The author is greatly indebted to an anonymous referee who made detailed comments on the previous versions of this paper. Thanks are also given to Professors H. Itoh, R. Komiya, and H. Okuno for many useful comments. All errors, of course, are my own.

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Abstract

The purpose of this paper is to refute the generally accepted idea that a cartel raises producers' surplus at the expense of consumers' surplus. If firms in an industry expect that a cartel in which the output is allocated on the basis of the capital equipment will be formed, then each firm tries to make a larger capital investment than before, which may lead to larger consumers' surplus. Through this argument, I shall present one of the reasons why heavy industries had excess capacity in postwar Japan.
1. INTRODUCTION

The purpose of this paper is to refute the generally accepted idea that a cartel raises producers' surplus at the expense of consumers' surplus. Through this argument, I shall present one of the reasons why heavy industries have had excess capacity in postwar Japan.

In the postwar period, the Japanese government focused its attention on protection and promotion of growing industries. From 1950s to 1960s when Japanese industries were less competitive than those of other developed countries, the administration took various methods to protect them: examples are import restrictions and the selective allocation of funds to heavy industries. When the performance of an industry became worse off because of business fluctuation, the government not only allowed firms in the industry to form a recession cartel but also suggested that they should do so.

Many observers have discussed the effects of recession cartels. Here we mention only two of them on which we focus our attention in this paper. The first is the effect on capital investment. It has been often said that there is "excessive competition in investment" in postwar Japan. Some observers argued that "excessive competition in investment" was caused by the fact that output under a cartel was allocated on the basis of the amount of capital equipment (see e.g. Komiyama[6] and Imai[5]). 1/

Unfortunately, the concept "excessive competition in investment" has not been well defined as pointed out by Tsuruta[11]. It is relatively easy to define the notion in a model. For example, consider the cost function such
that the marginal cost remains constant till it reaches some critical point and then becomes vertical. We may call this critical point the capacity and say that there exists "excessive competition in investment" if the output level in an equilibrium is below the capacity. This type of cost function is not realistic, nor can the definition be applied to general cost functions.

Nevertheless, the term "excessive competition in investment" has been often referred to because the competition in investment in postwar Japan seemed to be harder than normal competition. The normal competition we have in mind may be that under unrestricted situation. I define competition as unrestricted if there is neither collusive behavior nor any other restrictive method. Then we say that competition in investment is excessive if capital investment is greater than that under unrestricted competition. We use the term "excessive" without any normative sense. Note that the above definition can be applied to a general cost function.

The second effect of recession cartels is the effect on consumers' surplus and social welfare. Many observers are concerned with the effect of cartels and other protection policies on consumers' surplus and social welfare. There are three levels of arguments about this. The first, and the least sophisticated, is that output restriction increases producers' surplus at the expense of consumers' surplus and incurs dead weight loss which leads to smaller social welfare. The second is that those policies would make Japanese industries more competitive, which would lead to larger social welfare of Japan in the future. This point was insisted by those who approved protection and promotion policies. The third, and the most sophisticated, was presented by Negishi[8]. He argued that social welfare
should be measured in terms of the present value, which takes into account the loss during the restriction period. Those discussions described above, however, have a common assertion admitting that the existence of a cartel lowers consumers' surplus at the very time when the cartel is organized. The purpose of this paper is to refute this assertion.

When firms in a market intend to raise their profit by forming a cartel, in order to reach an agreement, it is necessary for every member of the cartel to gain larger profit through the cartel than that in the unrestricted situation. Competition is restricted by organizing a cartel, and other things being equal, cartelization raises the price of the products and reduces consumers' surplus. Furthermore, since the increase in producers' surplus is dominated by a decrease in consumers' surplus, social welfare declines.

In the case when there are more than one strategic variable that a firm can choose, the situation may vary. If only one variable is restricted, it is plausible that each firm competes harder than ever in the other variables which are not restricted. As is often argued, if the restricted variable is fixed to a certain value, other variables may change so as to increase consumers' surplus. Even in this case, the restriction usually leads to smaller consumers' surplus (see e.g. Walsh[13]). If the level of the restricted variable is determined by the choice of the unrestricted variables, however, consumers' surplus does not necessarily decline.3/ I shall present a model in which each firm has two strategic variables, namely output and capital investment, and show that the output restriction made on the basis of capital equipment causes firms to make larger capital investment, which may lead to larger consumers' surplus. This phenomenon
occurs not only because there is the unrestricted variable which determines the level of the restricted variable, but also because the number of firms in the market is fixed, and no other firm can enter the market even if there are excess profits. Unlike arguments in standard textbooks, such cartels may increase consumers' surplus at the expense of producers' surplus.

In postwar Japan, quite a few cartels have been organized as recession cartels supported by administrative guidance. Most of them were in the forms of price and quantity cartels. Since these cartels were not only legal but also supported by the government, each firm could expect even in an expansion phase that a cartel would be formed in a recession. If a firm with larger capital equipment gains larger profits in a cartel, each firm may have an incentive to make larger capital investment beforehand to get larger profits in comparison with unrestricted competition.

"Among those who are accustomed to the standard theory of neoclassical welfare economics and industrial organization, 'a widespread belief that increasing competition will increase welfare (Stiglitz)' strongly prevails (Okuno and Suzumura[9]).' It is also believed that a cartel decreases consumers' surplus. If the phenomenon mentioned above occurs, however, a cartel will cause 'excessive competition in investment,' and that effect may dominate the distortion caused by the output reduction, which leads to larger consumers' surplus.

