Discussion Paper No. 802

THE LIMITS OF MONOPOLIZATION THROUGH ACQUISITION[1]

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October 1988

(This paper is a revised version of Discussion Paper No. 754, December 1987)

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[1]The authors wish to acknowledge valuable discussions with Ehud Kalai and Ehud Lehrer, and remarks by the referees.

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Abstract

We address the question of whether competitive acquisition of firms by their rivals can result in complete or partial monopolization of a homogeneous product industry. This question is modelled in terms of two distinct three-stage noncooperative games. Analysis of subgame perfect pure strategy Nash equilibria of these games discloses that, under general weak assumptions, monopolization of an industry through acquisition is limited to industries with relatively few firms. For industries with a large number of firms, complete monopolization is impossible while partial monopolization is either impossible or limited in scope and can be completely eliminated by prohibiting any owner from acquiring over fifty percent of the firms in the industry. Moreover, there is always an equilibrium outcome in which the industry is not even partially monopolized and the original oligopolistic structure is retained.

The Limits of Monopolization Through Acquisition

by

Morton I. Kamien and Israel Zang

I. Introduction

A conventional view of how an industry is monopolized is through the acquisition by one firm of its rivals. This view appears to underlie the antitrust authorities efforts to inhibit such behavior through the issuance of merger guidelines. A firm that violates them risks an attempt by the Justice Department or the Federal Trade Commission to block the merger through the Federal Courts. A tacit assumption in the conventional view is that the firms being acquired react passively, perhaps out of fear that they will be victimized by predatory acts if they fail to sell out, or because they are unaware of what their buyer is attempting to accomplish. Selten (1978) has called the credibility of predatory pricing into question while McGee (1980) has questioned its actual role in the acquisition of rivals on favorable terms. The supposition that firms are unaware of what a rival seeking to acquire them is attempting to accomplish is belied in reality by their common appeal to the antitrust laws to ward off takeover.

Our purpose is to determine the limits of monopolization through acquisition in the absence of any legal barriers to such activity but in the presence of firms fully aware of the consequences of acquiring or being acquired by rivals, not susceptible to incredible threats, and behaving strategically with respect to this activity. In order to focus attention solely on this issue, we assume that the industry is composed of n identical firms with regard to the product they sell and their costs of production, which are assumed to be linearly increasing in the quantity produced, and that entry into the industry is difficult. The functional form of the industry's inverse demand function is assumed to be arbitrary but downward sloping with a strictly concave revenue function. The interaction among owners of firms is supposed to be describable as a Cournot-Nash oligopoly. It is also assumed that all the relevant variables and strategies available to the owners are common knowledge. Under these mild assumptions we model the strategic behavior of the firms' owners, in the formation of coalitions via acquisition as two distinct three-stage noncooperative games. In these games we allow owners to profit both from selling and buying firms and from operating them. Initially each owner possesses only one firm and can only sell it in its entirety. We characterize possible and impossible pure strategy subgame perfect Nash equilibria (SPNE) of these games, and show (Theorems 4, 6, 7, and Corollary 4) that for large industries there are substantial limits to the extent of industry monopolization via acquisitions.

The effects and desirability of horizontal mergers have been addressed by Salant, Switzer and Reynolds (1983) in the context of a Cournot oligopoly with a homogeneous product and linear demand and cost functions. They conclude that any coalition of firms, behaving as a merged firm, that consists of fewer than eighty percent of the industry's members will be disadvantageous. That is, members of the coalition would be better off abandoning it than staying in.[1] Underlying this result is the observation that, since production cost is linear, any coalition of firms will be

[1] Deneckere and Davidson (1985) on the other hand find merger advantageous when the firms produce differentiated products and engage in price competition.

indifferent with respect to how to split its total production among the members of the coalition. Hence it may behave in a centralized manner. Indeed, Salant, Switzer and Reynolds (1983) assume that every coalition of firms behaves as if it were a single firm. In one of our two games, the centralized game, we explore this possibility by assuming that an owner who acquired several firms behaves as one entity and, since cost is linear, this is equivalent to him operating only one of them. In a second, the decentralized game, we adopt a different approach. There we assume that an owner, possessing several firms, may wish to operate any number of them in competition with each other. This is achievable, say, by instructing or motivating the managers of the firms he has decided to operate, to maximize their individual firm profits. Consequently, such an owner follows a bi-level decision process, where at the first level he decides how many firms to operate and at the second level, knowing how many firms are operated by all other owners, his managers decide the optimal output level of every firm he chose to operate. Hence, in deciding how many of his firms to operate, he takes into account the resulting Cournot Nash equilibrium production decisions of all other owners. As a result, this owner may be better off than by operating all his firms in a centralized manner, i.e., as one unit.[2] The automobile industry provides a prominent example of divisionalized firms in which divisions compete with each other. While the divisions seek to distinguish their products through advertising and styling, the differences are more apparent than real.

As already mentioned, the strategic behavior of the firms' owners is

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[2]The idea that an owner of several firms may find it beneficial to have them compete against each other, has been suggested by Schwartz and Thompson (1986) in the context of entry deterrence.

posed in terms of two distinct three-stage noncooperative games in pure strategies. In the first stage of both games each owner makes offers or bids for every other firm and announces an asking price at which he would sell out. Once all bids and asking prices are known, firms are assigned to owners, at prices equal to the new owner's bids, following a general allocation scheme. An important feature of the allocation rule is that a firm can be sold to a new owner only if his bid for it is a maximal one and is not below its owner's asking price. In the second stage of the centralized game each owner, possessing one or more firms, operates only one of them. In the second stage of the decentralized game each owner decides how many of his firms to activate, i.e., operate at a positive level. He does this assuming that the managers of active firms seek their individual maximum profits even if several of them belong to a single owner. Finally, in the last stage of both games the active firm's output levels are decided. These output decisions, as those in the other stages of the game, are made under the usual Cournot-Nash assumptions. The final profit realized by each owner includes the net first stage ownership trading profits plus the last stage operating profits from all his active firms. We employ the SPNE as the solution concept for these games and therefore develop them by proceeding backwards from the Nash equilibrium of the last stage to that of its first stage.

A few remarks regarding the intuition of our analysis and results are in order as its formal phase involves a considerable amount of algebraic computations. Let us begin with a perhaps counter-intuitive result for the decentralized game, namely that an owner of several firms might optimally choose to operate more than one of them at a positive level. This may appear especially surprising in the presence of constant marginal costs. And in fact when one owner does purchase all his rivals he only operates one firm, as intuition suggests. However, if he does not own all the firms, then operating more than one of those he controls at a positive level, in response to competition from the others, may be optimal. For while he does compete against himself by doing so, the effect of this internal competition is diluted by the presence of active rivals. That is, by competing against himself he captures some sales from his rivals and thereby gains market share. Thus, while total industry profit declines his share of it enlarges enough to increase his total profit.

Relying on the above considerations we show in Theorem 1 that in any merged SPNE of the decentralized game, where by merged we mean that the number of firms operated by all owners is fewer than the initial n, there is one and only owner operating fewer firms than he owns. Furthermore, this owner must possess over fifty percent of the industry's firms. Based on this, we show that the same result applies to the centralized game as well. Hence, in both games, a prohibition against any single owner acquiring more than fifty percent of the industry's firms suffices to prevent monopolization via acquisition. In addition, we show in Theorem 4 and Corollary 4 that for numerically large industries, even this prohibition may be unnecessary. Thus, the common wisdom that numerically large industries are more difficult to monopolize than small ones does emerge from our analysis but not for the traditional reasons that they are more difficult to coordinate or that departure from agreed upon output levels is more difficult to monitor.

