assuming $p_1\le 6p_2$ and $p_3\le \min(2/3, 2p_2)$, both kinds of equilibrium exist, with $x_0=t_2=4$ proposed in the size one equilibrium and r=3 and $x_2=t_3=8$ proposed in the size two equilibrium. Type t_1 prefers the size one equilibrium, since $v(x_0,t_1)>v(r,t_1)$. Type t_2 prefers the size two equilibrium, and type t_3 is indifferent. Thus, one should expect that before the chooser knows his type, he will prefer the size one equilibrium if p_1 is large relative to p_2 . Indeed, it is easily verifed that the chooser <u>ex ante</u> prefers the size one equilibrium if p_1 is large relative to p_2 , and that he prefers the size two equilibrium otherwise. ¹⁵

This example is robust. It can be altered so as to satisfy the initial continuity and differentiability assumptions, and then arbitrarily perturbed without changing its basic nature. We conclude, therefore, that the <u>ex ante</u> preference of the chooser depends upon the distribution of types as well as the nature of the utility functions, even when the latter are regular and nicely ordered.

REMARK 4. Crawford and Sobel [1982] show in a related model that <u>ex ante</u>, the chooser (their Sender) prefers equilibria of larger size if certain monotonicity assumptions are

satisfied (as would be the case here if $v_{12}>0$, $u(\cdot)=v(\cdot,t')$ for some $t'\leq \underline{t}$, and (11) and (12) hold). In their model such assumptions imply that conditional on the chooser's type being in an interval of the form $[y,\overline{t}]$, the optimal proposal of the chooser is larger than that of the proposer. This need not be the case here; the option to veto and thereby get the status quo, a relatively large outcome, decreases the chooser's \underline{ex} ante preference for large proposals. Proposal x_2 can be so close to s that the ability to elicit it in a size two equilibrium is not worth very much to most of the chooser types.

Nevertheless, the chooser does tend to prefer a size two equilibrium. In a size two equilibrium he has the option of choosing between two proposals, r and x_2 . As long as they are not too remote from the x_0 that they straddle, having this choice is preferable to having x_0 for sure. In a series of numerical examples with utility functions that are quadratic and distributions that are uniforms raised to a power, the chooser always preferred a size two equilibrium. We conclude that communicative rhetoric tends to be beneficial to the chooser, just as it certainly is to the proposer.

Another question to ask is whether size one (or size two) equilibria can be ruled out by any of the recently proposed equilibrium refinements. Unfortunately, most of these are based on dominance relations and hence, because talk is "cheap," do not rule out any equivalence class of equilibria in a cheap talk game. Farrell [1985, 1988] introduces another kind of refinement especially for cheap talk games, that of "neologism-proof equilibrium." Unfortunately, it too does not generally distinguish between size one and size two equilibria. Matthews [1987] shows that if a size two equilibrium exists in which $x_2 < \overline{t}$, then neither it nor any size one equilibrium is neologism-proof. Since $x_2 < \overline{t}$ is the usual case, most often no equilibrium is neologism-proof if a size two equilibrium exists.

Comparing Equilibria with Different Status Quo Outcomes

Changing the status quo has some interesting and perhaps unexpected effects. We consider only size one equilibria; size two equilibria behave similarly.

The main result is that an increase in the status quo may either hurt or help the proposer. There are two opposing forces. An increase in the status quo decreases its utility to the proposer and hence hurts him when his proposal is rejected. But an increase in the status quo also helps the chooser by causing his proposal to be accepted more often because previously indifferent chooser types now dislike the status quo more.

