Discussion Paper No. 773

PRICE LEADERSHIP

by

Raymond Deneckere*

and

Dan Kovenock**

March 1988

*Department of Managerial Economics and Decision Sciences, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, IL 60201

**Department of Economics, Krannert School of Management, Purdue University, West Lafayette, IN 47907.

The idea for this paper was conceived while Raymond Deneckere was visiting the University of Bonn and Dan Kovenock was visiting the University of Bielefeld. The first author gratefully acknowledges the hospitality and financial assistance of the University of Bonn, as well as support through National Science Foundation Grants IST-86-09128 and SES-86-19012, and the Kellogg Graduate School of Management's Esmark Chair. The second author gratefully acknowledges the hospitality of the University of Bielefeld and the financial support of the Purdue Research Foundation, The Krannert Endowment and the Ford Motor Company Fund. The opinions expressed in this paper are those of the authors who bear full responsibility for any errors or omissions.

Price Leadership

Abstract: This paper analyzes duopolistic price leadership games in which firms have capacity constraints. We provide a complete characterization of price leader equilibria under quite general assumptions on demand and for arbitrary capacities. We show that when capacities are in the range where a simultaneous move price setting game (with residual demand specified à la Levitan-Shubik and Kreps-Scheinkman) yields a mixed strategy solution the large capacity firm is indifferent between being a leader, a follower, or moving simultaneously. The small capacity firm, while indifferent between being a leader and moving simultaneously, strictly prefers to be a follower. This motivates the discussion of several games of timing with ex-post inflexible prices in which the high capacity firm becomes an endogenously determined price leader. We thus provide a game theoretic model of dominant firm price leadership.

The traditional industrial organization literature is very fond of the dominant firm model of price leadership. In this model it is generally assumed that there is one large producer and many small producers - no one of which produces a high enough output to affect price. According to Scherer (1970, p. 164), "Dominant firm price leadership occurs when an industry consists of one firm dominant in the customary sense of the word i.e., controlling at least 50% of the total industry output - plus a 'competitive fringe' of firms, each too small to exert a perceptible influence on price through its individual output decisions." With firms in the fringe acting as price takers, the dominant firm is left as the only agent able to set price, and does so by maximizing profit subject to its residual demand curve. It, thus, by necessity becomes a "price leader." As Markham (1951, p. 895) concluded, "Price leadership in a dominant firm market is not simply a modus operandi designed to circumvent price competition but is instead an inevitable consequence of the industry's structure."

Stigler, in his paper "The Kinky Oligopoly Demand Curve and Rigid Prices" (1947, pp. 444-446), attaches major importance to the dominant firm model of price leadership. Stigler distinguishes between two types of price leadership, dominant firm leadership and barometric price leadership. In the former a dominant firm "sets the price, allows the minor firms to sell what they wish at this price . . ., and supplies the remainder of the quantity demanded." Barometric price leadership "refers to the existence of a firm that conventionally first announces price changes that are usually followed by the remainder of the industry, even though this firm may not occupy a dominant position." The latter "commands adherence of rivals to this price only because, and to the extent that, this price reflects market conditions with tolerable promptness." Stigler classifies as industries with dominant firm price leaders "those in which there is a relatively large firm, producing, say, 40% of the output of the industry at a minimum, and more if the second largest firm is large (because otherwise the situation approaches classical duopoly)." Of the nineteen industries Stigler analyzed, seven had dominant firm price leaders.

Despite its prominence in Stigler's analysis, it is not clear that it is appropriate to use the dominant firm model of price leadership to describe oligopolistic markets with more than one large firm. Large firms cannot be assumed to act as price takers, taking as given the price set by the leader. They should act strategically in their choice of price.[1] This somewhat limits the applicability of the traditional dominant firm model. Indeed, as noted by Bain in his critique of the Markham and Stigler papers (1960, p. 197), the dominant firm model "has scant theoretical interest or practical application." Empirically, there do not appear to be many industries fitting the structure described in the traditional model. Theoretically, if we take the fringe to be nonatomic, so that firms are behaving rationally, the dominant firm equilibrium coincides with the Cournot equilibrium. Bain went on to describe what he called "oligopolistic price leadership," requiring at least two large firms in the market, which realize their interdependence. There have been many observed cases of price leadership in markets with an oligopolistic structure. Standard examples include episodes in the histories of the automobile, breakfast cereal, cigarette, and steel industries.

In this paper, we construct a model of price leadership in a duopolistic market that maintains the spirit of the traditional model, while not assuming price taking behavior by a competitive fringe. We analyze duopolistic price setting games in which firms have capacity constraints and are allowed to choose the timing of their price announcements. We show that with residual demand specified à la Levitan-Shubik and Kreps-Scheinkman and when capacities are in the range where a simultaneous move price setting game yields a mixed strategy solution, there are several reasonable specifications of games of timing with ex-post inflexible prices in which the high capacity firm becomes a price leader. We thus provide a game theoretic model of dominant firm price leadership.

Our motivation for focusing on price setting games with capacity constraints is twofold. Clearly, in formulating a model of price leadership, firms should set price as the strategic variable. We incorporate capacity constraints because it is a natural way to model the size of firms. In the traditional literature size has been viewed as an important determinant of the existence and identity of price leaders. For instance, Markham noted that "in a large number of industries which do not contain a partial monopolist, the price leader is frequently but not always the largest firm." Oxenfeldt (1951, pp. 296-297) stated it more forcefully when he argued, "Price leadership probably works best and arises most frequently in industries in which a single firm is outstanding by virtue of large size or recognized high quality of management,"[2] and "The existence of a large firm facilitates price leadership in several ways. First, a large firm's price policy exerts a great influence on the sales of its smaller rivals; consequently the large firm must consider the probable responses of its rivals before setting its own price. In effect, the large firm must think in terms of a price policy for the entire industry."