To explain these results, note first that if each owner sets a sufficiently high asking price, say above the monopoly profit, as well as sufficiently low bids for every other firm, say below the single firm profit in an n-firm oligopoly, then no transaction will occur and, given the other bids and asking prices, no owner can be made better off by changing his bids and/or asking price. Consequently, the initial n-firm oligopoly structure will be retained and this will constitute an equilibrium for both games discussed (Theorem 3). Moreover, it is intuitively clear that under potentially complete monopolization of the industry by one owner acquiring all his rivals, the buyer will be ready to pay the n - 1 sellers altogether no more than the difference between the monopoly and the single firm profit in an n-firm oligopoly. This overall payment is, of course, bounded from above by the industry monopoly profit. The problem is that, in this case, in both games considered, each seller may refuse to sell his firm by deviating in the first stage of the game and raising his asking price above the highest bid he was offered, thus becoming a nonseller. If such deviation occurs in the centralized game, then the buyer will own only n - 1 firms, operate one and, subsequently, the industry will become a duopoly. It follows that, to prevent such a deviation, the buyer has to pay each of the n - 1 sellers at least the single firm profit in a duopoly. Altogether, for sufficiently large n, the total payment will exceed the monopoly profit, which is the upper bound on the buyer's readiness to pay. Hence complete or partial monopolization become impossible equilibria for the centralized game as the number of firms in the industry becomes sufficiently large. Instead, it is only possible to have an unmerged equilibrium in which the original oligopoly structure is retained (Corollary 4). For the decentralized game, similar but somewhat weaker results (Theorem 4, Corollary 3) follow. In this case, it is impossible for one owner to possess all but a given number of firms, while operating fewer of them as the number of the industry's firms increases. Hence, a monopoly equilibrium is impossible for sufficiently large n. But, in addition to the unmerged oligopoly equilibrium, it is possible to have, in this case, some merged equilibria in which one owner possesses more than one-half, say K, of the industry's firms (including his own) and operates fewer, say $r < K$. We also show that for both games considered, the above results still hold even if acquisition by an outsider as well as by the incumbent owners is allowed.

In a recent paper, Gal-Or (1987) also considered the question addressed here, employing different assumptions. In her two-stage game, owners are allowed to purchase and sell fractions of firms, the initial owner of a firm controls its output even when he becomes a minority shareholder, owners do not profit from buying and selling firms, and only interior symmetric solutions are considered at the Cournot production stage. Thus, collusive equilibria in which each owner purchases equal shares in every firm, making the industry a "monopoly in disguise" (all firms are operated but the monopoly price can prevail) are possible. However, Gal-Or considered only symmetric ownership level outcomes. Thus, the possibility, discussed in this paper, of merger by acquisition is not allowed in her model. Gal-Or established that a symmetric equilibrium in which each owner possesses 1/n of every firm, while industry output and prices are those of a monopoly, is possible only if there are two firms in the industry.

This paper is organized as follows. In section II we illustrate our main results by means of simple linear demand examples and industries consisting of two, three and five initial owners and firms. The general models are presented in section III and their analysis in sections IV and V. The possibility and impossibility of some equilibria is established in section VI. Acquisition by an outside buyer is discussed in section VII. Section VIII is devoted to a short summary. Proofs of results appear in an appendix.

II. Examples

To crystalize our results we offer the following example. Suppose the inverse demand function is P = 20 - Q, where Q refers to total quantity sold, and that production is costless. It is not difficult to establish that a monopolist's profit in this case would be 100, a duopolist's profit would be 44.44, that each firm in a three-firm oligopoly would realize a profit of 25 at the Cournot equilibrium, 16 if there were four firms, and 11.11 if there were five. Let us begin with the supposition that the industry initially consists of two firms and consider the three stage centralized game in which an owner possessing several firms operates them as one entity. We illustrate the two equilibria that can obtain in this case, i.e., one in which the industry remains a duopoly and the other in which the industry is monopolized by a single owner.

First, we note that if each owner's asking price is 55.55 or more, while his bid for the other firm is 44.44 or less, then no sales occur and hence there will be a duopoly with each owner realizing 44.44. No owner will have the incentive to raise his bid for the other's firm since, if he purchased it, he would realize the monopoly profit of 100 while paying over 55.55. His net profit will then be 44.44 or less, which is no more than his current profit. Moreover, in these circumstances, no owner has an incentive to lower his asking price below the other's bid in order to sell out, as he will then realize no more than his current profit of 44.44. Thus, the above asking prices and bids constitute a SPNE in pure strategies in which the original oligopoly structure is maintained.

The other possible equilibrium for the centralized game, when n = 2, is obtained if the first owner makes a bid B_2^1 for the second owner's firm, satisfying $55.55 \geq B_2^1 \geq 44.44$, while setting an asking price B_1^1 for himself, satisfying $B_1^1 \geq 100 - B_2^1$. Simultaneously, the second owner asks B_2^2 for himself, where $B_2^2 = B_2^1$, while bidding B_1^2 for the first owner's firm, where $B_1^2 < B_1^1$. In this case a monopoly equilibrium, in which the first owner purchases the other's firm, is obtained. For example, if the first owner's asking price is 51 and his bid for other firm is 50 while the second owner's asking price and bid are both 50, then a profit sharing monopoly equilibrium in which the first owner purchases the other's firm obtains. In this equilibrium, both the seller's and buyer's profit is 50. To show that this is indeed an equilibrium note that the buyer is indifferent to raising his asking price. Moreover, he will not have an incentive to deviate and lower his asking price below the bid of 50, since if he does so he will end up owning only one firm (the second) and realizing 50 - 50 + 44.44 in the resulting duopoly. Also, the buyer will not raise his bid for the second firm, since this means paying more for it than he can get it for (i.e., 50), and he will not lower his bid since this will leave him with only one firm, making 44.44 in the resulting duopoly. On the other hand, the seller, for similar reasons, will have no incentive to either raise his bid for the buyer's firm or his asking price.

We now turn to the possible outcomes for the centralized game if there are three firms (n = 3) in the industry. First note that, as in the case of n = 2, if the asking prices of all three owners are sufficiently high, say 37.5 or above, and their bids on all the other's firms are sufficiently low, say 25 or below, then no player will have an incentive to individually change his asking price and/or bids, and consequently a three firm oligopoly will prevail. Consider now the possibility of a monopoly equilibrium in this game. More specifically, suppose that the first owner acquired the firms owned by the other two. Note that the first owner, by lowering his bids for the two other firms below their asking prices, can refrain from buying them. Thus, he can guarantee himself at least the single firm profit of 25 in a triopoly. If he acquires the two other firms and becomes a monopolist, he will make 100. Hence he will not be ready to purchase the other two firms for more than the difference, of 75, between the monopoly profit and the single firm profit in a three firm oligopoly. Consider now the situation of a seller. If the first owner seeks to purchase all three firms, an owner, say the third, can raise his asking price above the bid he received from the first owner. If he does so individually then the first owner will purchase only one firm, own two, and operate only one of them. The industry will then turn into a duopoly. Thus, each seller can individually assure himself in this way 44.44, the single firm profit in a duopoly. To prevent this and achieve a monopoly, the buyer has to pay each seller at least 44.44 or altogether 88.88 to the two of them. This is more than the 75 he can afford to pay. Thus, a monopoly equilibrium is impossible in this case. Using a similar argument, it is possible to show that an equilibrium, in which one owner purchases only one other firm turning the industry into a duopoly, is impossible. Indeed, in such a case the buyer can afford to pay the seller no more than 44.44 - 25, while the seller can make 25 by deviating and raising his asking price.

