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ADVERTISING, COORDINATION, AND SIGNALING

by

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Abstract

When economies of scale exist and price information is difficult to communicate explicitly, dissipative advertising can direct buyers to low prices. The potential to advertise ensures that buyers coordinate and purchase from the most efficient firm, while observed advertising is used to signal the identity of the most efficient firm. The technique of eliminating equilibrium dominated strategies, most often applied to classic signaling games, is shown to have great selection power in coordination games as well.

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1. Introduction

A great deal of advertising contains little or no direct information. Yet advertisements are quite costly to produce and must therefore have some effect on consumer behavior. A challenge for economists is to reconcile the observation of dissipative or wasteful advertising with rational consumer behavior.

We offer here a theory of advertising for markets in which price information is difficult to transmit and returns to scale exist. Examples include the market for phone services, where complex nonlinear schedules may be difficult to communicate, and multi-product retail stores, which are typically unable to advertise prices for all of their products. Since costs are decreasing, efficiency requires all consumers to go to one firm. Indeed, the "natural monopoly" solution would be the most plausible outcome were firms able to compete directly in price. When price information is difficult to communicate, however, there arises the problem of coordinating consumer purchase decisions. It is in the resolution of this problem that we find a role for dissipative advertising.

The intuition is easily expressed. When costs are declining, firms' profits increase in the number of buyers they serve. Furthermore, under reasonable conditions (e.g., downward sloping demand curves), firms optimally offer lower prices when they expect more buyers. By committing resources to observable wasteful advertising, a firm communicates that it expects to make large profits through obtaining large market share, and this convinces consumers that the firm will offer a better deal. This coincidence of interests between the firm and consumers allows advertising to play a key role in coordinating consumer purchase decisions.

We study this intuition by means of a duopoly model in which one firm is known to be more efficient. Equilibria of this model may involve the inefficient firm capturing the market, or market splitting between the firms, which is even less efficient. Moreover, a continuum of advertising levels are possible. These equilibria are supported by consumer conjectures in which consumers ignore the firms' incentives to choose advertising, and thus advertising communicates nothing. For communication to occur, consumers must recognize why advertising is chosen.

We refine consumer beliefs by requiring that consumers never believe the firm would choose a combination of observable advertising and unobservable prices which could not increase its profits. In particular, consumers place no weight on advertising - price pairs which are <u>equilibrium dominated</u>, that is, which are incapable of giving the firm profits at least as great as those obtained through its equilibrium choices. Thus, advertising can only communicate that the firm intends to increase its profits, and this attracts consumers precisely because of the coincidence of interests.

<u>Dominant equilibria</u> are those which can be supported by beliefs which are not equilibrium dominated, and which survive elimination of weakly dominated strategies. We show that any inefficient equilibrium fails to be dominant, because the efficient firm can always use advertising to communicate that it intends to offer a better deal and capture the market. Thus, inefficient equilibria are never focal. The unique dominant equilibrium has the efficient firm capturing the market and choosing zero advertising, since once it expects to capture the market, there is no need to expend resources

to improve coordination. Thus, advertising is an extremely efficient coordinating device, since coordination is brought about simply by the possibility of advertising.

One nevertheless observes a great deal of dissipative advertising in actual markets. We argue that incomplete information gives an explanation for the observation of positive advertising. The duopoly model is augmented by designating one firm as an entrant with privately known costs. The incumbent and consumers are then unsure as to whether the entrant is more efficient. To deal with the expanded possibilities introduced by incomplete information, we restrict consumer beliefs by _iterated equilibrium dominance_, which eliminates any strategies for the firms or consumers which become equilibrium dominated due to elimination of other strategies. An _iterated dominant equilibrium_ survives both iterated equilibrium dominance and elimination of weakly dominated strategies.

We show that only one iterated dominant separating equilibrium exists, in which the most efficient firm always captures the market, and the entrant chooses positive advertising when it is more efficient than the incumbent. Advertising now simultaneously plays two communication roles. First, a positive level of advertising is used to signal the entrant's cost type. Positive advertising arises not from dissipative rivalry between firms, but rather from "informational rivalry" implicit in the entrant's communication of its costs. This is because communication of low cost through advertising is credible only if advertising dissipates all of the rents that would be available if costs were high. Second, the possibility of communicating a better deal through advertising continues to direct consumers to the most efficient firm. This coordination role for advertising does not require any additional advertising beyond that used for signaling in equilibrium.

There are also iterated dominant pooling equilibria which exist if and only if the entrant offers greater utility on the average, when the consumers do not learn its costs. Here, the only role of advertising is coordination. The firms choose zero advertising under the additional condition that firms' strategies are robust to low-probability "trembles" by consumers. The entrant prefers this equilibrium to separation, but the incumbent and consumers prefer the fully informative equilibrium, because signaling generates an entry barrier which keeps out the inefficient entrant.[1] Thus, advertising may be viewed as a screen on entrants which ensures efficiency.

Our ideas relate to a broad literature on the economics of advertising. In a variety of models, Kihlstrom and Riordan (1984), Klein and Leffler (1981), Milgrom and Roberts (1986), Nelson (1970, 1974), Ramey (1987), and Rogerson (1986) have explored the role of advertising in signaling quality for an experience good. We instead focus on markets in which advertising communicates search attributes. In addition, we draw an important distinction between the costless communication of imperfect information (price) and the costly communication of incomplete information (costs).[2] Finally, our theory of advertising requires returns to scale. This is consistent with the usual textbook logic, in which advertising stimulates demand enabling firms to exploit scale economies.

A second literature on advertising has focused on the direct informational content of advertisements. For example, Butters (1977), Chan and Leland (1982), Grossman and Shapiro (1984), Peters (1984), Rogerson (1988), and Schmalensee (1983) have examined markets in which advertisements contain direct price information. By contrast, we assume that price information is difficult, or even illegal, to advertise. The work of Benham (1972) is relevant here. He found the price of eyeglasses in states prohibiting price advertising--but not advertising in general--was only slightly above that in states which allowed price advertising and was significantly below that in states banning all advertising. This led Benham to conclude that "even 'non-price' advertising may lower prices," a conclusion which is of course consistent with our own analysis.[3]

A considerable volume of research has also explored the effect of advertising on industry structure. A prominent idea, due to Bain (1956), is that incumbent advertising deters entry, to the detriment of consumers.[4] In our separating equilibrium, however, advertising is not used by the incumbent to deter entry; rather, it is the necessity of entrant advertising that deters inefficient entry and improves consumer welfare. We also note that our results are consistent with Benham's observation that concentration correlates positively with the ability to advertise.

In addition to the literature on advertising, our work is related to research on rent dissipation. Posner (1975) has argued that competition to become a monopoly leads the eventual victor to dissipate all of its rents with wasteful activities. Posner's "wastefulness postulate" has been challenged by the contestability model, developed by Baumol, Panzar, and Willig (1982) and further studied by Maskin and Tirole (1988), and the "war of attrition" model of Fudenberg and Tirole (1986a, 1986b). In these models, all rent is dissipated, but the dissipation takes the useful form of low prices. In our model, rent dissipation is also not wasteful, as advertising plays a key role in directing buyers to low prices. We do differ from previous work, however, in not predicting total rent dissipation. Even in the separating equilibrium, where some rent is dissipated, the amount of rent dissipation is determined by the rival high cost state and not the rival firm.

Finally, our work draws heavily on the recent game theory literature. All of the refinements we employ are implied, in a finite context, by the stable equilibrium concept of Kohlberg and Mertens (1986).[5] Recent work of Ben-Porath and Dekel (1987) is closely related to our initial model. In particular, our model can be thought of as a combination of two "games of mutual interest" each between one firm and the consumers. The firms control the "money burning device" of dissipative advertising in their respective games. Our results show that rivalry to establish the firms' desired equilibria is resolved in favor of the firm capable of offering consumers the best deal. Moreover, our theorem relies on one round of equilibrium dominance. Ben-Porath and Dekel, in contrast, employ iterated elimination of strongly dominated strategies to obtain their coordination result, and make strong use of finiteness of strategy spaces.[6] Finally, van Damme (1987) and Glazer and Weiss (1987) apply equilibrium dominance to study the role of advertising in coordinating oligopoly behavior, and Cho and Kreps (1987) study equilibrium dominance in signaling games.

The paper proceeds with four more sections and an appendix. The basic framework is developed in Section two. Our results are stated and proved in Sections three and four. Section five then concludes. Explicit examples which fit into our framwork are provided in the Appendix.