The basic intuition for the economic forces that drive our results is that small firms, ceteris paribus, stand to lose more from being undercut than large firms. Consequently, small firms have a stronger preference than large firms for assuming a followership role in the industry, and choose not to lead or simultaneously set prices. More specifically, in our model, consider the simultaneous move price setting game when capacities are in the range where a nondegenerate mixed strategy equilibrium exists. Recall that the firm with the greatest capacity (firm 2 in our model) receives in equilibrium the same expected profit as it would receive if it chose the best possible price knowing that the low capacity firm (firm 1) would undercut it. It is not surprising, therefore, that the high capacity firm makes the same expected payoff in the simultaneous move game as it would receive if it were an exogenously specified price leader. The low capacity firm makes a strictly lower expected profit in the simultaneous move game than if it were an exogenously specified price follower. On the other hand, over the same range of capacities, in the game in which the low capacity firm is the price leader, both firms obtain the same profit as in the simultaneous move game (except possibly in one of two equilibria that exist when capacities are equal).

This fact leads one to expect that it may be possible to specify a game of timing of price announcements in which the high capacity firm becomes an endogenous price leader and the low capacity firm an endogenous price follower. Since the high capacity firm is indifferent as to which of the three potential price setting games it plays, whereas the small capacity firm strictly prefers to be a price follower, it is reasonable to expect that the small firm would be willing to outwait the large firm in announcing its price (as long as the periods at which the firms can announce prices are not too far apart) and that the large firm, knowing this, would announce price immediately.

Several implications of our model are amazingly consistent with observations made in the traditional literature, even though each firm is large enough to affect price. First, when firms are allowed to choose the timing of their announcements, leadership - when it exists - is an inevitable consequence of industry structure. In our model (in which variable costs are identical) leadership is entirely determined by capacity. Second, the outcome under price leadership is more collusive than the outcome under simultaneous price setting. The expected profit of the high capacity firm is the same in each game, but the low capacity firm benefits strictly from price leadership. Third, to the extent that price leadership leads to a deterministic solution (rather than the mixed strategy solution from the simultaneous move game) price leadership appears to induce more stable prices.

The welfare implication of our model, that price leadership leads to higher prices than simultaneous price setting, addresses a point of some dispute in earlier writings (see Bain's (1960) critique of Stigler and Markham). We demonstrate that price leadership can lead to less competitive outcomes without being either implicitly or explicitly collusive. Even in the absence of exogenous shocks to cost or demand, price leadership can increase price by coordinating the otherwise random actions of the firms. Although purely noncollusive in nature, our model can be used to construct a model of collusive price leadership by infinitely repeating our stage-game. We discuss this extension in the conclusion, but omit a detailed analysis. An alternative model of collusive price leadership, based on informational asymmetries in an infinitely repeated game, is presented in Rotemberg and Saloner (1986). These authors ignore the potential effect of size in their analysis; an interesting extension would incorporate the effect of both size and informational differences on price leadership.

In section 2 we present the basic model of price setting duopoly with capacity constraints and contingent demand specified à la Levitan-Shubik and Kreps-Scheinkman. We review the characteristics of the simultaneous move equilibrium with strictly decreasing, concave demand, and provide a complete characterization of the equilibria obtained both in the case where the large firm is a price leader and when the small firm is a price leader. This extends some previous work by Levitan and Shubik and provides a complete characterization of the price leader equilibria for a fairly general class of demand and arbitrary capacities.[3] In section 3 we construct several games of timing in the announcement of price in which the large firm becomes an endogenously determined price leader when capacities are in the range where the simultaneous move equilibrium is in nondegenerate mixed strategies. Section 4 concludes with the implications of our one-shot model for repeated games and also discusses why dominant firm price leadership, in reality, seems to require a much greater asymmetry in capacity than is indicated in our simple model. Further extensions incorporating different unit costs up to capacity and Beckmann contingent demand are also considered.

II. The Model

Consider a market shielded from entry, in which two firms produce a homogeneous good. Firm i has capacity k_i which is exogenously given; we assume throughout that $k_2 \geq k_1 > 0$. Each firm may produce up to capacity at the same unit cost, which we will normalize to be zero. The aggregate demand for the output of the firms as a function of price is $d(p)$: $R_+ \to R_+$. We make the following assumptions on $d(p)$:

A1. There exists a $p_o > 0$ such that $d(p) = 0$ for every $p \geq p_o$ and $d(p) > 0$ if $p < p_o$. $d(p)$ is twice continuously differentiable, strictly decreasing and concave on $[0, p_o)$.[4]

Let p^m be the unique maximizer of $p \cdot d(p)$.

We assume that price is the strategic variable; thus firm i chooses a price $p_i \varepsilon [0,\infty)$. If the firms charge different prices customers buy first from the cheapest supplier. When the low price supplier cannot satisfy all demand at that price, some customers will be left for the remaining firm. How much this firm will actually sell depends upon how the output of the low priced firm is rationed. We make the following simplifying assumption on the contingent demand of the high priced firm: the high priced firm faces the industry demand at its price less the quantity sold by the lower priced firm. Thus, if $p_i < p_j$ firm j faces a contingent demand of

(1) $q(p_j) = \max(0, d(p_j) - k_i)$

The above contingent demand, termed Compensated Contingent Demand (CCD) by Dixon (1987a,b) was first used by Levitan and Shubik (1972), and has since appeared, inter alia, in Benoit and Krishna (1987), Brock and Scheinkman (1985), Davidson and Deneckere (1986a), Kreps and Scheinkman (1983), and Osborne and Pitchik (1986). As noted by Dixon, such a contingent demand would obtain if there was only one consumer in the market and any income effects of low priced purchases were compensated. With many consumers, each with the same demand curve, it can be obtained from an "equal shares" rationing rule in which each consumer receives an equal share of the output available when excess demand exists. Alternatively, and perhaps most naturally, it can be obtained in a model where consumers have inelastic demand for one unit of the good, and the consumers with the highest reservation prices are served by the low priced firm ("reservation price rationing"), either directly or through resale of the product by consumers with low reservation prices. For a further discussion of the importance of this assumption see Davidson and Deneckere (1986b).[5]