In general, when the number of firms in the industry is sufficiently large, a monopoly equilibrium is impossible in the centralized game. The main reason for this is the combination of the potential buyer's inability to pay more than the monopoly profit with the ability of each potential seller to guarantee himself the duopoly profit if a monopoly is to be established. In Corollary 4 we prove an even stronger result, namely, if the number of firms in the industry is large enough, then "merged" equilibria, that is, ones in which the number of firms being actively operated is fewer than the initial number n, are impossible in the centralized game.

We now turn to the possibility and impossibility of equilibria for the decentralized game, that is, the one in which an owner possessing several firms decides how many of them to operate, anticipating the resulting Cournot equilibrium. First we note that, as in the centralized game, sufficiently high asking prices together with sufficiently low bids will result in a SPNE in pure strategies in which the original oligopoly structure is maintained. Consider now the decentralized game for the case of a three firm ($n = 3$) industry.

The third stage of the game is characterized by the per firm Cournot equilibrium payoffs indicated above when there are one, two, or three active firms. The second stage of the game is analyzed by observing that if all three firms were owned by a single owner he would operate only one and realize a payoff of 100; if he owned only two firms he would operate both as his payoff would be 50, which exceeds 44.44, the payoff from operating only one. Last, it is obvious that each individually owned firm will be operated, realizing a profit of 25.

It is immediately apparent that no owner will sell his firm for less than 25, the payoff he can realize in the original three firm oligopoly, and regardless of whether each firm remains individually owned or two of them are owned by a single owner, the industry will remain an oligopoly with three active firms. Thus, we focus on the SPNE in pure strategies in which one owner possesses all three firms and the industry is completely monopolized. To demonstrate this, we posit bids of 33.33 made by the first owner for the two other firms, bids smaller than 33.33 made by the two other owners for each other's firm, and asking prices of B_1^1, 33.33, 33.33, for the first, second, and third firms, respectively, where $B_1^1 \geq 33.33$. We show that these prices result in a profit sharing monopoly equilibrium in which the owner of the first firm purchases the other two and each realizes a payoff of 33.33. To establish this we must show that no owner has an incentive to deviate from his asking price and bids. Consider the first owner, namely, the buyer. Raising his asking price will not make any difference for him while decreasing it may result in his firm being bought for a price below 33.33. In the latter case, he ends up owning only two firms, paying a total of 66.67 for both, while operating the two of them and making 50 at the production stage. Hence, his net profit will be $50 - 55.67 + 33.33 = 16.67$ or less, which is below his current profit of 33.33. If he considers changing his bids, then he will not raise them as he will end up paying more for firms he can get for less. Lowering his bids on either of the firms he is buying, or both, will make them nonsellers.

Consequently he will end up either owning two firms while operating both and making 50 - 33.33, or owning one firm and making 25. In both cases his profit will fall below his current 33.33. Now consider a seller. Lowering his asking price will not change his profit, while raising it will make him a nonseller. But then, the first owner will end up possessing two firms and operate both. Consequently, the seller, who becomes a nonseller, will make 25, less than his current profit of 33.33. If a seller deviates by raising his bids for one or both of the other firms, he will end up owning one or two firms, making 25 or 50, respectively, while paying over 33.33 for each. If he raises his asking price and bids on the two other firms he will own all three firms, and make 100 while paying over 66.67. His net profit then will fall short of his current profit of 33.33.

Let us now turn to the case of a five firm oligopoly and demonstrate the absence of a monopoly equilibrium in the decentralized game. We show that in this case, in addition to the equilibrium in which the original oligopoly structure is maintained, another equilibrium in which one owner possesses four firms and actively operates two exists. In the latter equilibrium, therefore, the industry is reduced to a three firm oligopoly.

We begin by recalling that in a five firm oligopoly each firm realizes a profit of 11.11 at the Cournot equilibrium. Hence, an owner of two firms will actively operate both of them as 22.22 exceeds 16, the profit he would realize by operating only one. An owner of three firms will operate all of them as he will then realize a payoff of 33.33, which exceeds 32 and 25, the payoffs from operating two and one only, respectively. Finally, an owner of four firms will operate only two actively, realizing 50 which is more than

44.44, 48, and 44.44, the payoffs from operating four, three, and one only, respectively. An owner of all five firms will operate only one, realizing 100.

Let us now demonstrate the impossibility of a complete monopoly equilibrium in which one owner, say the first, purchases all the other firms and operates only one. For such an equilibrium to exist, the first owner will have to pay all the other owners no more than 100 - 11.11 = 88.89, his monopoly profit less what he can obtain by lowering all his bids below the other owner's asking prices, respectively, and becoming a nonbuyer. Consider now a seller in this potential equilibrium. By raising his asking price above the first owner's bid he becomes a nonseller. Then the first owner will own four firms, and operate only two. Thus, altogether three firms will be operated in the industry and the seller who deviated will make the single firm profit of 25 in a triopoly. It follows that, to become a monopolist, the buyer must pay each seller at least 25. Since there are four potential sellers, the first owner will have to pay at least 100 to become a monopolist. This is more than the 88.89 he can afford. Hence, a monopoly equilibrium is impossible for the decentralized game when n = 5.

In general, the inability of the buyer to pay more than the monopoly profit together with the ability of every seller to guarantee himself the profit of a single firm in a numerically small industry prevents the industry from becoming a monopoly in the decentralized game for sufficiently large n. Indeed, if the demand function is linear, the only possible merged equilibria, that is, ones in which fewer than the original n firms are operational, are those wherein the buyer owns slightly over one-half of the industry's original firms and operates almost all of them. This results in an insignificant degree of reduction in competition whenever the number of firms in the industry is large. As an example, it can be shown that asking prices of 51, 12.5, 12.5, 12.5, 51, by the five owners, respectively, bids of 12.5 made by the first owner to the second, third, and fourth owner, while all other bids are 11.11 result in an equilibrium in which the first owner purchases the second, third, and fourth firms. In this equilibrium only three out of the initial five firms will be active, and the owners of the first four firms will share the profit of 50 made by the two out of the four firms being operated.

III. The Model

We now turn to the formal description of our three-stage games. We posit an industry consisting of n identical firms producing a single good whose total quantity supplied is denoted by Q, facing an inverse demand function P(Q). Every firm has the same constant marginal cost technology and there is no fixed cost, i.e., if firm i produces a quantity q_i, then $C(q_i) = Cq_i$. We assume that the following properties hold:

(I) P is twice continuously differentiable, P(0) and P'(0) are finite, and $P'(Q) < 0$ for all $Q \geq 0$.

(II) $P(0) > C$ and for some $\hat{Q} > 0$, $P(\hat{Q}) < C$ holds.

(III) The industry total revenue function QP(Q) possesses a negative second derivative which is bounded from below, i.e., there exists a real number $\beta > 0$ such that $(QP(Q))'' \geq -\beta$ for all $Q \geq 0$. Note that this assumption implies strict concavity of the industry total revenue function.

We now proceed to describe the two distinct "acquisition games." In both, each firm is owned and controlled by a single owner. These owners are the players of the games in which each owner can purchase other firms or sell his. Naturally, if a firm is sold it becomes controlled by its buyer. Let $N = \{1,2,\ldots,n\}$.

The Decentralized Game G_c

Stage 1: Each owner $j \in N$ simultaneously announces a vector $B^j = (B_1^j, B_2^j, \ldots, B_n^j) \in R^n$ of bids for the entirety of each firm. The bid B_j^j is the j-th owner's bid or asking price for his own firm. Let $B = (B^1, B^2, \ldots, B^n)$ denote the $n \times n$ matrix of bids.