2. Framework

We will work with the following basic framework. There are two firms which sell a homogenous product to a large group of consumers. The firms are denoted Firm 1 and Firm 2, and there are a continuum of consumers with total mass one. The structure of trading is as follows. Each firm makes an observable choice A_j and an unobservable choice P_j, where the j subscript denotes the choice of Firm j. The firms make their choices simultaneously, and A_j and P_j are chosen from the nonnegative real numbers. Consumers observe A_1 and A_2, but not P_1 or P_2, and choose to purchase from one firm or the other, or to make no purchases. Trading occurs for one period only.

The profits of Firm j depend on A_j and P_j, and also on the mass of consumers choosing to purchase from it, given by M_j, and a real-valued cost parameter C_j with larger C_j denoting higher variable costs. Let fixed costs be written $F(C_j) \geq 0$. All fixed costs are assumed to be sunk.[7] Net operating revenues are given by the continuous function $\Pi(P_j,M_j|C_j)$, which satisfies $\Pi(P_j,0|C_j) = 0$ for all P_j and C_j. Let $\Pi(P_j,M_j|C_j)$ be strictly concave in P_j, with unique maximizer $P(M_j|C_j) > 0$ for $M_j > 0$, possessing finite right-hand limit $P(0^+|C_j)$. Assume $\Pi(P(M_j|C_j),M_j|C_j) \geq 0$ for all M_j and C_j. Total profits including advertising expense are:

$$\Pi(P_j,M_j|C_j) - F(C_j) - A_j$$

Consumers are assumed to be identical. Let the utility of a consumer purchasing from Firm j when P_j has been chosen be given by $U(P_j)$, which is strictly decreasing in P_j. Let the utility of not purchasing be zero. Assume

$U(P(M_j|C_j)) \geq 0$ for all M_j and C_j, reflecting the notion that purchase is voluntary.

In this model we seek to capture the idea that both the firm and its customers benefit when the customer base is larger. This may be due, for example, to increasing returns or network externalities. The following general properties are assumed here:

<u>Better Deal Property</u>: $P(M_j|C_j)$ strictly decreases in M_j

<u>Better Profit Property</u>: $\Pi(P_j,1|C_j) > \Pi(P_j,M_j|C_j)$ for $0 < M_j < 1$ whenever $\Pi(P_j,1|C_j) > 0$.[8]

From the better deal property it follows that customers are best served when a single firm captures the entire market. By the better profit property, a firm benefits as well when its market share expands. Together, the two properties establish the gains from coordination available to customers and a firm.

Customers also benefit when the firm's variable costs are lower. We assume:

<u>Cost Effects</u>: (a) $P(M_j|C_j)$ strictly increases in C_j

(b) $P(0^+|C_j) > P(1|C_j')$ for every C_j and C_j'

Thus, as shown in Figure 1, while a lower cost firm offers a better deal for a given market share, a higher cost firm is still best if it fully exploits the advantages of larger scale.

We can interpret A_j as dissipative advertising expenditure which neither communicates information directly nor adds to consumer utility. The variable P_j reflects the pricing policies of the firm which consumers cannot observe prior to making some commitment to purchase. For example, the firms may be retail outlets selling a wide variety of products, with P_j indexing the overall pricing policy. Due to the difficulty of communicating an extensive menu of prices, the consumers are required to make a partial commitment by visiting the store before the price menu can be observed.[9] The Appendix presents two examples of economic environments which fit into the basic framework.

3. Advertising and Coordination

We will suppose first that the cost parameters are publicly observable, and Firm 1 has the lower costs; thus, $C_1 < C_2$. The problem for each consumer is to determine which firm gives the lowest P_j, but this will depend on the purchase decisions of other consumers. Therefore, coordination of consumer choices becomes a key issue.

In a sequential equilibrium (Kreps and Wilson (1982)) of this model, firms choose strategies $\hat{A}_j$ and $\hat{P}_j$, while consumers make a conjecture $\hat{U}_j(A_j)$ as to the expected utility obtainable from Firm j, conditional on the observed A_j; it is assumed that consumer conjectures are held in common. Let $\hat{M}_j(A_j,A_i)$ denote the equilibrium market share obtained by Firm j. The equilibrium conditions are, for j = 1,2 and $i \neq j$:

(A) Profit Maximization.

$$\hat{A}_j,\hat{P}_j \in \underset{A_j,P_j}{\operatorname{argmax}}\ \Pi(P_j,\hat{M}_j(A_j,\hat{A}_i)|C_j) - F(C_j) - A_j$$

(B) Utility Maximization.

$$\hat{M}_j(A_j, A_i) \in \begin{cases} \{1\}, & \hat{U}_j(A_j) > \hat{U}_i(A_i) \\ [0,1], & \hat{U}_j(A_j) = \hat{U}_i(A_i) \\ \{0\}, & \hat{U}_j(A_j) < \hat{U}_i(A_i) \end{cases}$$

(C) Correct Conjectures.

$$\hat{U}_j(\hat{A}_j) = U(\hat{P}_j)$$

Let us first consider the benchmark case in which the consumers ignore the dissipative advertising. Since $\hat{M}_j$ is then independent of A_j, (A) implies $\hat{A}_1 = \hat{A}_2 = 0$. There are three equilibria of this form. First, Firm 1 may capture the entire market. By (A) we have $\hat{P}_1 = P(1|C_1)$, and consumers optimally purchase from Firm 1 as long as $\hat{P}_2 \geq P(1|C_1)$. We will call this the <u>efficient equilibrium</u>.

The advantages of scale also allow Firm 2 to offer a better deal, however, and this gives rise to an equilibrium in which it captures the entire market. The equilibrium is supported by the consumers' anticipation that no other consumers will purchase from Firm 1, leading it to offer a less favorable deal. Thus, the inefficient firm may capture the market due to failure of consumers to coordinate. Finally, there is an equilibrium in which the firms each receive positive market share, and offer consumers equally attractive deals. For this to be true, we must have $P(\hat{M}_1|C_1) = P(\hat{M}_2|C_2)$, or $\hat{M}_1 < \hat{M}_2$; the inefficient firm must receive greater market share. Consumer utility is the lowest in this equilibrium.

Consider now the possibility that consumers make their purchase decisions conditional on the observed advertising. Consumers are then free to interpret advertising in a variety of ways, and correspondingly an immense set of equilibria is generated. In general, for each of the three no-advertising equilibria derived above, there are infinitely many equilibria in which the firm(s) gaining positive market share choose positive levels of advertising. These equilibria may be supported by beliefs of the form $\hat{U}_j(A_j) = U(P_j)$ for some $P_j > P(0^+|C_2)$, for all $A_j < \hat{A}_j$, where $\hat{A}_j$ may be chosen arbitrarily from some non-negligible interval. In other words, consumers can threaten to have very pessimistic conjectures as to P_j when they observe lower-than-expected levels of advertising, which deters the firms from choosing the low levels.

This makes it important to consider restrictions on consumers' conjectures which allow us to test the reasonableness of such threats. For a given equilibrium, consider an advertising level $A_j \neq \hat{A}_j$. The strategy (A_j,P_j) is called <u>equilibrium dominated</u> if:

$$\text{(1)} \qquad \Pi(\hat{P}_j,\hat{M}_j(\hat{A}_j,\hat{A}_i)|C_j) - \hat{A}_j > \Pi(P_j,1|C_j) - A_j$$

That is, (A_j,P_j) is equilibrium dominated if under the most favorable consumer response it generates less profit than is provided by the equilibrium strategy. A conjecture $\hat{U}_j(A_j) = U(P_j)$ is then <u>unreasonable</u> if (A_j,P_j) is equilibrium dominated, and there is some P_j' such that (A_j,P_j') is not equilibrium dominated. This is because Firm j has no incentive to choose A_j in order to convince consumers that P_j is the unobserved choice, since the equilibrium would necessarily provide greater profits. When A_j is observed, the more reasonable conjecture is that the unobserved choice can make the firm at least as well off as staying with the equilibrium.

Consumer beliefs are not the only source of unreasonable threats, however. A firm which receives zero market share can offer a very favorable deal, since its operating profits are zero in any case, and this may serve as a threat to the firm which does capture the market. Such a threat will create no difficulties if we rule out strategies which are dominated by some other strategy in each contingency, i.e. eliminate weakly dominated strategies. In the present case, this requires $P_j \in [P(1|C_j), P(0^+|C_j)]$.[10] With prices thus restricted, we now assume directly that $\Pi(P_j, 1|C_j) > 0$.

A dominant equilibrium is one which can be supported by consumer conjectures which are reasonable, that is, robust to elimination of equilibrium dominated strategies for the firm, and which survives elimination of weakly dominated strategies. These restrictions are quite powerful in the present context, as we have:

Theorem 1: There exists one and only one dominant equilibrium, that being the efficient equilibrium in which both firms choose zero advertising.