To show this, the normalization (A4) must be abandoned, since chooser type t can only be indifferent between t and s for at most one value of s. (Maintaining (A4) while shifting s to the right would also shift the utility function of each type to the right, certainly hurting the proposer.) To keep things simple, replace (A4) by the assumption that each $v(\cdot,t)$ is symmetric around the ideal outcome z(t). The probability that a proposal $x \in (r,s)$ is accepted is then the probability that z(t) < (x+s)/2. Letting $\psi = z^{-1}$, the expected utility of the proposer when he proposes x is

(13)
$$U(x,s) = [u(x)-u(s)]F(\psi(\frac{x+s}{2})) + u(s).$$

Let $x_0(s)$ be a maximizer of U(x,s), the proposal that the proposer makes in a size one equilibrium when the status quo is s. From the envelope theorem,

(14)
$$\frac{d}{ds}U(x_0(s),s) = u'(s)[1-F(\psi)] + (\frac{1}{2})[u(x_0)-u(s)]F(\psi)\psi'$$
$$= u'(x_0)[1-2F(\psi)] + [u'(s)-u'(x_0)][1-F(\psi)]$$

(the first order condition is used to obtain the second equality). The first line shows the two opposing forces mentioned above (the first term is negative, the second is positive). In the second line of (14), the nonpositivity of the second term (assuming u is concave) can be overwhelmed by a positive first term if the indifferent type $\psi(\frac{x_0+s}{2})$ is sufficiently greater than the median chooser type. Thus, as ψ is increasing, the proposer's utility may increase (decrease) with an increase in s if s is large (small) relative to the type distribution.¹⁸

The proposal $x_0(s)$ can also either increase or decrease in s. It decreases in s in the corner case $x_0(s)=\overline{t}$, which occurs if s is large. In the interior case $x_0(s)<\overline{t}$, $x_0(s)$ can be shown

to increase in s if u is concave and $f'(\psi(\frac{x_0+s}{2}))>0$, but it decreases in s if $f'(\psi(\frac{x_0+s}{2}))<0$ and u is not too concave.

VIII. CONCLUSIONS AND CAVEATS

The proposer-chooser model gives one explanation for how Presidential rhetoric influences the nature of Congressional legislation. The President's rhetoric can convey real information about his preferences, which legislators care about because of his veto power. In the equilibria in which the President does influence Congress, he will veto only compromise bills that have been proposed after he has threatened to veto Congress's favorite legislation; the President never vetoes bills that embody Congress's favorite legislation.

The model can be criticized for neglecting many political realities. However, some can be put into the model without difficulty. Others cannot.

One political reality that can, to some extent, be put into the model is that Congress is actually a collection of entitites operating through a complicated social choice process. The most successfully modeled social choice processes, other than dictatorships, are simple majority rules on single-peaked preference domains. Replacing the proposer by a group of proposer-voters operating by majority rule need not change the model at all. Note first that if a proposer-voter learns that the chooser's type lies in an interval (a,b), then his expected utility when his utility function is $u(\cdot)$ and some $x \in (a, \min(s,b))$ is proposed is

$$U(x \mid a,b) = \frac{[u(x)-u(s)][F(x)-F(a)]}{[F(b)-F(a)]} + u(s).$$

Assuming the regularity condition (11), U is quasiconcave in x. The single-peakedness condition is therefore satisfied, the median voter theorem holds, and Propositions 1-3 continue to hold with x_0 , x_1 , and x_2 reinterpreted as the optimal proposals of the median voter. The model consequently applies equally to a Congress that resembles either polar extreme, a dictatorship or a majority rule body. Of course, to the extent that Congress is bicameral with a committee structure, a different model of the proposer should be adopted. Still, the basic results regarding rhetoric should remain.

The model can also be criticized on the basis that Presidential rhetoric seems to be aimed more at the electorate than the Congress. Although this may be true, the model still applies to the extent that a legislator's preferences reflect those of his constituency. The model does not apply if the electorate is less likely to vote for the President in the next election after he is observed to renege on his veto threats (the "wimp factor"). In this case making a veto threat increases the President's utility for vetoing, enhancing the credibility of the threat. Explanations along this line have evident merit, at least for first-term Presidents.

