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INFORMATION AND TIME-OF-USE DECISIONS IN STOCHASTICALLY CONGESTABLE FACILITIES

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Abstract

In this paper we investigate the impact of information on individuals' time of use of a congestable facility subject to stochastic fluctuations in demand and capacity. We solve for the continuous time equilibrium rate of use and for optimal (no-toll) design capacity for three information regimes: Full information (in which users know both demand and capacity when deciding whether and when to use the facility), zero information (in which users know only the joint probability distribution of demand and capacity), and partial information.

When capacity alone is stochastic, optimal design capacity is unambiguously greater with zero than with full information, and expected welfare is lower. However, when demand alone is stochastic, the rankings are parameter-dependent. For partial information we show via an example that, even with capacity alone stochastic, a marginal improvement in information may lower expected welfare, despite the fact that complete information is preferable to zero information. The results suggest that dissemination of information (e.g. radio traffic reports on weather and accidents) may be counterproductive for management of congestable facilities.

1. Introduction

In the standard peak-load pricing model of a congestable facility (Steiner [1957], Williamson [1966] *inter alios*) the timing and intensity of peak and off-peak demands are assumed to be predictable and known. Predictable fluctuations of a seasonal, weekly or daily frequency often do account for much of the total fluctuation in demand. But unpredictable variations may also be important.[1]

Beginning with Brown and Johnson [1969], the peak-load model has been extended to allow for stochastic demand fluctuations. Attention has been focussed on the optimal pricing and capacity of a public utility, as well as the profit-maximizing policy of a monopolist. This work has enriched the peak-load model and improved its practical applicability. However, the model remains incomplete in two respects.

First, while fluctuations in demand have been widely considered, fluctuations in capacity have not.[2] In reality, most services (electricity, water, gas, telephone, computer, transport *etc*.) are subject to periodic interruptions. In the case of road traffic, disruptions due to inclement weather, accidents, breakdowns and road repairs are quite frequent. Such disruptions can have an appreciable effect on system performance, and on user welfare.

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[1]For example, Ju *et al*. [1987, p.520] report that random events account for 57% of total traffic congestion on Los Angeles freeways. Stochastic variations in air travel demand are a principal determinant of airline booking policies and fleet capacity (*e.g.* Douglas and Miller [1974]). And significant nonsystematic load variations are observed in computer systems (*e.g.* Gale and Koenker [1984]).

[2]Naor [1969] and DeVany [1976] have developed queuing models in which the service time of individuals is random. However, they assume that both the long-run average service rate and average demand do not vary over time, thereby abstracting from the peak-load problem.

A second problem with the conventional peak-load model is a failure to treat properly users' time-of-use decisions. Periods of peak and off-peak demand are usually specified exogenously, with constant or even zero cross-price demand elasticities. The limitations of this approach can be illustrated with rush-hour traffic. Because of common work schedules and preferred hours for non-work activities, commuters travel to and from work at similar times. Those who travel at the peak of the rush hour in order to arrive 'on time' experience congestion and prolonged trips. Those who travel on the tails of the rush hour to avoid the worst congestion enjoy shorter trips, but suffer schedule delay: the psychic or monetary penalty from arriving either early or late. In equilibrium, the aggregate departure rate must be such that no commuter can reduce his travel costs by altering his departure time. Similar tradeoffs between convenience and congestion govern time-of-use decisions of air travel, computers and other facilities. The point of this example is that the time-of-use decisions of individuals are <u>endogenous</u>. If the pricing policy or capacity of a facility is changed, the time pattern of consumption throughout the demand cycle will be affected. Reduced-form demand functions with parametric cross-price elasticities cannot adequately capture such adjustments.[3]

The first systematic treatment of individual time-of-use decisions in the peak-load context was undertaken by Vickrey [1969] to describe the departure time of morning commuters. While Vickrey's work has been

[3] This issue is discussed in greater detail in Arnott <u>et al</u>. [1987b].

extended by a number of others[4], attention has been focussed largely on deterministic models which ignore fluctuations in demand or capacity.[5]

The purpose of this paper is to extend the peak-load model to allow fluctuations in both demand and capacity and to study the impact of information on the time-of-use decisions of consumers.[6] To fix ideas, we cast the problem in terms of traffic congestion during the morning rush hour. Although the nature of congestion differs with the type of facility, the analysis should provide a guide to understanding the impact of information on usage of other congestable facilities such as airlines, telephones and computers.

A number of experimental studies have been conducted on the impact of information on congestion and accidents in dense transport systems. Among these can be mentioned the Comprehensive Automobile Traffic Control (CACS) study carried out by MITI in Japan, the ALI-SCOUT Destination Guidance System in West Germany, the European PROMETHEUS project and the U.S. ETAK system introduced in the San Francisco and Los Angeles areas (see Boyce [1988]). Information made available to users can be static,

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[4] Henderson [1977, 1981], Hendrickson and Kocur [1981], Hurdle [1981], Fargier [1983], Mahmassani and Herman [1984], de Palma [1986], Newell [1987], de Palma and Arnott [1987], Braid [1987a,b], Arnott et al. [1987a, 1987b].

[5] Some work has been done developing probabilistic choice models of departure time (e.g. Alfa and Minh [1979], de Palma et al. [1983, 1987]). Multinomial logit demand specifications have been estimated by Cosslett [1977], Abkovitz [1981a,b], Small [1982], Hendrickson and Plank [1984] and Moore, Jovanis and Koppelman [1984]. However, these models deal only with stochasticity at the level of the individual, rather than in the aggregate. Aggregate demand fluctuations have been considered by DeVany and Saving [1980], but in a steady-state framework which abstracts from the peak-load problem.

[6] Preliminary research on this question has been conducted by Ben-Akiva et al. [1986] using a simulation model.

i.e. independent of current road conditions, or dynamic, i.e. updated on a daily or more frequent basis. Given the difficulties of analyzing the effect of information in a network, we limit attention in this paper to a single route. Demand and capacity are assumed to be remain constant over the course of a day, but to vary from day to day.

The analysis consists of three parts. In the first part, fluctuations in demand and capacity are assumed to be fully predictable. Each day the departure rate over the course of the rush hour adjusts so that no individual can reduce his travel cost by departing at a different time. The optimal design capacity of the road is determined by the condition that the expected marginal benefit from capacity expansion equals the marginal cost. While the derivation of this condition is conceptually straightforward it has important implications for cost-benefit analysis. For example, traffic engineers have employed rules of thumb in designing highway capacity, such as to limit congestion to a particular level in say the fiftieth busiest hour of the year on the assumption that road conditions are ideal.[7]

Such rules of thumb are questionable when actual highway capacity falls below its design level for a significant portion of the year, either because of accidents or ice and snow. We show that the appropriate rule for determining optimal capacity is characterized by a certainty-equivalent rush-hour flow that allows for fluctuations in both

[7]Analogous rules have been used to choose reserve levels for storable outputs. For example, British Gas holds sufficient gas reserves to meet demand in a cold winter occurring once in 50 years (Cannon [1987]). Similarly, water utilities may construct sufficient reservoir capacity to meet a once-in-50-years drought (Crew and Kleindorfer [1986, p.260]).

demand and capacity. We also show that if the elasticity of travel demand is less than unity (as it is in most commuting contexts) then optimal capacity is an increasing function of the variability of both demand and capacity and a decreasing function of the correlation between them.

In the second part of the analysis, users are assumed to know the joint probability distribution of demand and capacity, but not their realizations on a given day. The time pattern of departures must then be the same each day (in a sense made precise below), and such that individuals cannot reduce their <u>expected</u> costs by altering their departure time. Whether aggregate costs are higher or lower with zero information than full information is not clear <u>a priori</u> because there is an uninternalized congestion externality. It is possible that with zero information, users may spread out their departures sufficiently to reduce congestion and improve efficiency. We show that, if capacity alone is stochastic, travel costs are unambiguously greater with zero than full information, and that if the elasticity of demand is less than unity optimal capacity is greater. However, if demand alone is stochastic, the rankings of capacity and expected travel costs in the two information regimes are parameter-dependent. Public information about demand may thus have the perverse effect of decreasing welfare.

The two extremes of zero and full information are straightforward to model. The intermediate case of partial information is conceptually more difficult. Individuals will decide whether to travel, and if so when, on the basis of the perceived joint probability density of demand and capacity, conditional on the information they have. What this information is, and how it is acquired, will depend on the context. We consider a simple example in which capacity alone fluctuates between a high and a low level. We show that a marginal improvement in information can lower welfare, despite the fact that with fluctuations in capacity alone, full information is preferable to zero information. Limited information can thus have perverse effects in situations where full information is welfare-improving.

While we hesitate, given the simplicity of the model and the example, to draw policy implications from our results, they do raise some doubt about the benefits of information services such as morning radio reports on weather and traffic.[8]

In Section 2 we review the deterministic peak-load model on which our paper builds. Sections 3 and 4 concern respectively the polar cases of full information and zero information about demand and capacity fluctuations. Efficiency and optimal capacity in the two information regimes are compared in Section 5. Section 6 considers partial information by way of an example. A summary and directions for future research are provided in Section 7.

2. Review of the Deterministic Model

The analysis is based on a model of queueing congestion introduced by Vickrey [1969] and extended by Hendrickson and Kocur [1981], Fargier [1983] and Arnott et al. [1987a]. N identical commuters travel each morning from home in the suburbs to work downtown. There is a single

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[8] Whether traffic reports which influence commuters' choice of route could also be welfare-reducing is a subject for a separate investigation.

road along which each individual commutes in his own car. Travel is uncongested except at a single bottleneck (a bridge, tunnel, intersection etc.) which at most s cars can traverse per unit of time. If the arrival rate at the bottleneck exceeds s, a queue develops. Travel time is

$$T(t) = \bar{T} + T^{v}(t), \tag{1}$$

where $\bar{T}$ is travel time in the absence of a queue, $T^{v}(t)$ is waiting time at the bottleneck and t is departure time from home. Without loss of generality we set $\bar{T} = 0$, so that an individual reaches the bottleneck as soon as he leaves home, and arrives at work upon exiting the bottleneck. The length of the queue, D(t), is:

$$D(t) = \int_{\hat{t}}^{t} r(\tau)d\tau - s(t-\hat{t}), \tag{2}$$

where $\hat{t}$ denotes the time at which the queue was last zero and r the departure rate. Waiting time at the bottleneck for an individual who departs at time t is simply:

$$T^{v}(t) = D(t)/s. \tag{3}$$

Individuals are assumed to have a preferred arrival time, t^*, which can be thought of as their official starting time at work. Individual variable travel costs are given by the linear function

$$C(t) = \alpha T^{v}(t) + \beta(\text{time early}) + \gamma(\text{time late}), \tag{4}$$

where for individuals who arrive early (before t^*), time late = 0, and for those who arrive late (after t^*), time early = 0. The parameter α measures the cost or disutility of time spent in transit. β measures the cost of arriving an extra minute early at work and γ the cost of arriving an extra minute late.

Finally, t_n is defined to be the departure time for which an individual arrives on time, defined implicitly by the condition:

$$t_n + T^v(t_n) = t^*. \tag{5}$$

Henceforth, we take 'depart early' to mean depart so as to arrive early and use the term 'depart late' accordingly.

Equilibrium

In choosing when to leave home, individuals face a trade-off between travel time and schedule delay. Individuals are assumed to have full information about the departure time distribution. In equilibrium no one can reduce his costs by altering his departure time. With identical individuals this means that travel costs are constant over the rush hour.

Equilibrium is depicted in Figure 1.[9] Queue length is measured by the vertical distance between the cumulative departures and cumulative arrivals schedules. Travel time is measured by the horizontal distance. From the beginning of the rush hour at t_q until t_n the queue builds up at a constant rate. Once past t_n the queue dissipates, again at a constant rate, reaching zero at the end of the rush hour at t_q'.

Over the interval $[t_q, t_n]$ the equal cost condition implies from (4) that:

$$C(t) = \alpha T^v(t) + \beta(t^* - t - T^v(t)) \tag{6}$$

is constant. Differentiating (6) and using (2) and (3) the departure rate for individuals departing early is found to be

$$r(t) = \frac{\alpha s}{\alpha - \beta}, \qquad t \in (t_q, t_n). \tag{7}$$

By similar reasoning, the late departure rate is:

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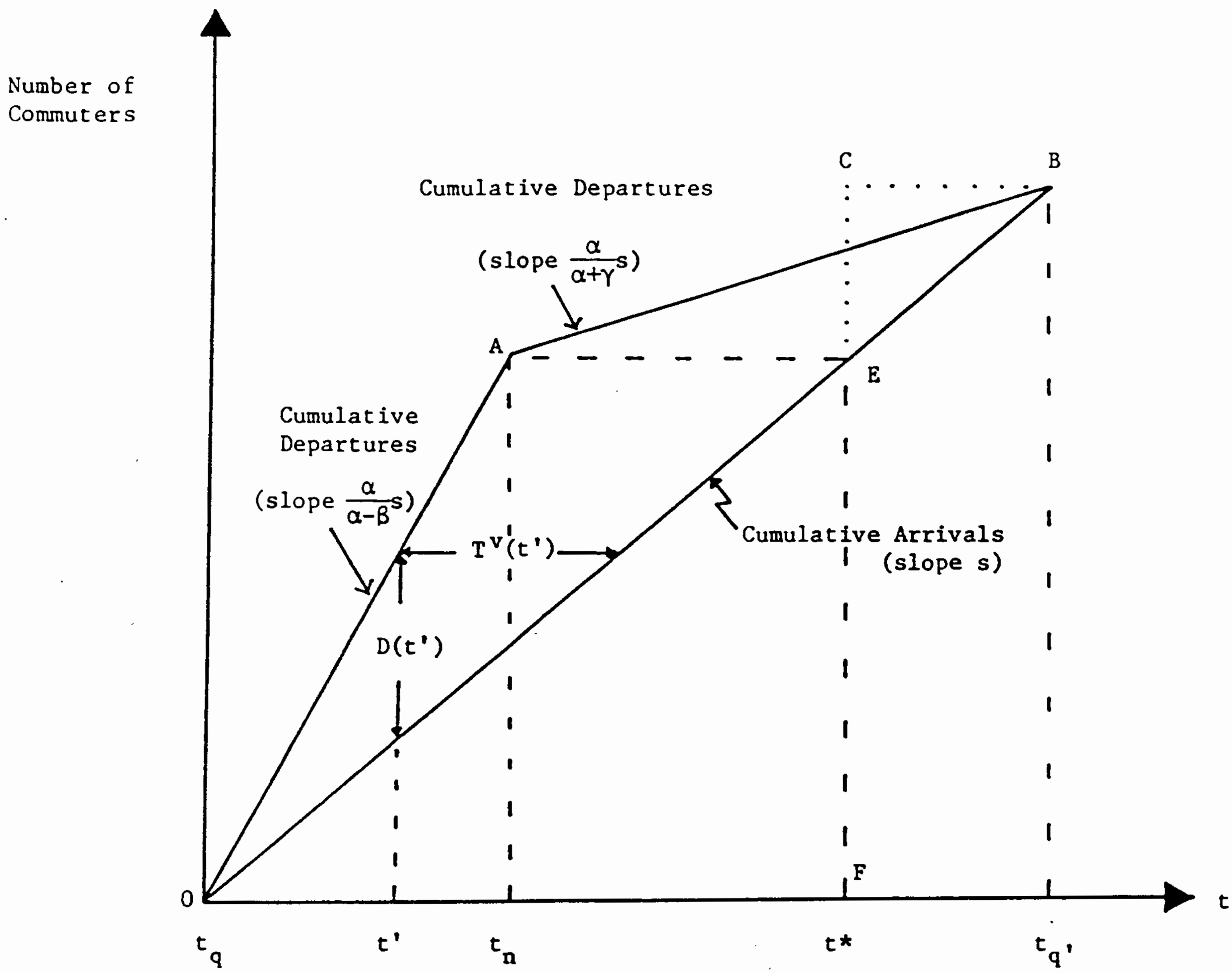
[9] It is assumed that $\alpha > \beta$. The case $\alpha \leq \beta$ is discussed in Arnott et al. [1985].

