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MARKET FRICTIONS AND POSTED PRICES*

by

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Abstract

A lot of recent literature has discussed the possibility of obtaining the Walrasian outcome with decentralized trading. I look at a variant of this question in the context of a commonly observed trading institution--that where one side posts prices. More precisely, I look at a dynamic Bertrand maodel, where market frictions consist of slow or incomplete adjustments to change in relative prices. I describe equilibria of such markets and show that outcomes converge towards the Walrasian equilibrium as adjustment speeds go up and as adjustment becomes more complete.

1. Introduction

In a series of papers, Rubinstein and Wolinsky (1985), Gale (1986, 1987), Shaked (1987), and others have discussed the possibility of obtaining the Walrasian equilibrium without an auctioneer. In particular, these authors looked at equilibria of large markets with decentralized bargaining. The result of this literature is that prices, depending on the bargaining rules, may approach the Walrasian level as frictions in the market vanish.

The present paper is concerned with a variant of the same question in a different institutional context. I look at a dynamic Bertrand model where buyers adjust slowly or imperfectly to changes in relative prices. I describe equilibria of such markets and show how outcomes converge to the Walrasian equilibrium as adjustment speeds go up and as adjustments become more complete. Given that price posting is a very commonly observed institution, these results seem to be an important contribution to the literature on decentralized trading.

The literature in the area usually defines market frictions in terms of search time, the idea being that it takes time to locate alternative trading partners. When sellers post prices this means that they have "dynamic monopoly power" (Arrow, 1959). I here follow Mortensen (1986) and Wernerfelt (1988) in explicitly modeling the search of individual buyers as a point process.

My model can also accommodate a second type of market frictions, incomplete adjustments, which recently have received much attention in the literature on switching costs (Shilony, 1977; Klemperer, 1986). To avoid the complications arising from endogenous switching costs I here model them as results of automatic, seller specific, learning-by-using on the part of buyers.

The phenomena are analyzed in a continuous time economy in which overlapping generations of buyers trade continuously with a given measure of sellers. With less than instantaneous, but complete adjustments, the equilibrium price distribution can have no mass points. So the usual single price equilibrium does not exist, while instead an asymmetric steady state equilibrium exists. However, I show that the price distribution converges to a single point at the Walrasian price as adjustment speed goes up. When well-behaved switching costs prevent complete adjustment, both asymmetric and single price equilibria are feasible. I show that the single price equilibrium converges to the Walrasian equilibrium as adjustment speeds go up or switching costs vanish. I finally discuss models with a finite number of firms. Throughout, various comparative statics and examples are given.

2. Dynamic Monopoly Power

I look at an atomless market with a unit measure of consumers who are born and die with intensity $\tau > 0$, while a measure n (<< 1) of firms exist in perpetuity. A homogeneous, nonstorable good is traded in the market and for simplicity we abstract from production costs.

The instantaneous utility function of a consumer (say, i) is $U(1 - p_{it}y_{it}, y_{it})$, where p_{it} is the price charged by i's current supplier at t, and y_{it} is the rate of consumption. $U: R \times R_+ \to R$ is C^3 and has sufficient curvature to make the optimal consumption rates $y^*(p)$ declining C^2 functions for which $2y^{*\prime} + py^{*\prime\prime} < 0$ (such that the monopoly price p^m is finite). For notational simplicity, I will generally operate with u: $R_+ \rightarrow R$, the indirect utility function.

A "newborn" consumer is assigned to a randomly chosen firm and starts purchasing from that firm while receiving offers from other firms with intensity $\lambda > 0$. Let ω be a realization of this process. Consumers find switching strategies as measurable functions of the price offer in question, $p_i^0(t,\omega)$; the price of their current seller $p_i(t,\omega)$, the marginal distribution of firms over prices, G, and time. So in steady state equilibrium, where firms charge constant prices, consumer i is looking for $\delta_i^*(p_i^0,p_i,G,t,\omega)$ to maximize the expectation of expected lifetime utility, discounted at $r > 0$:

$$(1) \qquad \int_0^\infty e^{-(r+\tau)t}\, u[p_i(t,\omega)]dt$$

given

$$(2) \qquad dp_i(t,\omega) = dM[p_i(t,\omega)|\delta_i(\cdot)]$$

and a set of initial conditions, where dM governs the jumps in prices between suppliers. It is easy to see (by contradiction) that $\delta^*(\cdot)$ is of the reservation price type and that $\delta^*(\cdot) = p_i$.

