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RESALE-PROOF TRADES OF INFORMATION

by

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Abstract

This paper presents a model of a trade of an information good such that resales of it are freely allowed; nevertheless, no agent has the incentive to resell after he acquired the information. Such a trade is called resale-proof. We give the definition, examples and some properties of resale-proof trades.

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1. Introduction.

Licensing a patent is a common arrangement to enforce a proper trade of an information good such as a technological innovation by conferring the property right to the original information holder (the innovator). Without such legal protection, an information good will not be traded in the market (Arrow [1]). Therefore, a significant portion of the existing literature on information trading deals with the trades by licensing (see, eg., [3, 4, 5, 6, 7]).

It is clear that the trade by licensing can be enforced only if the patent protection is perfect; that is, licensing should involve the agreement not to relicense the patent; and any violation of this agreement can be detected and effectively punished. However, it is often difficult or costly to verify the violation, especially when the information good is one that is easily kept secret. Therefore, such an agreement cannot be perfectly binding in general; there may still remain incentives in the licensees to get an additional profit at the risk of violating the agreement.

The purpose of this paper is to consider whether it is possible to protect a patent in a self-binding way. That is, we ask if it is possible to consider a trade such that resales of the information are freely allowed; nevertheless, no agent has the incentive to resell the information after he acquired it. Of course, the information would spread over all the agents if there were always the incentive to resell. Therefore, we restrict the information good to one with the external effect such that the benefit from the information of every holder may decrease as the number of the holders increases. A typical example of such an information good is a technological innovation traded in an oligopolistic industry (see, Kamien and Tauman [4] and Muto [8, 9]). We suppose that there is only one initial information holder, which we call the seller, trying to sell the information to a finite number of buyers. The seller must assure himself that no resale will take place after the sale by himself. To which buyers, then, should he sell the information?

An earlier treatment of this problem can be found in Noguchi [10]. His approach is based on the notion of the core. Specifically, a trade was said to be blocked if for some buyers participating in the trade, there existed a prospect to gain by reselling the information to some of the buyers not participating in the trade. A trade being not blocked was then called self-binding. In our view, however, this definition of a self-binding trade is too strong. Indeed, the threat of reselling is not fully convincing and enforceable because even if there existed a prospect to gain by reselling to some buyers, the latter, after acquiring the information, may also resell it further to other buyers, thereby destroying the original prospect through the external effect of the information good. Therefore, the requirement that all such resales be non-existent would be unduly strong. (See Example 2.2).

The point of departure of our approach is to give a stricter notion of being able to resell. The basic idea is that a resale is enforceable when not only there exists a prospect to gain by reselling the information but also the gain can be assured by the fact that after the resale no further resale is enforceable. Notice the recursive structure of this argument, which is what we will precisely formalize later. If some buyers can carry out a resale in this way, the original holder will not sell the information to these buyers, since the information will surely spread out contrary to his original intention to protect the patent. The information will be sold, therefore, only to those buyers who cannot resell it in this way. Such a trade will be called resale-proof.

Although our model is not a game in strategic form, it may be useful to note the similarity at least in the spirit between the resale-proofness and the general notion of coalition-proof Nash equilibria introduced recently by Bernheim, Peleg and Whinston [2]. The coalition-proofness restricts the strategic deviations by a coalition to those which are self-enforcing; while in ours, the deviations meant by the resales are restricted to those which are enforceable in the sense to be made precise later.

The paper is organized as follows. In the next section, the model and the definitions are presented, together with examples illustrating our basic idea. In section 3, several conditions for the resale-proofness will be given. In particular, it will be shown that a trade with a proper subset of buyers can be resale-proof even if any buyer in this set has a prospect to gain by reselling to any proper subset of the remaining buyers (Theorem 3.5). This is a natural conclusion from the definition of resale-proofness. Section 4 considers the number of resale-proof sets, and the upper bound of it will be identified (Theorem 4.2). In section 5, it will be shown that the minimal resale-proof set is a unique optimum to the seller in the resale-proof sets (Theorem 5.1), which might be of a particular interest. Finally, in section 6, we conclude with some remarks. Some of the proofs will be given in the appendix.