Following the announcement of B, each firm may be sold to one of its bidders at the bid price, or it may remain with its original owner. We now describe a general rule that allocates firms to owners. Let

$$S(i) = \{j \in N: j \neq i, B_i^j \geq B_i^k \ \forall \ k \neq j\}$$

be the set of owners, other than i, whose bid on i satisfies two properties: (i) it is not smaller than the i'th asking price; and (ii) it is the highest bid on i. It is natural to expect that the firm owned by i may be sold only to a member of S(i) or remain it with its original owner, i. If S(i) is empty or a singleton, then the allocation is obvious. However, we need to specify the allocation if S(i) contains more than one element. To that end we employ a general tie-breaking rule, namely, a function $f: N \times 2^N \rightarrow N$ satisfying

$$f(i,S) \in S \cup \{i\} \ \forall \ i \in N, \ S \subseteq N.$$

Thus, $S(i,S(i))$ uniquely determines the allocation of firms to owners for any given matrix B of bids.

Some possible tie breaking rules conforming with the above framework are:

1. Global priorities. Priorities are assigned to owners, say, by their numbering, and $f(i,S(i)) = \min \{j: j \in S(i)\}$ if $S(i) \neq 0$.
2. Individual priorities. Owner i will sell his firm to one of the highest bidders according to some known priorities unique to him.
3. No deal. In the presence of a tie, firm i is not sold, i.e., $f(i,S(i)) = i$ if $|S(i)| \geq 2$.

An important property of our general allocation rule is that once the i-th owner's asking price and the bids he received are known, the allocation of firm i is independent of the asking prices of and bids received by every other firm. We assume that the allocation rule is known to all the participants in the game.

Applying the allocation rule, the ownership of firm i, indexed by ε_{ij}, is determined by

$$\varepsilon_{ij} = \begin{cases} 1, & \text{if } f(i,S(i)) = j, \\ 0, & \text{otherwise.} \end{cases}$$

Let ε be the $n \times n$ matrix whose (i,j) entry is ε_{ij}. Note that $\varepsilon = \varepsilon(B)$. For $j \in N$ let

$$K_j = \Sigma_{i \in N}\, \varepsilon_{ij}$$

denote the number of firms owned (and controlled) by j, and by the vector $K = (K_1, K_2, \ldots, K_n) = K(B)$, the final number of firms possessed by each owner. Since all firms are identical only knowledge of K is required in the next stage.

<u>Stage 2</u>: Given K, each owner $j \in N$ decides simultaneously how many of his firms, $0 \leq r_j \leq K_j$, will be active (in competition with each other). Let

$$r = (r_1, r_2, \ldots, r_n) = r(B)$$

and denote by

$$m = \Sigma_{i \in N}\, r_i \tag{1}$$

the number of active firms, and by $M \subseteq N$ the set of active firms.

<u>Stage 3</u>: Given r, each owner $j \in N$ simultaneously chooses the production level of each of his r_j active firms, i.e., by the manager of each active firm independently seeking to maximize its profits. Letting $q_i = 0 \;\forall\; i \notin M$, we denote by

$$q = (q_1, q_2, \ldots, q_n) = q(r, B),$$

the vector of quantities produced by all firms in N as a result of this decision and let $Q = \Sigma_{i \in N}\, q_i$.

The payoff to each player (initial owner) is the sum of the stage 3 operating profits of all the firms he controls plus the net trade cash flow in stage 1.

The Centralized Game G_C

Stage 1 of the game is the same as in G_D. However, as the ownership distribution becomes known following this stage, we make use of the common assumption (see, e.g., Salant, Switzer and Reynolds, 1983) that each owner of at least one firm will behave as a single entity. That is, he will operate one firm regardless of how many he owns. This, of course, eliminates Stage 2 of the game. However, to conform with the stages numbering of G_D, we introduce a redundant Stage 2.

Stage 2: Owner $j \in N$ decides to operate one firm (if he owns any). That is,

$$r_j = \begin{cases} 1, & \text{if } K_j \geq 1. \\ 0, & \text{otherwise.} \end{cases}$$

Again, let the number of active firms be given by (1) and let $M \subseteq N$ denote the set of active firms.

Stage 3: Every owner $j \in N$ for which $r_j = 1$ simultaneously chooses the production level of his active firm.

The output vector q and the payoffs to the players are the same as in the decentralized game G_D.

We are concerned with characterizing properties of subgame perfect Nash equilibria in pure strategies of the games G_D and G_C.

Definition: A SPNE in an acquisition game is said to be merged if the number m of firms operated by all owners is fewer than the initial number, n. If m = n, in a SPNE, then this equilibrium is <u>unmerged</u>. If m = 1 in a SPNE then we have a <u>monopoly equilibrium</u>.

In the sequel we will characterize possible SPNE's of the acquisition games. In particular, we will consider existence of unmerged SPNE and identify instances in which a merged equilibrium cannot exist. We will also suggest possible regulatory rules that prevent merged equilibria. Such a rule will be given as a corollary to Theorem 1 below.

IV. <u>Analysis of the Decentralized Game</u> G_D

<u>Analysis of Stage 3</u>: For $i \in M$, q_i is determined via a manager seeking to

$$\text{(2)} \qquad \max_{q_i \geq 0} \pi(q,m) = q_i[P(\Sigma_{j\in M}\, q_j) - C].$$

The first order necessary conditions for this problem are

$$\text{(3.1)} \qquad \partial\pi/\partial q_i = P(Q) - C + q_i P'(Q) = 0, \text{ if } i \in M,\ q_i > 0,$$

$$\text{(3.2)} \qquad \partial\pi/\partial q_i = P(Q) - C + q_i P'(Q) \leq 0, \text{ if } i \in M,\ q_i = 0.$$

First note that whenever (3) holds then $P(Q) > C$ is satisfied, since otherwise, if $P(Q) \leq C$ the assumption that $P' < 0$ will imply through (3) that $Q = 0$ and hence $P(0) \leq C$ contradicting Assumption II. This however implies that $q_i = 0$ for some $i \in M$ cannot hold, since (3.2) will then imply $P(Q) \leq C$. It follows that at a stage 3 equilibrium of G_D, $q_i > 0\ \forall\ i \in M$.

But then (3.1) implies that $q_i = q_j = \bar{q}\ \forall\ i,j \in M$. Or,

$$(4) \qquad \bar{q} = -(P - C)/P',$$

where for convenience we use the notation $P = P(Q)$, $P' = P'(Q)$. Consequently, $Q = m\bar{q} = -m(P - C)/P'$, or

$$(5) \qquad m[P(Q) - C] + QP'(Q) = 0.$$

<u>Lemma 1</u>: For every $1 \leq m \leq n$ the game has a unique stage 3 equilibrium. In this equilibrium each active firm produces a positive quantity given by (4).

For proofs of all our results, see the Appendix.

Let $Q(m)$ denote the unique, by Lemma 1, solution to (5). Equation (5) describes the relationship between total output, $Q(m)$, and the number of active firms, m, operated by all owners collectively.

Upon substituting $q_i = \bar{q} = Q/m$ into (2) we obtain the individual firm's stage 3 profit:

$$(6) \qquad \pi(m) = (1/m)Q[P(Q) - C],$$

where $Q = Q(m)$ solves (5). An interesting property of $Q(m)$ is obtained by differentiating (5) with respect to m,

$$(7) \qquad P - C + [mP' + P' + QP'']Q'(m) = 0$$

or,

$$(8)\qquad Q'(m) = -\frac{P - C}{(m + 1)P' + QP''},$$

where the denominator of (8) cannot vanish as this would imply by (7) that $P(Q) = C$. However, (5) would then imply $Q = 0$. Hence, $P(0) = C$ which is impossible by Assumption II. Since

$$(9)\qquad [QP(Q)]'' = 2P' + QP'',$$

we obtain from (8)

$$(10)\qquad Q'(m) = -\frac{P - C}{(m - 1)P' + (QP)''}.$$

Assumptions (I) and (III) and (10) immediately imply that aggregate industry production increases with the number of active firms:

<u>Proposition 1</u>: For $m \geq 1$, $Q'(m) > 0$ holds.