FIGURE 1

Cumulative Arrivals and Departures, Queue Length, Total Travel Time, Total Time Early and Total Time Late in User Equilibrium



Total Travel Time = OABO

Total Time Early = OEFO

Total Time Late = BECB

$$r(t) = \frac{\alpha s}{\alpha+\gamma}, \qquad t \in (t_n, t_q'). \tag{8}$$

Equating the costs of the first and the last individuals to depart:

$$\beta(t^* - t_q) = \gamma(t_q' - t^*). \tag{9}$$

Since the bottleneck operates at capacity throughout the rush hour:

$$t_q' = t_q + N/s. \tag{10}$$

Combining (9) and (10) and defining $\delta = \beta\gamma/(\beta+\gamma)$, one obtains:

$$t_q = t^* - \frac{\delta}{\beta}\frac{N}{s} \quad , \; t_q' = t^* + \frac{\delta}{\beta}\frac{N}{s}. \tag{11a,11b}$$

Finally, using (5) and (7) one has:

$$t_n = t^* - \frac{\delta}{\alpha}\frac{N}{s}. \tag{12}$$

Total travel time and total schedule delay are identified in Figure 1. Aggregate Travel Time Costs, TTC, Schedule Delay Costs, SDC, and Travel Costs, TC, are:

$$TTC = SDC = \frac{\delta}{2}\frac{N^2}{s}, \tag{13}$$

$$TC = TTC + SDC = \delta\frac{N^2}{s}. \tag{14}$$

It is noteworthy that neither the timing of departures nor aggregate costs depend on the unit cost of travel time, α.

Optimal Capacity

In this section we determine the equilibrium number of users of the road. Following Arnott et al. [1987b] demand for trips is assumed to be:

$$N = np^{-\epsilon}, \tag{15}$$

where n is a parameter characterizing the intensity of demand, p is the 'price' (variable travel cost defined in (4)) of a trip and ϵ is the elasticity of demand with respect to p. The elasticity of demand depends amongst other factors on the availability of substitutes (in the commuting context, alternative transport modes in the city); assuming it to be constant simplifies the analysis. From (14)

$$p = \frac{TC}{N} = \frac{\delta N}{s}. \tag{16}$$

Solving (16) and (15) for N and p yields

$$N(n,s) = n^{\frac{1}{1+\epsilon}} \left\{\frac{s}{\delta}\right\}^{\frac{\epsilon}{1+\epsilon}}, \text{ and} \tag{17a}$$

$$p(n,s) = p(s/n) = \left\{\frac{n\delta}{s}\right\}^{\frac{1}{1+\epsilon}}. \tag{17b}$$

Consumers' surplus is

$$CS(n,s) = \int_{p(s/n)}^{\infty} N(p)dp. \tag{18}$$

From (15), (17b) and (18), the increase in consumers' surplus from a marginal capacity expansion, which we term marginal consumers' surplus, is

$$MCS(n,s) = \frac{dCS}{ds} = \frac{1}{1+\epsilon} \left\{\frac{n}{s}\right\}^{\frac{2}{1+\epsilon}} \delta^{\frac{1-\epsilon}{1+\epsilon}}. \tag{19}$$

If capacity costs are linear[10], with marginal cost k, then optimal capacity is given by

$$s^* = \text{argmax } [CS(n,s) - ks]. \tag{20}$$

Using the first-order condition MCS(n,s) = k and (19) one obtains

$$s^* = \delta^{\frac{1-\epsilon}{2}} [(1+\epsilon)k]^{-\frac{1+\epsilon}{2}} n. \tag{21}$$

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[10]Kraus [1981] has argued that there is a substantial fixed cost to highway construction, implying that costs are affine rather than linear.

3. Time-of-Use Decisions with Full Information

In this section we assume that users learn the precise values of capacity (s) and demand (n) sufficiently early in the day that they can adjust their departure times accordingly. If demand is elastic, some individuals may decide to use an alternative mode of transit rather than travel by car.

Under full information, equations (7), (8), (11a,b), (12), (13), (14), (17a,b) and (18) continue to apply for the realized values of n and s. Let $\hat{s}$ denote the design capacity of the bottleneck and $\sigma \equiv s/\hat{s} \leq 1$ the ratio of actual to design capacity. Design capacity may be thought of as the maximum feasible traffic flow under ideal conditions. Given (15) and (18), consumers' surplus with a given n and σ is:

$$CS(n,\hat{s}\sigma) = \int_{p(\hat{s}\sigma/n)}^{\infty} np^{-\epsilon}dp. \tag{18'}$$

We assume that the joint probability density of n and σ is $f(n,\sigma)$[11], which is independent of design capacity.[12] Expected consumers' surplus (expected values are hereafter denoted by a bar) is thus:

$$\overline{CS}^F(\hat{s}) = \int_0^1 \int_0^\infty \left\{\int_{p(\hat{s}\sigma/n)}^{\infty} np^{-\epsilon}dp\right\} f(n,\sigma)dn\, d\sigma, \tag{22}$$

and marginal expected consumers' surplus

$$\overline{MCS}^F(\hat{s}) = \int_0^1 \int_0^\infty \frac{1}{1+\epsilon} \left\{\frac{\delta}{\sigma}\right\}^{\frac{1-\epsilon}{1+\epsilon}} \left\{\frac{n}{\hat{s}}\right\}^{\frac{2}{1+\epsilon}} f(n,\sigma)dn\, d\sigma, \tag{23}$$

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[11] n and σ may be correlated because, for example, traffic accidents are more likely on busy days.

[12] This can only be approximately true if capacity comes in discrete units (e.g. traffic lanes) and if each unit is working either fully or not at all. This problem is bypassed here by the assumption that capacity is a continuous variable.

where the superscript F denotes the full information regime. Optimal design capacity is

$$\hat{s}* = \text{argmax}\ [\overline{CS}^F(\hat{s}) - k\hat{s}],$$

which, given (23), is

$$\hat{s}* = \delta^{\frac{1-\epsilon}{2}} [(1+\epsilon)k]^{-\frac{1+\epsilon}{2}} \nu, \tag{24a}$$

where

$$\nu = \left\{\int_0^1 \int_0^\infty \sigma^{\frac{\epsilon-1}{1+\epsilon}} n^{\frac{2}{1+\epsilon}} f(n,\sigma)dn\ d\sigma\right\}^{\frac{1+\epsilon}{2}}. \tag{24b}$$

It is easily verified that if demand and capacity are certain, (24a) reduces to (21).

Equation (24a) provides a prescription for computing optimal design capacity. To use it the demand parameters β, γ and ϵ must be known (but not α), as well as the joint probability density function $f(n,\sigma)$. The latter could be estimated by recording s and N over a large number of days, computing n from (17a) and $\sigma = s/\hat{s}$, and constructing the corresponding frequency distribution.

Besides the assumption that demand and capacity are known each day, (24a) was derived on the assumption that demand is served on a single bottleneck. At least with urban commuting, however, individuals generally have a choice of more than one route. The analysis of a single bottleneck cannot legitimately be applied to a link on a road network, because with unpriced congestion the expansion of one link affects the deadweight loss from congestion elsewhere on the network. Prior to practical application it will be necessary to extend the theory to a network with multiple routes (and perhaps multiple origins and destinations).

While the analysis is clearly simplified it is still conceptually superior to earlier methodology which assumed uniform traffic flow and ignored schedule delay costs. Equation (24a) also improves on other formulas for computing optimal design capacity by capturing the effect of variability in both demand and capacity. For example, according to the Highway Research Board [1985], traffic engineers have employed rules of thumb in designing highway capacity, such as to achieve a target level of congestion on the 50th busiest hour of the year assuming that road conditions are ideal. Besides omitting economic variables such a rule neglects capacity variations.

Since expression (24a) is the same as (21) for the nonstochastic case, but with ν in place of n, ν may be interpreted as a certainty-equivalent intensity of demand (i.e. the constant demand intensity for which under ideal conditions design capacity is $\hat{s}*$). This suggests an improved rule of thumb for constructing optimal capacity: Equate the marginal cost of capacity expansion with the expected marginal benefit, taking demand at its certainty-equivalent value and capacity at its design value. Using equation (17a) the certainty-equivalent level of demand can be written

$$\tilde{N} = \nu^{\frac{1}{1+\epsilon}} \left\{\frac{\hat{s}*}{\delta}\right\}^{\frac{\epsilon}{1+\epsilon}} . \tag{25a}$$

Mean demand, meanwhile, is

$$\overline{N} = \int_0^1 \int_0^\infty N(n,\sigma) f(n,\sigma) dn d\sigma \quad = \int_0^1 \int_0^\infty n^{\frac{1}{1+\epsilon}} \left\{\frac{\hat{s}*\sigma}{\delta}\right\}^{\frac{\epsilon}{1+\epsilon}} f(n,\sigma) dn\, d\sigma . \tag{25b}$$

Let $y = N/\bar{N}$ be the ratio of certainty-equivalent demand to mean demand. Then combining (25a) and (25b) one obtains

$$y = \nu^{\frac{1}{1+\epsilon}} / \{\int_0^1 \int_0^\infty n^{\frac{1}{1+\epsilon}} \sigma^{\frac{\epsilon}{1+\epsilon}} f(n,\sigma) dn\, d\sigma\}.$$

The effect of demand and capacity fluctuations on optimal design capacity is summarized in the following proposition (for a precise statement and proof see Appendix 1):

Proposition 1

If $\epsilon < 1$ then, for any mean demand intensity, optimal design capacity is greater with random than with certain capacity. Furthermore, optimal design capacity is the larger: a) the greater the variability in demand, b) the greater the variability in capacity, c) the lower the ratio of mean capacity to design capacity, and d) the lower the correlation between demand and capacity. The opposite is true of all the above when $\epsilon > 1$.

Proposition 1 establishes that if demand for trips is relatively inelastic (which is likely the case in most commuting contexts[13]) greater investment in capacity is warranted if loss of capacity can occur. There are two opposing forces at work. On the one hand, since only a fraction of design capacity is sometimes available, the marginal cost of constructing working capacity is greater in the stochastic case. On the

[13] Work by McFadden [1974], Pucher and Rothenberg [1976] and Small [1983] suggests that the elasticity is about 0.2.

other hand, the expected marginal benefit from actual capacity is also greater as long as demand for trips is price-sensitive. If $\epsilon < 1$ the second factor dominates, and optimal design capacity is greater in the stochastic case.

Proposition 1 also states that, if $\epsilon < 1$, optimal design capacity is increased by a (mean-preserving) spread in the distribution of demand intensity. This result may be compared with that of Brown and Johnson [1969], who considered a public utility which sets price before the intensity of demand is known. In their model, output can be adjusted to meet demand at the set price, but only up to the capacity limit. If demand exceeds capacity, rationing necessarily occurs since the period over which consumption takes place is fixed. On the assumption of a linear demand curve, and that supply is rationed to users with the highest willingness to pay, Brown and Johnson showed that optimal capacity is unambiguously increased by demand uncertainty.[14]

While our findings are similar to Brown and Johnson's, our model differs from theirs in two respects. First, since all users are served in our model there is no rationing, but the cost of the service varies with the amount of queueing. Second, the quality of the service deteriorates when demand is high and users are forced to consume at inconvenient times.[15]

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[14] Visscher [1973] later showed that the Brown-Johnson result is sensitive to the method of rationing. With random rationing, or rationing on the basis of lowest willingness to pay, optimal capacity may be lower than in the nonstochastic case.

[15] Our results may also be compared with those of Kraus [1982], who considered a situation in which travel demand is constant from day to day but known only imperfectly by the planner. Kraus assumed travel time depends on the total number of trips, but ignored time-of-use

4. Time-of-Use Decisions with Zero Information

In Section 3 it is assumed that users know the precise levels of demand and capacity each day. In this section we suppose contrarily that users have no current information on either one, so that they must base their travel mode and departure time decisions on experience. For simplicity we assume that users know the true joint probability distribution of demand and capacity. This we shall refer to as the zero information regime.

With demand and capacity unknown on a given day, individual travel costs will in general not be constant over the rush hour. We shall define equilibrium by a normalized departure rate, $\rho(t)$, where $\rho(t)dt$ is the fraction of users who depart between t and t+dt, such that <u>expected</u> travel costs are independent of t. If there are N users on a given day the departure rate at time t is $N\rho(t)$. The assumption that $\rho(t)$ is invariant from day to day can be justified by the law of large numbers if there are many identical users acting independently, none of whom has information about N.[16]

To describe the equilibrium some additional notation is necessary. Let R(t) be the cumulative distribution of departures, and t_q and t_r the time of first and last departures respectively, so that $R(t_q) = 0$ and

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preferences. On the assumption of a unitary demand elasticity he showed that optimal road capacity is unambiguously increased by the planner's uncertainty about demand.

[16] In reality, some individuals may be intermittent users lacking strong preferences as to when to travel (e.g. those who occasionally shop downtown). Such users are likely to avoid peak congestion. If they tend to travel on the same days, the departure rate in the tails of the rush hour will be relatively high when demand intensity is high, contrary to assumption.

$R(t_r) = 1$. Let $\phi = N/s$ be the ratio of demand to capacity on a given day. ϕ is assumed to have an upper-hemicontinuous c.d.f. $J(\phi)$, and a finite maximum value, ϕ_M. Let $\phi(t)$ be the largest ϕ such that there is no queue at time t. An individual departing at t thus experiences a queue if and only if $\phi > \phi(t)$. Let $\phi^*(t)$ be the value of ϕ such that a user departing at t arrives at t*. ($\phi(t)$ and $\phi^*(t)$ may or may not be elements of the support of $J(\phi)$.) Finally, let t_n be the latest departure time at which users never arrive late.

The qualitative characteristics of equilibrium are shown in Figure 2. There are 4 regions, defined by whether users arrive early or late and by whether or not they have to queue. In regions I and II users arrive early, in regions III and IV they are late. In regions I and III users experience no queue, in regions II and IV they do. If there is no queue a user arrives early if $t < t^*$ and late if $t > t^*$. The boundary between regions I and III thus occurs at $t = t^*$.[17] The boundary between regions II and IV in which queuing occurs is defined by the locus $\phi^*(t)$. Regions I and III are separated from II and IV by the locus $\phi(t)$.[18] $\phi(t) < \phi_M$ for all $t \in (t_q, t_r)$, since otherwise expected travel costs would depend on departure time. Moreover, $\phi^*(t) > \phi(t)$ for $t < t^*$ and $\phi^*(t) < \phi_M$ for $t > t_n$.

