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COMPETITIVELY COST ADVANTAGEOUS MERGERS AND MONOPOLIZATION[1]

by

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1. Introduction

The common incentive for horizontal mergers is to reduce competition. If in addition a merged firm can realize a production cost advantage over its unmerged rivals, then the incentive for merger is reinforced. Moreover, if relative production costs can be continually reduced through additional acquisitions, then complete monopolization of an industry would appear to be the ultimate outcome.

We address two major questions here. The first is whether or not the potential for lower production costs does lead to complete or partial monopolization of an industry through acquisition by one firm of all its rivals? The second is whether or not competitively advantageous cost reducing mergers necessarily result in a lower equilibrium market price? These questions are addressed in terms of a two stage noncooperative game. We posit an n-firm industry into which entry is difficult. Its firms produce a homogeneous good with identical convex cost functions and compete by choosing output levels in accordance with the standard Cournot assumptions. In the game's first stage each owner of a single firm sets a price at which he would be prepared to purchase each of his n - 1 rivals and a price at which he is prepared to sell his own firm. A firm is sold to its highest bidder, provided the highest bid is at least as high as its asking price. In the second stage, each owner of one or several firms determines how much to produce in each of them so as to maximize his total operating profits. The net payoff to each owner equals the sum of his receipts or payments from the game's first stage plus his second stage operating profits.

The opportunity to lower production costs through the acquisition of another firm obtains in our model through the supposition that each firm operates with a convex cost function. Thus, if an owner of a single firm acquires another, he can produce that same output at a cost no higher than if he possessed only the one firm. Moreover, if the cost functions, assumed to be identical, are strictly convex, then a two firm owner's cost of producing any given quantity is strictly below the owner's of a single firm, because he can split production between two facilities. An owner of three firms with identical strictly convex cost functions can reduce the cost of producing any fixed quantity further still relative to an owner of fewer firms, and so on for an owner of more firms. However, it should be kept in mind that a merged firm with two facilities cannot produce a given quantity at a lower cost than two individual firms producing the same quantity divided equally between them. If the industry were completely monopolized, its production costs would be identical to that of the original oligopolistic industry. Thus, merger in our setting does not improve an industry's overall production efficiency. It only affects the relative productive efficiency of some owners as compared with the others.

Despite the double incentive for merger, reduction of cost, and number of competitors, our analysis discloses that complete (or partial) monopolization of an industry with sufficiently many firms will not occur. This is essentially because nonsellers profits rise as competition declines when an additional firm is purchased. Therefore, each owner has an incentive to raise the asking price for his firm to the profit level he can enjoy from reduced competition. If the initial number of firms in the industry is large this pecuniary externality effect makes the purchase of all of them unprofitable for any single buyer. It is unprofitable in the sense that the buyer's net profit would fall below the level he could realize at the Cournot equilibrium in the original oligopolistic market. However, mergers short of complete monopolization are possible. These mergers do reduce the number of owners in the industry but not the number of active firms because an owner of several firms has to operate all of them to realize the cost savings from splitting production. Such an owner will, of course, operate his firms cooperatively rather than competitively. That is, he will determine how much to produce in each of his production facilities, hence his optimal total output given the outputs of his rivals, according to the usual Cournot oligopoly model. Our analysis indicates that, despite the competitively advantageous cost reduction gains from merger, the market price of the product cannot decline and consumers are no better off than with the original market structure. This is essentially because there is no overall industry-wide reduction in the cost of production. Thus, to the extent that the antitrust authorities relax their generally negative posture toward horizontal mergers if they appear to lead to cost reductions, they must carefully distinguish between competitively advantageous cost reductions and overall industry-wide ones.

In section 2 our model in the form of a two stage noncooperative game is described. We employ the subgame perfect Nash equilibrium (SPNE) in pure strategies as our solution concept. Our analysis proceeds backwards from the game's second stage to its first. In section 3 we discuss some possible and impossible SPNE's of the game. Section 4 contains the analysis of the effect of acquisition and mergers on prices and quantities. Proofs of propositions, corollaries and lemmas are provided in the Appendix.

The effects and desirability of horizontal mergers were addressed by Salant, Switzer and Reynolds (1983) in the context of a Cournot oligopoly with a linear demand function for a homogeneous product, and identical linear cost functions. They concluded that any coalition of firms, behaving as a merged firm, consisting of fewer than 80 percent of the industry's members is disadvantageous. However, they do not discuss how the merger is achieved. In Kamien and Zang (1988) we modeled the merger process as two possible acquisition games, one of which is similar to the one employed here. However, in that analysis, reduction in the number of active competitors is the only incentive for merger. Broadly speaking, our analysis disclosed that complete or substantial monopolization of the industry with a sufficiently large number of firms will not occur. However, some partial monopolization is, in general, possible.