Turning now to the firms, I assume that they cannot discriminate among consumers and only have the same aggregate information and set prices as measurable functions of G, the marginal distribution of reservation prices K, their own market shares, and time. Denoting the market shares of firm j by $b_j(t,\omega)$, we look for functions $p_j^*(b_j,G,K,t,\omega)$ to maximize the r-discounted expectation of profits

$$(3) \qquad \int_0^{\infty} e^{-rt} y^*(p_{jt}) p_{jt}\ b_j(t,\omega) dt$$

subject to

$$(4) \qquad db_j(\bullet)/dt = \tau\{1/n - b_j(\bullet)\} + \lambda(1 - K(P_{jt}))(1/n) - \lambda b_j(\bullet) G(p_{jt})$$

and initial conditions.

I now need the following:

Definition 1: A steady state equilibrium is a pair of a distribution of posted prices and a distribution of consumer states (the price they currently pay) such that:

(a) the distribution of consumer states is in a steady state;

(b) any posted price in the support of the firms' distribution is optimal given the two distributions.

Given this, we can show:

Lemma 1: In any steady state equilibrium, G has connected support and no mass points.

Proof: (By contradiction) Since only the ordering of prices matter, firms would move up in any gaps in G, so unconnected support is not compatible with equilibrium. Further, suppose there is a mass point above zero, say at p^0. In this case, there exists a $p' \in (0,p^0)$ such that $p^*(\bullet) = p'$ yields higher net present value than $p^*(\bullet) = p^0$. Finally, if there is a mass point at zero, then $p^*(\bullet) = p''$, $p'' \in R_+$, yields positive profits, while $p^*(\bullet) = 0$ yields zero net present value. Q.E.D.

Instead we get:

Lemma 2: There exists an asymmetric steady state equilibrium.

Proof: By the fixed point theorem of Fan (1952) and Glicksberg (1952). The object is to find a K such that steady state prices are identical to optimal prices for all market shares. Define M as the space of integrable functions from $[0,p^m]$ to R_+ and let M_C be the subset of M which is continuous. Now take any member of M, say k^0, and define $K^0(p) = \int_0^p k^0(x)dx$. If we insert this into (4), using $K(\bullet) = G(\bullet)nb(\bullet)$, we get the steady state condition:

$$(5) \qquad \tau(1/n - b(\bullet)) + \tau(1 - 2K(\bullet))(1/n) = 0$$

This identifies a unique, monotonic $\bar{b}(p|k^0) \in M_C$. The inverse of this function, $\bar{p}(b|k^0)$, is continuous in k^0.

Given $k \in M_C$, (3)-(4) is a control problem and standard results tell us that a solution $p_t^*(b_t|k)$ exists and is continuous in k. At this point

$$(6) \qquad \bar{p}(b) = p^*(b|k)$$

is an ordinary differential equation in k with solution $k^* \in M$. So we can define C as the correspondence from $\bar{b}(p)$ to k^*. Any fixed point of $C \bullet \bar{b}(p|k)$ thus identifies a k for which the optimal actions of each player (a) keeps the state of that player constant and (b) preserves the aggregate state distribution.

From the second order conditions and the monotonicity of $\bar{b}(p)$ we have that C is upper semicontinuous. It is straightforward to verify nonemptiness and that C is closed and convex valued. To check that $C \bullet \bar{b}(p|k)$ maps M into M, start in M and apply $\bar{b}$ to get into M_C from which C maps into M again. Since M is convex, we are done. Q.E.D.

<u>Remark 1</u>: Lemmas 1 and 2 are proved in Mortensen (1986) for the case where firms are restricted to time invariant prices and $r = 0$, while Wernerfelt (1988) proved general equilibrium versions.

To investigate the link to the Walrasian equilibrium, we ask what happens when $\lambda \rightarrow \infty$. Fortunately, orthodoxy is restored. If $\bar{K}$ is a steady state:

<u>Proposition 1</u>: $\forall\ \varepsilon_1, \varepsilon_2 > 0\ \exists\ \bar{\lambda}\ \forall\ \lambda > \bar{\lambda}: \quad \bar{K}(\varepsilon_1) > 1 - \varepsilon_2$.

<u>Proof</u>: From (5) we get

$$\bar{b}(p) = (1/n)[1 + (\lambda/\tau)(1 - 2K(p))] \tag{7}$$

Since $\bar{b}(p) \geq 0$, we need $K(p) < (\lambda + \tau)/2\lambda$. Q.E.D.