2. Definitions and Examples.

Let $N=\{1,...,n\}$ be the set of all agents. Agent 1 is the sole seller of an information good, and the rest are the potential buyers. The seller is the initial holder of the information, and any buyer who acquired it will be also called a holder. The information is replicatable without costs and resales of it are freely allowed. Each holder obtains a monetary profit by utilizing the information, and non-holders obtain nothing. We assume that the individual profit is same for all the holders, depends only on the number of the holders, and is non-increasing; namely, denoting by $E(k)$ the profit when there are k holders:

Assumption 2.1 $E(1)\geq E(2)\geq...\geq E(n)>0$.

This expresses the external diseconomy to be induced by the diffusion of the information. We assume the function $E(\bullet)$ is known to all agents.

We now proceed to the definition of the resale. In the sequel, $\subset$ will stand for the proper set-inclusion and $|S|$ will mean the cardinality of the set S. A resale of the information from $S\subset N$ to $T\subseteq N-\{1\}$, where $S\cap T=\Phi$, is simply an agreement to transfer the information from S to T under the assumption that S has the information. When we speak of a resale by S, we will always be assuming that S has the information.

Definition 2.1 Let $B\subseteq N-\{1\}$, and assume that N-B has the information. A resale from $S\subseteq N-B$ to $T\subseteq B$ is said to be profitable iff $S\neq\Phi$, $T\neq\Phi$ and $(|S|+|T|)E((n-|B|)+|T|) > |S|E(n-|B|)$. The resale will be called a sale if $S=\{1\}=N-B$.

Behind this definition is the assumption that side payments are allowed so that the profitability can be judged solely by the difference of the total profits before and after the resale. Notice also that the profitability is only <u>tentative</u>, because some buyers in T may further resell the information after they acquired it. The problem is therefore to formulate a condition that makes the profitability sure.

For better understanding our motivation, we provide two examples before giving the formalization.

<u>Example 2.1</u>. $n=3$; $E(1)=30$, $E(2)=16$ and $E(3)=9$.

The seller has a tentatively profitable sale to any one buyer because $E(1)<2E(2)$. But, since $E(2)<2E(3)$, any one buyer has also a profitable resale to the last buyer. This latter profitability is sure because there are no other buyers. Since the seller does not have a profitable sale to the two buyers; that is, $E(1)>3E(3)$, we must conclude that the seller cannot sell the information at all.

One might object to this conclusion insisting that the seller could require a high enough payment from the first buyer on the basis of the fact that the buyer surely resells to the second buyer. But, the price p cannot exceed $E(3)+E(3)$. This is because $E(3)$ is the maximum amount that the second buyer can pay to the first buyer, and $E(3)$ is also the amount that the first buyer will obtain after the information is acquired by all the agents. Hence, the seller's profit will be $p+E(3)\leq 3E(3)<E(1)$. We will state this fact more generally in Lemma 2.1.

<u>Example 2.2</u>. $n=4$; $E(1)=30$, $E(2)=16$, $E(3)=9$ and $E(4)=5$.

Let $M=\{1,i\}$, $(i=2,3,4)$. The seller has a profitable sale to i, and buyer i has a profitable resale to buyer $j \neq i$. But, then, buyer j also has a profitable resale to the last buyer and this profitability is sure because there are no other buyers. Since buyer i does not have a profitable resale to the buyers $\{j,k\}$, $(j \neq i, k \neq i)$, we must conclude that buyer i cannot resell the information at all. Therefore, the seller may sell the information to i, since there is no possibility for the information to spread further. The trade in M can be considered as self-binding, which we will call resale-proof. Notice that there is no need to require that the buyer i do not have a profitable resale. The self-bindingness given by Noguchi [10] do require this assumption.

As is seen in these examples, there is a case in which the profitability of a resale is automatically sure if the resale is profitable; namely, the case of a resale to all the rest of buyers. Noting this fact, we now define inductively an enforceable resale.

Definition 2.2. Let $B \subseteq N-\{1\}$, and assume that N-B has the information.

(i) For $|B|=1$, we say $S \subseteq N-B$ has an enforceable resale to $T \subseteq B$ iff the resale is profitable.

(ii) Suppose that the definition is complete for $|B|=1,\ldots,k < n-1$. Then, for $|B|=k+1$, we say $S \subseteq N-B$ has an enforceable resale to $T \subseteq B$ iff the resale is profitable and there is no $T' \subseteq (N-B) \cup T$ which has an enforceable resale to some $P \subseteq B-T$.