Our next proposition shows that aggregate industry profits decrease as the total number of active firms operated increases. Let

$$(11)\qquad \Pi(m) = m\pi(m)$$

denote aggregate industry profit. By (6)

(12) $\Pi(m) = Q[P(Q) - C]$.

Thus

(13) $\Pi'(m) = [P(Q) - C + QP'(Q)]Q' =$

$$[P(Q) - C + \bar{q}P'(Q)]Q' + (m - 1)\bar{q}P'(Q)Q' = (m - 1)\bar{q}P'(Q)Q' < 0,$$

where the second equality in (13) follows from $Q = m\bar{q}$ and the last one follows (3.1). By Lemma 1, $\bar{q} > 0$, by (10), $Q' > 0$, and since $P' < 0$ holds, it follows that $\Pi'(m) < 0$ for $m > 1$. Thus:

Proposition 2: Aggregate profits are a declining function of the total number of active firms.

It follows as an immediate corollary to Proposition 2 that an individual firm's profit increases as the number of active firms declines.

Corollary 1: $\pi(m)$ is a decreasing function of m.

Analysis of Stage 2: In this stage, owner $j \in N$ solves

(14) $\max_{0 \le r_j \le K_j} r_j \pi(\Sigma_{i \in N} r_i)$,

where π is given by (6). Note that the above objective function depends on r_j and on the sum of r_i, $i \neq j$. Hence we let $t_j = \Sigma_{i \neq j} r_i$ be the number of firms operated by all other owners except j and denote by

(15) $$T(r,t) = r\pi(r + t) = [r/(r + t)]Q(r + t)(P[Q(r + t)] - C).$$

Thus, problem (14) becomes

(16) $$\max_{0 \le r_j \le K_j} T(r_j, t_j).$$

The next two lemmas present the intuitive properties of the stage 2 profit function T.

Lemma 2: An owner possessing at least one firm ($K_j \geq 1$) will operate at least one ($r_j^* \geq 1$).

Lemma 3: An owner possessing $K_j = n$ firms will operate only one.

Let us now derive an expression for the derivative of T with respect to r, $T'(r;t)$. From (15) and by definition of $m = r + t$,

(17) $$T(r,t) = r\pi(m),$$

and therefore

(18) $$T'(r,t) = \pi(m) + r\pi'(m).$$

Differentiating (11) and substituting into (18) yields

$$(19) \qquad T'(r,t) = \pi(m) + (r/m)[\Pi'(m) - \pi(m)] = (\pi/m)[t + r\Pi'/\pi].$$

Substituting (6) and (13) into (19) yields

$$T'(r,t) = (\pi/m)[t + r(m - 1)P'Q'/(P - C)],$$

and by substituting (10) and then rearranging

$$(20) \qquad T'(r,t) = \frac{\pi}{m}\left[\frac{t(QP)'' + (m - 1)(m - 2r)P'}{(QP)'' + (m - 1)P'}\right].$$

It follows immediately that for $r \leq m/2$, $T' > 0$. Moreover, since $m = r + t$, we have that $r \leq m/2$ is equivalent to $r \leq t$. Thus,

Lemma 4: If $t \geq r$, then $T'(r,t) > 0$.

The implications of Lemma 4 are straightforward. Assuming r_j to be a continuous variable,[3] if the solution to (16) is obtained at r_j^* satisfying $r_j^* < K_j$ then $T'(r_j^*,t_j) \leq 0$ must hold, implying, in view of Lemma 4, that $t_j < r_j$ holds. In other words:

Proposition 3: If an owner finds it optimal to operate fewer firms than he owns (i.e., $T'(r_j,t_j) \leq 0$), then it must be that the number of firms he operates exceeds the number of firms operated by all other owners combined.

We mentioned already that when an owner of several firms decides how many more firms to actively operate, he evaluates the trade-off between the

[3]This assumption is not needed if the demand function is concave.

cost of increasing competition with himself and the profit he gains by taking sales from his rivals. Naturally the cost side will dominate when he owns many firms and the others operate a few, while the profit side will dominate when he owns a few firms and the others operate many. Hence he will tend to operate fewer firms of those he owns in the former case and more of them in the latter. The striking implication of Proposition 3 is that if he owns fewer firms than the others operate, he will choose to operate all of them. An immediate consequence of this is that in any stage 2 equilibrium outcome, there cannot be more than one owner operating fewer firms than he owns.

Proposition 4: In any stage 2 SPNE of G_D (and hence any SPNE of G_D) there can be at most one owner for which $r_j^* < K_j^*$.

A direct consequence of Proposition 4 is:

Theorem 1: If there is a merged SPNE to the game G_D, then it has only one owner, say j, for which $K_j \geq (n + 1)/2$. This owner operates fewer firms than he owns and all other firms are operated by their owners regardless of their ownership.

Hence, if one seeks a regulatory constraint that will prevent merged outcomes to the game G_D, then the following corollary applies.

Corollary 2: Under the restriction (regulatory constraint) that no owner can possess $(n + 1)/2$ or more firms, the game cannot have a merged SPNE.

Let $r_j(K_j)$ be an optimal solution to

(21) $$\max_{0\le r_j \le K_j} T(r_j, n - K_j),$$

and $r^* = r^*(K) = (r_1^*, r_2^*, \ldots, r_n^*)$ be a stage 2 SPNE, i.e., for all $j \in N$, r_j^* solves (16) when $t_j = \sum_{i\neq j} r_i^*$. Also, let

$$R_j(K) = T(r_j^*, \textstyle\sum_{i\neq j} r_i^*),\ \forall\ j \in N$$

denote the j-th owner's operating profit.

Analysis of Stage 1: For $j \in N$ let

$$W_j^D(B) = R_j(e^T\varepsilon) - \sum_{i\neq j} B_i^j\varepsilon_{ij} + \sum_{i\neq j} B_j^i\varepsilon_{ji}$$

where

$$\varepsilon_{i\ell} = \begin{cases} 1, & \text{if } f(i,S(i)) = \ell, \\ 0, & \text{otherwise}, \end{cases} \qquad \forall\ i,\ell \in N$$

and $e = (1,1,\ldots,1)^T$.

Thus, the j-th owner's total wealth, $W_j^D(B)$ at the end of the game, consists of his total operating profit, the first term on the right, less his payments for the firms he purchased, the second term, plus the payment he received for his firm if it was sold. Owner $j \in N$ then solves

$$\max_{B^j} W_j^D(B).$$

We denote by $B_D = (B_D^1, B_D^2, \ldots, B_D^n)$, a SPNE of G_D.

V. Analysis of the Centralized Game G_C

Stage 3 of the game is exactly the same as in the decentralized game G_D. It follows that Lemma 1, Propositions 1 and 2, and Corollary 1 are satisfied for this game as well.

We now turn to a formal description of Stage 1 of G_C. In this stage the j-th owner's objective is to maximize the function

$$W_j^C(B) = \pi(m) - \Sigma_{i \neq j} B_i^j \varepsilon_{ij} + \Sigma_{i \neq j} B_j^i \varepsilon_{ji}$$

where

$$\varepsilon_{i\ell} = \begin{cases} 1, & \text{if } f(i,S(i)) = \ell, \\ 0, & \text{otherwise}, \end{cases} \qquad \forall\ i,\ell \in N$$

$$m = \text{Number of positive components of } e^T\varepsilon$$

where $e = (1,1,\ldots,1)^T$. Hence, owner $j \in N$ solves

$$\text{Max}_{B^j} W_j^C(B).$$

We denote by $B_C = (B_C^1, B_C^2, \ldots, B_C^n)$ an SPNE of G_C. As in the decentralized game, the owner's wealth at the end of the game consists of his operating profit minus acquisition costs plus revenue from sale of assets.