So even though the price distribution remains nondegenerate, eventually almost all trading takes place at prices very close to zero, the Walrasian price. It is further true that:

Corollary 1: Profits in asymmetric steady state equilibria are higher for slower search, faster turnover, and higher market share.

Proof: Obvious. Q.E.D.

3. Incomplete Adjustments

To investigate another aspect of the equilibrium correspondence, I now allow consumers to automatically accumulate seller specific user skills over their tenure with a particular seller. So the indirect, instantaneous utility function of consumer i, now has the form $u(p_{it}, z_{it})$, where z_{it} is the age of current trading relationship. Of course, $u(\cdot)$ is C^2, increasing and concave in $z \geq 0$, and yields a finite p^m for all z.

In this model we allow the switching strategies to depend on the age of the trading relationships, so, in steady state equilibrium, consumer i is looking for a measurable function $\delta_i^*(p_i^0, p_i, G, z_i, t, \omega)$ to maximize

$$(8) \qquad \int_0^\infty e^{-(r+\tau)t}\, u[p_i(t,\omega), z_i(t,\omega)]$$

given

$$(9) \qquad dz_i(t,\omega) = dt + dN[z_i(t,\omega) | \delta_i(\cdot)]$$

$$(10) \qquad dp_i(t,\omega) = dM[p_i(t,\omega) | \delta_i(\cdot)]$$

and a set of initial conditions, where dN ensures that z_{it} jumps back to zero if the consumer switches.

Since this is formulated as a piecewise deterministic Markov process (Davis, 1984), it follows from Vermes (1985, Theorem 1) that a solution to (8)-(10) exists.[1] Further, it is easy to see that the optimal $\delta^*(\bullet)$ is of the reservation price type and that $\delta^*(p_i,\bullet)$ declines continuously from p_i as z_i grows above zero.

<u>Remark 2</u>: It is possible to generalize the model further by making the search intensity λ, a decision variable. Suppose there is a suitable utility cost of searching faster than $\lambda^0 > 0$, and consumers seek measurable functions $\lambda_i^*(p_i^0, p_i, G, z_i, t, \omega)$. In this case, Jacod and Protter (1982) and Protter (1982), after I asked them for help, proved the existence of a solution to (10) in which, now, the intensity is state dependent.

Returning to the simple model, I here assume that firms know the joint distribution of consumers over prices and reservation prices $K(p,\delta)$. In this case the firm is looking for a measurable function $p_j^*(b,G,K,t,\omega)$ to maximize

$$(11) \qquad \int_0^\infty e^{-rt} y^*(p_{jt}) p_{jt} b_j(t,\omega) dt$$

subject to initial conditions and

$$(12) \qquad db_j(\bullet)/dt = \tau[1/n - b_j(\bullet)] + \lambda \int_p^\infty [1 - k_p(x,p)] dx (1/n) - \lambda b_j(\bullet) \int_0^p [1 - k_\delta(p,x)] G(x) dx$$

where k_p and k_g are the marginal densities of K on p and δ, respectively.

For this model I define:

Definition 2: A single price steady state equilibrium is a price p* and a distribution of consumer states (the length of staying at their current firm) such that:

(a) the consumers' distribution is in a steady state, and

(b) no firm will do better by altering the posted price.

Given this we get:

Lemma 3: A steady state single price equilibrium exists.

Proof: By construction. Assume that all firms have market shares 1/n and charge $p^* > 0$ indefinitely. In that case, (11)-(12) is a control problem. Because of the assumptions on $u(\bullet,z)$, we avoid the discontinuities from the proof of Proposition 1 and, instead, have sufficient curvature to satisfy the second order conditions for existence. Since all firms face the same problem, a symmetric equilibrium exists. Q.E.D.

The equilibrium price is defined by

$$(13) \qquad py' + y = py(2\lambda/(r + \tau))k$$

where k is $\partial K(p,\delta)/\partial\delta|_{\delta = p}$. So as in other switching cost models only the marginal consumers matter. We further get:

Corollary 2: The symmetric equilibrium price is higher for higher r, higher τ, lower λ, and lower exponential market growth.

Proof: Implicit differentiation of (13), thinking of r as containing negative growth. Q.E.D.

Again, here the equilibrium approaches the Walrasian equilibrium as either type of market friction disappears.

Proposition 2: The symmetric equilibrium price goes to zero as $\lambda \to \infty$ or $\partial u/\partial z \to 0$.

Proof: From (13). Q.E.D.

Note further that (13) gives the monopoly price as $\lambda \to 0$ or $\partial u/\partial z \to \infty$.