Proof: Consider the class of equilibria in which Firm 2 captures the entire market; clearly $\hat{A}_1 = 0$ in such an equilibrium. Fix $\varepsilon > 0$ sufficiently small to ensure:

(2) $$\Pi(\hat{P}_2, 1|C_1) + \varepsilon < \Pi(P(1|C_1), 1|C_1)$$

which is possible since $\hat{P}_2 = P(1|C_2) > P(1|C_1)$. Further, since

$P(0^+|C_1) > \hat{P}_2 > P(1|C_1)$, we can be sure under the better profit property that $\Pi(\hat{P}_2,1|C_1) > 0$. Thus, there exists $A_1 > 0$ which satisfies:

(3) $$0 = \Pi(\hat{P}_1,0|C_1) = \Pi(\hat{P}_2,1|C_1) + \varepsilon - A_1$$

Combining (1) and (3), (A_1,P_1) is equilibrium dominated if and only if:

(4) $$\Pi(\hat{P}_2,1|C_1) + \varepsilon > \Pi(P_1,1|C_1)$$

Thus, as shown in Figure 2, there is some $P_1' < \hat{P}_2$ such that (A_1,P_1) is equilibrium dominated for all $P_1 > P_1'$, while (2) implies $(A_1,P(1|C_1))$ is not equilibrium dominated. Reasonable conjectures therefore require $\hat{U}_1(A_1) > U(\hat{P}_2) = \hat{U}_2(\hat{A}_2)$, or $\hat{M}_1(A_1,\hat{A}_2) = 1$. By (2), Firm 1 prefers to deviate to A_1, and the equilibrium is overturned.

Now consider the class of equilibria in which both firms obtain positive market share; in this case we have $\hat{P}_1 = \hat{P}_2$. Define ε as in (2), and choose $A_1 > \hat{A}_1$ to satisfy:

$$\Pi(\hat{P}_1,\hat{M}_1(\hat{A}_1,\hat{A}_2)|C_1) - \hat{A}_1 = \Pi(\hat{P}_2,1|C_1) + \varepsilon - A_1$$

and (A_1,P_1) is equilibrium dominated if and only if (4) holds. As above, reasonable conjectures imply $\hat{U}_1(A_1) > \hat{U}_2(\hat{A}_2)$, or $\hat{M}_1(A_1,\hat{A}_2) = 1$. By (2), Firm 1 overturns the equilibrium.

It is immediate that $\hat{A}_2 = 0$ in any equilibrium in which Firm 1 captures the market. Suppose $\hat{A}_1 > 0$ in such an equilibrium. Eliminating weakly dominated strategies gives $\hat{P}_2 \geq P(1|C_2)$, so we may choose $P_1' \in (P(1|C_1),\hat{P}_2)$ and $\delta \in (0,\hat{A}_1)$ to satisfy:

$$\Pi(P_1',1|C_1) = \Pi(P(1|C_1),1|C_1) - \delta$$

$(\hat{A}_1-\delta, P_1)$ is equilibrium dominated for all $P_1 > P_1'$, so that reasonable conjectures imply $\hat{U}_1(\hat{A}_1-\delta) > U(\hat{P}_2) = \hat{U}_2(\hat{A}_2)$, or $\hat{M}_1(\hat{A}_1-\delta, \hat{A}_2) = 1$. Clearly, Firm 1 prefers $\hat{A}_1-\delta$.

It remains to show that the efficient equilibrium with $\hat{A}_1 = \hat{A}_2 = 0$ is reasonable. For any $A_1 > 0$, (A_1, P_1) is equilibrium dominated for every P_1, so reasonable conjectures place no restriction. For $A_2 > 0$, whenever any (A_2, P_2) is not equilibrium dominated, $(A_2, P(1|C_2))$ will not be, so $\hat{U}_2(A_2) = U(P(1|C_2))$ is never an unreasonable conjecture. The result then follows since $\hat{U}_2(A_2) = U(P(1|C_2)) < U(P(1|C_1)) = \hat{U}_1(0)$. Q.E.D.

The key idea is that all equilibria save the efficient one fail to be reasonable because of the opportunity to use advertising to achieve better coordination. Advertising makes possible implicit communication between the efficient firm and consumers, whereby the efficient firm credibly informs consumers of the opportunity to benefit collectively from the low cost technology. The opportunity for such communication means that inefficient equilibria cannot be focal.

The theorem shows, moreover, that the coordination role of advertising can be realized without any actual dissipation of rents. Once the efficient firm anticipates capturing the market, there is no need for positive amounts to be spent in order to convince consumers of the opportunity for mutual gain. Advertising is a very efficient coordination mechanism because it is simply the possibility of advertising that makes the efficient equilibrium uniquely focal.

It is important to note that the elimination of weakly dominated strategies is not sufficient to predict the efficient equilibrium. For example, consider the equilibria in which the firms share the market at some price $\hat{P}_1$. Dominant strategy removal assures consumers a better deal only if $A_1 > \Pi(\hat{P}_1, 1 | C_1)$ is observed. The gain of then acquiring the market may be too small to compensate for the large loss in advertising.[11] As shown in the proof, when equilibrium dominated strategies are removed, Firm 1 can profitably break this equilibrium with a deviant advertising level below $\Pi(\hat{P}_1, 1 | C_1)$.

4. Advertising, Coordination, and Signaling

While advertising may not necessitate rent dissipation to fulfill its role as a coordinating device, there may be other roles carried out in conjunction with coordination which require actual expenditure of resources. One of these is signaling. It may be unreasonable to assume that consumers have absolute certainty as to the identity of the lowest-cost firm. In this instance, advertising may play the additional role of signaling which firm is in fact efficient.

We will focus on the case of a new entrant facing an established firm with known costs. From the preceding section we know that the entrant will prevail if its costs are observably lower, but it is more plausible to suppose that consumers are uncertain as to whether the new firm will offer a better deal. Let Firm 1 be the entrant, and suppose that its costs may assume two possible levels, C_1^L and C_1^H. The value of C_1 is the private information of Firm 1. Let the prior probability of C_1^L be given by $\rho \in (0,1)$. Firm 2 is the incumbent, with cost level C_2. Assume $C_1^L < C_2 < C_1^H$, so it is unclear *ex ante* which firm is most efficient. Figure 3 illustrates the optimal pricing functions in the expanded model.

The strategy of Firm 1 now depends on its privat information, and is written $\hat{A}_1^L, \hat{P}_1^L$ for $C_1 = C_1^L$, and $\hat{A}_1^H, \hat{P}_1^H$ for $C_1 = C_1^H$. The equilibrium conditions become:

(A) Profit Maximization.

$$\hat{A}_1^L, \hat{P}_1^L \in \underset{A_1, P_1}{\text{argmax}} \quad \Pi(P_1, \hat{M}_1(A_1, \hat{A}_2) | C_1^L) - F(C_1^L) - A_1$$

$$\hat{A}_1^H, \hat{P}_1^H \in \underset{A_1, P_1}{\text{argmax}} \quad \Pi(P_1, \hat{M}_1(A_1, \hat{A}_2) | C_1^H) - F(C_1^H) - A_1$$

$$\hat{A}_2, \hat{P}_2 \in \underset{A_2, P_2}{\text{argmax}} \quad [\rho\Pi(P_2, \hat{M}_2(A_2, \hat{A}_2^L) | C_2) + (1-\rho)\Pi(P_2, \hat{M}_2(A_2, \hat{A}_1^H) | C_2)] - F(C_2) - A_2$$

(B) Utility Maximization. As above.

(C) Correct Conjectures.

$$\hat{U}(\hat{A}_2) = U(\hat{P}_2)$$

$$\text{If } \hat{A}_1^L \neq \hat{A}_1^H: \quad \hat{U}_1(\hat{A}_1^L) = U(\hat{P}_1^L) \quad \text{and} \quad \hat{U}_1(\hat{A}_1^H) = U(\hat{P}_1^H)$$

$$\text{If } \hat{A}_1^L = \hat{A}_1^H: \quad \hat{U}_1(\hat{A}_1^L) = \rho U(\hat{P}_1^L) + (1 - \rho) U(\hat{P}_1^H)$$

In (A), the profit maximizing choices of Firm 1 depend on its private information, while those of Firm 2 reflect Firm 2's uncertainty about the costs of its rival. (C) differs from above only in that consumers may be uncertain as to Firm 1's costs even after observing its level of advertising. Uncertainty will persist in equilibrium if Firm 1 chooses the same amount of advertising under either cost level, while if the amounts are different, consumers will correctly infer costs. Thus, advertising may serve to signal the cost level.