Let $D(t,\phi)$ denote the length of the queue at time t when $N/s = \phi$. Travel costs in each regime can then be written:

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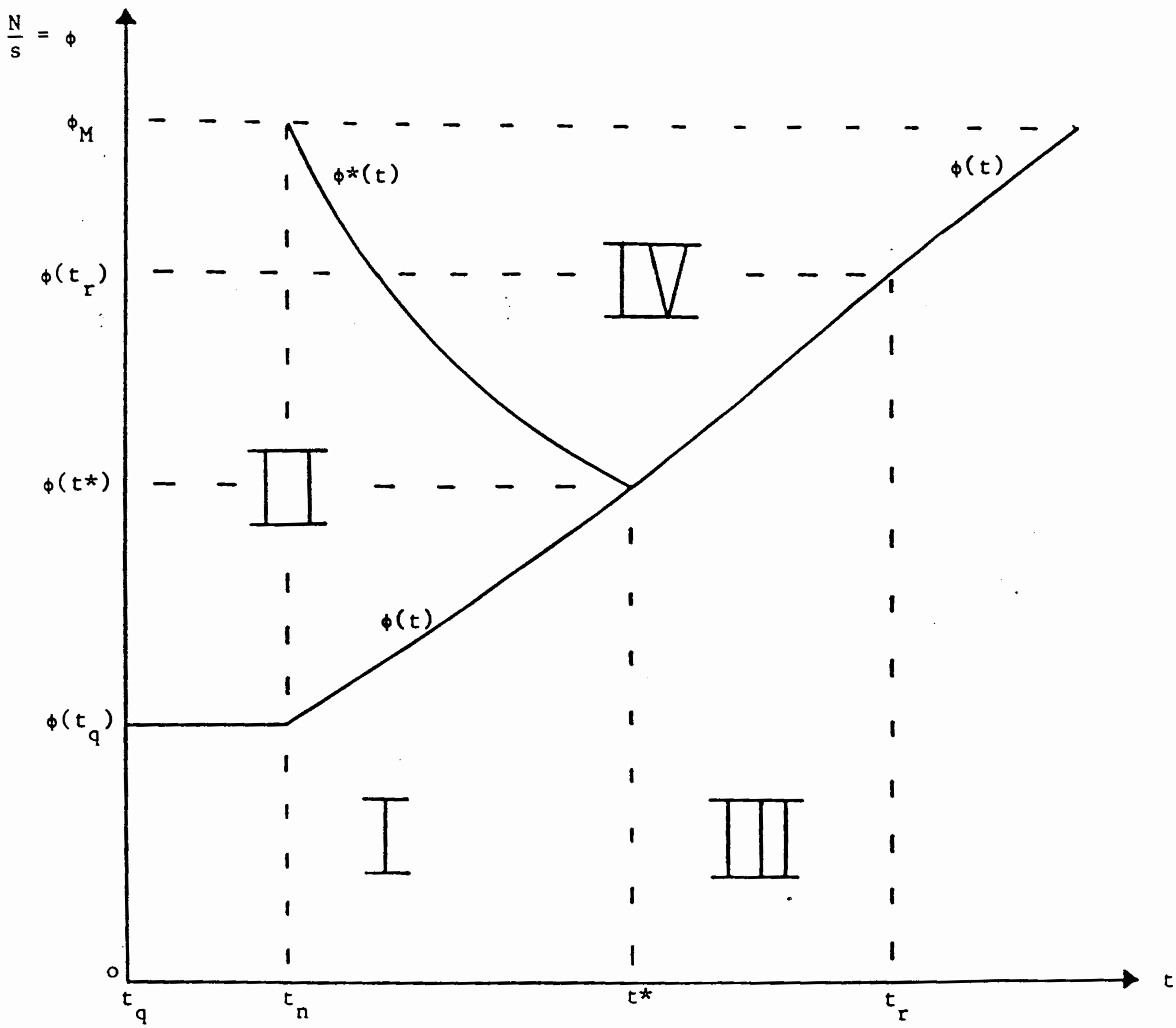
[17] In Figure 2 it is assumed that $t_r > t^*$, although we shall show that $t_r = t^*$ is also possible.

[18] In footnote 19 we show that this locus is horizontal for $t \in (t_q, t_n)$. We also show that $\phi(t)$ is upward-sloping for $t > t_n$ and that $\phi^*(t)$ is downward-sloping for $t \in (t_n, t^*)$.

FIGURE 2

Travel Experience in Equilibrium
as a Function of Departure Time

Key:	I	No queueing,	early arrival
	II	Queueing,	early arrival
	III	No queueing,	late arrival
	IV	Queueing,	late arrival



$$C^{I}(t,\phi) = \beta(t^*-t), \tag{26a}$$

$$C^{II}(t,\phi) = \beta(t^* - t - D(t,\phi)/s) + \alpha D(t,\phi)/s, \tag{26b}$$

$$C^{III}(t,\phi) = \gamma(t-t^*), \tag{26c}$$

$$C^{IV}(t,\phi) = \gamma(t + D(t,\phi)/s - t^*) + \alpha D(t,\phi)/s. \tag{26d}$$

As seen in Figure 2 there are three departure time intervals to consider:

a) $[t_q,t_n)$ in which users always arrive early,

b) $(t_n,t^*]$ in which, depending on ϕ, users may arrive early or late,

c) $(t^*,t_r]$ in which users always arrive late.

In equilibrium, expected travel costs must be independent of departure time. To derive the equilibrium we consider each interval in turn.

a) $[t_q,t_n)$

To see that this interval is non-empty note that the first individual to depart must do so before t*, since otherwise he could depart at t* and experience both zero schedule delay and zero travel time, which is inconsistent with equilibrium. Since he faces zero travel time, he always arrives strictly early. By continuity of R(t) there is a time interval over which arrival is never late.

Given (26a), (26b) and Figure 2, expected travel costs are (using the Lebesque-Stieltjes integral)

$$C(t) = \beta \left\{ \int_0^{\phi(t)} (t^* - t)dJ(\phi) + \int_{\phi(t)}^{\phi_M} (t^* - t - D(t,\phi)/s)dJ(\phi)\right\}$$

$$+ \alpha \int_{\phi(t)}^{\phi_M} D(t,\phi)/s \; dJ(\phi)$$

$$= \beta(t^*-t) + (\alpha-\beta) \int_{\phi(t)}^{\phi_M} D(t,\phi)/s \; dJ(\phi). \tag{27}$$

The time derivative of C(t) is

$$\dot{C}(t) = -\beta + (\alpha-\beta) \int_{\phi(t)}^{\phi_M} \dot{D}(t,\phi)/s \; dJ(\phi). \tag{28}$$

Setting $\dot{C}(t) = 0$ and using the relationships $D(t,\phi(t)) = 0$ and

$\dot{D}(t,\phi) = N\rho(t) - s$ for $\phi > \phi(t)$, we have

$$\dot{C}(t) = -\beta(1+Z(t)) + \alpha Z(t) = 0, \tag{29}$$

where

$$Z(t) = \int_{\phi(t)}^{\phi_M} (\phi\rho(t)-1) dJ(\phi). \tag{30}$$

Z(t) is the rate of increase in expected travel time from marginally postponing departure. Thus, when an individual departs dt later, he arrives on average (1+Z(t))dt later, and increases his expected travel time by Z(t)dt. Since arrival is always early for $t \in [t_q, t_n)$, leaving dt later decreases expected schedule delay costs by $\beta(1+Z(t))dt$, which is the first term on the RHS of (29). We show in Appendix 2 that (29) defines a constant value of $\rho(t)$. The departure rate is thus constant over the interval $[t_q, t_n)$, as was true of the deterministic model (viz. Figure 1).

b) $t \in (t_n, t^*]$

In this departure interval the individual is sometimes early and sometimes late. Given (26a), (26b), (26d) and Figure 2, expected travel costs are

$$C(t) = \beta \left\{ \int_0^{\phi(t)} (t^* - t) dJ(\phi) + \int_{\phi(t)}^{\phi^*(t)} (t^*-t-D(t,\phi)/s) dJ(\phi) \right\}$$

$$+ \gamma \int_{\phi^*(t)}^{\phi_M} (t + D(t,\phi)/s - t^*)dJ(\phi) + \alpha \int_{\phi(t)}^{\phi_M} D(t,\phi)/s \; dJ(\phi). \quad (31)$$

$$\dot{C}(t) = -\beta Z^e(t) + \gamma Z^1(t) + \alpha Z(t) = 0, \quad (32)$$

where

$$Z^e(t) = \int_{\phi(t)}^{\phi^*(t)} (\phi\rho(t)-1)dJ(\phi) + J(\phi^*(t)) \quad (33)$$

is the rate of decrease in expected early arrival time from delaying departure,

$$Z^1(t) = \int_{\phi^*(t)}^{\phi_M} (\phi\rho(t)-1)dJ(\phi) + 1 - J(\phi^*(t)) \quad (34)$$

is the rate of increase in expected late arrival time from delaying departure, and $Z(t)$ is as defined in (30). Equation (32) has an interpretation analogous to (29). We show in Appendix 2 that $\rho(t)$ is monotonically weakly decreasing for $t \in (t_n, t^*]$.

c) $t \in (t^*, t_r]$ (if $t_r = t^*$ this interval is degenerate)

In this departure interval the individual is always late. Given (26c), (26d) and Figure 2, expected travel costs are

$$C(t) = \gamma \left\{ \int_0^{\phi(t)} (t-t^*)dJ(\phi) + \int_{\phi(t)}^{\phi_M} (t + D(t,\phi)/s - t^*)dJ(\phi) \right\}$$

$$+ \alpha \int_{\phi(t)}^{\phi_M} D(t,\phi)/s \; dJ(\phi)$$

$$= \gamma(t-t^*) + (\alpha+\gamma) \int_{\phi(t)}^{\phi_M} D(t,\phi)/s \; dJ(\phi). \quad (35)$$

$$\dot{C}(t) = \gamma(1+Z(t)) + \alpha Z(t) = 0. \quad (36)$$

(36) has an interpretation analogous to (29) and (32). Again we show in

Appendix 2 that $\rho(t)$ is monotonically weakly decreasing.

Conditions (29), (32) and (36) ensure that expected travel costs are constant in each departure interval. Since $\phi^*(t_n) = \phi_M$, and $\phi(t^*) = \phi^*(t^*)$, expected travel costs are also continuous at t_n and t^*, and hence constant over the whole interval $[t_q, t_r]$. It is now possible to state:

Proposition 2

The normalized departure rate $\rho(t)$ is monotonically weakly decreasing over the departure time interval, and the cumulative departure distribution $R(t)$ is concave.

Proposition 2 follows immediately from the fact that, as stated earlier, $\rho(t)$ is constant on the interval (t_q, t_n) and monotonically decreasing thereafter. To see why, note that the later one departs after t_n the more likely one is to be late, and hence the more likely to experience an increase rather than decrease in schedule delay costs from marginally postponing departure. To compensate, expected travel time costs must increase at a decreasing rate and eventually decline, which requires that the departure rate decrease over time.

Given the concavity of $R(t)$ it follows that if queuing occurs on a given day it is over a continuous time interval beginning at t_q. Queuing time is

$$D(t,\phi)/s = \text{Max}\ [0,\ t_q + \phi R(t) - t]. \tag{37}$$

t_n is defined implicitly by the condition

$$t_q + \phi_M R(t_n) = t^*,$$

while

$$\phi(t) = (t-t_q)/R(t), \tag{38}$$

$$\phi^*(t) = (t^*-t_q)/R(t).^{19} \tag{39}$$

The departure rate function is shown in Figure 3, where as in Figure 2 it is assumed that $t_r > t^*$.)[20]

With regard to the timing of the departure interval it turns out that there are two possibilities, $t_r > t^*$ and $t_r = t^*$. To establish which of the two cases applies we first define the mean value of ϕ:

$$\phi = \int_0^{\phi_M} \phi dJ(\phi),$$

the fractile:

$$\bar{\phi} = J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\},$$

and the mean of ϕ for values greater than this fractile:

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[19]It is now possible to establish that the slopes of $\phi(t)$ and $\phi^*(t)$ are as shown in Figure 2. From (38) we have along $\phi(t)$

$$d\phi(t)/dt = [1-\phi(t)\rho(t)]/R(t) \geq 0 \text{ for } t > t_n$$

since the departure rate is less than capacity when a queue disappears. The second derivative may be positive or negative. For $t \in (t_q,t_n)$ the boundary between regimes I and II is horizontal since $\rho(t)$ and $\phi(t)$ are both constant.

From equation (39) we have along $\phi^*(t)$

$$d\phi^*(t)/dt = -\phi^*(t)\rho(t)/R(t) < 0.$$

The second derivative is

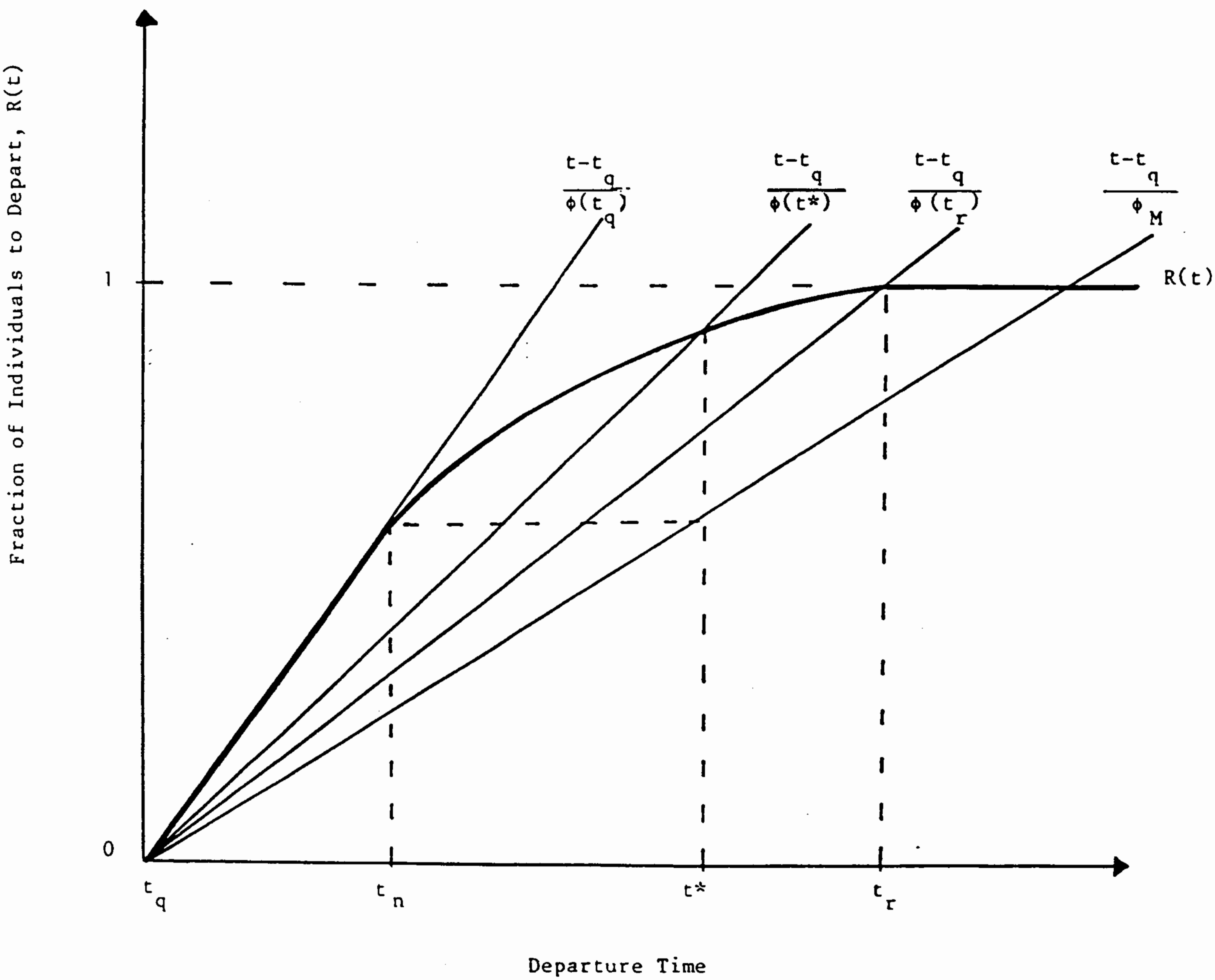
$$d^2\phi^*(t)/dt^2 = [2\rho^2(t) - R(t)d\rho(t)/dt]/R^2(t) \geq 0,$$

since $d\rho(t)/dt \leq 0$.