Condition (i) defines the enforceable resale when there is only one buyer left as a non-holder. If there are two buyers left, condition (ii) states that the resale to the two buyers is enforceable iff it is profitable, because there is no other buyer. The resale to one of the two buyers is enforceable iff it is profitable <u>and</u> after the resale, no holders have an enforceable resale to the remaining buyer, which is well-defined by (i). The induction goes through in this way to an arbitrary number $|B| < n$ of non-holders.

<u>Definition 2.3</u>. Let M be a subset of N with $1 \in M$. We say M is <u>resale-proof</u> iff no subset of M has an enforceable resale assuming that M has the information.

Under this definition, N is resale-proof because N has no resale. Notice also that if $M=\{1\}$ is resale-proof, then, under our convention, the seller cannot sell the information at all. We will say M is a <u>feasible</u> resale-proof set iff it is resale-proof and the seller has a profitable sale to $M-\{1\}$. Thus, the feasible resale-proof set is the one to the buyers of which the seller has an enforceable sale.

One might suspect here that the seller, instead of protecting the patent in the self-binding way, would try to sell the information to buyers with a high enough price fully knowing that the information will be further traded. However, as noted in Example 2.1, this will not occur in general. Specifically, we can state as follows:

<u>Lemma 2.1</u>. Let R, S and M be subsets of N such that $\{1\} \subseteq R \subset S \subset M$, and assume that S has an enforceable resale to M-S. Then, the profit of R by reselling to S-R with price p cannot exceed $|M|E(|M|)$; namely, $p+|R|E(|M|) \leq$

$|M|E(|M|)$.

Proof. Let q be the amount that M-S is willing to pay to S for the information. Then, $q \leq |M-S|E(|M|)$. Hence,

$$p \leq |S-R|E(|M|) + q \leq |M-R|E(|M|).$$

Therefore, any set of agents can do no better than considering directly a resale-proof trade, which by putting $R=\{1\}$, would justify our standpoint that the seller will try to make a resale-proof trade.

3. Conditions for Resale-Proof Sets.

In this section, we derive several conditions for a set of agents to be resale-proof. We first state a lemma which is convenient in simplifying the structure of resale-proof sets.

Lemma 3.1. Let $\phi \neq B \subset N$ and assume that N-B has the information. Then, there exists no $S \subseteq N-B$ having an enforceable resale to $T \subseteq B$ iff there exists no $i \in N-B$ having an enforceable resale to $T \subseteq B$.

The proof is given in the appendix. By this lemma, we may replace $T' \subseteq (N-B) \cup T$ in Definition 2.2 with $i \in (N-B) \cup T$, and this in turn will enable us to state the condition for resale-proof sets in a more tractable form.

Let b and r be integers such that $0 \leq b \leq n-1$ and $0 \leq r \leq b$, respectively, and define two propositions α and β as follows:

$\alpha(n-b;r)$: one holder has a profitable resale to r non-holders when there are n-b holders,

$\beta(n\text{-}b;r)$: one holder has an enforceable resale to r non-holders when there are n-b holders.

The logical conjunctions and disjunctions of k propositions $\gamma_1,\ldots,\gamma_k$ will be denoted by $\Pi_{h=1,\ldots,k}\ \gamma_h$, and $\Sigma_{h=1,\ldots,k}\ \gamma_h$, respectively. The negation will be denoted by putting ~. Notice that $\sim\beta(n\text{-}b;r)$ is always true if b=0 or r=0. Hereafter, the cardinality |S| of any subset S of N will be also denoted by the small letter s. Then, we can restate the Definition 2.2 as follows.

<u>Lemma 3.2</u>. For each b=1,...,n-1; and r=1,...,b,

$$\beta(n\text{-}b;r) \leftrightarrow \alpha(n\text{-}b;r) \text{ and } \Pi_{s=1,\ldots,b-r} \sim\beta(n\text{-}b+r;s).$$

The proof follows immediately from Definition 2.2 and Lemma 3.1. By this lemma, Definition 2.3 can be also restated immediately as follows.

<u>Theorem 3.3</u>. Let M be a subset of N with $1\in M$. Then, M is resale-proof iff $\sim\beta(m;r)$ is true for each r=1,...,n-m; that is, for b=n-m,

$$\Pi_{r=1,\ldots,b}[\sim\alpha(n\text{-}b;r) \text{ or } \sim(\Pi_{s=1,\ldots,b-r}\sim\beta(n\text{-}b+r;s))]$$

is true.