The contents of this paper are as follows. In Section 2, I shall make a sketch of the situation of cartels in postwar Japan. It is shown that since cartels were often organized in the postwar period, it was possible for private firms to expect that a cartel would be formed in recession. Section 3 describes the effect of a quantity cartel on capital investment and
consumers' surplus. There I shall show the logic of how a cartel expected beforehand increases consumers' surplus. Section 4 presents a model to clarify the logic presented in the previous section. It will be shown that a cartel under which consumers' surplus increases can be formed if the cost and the demand functions satisfy plausible assumptions. Section 5 concludes the paper.

2. CARTELS IN POSTWAR JAPAN

Based on a series of acts concerning exception from the application of the Antimonopoly Act, quite a few cartels were organized in many industries. Table 1 shows the number of cartels which were permitted by the Fair Trade Commission or the relevant authorities which have primary responsibilities for the industries in question. At the end of 1963, 1302 cartels were retained under the major acts which permit certain types of cartels to be formed. 5/ 78% of such cartels were promoted under the Small and Medium Enterprises Organization Law and the Export-Import Trading Act. Most cartels were formed among small and medium enterprises. Cartels based on Section 24 of the Antimonopoly Act which is concerned with exception from the application of the act, associations of exporters set up under the Export-Import Trading Act, or Ammonium Sulfate Industry Act are considered as cartels among big business, which amounted to 37 cases or only 7% of the total number of cartels. If, however, we count output reduction suggested by the government (kankoku sotan) and so on as a kind of cartels among big business (21 cases), shipment of the cartelize products produced by big business is considerably high.
523 cartels were formed in the manufacturing industry at the end of 1963. Though the cartels among big business defined as above were less than 10% of all cartels in number, the ratio of shipment is 58.3% of that of all cartels. The shipment of cartelized products amounted to 4 trillion 56 billion yen or 28% of total shipment. The ratio of cartelization which is measured in shipment was the highest in the textile industry, which amounted to 78.1%.

How frequent cartels have been organized can be seen in the data of a particular industry. In the paper and pulp industry, for example, cartels on cardboard were formed as many as ten times from 1957 to 1973. Those cartels were formed mainly in recessions. Though some of them lasted only one month or so, the average duration was about half a year. Simple calculation enables us to conclude that cartels have been organized for about 5 years out of 16 years. As described above, cartels have been organized widely and continuously in postwar Japan. In such a situation, there was a good reason for each firm to expect that a cartel would be formed if the rate of returns declined. More or less, allotments under a cartel were determined on the basis of capital equipment, each firm takes such tendency into account to determine the level of capital investment. Consequently, the amount of capital equipment may change in comparison with that under unrestricted competition. Typical examples are output reduction suggested by the government in the iron and steel industry and the import quota in the oil refinery industry.

3. THE EFFECT OF A QUANTITY CARTEL
As described in the previous section, there existed quite a few cartels in postwar Japan. How did they affect the economy at that time? According to standard textbooks, given other circumstances, output reduction raises the price of the products and increases producers' surplus at the expense of consumers' surplus. The economy incurs dead weight loss which leads to smaller social welfare. If we looked at the situation in the postwar period, however, the above explanation does not fully describe the effect of a cartel. Since recession cartels, output reduction suggested by the government and so on have been legally organized quite often, private firms could expect that there would be output reduction with certain rules if the market was in a slump. The important point is that the allocation of output under a cartel was often made based on the amount of capital equipment or past records of the output. Under such allocation mechanisms, a firm with larger capital equipment can get a larger allotment and gain a larger additional profit induced by the increase in price. Knowing this, each firm tries to make capital investment beyond the equilibrium level under unrestricted competition. Consequently, the "excessive competition in investment" prevails.

This phenomenon has two effects on consumers' surplus. The first is the direct effect of output reduction, which we shall call the distortion effect. The second is the indirect effect which is accompanied by increase in capital equipment. Other things being equal, larger capital equipment induces larger output and then larger consumers' surplus in an equilibrium, which we shall call the strategic effect. These two effects offset each other. Then the relative size of the two effects determines the direction in which consumers' surplus changes.
There is a remark about the above arguments. The output under a cartel is below the competitive level given the amount of capital equipment. Hence the firms have incentives to form a cartel because they have already committed to larger capital equipment and cannot change it costlessly.

4. MODEL

There are $n$ identical firms in a market. No other firm can enter the market. They produce homogeneous products, and each firm makes attempt to maximize its own profit.

We consider a two stage game. In the first stage, each firm commits itself to the fixed capital equipment, which determines its second stage cost function. We denote by $K_i$ the amount of capital equipment of the $i$-th firm. In the second stage, each firm produces goods. The $i$-th firm's output is denoted by $x_i$. The marginal cost of capital investment, which is made in the first stage, is $\beta$. We set $\beta=1$ without loss of generality. The second stage cost function for the $i$-th firm is $C(x_i, K_i)$.

Then the total cost that firm $i$ should pay is $C(x_i, K_i) + K_i$.

We assume that $C(x_i, K_i)$ is four times continuously differentiable with respect to the both arguments and use the following notations:

$$
\frac{\partial C(x_i, K_i)}{\partial x_i}, \quad \frac{\partial^2 C(x_i, K_i)}{\partial x_i^2}, \quad \frac{\partial C(x_i, K_i)}{\partial K_i}
$$

for all $i=1, 2, \ldots, n$. Likewise, other partial derivatives are denoted by $C_x(x_i, K_i)$, $C_{xx}(x_i, K_i)$, and $C_{kk}(x_i, K_i)$.
We assume that \( C(x_i, K_i) \) satisfies:

\[
C_x \geq 0, \quad C_K < 0, \quad C_{kk} > 0 \quad \text{and} \quad C_{xk} < 0
\]  
(1)

for all \( (x_i, K_i) \). The marginal cost is always nonnegative, and the capital investment reduces the second stage cost at a decreasing rate and decreases the marginal cost. Later, we will modify assumption (1) so that it should be satisfied only around the equilibrium output level. It will be clear that the assumption of cost-reducing investment is the one that captures the critical role of capacity investment.

