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BARGAINING WITH COMMON VALUES*

by

Daniel R. Vincent
J.L. Kellogg Graduate School of Management
Northwestern University
Evanston, IL 60208

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DANIEL R. VINCENT
M.E.D.S. Department,
The Kellogg Graduate School of Management,
Northwestern University,
Evanston, Illinois, 60201

ABSTRACT

B1 - Daniel R. Vincent --

B2 - Bargaining With Common Values

C2 - This paper examines a bargaining model with asymmetric information in which the private valuations of the two bargaining agents are correlated. It shows that equilibria in such models typically exhibit a significant probability of a significant delay to agreement. The paper characterizes the unique perfect Bayesian equilibrium to the game in which an uninformed buyer makes offers to a privately informed seller. It also shows, by example, that in this framework bargainers may rationally break off negotiations even in the presence of commonly known gains from trade.

B6 - J. Econ. Theory

B7 -

B8 -

C4 -

B4 - M.E.D.S. Department, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois, 60201.

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List of Symbols in Text

α - alpha

β - beta

δ - delta

ϵ - epsilon

$\int_a^b$ - the integral from a to b

$\leq$ - less than or equal to

$\geq$ - greater than or equal to

$<$ - less than

$>$ - greater than

Bargaining With Common Values

Since Rubinstein's paper in 1982, an important area of research in micro-economic theory has centred on strategic bargaining games. Recent work has focussed on bargaining games in which there is either one-sided or two-sided uncertainty about the preferences of the opponents who are bargaining. Although this is a natural extension, it is surprising to realize how restrictive these models are. Much of the current research on bargaining with incomplete information shares a common feature --each agent has perfect information about his own valuation of the object to be traded and, so, the models say little, for example, about behaviour when there is asymmetric information about the quality of the good to be traded. This paper takes the view that a more general specification of uncertainty is the appropriate method of modelling these games and shows that extending the models in this way alters some important conclusions of previous research. In particular, it is shown that allowing a more general form of private information re-establishes the possibility of significant delay to agreement even when bargaining offers can be made arbitrarily quickly. In addition, a common values model provides an explanation for the breakdown of negotiations even in the presence of commonly known gains from trade.

In previous models of bargaining under uncertainty an agent's valuation of a good is assumed to be independent of his opponent's valuation. Borrowing a term from the theory of auctions, these models might be called private values models. A common values model of bargaining incorporates the private values model as a special case but also allows the study of cases where values are correlated. If a player enjoys private information in this game, such information could include information which is relevant for his opponent's valuation as well. For example, one agent might have private information about the quality of the good to be traded. This feature of the information structure is recognizable in many well-known economic problems such as the market for lemons, job-market signalling or credit-rationing models. It is a natural environment in which to set incomplete information bargaining games.[1]

The bulk of the paper restricts attention to games where only the uninformed agent makes offers. This is a strong restriction. Solutions to the more complicated alternating offer games, however, still pose significant problems even in the private values models. The one-sided offer game has been examined in the private values case by Fudenberg, Levine and Tirole [5](henceforth FLT), and Gul, Sonnenschein and Wilson [8](henceforth GSW).[2] The first theorem of this paper shows that with slightly stronger assumptions on the parameters of the game (including the requirement that the uninformed agent be at least as patient as the informed agent), the uniqueness results of FLT and GSW carry over to the case of common values. That is, for a broad class of one-sided offer, one-sided uncertainty common values bargaining games there exists a unique perfect Bayesian equilibrium. The equilibrium is characterized and is shown to be similar to that of the FLT and GSW models.