For advertising to be an informative signal, Firm 1's incentives must vary systematically with its private information. In particular, we assume:

<u>Sorting Condition</u>: $\Pi(P(1|C_j),1|C_j) - \Pi(P(M_j|C_j),M_j|C_j)$ strictly decreases in C_j for $M_j < 1$

Thus, Firm 1 will have greater incentive to capture the market when its costs are lower.

A great many equilibria are possible when (C) is the only restriction on consumer conjectures. We will require a strengthened notion of dominance to deal with the expanded possibilities under incomplete information. In particular, suppose we have chosen an equilibrium and eliminated equilibrium dominated strategies for both firms and consumers. It is possible that some choice was not equilibrium dominated because it could improve on the equilibrium only for responses that have now been eliminated. With those responses ruled out, the equilibrium would guarantee greater profits or utility, and there would be no incentive to make the choice. In other words, elimination of equilibrium dominated choices may cause other choices to become equilibrium dominated, and the latter should be eliminated as well.

To formalize this idea, we must introduce notation to keep track of which choices have been previously eliminated. Fix an equilibrium and an advertising choice A_j which Firm j does not choose in the equilibrium.

For Firm j, let the sets $\mu_j^k(A_j,\hat{A}_i)$ and $\rho_j^k(A_j)$ give consumer purchase strategies and Firm j's price choices associated with A_j, which survive elimination of equilibrium dominated strategies up to stage k. They are defined inductively, as follows. Put $\mu_j^0(A_j,\hat{A}_i) = [0,1]$, $\rho_1^0(A_1) = [P(1|C_1^L),P(0^+|C_1^H)]$, and $\rho_2^0(A_2) = [P(1|C_2),P(0^+|C_2)]$; the latter two reflect elimination of weakly dominated strategies. Begin with consumer choices. If $U(P_j) < \hat{U}_i(\hat{A}_i)$ for all $P_j \in \rho_j^{k-1}(A_j)$, then $\mu_j^k(A_j,\hat{A}_i) = \{0\}$. If $U(P_j) > \hat{U}_i(\hat{A}_i)$ for all $P_j \in \rho_j^{k-1}(A_j)$, then $\mu_j^k(A_j,\hat{A}_i) = \{1\}$. Otherwise $\mu_j^k(A_j,\hat{A}_i) = [0,1]$.

Next consider the firms' choices. Let the equilibrium net operating revenues be given by:

$$\hat{\Pi}_1^L = \hat{\Pi}(\hat{P}_1^L,\hat{M}_1(\hat{A}_1^L,\hat{A}_2)|C_1^L)$$

and similarly for $\hat{\Pi}_1^H$ and $\hat{\Pi}_2$. Note that $\hat{\Pi}_2$ is expected equilibrium profit for Firm 2, with weights ρ and $1-\rho$. For Firm 1, $P_1 \in \rho_1^k(A_1)$ if and only if $P_1 \in \rho_1^{k-1}(A_1)$ and there is some $M_1 \in \mu_1^{k-1}(A_1,\hat{A}_2)$ such that:

$$(5) \quad \hat{\Pi}_1^L - \hat{A}_1^L \le \Pi(P_1,M_1|C_1^L) - A_1 \text{ or } \hat{\Pi}_1^H - \hat{A}_1^H \le \Pi(P_1,M_1|C_1^H) - A_1$$

For Firm 2, $P_2 \in \rho_2^k(A_2)$ if and only if $P_2 \in \rho_2^{k-1}(A_2)$ and there exist $M_2^L \in \mu_2^{k-1}(A_2,\hat{A}_1^L)$ and $M_2^H \in \mu_2^{k-1}(A_2,\hat{A}_1^H)$ such that:

$$(6) \quad \hat{\Pi}_2 - \hat{A}_2 \le \rho\Pi(P_2,M_2^L|C_2) + (1-\rho)\Pi(P_2,M_2^H|C_2) - A_2$$

A consumer conjecture $\hat{U}_j(A_j)$ is now said to be reasonable if $\hat{U}(A_j) = U(P_j)$ and $P_j \in \rho_j^k(A_j)$ for every k, that is, it can be based on a choice which is neither weakly dominated nor equilibrium dominated at any stage. If $\rho_j^k(A_j) = \emptyset$ for any k, then conjectures are unrestricted. An equilibrium is <u>iterated dominant</u> if it can be supported by consumer conjectures which are reasonable in this sense, and if it survives elimination of weakly dominated strategies.

As an immediate implication of iterated dominance, we have:

<u>Lemma</u>: C_1^L captures the market in any iterated dominant equilibrium.

<u>Proof</u>: Put $M_1 = \hat{M}_1(\hat{A}_1^L, \hat{A}_2)$ and suppose $M_1 < 1$. Using the sorting condition and the better profit property, we have:

$$\Pi(P(1|C_1^L),1|C_1^L) - \hat{\Pi}_1^L > \Pi(P(1|C_1^H),1|C_1^H) - \Pi(P(M_1|C_1^H),M_1|C_1^H) > 0$$

Thus, we can find $A_1 > \hat{A}_1^L$ such that:

$$\Pi(P(1|C_1^L),1|C_1^L) - A_1 > \hat{\Pi}_1^L - \hat{A}_1^L \tag{7}$$

$$\Pi(P(1|C_1^H),1|C_1^H) - A_1 < \Pi(P(M_1|C_1^H),M_1|C_1^H) - \hat{A}_1^L \le \hat{\Pi}_1^H - \hat{A}_1^H \tag{8}$$

Since $U(P(1|C_1^L)) > \hat{U}_2(\hat{A}_2)$, $1 \in \mu_1^k(A_1,\hat{A}_2)$ as long as $P(1|C_1^L) \in \rho_1^{k-1}(A_1)$, while (7) guarantees $P(1|C_1^L) \in \rho_1^k(A_1)$ whenever $1 \in \mu_1^{k-1}(A_1,\hat{A}_2)$; thus, the conjecture $\hat{U}_1(A_1) = U(P(1|C_1^L))$ is never eliminated by iterated equilibrium dominance. By (8), C_1^H always prefers the equilibrium choice to (A_1,P_1) for every level of P_1. Further, for given $P_1' \in (P(1|C_1^L),P(1|C_2))$, we can choose A_1 large enough to ensure that C_1^L prefers the equilibrium to (A_1,P_1) for every $P_1 > P_1'$; thus, $P_1 \notin \rho_1^1(A_1)$ whenever $P_1 > P_1'$. Iterated equilibrium dominance implies $\hat{U}_1(A_1) > \hat{U}_2(\hat{A}_2)$, or $\hat{M}_1(A_1,\hat{A}_2) = 1$, and by (7) type C_1^L deviates to A_1. Q.E.D.

The lemma shows that in any reasonable equilibrium the most efficient type will be able to use advertising to ensure that it captures the market. Advertising allows both coordination of consumer purchases, as in the preceding section, and signaling of costs to consumers. The sorting condition is essential for the latter, since for communication to be credible, the low cost type must gain relatively more from capturing the market. In the proof of the lemma, only one round of equilibrium dominance is required to eliminate conjectures which prevent the low cost firm from capturing the market; thus, the result holds for dominant equilibria as well.[12]

We must still ask whether the cost level is actually signaled in equilibrium, and what form of advertising is required. An equilibrium is called <u>separating</u> if $\hat{A}_1^L \neq \hat{A}_1^H$; consumers are then able to infer the entrant's costs from the observed advertising. Separating equilibria are characterized in:

<u>Theorem 2</u>: There exists one and only one separating equilibrium which is iterated dominant. In this equilibrium, the firm with the lower cost level always captures the market, and $\hat{A}_1^L = \Pi(P(1|C_1^H),1|C_1^H)$ while $\hat{A}_1^H = \hat{A}_2 = 0$.