2. The Model

The two stage acquisition game we employ here closely resembles the centralized game G_c of Kamien and Zang (1988), with the exception that total production cost is assumed to be convex rather than only linear--i.e., nonlinear convex and strictly convex cost functions are also allowed. In particular, we posit an industry consisting of n identical firms producing a single good with identical cost functions $C(q)$, whose total quantity supplied is denoted by Q, facing an inverse demand function $P(Q)$. We assume that the following properties hold:

(i) $P(Q)$ is twice continuously differentiable, $P(0)$ and $P'(0)$ are finite, and $P'(Q) < 0$ for all $Q \geq 0$.

(ii) $P(0) > C'(0)$ and for some $\hat{Q} > 0$, $P(\hat{Q}) < C'(0)$.

(iii) The industry total revenue function $QP(Q)$ possesses a negative second derivative, i.e., $(QP(Q))'' < 0$ for all $Q \geq 0$. Note that this assumption implies strict concavity of the industry total revenue function.

(iv) The cost function $C(q)$ is increasing and twice continuously differentiable, has no fixed cost, $C(0) = 0$, and is convex, $C''(q) \geq 0$ for all $q \geq 0$ denote the set of initial owners.

Each firm is initially owned and controlled by a single owner. The owners are the players of the game in which each can purchase other firms or sell his. Naturally, if a firm is sold it becomes controlled by its buyer. Let $N = \{1,2,\ldots,n\}$.

The Acquisition Game

Stage 1: Each owner $i \in N$ simultaneously announces a vector $B^i = (B^i_1, B^i_2, \ldots, B^i_n) \in R^n$ of bids for the entirety of each firm. The bid B^i_i is the i-th owner's bid or asking price for his own firm. Let $B = (B^1, B^2, \ldots, B^n)$ denote the $n \times n$ matrix of bids.

Following the announcement of B, each firm may be sold or remain with its original owner. We now describe a general rule that assigns firms to owners. Let

$$S(j) = \{i \in N: i \neq j,\ B^i_j \geq B^k_j\ \forall\ k \neq i\}$$

be the set of owners, other than j, whose bid for j satisfies two properties: (1) it is not smaller than j'th asking price; and (2) it is the highest bid for j. It is natural to expect that the firm owned by j may be sold only to a member of S(j) or remain with its original owner, j. If S(j) is empty or a singleton, then the assignment is obvious. However, we need to specify the assignment if S(j) contains more than one element. To that end we employ a general tie-breaking rule, namely, a function $f: N \times 2^N \to N$ satisfying

$$f(j,S) \in S \cup \{j\} \ \forall\ j \in N,\ S \subseteq N.$$

Thus, $f(j,S(j))$ uniquely determines the assignment of firms to owners for any given matrix B of bids.

Some possible tie breaking rules conforming with the above framework are:

1. Global priorities. Priorities are assigned to owners, say, by their numbering, and $f(j,S(j)) = \min \{i: i \in S(j)\}$ if $S(j) \neq 0$.
2. Individual priorities. Owner j sells his firm to one of the highest bidders according to some known priorities unique to him.
3. No deal. In the presence of a tie, firm j is not sold, i.e., $f(j,S(j)) = j$ if $|S(j)| \geq 2$.

An important property of our general assignment rule is that once the i-th owner's asking price and bids he received are known, the assignment of firm i is independent of the asking prices and bids for every other firm. We assume that all the game's participants know the assignment rule.

Applying the assignment rule, the ownership of firm j, indexed by ε_{ji}, is determined by

$$\varepsilon_{ji} = \begin{cases} 1, & \text{if } f(j,S(j)) = i, \\ 0, & \text{otherwise.} \end{cases}$$

Let ε be the $n \times n$ matrix whose (j,i) entry is ε_{ji}. Note that

$\varepsilon = \varepsilon(B)$. For $i \in N$ let

$$K_i = \{j \in N: \varepsilon_{ji} = 1\}$$
$$k_i = |K_i|$$

denote the subset and number of firms, respectively, owned (and controlled) by i, and by the vector $K = (K_1, K_2, \ldots, K_n) = K(B)$, the assignment of firms to owners. Also let

$$M = \{i \in N: K_i \neq \emptyset\}$$

denote the set of active owners, that is, those who own at least one firm following the game's first stage. Finally, let $m = |M|$ denote the number of active owners.

Stage 2: Given $K(B)$, each owner $i \in M$ independently and simultaneously chooses the production level q_j of each of his k_i firms, that is, for $j \in K_i$. We denote by

$$q = (q_1, q_2, \ldots, q_n),$$

the vector of quantities produced by all firms in N as a result of this decision and let $Q = \sum_{i \in N} q_i$. Note that while the number of owners may decline after stage 1, the number of firms does not. Each player's payoff is the sum of the stage 2 operating profits of all the firms he controls plus the net trade cash flow from stage 1.