The model also does not allow the Congress to override vetoes. However, sufficient votes for an override are generally difficult to obtain, so that the model may still be useful even if only applied to cases in which a veto obviously cannot be overriden. More generally, the basic conclusions should still hold if the proposer is allowed to override vetoes, so long as the proposer's preferences over outcomes and cost function for overriding vetoes are common knowledge. It might be more appropriate to give the proposer private information about his cost of overriding. In this case proposals would signal those costs, and equilibria might be found in which both parties -- in accordance with casual empiricism -- engage in rhetoric.

Another criticism is that real-world outcome spaces are generally multidimensional. Making the outcome space multidimensional would destroy the median voter argument used above to apply the model to a majority rule Congress. Furthermore, it is not clear how the results depend on the unidimensionality of X. For example, initial explorations assuming X and T are two-dimensional have (unsurprisingly) produced equilibria with more than two elicited proposals. Whether a tight bound on their number exists in the multidimensional case is still problematic.

A last and obvious observation is that the real-world is a repeated game. Specifically, the President and the Congress repeatedly talk to each other, propose bills, and veto bills. The proposer-chooser game should be made dynamic in a variety of ways to address questions such as whether rhetoric can speed up the ultimate passage of a bill, or whether rhetoric can influence sequences of policy decisions. ¹⁹

APPENDIX

Recall that

(6)
$$x_0 \in \underset{x}{\operatorname{argmax}} [u(x)-u(s)]F(x).$$

PROPOSITION 1. The elicited proposal in any size one equilibrium satisfies (6), which has a unique solution if

(7)
$$\frac{u'(x)}{u(x)-u(s)} + \frac{f(x)}{F(x)} \quad \text{decreases in } x \text{ on } (\max(\underline{t},r), \min(s,\overline{t})).$$

<u>Proof of Proposition 1</u>. As the second statement is obvious, consider the first. Let x_0 be the elicited proposal in a given size one equilibrium. By the first part of (E3), $x_0 \le s$. Let $M_0 = \{m \in M \mid \pi(m) = x_0\}$ be the set of messages that induce x_0 . By (3) and (E3),

$$x_0 \in \underset{x}{\operatorname{argmax}} [u(x)-u(s)] \int_{\rho(m+t)}^{\rho(m+t)} \rho(t) dt \text{ for each } m \in M_0.$$

Because x_0 is the only elicited proposal, $\rho(m \mid t)=0$ if $m \notin M_0$. Hence, using (1),

$$[u(x)\text{-}u(s)]F(x) \ = \sum_{m \in M_0} [u(x)\text{-}u(s)] \int\limits_{\underline{t}}^{x} \rho(m \mid t) f(t) dt.$$

As x_0 maximizes each term on the right, it maximizes that on the left. Q.E.D.

PROPOSITION 2. Only size one and size two equilibria exist. Furthermore, if x_1 and $x_2>x_1$ are elicited in an equilibrium, then $x_1=r$ and $x_2\in(r,s)$.

<u>Proof of Proposition 2</u>. Let $<(\rho,\alpha),\pi>$ be an equilibrium with n>1 elicited proposals. Enumerate them as $x_1,...,x_n$, with $x_i< x_{i+1}$. Note that (E3) implies $x_n\le s$. The three goals of the proof are to show that $x_n< s$, $x_{n-1}\le r$, and $x_1\ge r$, which together imply n=2, $x_1=r$, and $x_2\in (r,s)$. Each x_i is elicited by a type who weakly prefers it to any $x_j \neq x_i$. So by (A), types $\underline{t} = t_1 < t_2 < \dots < t_{n+1} = \overline{t}$ exist such that for each $i = 1, \dots, n$, the types in (t_i, t_{i+1}) strictly prefer x_i to each $x_i \neq x_i$. The types in (t_i, t_{i+1}) therefore elicit only x_i .