<u>Proof</u>: From the lemma we know $\hat{M}_1(\hat{A}_1^L,\hat{A}_2) = 1$. Suppose $\hat{M}_1(\hat{A}_1^H,\hat{A}_2) > 0$, which implies $\hat{P}_2 \geq \hat{P}_1^H$. Choose P_2' to satisfy $P(1|C_2) < P_2' < P(1|C_1^H)$. Since $\hat{P}_1^H \geq P(1|C_1^H)$, we must have $\Pi_2(\hat{P}_2,\hat{M}_2(\hat{A}_2,\hat{A}_1^H)|C_2) < \Pi(P_2',1|C_2)$, and there exists $A_2 > \hat{A}_2$ defined by:

$$(9) \qquad \hat{\Pi}_2 - \hat{A}_2 = (1-\rho)\Pi(P_2',1|C_2) - A_2$$

Since $\hat{P}_1^L = P(1|C_1^L)$ and $P_2 \geq P(1|C_2)$ for all $P_2 \in \rho_2^k(A_2)$, we have $\mu_2^k(A_2,\hat{A}_1^L) = \{0\}$ for $k \geq 1$. Further, $P(1|C_2) \in \rho_2^0(A_2)$ guarantees $1 \in \mu_2^1(A_2,\hat{A}_1^H)$. Using (6) and (9), we have $P_2 \in \rho_2^2(A_2)$ if and only if $P_2 \geq P(1|C_2)$ and:

$$\Pi(P_2',1|C_2) \leq \Pi(P_2,1|C_2)$$

or $\rho_2^2(A_2) = [P(1|C_2),P_2']$. An inductive argument establishes $\rho_2^k(A_2) = [P(1|C_2),P_2']$ for all $k \geq 2$, and reasonable conjectures thus require $\hat{U}_2(A_2) \geq U(P_2') > U(\hat{P}_1^H)$, or $\hat{M}_2(A_2,\hat{A}_1^H) = 1$. By (9), Firm 2 deviates to A_2.

It follows that $\hat{M}_1(\hat{A}_1^H,\hat{A}_2) = 0$ in an iterated dominant separating equilibrium. Profit maximization for C_1^H implies $\hat{A}_1^H = 0$, and also $\hat{A}_1^L \geq \Pi(P(1|C_1^H),1|C_1^H)$. Suppose the inequality is strict, and choose $P_1' \in (P(1|C_1^L),P(1|C_2))$ and $\delta \in (\hat{A}_1^L - \Pi(P(1|C_1^H),1|C_1^H),\hat{A}_1^L)$ to satisfy:

$$\Pi(P_1',1|C_1^L) = \Pi(P(1|C_1^L),1|C_1^L) - \delta$$

This implies $\rho_1^1(\hat{A}_1^L-\delta) = [P(1|C_1^L),P_1']$, and $P_1' < \hat{P}_2 = P(1|C_2)$ assures that this holds for all k. Thus, reasonable conjectures imply $\hat{M}_1(\hat{A}_1^L-\delta,\hat{A}_2) = 1$, and C_1^L deviates to $\hat{A}_1^L - \delta$. A similar argument establishes $\hat{A}_2 = 0$ in an iterated dominant separating equilibrium.

It remains to show that the specified equilibrium is iterated dominant. It is clear from (5) that $\rho_1^1(A_1) = \phi$ when $A_1 > \hat{A}_1^L$. For $A_1 \in (0,\hat{A}_1^L)$, we have $P(1|C_1^L) \in \rho_1^k(A_1)$ and $1 \in \mu_1^k(A_1,\hat{A}_2)$ for every k, which implies

$P(1|C_1^H) \epsilon \rho_1^k(A_1)$ for all k. Thus, $\hat{U}_1(A_1) = U(P(1|C_1^H))$ is a reasonable conjecture, and since $\hat{P}_2 = P(1|C_2)$, we have $\hat{M}_1(A_1,\hat{A}_2) = 0$. Finally, $\mu_2^k(A_2,\hat{A}_1^L) = \{0\}$ for all A_2 under reasonable conjectures, and it is clear that $\rho_2^k(A_2) = \phi$ for $A_2 > 0$, for all k. Q.E.D.

We see that advertising may simultaneously carry out the tasks of coordinating purchase decisions and signaling cost levels, with the implication that the most efficient firm will capture the market. For signaling to occur, however, positive advertising must be chosen by the efficient entrant to convince consumers that its costs are not actually high. Incomplete information generates an externality whereby the entrant's inefficient counterpart free rides on the efforts of the efficient entrant to achieve coordination. The efficient entrant overcomes this difficulty only by dissipating through advertising all the rents available under inefficiency. Rent dissipation is caused not by rivalry between active firms, but rather by "informational rivalry" arising from cost uncertainty.

Iterated dominance is needed to rule out consumer conjectures which prevent the incumbent from capturing the market when the entrant is inefficient. Because of iterated dominance, the incumbent recognizes that capturing the market is impossible when the entrant is efficient, and this places tighter restrictions on the prices which may profitably accompany a disequilibrium advertising choice.

The theorem makes clear the conditions which are needed for dissipative advertising to serve as an informative signal. Since the marginal signaling cost is the same for both types of the entrant, the source of differential incentives which support credible signaling must be differential preference for increased market share. Advertising arises as a signal only where there is a coincidence of interests between consumers and the firm: Consumers must gain higher utility from types which profit more from increased market share. Such a situation is quite natural in an increasing returns industry, where advertising might consequently be expected to play an important signaling role. In contrast, there is less reason to expect advertising to serve as a signal of product quality, for example, since the types with the differential preference for increased market share offer the low-quality, low-cost products which give lower consumer utility.

Theorem 2 shows that positive advertising is required if there is to be both coordination and signaling of cost information. But it is also possible that signaling does not occur; an equilibrium is called <u>pooling</u> if $\hat{A}_1^L = \hat{A}_1^H$, so that consumers learn nothing about the entrant's costs from observing advertising. Will advertising still serve as an efficient coordinating mechanism in this case?

To study pooling equilibria, we require one last restriction on equilibrium behavior. Eliminating weakly dominated strategies did not prevent a firm from choosing $P(1|C_j)$ if it anticipated obtaining zero market share. Suppose, however, that the firm believes consumers may err with some small probability in making purchase decisions, that is, consumers may "tremble." The firm would then prefer some price close to $P(0^+|C_j)$. We will say that an equilibrium is <u>robust to consumer trembles</u> if $\hat{P}_j = P(0^+|C_j)$ when Firm j obtains zero market share in equilibrium.[13] With this, we have:

Theorem 3: (a) There exists an iterated dominant pooling equilibrium if and only if:

$$\text{(10)} \qquad \rho U(P(1|C_1^L)) + (1-\rho)U(P(1|C_1^H)) \geq U(P(1|C_2))$$

In any pooling equilibrium, the entrant always captures the market, and $\hat{A}_2 = 0$.

(b) If in addition the equilibrium is robust to consumer trembles, then $\hat{A}_1^L = \hat{A}_1^H = 0$, and the pooling equilibrium is unique.

Proof: (a) From the lemma we know $\hat{M}_1(\hat{A}_1^L, \hat{A}_2) = 1$, so it follows at once that the entrant captures the market in any pooling equilibrium, and also that $\hat{A}_2 = 0$. To show existence, put $\hat{A}_1^L = \hat{A}_1^H = 0$, and observe that $\rho_1^1(A_1) = \phi$ for all $A_1 > 0$. For any $A_2 > 0$ and $P_2 \in \rho_2^0(A_2)$, (10) gives:

$$\hat{U}_1(0) = \rho U(P(1|C_1^L)) + (1-\rho)U(P(1|C_1^H)) \geq U(P(1|C_2)) \geq U(P_2)$$

If the inequality in (10) is strict, then $\mu_2^1(A_2,0) = \{0\}$ and so $\rho_2^k(A_2) = \phi$ for all $k \geq 2$. If instead (10) holds with equality, then $\mu_2^1(A_2,0) = [0,1]$, whence $\rho_2^k(A_2) = [P(1|C_2), P_2']$ for $k \geq 1$, where P_2' is defined by $\Pi(P_2', 1|C_2) = A_2$. In either case, $\hat{M}_2(A_2, 0) = 0$ for all $A_2 > 0$ is consistent with reasonable conjectures. For necessity, suppose (10) does not hold and fix $P_2' > P(1|C_2)$ sufficiently close to $P(1|C_2)$ to satisfy $\Pi(P_2', 1|C_2) > 0$ and:

$$U(P_2') > \rho U(P(1|C_1^L)) + (1-\rho)U(P(1|C_1^H))$$

Put $A_2 = \Pi(P_2', 1|C_2)$. It is easily seen that $\rho_2^k(A_2) = [P(1|C_2), P_2']$ for $k \geq 1$, and reasonable conjectures imply $\hat{M}_2(A_2, \hat{A}_1^L) = 1$. Firm 2 then deviates to A_2.

(b) Suppose $\hat{A}_1^L = \hat{A}_1^H > 0$. Fix $\delta \in (0,\hat{A}_1^L)$, $P_1^L > P(1|C_1^L)$ and $P_1^H > P(1|C_1^H)$ to satisfy:

$$\Pi(P_1^L,1|C^L) = \Pi(P(1|C_1^L),1|C_1^L) - \delta, \quad \Pi(P_1^H,1|C_1^H) = \Pi(P(1|C_1^H),1|C_1^H) - \delta$$

$$\min\{U(P_1^L),U(P_1^H)\} > U(P(0^+|C_2))$$

By the usual argument we have $\rho_1^k(\hat{A}_1^L-\delta) \subset [P(1|C_1^L),\max\{P_1^L,P_1^H\}]$ for $k \geq 1$. Under robustness to consumer trembles, $\hat{U}_1(\hat{A}_1^L-\delta) > U(\hat{P}_2) = U(P(0^+|C_2))$ when conjectures are reasonable, or $\hat{M}_1(\hat{A}_1-\delta,0) = 1$. Either type deviates to $\hat{A}_1^L-\delta$. Q.E.D.