It is also clear that M is a feasible resale-proof set iff $\sim\beta(m;r)$ is true for each r=1,...,n-m, and $mE(m)>E(1)$. We can now state a sufficient condition for the resale-proofness as a corollary of this theorem.

Corollary 3.4. Let M be a subset of N with $1 \in M$, and

$$E(m) \geq (1+r)E(m+r) \quad \text{for each } r=1,\ldots,n-m.$$

Then, M is resale-proof.

Proof. The assumption implies that for each $r=1,\ldots,n-m=b$, $\sim\alpha(n-b;r)$ is true. Hence, by Theorem 3.3, the result follows.

This is essentially the result of Noguchi [10], and is an extreme case in which no holder in M has a profitable resale at all. This will be the case when the external effect becomes intense enough beyond m.

Another extreme case would be one in which every holder in M has a profitable resale to every set of buyers except N-M; nevertheless, M is resale-proof. The next theorem indicates that this is indeed possible.

Theorem 3.5. Let M be a subset of N with $1 \in M$, and assume that

$$E(m) \geq (1+(n-m))E(n), \text{ and}$$

$$E(m) < (1+r)E(m+r), \text{ for each } r=1,\ldots,n-m-1.$$

Then, M is resale-proof iff for each proper subset $R \subset N-M$ there exists an $i \in R$ who has a profitable resale to N-M-R.

The proof is given in the appendix. The intuition of this theorem is the following. Suppose M is resale-proof under the assumption of the theorem. Then every proper subset $R \subset N-M$ must have an enforceable resale to some set $S \subseteq N-M-R$. But, S must not be proper, i.e., S=N-M-R, since, otherwise, every holder in M has an enforceable resale to the union $R \cup S$ by assumption, contradicting the resale-proofness of M.

In this theorem, if M={1} then the seller cannot sell the information in spite of the fact that he has a profitable sale to every set of buyers except $N-\{1\}$. Example 2.1 is such a case. The following example illustrates the case in which $M \neq \{1\}$.

Example 3.1. $n=5$; $E(1)=30$, $E(2)=17$, $E(3)=9$, $E(4)=6$, and $E(5)=4$.

In this example, $M=\{1,i\},(i=2,3,4,5)$ is the only feasible resale-proof set, and any one of the seller and the buyer i has a profitable resale to any set of remaining buyers except $N-M$.

Theorem 3.5 describes the case when the external effect is not so intense as in Corollary 3.4. In fact, it is mild enough for any buyer not in M to have a profitable resale to the rest of all non-holders; that is, $E(n-b)<(1+b)E(n)$ for each $b=1,\ldots,n-m-1$. For the sake of contrast with Corollary 3.4, we shall state this as the following corollary.

Corollary 3.6. Let M be a subset of N with $1 \in M \subset N$ such that

$$E(m) \geq (1+(n-m))E(n), \quad \text{and}$$

$$E(n-b) < (1+b)E(n) \quad \text{for each } b=1,\ldots,n-m-1.$$

Then, M is resale-proof.

Proof. This follows from the sufficiency part of the proof of Theorem 3.5.

4. Structure of Resale-Proof Sets.

In this section, we discuss about some of the structural properties of resale-proof sets. The main result is the upper bound of the number of feasible resale-proof sets.

Let λ_n be the number of different sizes of feasible resale-proof sets; namely,

$$\lambda_n = |\{|M| : 1 \in M \subseteq N,\ E(1) < |M| E(|M|) \text{ and } M \text{ is resale-proof}\}|.$$

The problem is then to identify the upper bound of λ_n under the monotonicity assumption 2.1 on the function $E(\bullet)$. The following intuitive fact provides the key to the solution.

<u>Lemma 4.1</u>. Let M and M' be any two resale-proof sets with $M \subset M'$. Then, $\sim\alpha(m;m'-m)$ is true.

<u>Proof</u>. Putting $n-b=m$ and $r=m'-m$ in Lemma 3.2, we have

$$\sim\beta(m;m'-m) \leftrightarrow \sim\alpha(m;m'-m) \text{ or } \sim[\Pi_{s=1,\ldots,n-m'} \sim\beta(m';s)].$$

Since M and M' are both resale-proof by assumption, it follows that $\sim\alpha(m;m'-m)$ is true.