As t_n is indifferent between x_{n-1} and x_n , and $x_{n-1} < x_n \le s$, assumptions (A2) and (A4) imply $t_n \le x_{n-1}$. We now show $t_n < x_{n-1}$, obtaining on the way our first goal, $x_n < s$. Choose $x \in (\max(r, t_n), s)$. Since

$$\sum_{m \in M} \int_{t_n}^{x} \rho(m \mid t) f(t) dt = F(x) - F(t_n) > 0,$$

some term in the sum, say the one corresponding to message m, is positive. Types in (t_n,x) prefer x to s, so that

$$\beta(x \mid m) = \frac{\int_{t_n}^{x} \rho(m \mid t) f(t) dt}{\frac{\overline{t}}{t_n}} > 0.$$

$$\int_{t_n}^{x} \rho(m \mid t) f(t) dt$$

Thus, since u(x)>u(s), the proposer's conditional expected utility if he were to propose x in response to m exceeds u(s). He actually proposes $\pi(m)$, so that $\pi(m)<s$. As the types in (t_n,\overline{t}) elicit x_n , $\pi(m)=x_n$. Hence our first goal: $x_n<s$. And $t_n< x_{n-1}$, since $x_{n-1}< x_n< s$ and t_n is indifferent between x_{n-1} and x_n .

Now let $m \in M$ be a message inducing x_{n-1} that has positive probability of being sent (it exists by an argument similar to that above). This message is sent only by types in (t_{n-1}, t_n) , and they would all accept proposals in (t_n, s) . So $\beta(x \mid m) = 1$ for all $x \in (t_n, x_{n-1})$. Since $\pi(m) = x_{n-1}$, this yields our second goal: $x_{n-1} \le r$ (otherwise, the proposer would prefer a proposal to the left of x_{n-1}).

Now consider x_1 . It is elicited by the types in (\underline{t},t_2) . As they prefer x_1 to x_n , which is less than s, $t_2 < x_1$. Let m be a message sent with positive probability that induces x_1 . Only the types in $(\underline{t},t_2]$ would send m, and they would accept proposals in (t_2,s) . Thus, any

 $x \in (x_1,s)$ is surely accepted conditional on m. Thus our final goal: $x_1 \ge r$ (otherwise the proposer would prefer a proposal to the right of x_1). Q.E.D.

Recall the definition

(8)
$$g(t) = \underset{x > t}{\operatorname{argmax}} [u(x)-u(s)] [F(x)-F(t)].$$

LEMMA 1. (i) $g(t) \subset [t,\overline{t}] \cap (r,\infty)$ for all $t \in T$;

- (ii) $g(t) \subset (r,s)$ for all t<s in T;
- (iii) g is upper hemicontinuous; and
- (iv) g is nondecreasing, increasing where it does not contain \bar{t} : if $\underline{t} \le t < t' < \bar{t}$, $x \in g(t)$, and $y \in g(t')$, then $x \le y$ and x = y only if $x = \bar{t}$.

<u>Proof of Lemma 1</u>. Entirely standard. It uses the differentiability of u at r (u'(r)=0) and the positivity of f on $T=(t,\overline{t})$.

PROPOSITION 3. If x_1 and $x_2 > x_1$ are the elicited proposals in a size two equilibrium, then $x_1 = r$, $x_2 \in (r,s)$, $x_2 \le \overline{t}$, and $t \in (\underline{t},r)$ exists such that $x_2 \in g(t)$ and

(9)
$$v(r,\tau) = v(x_2,\tau).$$

Types less than τ elicit only r, and types greater than τ elicit only x_2 . A size two equilibrium exists iff

(10)
$$v(r,t) \ge v(x,t)$$
 for some $t \in T$ and $x \in g(t)$.

Proof of Proposition 3. Given a size two equilibrium, Proposition 2 implies x_1 =r and $x_2 \in (r,s)$. Because the proposer does not prefer a proposal to the left of x_2 when he proposes it, $x_2 \le \overline{t}$. As some types prefer r to x_2 and some do not, some $\tau \in T$ is indifferent between r and x_2 . Hence (9), and $\tau < r$ because $v(\cdot,\tau)$ is unimodal. From (A3), types less than τ elicit only r and types greater than τ elicit only x_2 . Of the latter types, precisely those in (τ,x) would

accept an $x \in (\tau, s)$. Thus, by an argument like that in Proposition 1, proposing x_2 is optimal only if $x_2 \in g(\tau)$. Condition (10) is satisfied with $t=\tau$ and $x=x_2$.