A branch of the recent literature on bargaining with incomplete information has been concerned with the relationship between uncertainty and delay to agreement. In Rubinstein's [9] game of complete information, the unique subgame perfect equilibrium involved agreement in the first period of bargaining. However, the expectation was that in models with private information there would be realizations of the bargaining game where significant delay to agreement resulted as agents used delay to communicate their private information. Recent studies have suggested that this expectation was not justified. Work by GSW and Gul and Sonnenschein [7] on games with one-sided uncertainty showed that significant delay to agreement occurred only as a result of the inability to make offers quickly. Delay was generated by the technology of the game not by uncertainty. This paper shows that the conclusions of GSW and Gul and Sonnenschein depend on the formulation of uncertainty. A simple and natural generalization of the asymmetric information bargaining game to allow for correlated values re-establishes the possibility of delay no matter how quickly bargaining may occur. Delay results because when the buyer does not know his own valuation the possibility arises that the buyer may pay more for the good than it is worth to him. Suppose that the probability that the good is worth little to him is relatively high. Unless the buyer can ensure that he will pay a correspondingly lower price for the bad quality object he may suffer a net loss in the game. There is thus an additional incentive to keep prices paid to sellers of goods of different qualities far enough apart. In this game form, the only way trading prices can differ while maintaining self-selection constraints is if real time elapses in the bargaining process to induce low quality sellers to deal at significantly lower prices. This effect generates delay.

It would be desirable to examine what happens when players are allowed to alternate offers. Solutions to these more general games are typically plagued by the problems of multiple equilibria. It is known that bargaining games with two-sided uncertainty can exhibit delay. Section Four shows that in many common values models with one-sided uncertainty delay also occurs independent of the extensive form bargaining game and the time between offers.

Another result of the paper shows that equilibrium behaviour can involve no trade even when it is common knowledge that there are gains from trade. The characterization of the equilibrium in the game where the uninformed agent makes all the offers requires that the buyer be at least as patient as the seller. If this condition is violated, there are games with a perfect Bayesian equilibrium which involves the buyer making an offer which would extract all the surplus from a low quality seller. If the offer is rejected, the buyer breaks off negotiations. The model thus may suggest why breakdowns in bargaining processes occur even in the apparent presence of gains from trade.

Section One: The Model

A seller of a single, indivisible good and a buyer seek to agree on a price at which to trade. In standard formulations of bargaining games, outcomes are characterized by a pair, (p,t), where t is the period in which trade occurs, if ever, and p is the price agreed upon. For a game to be specified, then, preferences must be defined over the (p,t) space. It is assumed that preferences can be represented by the functions, $b^t(v(q) - p)$, for the buyer and $s^t(p - f(q))$ for the seller. b and s are the buyer's and the seller's discount factors and lie in the open interval, (0,1). The random variable q is determined by nature and distributed uniformly over the unit interval. In the paper, q will be interpreted as an index of the quality of the good or, more generally, an index of the 'type' of the seller. It is assumed that only one agent (here, the seller) observes the true value of q while the other agent knows only its prior distribution. The functions, $v(.)$ and $f(.)$, represent the valuations of the object for the buyer and the seller in money terms, $f(.)$ is assumed to be a non-decreasing, left-continuous function of q. The buyer's valuation function, $v(.)$, may also be a function of q. It is restricted to be non-decreasing and it will be required that $v(q) > f(q)$ for all q, that is there are always gains from trade to be realized, ex post. Agents maximize expected utility. All these features of the model are common knowledge. Note that since the seller observes q, he still enjoys full information about the relevant details of the model. The buyer, though, may be uncertain not only about the seller's valuation but about his own valuation as well.

Note that this model can incorporate a broad range of distributions concerning the seller's and buyer's types. For example, setting $f(q) = q$ yields the special case for which the seller's valuation is distributed uniformly on the unit interval. Setting $f(q) = 0$ for $q \leq 1/2$ and $f(q) = 4$ otherwise yields the special case for which the seller's valuation has a discrete distribution taking the values 0 and 4 each with probability one-half. Assuming that $v(q)$ is a constant $v \geq f(1)$ yields the private values models usually studied -- a seller has private information about his valuation of the object but the buyer's valuation is common knowledge. For the purposes of this paper, of course, we are interested in cases in which $v(.)$ is a non-trivial function of q.