Condition (10) means that the expected utility offered by the entrant is no less than that of the incumbent. Thus, advertising plays the same coordinating role as in Theorem 1, with the entrant being the efficient firm from the ex ante viewpoint. Incomplete information gives rise to the additional possibility that consumers associate disequilibrium advertising choices with the high cost entrant only. This does not threaten the entrant, however, as long as the incumbent's strategy is a best response to the slight possibility of consumer trembles, in which case the incumbent chooses the high price associated with infinitesimal market share.[14]

The pooling and separating equilibria cannot be Pareto ranked, since consumers and the incumbent always prefer the separating equilibrium, while both types of the entrant earn higher profits in the pooling equilibrium. The positive advertising which arises in the separating equilibrium may be viewed as an entry barrier which gives the incumbent increased profits, but it does so only by keeping out inefficient entrants. This entry barrier leads to greater utility for consumers.[15]

5. Conclusion

We have argued that dissipative advertising can actually improve buyer welfare by directing buyers to the most efficient firm. Our two key assumptions have been that production costs exhibit returns to scale and that price information is difficult to transmit.

Our work contrasts with the traditional Bertrand polar case, in which it is assumed that firms can costlessly communicate all prices to all buyers. This case seems inappropriate for firms employing complex nonlinear pricing schedules and/or selling many products. In addition, firms often face legal or professional restrictions on price advertising. Our goal has been to examine the relatively unexplored opposite polar case in which prices cannot be advertised *ex ante*. The natural next step would be to consider the middle ground, where partial price information can be communicated to buyers. In the case of a multi-product retail store, for example, advertised loss leaders represent an important form of partial price communication. With an appropriate specification in which the firms' products are differentiated, price advertisements may play both rent dissipation and coordination roles.

An intriguing area for future research concerns the transition path to equilibria. Implicit in our story is an unmodeled dynamic in which positive advertising is used to "break" inefficient equilibria. This form of advertising is instantaneous and unobservable in our original model, but it does suggest a short-term role for observed advertising in achieving a dynamic transition to an efficient equilibrium.

APPENDIX

Example 1 Uniform Prices

We provide here an example under which all of the model's assumptions hold. Let $q(P_j)$ be the common demand function for consumers and let $C_j\ h(q(P_j)M_j)$ be the cost for Firm j when it receives a measure M_j of consumers, where $C_j > 0$. Letting primes denote derivatives, assume $q'(P_j) < 0$, $h'(q(P_j)M_j) > 0$ and $h''(q(P_j)M_j) < 0$, with the latter inequality reflecting returns to scale. We now have

$$\Pi(P_j, M_j | C_j) = P_j q(P_j) M_j - C_j\ h(q(P_j)M_j)$$

The first order condition, which is solved by $P(M_j|C_j)$, is given by

$$M_j\ [P_j q'(P_j) + q(P_j) - C_j\ h'(q(P_j)M_j)\ q'(P_j)] = 0.$$

The second order condition holds, for example, if $q(P_j)$ is concave and $h''(q(P_j)M_j)$ is not too negative.

Implicit differentiation gives

$$\text{sign}\ \frac{dP(M_j|C_j)}{d\ M_j} = \text{sign}\ h''(q(P(M_j|C_j))M_j) < 0$$

and

$$\text{sign}\ \frac{d\ P(M_j|C_j)}{d\ C_j} = \text{sign}\ h'(q(P(M_j|C_j))M_j) > 0.$$

Thus, the better deal property holds as does the cost effects condition.

Next, it is straightforward to verify that the better profit property holds. Since $h''(q(P_j)M_j) < 0$, if P_j weakly exceeds average cost, then P_j exceeds marginal cost. Hence, if $\Pi(P_j, M_j|C_j) \geq 0$, then

$$\frac{d\,\Pi(P_j,M_j|C_j)}{d\,M_j} = [P_j - C_j\, h'(q(P_j)M_j)]\, q(P_j) > 0$$

and thus $\Pi(P_j,1|C_j) > \Pi(P_j,M_j|C_j) \geq 0$. Of course, if $\Pi(P_j,M_j|C_j) < 0$ and $\Pi(P_j,1|C_j) > 0$, then $\Pi(P_j,1|C_j) > \Pi(P_j,M_j|C_j)$. The better profit property therefore holds.

Finally, we establish that the sorting condition holds. Using the envelope theorem,

$$\frac{d\{\Pi(P(1|C_j),1|C_j) - \Pi(P(M_j|C_j),M_j|C_j)\}}{d\,C_j}$$
$$= h(q(P(M_j|C_j))M_j) - h(q(P(1|C_j))) < 0,$$

for $M_j < 1$, since $q'(P_j) < 0$, $h'(q(P_j)M_j) > 0$, and the better deal property holds.

It is worth noting that this analysis extends readily to multi-product firms. A novel factor arising in that case is "economies of scope" whereby extra production of one good lowers the marginal cost of other goods as well. This effect acts to reinforce the better deal property.

<u>Example 2</u> Two Part Tariffs

We now demonstrate that the assumptions continue to hold in a more complex pricing environment. Let there be two types of consumers, θ_1 and θ_2, with $\theta_2 > \theta_1 > 0$. Let λ be the proportion of consumers of type 1. Utility of a type i consumer paying T for q units of output is:

$$\theta_i\ [q-q^2/2] - T$$

Each firm offers a two part tariff, $T_j(q) = E_j + P_jq$. A type i consumer who purchases from Firm j will have the following demand:

$$D_i(P_j) = (1-P_j/\theta_i).$$

Suppose Firm j obtains proportion M_j^i of type i consumers, and $M_j^1 > 0$. If θ_1 and θ_2 are sufficiently close, it is optimal for the firm to induce consumers of both types to buy. E_j is therefore set to appropriate all of the surplus of type 1 consumers:

$$E_j = (\theta_1-P_j)^2/(2\theta_1)$$

Firm j's profits are then given by

$$(\lambda M_j^1 + (1-\lambda)M_j^2)(\theta_1-P_j)^2/(2\theta_1) + P_j[\lambda M_j^1(1-P_j/\theta_1) + (1-\lambda)M_j^2(1-P_j/\theta_2)]$$
$$- C_jh(\lambda M_j^1(1-P_j/\theta_1) + (1-\lambda)M_j^2(1-P_j/\theta_2)),$$

where again $C_j > 0$, $h'(\cdot) > 0$, and $h''(\cdot) < 0$.

The profit maximizing P_j is well defined if $h''(\cdot)$ is not too negative. It is easily shown that:

$$\frac{d\ P_j(M_j^1,M_j^2|C_j)}{d\ M_j^2} < 0 \qquad \text{and}$$

$$\frac{d\ P_j(M_j^1, M_j^2 | C_j)}{dC_j} > 0 .$$

Moreover, the surplus of type 2 consumers is given by

$$(\theta_2 - P_j)^2/(2\theta_2) - (\theta_1 - P_j)^2/(2\theta_1)$$

Since this decreases with P_j, the better deal property holds for type 2 consumers. Further, it is easily shown that the better profit property and sorting condition hold for this example, and cost effect (B) holds if C_j and C_j' are restricted to a sufficiently small interval.

The example departs from the basic framework in that consumer utility is now determined by a pricing rule involving two parameters, with type 1 consumers always having all surplus extracted, so that the better deal property does not hold for them. All of our results go through, however, if we assume that the firms always evenly split the indifferent type 1 consumers, and if the resulting optimal two part tariff as a function of M_j^2 and C_j is substituted into the utility function of the type 2 consumers. The framework may then be renormalized by taking a half share of type 1 to be zero (in fact, the theorems will no longer require elimination of weakly dominated strategies and robustness to consumer trembles).

REFERENCES

Bagwell, K., "Introductory Price as a Signal of Cost in a Model of Repeat Business," The Review of Economic Studies, Vol. 54 (1987), pp. 365-84.

Bagwell, K. and M. Peters, "Dynamic Monopoly Power When Search Is Costly," Northwestern D.P. 772, 1988.

Bagwell, K. and G. Ramey, "Advertising and Limit Pricing," The Rand Journal of Economics, Vol. 19 (1988), pp. 59-71.