Given the above rationing rule the profit of firm i as a function of both firms' prices and capacities is given by

$$(2)\quad \pi_i(p_i,\, p_j \mid k_i,\, k_j) = \begin{cases} L_i(p_i) \equiv p_i \min(k_i, d(p_i)) & \text{if } p_i < p_j \\ T_i(p_i) \equiv p_i \min(k_i,\, \max(0, d(p_i) - I_\ell^i\, k_j)) & \text{if } p_i = p_j \\ H_i(p_i) \equiv p_i \min(k_i,\, \max(0, d(p_i) - k_j)) & \text{if } p_i > p_j \end{cases}$$

where I_ℓ^i is an indicator which takes on the value 1 if i is a leader and 0 if i is a follower. Here $L_i(p_i)$ refers to the profit from being the low priced seller at p_i, $H_i(p_i)$ the profit from being the high priced seller at p_i, and $T_i(p_i)$ the profit to firm i when it charges $p_i = p_j$. For leader-follower games we will assume that in the event of a tie the second firm to set price sells its capacity first. We make this assumption for the purely technical reason of avoiding the need to have a follower charge a price arbitrarily close to, but below, the leader's price. In examining the simultaneous move game we will assume that $T_i(p_i) = p_i \min(k_i, \frac{k_i d(p_i)}{k_1 + k_2})$, although a wide range of other allocation rules would yield the same results. Thus, in the case of a tie in the simultaneous move game we assume that demand is allocated in proportion to capacities.

Let P_i^L be the set of prices which maximize $L_i(p_i)$ and P_i^H be the set of prices which maximize $H_i(p_i)$. By (A1) there exists a unique $p_i^L \in P_i^L$ and $L_i(p_i)$ is continuous and strictly increasing in p_i for $p_i < p_i^L$. Let $H_i^* = H_i(P_i^H)$. If H_i^* is nonzero, (A1) implies that there is a unique element p_i^H in P_i^H. Furthermore, $H_i(p_i)$ is continuous and strictly increasing in p_i for $p_i < p_i^H$. If $H_i^* = 0$, we define $p_i^H = 0$. Let

$$(3)\quad \underline{p}_i = \min \{p_i\colon\ L_i(p_i) = H_i^*\} \qquad i = 1,2.$$

Note that $\underline{p}_i \leq p_i^H < p_i^L$, i = 1,2, and that $p_1^L \geq p_2^L$ (see figure 1). In the analysis that follows we shall sometimes write p_i^L, p_i^H, and $\underline{p}_i$ as functions of (k_1, k_2) to indicate the dependence of these prices on capacities. Also, it will sometimes be useful to refer to the inverse demand function P(q), defined by: $P(q) = d^{-1}(q)$ for $0 < q \leq d(0)$, $P(0) = p_0$, and $P(q) = 0$ for $q > d(0)$. An immediate consequence of the above definitions is that $p_i^L \geq P(k_i)$, i = 1,2. Also, $P(k_1 + k_2) \leq \underline{p}_i \leq p_i^H \leq P(k_j)$ i = 1,2, $j \neq i$, with equality on the right-hand side if and only if $P(k_j) = 0$.

Before proceeding with the characterization of equilibria when each of the firms is exogenously specified as a price leader, we characterize the equilibrium of the simultaneous move price setting game. This game was first analyzed by Levitan and Shubik (1972) for linear demand and special cases of capacity and in more generality by Kreps and Scheinkman (1983) and Osborne and Pitchik (1986). The following theorem is a special case of Osborne and Pitchik (1986, Theorem 1):

Theorem 1: For each pair (k_1, k_2) with $k_2 \geq k_1$, the simultaneous move price setting game with capacity constraints has a unique Nash equilibrium:

(a) If $P(k_1) = 0$ the equilibrium is in pure strategies, both firms charge p = 0 and earn zero profit.

(b) If $0 < p_2^H(k_1, k_2) = P(k_1 + k_2)$ then the equilibrium is in pure strategies, both firms charge $p = P(k_1 + k_2)$ and profits are given by $\pi_i = k_i P(k_1 + k_2)$.

(c) If $p_2^H(k_1, k_2) > P(k_1 + k_2)$ the equilibrium is in nondegenerate mixed strategies with common support $[\underline{p}_2, p_2^H]$, where $\underline{p}_2$ is defined in (3). Equilibrium payoffs are $\pi_i = L_i(\underline{p}_2)$.

A proof of this theorem, as well as a detailed description of the derivation of the mixed strategy equilibria may be found in Kreps and Scheinkman (1983) or Osborne and Pitchik (1986).

Equilibrium in the price setting game with an exogenously specified leader also has a simple characterization. As long as $k_1 < d(0)$ firms will charge only two prices in equilibrium, $\underline{p}_2$ or p_2^H. Suppose $k_2 > k_1$. If firm 2 is a leader it sets p_2^H and firm 1 matches that price (and sells its capacity). If firm 1 is a leader, it sets $\underline{p}_2$ and firm 2 follows with p_2^H. When capacities are in the range where a mixed strategy equilibrium exists in the simultaneous move game, $p_2^H > \underline{p}_2$ and firm 1 sets a strictly lower price as a leader than as a follower. When $k_1 = k_2$, there are two pure strategy equilibria. In one of them, the leader sets price equal to $\underline{p}_2$ and the follower sets p_2^H; in the other the leader sets p_2^H and the follower matches that price. The follower strictly prefers the second equilibrium to the first.

In the proofs of these assertions the prices $\underline{p}_1$ and $\underline{p}_2$, defined in (3), play an important role. When firm j is a price leader it must charge a price less than or equal to $\underline{p}_i$ to induce firm i to set a higher price as follower. For prices above $\underline{p}_i$ firm i would prefer to match (or undercut if $p_j > p_i^L$) and, as follower, sell all its capacity.[6] In order to show that the price leader equilibria are indeed of the form indicated above, we first provide a lemma ordering $\underline{p}_1$ and $\underline{p}_2$.

<u>Lemma 1</u>: For each pair (k_1, k_2) with $k_2 > k_1$, $\underline{p}_2 \geq \underline{p}_1$ with equality if and only if there is a pure strategy equilibrium in the simultaneous move game.