[20]If the distribution function $J(\phi)$ is everywhere continuous then so is $\rho(t)$, and $R(t)$ is smooth. If $J(\phi)$ is discontinuous at some ϕ (i.e. there is a probability mass at ϕ) then $R(t)$ is kinked downward. The departure distribution in Figure 1 is a special case with $J(\phi) = H(\phi-N/s)$, where H is the Heaviside function. Here, $R(t)$ has a single kink at t_n.

FIGURE 3

Equilibrium Departure Rate Schedule



$$\phi = \int_{\tilde{\phi}}^{\phi_M} \phi dJ(\phi) \,/(1-J(\phi)) = \frac{\alpha+\gamma}{\gamma} \int_{\tilde{\phi}}^{\phi_M} \phi dJ(\phi).$$

The case $t_r > t^*$ is described in Lemma 1 (for the proof, see Appendix 3):

<u>Lemma 1</u>

If $\tilde{\phi} > \frac{\alpha+\gamma}{\beta+\gamma} \hat{\phi}$ (40)

then the first departure occurs at

$$t_q = t^* - \frac{\gamma}{\beta+\gamma} \hat{\phi} \qquad (41)$$

and the last departure at

$$t_r = t_q + \tilde{\phi} > t^*. \qquad (42)$$

Expected travel costs equal those of the first user to depart:

$$C = \beta(t^*-t_q) = \frac{\beta\gamma}{\beta+\gamma} \hat{\phi}. \qquad (42')$$

The case $t_r > t^*$ is similar to the full information regime in that departures occur both before and after t^*. However, since $\hat{\phi} > \bar{\phi}$ it follows from (11a) and (41) that the first departure occurs earlier in the zero information regime than in the full information regime with $\phi = \bar{\phi}$. Moreover, it follows from (14) and (42') that individual expected travel costs are greater (in the same sense) in the zero than the full information regime.

The case $t_r = t^*$ is described in Lemma 2 (for the proof see Appendix 4):

<u>Lemma 2</u>

If $\tilde{\phi} \le \frac{\alpha+\gamma}{\beta+\gamma} \hat{\phi}$ (43)

then the first departure occurs at t_q, defined implicitly by the equation

$$(t^*-t_q)\{\beta + (\alpha+\gamma)[1-J(t^*-t_q)]\} - (\alpha+\gamma)\int_{t^*-t_q}^{\phi_M} \phi dJ(\phi) = 0, \qquad (44)$$

and the last departure occurs at

$$t_r = t^*. \qquad (44')$$

Expected travel costs are

$$C = C(t_q) = \beta(t^*-t_q). \qquad (45)$$

Since conditions (40) and (43) in Lemmas 1 and 2 are complementary, the cases $t_r > t^*$ and $t_r = t^*$ are exhaustive. There are no equilibria with $t_r < t^*$, at least in the absence of a mass of departures at t_r.[21] To see why, suppose $t_r < t^*$ and consider an individual departing at $t = t_r+\epsilon < t^*$. If $\phi < \phi(t)$, the individual is better off departing at t than t_r, since he arrives less early and incurs no travel time costs. If $\phi > \phi(t_r)$ the individual is also better off, since he arrives at the same time as when departing at t_r, but spends less time queueing. Departure after t_r would thus be preferable to departure at t_r, which would be inconsistent with equilibrium. Combining this result with Lemmas 1 and 2 yields

Proposition 3

In the zero information regime the last user to depart does so either at t* or after t*.

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[21] An equilibrium with a mass of departures is shown in Arnott et al. [1985] to exist in the nonstochastic case with $\alpha \le \beta$.

Whether $t_r > t^*$ or $t_r = t^*$ turns out to be a determining factor of the welfare effects of information, as will be shown in Section 6.

In the next section we compare the efficiency and optimal capacity of the full information and zero information regimes. To do so we require expected consumers' surplus in the zero information regime. Since individuals are by assumption risk-neutral, they decide whether or not to travel on the basis of the expected price $\bar{p}$ (= average travel cost C). For a given realization of demand intensity the demand for trips is thus given by

$$N = n(\bar{p})^{-\epsilon}.$$

Total benefits for given n equal the area under the inverse demand curve:

$$TB^0(n) = \int_{\bar{p}}^{\infty} n(p)^{-\epsilon} dp + n(\bar{p})^{1-\epsilon}, \tag{46}$$

where the superscript 0 denotes the zero information regime. Total expected benefits are obtained by integrating (46) over n and σ:

$$\overline{TB}^0 = \int_0^1 \int_0^{\infty} \left\{ \int_{\bar{p}}^{\infty} np^{-\epsilon} dp + n(\bar{p})^{1-\epsilon} \right\} f(n,\sigma) dn\, d\sigma$$

$$= \int_0^1 \int_0^{\infty} \left\{ \int_{\bar{p}}^{\infty} np^{-\epsilon} dp \right\} f(n,\sigma) dn\, d\sigma + \bar{n}\, (\bar{p})^{1-\epsilon}. \tag{47}$$

Now, total expected costs are

$$\overline{TC} = \bar{N}\, \bar{p} = \bar{n}\, (\bar{p})^{1-\epsilon}. \tag{48}$$

Subtracting (48) from (47) one obtains expected consumers' surplus:

$$\overline{CS}^0(\hat{s}) = \int_0^1 \int_0^{\infty} \left\{ \int_{\bar{p}(\hat{s})}^{\infty} np^{-\epsilon}\, dp \right\} f(n,\sigma) dn\, d\sigma, \tag{49}$$

with $\bar{p}(\hat{s})$ given by (42') or (45). The following additional relationships are derived in Appendix 5:

$$\overline{MCS}^0(\hat{s}) = -\bar{N}\frac{d\bar{p}}{d\hat{s}}, \tag{50}$$

$$\frac{d\bar{p}}{d\hat{s}}\frac{\hat{s}}{\bar{p}} = -\frac{1}{1+\epsilon}, \tag{51}$$

$$\frac{d\overline{MCS}^0(\hat{s})}{d\hat{s}}\frac{\hat{s}}{\overline{MCS}^0(\hat{s})} = -\frac{2}{1+\epsilon}. \tag{52}$$

5. Comparison of the Full Information and Zero Information Regimes

In this section we compare the efficiency and optimal capacities of the two polar information regimes. One regime will be said to be more efficient than the other if, for all levels of design capacity, expected consumers' surplus is higher. To rank the efficiency of the two regimes we shall make use of the following proposition (proved in Appendix 6):

Proposition 4

If $\epsilon < 1$ the more efficient regime has the lower marginal consumers' surplus for any level of design capacity. The opposite is true when $\epsilon > 1$.

Since a lower marginal consumers' surplus implies a lower optimal design capacity, with $\epsilon<1$ (resp. $\epsilon>1$) the more efficient regime has the lower (resp. higher) design capacity. Both the efficiency and optimal capacities of the two regimes can thus be ranked by comparing the respective marginal consumers' surpluses from capacity expansion. For the full information regime (using equations (23) and (24b))

$$\overline{MCS}^F(\hat{s}) = \frac{1}{1+\epsilon}\,\delta^{\frac{1-\epsilon}{1+\epsilon}}\,\hat{s}^{-\frac{2}{1+\epsilon}}\,\nu^{\frac{2}{1+\epsilon}}, \tag{53}$$

where ν is given by (24b). For the zero information regime (using equations (50) and (51))

$$\overline{MCS}^0(\hat{s}) = \frac{\bar{n}\,\bar{p}(\hat{s})^{1-\epsilon}}{\hat{s}(1+\epsilon)} \tag{54}$$

with $\bar{p}(\hat{s})$ given by (42′) or (45).

It turns out that a definitive comparison of the two regimes can be obtained when fluctuations occur only in capacity:

THEOREM 1

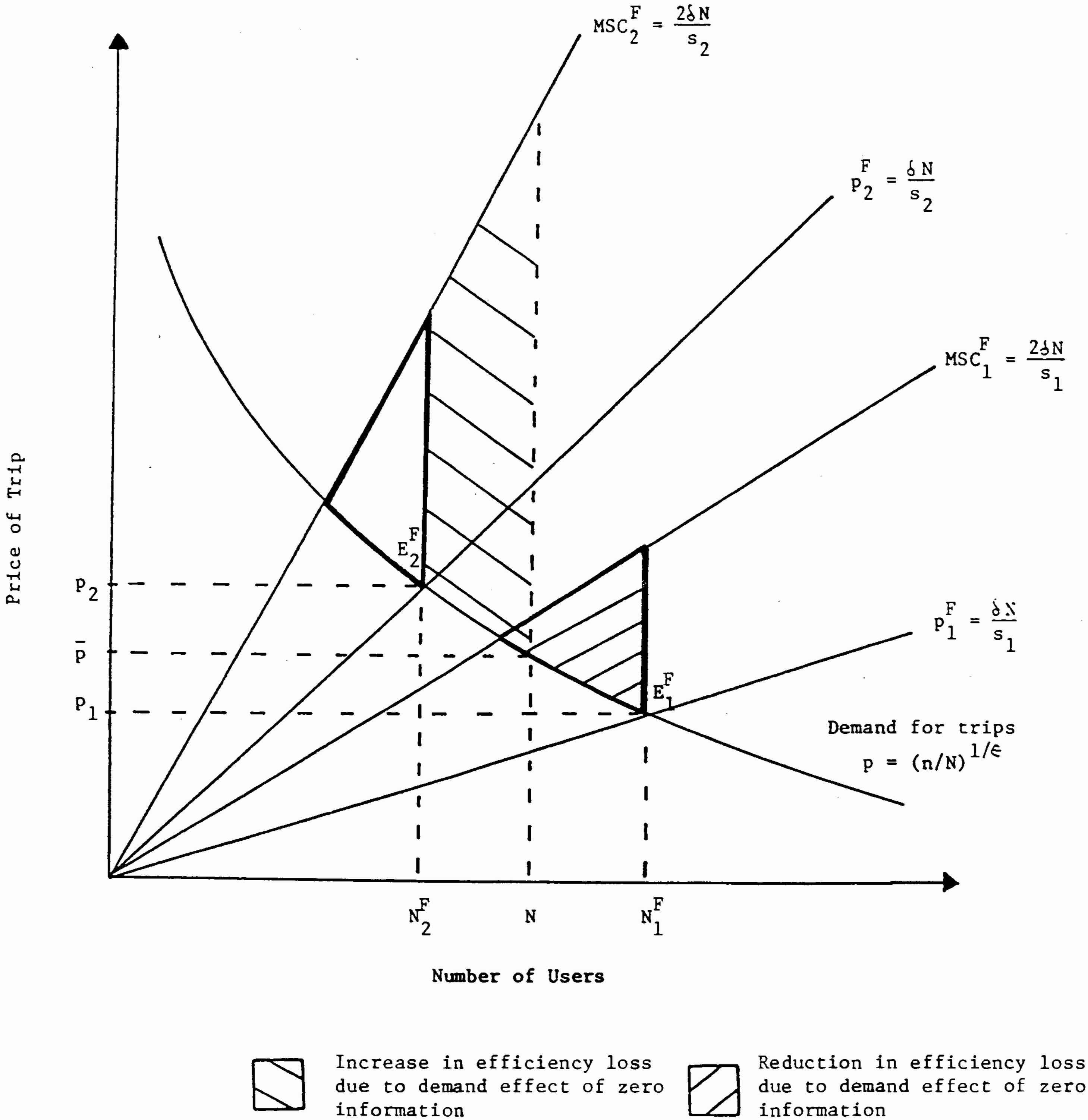
> If fluctuations occur in capacity, but not in demand, then the full information regime is more efficient than the zero information regime. Furthermore, if $\epsilon < 1$ (resp. $\epsilon > 1$) then optimal design capacity is larger (resp. smaller) in the zero information regime.

Theorem 1 is proved in Appendix 7. Some intuition for the proposition can be gleaned from Figure 4.[22] Suppose that there are two equiprobable capacity levels, s_1 and $s_2 < s_1$. Consider first the case of full information about capacity. From (16) the 'supply' curve for trips as a function of N with $s = s_i$ is $p_i^F = \delta N/s_i$, and the demand curve $p = (n/N)^{1/\epsilon}$. Equilibrium occurs at the intersection, E_i^F, of the supply and demand curves. The marginal social cost of an extra trip is given by the

[22] This figure is adapted from Arnott et al. [1987b].

FIGURE 4

Comparison of the Full Information and Zero Information
Regimes with Fluctuations only in Capacity
Example with Two Possible Capacities



schedules labelled MSC_i^F. Because there is a queuing externality, too many trips are taken when $\epsilon>0$. The efficiency loss is shown by the heavily bordered triangular areas.

There are two essential differences between the full information and zero information regimes. First, in the full information regime supply equals demand for each capacity level. In the zero information regime, drivers choose the number of trips on the basis of the average price, so that the number of trips is independent of the capacity on a given day. Second, the 'supply' curves are different for the two regimes at each capacity level since the departure rates differ.

These differences can be decomposed diagramatically. First, imagine holding the supply curves fixed at their position in the full information regime, but imposing the constraint that the number of trips, N, be the same for the two capacity levels, and such that 'average' supply equals demand. This first effect generates a change in the efficiency loss indicated by the shaded areas. When $s=s_2$ the efficiency loss is increased because of greater demand. When $s=s_1$, however, the efficiency loss due to queuing is reduced (although if demand falls sufficiently, it may end up below its efficient level).

The second effect comes about from shifts in the supply curves. How they shift, and what effect the shifts have on efficiency loss, is difficult to determine. In both regimes travel is inefficient because of queuing, and it is unclear <u>a priori</u> which is more efficient. Theorem 1 reveals, however, that with fluctuations in capacity alone the sum of the two effects results in the full information regime being more efficient.

Suppose now that capacity is fixed, but that demand is stochastic.

Here, the results are less definitive:

THEOREM 2

If fluctuations occur in demand, but not in capacity, then whether the full information regime or the zero information regime is the more efficient, and which has greater optimal capacity, depends on parameter values.

Theorem 2 is proved in Appendix 8. As in the stochastic capacity case, the difference between the full and zero information regimes can be decomposed. First there is an 'average price effect' that, in the zero information regime, the number of trips is taken on the basis of the average price, while in the full information regime demand is determined by the day-specific price. The effect on efficiency is shown in Figure 5 by the shaded areas. There is a reduction in the efficiency loss due to excessive trips on days with low demand, but an increase in loss on days with high demand.