We are concerned with characterizing properties of subgame perfect Nash equilibria in pure strategies of this game.

Definition: A SPNE in an acquisition game is said to be *merged* if the number m of active owners is fewer than the initial number, n. If $m = n$, in a SPNE, then it is *unmerged*. If $m = 1$ in a SPNE then we have a *monopoly equilibrium*.

In the sequel we characterize possible SPNE's of the acquisition game. In particular, we consider existence of unmerged SPNE and identify instances in which a merged equilibrium cannot exist.

Analysis of Stage 2

Without loss of generality, suppose $k_1 \geq k_2 \geq \ldots \geq k_m$, that is, $k_{m+1} = k_{m+2} = k_n = 0$ holds. Owner $i \in M$ selects q_j, $j \in K_i$ so as to maximize the function $\pi_i : E^m \to E$ given by

$$(1) \qquad \pi_i(q) = \Sigma_{\ell \in K_i} [q_\ell P(Q) - C(q_\ell)].$$

The next lemma is needed to establish existence of stage 2 equilibria:

Lemma 1: π_i is concave in q_j, $j \in K_i$.

The first order necessary conditions for a stage 2 equilibrium are for every $i \in M$ and $j \in K_i$

$$(2a) \qquad P(Q) + \Sigma_{\ell \in K_i} q_\ell P'(Q) - C'(q_j) = 0 \text{ if } q_j > 0,$$

(2b) $\qquad P(Q) + \sum_{\ell \in K_i} q_\ell P'(Q) - C'(q_j) \le 0$ if $q_j = 0$.

From (2a) (2b) and the convexity of C (i.e., C' is nondecreasing), it trivially follows that if $q_j > 0$ for some $j \in K_i$ holds then $q_\ell = q_j$ for all $\ell \in K_i$. If alternatively $q_j = 0$ for all $j \in K_i$, then (2b) implies

(3) $\qquad P(Q) - C'(0) \le 0$.

Consider now another owner, $\overset{t}{\ell} \in m$, $\overset{t}{\ell} \neq i$ for whom $q_j > 0$ for all $j \in K_{\underset{t}{\ell}}$. Since by convexity of C, $C'(q_j) \ge C'(0)$, then (3) implies

$$P(Q) - C'(q_j) \le 0, \ \forall\, j \in K_{\underset{t}{\ell}}.$$

and hence (2b) holds for $\overset{t}{\ell} \in M$ as well. It follows that $q_j = 0$ for some $j \in N$ implies $q_j = 0$ for all $j \in N$. But then (2b) implies $P(0) \le C'(0)$, contradicting Assumption ii. Hence in a stage 2 equilibrium $q_j > 0 \ \forall\, j \in N$. Also note that (2a) and (1) now imply

(4) $\qquad \pi_i = -(\sum_{\ell \in K_i} q_\ell)^2 P'(Q) + \sum_{\ell \in K_i} [q_\ell C'(q_\ell) - C(q_\ell)]$.

Convexity of C(q) implies

(5) $\qquad 0 = C(0) \ge C(q_\ell) - q_\ell C'(q_\ell)$.

Substituting (5) in (4)

$$\pi_i \geq -(\Sigma_{\ell \in K_i} q_\ell)^2 P'(Q) > 0$$

follows. We summarize the above discussion by:

Proposition 1: In a stage 2 equilibrium of the acquisition game, all firms will be operated. Every owner possessing at least one firm will produce the same quantity in each and realize positive profits.

Note that no owner will operate his firm in the range where $P(Q) < C'(0)$ holds, since the convexity assumption on $C(q)$ will imply negative profits through (1). Assumption iv then implies that the feasible set of production decisions for owner i is given by

$$q_\ell \geq 0, \ \forall \ \ell \in K_i,$$
$$\Sigma_{\ell \in K_i} q_i \leq \hat{Q},$$

where $\hat{Q}$ was given in Assumption (ii). That is, owner i maximizes a continuous convex function over a compact convex set. Kakutani's fixed point theorem then implies:

Proposition 2: For every possible ownership configuration, a stage 2 Cournot equilibrium of the acquisition game exists.

3. Analysis of SPNE's in Pure Strategies

We now turn to the question of existence of SPNE in pure strategies to the acquisition game and to their character. Our analysis discloses that a SPNE in pure strategies in which the original industry structure is retained is possible while one in which the industry is monopolized is impossible for sufficiently large n. As in Kamien and Zang (1988), it is easy to show that if all the owners' bids for other firms are sufficiently low while the asking prices for their own firms are sufficiently high, then no transactions will occur in the game's first stage and no profitable deviations are possible. Hence,

Theorem 1: An unmerged SPNE to the acquisition game exists.