Let the inverse demand function be

\[
p = P(X)
\]

where \( p \) is the price and \( X = \sum_{i=1}^{n} x_i \) is the aggregate output. It is assumed that \( P \) is four times differentiable and satisfies \( P'(X) > 0 \) so that the price elasticity of demand \( \eta(X) = -P(X)/XP'(X) \) is positive. Furthermore, we assume that

\[
P'(X) - C_{xx}(x_i, K_i) < 0.
\]  
(2)

The marginal cost may be downward sloping, but its slope is not steeper than that of the demand curve.

Given \( K_i \) and the output of the other firms, the second stage rent of firm \( i \) is given by

\[
R_i(x, K_i) = P(X)x_i - C(x_i, K_i)
\]  
(3)

where \( x = (x_1, x_2, \ldots, x_n) \), the output configuration. Under the Cournot conjecture hypothesis, which we assume throughout the paper, the first-order conditions for interior profit maximization are given by

\[
\frac{\partial R_i}{\partial x_i} = P(X) + P'(X)x_i - C_x(x_i, K_i) = 0, \quad i = 1, 2, \ldots, n
\]  
(4)

where we assume that the following second-order conditions are satisfied:
\[
\frac{\partial^2 B_i}{\partial x_i^2} = 2P'(X) + P''(X)x_i - C_{x_i}(x_i, K_i) < 0, \quad i = 1, 2, \ldots, n
\]  \hspace{1cm} (5)

where \( x_i \) is evaluated at the value satisfying (4).

Given the first-stage capital commitment \( K_i \), (4) gives the reaction function of firm \( i \) which we express as \( x_i = r(X, K_i) \). By definition, it should satisfy (4), i.e.,

\[
P(X) + P'(X)r(X, K_i) - C_{x_i}(r(X, K_i), K_i) = 0,
\]  \hspace{1cm} (6)

where one should note that the reaction function of each firm is of an identical form.

Given the capital configuration \( K = (K_1, K_2, \ldots, K_n) \), the second stage Cournot-Nash equilibrium output configuration, which is assumed to be unique, is denoted by \( x^*(K) = (x_1^*(K), x_2^*(K), \ldots, x_n^*(K)) \) satisfying (4) for all \( i = 1, 2, \ldots, n \) simultaneously. The second stage Cournot-Nash equilibrium, \( x^*(K) \), should satisfy

\[
x_i^*(K) = r(x^*(K), K_i), \quad i = 1, 2, \ldots, n.
\]  \hspace{1cm} (7)

Adding up (7) for all \( i \), we obtain

\[
x^*(K) = \sum_{i=1}^{n} r(x^*(K), K_i).
\]  \hspace{1cm} (8)

Let \( \partial r(X, K_i)/\partial X \) and \( \partial r(X, K_i)/\partial K_i \) be denoted by \( r_X(X, K_i) \) and \( r_K(X, K_i) \) respectively. Then

\[
r_X(X, K_i) = - \frac{[P'(X) + P''(X)r(X, K_i)]/[P'(X) - C_{x_i}(r(X, K_i), K_i)]}{P''(X)}
\]  \hspace{1cm} (9)

and

\[
r_K(X, K_i) = \frac{C_{x_i}(r(X, K_i), K_i) / [P'(X) - C_{x_i}(r(X, K_i), K_i)]}{P''(X)}
\]  \hspace{1cm} (10)

hold. The sign of (9) depends on that of the numerator. If \( P' + P''r(X, K_i) \) is positive, the sign is positive and the goods in question are strategic complements. If the numerator is negative, (9) is negative and the goods are strategic substitutes.

Here it is easy to verify that

12
\[ \sum_{J=1}^{B} r_{x}(X, K, J) < 1 \] at \( X = X^*(K) \) for any \( K \) (11)

gives a local stability condition for the equilibrium, which we assume throughout the paper.

How does the equilibrium output change as capital investment increases?

Differentiating \( X^*(K) \) and \( x^*_j(K) \) with respect to \( K \), we obtain

\[
\begin{align*}
\frac{\partial x^*_j(K)}{\partial K_j} &= r_{x}(X^*(K), K, j)/\left(1 - \sum_{J=1}^{B} r_{x}(X^*(K), K, J)\right) > 0, \\
\frac{\partial x^*_j(K)}{\partial K_j} &= r_{x}(X^*(K), K, j)\left(\frac{\partial x^*_j(K)}{\partial K_j}\right) - r_{x}(X^*(K), K, j) \quad \text{if } j = 1, \\
&= \frac{r_{x}(X^*(K), K, j)}{\partial x^*_j(K)/\partial K_j} \quad \text{if } j \neq 1.
\end{align*}
\] (12)

If the goods in question are strategic substitute, i.e., \( r_{x}(X^*(K), K, j) < 0 \), then \( \partial x^*_j(K)/\partial K_j \) is positive if \( j = 1 \) and is negative if \( j \neq 1 \) by virtue of (9), (10) and (11). So in this case, by expanding its capital investment, each firm can increase its market share.