Given this common values specification it remains to describe an extensive form game which characterizes the process of coming to agreement. The canonical extensive form bargaining game is that described by Ståhl [13] and Rubinstein [9] in which players alternate making and responding to offers sequentially. The game ends when an offer made by one agent is accepted by the other. It is well-known that with models of incomplete information such games yield a multiplicity of perfect and perfect Bayesian equilibria.[3]

Restricting attention to games where only the uninformed agent makes offers avoids this problem. FLT showed that in the one-sided offer, one-sided uncertainty bargaining game, as long as the seller's valuation is bounded away from the buyer's, there exists a unique perfect Bayesian equilibrium in mixed strategies and that bargaining ends in a finite number of periods. GSW strengthened this result to allow for more general types of distributions for the seller. They also showed that along the equilibrium path, the buyer follows a strategy of determinate offers -- that is, equilibrium behaviour involved no mixing on the part of the uninformed agent.

In view of these results the paper restricts attention to the one-sided offer bargaining game. The next section characterizes the perfect Bayesian equilibria to the extensive form game in which the uninformed buyer makes successive offers to an informed seller. The first theorem of the paper shows that this equilibrium is generically unique and that, with some modifications, the FLT-GSW results carry over.[4]

Section Two: Perfect Bayesian Equilibria to the One-sided Offer Bargaining Game

Impose the following further restrictions on the model of Section One:

1) f satisfies a Lipschitz condition at $q = 1$;

2) b, the discount factor of the buyer is not less than that of the seller -- $s \leq b$;

3) There is an $\epsilon > 0$ such that for all q, $v(q) - f(q) \geq \epsilon$.[5]

Condition 2) is problematic. It requires that the buyer be at least as patient as the seller. An example in Appendix Two shows that if the condition does not hold, then the equilibrium proposed by the theorem involves strategies which are dominated. Furthermore, an equilibrium is derived in which bargaining breaks off in the presence of known gains from trade. This condition is explored further in Section Five.

Given these conditions, Theorem One shows that there is a unique perfect Bayesian equilibrium.

Theorem One: (similar to that of GSW) Let the conditions 1) - 3) along with those of Section One be satisfied. In the game in which the uninformed agent makes all the offers there is a 'generically' unique perfect Bayesian equilibrium.[6] The equilibrium strategies of the seller are stationary and the equilibrium path of the buyer involves a determinate, monotonic sequence of price offers. Furthermore, bargaining ends after a finite number of periods.

Proof: The proof of this theorem is provided in an earlier version of this paper and is available from the author on request [14].

The determinate nature of the buyer's equilibrium behaviour enables us to characterize the equilibrium path by backward programming for many specifications of the game. Furthermore, the fact that there is a unique equilibrium allows us to analyze how payoffs and behaviour change with changes in the parameters of the game.

Consider the following simple example of the bargaining game. With probability three-quarters, the seller has an object which is of good quality and is worth 4 to him and 5 to the buyer. Otherwise the object is of bad quality and is worth 0 to the seller and 1 to the buyer. Discount factors are equal and set at 1/2. The highest (and final) price offered in equilibrium by the buyer is $p = 4$. If there is a period before the final period, the price must be such that the low quality seller is indifferent between accepting and waiting for the higher price next period. With the discount rate at 1/2, this price is 2. Similarly, the price before that if there is an earlier period is 1, before that, one-half, and so on. In fact, the equilibrium strategies for this game call for only two periods of offers. The equilibrium strategies are:

For the seller: In every period,

if $q \leq 1/16$ accept all price offers $p \geq 1$;

if $q \leq 1/4$ accept all price offers $p \geq 2$;

if $q > 1/4$ accept all price offers $p \geq 4$.

For the buyer: Given beliefs on q such that

if $q \in (1/4,1]$ offer $p = 4$;

if $q \in (1/16,1]$ offer $p = 4$ with probability β

$= 2$ with probability $1 - \beta$

where, if last period price was p', β satisfies

$p' = 1/2(\beta 4 + (1 - \beta)2)$;

if $q \in [0,1]$ offer $p = 2$.