Consider now the possibility of a merged SPNE to the game in which one owner, say the first, possesses k_1 firms, where $k_1 = n - d$ for some fixed $d \geq 0$. Let π_1 denote the first owner's stage 2 profits in this case and let $\hat{\pi}_1$ be his stage 2 profits if he lowered his bids to the $k_1 - 1$ sellers and became a nonbuyer. Certainly, the first owner is unwilling to pay the $k_1 - 1$ sellers more than $\pi_1 - \hat{\pi}_1$, as $\hat{\pi}_1$ is his opportunity cost. Consider next one of the $k_1 - 1$ sellers, say the second owner. If he deviates and raises his asking price above the first owner's bid he will not be bought, and then realize a profit of $\hat{\pi}_2$. It follows that the first owner has to pay each seller at least $\hat{\pi}_2$ or at least $(n - d - 1)\hat{\pi}_2$ to all of them. Consequently, such an equilibrium would be impossible if

$$\pi_1 - \hat{\pi}_1 < (n - d - 1)\hat{\pi}_2,$$

or, since $\hat{\pi}_1 \geq 0$, if

$$\pi_1/(n - d - 1) \leq \hat{\pi}_2 \tag{6}$$

holds. We establish in Theorem 2 that, for every given $d \geq 0$, (6) holds for sufficiently large n.

By Proposition 1, each owner $i \in M$ produces the same quantity q^i in each of his k_i firms. Equation (2a) then implies

$$(7) \qquad P(Q) + k_i q^i P'(Q) - C'(q^i) = 0, \ \forall\, i \in M.$$

Since the industry's total production, Q, is

$$Q = \Sigma_{i \in M}\, k_i q^i,$$

(7) implies

$$(8) \qquad mP(Q) + QP'(Q) = \Sigma_{i \in M}\, C'(q^i).$$

The Mean Value Theorem implies

$$(9) \qquad C'(q^i) = C'(0) + q^i C''(\bar{q}^i), \ \forall\, i \in M,$$

where $\bar{q}^i \in [0,q^i]\ \forall\ i \in M$. Substituting (9) in (8), we obtain

$$(10) \qquad m[P(Q) - C'(0)] + QP'(Q) = \Sigma_{i \in M}\, q^i C''(\bar{q}^i).$$

Note that the right side of (10) is nonnegative by Assumption iv. Consider now the solution $Q(\alpha,m)$ to the equation

(11) $\quad m[P(Q) - C'(0)] + QP'(Q) = \alpha.$

Note that industry total output determined by (10) is $Q(\sum_{i\in M} q^i C''(\bar{q}^i);m)$ and that $Q(0;m)$ is total industry output if production cost were linear, with constant marginal cost $C'(0)$ and m owners in the game's second stage.

Lemma 2: For every m and every $0 \le \alpha \le m[P(0) - C'(0)]$, (11) has a unique solution $Q(\alpha;m)$ and $\partial Q(\alpha;m)/\partial\alpha < 0$ holds.

Corollary 1: In a merged SPNE to the game, with m owners, total production Q, satisfying (10), does not exceed the quantity $Q(0;m)$, the solution to (11) for $\alpha = 0$, that would be produced under the same circumstances but with a linear total cost function with marginal cost ~~does~~ not exceeding $C'(0)$.

Corollary 1 implies that for the identical ownership configuration, convexity of the cost function will generally lead to lower quantities and higher prices in the SPNE than those that would obtain with a linear cost function for which marginal cost is at or below $C'(0)$. However, this result does not necessarily follow if the ownership configurations of an industry with linear costs differ at their respective merged equilibria.

We now proceed to establish the impossibility of complete monopolization of an industry with a sufficiently large number of firms. This is done by showing that the inequality (6) must hold for sufficiently large n. That is, that no single buyer will find it profitable to buy out all his rivals if they are sufficiently numerous.

Theorem 2: For a given $d \geq 0$, and sufficiently large n there is no SPNE in the acquisition game in which one owner possesses $n - d$ or more of the industry's firms.

We may contrast the result in Theorem 2 with a similar result obtained in Kamien and Zang (1988) under the supposition that the total cost function is linear. The result demonstrated here is weaker in the sense that it allows for merged equilibria for industries with a larger number of firms. In other words, the assumption that the total cost function is convex enables merged SPNE to exist for industries in which the same number of firms would preclude the existence of a SPNE if the total cost function were linear. Thus, it is here that the competitively cost advantageous incentive for merger has an impact on the SPNE of the game.

For the special case where $d = 0$, we have:

Corollary 2: For sufficiently large n, a monopoly SPNE to the acquisition game is impossible.