<u>Proof</u>: If there is a pure strategy equilibrium in the simultaneous move game it is clear that $\underline{p}_2 = p_2^H = P(k_1 + k_2) = p_1^H = \underline{p}_1$. Assume, then, that no pure strategy equilibrium exists in the simultaneous move game, and thus $P(k_1) > 0$.

We first demonstrate that this implies that $\underline{p}_1 < P(k_1)$.

Suppose not; then $\underline{p}_1 \geq P(k_1)$, implying $\underline{p}_1 > P(k_2)$. The strict inequality follows from the fact that, from our assumption on the equilibrium of the simultaneous move game, $P(k_1) > 0$. With $\underline{p}_1 > P(k_2)$, $H_1(\underline{p}_1) = 0$, since firm 2 is not capacity constrained at $\underline{p}_1$. Since $\underline{p}_1 \leq p_1^H$ this implies that $\underline{p}_1 = 0$, a contradiction to the fact that $P(k_1) > 0$. Thus, $\underline{p}_1 < P(k_1)$.

The remainder of the proof is divided into two cases, depending on the relationship between $\underline{p}_2$ and $P(k_2)$.

(i) Suppose $\underline{p}_2 < P(k_2)$. Thus, if firm 2 is the low price firm at $\underline{p}_2$ it is capacity constrained. With $\underline{p}_i < P(k_i)$, $i = 1,2$, by definition of $\underline{p}_i$,

$$(4) \quad \underline{p}_i k_i = \max_p \, p \min(k_i, \max(0, d(p) - k_j)) \equiv H_i^* \qquad i = 1, 2 \quad j \neq i$$

Define $r(k) = \operatorname{argmax}_x xP(x + k)$. That is, $r(k)$ is the Cournot best response function when the rival firm puts output k on the market and firms have zero cost of production. We divide the analysis of this case into two subcases, depending upon the relationship of k_1 to $r(k_2)$. Suppose, first, that $k_1 > r(k_2)$. Then $P(k_1 + k_2) < P(k_2 + r(k_2))$, which implies that the price that satisfies the right hand side of (4) for $i = 1$ solves $\max_p p \cdot \max(0,d(p) - k_2)$. Thus,

$$(5) \quad \underline{p}_1 = \frac{1}{k_1} \max_p \, p \cdot \max(0,d(p) - k_2)$$

With $k_1 > r(k_2)$ and $k_2 > k_1$ by assumption, lemma 1 of Kreps and Scheinkman (1983) shows that $k_2 > r(k_1)$, so $\underline{p}_2 k_2 = \max_p p \cdot \max(0,d(p)-k_1)$ and

$$(6) \quad \underline{p}_2 = \frac{1}{k_2} \max_p \, p \cdot \max(0,d(p)-k_1).$$

Define $\theta(k_i) \equiv k_i \max_p p \cdot \max(0,d(p)-k_i)$, $i=1,2$.

From (5) and (6)

$$\underline{p}_2 > \underline{p}_1 \iff \theta(k_1) > \theta(k_2).$$

The right hand inequality is demonstrated in the course of the proof of Kreps and Scheinkman's lemma 5 (1983, page 332). Thus, $\underline{p}_2 > \underline{p}_1$.

Next let $k_1 \leq r(k_2)$; then $\underline{p}_1 = p_1^H = P(k_1 + k_2)$. We know that $\underline{p}_2 \geq P(k_1 + k_2)$ with equality if and only if there exists a pure strategy equilibrium (i.e., when $\underline{p}_2 = p_2^H$). So, $\underline{p}_2 > P(k_1 + k_2) = \underline{p}_1$.

(ii) Suppose $\underline{p}_2 \geq P(k_2)$. Then $H_1(\underline{p}_2) = 0$, which implies that $p_1^H \leq \underline{p}_2$ with equality only if $\underline{p}_2 = 0$. But then $\underline{p}_1 \leq \underline{p}_2$, with equality only if $\underline{p}_1 = \underline{p}_2 = 0$, i.e. when a pure strategy equilibrium exists in the simultaneous move game. #

Let us now turn to the equilibrium in the game where firm 2, the large firm, is an exogenously determined price leader.

<u>Theorem 2</u>: For each pair (k_1,k_2) with $k_2 \geq k_1$ the price setting game with capacity constraints and firm 2 as a price leader has the following equilibria:

(a) If $P(k_1) = 0$ there is a continuum of equilibria. The equilibrium set of prices is given by: $\{(p_1, p_2)\colon\ p^m \geq p_2 \geq 0,\ p_1 = p_2\} \cup \{(p_1, p_2)\colon p_2 > p^m,\ p_1 = p^m\} \cup \{(p_1, p_2)\colon p_2 = 0\}$. Furthermore, equilibrium profits for firm 2, π_2, equal zero and equilibrium profits for firm 1, π_1, depend on the particular equilibrium chosen.[7]

(b) If $0 < p_2^H(k_1, k_2) = P(k_1 + k_2)$ then both firms set $p = P(k_1 + k_2)$ and profits are given by $\pi_i = k_i P(k_1 + k_2)$, i=1,2.

(c) If $p_2^H(k_1, k_2) > P(k_1 + k_2)$ then $p_2 = p_2^H$, $p_1 = p_2$, $\pi_2 = H_2(p_2^H)$ and $\pi_1 = L_1(p_2^H)$. When, in addition, $k_1 = k_2 = k$ there is another equilibrium (and thus a continuum of mixed strategy equilibria for firm 2) in which $p_2 = \underline{p}_2$, $p_1 = p_2^H$, and $\pi_1 = \pi_2 = H_2^*$.