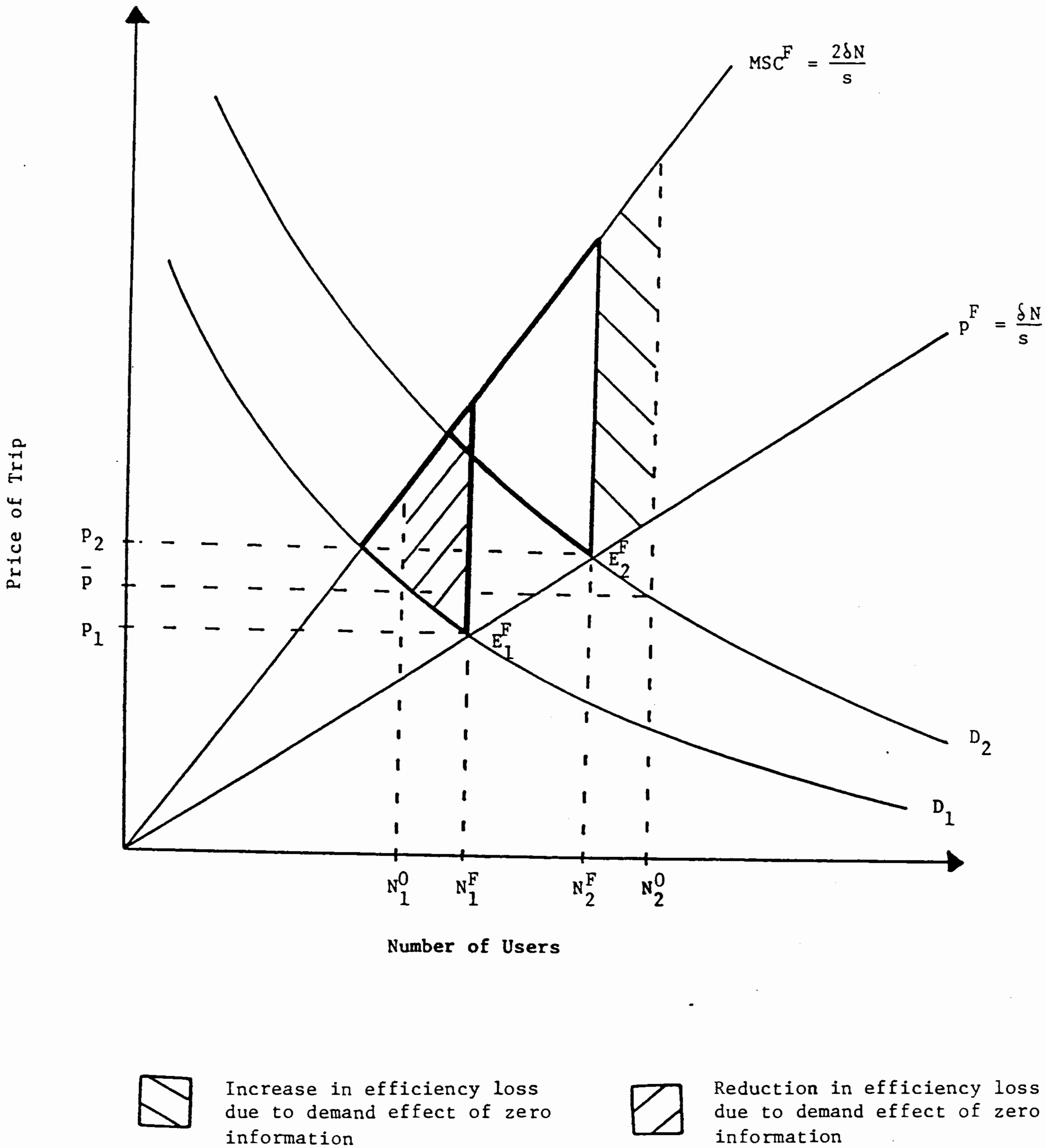
Second, there is a 'departure rate effect': for a given number of drivers the departure schedule differs between the regimes and so therefore do the supply curves. As true of stochastic capacity the net result of the two effects is difficult to determine. Theorem 2 reveals that the relative efficiency of the two regimes is parameter-dependent.

Example

Additional insight can be obtained by considering an example in which $\epsilon = 0$ and demand intensity has a two-point distribution:

FIGURE 5

Comparison of the Full Information and Zero Information Regimes with Fixed Capacity and Two Possible Demand Intensities



$$n = \begin{cases} n_1 & \text{with probability } 1-\pi \\ n_2 > n_1 & \text{with probability } \pi \end{cases} . \tag{55}$$

This example can be thought of as illustrating a situation where there is an occasional unanticipated surge in demand.[23] The relationship between marginal consumers' surplus from capacity expansion in the two regimes is shown in Figure 6. (Derivations are found in Appendix 8.) For $\pi < \gamma/(\alpha+\gamma)$ the downward-sloping locus divides the parameter space into a lower region where $t_r > t^*$ and an upper region where $t_r = t^*$. (For $\pi > \gamma/(\alpha+\gamma)$ $t_r > t^*$ always.) The upward-sloping locus defines a lower region where consumers' surplus is higher in the full information regime and an upper region in which the reverse is true. Figure 6 shows that the zero information regime is more efficient when there is a small probability of a relatively large demand shock.

Because the elasticity of demand is zero in this example the 'average price effect' is absent and only the 'departure rate effect' plays a role. A tentative explanation for why the zero information regime is more efficient in the upper left region in Figure 6 is as follows. On days of low demand, individuals in the zero information regime spread out their departures more than in the full information regime because of their perception that it may be a high demand day. Because demand is much greater on high demand days (n_2/n_1 is large) the effect on the low demand day departure schedule is appreciable. This

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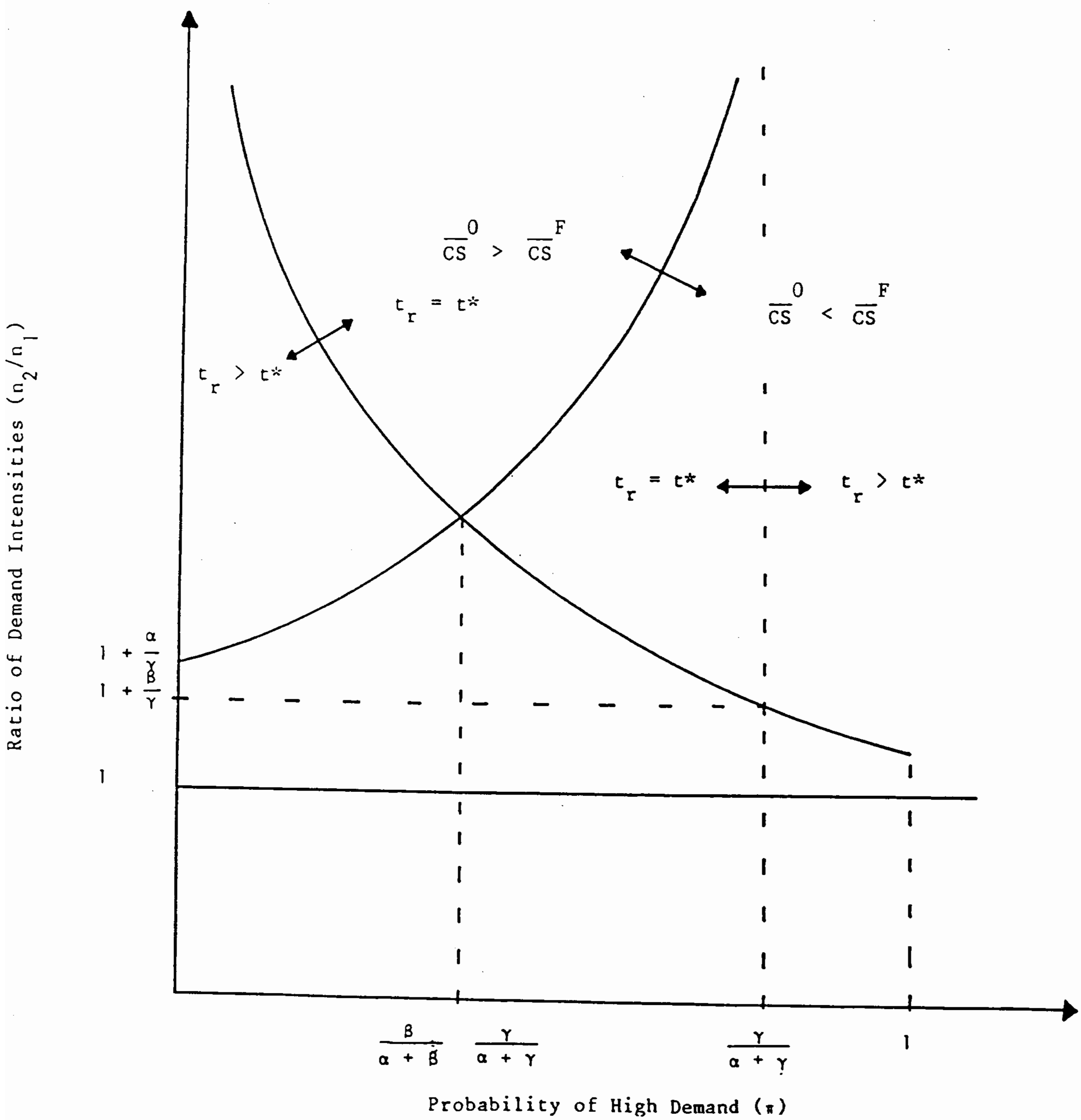
[23]The example is not appropriate for a situation in which a particular group of individuals consistently travels only during peak demand periods, since they will presumably employ a different time-of-use decision rule than regular users.

FIGURE 6

Comparison of Consumers' Surplus in the Full Information and Zero Information Regimes with Fixed Capacity and Two Possible Demand Intensities



reduces the amount of time spent queuing, and hence improves efficiency relative to the full information regime on low demand days. The opposite is true on high demand days. But because the probability of high demand is relatively low (π is small) the efficiency gain on low demand days outweighs the efficiency loss on high demand days, resulting in the zero information regime being overall more efficient.

6. Partial Information

In the two preceding sections we considered two polar information regimes: one of full information, in which users know demand and capacity precisely each day, and one of zero information, in which users know only the unconditional joint probability distribution of capacity and demand. Whereas these extremes serve as useful benchmarks, situations in which users have some, but imperfect current information are likely to be more common.

Since a general analysis of partial information would be conceptually and analytically difficult we focus here on a simplified situation in which demand is fixed and capacity fluctuates between two levels. In the commuting context one can imagine road capacity varying between a normal value in good weather and a lower value under adverse weather conditions such as rain and snow. Each day, weather forecasts provide commuters with updated probability estimates as to whether capacity wil be normal or reduced. Using this example we show that an incremental gain in information can actually be welfare-reducing, despite the fact that when capacity alone is stochastic full information yields higher welfare than zero information. The example suggests that whether positive benefits accrue from more information has to be tested case by case.

Example

For the example in question we assume $\epsilon = 0$ and take the unconditional probability distribution of capacity to be:

$$s = \begin{cases} s_1 & \text{with probability } 1-\pi \\ s_2 < s_1 & \text{with probability } \pi \end{cases} . \tag{56}$$

The parameter π can be interpreted as the probability of an event such as bad weather or an accident that reduces capacity below its design level, s_1. The expected price of a trip is derived as a function of the probability π in Appendix 9. Figure 7 shows one of the possibilities in which the expected price is a concave function over most of its range except at a critical probability $\pi_c = \beta(s_2/s_1)/[(\alpha+\beta\gamma)(1-s_2/s_1)]$.

Now, in the zero information regime the probability of s_2 is π every day. In the full information regime it is either 0 or 1. From Figure 7 it is clear that with full information (whereby the probability of capacity reduction is 0 a proportion $1-\pi$ of the time and 1 a proportion π of the time) the average price of a trip, and hence expected travel costs, are lower than with zero information.[24]

With partial information, however, the probability of reduced

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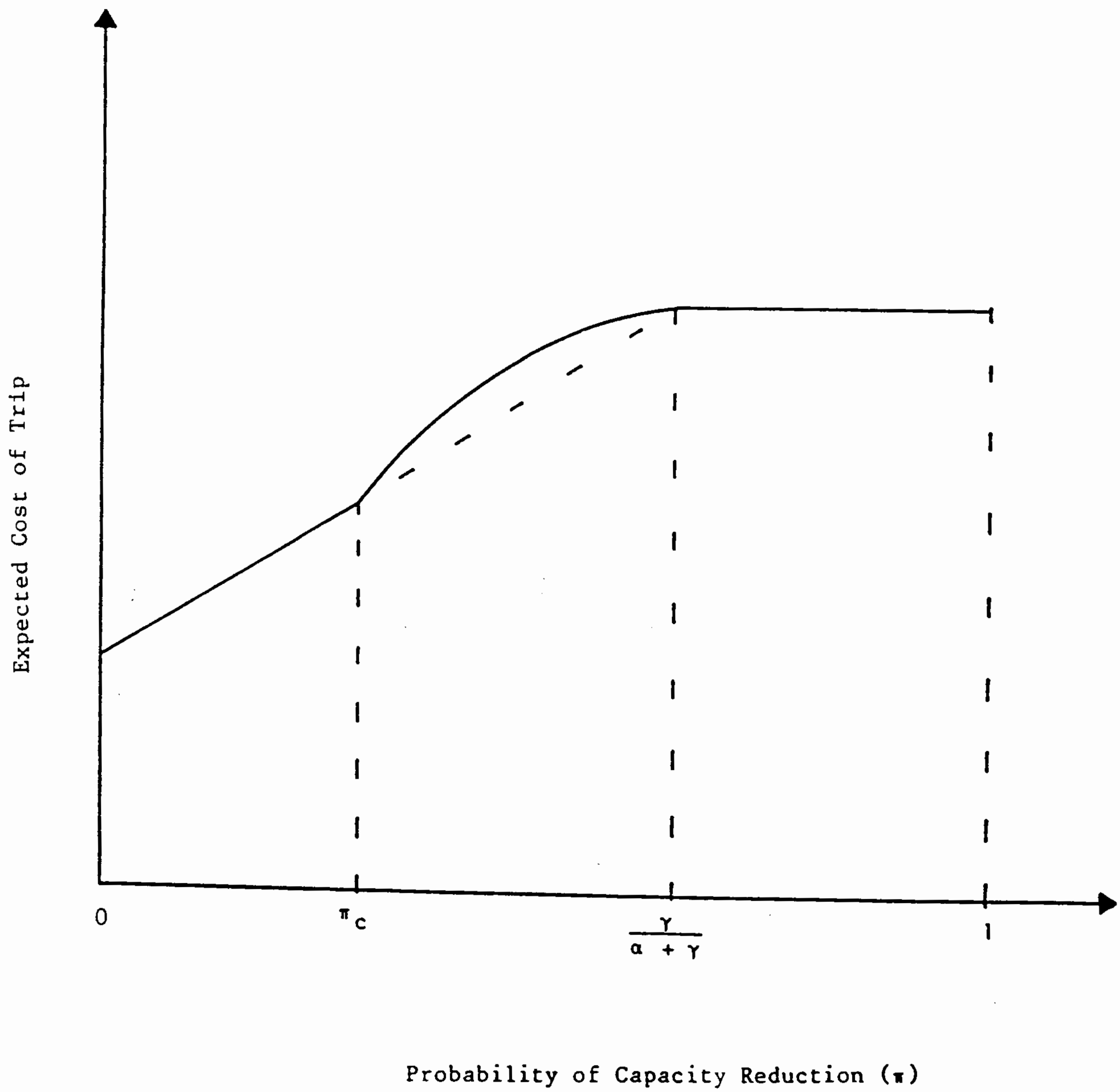
[24]For $\pi > \gamma/(\alpha+\gamma)$ the expected cost of a trip is the same as if capacity were s_2 with certainty. Within this interval neither reducing the probability of a capacity loss nor increasing design capacity (s_1) yields an expected gain in welfare.