4. Effects of Acquisition and Merger on Prices and Quantities

We turn next to the production characteristics of individual owners in a SPNE of the acquisition game and to the comparison between the total output of a merged industry and an unmerged one. It turns out that an owner of more firms will in total produce no less than an owner of fewer firms, but the former will produce less in each of his firms than the latter. Thus, for example, an owner of two firms in a three firm industry will produce less in each of them than the single firm owner will, but not less in the two combined. Production in each of his two firms will, therefore, exceed one-half of the single firm owner's production. This is the essence of Proposition 3. However, under this ownership structure, the three firm's total production will not exceed the total production of the three individual firms in the Cournot equilibrium of the original unmerged industry. Thus, despite the competitive cost advantages of mergers total industry output will decline relative to its pre-merger level. This is the essence of Proposition 4 and Corollary 3.

Proposition 3: In a SPNE of the acquisition game in which $k_i > k_j$ for some $i, j \in m$, the overall quantity $k_i q^i$ produced by the i-th owner in all his k_i firms is not below the quantity $k_j q^j$ produced by the j-th owner in all his firms. Furthermore, the quantity q^i produced by i in each of his firms is smaller than the quantity, q^j, produced by j in each of his firms. If $k_i = k_j$, then $k_i q^i = k_j q^j$ and $q^i = q^j$ holds.

Proposition 4: The overall quantity produced at the Cournot equilibrium with m active owners in the game's second stage does not exceed the quantity that would have been produced at the Cournot equilibrium of the original n firm oligopoly.

Corollary 3: In a merged SPNE of the acquisition game, the overall quantity produced does not exceed and the price is not below the corresponding quantity and price that obtains at the Cournot equilibrium of the original unmerged industry.

Note that although Corollary 3 only implies that acquisition cannot result, in a lower product price in equilibrium if the game under consideration is the one discussed above, Proposition 4 is even stronger. Actually, it implies that prices will not decline for any mechanism in which the final possible result is that total ownership and control of firms is allocated to other owners.

5. Summary

We have investigated the merger propensities, through acquisition, of an industry in which merger leads to both reduction of competition and competitively advantageous cost reduction. Despite these twin incentives for merger, complete monopolization does not occur if the number of firms in the industry is sufficiently large. This provides a mixed message for the behavior of the antitrust authorities towards mergers. While they need not fear complete monopolization of an industry with sufficiently numerous firms they must be sure that this is in fact the case. Moreover, even partial monopolization of the industry, in the form of reduction of the number of firm owners, tends to lead to a rise in the product price in equilibrium.

References

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Appendix

Proofs of the Results

Proof of Lemma 1: It was shown in Kamien and Zang (1988, Lemma 1) that $h(q) = qP(q + q^C)$ is strictly concave in q for a given q^C. If we consider now $H(q_j: j \in K_i) = h(\sum_{j\in K_i} q_j)$ it can be easily shown that all the second order partial derivatives of H are $h''(\sum_{j\in K_i} q_j)$ and are negative. Thus the Hessian of H is a matrix of rank 1 with one negative eigenvalue. Hence it is negative semidefinite and H is concave. Since C is convex, π_i is concave. ||

Proof of Lemma 2: The existence and uniqueness of $Q(\alpha;m)$ follows the proof of Lemma 1 in Kamien and Zang (1988). Differentiating (11) with respect to α we obtain

$$[(m + 1)P' + QP''](\partial Q(\alpha;m)/\partial\alpha) = 1,$$

or

$$\frac{\partial Q(\alpha;m)}{\partial\alpha} = \frac{1}{(m + 1)P' + QP''} = \frac{1}{(m - 1)P' + (QP)''} < 0$$

by Assumptions (i) and (iii). ||

Proof of Corollary 1: Immediate from Lemma 2 and observing that the right side of (10) is nonnegative. ||

Proposition A.1: There exists a $\delta_1 > 0$ such that

$$\pi_1 < \delta_1$$

holds for all n.

Proof:

$$\text{(A.1)} \qquad \pi_1 = k_1[q^1 P(Q) - C(q^1)] \le k_1 q^1 P(Q) \le QP(Q).$$

Now Corollary 1 implies $Q \le Q(0;m)$. Hence, the left side of $QP(Q)$ is bounded from above by

$$\delta_1(m) = \max\{QP(Q)\colon 0 \le Q \le Q(0;m)\}.$$

Substituting $\alpha = 0$ in (11), we note that the solution $Q(0;m)$ to (11) must be bounded from above in m since otherwise, by Assumption (ii), the left side of (11) becomes negative as m increases. Hence $Q(0;m) \le \bar{Q}$ $\forall$ m and thus, $QP(Q)$ is bounded from above by

$$\text{(A.2)} \qquad \delta_1 = \max\{QP(Q)\colon 0 \le Q \le \bar{Q}\}.$$

Hence, the proposition follows (A.1) and (A.2). ||

Proposition A.2: For a given $d \ge 0$, there exists a $\delta_2(d) > 0$ such that

$$\hat{\pi}_2 > \delta_2(d)$$

holds for all n.