Proof: (a) and (b) are straightforward and are left to the reader. We now prove (c). Observe that since $\underline{p}_1 < p_1^L$ and since $L_1(p)$ is strictly increasing in p on $[0, p_1^L)$, $L_1(p) < H_1^*$ for all $p < \underline{p}_1$. Thus, if firm 2 sets $p_2 \leq \underline{p}_1$ firm 1 responds optimally by setting $p_1 = p_1^H$.[8] For $p_2 > \underline{p}_1$ firm 1 sets p_1 equal to $\min(p_1^L, p_2)$, i.e. it either matches or undercuts firm 2. This follows from the fact that, since $L_1(p)$ is strictly increasing on $[0, p_1^L)$ and strictly decreasing on $[p_1^L, p_0)$, $L_1(\min(p_1^L, p_2)) > H_1^*$ for $p_2 > \underline{p}_1$. Firm 2, in deciding which price to set as leader, will find it optimal to set either the price in $[0, \underline{p}_1]$ which maximizes $L_2(p)$, or the price greater than $\underline{p}_1$ which maximizes $H_2(p)$. The former price is $\underline{p}_1$ since $p_2^L > \underline{p}_2 \geq \underline{p}_1$ and $L_2(p)$ is strictly increasing on $[0, p_2^L)$; the latter price is p_2^H because $p_2^H > \underline{p}_2 \geq \underline{p}_1$, since we are in case (c) of Theorem 1. If $k_2 > k_1$, then by lemma 1, $\underline{p}_2 > \underline{p}_1$. Setting $p_2 = \underline{p}_1$ yields $L_2(\underline{p}_1) < L_2(\underline{p}_2) = H_2^* = H_2(p_2^H)$. Thus, firm 2 as a leader optimally chooses p_2^H, and firm 1 matches since $p_1^L \geq p_2^L > p_2^H$. If $k_2 = k_1$, $\underline{p}_2 = \underline{p}_1$ and firm 2 is indifferent between setting $p_2 = \underline{p}_1$, whereupon firm 1 charges $p_1 = p_1^H > \underline{p}_1$, and setting $p_2 = p_2^H$, whereupon firm 1 will match.#

When the low capacity firm is exogenously specified as a price leader the equilibrium outcome is as given in the following theorem:

Theorem 3: For each pair (k_1, k_2) with $k_2 \geq k_1$ the price setting game with capacity constraints and firm 1 as a price leader has the following equilibria:

(a) If $P(k_1) = 0$ there is a continuum of equilibria. The set of (pure strategy) equilibrium prices is given by: $\{(p_1, p_2)\colon\ p^m \geq p_1 > 0,\ p_2 = p_1\} \cup$

$\{(p_1, p_2): p_1 > p^m, p_2 = p^m\} \cup \{(p_1, p_2): p_1 = 0\}$. In equilibrium $\pi_1 = 0$ and π_2 depends upon the particular equilibrium chosen.

(b) If $0 < p_2^H(k_1, k_2) = P(k_1 + k_2)$ then both firms set $p = P(k_1 + k_2)$ and profits are given by $\pi_i = k_i\, P(k_1 + k_2)$, $i = 1, 2$.

(c) If $p_2^H(k_1, k_2) > P(k_1 + k_2)$ then $p_1 = \underline{p}_2$, $p_2 = p_2^H$, $\pi_1 = L_1(\underline{p}_2)$ and $\pi_2 = H_2(p_2^H)$. When, in addition, $k_1=k_2=k$ there is another equilibrium (and thus a continuum of mixed strategy equilibria for firm 1) in which $p_1 = p_1^H$, $p_2 = p_1$, $\pi_1 = H_1(p_1^H)$ and $\pi_2 = L_2(p_1^H)$.

Proof: Again, (a) and (b) are easily verified and are left to the reader. We now prove (c). By reasoning similar to that given in the proof of theorem 2 firm 2's best response as a follower is to set $p_2 = \min(p_2^L, p_1)$ for $p_1 > \underline{p}_2$ and set $p_2 = p_2^H$ for $p_1 \leq \underline{p}_2$. Firm 1, as leader, finds it optimal to set either the price in $[0, \underline{p}_2]$ which maximizes $L_1(p)$, or p_1^H. The former price is $\underline{p}_2$ since $p_1^L \geq p_2^L > \underline{p}_2$ and $L_1(p)$ is strictly increasing on $[0, p_1^L)$. If $k_2 > k_1$, then by lemma 1 $\underline{p}_2 > \underline{p}_1$, and thus $L_1(\underline{p}_2) > L_1(\underline{p}_1) = H_1^* = H_1(p_1^H)$. Firm 1's optimal choice as leader is then to set $p_1 = \underline{p}_2$, and firm 2 responds with $p_2 = p_2^H$. If $k_1 = k_2$, $\underline{p}_1 = \underline{p}_2$ and firm 1 is indifferent between setting $p_1 = \underline{p}_2$, whereupon firm 2 charges $p_2 = p_2^H > \underline{p}_2$, and setting $p_1 = p_1^H$, whereupon firm 2 will match.#

Theorems 1-3 allow us to compare the returns to the two firms in the simultaneous move game and the two leader follower games. Let π_i^S be the payoff to firm i in the simultaneous move game, π_i^L the payoff to i as a leader and π_i^F the payoff to i as a follower. When $0 < p_2^H(k_1, k_2) = P(k_1 + k_2)$ it is easily seen that all three games yield the same payoffs to each firm. If $P(k_1) = 0$ then $0 = \pi_i^L = \pi_i^S$ i=1,2, but π_i^F depends upon the particular equilibrium played. There is an equilibrium in which this payoff is 0, but in all other possible equilibria it is positive. Which type of equilibrium seems most natural depends upon the particular game of timing being played. When k_1 and k_2 are in the range where the simultaneous move game has a mixed strategy equilibrium the results appear to be more interesting. With $k_2 > k_1$,

$$\pi_2^S = \pi_2^L = \pi_2^F$$

whereas

$$\pi_1^S = \pi_1^L < \pi_1^F.$$

The strict inequality follows because $L_1(p)$ is strictly increasing on $[0, p_1^L)$, and $p_1^L \geq p_2^L > p_2^H > \underline{p}_2$. We conclude that firm 2 is indifferent as to which game it plays, while firm 1 is indifferent between being a leader and moving simultaneously, but strictly prefers to be a follower.[9] This motivates the discussion of several games of timing of price announcements in the section to follow.