Consider the first stage of the game. Each firm knows what will be likely to occur in the second stage and makes capital investment to maximize its expected profit. Since each firm knows that the second stage output will be \( X^*(K) \), the profit of firm \( i \) which is expected in the first stage is

\[ \pi_i^*(K) = r_{i}(x^*_i(K), K) - K_i. \quad i = 1, 2, \ldots, n. \] (14)

The first-order conditions for interior profit maximization are given by

\[
\begin{align*}
\frac{\partial \pi_i^*(K)}{\partial K_j} &= \sum_{J=1}^{B} r_{i}(X^*(K), K, J) \frac{\partial x^*_i(K)}{\partial K_j} - (1 - \sum_{J=1}^{B} r_{i}(X^*(K), K, J)) < 0, \\
&= \sum_{J=1}^{B} r_{i}(X^*(K), K, J) \frac{\partial x^*_i(K)}{\partial K_j} - 1 < 0, \quad j = 1, 2, \ldots, n.
\end{align*}
\] (15)

where we assume that the following second-order conditions should hold:

\[
\begin{align*}
\frac{\partial^2 \pi_i^*(K)}{\partial K_j \partial K_l} &= -P''(X^*) \frac{\partial x^*_i(K)}{\partial K_j} x^*_i(K) - P''(X^*) \frac{\partial x^*_i(K)}{\partial K_l} x^*_i(K) - 2P'(X^*) \frac{\partial x^*_i(K)}{\partial K_j} \frac{\partial x^*_i(K)}{\partial K_l} < 0, \\
&= -P''(X^*) \frac{\partial x^*_i(K)}{\partial K_j} x^*_i(K) - P''(X^*) \frac{\partial x^*_i(K)}{\partial K_l} x^*_i(K) - 2P'(X^*) \frac{\partial x^*_i(K)}{\partial K_j} \frac{\partial x^*_i(K)}{\partial K_l} < 0, \quad j \neq l.
\end{align*}
\] (16)
for $i = 1, 2, \ldots, n$ and all $K$. We make use of the notational convention $X^* = X^*(K)$ and $x^*_i = x^*_i(K)$. We denote by $K^* = (K^*_1, K^*_2, \ldots, K^*_n)$ the capital configuration in an equilibrium. In the following, we restrict our attention to the symmetric equilibrium capital configuration $(K^*_1, \ldots, K^*_n)$ which is assumed to be unique. Let $K^*_i(K_{-i})$ be the reaction function of firm $i$ to $K_{-i} = (K_1, \ldots, K_{i-1}, K_{i+1}, \ldots, K_n)$. Indeed, the best response correspondence is unique-valued since we have assumed (16). Then at a symmetric equilibrium,

$$K^*_i(K^*_1) = K^*$$

holds. It is easy to verify that

$$\sum_{j \neq i} \frac{\partial x^*_i(K)}{\partial K_j} < 1$$

(17)

gives a local stability condition for the first stage game. We assume that (17) holds.

Now we can modify assumption (1) so that it should hold only in the neighborhood of the symmetric equilibrium, which implies that the second stage cost function may be the same between firms with different levels of capital equipment until output reaches a critical point, which may be called capacity, and the only difference appears in the level of capacity. Figure 2 shows an example.

**Cartel**

A cartel is formed in the second stage. It is enforced by means of government intervention as well as mutual monitoring activity by the firms. The output of each firm is restricted by cartelization, and it is assumed that no other means of competition is restricted. The output of $i$-th firm
is given by

$$\bar{x}_1 = \bar{x}_1(K), \quad i=1,2,\ldots,n,$$

which satisfies

C-1) \( R_i(\bar{x}(K),K)^* > R_i(x^*(K),K) \), \( i=1,2,\ldots,n \),

for all \( K \) where \( \bar{x} = (\bar{x}_1,\ldots,\bar{x}_n) \). Since \( \frac{\partial R_i}{\partial x_j} = P'x_j < 0 \) for \( j \neq i \),

$$\frac{dR_i}{dK} = \sum_{j=1}^{n} \frac{\partial R_i(x^*,K)}{\partial x_j} dx_j = \sum_{j=1}^{n} \frac{\partial R_i(x^*,K)}{\partial x_j} dx_j < 0$$

if \( dx_j > 0 \) where we made use of (4). Then there exists a cartel rule satisfying C-1. It is attained by reducing the output of every firm to some extent, which we assume hereafter, i.e.,

C-2) \( \bar{x}(K) < x^*(K) \)

for all \( K \). Furthermore, we assume that

C-3) \( \frac{\partial \bar{x}_i}{\partial K_j} > 0 \)

for all \( K \) and \( i \). Note that the requirements which the rule should satisfy are limited to realistic ones.

Under this rule, firms may change their strategies in the first stage. If they do not know the possibility of cartelization and act as if Cournot-Nash equilibrium would be realized in the second stage, strategies the firms take in the first stage do not change as before. Then the amount of capital they have in the second stage remains \( K^* \). In comparison with the unrestricted competition where a cartel is not formed, the profit of \( i \)-th firm is increased for all \( i = 1,2,\ldots,n \), and consumers' surplus is decreased.