Now assume (10) holds. Then

$$B = \{t \in T \mid v(r,t) \ge v(x,t) \text{ for some } x \in g(t)\}$$

is nonempty. Let $\tau = \sup(B)$. We need only show that $\tau \in T$, and that $v(r,\tau) = v(x,\tau)$ for some $x \neq r$ in $g(\tau)$. For then a size two equilibrium exists with $x_2 = x$: let some messages induce r, all others induce x_2 , all types less than τ send with equal probability only messages that induce r, and all types greater than τ send with equal probability only messages that induce r.

From Lemma 1(i) and the unimodality of $g(\cdot,t)$, t < r for each $t \in B$. Hence $\tau < \overline{t}$ (recall $r < \overline{t}$), implying that $\tau \in T$. From Lemma 1(ii), $x \ne r$ for all $x \in g(\tau)$. Thus, it remains only to show that $v(r,\tau)=v(x,\tau)$ for some $x \in g(\tau)$. Let $x=\max(g(\tau))$. From Lemma 1(iii) and the continuity of v, B contains τ , so that $v(r,\tau) \ge v(y,\tau)$ for some $y \in g(\tau)$. Thus, as $g(\cdot,\tau)$ is unimodal and $y \le x$, $v(r,\tau) \ge v(x,\tau)$. Let $\{t_n\}$ be a sequence converging to τ from the right, and let $x_n \in g(t_n)$. By Lemma 1(iii,iv), $x_n \to x$. As each $t_n \notin B$, $v(r,t_n) < v(x_n,t_n)$. Taking limits gives $v(r,\tau) \le v(x,\tau)$. Hence, $v(r,\tau) = v(x,\tau)$.

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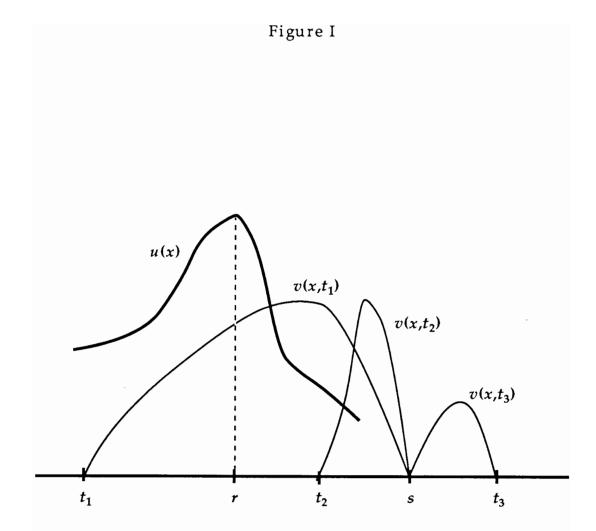
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FOOTNOTES

- 1. Models like this of the legislative process are in Kiewiet and McCubbins [1986], MacKay and Weaver [1985], and Shepsle and Weingast [1981].
- 2. See, e.g., Crawford and Sobel [1982], Farrell [1985,1988], and Myerson [1989] for abstract treatments, and Farrell and Saloner [1985], Forges [1987], Farrell and Gibbons [1986], and Matthews and Postlewaite [1989] for applications.
 - 3. Also Green and Stokey [1980].
- 4. The function u must be unimodal if it is derived from a strictly quasiconcave, monotonic utility function defined on a two-dimensional outcome space (e.g., military vs. social spending), and choice is restricted to a budget line. More generally, satiation seems likely in political contexts.
- 5. This is referred to as the "single-crossing of utility curves" property in Matthews and Moore [1987]. It is implied by $v_{12}>0$.
- 6. The range of $e(\cdot)$ may need to be the extended reals. For example, $e(t)=-\infty$ if v(x,t)>v(s,t) for all x<s.
- 7. We could generalize by letting $\alpha(x,t)$ be the probability that x is accepted by type t. But this would be to no advantage because this probability could be nontrivial for at most one, probability zero type.
- 8. Examples of such equilibria are presented and discussed in Appendix B of the discussion paper version, Matthews [1987].
- 9. Conceivably, many bills that could be passed by Congress might be supported by powerful backers of the President who can persuade him not to veto.
 - 10. The proofs use the assumptions that t < s, $\bar{t} > r$, u is unimodal at r, and u'(r) = 0.