Thus, if $|M'|=m+1$ in this lemma, then no holder in M must have a profitable resale to one of the non-holders; that is, $E(m) \geq 2E(m+1)$. Therefore, if there exist p consecutive sizes of resale-proof sets, which is clearly the requirement for the upper bound, then the function $E(\bullet)$ must be decreasing at least by half for each of the consecutive sizes. This fact leads to the following result.

<u>Theorem 4.2</u>. Let $2^{p-1} < n \leq 2^p$. Then,

(i) $\lambda_n \leq p$.

(ii) For each p' with $1 \leq p' \leq p$, there is at least one function

E(•) such that $\lambda_n = p'$.

The proof is given in the appendix. Let us say the resale-proof set is _pure_ if some holder in it has at least one profitable resale. Then, the upper bound p can be attained only if every resale-proof set is non-pure. If some of the feasible resale-proof sets are pure, then the number λ_n will be much smaller in general. As a special case, we may consider the structure with every feasible resale-proof set being pure. A typical example would be the following.

Example 4.1. n=6; E(1)=100, E(2)=90, E(3)=50, E(4)=30, E(5)=18 and E(6)=10.

The two sets $M_1=\{1,i\}$, and $M_2=\{1,i,j,k\}$ are resale-proof which are feasible and pure. Each of these sets has a profitable resale to any one buyer. Notice the regular structure that every set M with |M|=n, n-2, n-4 is resale-proof in this example. It is not difficult to show that under this structure, if n≥6 and $3^{p-1}<n-2\leq 3^p$, then there is a function E(•) such that $\lambda_n=p$ under the restriction that every feasible resale-proof set is pure. However, this structure does not give the upper bound. The following 10-person example has three different sizes of resale-proof sets which are feasible and pure; whereas, the above structure gives only two when n=10.

Example 4.2. n=10; E(1)=...=E(4)=60, E(5)=32, E(6)=20, E(7)=10, E(8)=7, E(9)=3 and E(10)=2.5 .

Three sets M_1, M_2 and M_3 such that $1\in M_1\subset M_2\subset M_3$, $|M_1|=4$, $|M_2|=6$ and $|M_3|=7$ are resale-proof which are feasible and pure. M_1 has a profitable resale to any one buyer, M_2 to any two buyers, and M_3 to any one buyer.

We believe a certain repetition of the structure of Example 4.2 will give the maximum number when $n \geq 10$ is given arbitrary. But, to show this will require a long specific argument and is beyond the scope of this paper.

5. Unique Optimum.

As we have seen in the last section, there will be many resale-proof sets in general. Therefore, the seller will have to make a choice among these feasible resale-proof sets. The following theorem provides a natural answer to this problem.

Theorem 5.1. Let M and M' be any two resale-proof sets such that $\{1\} \subset M \subset M'$. Then, $mE(m) > m'E(m')$.

This result states that the minimal resale-proof set guarantees the maximal profit to the seller in the resale-proof sets, so that the seller has only to choose the minimal feasible resale-proof set. Note that the maximality is not necessarily the overall one. In Example 2.1, the set with maximal profit is not resale-proof.

Proof of Theorem 5.1. Suppose that $mE(m) \leq m'E(m')$. By Assumption 1, $E(m) \geq E(m')$. Hence, noting that $m > 1$,

$$E(m) + (m-1)E(m') \leq mE(m) \leq m'E(m'),$$

which implies

$$E(m) \leq (1+m'-m)E(m').$$

If $E(m)=(1+m'-m)E(m')$, then $E(m)>E(m')$ because $m'>m$. Then, by this equality, we have

$$m'E(m') = E(m) + (m-1)E(m') < mE(m),$$

which is a contradiction. Hence we must have the inequality:

$$E(m) < (1+m'-m)E(m').$$

This implies that $\alpha(m;m'-m)$ is true, which, by Lemma 4.1, contradicts the assumption that M and M' are both resale-proof.

As for the uniqueness of the resale-proof set itself, two sufficient conditions can be stated.

Corollary 5.2. Assume that $mE(m) \leq nE(n)$ for all $m=1,\ldots,n$. Then, N is the only feasible resale-proof set iff $E(1) < nE(n)$.

Proof. By definition, N is a feasible resale-proof set iff $E(1)<nE(n)$. No proper subset M with $\{1\}\subset M$ can be resale-proof by Theorem 5.1. The set $M=\{1\}$ cannot be feasible by definition.