III. Game Forms Generating an Endogenous Leader

Suppose $k_2 > k_1$ and capacities lie in the range where the simultaneous move game has an equilibrium in mixed strategies. We now examine several games of timing that yield firm 2 as an endogenous price leader. The first game examined is chosen for its simplicity. Take the time period under consideration to be the unit interval [0, 1]. Divide the interval up into T (T even) periods each of length $\tau = 1/T$, labelled $t = 0,\ldots,T-1$. Assume that firm 1 may announce price at the beginning of intervals having an even index and firm 2 may announce price at the beginning of intervals having an odd index. Suppose that until both firms announce prices, both firms earn zero profits, but that the demand d(p) remains the total demand curve independently of the point at which prices are announced.[10] Thus, the profits given in equation (2) remain the relevant profit expressions once both prices are announced. We assume that firms discount profit earned continuously with discount rate r, and let $\delta = e^{-r\tau}$. To solve for the subgame perfect equilibrium of the game we proceed by backwards induction. Suppose that in this game, firm 1 has made it to the time period T-2 and no price has been announced. Then clearly it will set the leader price, since if it doesn't announce a price it will receive a payoff of zero for the game. Firm 2 will follow with the follower price as given in Theorem 3. Thus, firm 1's payoff is $\delta^{T-1}\Pi_1^L$ and firm 2's is $\delta^{T-1}\Pi_2^F$. Now suppose firm 2 is faced with a situation at T-3 where neither firm has set a price. Then it must weigh the returns $\delta^{T-2}\Pi_2^L$ with $\delta^{T-1}\Pi_2^F$. The former is the return it obtains if it sets its leader price at T-3, and the latter is the return it obtains if it fails to set a price at T-3 and responds optimally to firm 1's optimal strategy at T-2. Since $\Pi_2^L = \Pi_2^F$, firm 2 will set its price in T-3 at the leader price. Firm 1, in period T-4 will anticipate this and not set its price if $\delta^{T-3}\Pi_1^L <$ $\delta^{T-2}\Pi_1^F$. This is equivalent to $\Pi_1^L/\Pi_1^F < \delta$. Thus, if δ is close enough to 1, firm 1 will wait for firm 2 to announce its price first. More generally, suppose firm 2 is faced with a situation in period t, t = 1,3,...,T-3, where neither firm has set a price. Then it must weigh the return $\delta^{t+1}\Pi_2^L$, if it sets a leader price at that time, with either $\delta^{t+j+1}\Pi_2^L$ if it is a leader in t+j or $\delta^{t+j}\Pi_2^F$ if it is a follower in t+j, where $j \geq 2$. Since $\Pi_2^L = \Pi_2^F$ firm 2 will move at t. Thus, firm 2 always sets the leader price at the first opportunity. Suppose firm 1 at time t, t = 0,...,T-4 is faced with a situation in which neither firm has set a price. It knows that firm 2 will set the leader price in the next period if it does not move first so it must compare the return to leading, $\delta^{t+1}\Pi_1^L$, with the return to following at t+2, $\delta^{t+2}\Pi_1^F$. If $\Pi_1^L < \delta\Pi_1^F$ firm 1 will wait and follow. Thus, if $\Pi_1^L/\Pi_1^F < \delta$ firm 1 will not set price in period 0 and firm 2 will set its leader price in period 1. As τ approaches zero the firms will play a game in which firm 2 is a price leader with a delay that converges to zero.

Clearly, if firm 2 has the first move in this game it will set its price first at the leader price and no lag occurs. Thus, no matter what the sequencing of moves, as the period between moves becomes shorter we obtain a leader-follower equilibrium with firm 2 as a price leader. While this type of game does not address the question of when, if ever, simultaneous move games will be played, it does show the robustness of the tendency of endogenous timing to force firm 2 to be a price leader.[11]

When we move to games in which the two firms can choose to move simultaneously a problem arises; the leader-follower equilibria are not Nash equilibria. When the players discount time, if firm 2 were to set a leader price at some time t, firm 1 would want to set the follower price at t rather than waiting until time t+1. But then firm 2 is not playing a best response to firm 1's strategy, and the equilibrium is disrupted. The problem arises for the following reason: unless firm 1 strictly follows firm 2, the latter will not set the leader price, but if it does set the leader price, the former would like to move simultaneously with the follower price. Thus, the best one could hope for in the case where the firms may move simultaneously is to obtain the leader-follower equilibrium as a subgame perfect ε-equilibrium in the game of timing as the length between the times at which prices can be announced goes to zero. Several specifications of such games yield precisely this result, the simplest of which is that obtained by taking the game outlined above but allowing simultaneous moves.[12]

One way to avoid the problem above is to require that firms commit themselves to setting a price at a point in time strictly before the actual price is chosen. In this way firm 2 can determine at any point in time whether it is setting price simultaneously with firm 1 or not. If the discount rate is not too high, firm 1 will want to commit itself to follow firm 2 in order to induce him to set a leader price. Formally, again assume that the time period under consideration is the unit interval [0, 1] and divide the interval up into T periods, each of length $\tau = \frac{1}{T}$, labelled $t = 0,\ldots,T-1$. Suppose that either firm can announce a price at the midpoint of any interval, and that once the price is announced it must remain in force until time 1. As before, we assume that until both firms announce prices, both firms earn zero profit, and that the demand $d(p)$ remains the total demand curve independent of the point at which prices are announced. Again, firms discount their profit earned continuously with discount rate r. In order to announce a price at the midpoint of an interval t, a firm must commit itself to do so at the first point in the interval. Thus, we can think of the time period t as divided into two subperiods t^+ and t^-. Prices can only be announced at t^+ and in order to announce a price the firm has to commit itself to do so at time t^-. One can think of this commitment as hiring an advertising agency at zero cost. The commitment decision at t^- is made by both firms simultaneously and price setting at t^+ takes place with knowledge of the commitments made. Thus, if one firm does not commit itself to set a price at t^- and the other firm does, the latter can set its price at t^+ knowing that the former must wait until the next period $(t+1)^+$ to set its price. We assume that if a firm has committed itself to announce a price at t^+ it must do so; if both firms commit themselves at t^- to set a price then at t^+ they know that they are playing a simultaneous move price setting game. We conjecture that this restriction is unnecessary, but maintain it so that we might carry out the analysis without having to worry about strategies mixing over both prices and the time period of announcement. In the model as specified, for a small enough discount rate, along a path in which neither firm has set a price, firm 1 will find it in its best interest not to commit itself to announce a price in order to signal to firm 2 that it will be a follower in the next period. Firm 2, no matter what the choice of commitment by 1, will commit itself at $\bar{t}$ to set a price, and will set the leader price if firm 1 has not committed and play the mixed simultaneous move equilibrium strategy if firm 1 has committed.[13] This game of timing, which allows firms to move simultaneously, generates a subgame perfect equilibrium in which firm 2 sets price first as long as the time period between points at which price announcements can be made is "short enough". More precisely, "short enough" means that $\tau < -\frac{1}{r}\log(\Pi_1^S/\Pi_1^F)$.[14]