Bain, J., Barriers to New Competition, Cambridge: Harvard University Press, 1956.

Baumol, W., J. Panzar, and R. Willig, Contestable Markets and the Theory of Industry Structure, New York: Harcourt Brace Jovanovich, 1982.

Benham, L., "The Effect of Advertising on the Price of Eyeglasses," Journal of Law and Economics, Vol. 15 (1972), pp. 337-52.

Ben-Porath, E. and E. Dekel, "Coordination and the Potential for Self Sacrifice," unpublished manuscript, 1987.

Butters, G., "Equilibrium Distributions of Sales and Advertising Prices," The Review of Economic Studies, Vol. 44 (1977), pp. 465-91.

Chan, Y. and H. Leland, "Prices and Qualities in Markets With Costly Information," The Review of Economic Studies, Vol. 49 (1982), pp. 499-516.

Cho, I. and D. Kreps, "Signaling Games and Stable Equilibria," The Quarterly Journal of Economics, Vol. 102 1987), pp. 179-221.

Cho, I. and J. Sobel, "Strategic Stability and Uniqueness in Signaling Games," University of California, San Diego, D.P. 87-10, 1987.

Fudenberg, D. and J. Tirole, "A Theory of Exit in Duopoly," Econometrica, Vol. 54 (1986a), pp. 943-60.

Fudenberg, D. and J. Tirole, "Understanding Rent Dissipation: On the Use of Game Theory in Industrial Organization," American Economic Association Papers and Proceedings, Vol. 77 (1986b), pp. 176-83.

Glazer, J. and A. Weiss, "Pricing and Coordination: Strategically Stable Equilibria," unpublished manuscript, 1988.

Grossman, G. and C. Shapiro," Informative Advertising with Differentiated Products," The Review of Economic Studies, Vol. 51 1984), pp. 33-52.

Kihlstrom, R. and M. Riordan, "Advertising as a Signal," The Journal of Political Economy, Vol. 92 (1984), pp. 427-50.

Klein, B. and K. Leffler, "The Role of Market Forces in Assuring Contractual Performance," The Journal of Political Economy, Vol. 89 (1981), pp. 615-41.

Kohlberg, E. and J. Mertens, "On the Strategic Stability of Equilibria," Econometrica, Vol. 54 (1986), pp. 1003-38.

Kreps, D. and R. Wilson, "Sequential Equilibria," Econometrica, Vol. 50, pp. 863-94.

Maskin, E. and J. Tirole, "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition With Large Fixed Costs," Econometrica, Vol. 56 (1988), pp. 549-69.

Milgrom, P. and J. Roberts, "Price and Advertising Signals of Product Quality," The Journal of Political Economy, Vol. 94 (1986), pp. 796-821.

Nelson, P., "Information and Consumer Behavior," The Journal of Political Economy, Vol. 78 (1970), pp. 311-29.

Nelson, P., "Advertising as Information," The Journal of Political Economy, Vol. 81 (1974), pp. 729-54.

Peters, M., "Restrictions on Price Advertising," The Journal of Political Economy, Vol. 92 (1984), pp. 472-85.

Posner, R., "The Social Costs of Monopoly and Regulation," The Journal of Political Economy, Vol. 83 (1975), pp. 807-27.

Ramey, G., "Product Quality Signaling and Market Performance," Technical Report 504, IMSSS, Stanford University, 1987.

Ramey, G., "Intuitive Signaling Equilibra with Multiple Signals and a Continuum of Types," University of California, San Diego, D.P. 88-35, 1988.

Rogerson, W., "Advertising as a Signal When Price Guarantees Quality," Northwestern D.P. 704, 1986.

Rogerson, W., "Price Advertising and the Deterioration of Product Quality," The Review of Economic Studies, Vol. 55 (1988), pp. 215-29.

Salop, S., "Strategic Entry Deterrence," The American Economic Review, Vol. 69 (1979), pp. 335-38.

Schmalensee, R., "Advertising and Entry Deterrence: An Exploratory Model," The Journal of Political Economy, Vol. 91 (1983), pp. 636-53.

van Damme, E., "Stable Equilibria and Forward Induction," Bonn D.P. 128, 1987.

Notes

1. The comparison in terms of *ex ante* producer surplus, and thus *ex ante* total surplus, is less clear. The separating equilibrium does concentrate all production at the most efficient firm, but it also involves the burning of resources. Of course, if advertising is interpreted as an observable transfer of resources (e.g., to a charity), then total welfare is unambiguously higher in the separating equilibrium.

2. A somewhat related distinction is also made by Rogerson (1986), who argues that advertisements signal production costs and thereby quality choice. See also Bagwell (1987), where it is argued that early prices signal costs and thus future price choices.

3. This conclusion is reinforced, if anecdotally, by a recent television advertisement by the Builder's Square hardware store chain, in which the gains of coordination are explicitly lauded with the phrase "the more we sell, the lower the price; the lower the price, the more we sell."

4. See Salop (1977) for related themes. For other views, see Bagwell and Ramey (1987) and Schmalensee (1983).

5. In particular, equilibrium dominance and iterated equilibrium dominance are equivalent to Kohlberg and Mertens' criterion that the outcome of a stable set of equilibria be robust to the deletion of a strategy which is an *inferior* response in all equilibria of the set, i.e., is not a best response in any of the equilibria of the set.

6. Their result is obtainable as well by applying one round of equilibrium dominance in the manner of Theorem 1 below, as long as arbitrarily small advertising increments are permitted.

7. The model is slightly more complex if some fixed costs are avoidable, in which case non-active firms might choose to exit. Our basic results, however, survive this extension.

8. The requirement that $\Pi(P_j, 1|C_j) > 0$ captures the fact that the better deal property will fail in examples for prices below the marginal cost associated with $M_j = 1$ (e.g., $P_j = 0$). We will soon eliminate weakly dominated strategies, however, and this requirement will become irrelevant, as all remaining prices exceed the full-market marginal cost.

9. A related possibility is that P_j corresponds to a pricing policy and also a variety choice. For example, television advertisements are often observed in which a car dealer claims to have "the lowest prices and the best selection." This situation would complement our framework if, as seems likely, customers prefer greater variety and firms choose greater variety when more customers are expected. Our arguments suggest that the costs of such advertisements may actually make credible the claims made therein.

10. We do not eliminate $P(0^+|C_j)$, however, because it is a limit point of strategies which are not weakly dominated. Also, we note that the elimination of weakly dominated strategies imposes stronger conditions than the requirement that $P_j \epsilon [P(1|C_j), P(0^+|C_j)]$. We prefer, however, to impose further restrictions via the equilibrium dominance restriction, for reasons discussed below.

11. A sufficient condition under which the elimination of weakly dominated strategies gives the efficient equilibrium is $\Pi(P(1|C_1),1|C_1) - \Pi(\hat{P}_1,1|C_1) > \Pi(\hat{P}_1,\hat{M}_1(\hat{A}_1,\hat{A}_2)|C_1)$. In general, because our model has a continuum of strategies, even iterated elimination of dominated strategies will not suffice to achieve coordination. See Ben-Porath and Dekel for discussion.

12. This lemma does not actually require the elimination of weakly dominated strategies.

13. See Bagwell and Peters (1988) for an illustration of the importance of robustness to trembles in a repeated game with decreasing returns and no advertising. As our second example in the Appendix illustrates, other formulations of the present model make unnecessary the elimination of dominated strategies and robustness to consumer troubles.

14. In many games, pooling equilibria are not robust to the incentive of a strong pooling type to communicate its type. For example, criterion D1 (Cho and Sobel (1987); Ramey (1988)) eliminates pooling equilibria for signaling games. In our game, however, the strong, low-cost type captures the entire market without advertising in the pooling equilibrium, and thus has no incentive to communicate its type. Our pooling equilibrium is therefore robust to further refinement.

15. We have concentrated on pure strategy equilibria. There may also exist mixed-pooling equilibria. These equilibria can be eliminated by extending the sorting condition to cover randomized market shares, in the following way: For any nonnegative random variable $M_j \in [0,1]$: $\Pi(P(1|C_j),1|C_j)$ - Max E $\Pi(P_j,M_j|C_j)$, with the maximization taken over P_j, strictly decreases in C_j. This condition is made necessary by the fact that Firm 2 may play a randomized strategy in a mixed pooling equilibrium. The high cost entrant might then differentially prefer capturing the market with certainty, if higher costs are associated with greater concavity of profits in market share.