We now turn to show that other results, obtained so far for the decentralized game, apply for the centralized game as well. Indeed, in a merged SPNE of G_C, a player who, following Stage 1 ended up owning $K \geq 2$ firms, must realize in Stage 3 (when all other owners operate t firms) at least K times the operating profit he could by operating all of the K firms independently (when all other owners continue to operate the same number, t, of firms). This is true because he, as well as each of the K - 1 firms he purchased, could have earned at least the profit of a single firm in a K + t firm oligopoly. They could achieve this profit by either the buyer lowering his bids sufficiently to become a nonbuyer or the sellers raising their asking prices sufficiently to become nonsellers.

It follows that, in a merged equilibrium, a buyer, when operating only one of his K firms, is at least as well off as he would be by operating all his K firms in competition against one another, as in the game G_D. Hence, if this owner were to play Stage 2 of the decentralized game, then under the same circumstances operating fewer firms than he owns would be a Stage 2 best response (in G_D) for him. Hence, Propositions 3 and 4, and Theorem 1 have their counterparts for the centralized game G_C. In particular, in a SPNE of G_C, if a player owns more than one firm, then the number of firms he would have operated under the same circumstances in Stage 2 of G_D should have been greater than the number of firms operated by all other owners combined (using Proposition 3). Hence, there can be at most one such owner (Proposition 4), and:

<u>Theorem 2</u>: If there is a merged SPNE to the game G_C, then in it there is only one owner, say j, for which $K_j \geq (n + 1)/2$ and all other players can

own at most one firm.

A counterpart of Corollary 2 also follows. That is, under the regulatory constraint, that no owner may possess $(n + 1)/2$ or more firms, the game G_C cannot have a merged SPNE.

Theorem 2 is a generalization to the case of a general nonlinear demand of the result due to Salant, Switzer and Reynolds (1983), who showed that, for linear demand functions, the bound $K_j \geq (n + 1)/2$ in Theorem 2 can be replaced by the tighter bound $K_j \geq 0.8n$.

VI. Analysis of Equilibria

We will now discuss some possible and impossible equilibrium outcomes of the acquisition games. We begin with the question of the existence of unmerged equilibria. Indeed, if every owner's asking price B_j^j, $j \in N$, is sufficiently high, say, above the monopoly profit $\pi(1)$, and each bid is sufficiently low, say, below the single firm profit in an n firm oligopoly $\pi(n)$, then in neither game can an owner become better off by either lowering his asking price and becoming a seller, or by raising his bids and becoming a buyer, or both. Thus:

Theorem 3: For every n there is an unmerged SPNE to both G_D and G_C.

We now turn to the possibilities and impossibilities of existence of merged SPNE in both games. We will first focus our attention on the decentralized game G_D. Consider a merged equilibrium to this game. Then by Theorem 1, there is one and only one owner, say the first, owning K_1^* firms and operating $r(K_1^*) < K_1^*$ of them. Without loss of generality, assume that the $K_1^* - 1$ sellers are $2, 3, \ldots, K_1^*$. Certainly, the sum $\sum_{i=2}^{K_1^*} B_i^1$ of the first owner's bids (and hence payments) to the $K_1^* - 1$ sellers cannot exceed the difference between his Stage 3 operating profit $T(r(K_1^*), n - K_1^*)$ and the single firm profit in an n firm oligopoly $T(1, n - 1)$. That is

$$(22) \qquad \sum_{i=2}^{K_1^*} B_i^1 \leq T(r(K_1^*), n - K_1^*) - T(1, n - 1).$$

Indeed, if (22) is violated, then the first owner's overall wealth $W_1^D(B)$ is below $T(1, n - 1)$. But he can always guarantee himself at least $T(1, n - 1)$ by lowering all his bids to zero and becoming a nonbuyer. Inequality (22) implies that there is at least one seller, say the second, who sold his firm for a bid B_2^1 not greater than the average maximum possible payment. That is,

$$(23) \qquad B_2^1 \leq [T(r(K_1^*), n - K_1^*) - T(1, n - 1)]/(K_1^* - 1).$$

For this to be a merged equilibrium, each of the $K_1^* - 1$ sellers, and in particular the second, cannot become better off by raising his asking price B_2^2 above the bid B_2^1, offered by the first owner. Indeed, if the second owner deviates in this way, then, since B_2^1 is the highest bid he received, his firm will not be sold and the first owner will end up with $K_1^* - 1$ firms, while operating $r(K_1^* - 1)$ firms in Stage 2 of the game. Consequently, the number of operated firms in the industry will be $r(K_1^* - 1) + n - K_1^* + 1$ and the second owner will make $T(1, r(K_1^* - 1) + n - K_1^*)$ as a result of his deviation. If

(24) $$[T(r(K_1^*),\ n - K_1^*) - T(1,\ n - 1)]/(K_1^* - 1)$$
$$< T(1,\ r^*(K_1^* - 1) + n - K_1^*)$$

holds, then, in view of (23), the second owner will have the incentive to deviate and raise his asking price, anticipating higher profits. Hence, if (24) holds, then a merged SPNE in G_D in which one owner possesses K_1^* firms while operating fewer is impossible. In our next theorem we show that if the number of firms not purchased by the buyer is held constant, then for sufficiently large n, such an equilibrium becomes impossible.

Theorem 4: For any given $d \geq 0$, a merged SPNE to G_D in which $K_1^* = n - d$ is impossible for sufficiently large n.

Letting $d = 0$, we obtain a special case of Theorem 4:

Corollary 3: For sufficiently large n, a monopoly SPNE is impossible in the game G_D.

It is further possible to prove that if the demand function is linear, then a monopoly equilibrium is impossible in G_D if $n \geq 5$. This conforms with the example presented in Section II.

A few words, with respect to the implications of Theorem 4, are in order. The theorem indeed implies that there cannot be a merged equilibrium in which one owner possesses all but a few firms and operates fewer of them, if n is large enough. However, it does not exclude the theoretical possibility that for every n there is one owner possessing all but $n^{1/2}$ firms, that is, $n - n^{1/2}$ firms, while operating fewer. This can occur because for any given $d \geq 0$, $d < n^{1/2}$ holds for sufficiently large n. In this case the fraction $(n - n^{1/2})/n$ of the industry's firms held by this particular owner approaches unity as $n \to \infty$. That is, although a monopoly is impossible, the theorem does not exclude the possibility of one owner asymptotically possessing one hundred percent of the industry's firms. It should be pointed out, however, that with a linear demand function, a stronger version of Theorem 4 applies, namely, for each given fraction $.5 < \theta \leq 1$ of the industry's firms, for sufficiently large n it is impossible to have an owner possessing a proportion greater than θ of them while operating fewer.