Corollary 5.3. Let M be a proper subset of N with $1\in M$ such that

$$E(m) \geq (1+(n-m))E(n),$$

$$E(n-b) < (1+b)E(n) \quad \text{for each } b=1,\ldots,n-m-1, \quad \text{and}$$

$$mE(m) \geq sE(s) \quad \text{for each } s=1,\ldots,m.$$

Then, m is the only size of feasible resale-proof sets iff $nE(n) \leq E(1) < mE(m)$.

Proof. By Corollary 3.6, M is a feasible resale-proof set iff $E(1)<mE(m)$. No proper subset of M can be a feasible resale-proof set, which

can be verified in a similar way to Corollary 5.2. Finally, no proper superset of M can be a feasible resale-proof set by assumption.

6. Concluding Remarks.

When an information good can be replicated and freely resold, and the information has the property that the profit from it may decrease as it diffuses, the information good can be sold to the buyers who do not have profitable resales at all when the external diseconomy is strong enough. But, this is not the only possibility of the trade: even if the external effect is weak, so that the buyers have profitable resales, they cannot resell the information if they know that further resales will surely occur. In this case, the seller can sell the information to these buyers since he can assure himself that no further trades will take place. This is what we have formalized in this paper, and the notion of the resale-proofness seems to capture in a more satisfactory way the nature of the trade of the information under free resales.

We have assumed for simplicity that the agents are symmetric in their profits. If this symmetry assumption is relaxed, the resale-proof set will depend not only on the cardinality of the set but also on who are the members of the set. There may, for example, exist an agent to whom the external effect is extremely weak. The set with such an agent will not be resale-proof if the agent has a profitable resale to all the rest of buyers. Therefore, a more elaborate model will be necessary to consider the general case. This issue would merit further study.

Another problem, which is of its own interest, would be to determine the upper bound of different sizes of resale-proof sets which are both feasible and pure. Example 4.1 has an interesting structure with every pure resale-proof set having a profitable resale to exactly one buyer. To obtain the upper bound, however, it will be necessary to reconstruct the structure based on Example 4.2. This would also be worth studying.

APPENDIX

Proof of Lemma 3.1. The necessity part is self-evident.

(sufficiency). Suppose there existed an $S \subseteq N-B$ such that S has an enforceable resale to $T \subseteq B$. If $|S|=1$, the proof is complete. Assume that $|S|>1$. Then, by definition, we have

$$(s+t)E(n-b+t) > sE(n-b),$$

where and hereafter the small letters s,t,.. denote the cardinalities of the sets S,T,.. . This implies

$$(1+t)E(n-b+t) > E(n-b),$$

since we would otherwise have a contradiction, because

$$(1+t)E(n-b+t) \leq E(n-b)$$

and

$$(s-1)E(n-b+t) \leq (s-1)E(n-b)$$

sum to

$$(s+t)E(n-b+t) \leq sE(n-b).$$

Therefore, any $i \in N-B$ has a profitable resale to T. But, since S has an enforceable resale to T, no subset of $(N-B) \cup T$ has an enforceable resale, which by Definition 2.2 implies that $i \in N-B$ has an enforceable resale to T.

Proof of Theorem 3.5. (sufficiency). Since $E(m) \geq (1+n-m)E(n)$, it follows that $\sim\beta(n-b;b)$ is true for b=n-m. Also, by assumption, $\beta(n-b+r;b-r)$ is true for each r=1,...,b-1. Hence, by Lemma 3.2,

$$\sim\beta(n-b;r) \text{ is true for each } r=1,\dots,b;\ b=n-m,$$

which, by Theorem 3.3, implies that M is resale-proof.

(necessity). By assumption, $\alpha(n-b;r)$ is true for each r=1,...,b-1, where n-b=m. Hence, for each r=1,...,b-1, we have by Lemma 3.2,

$\sim\beta(n-b;r) \leftrightarrow \Sigma_{s=1,\ldots,b-r}\beta(n-b+r;s)$

$\leftrightarrow \Sigma_{s=1,\ldots,b-r}[\alpha(n-b+r;s)$ and $\Pi_{t=1,\ldots,b-r-s}\sim\beta(n-b+r+s;t)]$

$\leftrightarrow \Sigma_{s=1,\ldots,b-r-1}[\alpha(n-b+r;s)$ and $\Pi_{t=1,\ldots,b-r-s}\sim\beta(n-b+r+s;t)]$

or $[\alpha(n-b+r;b-r)$ and $\Pi_{t=1,\ldots,b-r-s}\sim\beta(n;t)]$

$\leftrightarrow \Sigma_{s=1,\ldots,b-r-1}[\alpha(n-b+r;s)$ and $\beta(n-b;r+s)]$

or $\alpha(n-b+r;b-r)$.