The above assertion, however, neglects that the firms have incentives to invest on cost-reducing capital strategically so as to change the capital configuration and to obtain the more favorable output allotment in a newly formed cartel. In the case when each firm knows in the first stage what
will be likely to occur in the second stage, the situation will vary. If every firm knows that the cartel of a rule which satisfies 5-1 through 6-3 will be formed in the second stage, the expected profit of $i$-th firm in the first stage is

$$\hat{\pi}_i(K) = R_i(K_1) - K_i, \quad i = 1, 2, \ldots, n. \quad (18)$$

The first order condition for interior profit maximization is given by

$$\frac{8\hat{\pi}_i(K)}{8K_i} = \frac{8R_i(K_1)}{8K_i} - 1 = 0, \quad i = 1, 2, \ldots, n. \quad (19)$$

where $\hat{R}_i(K) = R_i(K_1)$. (and $R_i^*(K) = R_i(\hat{K}_i(K), K_1)$). Let $\hat{K}_i(K_{i-1})$ be the best response correspondence of firm $i$ to $K_{i-1} = (K_1, \ldots, K_{i-1}, K_{i+1}, \ldots, K_n)$. Then $\{\hat{K}_i(K_{i-1}), K_{i-1}\}$ satisfies (19). A symmetric equilibrium $\hat{K}$ to which we restrict our attention satisfies

$$\hat{K} = \hat{K}_i(K_{i-1})$$

for all $i = 1, 2, \ldots, n$. We assume that the following conditions should hold:

C-4) $\frac{8^2\hat{\pi}_i(K)}{8K_i^2} < 0$ for $i = 1, 2, \ldots, n$ and all $K$.

C-5) $\frac{8\hat{\pi}_i(K_i)}{8K_i} < 1$ at $K = \hat{K}$.

and

C-6) there is a unique symmetric equilibrium in the first stage game.

Now we present the following proposition.

Proposition: $K$ is increased by cartelization if and only if

$$\frac{8\hat{R}_i(K_1)}/8K_i > \frac{8R_i^*(K_1)}{8K_i} \quad (20)$$

for all $j = 1, 2, \ldots, n$.

Proof: Suppose (20) holds for all $i = 1, 2, \ldots, n$. Evaluating (19) at $K = \hat{K}$, we obtain

$$\frac{8\hat{\pi}_i(K_1)/8K_i = \frac{8R_i(K_1)}{8K_i} - 1 > \frac{8R_i^*(K_1)}{8K_i} - 1 = 0, \quad i = 1, 2, \ldots, n$$

which implies that each firm has an incentive to make further investment beyond $K^*$. The "only if" part is similar. Q.E.D.
Condition (20) means that the amount of additional profit, the difference in profit between under unrestricted competition and restricted competition, is allocated more to a larger firm than to a smaller firm. In that case, the marginal revenue of capital equipment becomes larger. Hence each firm comes to have an incentive to make a larger investment than before. It is easy to derive that each firm increases the amount of capital as $\frac{\partial P_1(K)}{\partial K_1}$ increases.

Thus the greater the marginal profit of capital becomes, the larger the investment each firm tries to make.

Consumers' surplus

Consumers' surplus is increased by cartelization if and only if the amount of output is increased, i.e.,

$$\tilde{x}(\bar{K}) - x^*(K^*) > 0.$$ 

Rewriting the left hand side of the above inequality, we obtain

$$\{\tilde{x}(\bar{K}) - x^*(\bar{K})\} + \{x^*(\bar{K}) - x^*(K^*)\}.$$ 

The former bracket is the distortion effect, the direct effect of output reduction, which is unambiguously negative. While the latter is the strategic effect, the indirect effect through increase in investment, which will be positive if (20) holds. Hence it can be said that consumers' surplus is increased by cartelization if and only if the strategic effect is positive and dominates the distortion effect. Here we show the main theorem stating that there exists a cartel rule satisfying C-1 through C-6 such that consumers' surplus is increased by cartelization.

17
Theorem: There exists a cartel rule satisfying C-1 through C-6 such that the strategic effect is positive and dominates the distortion effect.

Before we present the proof of the theorem, we derive the following lemma in which we show that a particular type of cartel rules satisfy C-1 through C-6. At first, we define the following twice differentiable rationing function \( \Psi: \mathbb{R} \to \mathbb{R} \) satisfying:

i) \( \Psi(z) \in (0,1) \) for all \( z \in \mathbb{R} \).

ii) \( \Psi'(z) \geq 0 \) for all \( z \in \mathbb{R} \).

Let \( \Psi^* = \sup \Psi(z) \) and \( \psi^* = \inf \Psi(z) \). We assume that

iii) \( (1-\psi^*)/(1-\Psi^*) \leq M \)

where \( M \) is a finite number. Based on the above rationing function \( \Psi \), we consider the following second stage output rationing rule:

\[
R-1. \quad x_i(K) = \Psi(\Delta x_i) + \epsilon_i(K), \quad i = 1, 2, \ldots, n,
\]

where \( \Delta x_i = x_i - \frac{1}{n} \sum_{j=1}^{n} x_j \). Now we can state the following lemma, the proof of which will be presented in the appendix.

Lemma: Suppose a rationing function \( \Psi \) satisfying i)-iii). If \( \Psi(\cdot) \) is sufficiently close to one and if \( \Psi'(\cdot) \) and \( \Psi''(\cdot) \) are sufficiently close to zero, then the associated rationing rule R-1 satisfies C-1 through C-6.

Proof. See Appendix.

Condition C-1 is the main feature that the cartel should satisfy. The logic behind this is that first-order reduction in own output causes only second-order loss to that firm, while first-order gains to the others, so the first-order reduction by all firms leads to first-order increase in profits of all firms. Now we confine our attention on rule R-1 satisfying i)-iii) and present the proof of the theorem.
Proof of the Theorem: Let $\tilde{K} = (K^0, \ldots, K^n)$ satisfy

$$x_1^i(K^0) = \psi(0)x_1^i(K^0) = x_1^i(K^*), \quad i = 1, 2, \ldots, n.$$  

It suffices to show that there exists a cartel rule satisfying C-1 through C-6 such that (22) evaluated at $K = K^0$ is positive.