(Note that $q \in [a,b]$ should be read as the distribution of q conditional on q in the interval [a,b], etc. See Appendix One for a description of how the strategies are computed.) The equilibrium path involves an offer of two in the first period and if that is rejected an offer of four in the second. All sellers will have accepted by the end of the second period. The buyer's expected payoff from playing the game is $1/4(-1) + 1/2(3/4)(1) = 1/8$.

Notice that there may be periods where the buyer makes offers which he knows will be accepted only by the low-quality seller. In the example above, the offer of two attracts only sellers of a good that is worth one to the buyer. If the buyer hears an acceptance, he would like to withdraw his offer. It is important to emphasize, then, the fact that an offer implies a true commitment on the part of the buyer. The reason the buyer is willing to make such offers is that if he hears a rejection, he knows that he can go after the owner of the high-quality object and extract the maximum surplus. The buyer is taking a gamble in order to acquire information that is of some positive value.[7]

Notice, also, the role of mixing in the example. Along the equilibrium path, the behaviour of both the buyer and seller involves only pure strategies. While this is true in general for the buyer it is not always the case for the seller. As observed in note three, mixed strategies by the seller are to be interpreted as pure strategies of sellers who have the same valuation but observe different values of q. Off the equilibrium path, both the buyer and the seller may, in general, mix. In the example, a seller of 'type' 0 mixes his acceptance of non-equilibrium price offers between one and two with probability one-quarter. If this offer is rejected, then the buyer is indifferent between offering two or four and mixes between the two in such a way as to justify the indifference of the seller.

The next Section uses our ability to characterize equilibria to show what happens when it becomes possible to make offers arbitrarily quickly.

Section Three: Bargaining With One-sided Uncertainty Causes Delay

Typically, in economic contexts where there is incomplete information, it is necessary to use up resources to enable trade. This is because a cost of transaction is screening. In bargaining games, this cost should be represented by delay to agreement. In view of this conventional wisdom, the GSW and Gul and Sonnenschein results were striking. When the degree of commitment is insignificant, in the models they examine, practically no surplus is consumed by the trading process. If the time between offers is very small, the probability that bargaining will continue past any given time becomes arbitrarily small as well. The belief that asymmetries of information impose social costs was not borne out in their models.

It should be noted that other researchers have shown the existence of bargaining games which exhibit delay. Admati and Perry [1] show that if the informed agent can delay his response to the move of an uninformed agent there exist specifications of private values games which have equilibria with significant delay to agreement. Ausubel and Deneckere [2] also show that delay may occur as the outcome of some perfect Bayesian equilibrium if the restriction that the buyer's valuation be strictly greater than the seller's valuation is relaxed. In this case, there also exist equilibria in which there is no delay. The possibility of delay to agreement, independent of the speed of offers is present in a common values game as well and in a stronger form. For many specifications of a common values bargaining model <u>all</u> equilibria must exhibit delay to agreement.

In order to analyze the effects of shorter time between offers it is helpful to respecify the earlier model. Let the preferences over time be generated by a continuous discounting process of the form $e^{-rDt}(v - p)$ for the buyer where the discount factor, b, comes from the loss from waiting for the length of period, D, discounted at the continuous rate, r. That is,

$b = e^{-rD}$.

A similar representation holds for the seller where σ is the seller's continuous time discounting factor. With this specification, it is now possible to parametrize equilibria by the length of time between offers, D.

The result in Section Two showed that for any given bargaining game, bargaining ended within a finite number of periods. For some game with length of period D let the maximum number of periods in equilibrium be n(D) + 1. As D goes to zero, one might expect two opposing effects to occur. A shorter time between offers allows the buyer to screen finely at a lower delay cost and so the number n(D) may become very large. On the other hand, D itself becomes very small so the effects on total bargaining time, Dn(D) may also be very small. In the private values case, this is in fact what happens. D goes to zero faster than n grows so we get the result that, if the extensive form game allows arbitrarily fast offers, bargaining ends arbitrarily quickly. This result was first shown by GSW and was later extended by Gul and Sonnenschein [7] to many of the sequential equilibria of two-sided offer games as well.