FIGURE 7

Expected Trip Cost as a Function
of the Probability of Capacity Reduction



capacity on a given day fluctuates around π. If π is close to π_c, and if the variation in probability about π is relatively small, then the expected price of a trip is <u>greater</u> than with zero information.[25] Imperfect information can thus be welfare-reducing.

7. Concluding Remarks

In this paper we have investigated the impact of information on individuals' time of use of a congestable facility which is subject to stochastic fluctuations in demand and capacity. We showed in the case of full information that, with a demand elasticity less than unity, optimal design capacity is greater: a) the lower is mean capacity relative to design capacity, b) the greater the variability in demand and capacity for given mean values and c) the more negatively correlated are demand and capacity.

We showed that when capacity alone is stochastic, expected welfare is lower with zero information than full information, while with a demand elasticity less than unity optimal capacity is greater. But if demand alone is stochastic, capacity and welfare rankings are parameter-dependent. Moreover, a marginal improvement in information can be welfare-reducing even if capacity alone is stochastic. These results raise doubts about the benefits of providing public information, at least absent tolls or other means of regulation.

There are several directions in which the analysis could be extended. Perhaps the most important next step in the analysis is to

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[25] This is true whether capacity is positively or negatively serially correlated.

derive the socially optimal time-of-use schedule and to see whether it is decentralizable with tolls or other regulatory instruments. In the special case of nonstochastic and price-insensitive demand and a two-point capacity distribution it can be shown that the optimal departure rate is a <u>convex</u> function of time, in contrast to the equilibrium rate which is concave. The optimum can be decentralized by a time-varying and capacity-independent toll paid at the head of the queue. Whether these results hold true for other distribution functions, or for demand fluctuations, remains to be determined. There is also the question how to combine information dissemination with tolls to design the most efficient scheme for relieving congestion.

We have assumed no balking once the decision to make a trip has been made. In practice, road users may decide to cancel a trip or choose an alternate route if traffic turns out to be particularly heavy. With other congestable facilities such as the telephone, the cost of initiating usage may be quite low, so that users can abort an attempt and try again later.

We have assumed that the probability distribution of capacity (and demand) is independent of design capacity. In practice, there may exist a tradeoff between the capacity at which a facility is operated and the probability or magnitude of loss of capacity. For example, if highway shoulders are used as travel lanes during peak hours they will be unavailable for stalled vehicles or traffic enforcement (Ju <u>et al</u>. [1987]). A proper definition of capacity would thus include a reliability coefficient which would play a role in determining optimal design capacity, as well as maintenance and repair policy.[26]

As discussed earlier 3 the analysis of a single bottleneck in isolation is unsatisfactory if it is part of a system because, with unpriced congestion, policies adopted on one link will affect the efficiency loss due to congestion elsewhere on the network. For practical applications the model needs to be extended to multiple bottlenecks and multiple origins and destinations. On a network, information will affect users' choice of route as well as their decisions whether and when to depart. Route choice may also be affected by information received <u>after</u> the user is in the system. The welfare effects of such information are another possible subject for investigation.

Finally, we have asumed that all users have the same information. A useful extension of the analysis would be to situations in which some users are better informed than others. Given that more information is not necessarily welfare-improving a policy issue is raised as to the optimal fraction of users to inform.[27]

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[26] In the case of roads the effects of damage are generally recognized but have not yet been widely incorporated into traffic congestion models.

[27] For example, electronic road guidance systems are currently being developed. The question arises whether, costs aside, it is more efficient for all vehicles to be equipped or only a portion.

Notational Glossary

(in alphabetic order)

Greek Characters

α	Unit cost of travel time
β	Unit cost of arriving early
γ	Unit cost of arriving late
δ	$\beta\gamma/(\beta+\gamma)$
ϵ	Elasticity of demand
ν	Certainty-equivalent intensity of demand
$\rho(t)$	Density function of departure times
σ	Ratio of actual to design capacity of bottleneck
ϕ	N/s
$\phi(t)$	Largest ϕ such that no queue occurs at time t
$\phi^*(t)$	ϕ such that user departing at time t arrives on time

Arabic Characters

C(t)	Travel cost when departing at time t
CS	Consumers' surplus
D(t)	Length of queue at time t
$D(t,\phi)$	Length of queue at time t with $N/s = \phi$
$f(n,\sigma)$	Joint p.d.f. of n and σ
F	(superscript) denotes the full information regime
g(n)	probability density function of n
G(n)	Cumulative density function of n
$h(\sigma)$	probability density function of σ
$H(\sigma)$	Cumulative density function of σ
$J(\phi)$	Cumulative density function of ϕ
k	Unit cost of capacity expansion
MCS	Marginal consumers' surplus from expanding design capacity
n	Intensity of demand
N	Number of commuters
p	Price (= cost) of trip
$\bar{p}$	Expected price of trip
r(t)	Departure rate
R(t)	Cumulative distribution function of $\rho(t)$
s	Capacity of bottleneck
$\hat{s}$	Design capacity of bottleneck
SDC	Aggregate Schedule Delay Costs
t	Departure time
t^*	Desired arrival time at work
t_n	Departure time for which commuter arrives at t^* (full information regime)

t_n	Latest departure time for which commuter never arrives late (zero information regime)
t_q	Earliest departure time
t'_q	Latest arrival time
t_r	Latest departure time
$T(t)$	Travel time when departing at time t
$\bar{T}$	Travel time in absence of queue
$T^v(t)$	Time spent queuing if departing at time t
TB	Total Benefits from travel
TC	Aggregate Travel Costs
TTC	Aggregate Travel Time Costs
0	(superscript) denotes the zero information regime
y	Certainty-equivalent number of commuters as a percentage of mean number for determining optimal capacity.
z	σ/n
$Z(t)$	Rate of increase in expected travel time from delaying departure
$Z^e(t)$	Rate of decrease in expected early arrival time from delaying departure
$Z^l(t)$	Rate of increase in expected late arrival time from delaying departure

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Appendix 1

Proof of Proposition 1

Proposition 1 concerns the ratio of optimal design capacity with anticipated fluctuations in demand and capacity to optimal design capacity in the absence of fluctuations, with demand intensity fixed at its mean value. Using equations (21), (24a) and (24b) in the text this ratio can be written:

$$\hat{s}^*/s^* = \nu/\bar{n},$$

where $\bar{n}$ is mean demand intensity in the stochastic case. To establish that, under given conditions, the ratio of optimal capacities is increased (or decreased) by stochasticity it suffices to establish it for $\nu/\bar{n}$.

Lemma 1

$$\nu \gtreqless \bar{n} \quad \text{as } \epsilon \lesseqgtr 1.$$

Proof By Holder's inequality

$$\int\int x^{\alpha}y^{1-\alpha}f(x,y)\ dxdy \lesseqgtr (\bar{x})^{\alpha}(\bar{y})^{1-\alpha} \quad \text{as } \alpha \begin{cases} \epsilon\ (0,1) \\ =\ 0,\ 1 \\ <0 \text{ or } >1, \end{cases}$$

where f(x,y) is an arbitrary nondegenerate joint p.d.f. Replacing x with σ, y with n, α with $(\epsilon-1)/(\epsilon+1)$ and f(x,y) with $f(n,\sigma)$ yields

$$\nu \gtreqless (\bar{\sigma})^{\frac{\epsilon-1}{2}}\ \bar{n} \gtreqless \bar{n} \quad \text{as } \epsilon \lesseqgtr 1. \quad \text{QED.}$$

The relative magnitude of ν and $\bar{n}$ depends on the concavity or convexity of marginal consumers' surplus as a function of n and σ. If $\epsilon > 1$, marginal consumers' surplus is concave, whereas if $\epsilon < 1$ it is convex.

Lemma 2

Let $H(\sigma)$ be the c.d.f. of σ and $G(n|\sigma)$ be the c.d.f. of n conditional on σ. Then if $\epsilon < 1$, $\nu/\bar{n}$ will increase with a mean-preserving spread to $G(n|\sigma)$ for any set of σ of positive probability measure. The opposite is true if $\epsilon > 1$.

Proof From equation (24b) in the text we have:

$$\nu = \left\{ \int_0^1 \sigma^{\frac{\epsilon-1}{1+\epsilon}} m(\sigma) dH(\sigma) \right\}^{\frac{1+\epsilon}{2}} \quad , \text{ where } m(\sigma) = \int_0^\infty n^{\frac{2}{1+\epsilon}} dG(n|\sigma) dn.$$

Let i_1 and i_2 be indexes. Assume that, for all values of σ, $G(n|\sigma,i_1)$ either coincides with $G(n|\sigma,i_2)$ or can be obtained by a mean-preserving spread, with the latter true for a subset of σ of positive measure.

If $\epsilon < 1$ then $n^{\frac{2}{1+\epsilon}}$ is a convex function of n, so that by Rothschild and Stiglitz [1970], $m(\sigma,i_1) \geq m(\sigma,i_2)$ for all σ and hence $\nu(i_1) > \nu(i_2)$. The proof is completed by noting that the construction holds $\bar{n}$ fixed. QED.

Lemma 3

Let $G(n)$ be the c.d.f. of n and $H(\sigma|n)$ the c.d.f. of σ conditional on n. Then if $\epsilon < 1$, $\nu/\bar{n}$ will increase with a mean-preserving spread to $H(\sigma|n)$ for any set of n of positive probability measure. The opposite is true if $\epsilon > 1$.

Proof The proof is analogous to that of Lemma 2, since with $\epsilon < 1$ $\sigma^{\frac{\epsilon-1}{1+\epsilon}}$ is a convex function of σ. QED.

To establish the two remaining lemmas some preliminaries are required. Let x and y be two arbitrary variates. Let f(x,y) be the joint p.d.f. of x and y and $h_x(x)$ and $h_y(y)$ be their respective marginal distributions. Define

$$M_\alpha(x) = \left\{ \int x^\alpha h_x(x)dx \right\}^{\frac{1}{\alpha}},$$

$$M_\alpha(x,y) = \int \int x^\alpha y^{1-\alpha} f(x,y)dxdy,$$

and

$$\rho_\alpha(x,y) = \{M_\alpha(x,y)/[(M_\alpha(x))^\alpha (M_{1-\alpha}(y))^{1-\alpha}]\} - 1, \tag{A1.1}$$

$M_\alpha(x)$ and $M_\alpha(x,y)$ are generalized means and $\rho_\alpha(x,y)$ is a generalized correlation coefficient. If x and y are independent then:

$$M_\alpha(x,y) = \int y^{1-\alpha} h_y(y)dy[\int x^\alpha h_x(x)dx] = (M_\alpha(x))^\alpha (M_{1-\alpha}(y))^{1-\alpha},$$

so that $\rho_\alpha(x,y) = 0$. From equation (24b) in the text and (A1.1)

$$\nu = [(M_\alpha(\sigma))^\alpha (M_{1-\alpha}(n))^{1-\alpha}(1 + \rho_\alpha(\sigma,n))]^{\frac{1}{1-\alpha}}, \tag{A1.2}$$

where

$$\alpha = \frac{\epsilon-1}{1+\epsilon} < 1.$$

We can now proceed to lemmas 4 and 5.

Lemma 4

Holding $M_{1-\alpha}(n)$, $\bar{n}$ and $\rho_\alpha(\sigma,n)$ fixed, $\nu/\bar{n}$ is larger the larger is $(M_\alpha(\sigma))^\alpha$, and hence with $\epsilon <$ (resp. $>$) 1 and $\alpha <$ (resp. $>$) 0 the smaller (resp. larger) is $M_\alpha(\sigma)$. QED.

It is in this sense that, with $\epsilon < 1$, $\nu/\bar{n}$ is larger the smaller the mean value of σ.

Lemma 5

Holding $M_{\alpha}(\sigma)$, $M_{1-\alpha}(n)$ and $\bar{n}$ fixed, (A1.2) reveals that $\nu/\bar{n}$ is larger the larger is $\rho_{\alpha}(\sigma,n)$. With ϵ < (resp. >) 1, α < (resp. >) 0, and hence $\nu/\bar{n}$ is larger the smaller the correlation coefficient between σ and n. QED.

Appendix 2

In this appendix we show that $\rho(t)$ is constant for $t \epsilon (t_q, t_n)$ and monotonically weakly decreasing for $t \epsilon (t_n, t_r)$.

a) $t \epsilon (t_q, t_n)$

Condition (29) in the text stipulates that Z(t) defined in (30) is constant. Differentiating (30) we find

$$\dot{\rho}(t) \int_{\phi(t)}^{\phi_M} \phi \; dJ(\phi) = (\phi(t)\rho(t)-1) dJ(\phi(t))/d\phi \; \dot{\phi}(t). \tag{A2.1}$$

(A2.1) has a solution $R = \rho(t-t_q)$ characterized by a constant departure rate.

QED.

b) $t \epsilon (t_n, t^*]$

Differentiating (32) and cancelling terms one obtains

$$\{(\alpha-\beta) \int_{\phi(t)}^{\phi^*(t)} \phi \; dJ(\phi) + (\alpha+\gamma) \int_{\phi^*(t)}^{\phi_M} \phi \; dJ(\phi)\} \; \dot{\rho}(t) =$$

$$(\alpha-\beta)[\phi(t)\rho(t)-1]\dot{\phi}(t) \; dJ(\phi(t))/d\phi \tag{A2.2}$$

$$+ (\beta+\gamma)\phi^*(t)\rho(t)\dot{\phi}^*(t) \; dJ(\phi^*(t))/d\phi.$$

The term in braces on the LHS of (A2.2) is strictly positive. The term $[\phi(t)\rho(t)-1]\dot{\phi}(t)$ can be shown graphically to be negative, so that the first RHS term of (A2.2) is negative if $dJ(\phi(t)) > 0$. Since $\dot{\phi}^*(t) < 0$ (this follows immediately from the definition of $\phi^*(t)$) the second RHS term is negative if $dJ(\phi^*(t))/d\phi > 0$.

This proves that $\dot{\rho}(t) \leq 0$, with strict inequality if either $\phi(t)$ or $\phi^*(t)$ has positive probability density. This result may be explained as follows. If a user delays departure and $dJ(\phi(t))/d\phi > 0$ he is more likely to escape queuing. Instead of decreasing travel time costs he may end up decreasing his early arrival cost instead. Since $\alpha > \beta$ the benefit from delaying departure is reduced. Hence to maintain constant expected travel costs $\rho(t)$ must fall over time.