$$\frac{\partial}{\partial K_i} \left( x_1^i(K^0) \right) = \left( P'(x_1^i(K^*))x_1^i(K^*) - P(x_1^i(K^*)) - C_k(x_1^i(K^*), K^0) \right) \frac{\partial K_i}{\partial K_i}$$

$$+ \sum_{j \neq i} P'(x_1^j(K^*))x_1^j(K^*) \frac{\partial x_1^i(K^0)}{\partial K_i} - C_k(x_1^i(K^*), K^0) - 1$$

$$= \left( C_k(x_1^i(K^*), K^*) - C_k(x_1^i(K^*), K^0) \right) \frac{\partial x_1^i(K^0)}{\partial K_i}$$

$$+ \sum_{j \neq i} P'(x_1^j(K^*))x_1^j(K^*) \frac{\partial x_1^i(K^0)}{\partial K_i} - \frac{1}{n} \frac{\partial x_1^i(K^0)}{\partial K_i}$$

$$- \left( C_k(x_1^i(K^*), K^0) - C_k(x_1^i(K^*), K^0) \right)$$

where we made use of (4) and (15). Let $q = K^0 - K^*$. Then by definition $q$ tends to zero as $\psi(0)$ approaches to unity. Taking the limit of $\psi(0)$, we have

$$\lim_{\psi(0) \to 1} \frac{\partial}{\partial K_i} \left( x_1^i(K^0) \right) = \sum_{j \neq i} \frac{P'(x_1^j(K^*))x_1^j(K^*)}{\psi(0) - \frac{1}{n}} - \frac{1}{n} \frac{\partial x_1^i(K^0)}{\partial K_i} - \psi(0) \frac{1}{n} x_1^i(K^0)$$

$$= \sum_{j \neq i} P'(x_1^j(K^*))x_1^j(K^*) \left( \frac{\partial x_1^i(K^0)}{\partial K_i} - \frac{1}{n} \frac{\partial x_1^i(K^0)}{\partial K_i} - \psi(0) x_1^i(K^0) \right) \geq 0 \quad (22)$$

If $\psi(0)$ tends to zero at sufficiently lower rate than $\psi(0)$ does to unity, and if $\psi(\cdot)$ and $\psi'(\cdot)$ converge to zero, it satisfies C-1 through C-6 as well as (22), which completes the proof of the theorem.

Given capital configuration, the cartel raises the profit of all the firms, which implies that every firm prefers output reduction through cartelization to unrestricted competition once they commit themselves to fixed capital equipment. Output reduction through the distortion is smaller than increase in output through the strategic effect, which leads to larger consumers'
surplus than that under unrestricted competition. This situation tends to occur if the rule of a cartel allocates more output to a larger firm than to a smaller firm. It makes the marginal profit of capital investment larger. Hence firms have incentives to make a larger investment.

**Example**

Suppose there are two firms in the market, i.e., $n=2$. Their second stage cost functions are identical and given by

$$C(x^*_1, K^*_1) = C(K^*_1)x^*_1, \quad i = 1, 2.$$  \hspace{1cm} (E1)

and satisfies (1) for all $\{x^*_1, K^*_1\}$. Let the inverse demand function be $p = a-bX$.

where $X = x^*_1 + x^*_2$. Given capital configuration $(K^*_1, K^*_2)$, the first-order condition for second stage profit maximization is

$$a-b(x^*_1 + 2x^*_2) - C(K^*_1) = 0, \quad i = 1, 2.$$  \hspace{1cm} (E2)

where $\{x^*_1, x^*_2\}$ is an equilibrium output configuration, and subscript $j$ denotes not $i$. Solving (E2), we obtain

$$x^*_1(K) = (1/3b)(a-C(K^*_2)-2C(K^*_1))$$
$$x^*_2(K) = (1/3b)(a+C(K^*_2)-2C(K^*_1)).$$  \hspace{1cm} (E3)

suppose a cartel would be formed in the second stage of which rule restricts the output of each firm to

$$R-2) \quad \bar{x}^*_i(K) = \Psi x^*_i(K), \quad \Psi \in (0, 1), \quad i = 1, 2.$$  \hspace{1cm} (R-2)

We assume that $\Psi$ is not far from unity so that rule R-2 satisfies C-1 through C-6. Under the rational expectation that a cartel will be formed, the first-order condition for the first stage profit maximization of firm $i$ is given by
\[(1-\psi)(2b\hat{x}_1+b\hat{x}_m)-(2/3b)C'(K_1^*) - \hat{b}^2 \frac{\hat{x}_1^2}{2} C'(K_1^*) - C'(K_1^*) \psi \hat{x}_1^* - 1 = 0 \quad (E4)\]

Differentiating (E4) with respect to \(\psi\) and evaluating it at \(\psi=1\), we obtain
\[C'(K_1^*) (2\hat{x}_1^* K^*) - \hat{x}_1^* (K^*) / 3 < 0, \quad 1 = 1.2. \quad (E5)\]

(E5) implies that \(\frac{\partial \hat{x}_1^*}{\partial K_1^*}\) is increased in the neighborhood of \(\psi=1\) as \(\psi\) decreases which ends in larger equilibrium capital configuration.