Proof: We need to consider two cases:

Case 1: The deviating player (the second) possesses no firm if he sells his. In this case, upon raising his asking price above the first owner's bid, he will own one firm and there will be m + 1 active owners in the industry. Equation (2a), written for the second owner will be

$$\text{(A.3)} \qquad P(\hat{Q}) + \hat{q}^2 P'(\hat{Q}) - C'(\hat{q}^2) = 0,$$

where $\hat{q}^2$ and $\hat{Q}$ are the quantities produced by the second owner and the whole industry, respectively, following his deviation. We also have

$$\text{(A.4)} \qquad \hat{\pi}_2 = \hat{q}^2 P(\hat{Q}) - C(\hat{q}^2).$$

From (A.3) we obtain an expression for $P(\hat{Q})$ which, when substituted into (A.3) yields

$$\text{(A.5)} \qquad \hat{\pi}_2 = -(\hat{q}^2)^2 P'(\hat{Q}) + \hat{q}^2 C'(\hat{q}^2) - C(\hat{q}^2) \geq -(\hat{q}^2)^2 P'(\hat{Q}),$$

where the inequality in (A.5) follows (5), namely, the convexity of C. Consider now Q(0;m + 1) that solves (11), that is

(A.6) $$(m + 1)[P(Q) - C'(0)] + QP'(Q) = \alpha$$

when $\alpha = 0$, and note that $Q(0;m + 1) > 0$ since otherwise $P(0) = C'(0)$ holds, contradicting Assumption (ii). Hence,

(A.7) $$P(Q(0;m + 1)) = C'(0) - Q(0;m + 1)P'(Q(0;m + 1))/(m + 1).$$

Observe now that, since d is a constant, m can range from 1 to d + 1, and let

(A.8) $$\gamma_1 = \min_{m+1 =2,\ldots,d+2} - [Q(0;m + 1)P'(Q(0;m + 1))]/(m + 1).$$

Note that $\gamma_1 > 0$ must hold since $Q(0;m + 1) > 0$. Combining (A.7) and (A.8) we obtain

(A.9) $$P(Q(0;m + 1) \geq C'(0) + \gamma_1.$$

Note that by the way we obtained (10), it can be established that $\hat{Q}$ satisfying (A.3) is a solution to (A.6) for a positive value of α, namely, $\hat{Q} = Q(\alpha;m + 1)$ for some $\alpha > 0$. This implies, in view of Corollary 1, that $\hat{Q} < Q(0;m + 1)$ and since price is decreasing in quantity, (A.9) implies

(A.10) $$P(\hat{Q}) > C'(0) + \gamma_1.$$

Moreover, the above discussion implies that

(A.11) $$0 < \tilde{Q} \leq \bar{Q}$$

regardless of m and n, where

$$\bar{Q} = \max_{m=1,\ldots,d-1} \{Q(0:m+1)\}.$$

Returning now to (A.3), we note that $P'(\tilde{Q})$ is, by (A.11), bounded and hence if $\tilde{q}^2 \to 0$ as n goes to infinity, $P(\tilde{Q}) - C'(\tilde{q}^2) \to 0$, contradicting (A.10). It follows that there exists a $\gamma_2 > 0$ such that

(A.12) $$\tilde{q}^2 > \gamma_2$$

holds for all n. A similar argument holds for the product $-\tilde{q}^2 P'(\tilde{Q})$, for if it approaches 0 as n goes to infinity, then $P(\tilde{Q}) - C'(\tilde{q}^2) \to 0$ contradicting (A.10). Hence, for some $\gamma_3 > 0$ and all n

(A.13) $$-\tilde{q}^2 P'(\tilde{Q}) > \gamma_3$$

holds for all n. Substituting (A.12) and (A.13) into (A.5), we obtain that there exists a $\delta_2 > 0$, $\delta_2 = \gamma_2\gamma_3$ such that

(A.14) $$\tilde{\pi}_2 > \delta_2$$

holds for all n.

Case 2: The deviating player possesses $d \geq k_2 \geq 1$ firms, through purchases.

upon selling his. It this case, there will be m active owners in the industry and expression (2a) will read

(A.15) $$P(\hat{Q}) + k_2\hat{q}^2 P(\hat{Q}) - C'(\hat{q}^2) = 0,$$

and

(A.16) $$\hat{\pi}_2 = k_2[\hat{q}^2 P(\hat{Q}) - C(\hat{q}^2)].$$

Substituting (A.15) into (A.16), and using the convexity of C', we obtain

$$\hat{\pi}_2 \geq -(k_2\hat{q}^2)^2 P'(\hat{Q}).$$

Following the derivation of (A.6) to (A.13), employing the property that k_2 is bounded from above by d, we obtain that there exists a $\tilde{\delta}_2 > 0$ such that

(A.17) $$\hat{\pi}_2 > \tilde{\delta}_2$$

holds for all n. Combining now (A.14) with (A.17), we have established the proposition with $\delta_2(d) = \min\{\bar{\delta}_2, \tilde{\delta}_2\}$. ||

Proof of Theorem 2: We need to establish (6) for sufficiently large n. Proposition A.1 implies that

(A.18) $$\pi_1/(n - d - 1) < \delta_1/(n - d - 1)$$

for all n. But since d is constant,

$$\delta^1(n - d - 1) < \delta_2(d)$$

holds for sufficiently large n. Proposition A.2 then implies (6).