A simple example shows that a similar conclusion does not hold for the general common values game. Consider why we might expect it not to hold. If the total time between the first and last offers in the bargaining game is short, then the incentive compatibility constraints which induce low valuation sellers to accept early will force the initial price offers to be close to the final price offers. In the case of the private values models where the buyer's valuation is always greater than the seller's, this rise in the first price offer has the effect of reducing the buyer's ability to extract surplus from the low valuation sellers. In a common values model, though, it is possible for a buyer to pay more than the object is worth to him and, so, there is a further incentive to extend bargaining over a period of time; that is, to keep from having to pay higher prices for goods of lower value. This effect leads to a lower bound on the maximum bargaining time for some specifications of the bargaining game.

The following example illustrates this point. Adapt the example in Section Two so that,

for $q \in [0,1/2]$, $f(q) = 0$ and $v(q) = 1$

and for $q \in (1/2,1]$, $f(q) = 4$ and $v(q) = 5$.[8]

Set the discount factors equal so that $s = b = e^{-Dr} = \delta$. For a fixed period between offers, D, there exists a unique perfect Bayesian equilibrium with a maximum time to agreement, n(D) + 1. The theorem in Section Two enables us to describe the path of offers in the equilibrium. It is of the form

$$(\delta^n 4,\ \delta^{(n-1)}4,\ \ldots,\ \delta 4,\ 4).$$

Since the buyer can always offer a price of 0, the expected value of following the equilibrium path must be non-negative. The value of the game to the buyer is easily calculated to be

$$0 \le u = [(1 - \delta^n 4)m_1 + \delta(1 - \delta^{n-1}4)m_2 + \ldots + \delta^{n-1}(1 - \delta 4)m_n]/2$$
$$+ \delta^n(1/2)$$

where m_i is the probability a seller accepts an offer $\delta^{n-i+1}4$ given that the seller is of low valuation. Therefore,

$$m_1 + m_2 + \ldots + m_n = 1,\ m_i \ge 0$$

and $u \le (1/2)[(1 - \delta^n 4) + \delta^n]$.

This yields

$$1 - 3\delta^n \ge 0 \text{ or}$$

$$1/3 \ge \delta^n = e^{-rDn}$$

$-\log 3 \geq -rDn$ or

$(\log 3)/r \leq Dn(D)$.

With $r > 0$, this then gives us a lower bound on $n(D)D$, the total elapsed time until the final possible offer of 4 is made. An equilibrium offer of four is made whenever the seller is of a good type, so, ex ante, with probability one-half bargaining lasts for at least $(\log 3)/r$ time units.

Section Four: Variations on the Extensive Form

While the one-sided offer game generates pleasingly concrete results, it is a very restrictive characterization of a bargaining process. One would like to extend the analysis to games where both parties are able to make offers. It is well known, however, that in the private value, two-sided offer game, one-sided uncertainty leads to many sequential equilibria.[9] To conduct analyses of these games it is usually necessary to refine out this multiplicity. This is the approach used in Grossman and Perry [6] and Rubinstein [10]. That approach is beyond the scope of this paper but it is an area where further research could be fruitful. It is possible, though, to extend the result of the previous section to include these more complicated games. This section shows that, for a broad class of extensive form games and a broad class of specifications of valuations, equilibrium behaviour requires delay to agreement.

To see this result, use the technique employed by Samuelson [11] for static trading mechanisms with common values. Consider any static mechanism which generates outcomes represented by a pair $(m(f),P(f))$ where $m(f)$ is the expected payment given that a seller reports f and $P(f)$ is the probability that trade occurs. Samuelson shows that the incentive compatibility (IC) and individual rationality (IR) constraints of any equilibrium require that the P(.) function satisfy the restriction

$$\int_0^g (v(f) - f - G(f)/g(f))P(f)df \geq 0 \qquad (1)$$

where G(f) is the continuous distribution function of the seller's type, f, and g(f) is its strictly positive density function on the range [0,g]. For simplicity let the buyer's valuation v = f + 1 and set G(f) = f/g -- the seller's type varies uniformly over the interval [0,g]. Equation (1) becomes

$$\int_0^g (1 - f)P(f)df \geq 0. \qquad (2)$$

In this case, any Pareto optimal mechanism should always ensure trade since v - f = 1. However, it is clear that as g becomes large, setting P(f) = 1 for all f violates the constraints implied in (2). Some expected gains from trade must be used up to enable trade.