Similarly, if a user delays departure and $dJ(\phi^*(t))/d\phi > 0$ he is more likely to arrive late. Instead of reducing his early arrival cost he may end up increasing his late arrival cost. Departing later is again less attractive; hence $\rho(t)$ must fall over time.

c) $t \in (t^*, t_r)$

Differentiating (36) one obtains:

$$\{\int_{\phi(t)}^{\phi_M} \phi dJ(\phi)\}\dot{\rho}(t) = [\phi(t)\rho(t)-1]\dot{\phi}\ dJ(\phi(t))/d\phi. \tag{A2.3}$$

Since the LHS term in braces of (A2.3) in braces is positive and the RHS term is nonpositive,

$\dot{\rho}(t) \leq 0$. QED.

Since by departing after t* a user is guaranteed to arrive late, the late arrival effect operating through changes in $\phi^*(t)$ which applies in the departure interval $(t^*, t_r]$ is absent, leaving only the queuing effect operating through changes in $\phi(t)$.

Appendix 3

Proof of Lemma 1

Using (35) and (37) in the text, and the fact that with $t > t_r$, $R(t) = 1$ and hence $\phi(t) = t - t_q$, expected travel costs for departure after t_r are

$$C(t) = \gamma \left\{ \int_0^{t-t_q} (t-t^*)dJ(\phi) + \int_{t-t_q}^{\phi_M} (t_q + \phi - t^*)dJ(\phi) \right\} \qquad (A3.1)$$

$$+ \alpha \int_{t-t_q}^{\phi_M} (t_q + \phi - t)dJ(\phi).$$

If t_r is indeed the last departure time then expected travel costs must increase after t_r. Since expected travel costs are a continuous function of departure time it suffices to examine the behaviour of $C(t)$ after t_r. From (36) and (30), the left-hand derivative (which is zero by construction) is:

$$\lim_{t\uparrow t_r} \dot{C}(t) = \gamma(1+ \lim_{t\uparrow t_r} Z(t)) + \alpha \lim_{t\uparrow t_r} Z(t)$$

$$= \gamma - (\alpha+\gamma)(1-\lim_{t\uparrow t_r} J(t-t_q)) + (\alpha+\gamma) \lim_{t\uparrow t_r} \int_{t-t_q}^{\phi_M} \phi\ \rho(t)dJ(\phi) = 0. \qquad (A3.2)$$

From (A3.1), the right-hand derivative is:

$$\lim_{t\downarrow t_r} \dot{C}(t) = \gamma - (\alpha+\gamma)(1-J(t_r-t_q)). \qquad (A3.3)$$

Comparison of (A3.2) and (A3.3) reveals that (A3.3) is negative, and thus inconsistent with equilibrium, unless the last term involving an integral in (A3.2) is nonpositive. Since this last term is nonnegative, a necessary condition for equilibrium with $t_r > t^*$ is that the integral term be zero. In that case (A3.3) is zero, so that

$$J(t_r-t_q) = \alpha/(\alpha+\gamma), \text{ or}$$

$$t_r = t_q + J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}, \tag{A3.4}$$

which is equation (42) in the text.

Since $\lim_{t\uparrow t_r} \dot{C}(t) = \lim_{t\downarrow t_r} \dot{C}(t) = 0$, to prove that expected travel costs are higher after t_r than before we must show

$$\lim_{t\downarrow t_r} \ddot{C}(t) \geq 0.$$

But from (A3.3)

$$\lim_{t\downarrow t_r} \ddot{C}(t) = (\alpha+\gamma) \lim_{t\downarrow t_r} dJ(t-t_q)/d\phi \geq 0.$$

A solution for t_q and t_r now follows directly. From (27)

$$C(t_q) = \beta(t^*-t_q), \tag{A3.5}$$

while from (35) and (37)

$$C(t_r) = \gamma(t_r-t^*) + (\alpha+\gamma)\int_{t_r-t_q}^{\phi_M} (t_q + \phi - t_r)dJ(\phi)$$

$$= \gamma(t_r-t^*) - (\alpha+\gamma)(t_r-t_q)(1-J(t_r-t_q)) + (\alpha+\gamma)\int_{t_r-t_q}^{\phi_M} \phi dJ(\phi)$$

$$= -\gamma(t^*-t_q) + (\alpha+\gamma)\int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi) \text{ (using (A3.4))}.$$

Setting $C(t_q) = C(t_r) \equiv C$ yields

$$C = \beta(t^*-t_q) = \beta\frac{\alpha+\gamma}{\beta+\gamma} \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi), \tag{A3.6}$$

which is equation (42′) in Lemma 1. Equation (A3.6) can be solved for t_q, and the result substituted into (A3.4) to yield t_r.

The initial premise that $t_r > t^*$ is satisfied provided:

$$t_r - t_q > t^* - t_q \Leftrightarrow J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\} > \frac{\alpha+\gamma}{\beta+\gamma} \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi),$$

which yields condition (40) and completes the proof of Lemma 1.

Appendix 4

Proof of Lemma 2

With $t_r = t^*$ we have

$$\phi(t_r) = \phi(t^*) = \phi^*(t_r) = \phi^*(t^*) = t^*-t_q, \text{ and } R(t^*) = 1. \tag{A4.1}$$

Equation (35) in the text specifies expected travel costs for departures after t^*. Equation (36) specifies the derivative. If $t_r = t^*$ a necessary condition for equilibrium is that costs be nondecreasing after t^*:

$$\lim_{t \downarrow t_r} \dot{C}(t) = \gamma - (\alpha+\gamma)(1-J(t_r-t_q)) \geq 0, \text{ or}$$

$$J(t^*-t_q) \geq \alpha/(\alpha+\gamma). \tag{A4.2}$$

Condition (A4.2) is also sufficient since

$$\lim_{t \downarrow t_r} \ddot{C}(t) = (\alpha+\gamma)dJ(t-t_q) \geq 0.$$

Using (A4.1) and (31) one also has

$$C(t_r) = C(t^*) = (\alpha+\gamma) \int_{t^*-t_q}^{\phi_M} (t_q+\phi-t^*)dJ(\phi). \tag{A4.3}$$

As before, $C(t_q) = \beta(t^*-t_q)$, which is equation (45) in the text. Equating $C(t_q)$ and $C(t^*)$ yields an implicit equation for t_q:

$$(t^*-t_q)\{\beta + (\alpha+\gamma)[1-J(t^*-t_q)]\} - (\alpha+\gamma)\int_{t^*-t_q}^{\phi_M} \phi dJ(\phi) = 0, \tag{A4.4}$$

which is equation (44) in the text. Now the LHS of (A4.4) is strictly increasing in t^*-t_q. Since $J(\ )$ is a monotonically increasing function of ϕ, condition (A4.2) is satisfied if the LHS of (A4.4) is nonpositive with $J(t^*-t_q) = \alpha/(\alpha+\gamma)$:

$$J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}(\beta+\gamma) - (\alpha+\gamma) \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi) \leq 0, \quad \text{or}$$

$$J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\} \leq \frac{\alpha+\gamma}{\beta+\gamma} \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi), \tag{A4.5}$$

which is condition (43) in Lemma 2. This completes the proof of Lemma 2.

Appendix 5

Derivation of Equations (50), (51) and (52)

Equation (50) follows directly from differentiating (49):

$$\overline{MCS}^0(\hat{s}) = -\bar{N}\,\frac{d\bar{p}}{d\hat{s}}\ . \tag{A5.1}$$

In the case $t_r > t^*$ we have

$$\bar{p} = C - \beta(t^*-t_q) - \beta\frac{\alpha+\gamma}{\beta+\gamma}\int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi\, dJ(\phi). \tag{A5.2}$$

Integrating by parts:

$$\bar{p} = \beta\frac{\alpha+\gamma}{\beta+\gamma}\left\{\phi J(\phi)\Big|_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} - \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} J(\phi)\,d\phi\right\},$$

and differentiating

$$\frac{d\bar{p}}{d\hat{s}} = -\beta\frac{\alpha+\gamma}{\beta+\gamma}\int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \frac{dJ(\phi)}{d\hat{s}}\,d\phi. \tag{A5.3}$$

Now $J(\phi) = \Pr\,\{N/s \le \phi\} = \Pr\,\{\frac{n(\bar{p})^{-\epsilon}}{\hat{s}\sigma} \le \phi\}$

$$= \Pr\,\{n \le \phi\hat{s}\sigma(\bar{p})^{\epsilon}\}.$$

Thus,

$$J(\phi) = \int_0^1\int_0^{\phi\hat{s}\sigma(\bar{p})^{\epsilon}} f(n,\sigma)\,dn\,d\sigma, \tag{A5.4}$$

$$dJ(\phi)/d\hat{s} = \int_0^1 \phi\sigma(\bar{p})^{\epsilon}\left[1 + \frac{\hat{s}\epsilon}{\bar{p}}\frac{d\bar{p}}{d\hat{s}}\right] f(\phi\hat{s}\sigma(\bar{p})^{\epsilon},\sigma)\,d\sigma \tag{A5.5}$$

From (A5.4)

$$J(\phi) = \int_0^1 \phi\hat{s}\sigma(\bar{p})^{\epsilon}\ f(\phi\hat{s}\sigma(\bar{p})^{\epsilon},\sigma)d\sigma. \tag{A5.6}$$

From (A5.2) - (A5.6) it follows that

$$\frac{d\bar{p}}{d\hat{s}}\ \frac{\hat{s}}{\bar{p}} = -\ \frac{1}{1+\epsilon}\ , \tag{A5.7}$$

which is equation (51) in the text.

And from (A5.1), (A5.7) and (15)

$$\frac{d\overline{MCS}^0(\hat{s})}{d\hat{s}}\ \frac{\hat{s}}{\overline{MCS}^0(\hat{s})} = -\ \frac{2}{1+\epsilon}, \tag{A5.8}$$

which is (52) in the text.

In the case $t_r = t^*$ we have from (44)

$$(t^*-t_q)\{\beta + (\alpha+\gamma)[1-J(t^*-t_q)]\} - (\alpha+\gamma)\int_{t^*-t_q}^{\phi_M} \phi dJ(\phi) = 0. \tag{A5.9}$$

Following the procedure for $t_r > t^*$ one obtains the counterpart to (A5.3)

$$\frac{d\bar{p}}{d\hat{s}} = -\ \beta\frac{\alpha+\gamma}{\beta+(\alpha+\gamma)[1-J(\tilde{\phi})]}\ \int_{\tilde{\phi}}^{\phi_M} \frac{dJ(\phi)}{d\hat{s}}\ d\phi, \tag{A5.10}$$

where $\tilde{\phi} = t^*-t_q$.

The rest of the derivation is similar.

Appendix 6

Proof of Proposition 4

From equation (22) in the text:

$$\overline{CS}^F(\hat{s}) = \int_0^1 \int_0^\infty \left\{ \int_{p(\hat{s}\sigma/n)}^\infty np^{-\epsilon}\, dp \right\} f(n,\sigma)dn\, d\sigma, \tag{A6.1}$$

while from (49)

$$\overline{CS}^0(\hat{s}) = \int_0^1 \int_0^\infty \left\{ \int_{\bar{p}(\hat{s})}^\infty np^{-\epsilon}\, dp \right\} f(n,\sigma)dn\, d\sigma. \tag{A6.2}$$

$p(\hat{s}\sigma/n)$ in (A6.1) is given by (17b), and $\bar{p}(\hat{s})$ in (A6.2) by (42′).

Define $z \equiv \sigma/n$, and let $z(\hat{s})$ be the value of z such that the price of a trip in the predictable regime equals the expected price in the unpredictable regime, i.e.

$p(\hat{s}z(\hat{s})) = \bar{p}(\hat{s})$. Then

$$z \gtrless z(\hat{s}) \Leftrightarrow \bar{p}(\hat{s}z) \lessgtr p(\hat{s}) \Leftrightarrow n \lessgtr \sigma/z(\hat{s}).$$

Thus

$$\overline{CS}^F(\hat{s}) - \overline{CS}^0(\hat{s}) = \int_0^1 \int_0^{\sigma/z(\hat{s})} \left\{ \int_{p(\hat{s}z)}^{\bar{p}(\hat{s})} np^{-\epsilon}\, dp \right\} f(n,\sigma)dn\, d\sigma,$$

$$- \int_0^1 \int_{\sigma/z(\hat{s})}^\infty \left\{ \int_{\bar{p}(\hat{s})}^{p(\hat{s}z)} np^{-\epsilon} dp \right\} f(n,\sigma)dn\, d\sigma.$$

$$= \int_0^1 \left\{ \int_0^{\sigma/z(\hat{s})} \left[\frac{n}{1-\epsilon}\, p^{1-\epsilon} \Big|_{p(\hat{s}z)}^{\bar{p}(\hat{s})} \right] - \int_{\sigma/z(\hat{s})}^\infty \left[\frac{n}{1-\epsilon}\, p^{1-\epsilon} \Big|_{\bar{p}(\hat{s})}^{p(\hat{s}z)} \right] \right\} \quad f(n,\sigma)dn\, d\sigma.$$

Since $\frac{dp}{d\hat{s}} \frac{\hat{s}}{p} = - \frac{1}{1+\epsilon}$ for both regimes (viz. equations (17b) and (51)),

$$\overline{MCS}^F(\hat{s}) - \overline{MCS}^0(\hat{s}) = - \frac{1-\epsilon}{\hat{s}(1+\epsilon)} \quad (\overline{CS}^F(\hat{s}) - \overline{CS}^0(\hat{s})).$$

If $\epsilon < 1$ the differences in marginal and total expected consumers' surplus have the opposite sign. If $\epsilon > 1$ they have the same sign. Furthermore, since

$$\frac{dMCS}{d\hat{s}} \quad \frac{\hat{s}}{MCS} = - \frac{2}{1+\epsilon}$$

for both regimes (equations (23) and (52)), their relative efficiency can be established by examining any level of design capacity. QED.

Appendix 7

Proof of Theorem 1

To compare the relative efficiency and optimal capacities of the full information and zero information regimes it suffices to compare the marginal consumers' surplus from capacity expansion for any level of design capacity. For simplicity, we set $\hat{s} = 1$.