Proof of Proposition 3: The first order necessary conditions for i and j are, by (7)

$$P(Q) + k_i q^i P'(Q) - C'(q^i) = 0$$
$$P(Q) + k_j q^j P'(Q) - C'(q^j) = 0$$

By subtraction:

$$(k_i q^i - k_j q^j)P'(Q) = C'(q^i) - C'(q^j). \tag{A.19}$$

Note that $P'(Q) < 0$ and $C'(q)$ is nondecreasing by our assumptions. Three cases are now possible:

(i) $k_i q^i > k_j q^j$. In this case the left side of (A.19) is negative and hence $C'(q^i) < C'(q^j)$ holds, implying $q^j > q^i$. Hence

$$k_i q^i > k_j q^j > k_j q^i,$$

which is possible only if $k_i > k_j$.

(ii) $k_i q^i < k_j q^j$. Then the left side of (A.19) is positive and hence $C'(q^i) > C'(q^j)$ implying $q^i > q^j$. Hence

$$k_j q^j > k_i q^i > k_i q^j$$

implying a contradiction since $k_i \geq k_j$ was assumed.

(iii) $k_i q^i = k_j q^j$. Then $q^i \leq q^j$ follows if $k_i = k_j$ and $q^i > q^2$ holds if $k_i > k_j$.

Proof of Proposition 4: The proof is by induction on m, the number of active owners in the resulting merged industry.

$m = 1$: Let Q and $\hat{Q}$ denote the total industry output before and following the merger, respectively. The first order necessary condition, before the merger, is by (2.a)

$$P(Q) + (Q/n)P'(Q) - C'(Q/n) = 0,$$

or

$$n(P(Q) - C'(Q/n)) + QP'(Q) = 0. \tag{A.20}$$

Following the merger, (2.a) gives

$$P(\hat{O}) - C'(\hat{Q}/n) + \hat{Q}P'(\hat{Q}) = 0. \tag{A.21}$$

Consider the equation

(A.22) $\qquad \ell[P(Q) - C'(Q/n)] + QP'(Q) = 0.$

and denote by $Q(\ell)$ the solution to (A.22). Note that $Q(n)$ and $Q(1)$ solve (A.20) and (A.21), respectively. Differentiating (A.20), w.r.t. ℓ we get

$$P(Q) - C'(Q/n) + [\ell P'(Q) - (\ell/n)C''(Q/n) + P'(Q) + QP''(Q)]Q'(\ell) = 0.$$

Hence

$$Q'(\ell) = -[P(Q) - C'(Q/n)] \;/\; [(\ell - 1)P'(Q) + (QP(Q))'' - (\ell/n)C''(Q/n)].$$

Note that $P(Q) \leq C'(Q/n)$ cannot hold since this would imply a contradiction. ($P'(Q) > 0$) in (A.22). Hence from our assumption, $Q'(\ell) > 0$, and since $\hat{Q} = Q(1)$, it follows that $Q > \hat{Q}$ holds.

Lemma A.1: Consider a subset $K \subseteq N$ of the industry's firms and let $Q_K(Q_{N-K})$ describe the reaction function of the subset K in an unmerged industry, namely, the overall quantity produced, in Cournot competition, by the firms in K if those remaining in N - K produce Q_{N-K} altogether. Then $0 > Q_K' = \partial Q_K(Q_{N-K})/\partial Q_{N-K} > -1$ holds.

Proof: From (2.a) the first order necessary conditions for $i \in K$ are

(A.23) $\qquad P(Q) + q_i P'(Q) - C'(q_i) = 0$

where $Q_K = \sum_{i \in K} q_i$ and $Q = Q_K(Q_{N-K})$. Expression (A.23) implies that $q_i = q_j = Q_K/k$ holds if $\forall$ i, j $\in$ K, where $k = |K|$. Hence,

$$(A.23) \qquad k[P(Q_K + Q_{N-K}) - C'(Q_K/k)] + Q_K P'(Q_K + Q_{N-K}) = 0.$$

Differentiating w.r.t. Q_{N-K} we get

$$kP'[Q_K' + 1] - Q_K' C''(Q_K/k) + P'Q_K' + Q_K P''[Q_K' + 1] = 0.$$

where, to simplify presentation, we omitted in (A.24) the arguments of P', Q_K' and P". Rearranging (A.24) we get

$$Q_K'[(k + 1)P' + Q_K P'' - C''(Q_K/k)] + kP' + Q_K P'' = 0.$$

Hence,

$$Q_K' = -[kP' + Q_K P''] / [(k + 1)P' + Q_K P'' - C''(Q_K/k)].$$

Obviously, the lemma will follow if in the numerator of the above expression, $kP' + Q_K P''$ is negative. This is indeed the case if $P'' \le 0$ holds. If $P'' > 0$, then two cases are possible:

Case a ($k \ge 2$): Then

$$kP' + Q_K P'' \le kP' + QP'' = (k - 2)P' + (QP)'' < 0.$$

where the first inequality follows $Q_K \le Q$ and $P'' > 0$, and the last one from our assumptions.

Case b ($k = 1$): Since $|N - K| \ge 1 = k$, we have by Proposition 3 that $Q_{N-K} \ge Q_K$, and since $Q = Q_K + Q_{N-K}$, the inequality $Q_K \le Q/2$ follows. Hence

$$kP' + Q_K P'' = P' + Q_K P'' \le P' + (Q/2)P'' = (QP)''/2 < 0. \qquad ||$$

We now proceed with the proof of the proposition. Suppose the induction holds for the case with m active owners in the game's second stage, and consider the case where m + 1 owners are active. Consider first the possibility that the overall production quantity of each owner's firms exceeds the quantity that these firms would have produced in an n firm oligopoly. Partition the set of owners to $\{1\}$ and $\{2,\ldots,m+1\}$, and let Q_{K_1}, Q_{N-K_1} and $\hat{Q}_{K_1}$ and $\hat{Q}_{N-K_1}$ be the production of the firms owned by the first and the rest of the owners before and following the merger, respectively. Also let Q and $\hat{Q}$ denote the corresponding overall quantities, respectively. In the case we are considering now, $\hat{Q}_{K_1} > Q_{K_1}$ and $\hat{Q}_{N-K_1} > Q_{N-K_1}$ hold. However, $\hat{Q}_{K_1} > Q_{K_1}$ implies by the lemma, since $Q'_{N-K_1} < 0$ that

(A.25) $$Q_{N-K_1}(\hat{Q}_{K_1}) < Q_{N-K_1}(Q_{K_1}) = Q_{N-K_1}$$

holds. Note that $\hat{Q}_{N-K_1}$ and $Q_{N-K_1}(\hat{Q}_{K_1})$ are the best response quantities produced by the firms owned by owners $2,\ldots,m+1$, following and before the merger, respectively, given that the remainder of the firms--that is, those in K_1 --produce altogether the same quantity $\hat{Q}_{K_1}$. However, when the firms in K_1 produce the constant quantity $\hat{Q}_{K_1}$ before and after the merger, the inverse demand function faced by the rest of the firms in $N - K_1$, and their m owners is $\tilde{P}(Q) = P(Q + \hat{Q}_{K_1})$. It is easy to verify that if $P(Q)$ is replaced by $\tilde{P}(Q)$ all our assumptions continue to hold. Hence, by the induction hypothesis,

$$\hat{Q}_{N-K_1} \le Q_{N-K_1}(\hat{Q}_{K_1}) \tag{A.26}$$

which, together with (A.25), implies $\hat{Q}_{N-K_1} < Q_{N-K_1}$, contradicting our hypothesis that all owners produce under the merged structure more than their firms would produce under the unmerged one.

It follows that there is one owner, say the first, producing in the merged structure no more than his firms would have produced under the unmerged structure. That is

$$\hat{Q}_{K_1} \le Q_{K_1}. \tag{A.27}$$

Moreover,

$$\hat{Q} = \hat{Q}_{K_1} + \hat{Q}_{N-K_1} \le \hat{Q}_{K_1} + Q_{N-K_1}(\hat{Q}_{K_1}) \tag{A.28}$$

where the inequality in (A.28) follows the induction hypothesis, i.e., (A.26). But, by (A.27), and since $Q'_{N-K_1} < 0$ holds, by Lemma A.1, we have

$$Q_{N-K_1}(\hat{Q}_{K_1}) \ge Q_{N-K_1}.$$

and since $Q'_{N-K_1} > -1$,

$$Q_{N-K_1}(\hat{Q}_{K_1}) - Q_{N-K_1} \le Q_{K_1} - \hat{Q}_{K_1}.$$

or

$$Q_{N-K_1}(\hat{Q}_{K_1}) \le Q_{K_1} + Q_{N-K_1} - \hat{Q}_{K_1} = Q - \hat{Q}_{K_1}. \tag{A.29}$$

Substituting (A.29) into (A.28) we obtain

$$\hat{Q} \le Q. \qquad \square$$