Now use the insight of Cramton [3],[4] which extends the static analysis to sequential mechanisms. For any f, a sequential mechanism generates a probability distribution, T_f, over the price and time of trade. If we restrict discount factors to be equal, the IC and IR constraints of a sequential mechanism can also be expressed as equation (2) where, now, the P(.) function represents the discounted probability of trade. That is,

$$P(f) = \int_0^\infty \delta^t \, dT_f(t).$$

Here, loss is generated not by the probability of no trade but by the probability of having to wait some period of time for trade.

Equation (2) tells us that if an equilibrium exists then, typically, delay must occur. A mechanism in which the probability of significant delay is arbitrarily small is one for which the P(.) function is arbitrarily close to one for all but an arbitrarily small set of f. For g large enough, however, any such P(.) function violates the constraints implied by IC and IR. Alternatively speaking, any mechanism which satisfies IC and IR given the specification of this model and given a large g, must use up surplus by imposing a significant probability of significant delay.

Notice that the argument applies to a broad class of examples. Let the seller's valuation range uniformly from 0 to g and the buyer's valuation always be one more than the seller. Give each identical rates of time preference. The theorem in Section Two shows that in the one-sided offer game, for given discount factors, there exists a unique perfect Bayesian equilibrium and that trade always occurs. The above argument shows that when g is large the equilibrium must exhibit a significant probability of significant delay to agreement.

Observe how general this result is in terms of game forms as well. As long as the trading mechanism of the game form yields equilibrium outcomes in price-time space and has an exit option (and discount factors are equal), the incentive compatibility constraints of any equilibrium to this mechanism imply the need for delay. Since the alternating offers game as well as the altered version of the game form described by Admati and Perry [1] fit into this framework, these games will often exhibit delay to agreement in common values models.

Section Five: The Endogeneity of Take it or Leave it Offers

It might seem reasonable to expect that as long as there are known gains from trade, if bargainers do not have the power to commit to breaking off negotiations, bargaining will continue until these gains are realized.

Indeed, one reason for examining infinite horizon games was to study what occurs when neither agent can credibly threaten to stop bargaining before a trade is consummated. However, it is important to recognize that a net gain from trade must include the costs of trading as well -- these include the costs of delay and, in these models, the risk of purchasing a poor quality object. When these costs are too high, the benefits from trade may not be great enough to induce an agent to continue bargaining. This section illustrates that the consideration of infinite horizon bargaining games does not ensure that trade occurs even in the presence of a known positive surplus.

The equilibrium characterized in Section Two required that the buyer be at least as patient as the seller -- that is, $b \geq s$. If this condition holds, the value of the bargaining game to the buyer is always strictly positive and the buyer can never credibly threaten to leave the game before trade occurs. When the condition does not hold, when the seller is more patient than the buyer, there are specifications of the model such that the buyer has an expected value of zero in the game. Equilibrium behaviour, therefore, can involve breaking off negotiations.

The example in Appendix Two illustrates this phenomenon. It is a variation of the example of Section Two. With probability 5/16 the object is worth 0 to the seller and 1 to the buyer and with probability 11/16 it is worth 4 to the seller and 5 to the buyer. Both the buyer and the seller know that there are gains from trade of one in the game. However, when the buyer's discount factor is 1/2 and the seller's is 3/4 the game exhibits an equilibrium in which the buyer makes an offer of zero throughout the game. If trade takes place, it must occur in the first period.

When the seller's discount factor is relatively high, the price a buyer must offer to ensure that low-type sellers are not in the continuation game is correspondingly higher. When, in addition, the buyer's discount factor is low, the value of any continuation game is low. The expected cost of eliminating low valuation sellers may exceed the expected value of the continuation game and buyers would then prefer to end negotiations as occurs in this example.