But, since $\sim\beta(n-b;r)$ is true for each $r=1,\ldots,b-1$ by assumption, $\sim\beta(n-b;r+s)$ must be true for each $s=1,\ldots,b-r-1$. Hence,

$\sim\beta(n-b;r) \leftrightarrow \alpha(n-b+r;b-r)$, for each $r=1,\ldots,b-1$.

This implies that for each $R\subset N-M$ with $|R|=r$, there is an $i\in R$ such that i has a profitable resale to N-M-R.

<u>Proof of Theorem 4.2</u>. (i). Assume that $\lambda_n = q > p$. Then, we may choose q resale proof sets $M_1,\ldots,M_q$ such that $1\in M_1\subset\ldots\subset M_q$. By Lemma 4.1, $\sim\alpha(m_k;m_{k+1}-m_k)$ is true for each $k=1,\ldots,q-1$. Hence,

$$E(m_k) \geq (1+m_{k+1}-m_k)E(m_{k+1}) \geq 2E(m_{k+1}).$$

By multiplying both sides for each $k=1,\ldots,q-1$, we have

$$E(m_1) \geq 2^{q-1}E(m_q).$$

But, since M_q is a feasible resale-proof set, we have

$$m_qE(m_q) > E(1) \geq E(m_1).$$

Hence, $m_q > 2^{q-1} \geq 2^p \geq n$, which is a contradiction.

(ii). Given p' such that $1\leq p'\leq p$, define $E(\bullet)$ by

$E(t) = E(1)$, for each $t=1,\ldots,n-p'$,

$E(t) = E(1)/2^{t+p'-n-1}$, for each $t=n-p'+1,\ldots,n$.

It will be sufficient to prove the following claims:

<u>Claim A</u>. $\alpha(1;t-1)$ is true for each $t=n-p'+1,\ldots,n$.

Claim B. $\sim\beta(t;r)$ is true for each $r=1,\ldots,n-t$; and $t=n-p'+1,\ldots,n$.

Claim C. $\beta(t;(n-p'+1)-t)$ is true for each $t=1,\ldots,n-p'$.

These claims together imply that $\lambda_n = p'$.

Proof of Claim A. Let $t \geq n-p'+1$. Note that $2^{p-1} < n$. Then,

$$
\begin{aligned}
tE(t) &= tE(1)/2^{t+p'-n-1} \\
&= (t/2^{t})E(1)/2^{p'-n-1} \\
&\geq (n/2^{n})E(1)/2^{p'-n-1} = (n/2^{p'-1})E(1) \\
&\qquad \geq (n/2^{p-1})E(1) > E(1),
\end{aligned}
$$

which implies that $\alpha(1;t-1)$ is true.

Proof of Claim B. Let t be given such that $n-p'+1 \leq t \leq n-1$. Then, for any r with $1 \leq r \leq n-t$, we have

$$
\begin{aligned}
(1+r)E(t+r) &= (1+r)E(1)/2^{t+r+p'-n-1} \\
&= ((1+r)/2^{r})E(1)/2^{t+p'-n-1} \\
&\leq E(1)/2^{t+p'-n-1} \\
&= E(t),
\end{aligned}
$$

which implies that $\sim\beta(t;r)$ is true because $\sim\alpha(t;r)$ is true. If $t=n$, then $\sim\beta(n;r)$ is true by definition.

Proof of Claim C. Let t be given such that $1 \leq t \leq n-p'$. By construction, we have

$$
\begin{aligned}
E(t) &= E(1) \\
&< (1+(n-p'+1)-t)E(1) \\
&= (1+(n-p'+1)-t)E(n-p'+1).
\end{aligned}
$$

Hence, $\alpha(t;(n-p'+1)-t)$ is true for each $t=1,\ldots,n-p'$. But, putting $t=n-p'+1$ in Claim B, it follows that $\sim\beta(n-p'+1;r)$ is true for each $r=1,\ldots,p'-1$. Hence, putting $n-b=t$ and $r=n-p'+1-t$ in Lemma 3.2, we conclude that $\beta(t;(n-p'+1)-t)$ is true for each $t=1,\ldots,n-p'$. Hence, Claim C follows.

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