Let us turn now to establish an impossibility of merged equilibria theorem for the centralized game G_C. It will be shown that arguments similar to those used to establish Theorem 4 will imply stronger results. Consider a merged SPNE in G_C in which, by Theorem 2, there is only one owner, say the first, possessing K_1^* firms while operating only one. Assume that the $K_1^* - 1$ sellers are $2,3,\ldots,K_1^*$ and note that $K_1^* \geq (n+1)/2$ must hold by Theorem 2. In this case, the sum $\sum_{i=2}^{K_1^*} B_i^1$ of the bids and payments made by the first owner cannot exceed the difference between the Stage 3 operating profit $\pi(n - K_1^* + 1)$ the first owner realizes, and the profit $\pi(n)$ of a single firm in an n firm oligopoly, when π is given by (6). That is,

$$\sum_{i=2}^{K_1^*} B_i^1 \leq \pi(n - K_1^* + 1) - \pi(n). \tag{25}$$

It follows that at least one seller, say the second owner, gets no more than the average maximal payment implied by (25), and hence the inequality

$$(26) \qquad B_2^1 \le [\pi(n - K_1^* + 1) - \pi(n)]/(K_1^* - 1)$$

holds. If the second owner raises his asking price B_2^2 above B_2^1 then his firm will not be bought and consequently the number of operated firms in Stage 3 of the game will increase by one to $n - K_1^* + 2$. The second owner will then make $\pi(n - K_1^* + 2)$. Thus the above cannot be an equilibrium if owner 2 expects to make more by increasing his asking price. In view of (26), a sufficient condition for such a deviation to be profitable is:

$$(27) \qquad [\pi(n - K_1^* + 1) - \pi(n)]/(K_1^* - 1) < \pi(n - K_1^* + 2)$$

Thus, if (27) holds, then a SPNE in G_C in which one owner possesses K_1^* firms is impossible.

Theorem 5: For sufficiently large n, the inequality (27) holds for all $(n + 1)/2 \le K \le n$.

As an immediate consequence of Theorems 2, 3 and 5, we have

Corollary 4: For sufficiently large n, there is only one SPNE to the centralized game G_C. In this SPNE the original oligopoly is retained.

It can be demonstrated that if the demand function is linear, then whenever there are three or more owners in the industry the only SPNE for G_C is the oligopoly equilibrium. This again conforms with the example presented in Section II.

A comparison of Theorem 4 with Corollary 4, as well as the examples of Section II, reveals the surprising result that the more centralized mechanism of G_C leads to outcomes which would be socially preferable over those obtained by the decentralized mechanisms of G_D. That is, G_D allows some restricted partial monopolization of the industry in cases where G_C would allow none. (See also the discussion following Corollary 3.) Thus, an owner seeking to restrict competition in an industry would prefer to play the decentralized game G_D to the centralized game G_C.

VII. Acquisition by an Outsider

So far we have discussed the possibility of acquisition only by incumbent owners. Since acquisition by an outsider is also possible, it is interesting to examine the effects of this possibility on the equilibrium outcomes of the games G_D and G_C. Hence we consider extended versions of the two games in which, in addition to the n incumbent owners, there is another potential purchaser, player zero, who is not currently in the industry. This change only modifies Stage 1 of the two games. Namely, in Stage 1, player 0 announces, simultaneously with the incumbents 1,...,n, a vector $B^0 \subseteq R^n$ of bids, where B_i^0 is his bid on the firm owned by the i-th player. Denote by $\bar{G}_D$ and $\bar{G}_C$ the resulting decentralized and centralized games, respectively.

It is fairly easy to see that Theorems 1-3 will continue to hold for the two extended games. Namely, in a merged SPNE there would be only one owner possessing over $(n + 1)/2$ of the industry's firms while operating fewer, and futhermore, for every n there is an unmerged SPNE to both games. We now turn to establish that our impossibility of merged equilibria results of Section VI hold for the extended games as well.

Consider first the extended decentralized game $\bar{G}_D$. If there is a merged SPNE in which the outsider owns K^* firms, then he will pay their K^* sellers altogether at most $T(r(K^*), n - K^*)$. Hence, there is at least one seller, say the first owner, for whom

$$B_1^0 \leq T(r(K^*), n - K^*)/K^*$$

holds. If, however, the first owner were to raise his asking price above B_1^0, he will not be bought and hence make $T(1, r(K^* - 1) + n - K^*)$. Thus, he will find such a deviation profitable if

$$(28) \qquad T(r(K^*), n - K^*)/K^* < T(1, r(K^* - 1) + n - K^*)$$

holds. Inequality (28) complements (24) in establishing the following impossibility of merged equilibria theorem for $\bar{G}_D$. The proof of the next theorem is almost identical to that of Theorem 4.

<u>Theorem 6</u>: For any given $d > 0$ a merged SPNE to $\bar{G}_D$ in which $K^* = n - d$ is impossible for sufficiently large n.

As a result, a monopoly equilibrium $(d = 0)$ in $\bar{G}_D$ is impossible for sufficiently large n. Moreover, if demand is linear then it can be shown that such an equilibrium is impossible for $n \geq 5$.

As for the extended centralized game, it is easy to see that the complement inequality to (27), implying that at least one of the K^* sellers will have the incentive to raise his asking price if his firm is bought by an outsider, is

$$(29) \qquad \pi(n - K^* + 1)/K^* < \pi(n - K^* + 2).$$

The method of proof of Theorem 5 can be applied to establish (29) for sufficiently large n and for all $(n + 1)/2 \leq K^* \leq n$. Hence, for sufficiently large n, a merged equilibrium in $\bar{G}_C$ cannot be sustained regardless of whether the buyer is an incumbent or an outsider.

Theorem 7: For sufficiently large n, the only SPNE to the centralized game $\bar{G}_C$ is an oligopoly equilibrium.

Again, if demand is linear, then it can be shown that the above result holds for $n \geq 3$.

VIII. Summary

We have shown that monopolization of an industry through acquisition by one owner of his rivals is limited. Indeed, complete monopolization of the industry is possible only if it is initially small. For industries with a large number of firms, only unmerged equilibria or restricted partial monopolization equilibria are possible. Moreover, since any partial monopolization equilibrium is characterized by one owner possessing over one-half of the industry's firms, a prohibition of this possibility would suffice to eliminate even this reduction in competition.

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Appendix

Proof of Lemma 1: First we show that the profit function faced by each producer is strictly concave. To that end all we have to show is that the revenue function faced by this producer is strictly concave. Let

$$q_i^C = \Sigma_{\substack{j\in m \\ j\neq i}} q_j .$$

Hence i's revenue function is $g(q_i) = q_i P(q_i + q_i^C)$. Now, suppressing the index i,

$$g''(q) = 2P'(q + q^C) + qP''(q + q^C).$$

By Assumption I, $g''(0) < 0$ and $g''(q) < 0 \ \forall\ q > 0$ iff $(q + q^C)g''(q)/q < 0$, $\forall\ q > 0$. That is, iff

$$2(q + q^C)P'(q + q^C)/q + (q + q^C)P''(q + q^C) < 0,$$

or, equivalently,

$$[2(q + q^C)/q - 2]P'(q + q^C) + 2P'(q + q^C) + (q + q^C)P''(q + q^C)$$
$$= (2q^C/q)P'(q + q^C) + \partial^2[QP(Q)]/\partial Q^2|_{Q=q+q^C} < 0.$$

The last inequality indeed holds by Assumptions I and III.