Note that an equivalent type of behaviour would be for the buyer to offer a price of zero in the first period and, if he is rejected, then to break off negotiations. The example suggests how we might explain failure to come to agreement even in the apparent presence of gains from trade. The buyer breaks off bargaining because the cost of acquiring further information needed to obtain a positive payoff is too high. It is clear that a slight modification of this example could provide cases where a number of offers are made before bargaining ends in disagreement.

Conclusion

The paper examines a natural extension of one-sided uncertainty bargaining games to include common values and shows that, for the one-sided offer game there is a unique perfect Bayesian equilibrium. It uses the characterization of equilibrium to show that such games typically exhibit delay to agreement as sellers of different types use time to signal their information. This result remains true when more general extensive form bargaining games are considered. The paper also provides an example of a one-sided offer bargaining game in which equilibrium behaviour may involve breaking off negotiations in the presence of gains from trade. It thus shows that the Akerlof lemons problem may persist in infinite horizon bargaining games and despite the common knowledge that there are gains from trade.

Appendix One

The equilibrium strategies to the example in Section Two are calculated in the following way. Let the mass of good quality sellers be fixed at 3/4 and let m be the mass of bad quality sellers. Determine, first, the point m_1 at which a buyer is indifferent between offering a price of four immediately and gaining acceptance by any seller type or offering two (accepted by low quality sellers) and then a price of four. At $m_1 = 3/16$, or if the relative weights are 1:4 the buyer is just indifferent. (This would correspond to the state $q \in [1/16,1]$.) At $m_2 = 9/32$, or $q \in [-1/32,1]$, the buyer is indifferent between the two period game and a three period game consisting of a price offer of one, then two, and then four. For states $[x,1]$ with $x \in (0,1/16)$, the buyer strictly prefers the two period game. If sellers in $[0,1/16)$ reject a price p in a given period and are expected to reject p they can expect a price of two in the next period which is worth one to them now. Thus $p < 1$ and any price greater than one is acceptable to them immediately. Similarly, if sellers in $(1/16,1/4)$ reject a price $p' > 1$ in the current period and are expected to do so, they can get a price of four in the next period. Therefore, $p' < 2$. For $x = 1/16$, the buyer is indifferent between prices of two and four and his ex post mixing justifies the behaviour of the low quality sellers with the appropriate definition of β. This reasoning shows that an offer of two in the first period, acceptance for sure by low-quality sellers and an offer of four in the second period is an equilibrium profile. By the theorem, this path is generically unique.

Appendix Two

One of the conditions required to prove the theorem in the paper is that the buyer's discount factor be no less than the seller's or that the buyer be at least as patient as the seller. An example in this section illustrates what may happen when the condition is not met and helps to explain the role of the requirement.

Consider the example described in Section Two modified as follows:

for $q \in [0,5/16]$ $f(q) = 0$, $v(q) = 1$;

for $q \in (5/16,1]$ $f(q) = 4$, $v(q) = 5$.

Let the discount factors be $b = 1/2$ for the buyer and $s = 3/4$ for the seller.

If m is the measure of low valuation sellers left in the game, note that $m^* = 11/48$ yields the buyer an expected utility of 0 if he offers a price equal to four and buys the good for sure. Any equilibrium of the form described in the theorem must end with an offer of 3 and then 4 if bargaining is to take more than one period. However, if $m < m^*$, the buyer will always prefer to offer 4 rather than to screen out some sellers by offering $p = 3$. Furthermore, if $m > 11/48$, for any strategy of the low-valuation seller, an offer of 3 and then 4 yields a strictly negative expected utility. At $m^* = 11/48$, if any low valuation seller accepts $p = 3$, the buyer would receive an expected utility of less than zero. Therefore, the only way the path described in the theorem could be an equilibrium path is if low-valuation sellers follow a strategy of always rejecting $p = 3$. If this is an equilibrium strategy, note that it is dominated for the buyer by offering either four or zero. Any tremble on the part of the seller would yield a negative payoff.