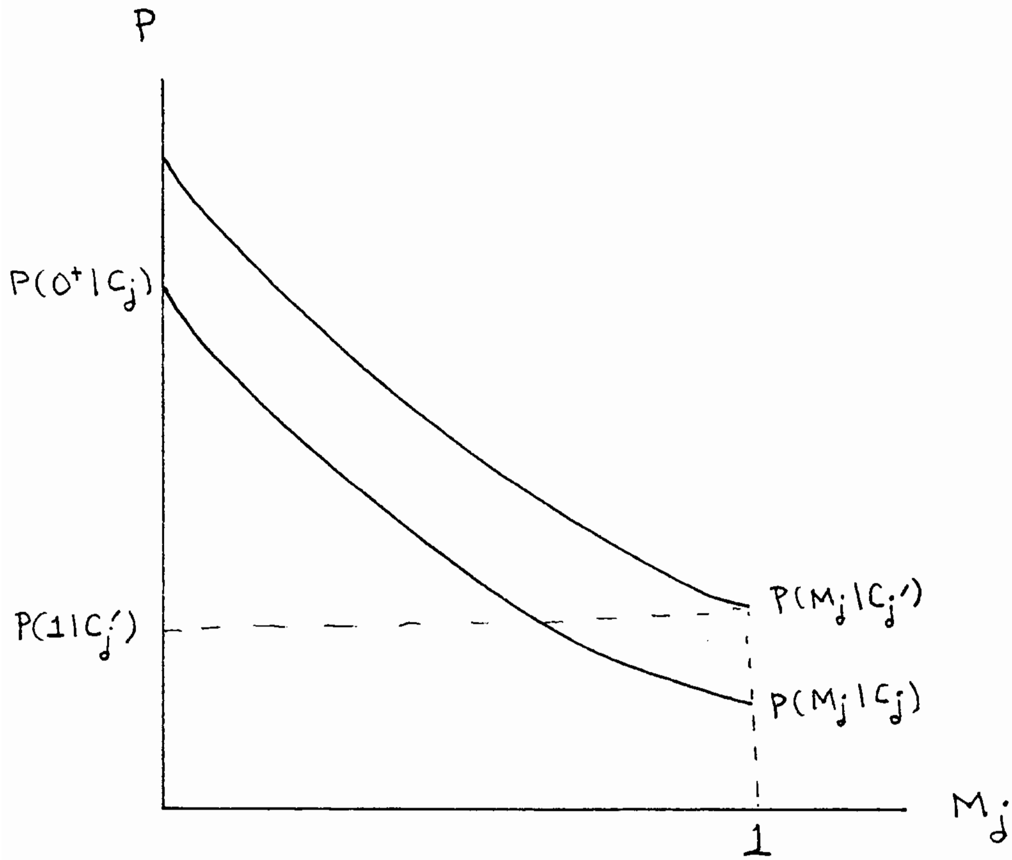

Figure 1

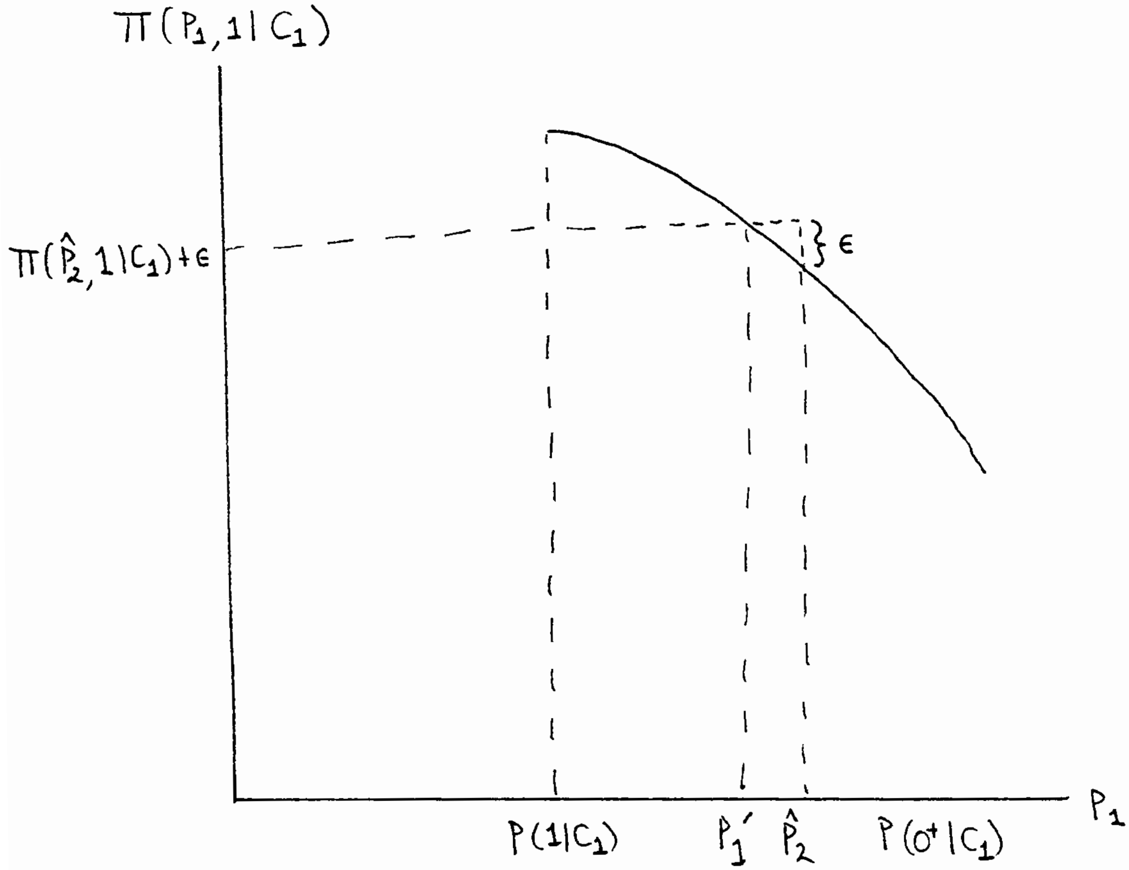

Figure 2

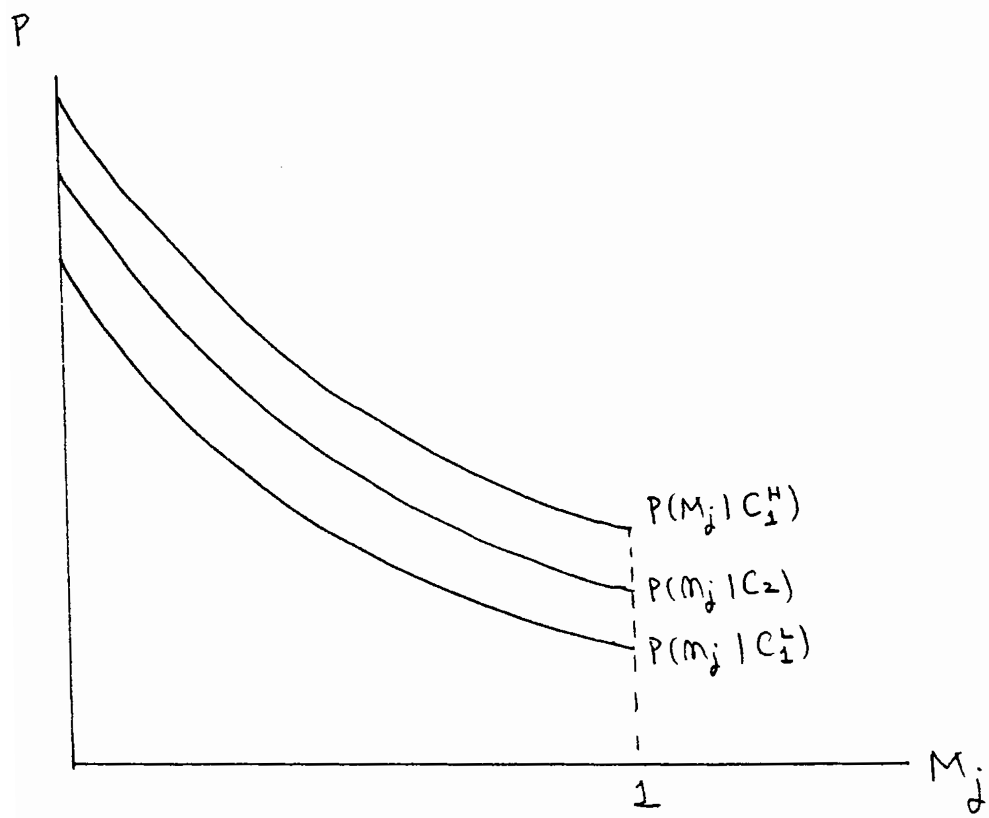

Figure 3