To see if output is raised by a decrease in \(\psi\), we check the sign of the following equation:
\[
\frac{\partial \hat{x}_1^*}{\partial \psi} \bigg|_{\psi=1} = \frac{\partial \hat{x}_1^*}{\partial \psi} + \frac{\partial K_1^*}{\partial \psi} \frac{\partial \hat{x}_1^*}{\partial K_1^*} + \frac{\partial K_1^*}{\partial \psi} \frac{\partial \hat{x}_1^*}{\partial K_1^*} \quad (E6)
\]

If the sign of (E6) is negative, we can conclude that output under the rational expectation is increased as \(\psi\) decreases, i.e., through cartelization. After rigorous calculation, we obtain
\[
\frac{\partial \hat{x}_1^*}{\partial \psi} \bigg|_{\psi=1} = \frac{1}{4} \left[ 1 + \left( 1 - \frac{3bC'(K_1^*) \hat{x}_1^*}{2(C'(K_1^*))^2} \right)^{-1} \right] \quad (E7)
\]

where we made use of (E4) and symmetry. The first term in the bracket is positive while the second term is negative because the second-order condition
\[
\frac{\partial \hat{x}_1^*}{\partial K_1^*} \bigg|_{\psi=1} = \frac{1}{3} \left( -C'(K_1^*) \hat{x}_1^* + \frac{2}{3b} C'(K_1^*) \hat{x}_1^* \right) < 0
\]

should hold. Then the sign of (E7) is ambiguous. If \(C'(K_1^*) \hat{x}_1^*\) is close to
\[\frac{2}{13b} C'(K_1^*) \hat{x}_1^* \]

then the second term dominates the first term, and (E7) is negative.

5. CONCLUSION

If there are more than one strategic variable that the firm can choose,
to restrict only one of them may affect other strategies. A quantity cartel has this effect. It affects firms' strategies on capital investment. Suppose output is assigned to a firm on the basis of its capacity. If a larger firm gains larger additional profit in a cartel, each firm plans to make investment beyond that in the unrestricted competition. Furthermore, there exists the case when consumers' surplus is increased by a cartel.\(^7\)

There is no doubt that in postwar Japan, private firms have behaved under the expectation that cartels would be formed in recession. It is reasonable to conclude that the existence of cartels induces 'excessive competition in investment.' As for consumers' surplus, the distortion effect and the strategic effect offset each other, and the relative size of the two effects determines the direction in which consumers' surplus changes. It cannot be said that a cartel always raises producers' surplus at the expense of consumers' surplus in postwar Japan. Of course, consumers' surplus might be decreased by a cartel in general. Even in that case, the welfare loss through the cartel will be overestimated unless one does not take the strategic effect into account.

There is an important remark about the arguments presented in this paper. We assume that no entry is available and every firm in the industry is the member of a cartel. The most serious problem for a cartel is its vulnerability. Cartels are often collapsed by the existence of outsiders who do not belong to them. Outsiders have desirable roles from the viewpoint of social welfare. The existence of outsiders, however, is undesirable for incumbents, and their entry was often restricted by the administration who was supporting cartels.\(^8\) In that case, the restrictive effect becomes stronger. Furthermore, the administration took various
methods to restrict excess capacity itself, which strengthened the restrictive effect, though in fact these methods did not work well. Without the other restriction, the "excessive competition in investment" caused by cartels is likely to prevent entrants from making capital investment since the excessive capacity has the effect of entry deterrence when it takes the form of sunk cost (see e.g. Dixit [2] and Ware [14]). Then the total capacity is less likely to increase enough to dominate the distortion effect of a cartel. Another effect of a cartel under free entry is the one on social welfare. If the number of firms in the market decreases as a result of a cartel, social welfare may increase according to the excess entry theorem proven by Suzumura and Kiyono [10], though nothing can be said in the framework of the present paper since their model is different from ours. It is an open question what will occur when there are potential entrants.
Proof of the lemma: The output rationing rule R-1 clearly satisfies C-2. Thus we next show that C-3 is satisfied. From the definition of the output rationing rule, given any capital configuration $K$ we have:

$$\bar{x}(K) = \sum_{i=1}^{n} \psi(\Delta K_i)x_i^*(K).$$

Differentiating this with respect to $K_j$, we obtain

$$\Delta x(K)/\Delta K_j = \sum_{i=1}^{n} \psi(\Delta K_j)x_i^*(K) = \psi(\Delta K_j)x_j^*(K).$$

Taking the limit of $\psi(\cdot)$ and $\psi(\cdot)$, we have

$$\lim_{\psi \rightarrow 0} \lim_{x_j \rightarrow 0} \Delta x(K)/\Delta K_j = \sum_{i=1}^{n} (x_i^*(K)/\Delta K_i) = \frac{\Delta x(K)}{\Delta K_j} > 0$$

by virtue of (12). Note that $\psi(\cdot)$ is a function, so its convergence is defined in terms of uniform convergence.

Next, we check if C-1 is satisfied. The second stage rent of firm $i$ under restricted competition can be written as

$$\bar{r}_i(K) = P(\sum_{j=1}^{n} \psi(\Delta K_j)x_j^*(K))x_i^*(K) - C(\psi(\Delta K_j)x_i^*(K),K_i)$$

$$\geq P(\sum_{j=1}^{n} (1-\psi(\Delta K_j))M_j)x_i^*(K) + C(\psi(\Delta K_j)x_i^*(K),K_i)$$

Now we treat $\psi(\Delta K_j)$ as a parameter. Differentiating the right hand side of the above inequality with respect to $\psi(\Delta K_j)$, and evaluating it at $\psi(\Delta K_j) = 1$, we obtain

$$\frac{d}{d\psi} \left[ P\left( \sum_{j=1}^{n} \psi(\Delta K_j)x_j^*(K) \right) \right] x_i^*(K) + P\left( \sum_{j=1}^{n} (1-\psi(\Delta K_j))M_j \right) x_i^*(K) - C(\psi(\Delta K_j)x_i^*(K),K_i) x_j^*(K)$$

Where we have used $\Delta x(K) = \frac{\Delta x(K)}{\Delta K_j}$. Then by slightly decreasing $\psi(\cdot)$ below unity, we have $\bar{r}_i(K) > R_i^*(K)$.