It is not known whether the path of the theorem can be supported as an equilibrium. However, it is possible to describe another, quite different, equilibrium. The following strategies form an equilibrium to the game above.

Equilibrium strategies:

For the buyer: $p_o = 0$, if $p_{i-1} = 0$ offer $p_i = 0$;

if $p_{i-1} \epsilon (0,3)$, offer 4 with probability β

or 0 with probability $1 - \beta$

where $p_{i-1} = s4\beta$;

if $p_{i-1} \geq 3$, offer $p_i = 4$.

For the seller: if $q \leq 4/48$, accept $p \geq 0$;

if $q \epsilon (4/48,5/16]$, accept $p \geq 3$;

if $q \geq 5/16$, accept $p \geq 4$.

In this equilibrium, a buyer extracts all of the surplus from the low-valuation seller four-elevenths of the time and none seven-elevenths of the time. He never trades with a high-valuation seller. If he tries to attract more low-valuation sellers by offering a little higher price, the expectation of his future mixing between four and zero prevents this. Note that his strategy, here, is undominated.

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1. For an early treatment of bargaining models with common values, see Samuelson [11].

2. This game has also been examined in Sobel and Takahashi [12] and Cramton [3].

3. Perfect Bayesian equilibrium is a slightly weaker concept than sequential equilibrium. It requires that beliefs and strategies be specified for all histories of the game, that strategies be optimal given the specified beliefs and that, wherever possible, beliefs are defined by Bayes' rule and the equilibrium strategies. The first definition of it that I am aware of is given in FLT.

4. Note one final aspect of this game. In general, a seller can follow a strategy of mixing responses. In what follows, sellers with the same valuation will be said to be of the same type. Thus the seller who observes q is the same type as the seller who observes q' if and only if $f(q) = f(q')$. When sellers of a certain type have probability measure zero, the effect of their mixing has no consequence for expected payoffs and so can be ignored. This is not so for sellers whose type have positive probability. Since mixed strategies can, in fact, form part of an equilibrium profile, it is not desirable to keep them from following such strategies by fiat. However, such strategies complicate the analysis considerably. The approach of this paper is to disallow mixed strategies on the part of individual sellers. When mixing is called for by sellers of types of positive measure, different sellers within that type group will be assumed to follow different pure strategies so that the final consequence will be as if a mixed strategy was followed. For example, when $f(q) = f$ for q in $[1/2,2/3]$, and a mixed strategy of accept price p with probability one-half is called for, it will be interpreted as the strategy:

Accept for q in $[1/2,7/12]$; and

Reject for q in $(7/12,2/3]$.

Thus, when the seller has valuation f, one-half of the time he is of the type that accepts and one-half of the time he is of the type that rejects.

5. Let $f(q) = q$ and $v(q) = 1.5q$, $q \in [0,1]$, buyer and seller have equal discount factors so 3) is violated. Samuelson [11] shows that the best possible mechanism for trade for the buyer is that for which the time of transfer and price of transfer is 0. That is, the only seller to trade if at all is the seller $q = 0$ at price $p = 0$. The proof of this is a simple generalization of the Samuelson static model.

6. 'Generically unique' is meant in the following sense. Let the lower end of the support of q be q_0 not necessarily zero. Define a class of games by fixing a pair of valuation functions. $(f(.), v(.))$, a pair of discount factors, (b,s), any $\epsilon > 0$, and any continuous probability density function over the support $[0,\epsilon]$, $g(q_0)$. A game in this class is parametrized by the lower end of the initial support of q, q_0. Therefore we can define a measure over this class of games by $g(q_0)$. The event that games in this class have more than one pBe occurs with probability zero.

7. Of course, the buyer would also like to renegotiate in the private values game as well. In that game, however, the buyer is always sure of gaining some non-negative surplus.

8. To correspond to the GSW framework, v(q) would be set to 5 for all q. Since this falls within the class of models they examine, their Theorem Three shows that no delay would result if the time between offers was arbitrarily small.

9. The same is also true of the common values game in which only the informed agent makes offers.