In each of the models outlined above production does not take place until both firms have announced price. A more complicated model would specify demand as a flow and remove the assumption that no sales take place until both prices are announced. In such a model, a price leader sells as the sole firm in the market until the other firm sets its price. Due to the lag in production of the following firm, theorem 2 no longer gives the equilibrium prices. Prices set will depend upon the length of time between price announcements. However, for the case where firm 2 is strictly larger than firm 1 (which rules out multiple equilibria), prices set converge to those of theorem 2 as the length of each time period approaches zero.

IV. Conclusion

The analysis of sections II and III provide a workable foundation for a game theoretic model of dominant firm price leadership. One way in which our results differ substantially from the traditional treatment of dominant firm price leadership, and somewhat less substantially from observed behavior, is that our model seems to require less of an asymmetry in capacity in order to generate a price leader. A possible reason for this is the nonrepeated nature of the game examined here. The game examined only covers one episode of price setting, whereas a repeated game framework, covering a large number (i.e. countable infinity) of price setting episodes, would be more realistic. In such a repeated game framework, collusive outcomes are more likely to prevail than the single period, purely noncooperative behavior described here. If there are set up costs involved in coordinating a collusive outcome, which may be dependent on the number of firms colluding, one would expect collusion to occur only in those cases where the returns to colluding are high. In our model this occurs only when both firms are relatively large. When the small capacity firm is small relative to the market, the returns to colluding are small and the timing considerations displayed here appear more likely. Thus, our model seems an accurate description of dominant firm price leadership despite the absence of purely price-taking firms.

Our theory nonetheless has implications for the case where the firms are large and fairly symmetric in size. In an equilibrium in grim trigger strategies of the supergame constructed by infinitely repeating the single period constituent game outlined here, punishment phases would take the form of the leader-follower equilibria rather than the simultaneous move equilibria analyzed by Brock and Scheinkman (1985). Since the small firm does better in the leader-follower equilibrium than the simultaneous move equilibrium, it may be more difficult to prevent the small firm from cheating in the model outlined here. Counteracting this tendency is the fact that, by having the small firm announce price first in the collusive phase, the large firm may respond more quickly to any cheating that may occur. In general, when the joint monopoly price (announced with no delay) is not sustainable as an outcome of the supergame, the most collusive outcome sustainable may involve a specific sequencing of price announcements rather than a lower price. The first firm to set a price in a period cannot cheat on the collusive outcome without drawing immediate retaliation from the price follower. The following firm, on the other hand, can play a best response to the leader's price without receiving punishment until the next price setting episode. Thus, in games where the joint monopoly outcome cannot be sustained as an immediate simultaneous move equilibrium in the repeated game, the most collusive sustainable outcome may take the form of delay on the part of one firm in the announcement of price rather than a decrease in price. This phenomenon, which will be investigated in more detail in future research, may provide the basis for a model of collusive price leadership.

The discussion of repeated games above involved the infinite repetition of the one shot game used in the paper. This game had the property that, no matter when a firm sets its price it remains fixed until the end of the period (time 1). Thus, the repeated game would always involve price rigidities of the following type: No matter when a firm sets it price in a given play of the game, both firms' prices remain inflexible until the same point in time. An alternative approach would be to construct a dynamic game in which price remained rigid for a fixed period of time from the moment that it is set. Such a model has been constructed by Maskin and Tirole (1986) to examine issues other than the endogeneity of price leadership. In the Maskin and Tirole paper firms were restricted to move either simultaneously or alternatingly, thus requiring either no lag between price announcements or equal spacing between the announcements of firms. An important extension of the current model would be to examine the endogeneity of leadership in a Maskin-Tirole type framework, possibly allowing a finer partition of the time line so that "following" allows for announcing a price with a lag after the announcement of the leader that is small relative to the period for which prices must remain flexible. This extension is being investigated in Deneckere and Kovenock (1988).

A further extension of the model would examine price leadership when firms have different (constant) unit costs of production up to capacity. For the case of linear demand, the equilibria of the simultaneous move game have been completely characterized in Deneckere and Kovenock (1987). For this case, the characterization of the equilibria in the price leadership games uses methods very similar to those utilized here. The crucial determinant of the nature of the leader-follower equilibria is the relationship between $\underline{p}_1$ and $\underline{p}_2$ defined in (3). When the large capacity firm, firm 2, has a strictly greater unit cost up to capacity than firm 1, $\underline{p}_2 > \underline{p}_1$ for ranges of capacity where the simultaneous move game has an equilibrium in which both firms play mixed strategies. This leads to qualitative results analogous to those in Theorems 2 and 3. When the large capacity firm has a strictly lower unit cost up to capacity, the ordering of $\underline{p}_2$ and $\underline{p}_1$ depends upon the particular capacity levels and unit costs. When firm i leads, it sets a price high enough to induce firm j to undercut or match if $\underline{p}_i > \underline{p}_j$. If $\underline{p}_i < \underline{p}_j$ firm i sets a price which deters j from undercutting. Thus, when firms have unit costs negatively related to their capacities, the roles of the two firms may be reversed.