To show the concavity of $\bar{r}_i(K)$ in $K_i$, we differentiate $\bar{r}_i(K)$ with respect to $K_i$. 

24
\[
\frac{\partial \overline{y}_i(K)}{\partial \overline{x}_1} = \left( P'(\overline{x}(K))\overline{x}_1(K) + P(\overline{x}(K)) - x_2(K)K_1 \right) \frac{\partial \overline{x}_1}{\partial \overline{x}_1} + \frac{\partial \overline{x}_1}{\partial \overline{y}_i(K)} + \frac{\partial \overline{y}_i(K)}{\partial \overline{x}_2} = \frac{\partial \overline{x}_1}{\partial \overline{y}_i(K)} - C_2(K_1, K_1) - 1
\]

(21)

Differentiating (21) with respect to \(K_1\) again, we obtain

\[
\frac{\partial^2 \overline{y}_i(K)}{\partial K_1^2} = P'(\overline{x}(K)) \left[ \frac{\partial^2 \overline{x}(K)}{\partial K_1^2} \right] \overline{x}_1(K) + P'(\overline{x}(K)) \frac{\partial^2 \overline{x}(K)}{\partial K_1^2} \overline{x}_1(K) + 2P'(\overline{x}(K)) \frac{\partial^2 \overline{x}(K)}{\partial K_1^2} \overline{x}_1(K) + 2P''(\overline{x}(K)) \frac{\partial \overline{x}_1}{\partial K_1} \frac{\partial \overline{x}_1}{\partial K_1} \overline{x}_1(K) + \frac{\partial^2 \overline{x}_1}{\partial K_1^2} \frac{\partial \overline{x}_1}{\partial K_1} \overline{x}_1(K) + \frac{\partial^2 \overline{x}_1}{\partial K_1^2} \frac{\partial \overline{x}_1}{\partial K_1} \overline{x}_1(K)
\]

We now use the fact that if \(\psi(*)\) converges to unity, and \(\psi'(*)\) and \(\psi''(*)\) to zero. \((\overline{y}_i(K)/\overline{x}_1)\) and \((\overline{x}_1(K)/\overline{x}_1)^2\) tend to \((\overline{y}_i(K)/\overline{x}_1)^2\) and \((\overline{x}_1(K)/\overline{x}_1)^2\) respectively. Then it is easy to verify that \((\overline{y}_i(K)/\overline{x}_1)^2\) tends to \((\overline{x}_1(K)/\overline{x}_1)^2\) by using four times differentiability of \(P(*)\) and \(C(*,*)\), which implies \(\overline{y}_i(K)\) is concave in \(K_1\) by virtue of (16).

Similarly, since \((\overline{y}_i(K)/\overline{x}_1)\) tends to \((\overline{y}_i(K)/\overline{x}_1)^2\), we have:

\[
\frac{\partial^2 \overline{y}_i(K)}{\partial \overline{y}_i(K)} = \frac{\partial^2 \overline{y}_i(K)}{\partial \overline{y}_i(K)} = \frac{\partial^2 \overline{y}_i(K)}{\partial \overline{y}_i(K)} = -1
\]

Note that C-5, together with C-4, implies local uniqueness of a symmetric equilibrium as well as local stability.

Finally, we show that C-6 holds. Similar to the above argument, \(\overline{y}_i(K)/\overline{x}_1\) tends to \(\overline{y}_i(K)/\overline{x}_1\) as \(\psi(*)\) goes to unity, and \(\psi'(*)\) converges to zero. Then since \(\overline{y}_i(K)/\overline{x}_1\) is not equal to zero at every symmetric capital configuration but \(K = K^*\), \(\overline{y}_i(K)/\overline{x}_1\) is distinct from zero except in the neighborhood of \(K^*\). \(K^*\) is in the neighborhood of \(K^*\), and from the local uniqueness of a symmetric equilibrium which is proved earlier \(K^*\) is the unique symmetric equilibrium which completes the proof of the lemma.
FOOTNOTES

1) Tsuruta [11] argued that prices of products in heavy industries were rather stable in the postwar period and concluded that there was no undesirable "excessive competition." He also admitted, however, that the amount of capital equipment is increased by means of a quantity cartel.

2) See e.g. Morozumi [7].

3) Yano [15] focused his attention on voluntary export restrictions (VERs) and concluded that VERs may benefit the importing country's consumers. The phenomenon occurs because export quota are allocated on the basis of the amount of exports during pre-VERs period, and firms compete for larger share during VERs period. In that case, the above conclusion may be derived. It is because one variable (output during VERs period) is restricted, while other (output during pre-VERs period) is not restricted, and the former is determined on the basis of the latter.


5) The major acts referred to here are Section 24 of the Antimonopoly Act, the Small and Medium Enterprises Organization Law, the Export-Import Trading Act, the Act concerning Improved Management of Business Dealing with Sanitation, the Marine Transportation Act, and other various acts concerning rationalization measures (ibid pp.156-161). The English titles are those used in Allen [1] and the Fair Trade Institute [4].

26
6) See Tokusa [12], pp.195.

7) This result might hold when the mode of competition in the second stage is price instead of quality, which this paper does not dwell with.

8) As for the measures against outsiders, nine acts have the sections concerning this. An example is the Export-Import Trading Act (see the Fair Trade Commission [3]). As is often mentioned, these measures did not work well. The Susitomo Kinzoku case is an example. At the same time, however, it cannot be neglected that there were quite a few cartels which were sustained owing to those measures against outsiders.
REFERENCES


[4] Fair Trade Institute, Antimonopoly Legislation, 19XX.


The number of cartels which were permitted by authorities
(at the end of 1963)

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**TABLE 1**
FIGURE 2