Remark 3: It is important that $u(\cdot,\cdot)$ be smooth at $z = 0$. If there is a fixed component to switching costs, the discontinuities reappear. Following Shilony (1977), one could conjecture that this can give mixed strategy equilibria, but it is unclear how one defines such strategies in differential games. Of course, a small amount of noise may smooth he aggregate discontinuities and eliminate this problem.

Remark 4: The asymmetric equilibrium still exists in this model. Further, I suspect that periodic (Gilbert, 1977) or chaotic equilibria may exist as well.

Example: Suppose $u(p,z) = u(pe^{-\alpha z})$, $\alpha > 0$. In this case $\delta(p,z) = pe^{-\alpha z}$ such that consumers are myopic in effective prices $\hat{p} \equiv pe^{-\alpha z}$. Now define $k(\bullet)$ as the density of such prices. If $g = G'$, we can write the dynamics of $k(\bullet)$ as

$$\dot{k}(x) = \tau[g(x) - k(x)] + \lambda^0 \int_x^\infty k(s)ds\,g(x) - \lambda^0 k(x) \int_0^x g(s)ds - [1 - \tau - \lambda^0 G(x)]\alpha x(dk/dx)$$

In the equilibrium described above, the steady state equations for k are

$$0 = \tau(1 - k(p)) + (1 - \tau)\alpha p\ (dk/dx)|_p,$$
$$0 = -\tau k(x) + (1 - \tau)\alpha x(\partial k/\partial x)|_x,\ x < p$$

in which $k(\bullet)$ has to satisfy

$$\int_0^p k(x)dx = 1 \text{ and } \lim_{x\to p} k(x) = k(p).$$

This gives $k(p) = 1 - p[\tau/[(1 - \tau)\alpha] + 1 + p]^{-1}$ such that (13) reduces to

$$(15)\qquad (1 - \frac{p}{\tau/[(1 - \tau)\alpha] + 1})(p\partial y/\partial p + y) = yp\,\frac{2\lambda^0}{r + \tau}$$

by implicit differentiation, this gives:

Finding 1: Faster learning gives higher prices.

4. A Finite Number of Firms

With a finite number of firms the anonymous formulation of the game is no longer satisfactory. The appropriate strategy space is for firms to seek measurable functions of the vector of market shares, G, K, t, and ω. It does seem possible to prove existence of equilibrium in this model, especially in the case with smooth consumer learning. I have, however, not been able to do so.[2] Instead, I have the following:

Lemma 4: If steady state single price equilibria exist, at least one is stable.

Proof: If a closed loop equilibrium exists, the market shares form a dynamical system. From the first order conditions we see that the vector field points in on the boundary of its support (one prices lower if one has no market share). So we can apply the Poincare-Hopf theorem (Guillemin and Pollack, 1974, p. 134) to show that the dynamical system has at least one stable point. Q.E.D.

Remark 5: Wernerfelt (1984) contains an analogous argument.

Corollary 3: Prices in stable loop equilibria are higher than those in open loop equilibria.

Proof: By direct calculation, the closed loop analogue of (13) is

$$p^*\partial y/\partial p + y(p^*) = \left(p^*y(p^*) + \frac{p'}{n}[p^*\partial y/\partial p + y(p^*)]\right)\left(\frac{2\lambda^0 k(p^*)(1 - 1/n)}{r + \tau + 2\lambda^0 k(p^*)p'/n}\right) \tag{15}$$

where $\partial p_j/\partial b_j = p'$ has been added in several places. Since this is positive in a stable equilibrium, the result follows. Q.E.D.

Intuitively, a price cut will decrease competitors' market shares and this will in turn lead them to cut prices, leaving overall incentives lower (much like conjectural variations).

Remark 6: The opposite result pertains for quantity games. There, volume expansion will lead to contraction by competitors, thus making it more attractive.

5. Discussion

Remark 7: I have shown how slow and incomplete adjustments to price changes affect the outcome in competitive markets. It turns out that the Walrasian outcome is the limit as adjustments grow fast and switching costs vanish.

Remark 8: In my opinion the results have interest beyond their limiting properties. Search lags and consumer learning pertain to a very large number of markets and these could profitably be analyzed in this framework.

Notes

[1]Vermes operates with a finite dimensional state space so I am cutting a corner here. One should think of a large but finite population of firms and consumers.

[2]In Wernerfelt (forthcoming), I have a verification result for the even more difficult case where consumers are finite also. The problem is that I have been unable to generate a set of candidate strategies. I have recently become aware of the nonstandard calculus approach of Judd (1985), which seems to present another avenue.

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