Next we show that for every $1 \le m \le n$, (5) has a unique positive solution Q, and hence $\bar{q} = Q/m$ must hold in a stage 3 equilibrium to G if an equilibrium exists. Let

$$f(Q) = m(P(Q) - C) + QP'(Q).$$

Then, by Assumptions I and II, for every $1 \leq m \leq n$, $f(0) > 0$ and since $P' < 0$ by Assumption I and $P(\hat{Q}) < C$ for some $\hat{Q} > 0$, by Assumption II, then $f(\hat{Q}) < 0$. By continuity of f, it follows that a positive solution to (5) must exist. To establish the uniqueness of this solution we show that f is decreasing. Indeed, by Assumptions I and III:

$$f'(Q) = (m + 1)P'(Q) + QP''(Q) = (m - 1)P'(Q) + (QP(Q))'' < 0$$

for all $Q \geq 0$ and $1 \leq m \leq n$. It follows that the only possible stage 3 equilibrium to G is the symmetric one. But if all $j \neq i$ set $q_j = \bar{q}$, then by strict concavity of i-th profit function, setting $q_i = \bar{q}$ is the unique best response possible for i. Hence a stage 3 equilibrium does exist. []

Proof of Corollary 1: From (11),

$$\text{(A.1)} \qquad \Pi'(m) = [m\pi(m)]' = m\pi' + \pi < 0,$$

which implies $\pi' < 0$ since $\pi \geq 0$. []

Proof of Lemma 2: Since $P(Q(m)) > C$ must hold $\forall\, m \geq 1$ (see the paragraph following (3)) then $\pi(m)$ given by (6) is positive for $m \geq 1$, so an owner can always make positive income by selecting $r_j \geq 1$ as compared to zero income when $r_j = 0$. [ ]

Proof of Lemma 3: For any choice of $1 \leq r_j \leq n$ this owner will realize the total industry profits (since $t_j = n - K_j = 0$ holds). By Proposition 2, he will maximize his profits by letting $r_j^* = 1$. [ ]

Proof of Proposition 4: From (20) it follows immediately that $T' \leq 0$ implies that $r > m/2$. But m is the total number of active firms and there cannot be more than one owner who operates more than one-half of them. [ ]

Proof of Theorem 1: Proposition 4 implies that only one owner, say j, will decide on $r_j < K_j$ and hence all other firms will be fully operated. It follows that $t_j = n - K_j$. By Proposition 3, $r_j > t_j$ must hold or altogether $K_j > r_j > n - K_j$ or $K_j > n - K_j$ and since K_j is an integer

$$K_j - 1 \geq n - K_j$$

must hold, implying $K_j \geq (n + 1)/2$. [ ]

Proof of Theorem 4: A sufficient condition for such an equilibrium not to exist is that (24) will hold or, substituting $K_1^* = n - d$

$$\frac{T(r(n - d), d) - T(1, n - 1)}{n - d - 1} < T(1, d + r(n - d - 1)) \tag{A.2}$$

Lemma A.1: There exists a $\alpha > 0$ such that $P'(Q(m)) \leq -\alpha$ for all $m \geq 1$.

Proof of Lemma A.1: Note that for every m, equation (5) that is

$$(A.3) \qquad m[P(Q(m)) - C] + Q(m)P'(Q(m)) = 0.$$

should hold. Suppose to the contrary that $\lim_{m\to\infty} P'(Q(m)) = 0$ for a subsequence of the natural numbers. Since $P'(Q) < 0 \; \forall \; Q \geq 0$ and P' is continuous this can only happen if $Q(m)$ is unbounded from above for the same subsequence. However, this will imply by Assumption II that $P(Q(m)) < C$ holds for sufficiently large m and since $P' < 0$ (A.3) cannot hold for these values of m. []

Let

$$(A.4) \qquad g(d,n) = \max\{r: 1 \leq r \leq n - d - 1, \; T'(r, d + 1) \geq 0\},$$

where T' is given by (20). In view of (20), Assumptions I, III, and (A.4), the numerator of $T'(g(d,n), d + 1)$ has to be nonpositive, i.e.,

$$(A.5) \qquad (d + 1)(QP)'' + [g(d,n) + d][d + 1 - g(d,n)]P' \leq 0,$$

where P' and $(QP)''$ are evaluated at $Q(m)$, $Q(m)$ solves (5) and $m = g(d,n) + d + 1$. If $g(d,n)$ is unbounded from above as $n \to \infty$, then for sufficiently large n the coefficient of P' in (A.5) is negative. In view of Assumption III and Lemma A.1, for these values of n the inequality,

$$\text{(A.6)} \qquad -(d + 1)\beta + [g(d,n) + d][g(d,n) - d - 1]\alpha \le 0$$

follows (A.5). But then, for sufficiently large n as g(d,n) increases, the left side of (A.6) becomes positive. It follows that there exists an $\tilde{r}(d + 1)$ and that

$$g(d,n) \le \tilde{r}(d + 1)$$

holds for all n. Thus, the maximizer $r(n - d - 1)$ of $T(r, d + 1)$ must satisfy

$$\text{(A.7)} \qquad r(n - d - 1) \le \bar{r}(d + 1) = \tilde{r}(d + 1) + 1$$

for all n.

Returning now to establishing (A.2), we observe that by Corollary 1 and (A.7), the right side of (A.2), $T(1, d + r(n - d - 1))$ is bounded from below by the positive number $T(1, d + \bar{r}(d + 1))$. As for the left side of (A.2), the numerator is bounded from above by $\max_{1 \le r \le \bar{r}(d)} T(r,d)$ while the denominator is unbounded in n. It follows that (A.2) holds for sufficiently large n. []

Proof of Theorem 5

Lemma A.2: There exists a positive number ρ and a positive integer $\bar{n}$ such that

$$\pi(n)/\pi(n + 1) < \rho$$

for all $n \geq \bar{n}$.

<u>Proof of Lemma A.2</u>: By (6),

$$\frac{\pi(n)}{\pi(n + 1)} = \frac{n + 1}{n} \frac{Q(n)[P(Q(n)) - C]}{Q(n + 1)[P(Q(n + 1)) - C]},$$

where $Q(n)$ solves (5). However, by (5) we have

$$\frac{\pi(n)}{\pi(n + 1)} = \left(\frac{n + 1}{n}\right)^2 \left[\frac{Q(n)}{Q(n + 1)}\right]^2 \frac{P'(Q(n))}{P'(Q(n + 1))},$$

and by Proposition 1

$$\leq \left(\frac{n + 1}{n}\right)^2 \frac{P'(Q(n))}{P'(Q(n + 1))}.$$

By Lemma A.1 and Assumption I, $P'(Q(n))/P'(Q(n + 1))$ is bounded from above in n. []

<u>Lemma A.3</u>: Let $d = n - K$. For every $d \geq 0$ there exists a positive integer n_d such that

(A.8) $$[\pi(d + 1) - \pi(n)]/(n - d - 1) \leq \pi(d + 2)$$

holds for all $n \geq n_d$.

Proof of Lemma A.3: Obvious, since the right side of (A.8) is a positive constant while the left side declines to zero as $n \to \infty$. []

We may now complete the proof of Theorem 5. The inequality (27) holds if

$$\text{(A.9)} \qquad \pi(n - K_1^* + 1)/\pi(n - K_1^* + 2) \leq K_1^* - 1$$

holds for sufficiently large n and all $(n + 1)/2 \leq K_1^* \leq n$. By Lemma A.2, there exists an $\rho > 0$ and $\bar{n}$ such that

$$\text{(A.10)} \qquad \pi(n - K_1^* + 1)/\pi(n - K_1^* + 2) < \rho$$

holds for all $n - K_1^* + 1 \geq \bar{n}$. Since $(n - 1)/2 \geq \rho$ will hold for sufficiently large n, say $n \geq n'$, then for $n \geq n'$ and $(n + 1)/2 \leq K_1^* \leq n$, the inequality $K_1^* - 1 \geq \rho$ holds. Thus, using (A.10), it follows that (A.9) (and hence (27)) holds for all $n \geq n'$, $n - K_1^* + 1 \geq \bar{n}$ and $(n + 1)/ \leq K_1^* \leq n$. If $n - K_1^* + 1 < \bar{n}$, that is, $n - K_1^* = d < \bar{n} - 1$, then Lemma A.3 implies (27) for all $n \geq n_d$. Since there is a finite number of values for d for which $0 \leq d < \bar{n} - 1$ hold, it follows that (27) is satisfied for all $n \geq \max\{n', n_0, n_1 \ldots n_{\bar{n}-2}\}$ and $(n + 1)/2 \leq K_1^* \leq n$. []