Theorem 1 concerns fluctuations in capacity alone. Let $H(\sigma)$ be the c.d.f. of σ. Now, with $\hat{s}=1$, $\phi = N/\sigma$. Making the change of variable from ϕ to σ, equation (53) in the text becomes

$$\overline{MCS}^F = \frac{n^{\frac{2}{1+\epsilon}}}{1+\epsilon} \quad \delta^{\frac{1-\epsilon}{1+\epsilon}} \int_0^1 \sigma^{\frac{\epsilon-1}{1+\epsilon}} \, dH(\sigma). \tag{A7.1}$$

To proceed further, the cases $t_r > t^*$ and $t_r = t^*$ must be considered separately.

a) $t_r > t^*$

From equation (42') in the text

$$\bar{p} = \beta\frac{\alpha+\gamma}{\beta+\gamma} \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi).$$

$$= \beta\frac{\alpha+\gamma}{\beta+\gamma} \int_0^{H^{-1}\left\{\frac{\gamma}{\alpha+\gamma}\right\}} \sigma^{-1} dH(\sigma).$$

Given $N = n\,(\bar{p})^{-\epsilon}$ this reduces to

$$\bar{p} = \left\{ \beta\frac{\alpha+\gamma}{\beta+\gamma}\, n \int_0^{H^{-1}\left\{\frac{\gamma}{\alpha+\gamma}\right\}} \sigma^{-1} dH(\sigma) \right\}^{\frac{1}{1+\epsilon}} . \tag{A7.2}$$

Using (54) and (A7.2)

$$\overline{MCS}^0 = \frac{n^{\frac{2}{1+\epsilon}}}{1+\epsilon} \left\{ \beta^{\frac{\alpha+\gamma}{\beta+\gamma}} n \int_0^{H^{-1}\left\{\frac{\gamma}{\alpha+\gamma}\right\}} \sigma^{-1} dH(\sigma) N. \right\}^{\frac{1-\epsilon}{1+\epsilon}} \tag{A7.3}$$

Comparing (A7.1) and (A7.3), and using the relationship $\delta = \beta\gamma/(\beta+\gamma)$, we have

$\overline{MCS}^0 \gtreqless \overline{MCS}^F \Leftrightarrow$

$$\frac{\alpha+\gamma}{\alpha} \int_0^{H^{-1}\left\{\frac{\gamma}{\alpha+\gamma}\right\}} \sigma^{-1} dH(\sigma) \gtrless \left\{ \int_0^1 \sigma^{\frac{\epsilon-1}{1+\epsilon}} dH(\sigma) \right\}^{\frac{1+\epsilon}{1-\epsilon}} \quad \text{as } \epsilon \lessgtr 1. \tag{A7.4}$$

Now, since σ^{-1} is a decreasing function of σ,

$$\frac{\alpha+\gamma}{\alpha} \int_0^{H^{-1}\left\{\frac{\gamma}{\alpha+\gamma}\right\}} \sigma^{-1} dH(\sigma) > \int_0^1 \sigma^{-1} dH(\sigma). \tag{A7.5}$$

Moreover (See Hardy, Littlewood and Polya [1934, Proposition 2.9.1])

$$\int_0^1 \sigma^{-1} dH(\sigma) \geq \left\{ \int_0^1 \sigma^{\frac{\epsilon-1}{1+\epsilon}} dH(\sigma) \right\}^{\frac{1+\epsilon}{1-\epsilon}}, \tag{A7.6}$$

for $\epsilon \geq 0$ (= if $\epsilon = 0$). Combining (A7.5) and (A7.6) with (A7.4) yields

$$\overline{MCS}^0 \gtrless \overline{MCS}^F \quad \text{as} \quad \epsilon \lessgtr 1. \tag{A7.7}$$

This proves the second statement in Proposition 3. Combining (A7.7) with (A6.1) in Appendix 6 we have

$$\overline{CS}^F(\hat{s}) > \overline{CS}^0(\hat{s}),$$

which proves the first statement. QED.

b) $t_r = t^*$

From equation (54) in the text

$$\overline{MCS}^0 = n(\overline{p})^{1-\epsilon}/(1+\epsilon), \tag{A7.8}$$

where from (45)

$$\bar{p} = \beta\tilde{\phi} \tag{A7.9}$$

with $\tilde{\phi}$ defined implicitly by

$$\tilde{\phi}\{\beta + (\alpha+\gamma)[1-J(\tilde{\phi})]\} - (\alpha+\gamma)\int_{\tilde{\phi}}^{\phi_M} \phi dJ(\phi) = 0.$$

Transforming variables, this can be written

$$\tilde{\sigma}\int_0^{\tilde{\sigma}} \sigma^{-1}\, dH(\sigma) - H(\sigma) = \frac{\beta}{\alpha+\gamma} \quad . \tag{A7.10}$$

Setting $\tilde{\sigma} = \bar{n}(p)^{-\epsilon}/\tilde{\phi}$,

and substituting into (A7.9)

$$\bar{p} = (\beta n/\tilde{\sigma})^{1/(1+\epsilon)}. \tag{A7.11}$$

Substituting (A7.11) into (A7.8):

$$\overline{MCS}^0 = \frac{n^{\frac{1}{1+\epsilon}}}{1+\epsilon}\, \beta^{\frac{1-\epsilon}{1+\epsilon}}\, \sigma^{-\frac{\epsilon-1}{1+\epsilon}} \quad . \tag{A7.12}$$

Thus, given (A7.1) and (A7.12),

$$\overline{MCS}^0 > \overline{MCS}^F \Leftrightarrow$$

$$\frac{\beta+\gamma}{\gamma\tilde{\sigma}} \gtrless \left\{\int_0^1 \sigma^{\frac{\epsilon-1}{1+\epsilon}}\, dH(\sigma)\right\}^{\frac{1+\epsilon}{1-\epsilon}} \quad \text{as } \epsilon \lessgtr 1. \tag{A7.13}$$

<u>Lemma</u>

$$\frac{\beta+\gamma}{\gamma} > \tilde{\sigma}\int_0^1 \tilde{\sigma}^{-1}\, dH(\sigma) \tag{A7.14}$$

<u>Proof of Lemma</u>

Using (A7.10) we have

$$\frac{\beta+\gamma}{\gamma} - \bar{\sigma} \int_0^1 \sigma^{-1} \, dH(\sigma) = 1 + \frac{\beta}{\gamma} - \frac{\beta}{\alpha+\gamma} - Y(\bar{\sigma}), \tag{A7.15}$$

where

$$Y(\bar{\sigma}) = H(\bar{\sigma}) + \bar{\sigma} \int_{\bar{\sigma}}^1 \sigma^{-1} \, dH(\sigma).$$

Since Y( ) achieves a maximum value of 1 at $\bar{\sigma} = 1$, the expression in (A7.15) is positive. QED.

Combining (A7.14), (A7.6) and (A7.13) one obtains (A7.7) as before. This completes the proof of Theorem 1.

Appendix 8

Proof of Theorem 2

The method of proof is similar to that for Theorem 1. Let G(n) be the c.d.f. of n. With $\hat{s}=1$ as before, $\phi = n(\bar{p})^{-\epsilon}$. Changing variable from ϕ to n, equation (53) becomes

$$\overline{MCS}^F = \frac{1}{1+\epsilon} \delta^{\frac{1-\epsilon}{1+\epsilon}} \int_0^\infty n^{\frac{2}{1+\epsilon}} dG(n). \tag{A8.1}$$

Again, the cases $t_r > t^*$ and $t_r = t^*$ must be considered separately.

a) $t_r > t^*$

From equation (42′) and the relation $N = n(\bar{p})^{-\epsilon}$

$$\bar{p} = \left\{ \beta \frac{\alpha+\gamma}{\beta+\gamma} \int_{G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\infty} n dG(n) \right\}^{\frac{1}{1+\epsilon}}. \tag{A8.2}$$

Substituting (A8.2) into (54)

$$\overline{MCS}^0 = \frac{1}{1+\epsilon} \bar{n} \left\{ \beta \frac{\alpha+\gamma}{\beta+\gamma} \int_{G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\infty} n dG(n) \right\}^{\frac{1-\epsilon}{1+\epsilon}}. \tag{A8.3}$$

Comparing (A8.1) and (A8.3) we have

$$\overline{MCS}^0 \gtrless \overline{MCS}^F \Leftrightarrow$$

$$\left\{\frac{\alpha+\gamma}{\gamma}\right\}^{\frac{1-\epsilon}{2}} (\bar{n})^{\frac{1+\epsilon}{2}} \left\{ \int_{G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\infty} n dG(n) \right\}^{\frac{1-\epsilon}{2}} \gtrless \left\{ \int_0^\infty n^{\frac{2}{1+\epsilon}} dG(n) \right\}^{\frac{1+\epsilon}{2}}. \tag{A8.4}$$

Here, there are two inequalities, working in opposite directions. On the one hand,

$$\left\{\frac{\alpha+\gamma}{\gamma}\right\}^{\frac{1-\epsilon}{2}} (\bar{n})^{\frac{1+\epsilon}{2}} \left\{ \int_{G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\infty} ndG(n)\right\}^{\frac{1-\epsilon}{2}} \gtrless \left\{ \int_0^{\infty} n \; dG(n)\right\}^{\frac{1-\epsilon}{2}} (\bar{n})^{\frac{1+\epsilon}{2}} = \bar{n} \text{ as } \epsilon \lessgtr 1. \quad (A8.5)$$

On the other hand, (Hardy, Littlewood and Polya [1934, Proposition 2.9.1])

$$\bar{n} = \int_0^{\infty} ndG(n) \lessgtr \left\{ \int_0^{\infty} n^{\frac{2}{1+\epsilon}} dG(n)\right\}^{\frac{1+\epsilon}{2}} \quad \text{as } \epsilon \lessgtr 1.$$

The sign of the inequality in (A8.4) is thus ambiguous, unless $\epsilon = 1$ in which case marginal consumers' surpluses in the two regimes are equal.

b) $t_r = t^*$

From (54)

$$\overline{MCS}^0 = \tilde{n}(\bar{p})^{1-\epsilon}/(1+\epsilon), \quad (A8.6)$$

where from (45)

$$\bar{p} = \beta\bar{\phi} = \beta(\bar{p})^{-\epsilon} \tilde{n},$$

and with $\tilde{n}$ defined implicitly by

$$G(\tilde{n}) + \frac{1}{\tilde{n}} \int_{\tilde{n}}^{\infty} ndG(n) = \frac{\alpha+\beta+\gamma}{\alpha+\gamma} . \quad (A8.7)$$

The relative magnitude of (A8.1) and (A8.6) is ambiguous, as true of the case $t_r > t^*$.

To conclude the proof we consider an example in which, depending on parameter values, the inequality in (A8.4) can go in either direction. Suppose that $\epsilon = 0$ and the intensity of demand has the p.d.f:

$$n = \begin{cases} n_1 & \text{with probability } 1-\pi \\ n_2 > n_1 & \text{with probability } \pi \end{cases} . \quad (A8.8)$$

We consider the two cases $t_r > t^*$ and $t_r = t^*$ in turn.

a) $t_r > t*$.

The condition for this to be the relevant case is given by equation (40) in the text:

$$J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\} > \frac{\alpha+\gamma}{\beta+\gamma} \int_{J^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\phi_M} \phi dJ(\phi). \tag{A8.9}$$

Making the change of variable from ϕ to n, (A8.9) becomes

$$G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\} > \frac{\alpha+\gamma}{\beta+\gamma} \int_{G^{-1}\left\{\frac{\alpha}{\alpha+\gamma}\right\}}^{\infty} ndG(n). \tag{A8.10}$$

There are two ranges of the probability π to consider.

(i) $\pi > \gamma/(\alpha+\gamma)$

In this case it is easily verified that condition (A8.10) is universally satisfied. Substituting (A8.8) into (A8.4), one finds, provided only that $n_2 > n_1$,

$$\overline{MCS}^0 > \overline{MCS}^F. \tag{A8.11}$$

In this case the full information regime is unambiguously more efficient than the zero information regime, and has a lower optimal capacity.

(ii) $\pi < \gamma/(\alpha+\gamma)$

Substituting (A8.8) into (A8.10) one finds that the case $t_r > t*$ is the relevant one if and only if

$$n_2/n_1 < 1 + \frac{\beta}{(\alpha+\gamma)\pi} . \tag{A8.12}$$

Substituting (A8.8) into (A8.4), one obtains after some manipulation

$\overline{MCS}^0 \gtrless \overline{MCS}^F$ as

$$[\pi(\alpha+\gamma)-\gamma](n_2/n_1)^2 + [\gamma+(1-2\pi)(\alpha+\gamma)](n_2/n_1) - (1-\pi)(\alpha+\gamma) \gtrless 0,$$

which reduces to

$$\overline{MCS}^0 \gtrless \overline{MCS}^F \quad \text{as} \quad n_2/n_1 \lessgtr 1 + \frac{\alpha}{\gamma-\pi(\alpha+\gamma)} \quad . \tag{A8.13}$$

The RHS of conditions (A8.12) and (A8.13) intersect at

$$\pi = \frac{\beta}{\alpha+\beta}\frac{\gamma}{\alpha+\gamma} \quad . \tag{A8.14}$$

b) $t_r = t*$

This case occurs only with $\pi < \gamma/(\alpha+\gamma)$. Substituting (A8.8) into (A8.7) one gets

$$\tilde{n} = \frac{(\alpha+\gamma)\pi}{\beta+(\alpha+\gamma)\pi} n_2. \tag{A8.15}$$

Substitution of (A8.15) into (A8.6) and comparison of the result with (A8.1) yields

$$\overline{MCS}^0 \gtrless \overline{MCS}^F \text{ as}$$

$$-\pi(1-f)(n_2/n_1)^2 + (1-\pi)f(n_2/n_1) - (1-\pi) \gtrless 0, \tag{A8.16}$$

where

$$f = 1 - \beta\frac{\gamma-\pi(\alpha+\gamma)}{\gamma[\beta+\pi(\alpha+\gamma)]}.$$

Figure 6 in the text is constructed by combining the above conditions.

Appendix 9

In this appendix we work through the example of a bivariate distribution of capacity illustrating the effects of autocorrelation on expected travel costs. We assume $\epsilon = 0$ and

$$s = \begin{cases} s_1 & \text{with probability } 1-\pi \\ s_2 = \sigma s_1 < s_1 & \text{with probability } \pi \end{cases} .$$

As with the example in Appendix 8 there are two ranges of the probability π to consider.

(i) $\pi > \gamma/(\alpha+\gamma)$

In this case it is easily verified that the condition for $t_r > t^*$ given by equation (40) in the text is universally satisfied. Applying the probability distribution given by (56) to (42′) one obtains:

$$\bar{p} = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s_2} , \tag{A9.1}$$

which is independent of s_1.

(ii) $\pi < \gamma/(\alpha+\gamma)$

In this case we have $t_r > t^*$ if and only if

$$\pi < \pi_c = \frac{\beta \quad \sigma}{(\alpha+\gamma)(1-\sigma)}. \tag{A9.2}$$

When (A9.2) is satisfied, application of (56) to (42′) yields

$$\bar{p} = \frac{\beta\gamma}{\beta+\gamma}\left[1 - \frac{(1-\pi)(\alpha+\gamma)-\alpha}{\gamma}(1 - \sigma)\right]\frac{N}{s_2} \tag{A9.3}$$

Finally, if (A9.2) is not satisfied, (44) and (45) apply and one obtains

$$\bar{p} = \frac{\beta(\alpha+\gamma)\pi}{\beta+(\alpha+\gamma)\pi}\ \frac{N}{s_2} , \tag{A9.4}$$

which is also independent of s_1.

In order of increasing π the appropriate functions of $\bar{p}$ are thus (A9.3), (A9.4) and (A9.1), although for some parameter values either (A9.3) or (A9.4) has a null range.

It is clear that (A9.3) is a linear function of π, that (A9.4) is an increasing and concave function of π and that (A9.1) is independent of π. Now the left-hand derivative of $\bar{p}$ at π_c, obtained by differentiating (A9.3) is

$$\left.\frac{d\bar{p}}{d\pi}\right|_{\pi\uparrow\pi_c} = \frac{\beta(\alpha+\gamma)(1-\sigma)}{\beta+\gamma}\,\frac{N}{s_2}\,. \tag{A9.5}$$

The right-hand derivative, obtained by differentiating (A9.4), is

$$\left.\frac{d\bar{p}}{d\pi}\right|_{\pi\downarrow\pi_c} = (\alpha+\gamma)(1-\sigma)^2\,\frac{N}{s_2}\,. \tag{A9.6}$$

As long as $\pi_c < \gamma/(\alpha+\gamma)$ we have (A9.6) > (A9.5) so that $\bar{p}$ has an upward kink at $\pi = \pi_c$. This establishes that Figure 7 in the text is applicable for a nondegenerate set of parameter values.