- 11. Only if $\bar{t} \in (r,s)$ and fairly close to r is $x_0 = \bar{t}$ likely.
- 12. The related game in Crawford and Sobel [1982] generally does have equilibria of largersize. Appendix B of Matthews [1987] contains a comparison of the models.
- 13. The recalcitrant types always lack influence: the elicited proposals and final outcomes of any equilibrium are unaltered by truncating the distribution of types above s. But this is not the point. Equilibria do exist in which non-recalcitrant types send credible veto threats, but such equilibria are still equivalent to maximal-pooling equilibria which do not have credible veto threats. (See footnote 14 of Matthews [1987] for an example.)
- 14. If and only if $x_0=x_2$ do all chooser types prefer the size two equilibrium. (Example: $\underline{t}=0$, r=3, $\overline{t}=4$, s=6, F(t)=t/4 on [0,4], $u(x)=-(x-3)^2$, and $v(x,t)=-(x-z(t))^2$, where z(t)=(t+6)/2. Then $x_2=x_0=\overline{t}=4$, with t=1.) A necessary condition for $x_0=x_2$ is $x_0=x_2=\overline{t}< s$. This is an unusual case in which (i) an outcome $x=\overline{t}$ exists that is preferred to the status quo by both the proposer and every type of chooser, and (ii) this outcome is elicited in both kinds of equilibria.
- 15. The chooser prefers the size one equilibrium if $(p_1,p_2,p_3)=(29/42,1/6,1/7)$, and he prefers the size two equilibrium if $(p_1,p_2,p_3)=(1/3,1/3,1/3)$.
- 16. The utility functions can be smoothed at their peaks to make them fully differentiable, concave in x, and with $v_{12}>0$. The distribution of types can be made continuous by having $v(\cdot,t)$ closely resemble $v(\cdot,t_1)$, $v(\cdot,t_2)$, or $v(\cdot,t_3)$ for $t\in(\underline{t},t_1)$, $t\in(t_1,t_2)$, or $t\in(t_2,s)$, respectively, and by having $F(t_1)=p_1$ and $F(t_2)=p_1+p_2$. The <u>ex ante</u> preference of the chooser then would still depend upon the magnitude of p_1/p_2 .
- 17. Cho and Kreps [1987] and Banks and Sobel [1987] review and propose dominance-based equilibrium refinements. Farrell [1985, 1988] describes their lack of power in cheap talk games. Matthews [1987] discusses their application to the present game.

- 18. For example, suppose z(t)=(t+1)/2, $\underline{t}=-1$, $\overline{t}=2$, and f(t)=1/3 on [-1,2]. Let u(x)=-|x-r| for some $r \in (-1,0)$. Then $x_0(s)=0$ for $s \in (0,3)$, $x_0(s)=3-s$ for $s \in (3,3-r)$, and $x_0(s)=r$ for s>3-r. For all s>r, $U(x_0(s),s)$ decreases in s if s<3/2 and increases in s if s>3/2.
- 19. The dynamics might resemble those of the buyer-seller bargaining games exemplified by Fudenberg, Levine, and Tirole [1986], or the repeated political models of Kramer [1977] or Ingberman [1985].



Chooser type t_1 is accomodating, t_2 is compromising, and t_3 is recalcitrant.