Finally, this paper has examined price leader equilibria under the assumption of Levitan-Shubik contingent demand. Preliminary results using Beckmann contingent demand indicate that, unlike the case examined here, the large firm, as leader, may not choose to form a price umbrella under which the small firm can live. This eliminates the small firm's advantage from following. The reason the large firm may not form an umbrella is that under

Beckmann contingent demand a more favorable contingent demand curve is left for the high-priced firm. Thus, a price leader need not set as low a price to deter undercutting as is needed under Levitan-Shubik contingent demand. If the low capacity firm is not too small this is sufficient to induce the high capacity firm to set a price as a leader which is low enough so that it is not undercut. The implications of this result for the determination of leader-follower roles will be examined in future work.

Footnotes

[1]This point is also made in Rotemberg and Saloner (1986).

[2]High quality management might be thought of as being related to informational advantages. For a model incorporating informational asymmetries to explain collusive price leadership see Rotemberg and Saloner (1986).

[3]Shubik and Levitan (1980) provide a characterization of price leader equilibria for linear demand and a very restricted range of capacities. The first draft of this paper extended their analysis to arbitrary capacities, while maintaining the assumption of linear demand.

[4]These assumptions are borrowed from Kreps and Scheinkman (1983). We make them to facilitate the proofs. All of our results remain true if the following less restrictive assumptions hold:

<u>A1'</u>. There exists a $p_0 > 0$ such that $d(p) = 0$ for every $p \geq p_0$ and $d(p) > 0$ if $p < p_0$. $d(p)$ is continuous and strictly decreasing on $[0, p_0)$. $p \cdot d(p)$ attains a maximum at a unique interior point p^m. $p \cdot d(p)$ is a strictly concave function on $[0, p^m]$ and a strictly decreasing function on $[p^m, p_0)$.

(A1') is sufficient to guarantee that in the quantity setting game with zero cost of production and inverse demand given by

$$P(q) = \begin{cases} p_0 & q = 0 \\ d^{-1}(q) & 0 < q \leq d(0) \\ 0 & q > d(0) \end{cases}$$

best response functions are continuous, and intersect only once.

[5]The results in this paper rely on our choice of contingent demand. An examination of price leadership games with Beckmann contingent demand is in progress.

[6]When firm j leads and sets $p_j = \underline{p}_i$ firm i, as follower, is indifferent between setting $\underline{p}_i$ and p_i^H. We assume in this case that (when $p_i^H > \underline{p}_i$) firm i sets p_i^H. This seems reasonable since firm i is indifferent only because of our technical assumption that it need only match price as the follower in order to sell its capacity.

[7]Whenever the leader has more than one equilibrium strategy it also has a continuum of mixed equilibrium strategies.

[8]See footnote 6.

[9]Boyer and Moreaux (1987), Dowrick (1986), Gal-Or (1985), and Ono (1978, 1982) analyze the incentives to be a leader or follower in contexts different from ours. Boyer and Moreaux investigate the choice of being a leader or follower in a game where the strategy space is price-quantity pairs. Dowrick and Gal-Or examine the role of the slopes of the firms' best response functions (in price or quantity space) in determining the desire to lead or follow. Ono

examines the incentives to be a leader or follower in a model in which the leader sets price and the follower decides how much to produce at that price.

[10]We can easily substitute a flow demand and obtain the same qualitative results.

[11]If $k_2 = k_1$, and capacities lie in the range where the simultaneous move game has an equilibrium in nondegenerate mixed strategies, there is a continuum of subgame perfect equilibria in the game of timing. The leader's price will be either $\underline{p}$ or p^H although, as long as δ is large enough, the firm that has the first opportunity to move need not become the price leader, and prices may be set with some delay (the upper bound to which depends upon δ).

[12]If δ is sufficiently small it is easily shown that the simultaneous move equilibrium of Theorem 1 must be played at time 0 in any subgame perfect equilibrium of this game of timing. As δ gets larger, subgame perfect equilibria generally involve mixing over both the price and the time period.

[13]This statement holds for all $t < T-2$. If $t = T-2$, then when firm 1 does not commit itself at $\bar{t}$ firm 2 is indifferent between committing, whereupon it sets the leader price, and not committing, whereupon it plays the simultaneous move game at $t = T-1$.

[14]If the two firms' capacities are equal and lie in the range where the simultaneous move game has an equilibrium in nondegenerate mixed strategies, there is a continuum of subgame perfect equilibria in this game of timing. The realized equilibrium path may involve firms simultaneously committing and

drawing from their simultaneous move price distributions or committing sequentially with the leader choosing p^H or $\underline{p}$ (or randomizing between them) and the follower setting p^H. As long as δ is large enough, commitment may occur with a delay, the length of which may be random with an upper bound depending on δ.

References

Bain, J. (1960), "Price Leaders, Barometers, and Kinks," Journal of Business 33 (3), 193-203.

Benoit, J. and V. Krishna (1987), "Dynamic Duopoly: Prices and Quantities" Review of Economic Studies 54 (1), 23-34.

Boyer, M. and M. Moreaux (1987), "Being a Leader or a Follower: Reflections on the Distribution of Roles in Duopoly," International Journal of Industrial Organization 5, 175-192.

Brock, W. and J. Scheinkman (1985), "Price-setting Supergames with Capacity Constraints," Review of Economic Studies 52, 371-82.

Davidson, C. and R. Deneckere (1986a), "Excess Capacity and Collusion," Northwestern University, CMSEMS Discussion Paper No. 675.

Davidson, C. and R. Deneckere (1986b), "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," The Rand Journal of Economics 17 (3), 404-415.

Deneckere, R. and D. Kovenock (1987), "Capacity-Constrained Price Competition When Unit Costs Differ," mimeo.

Deneckere, R. and D. Kovenock (1988), "Price Leadership in a Dynamic Oligopoly," in progress.

Dixon, H. (1987a), "The General Theory of Household and Market Contingent Demand," The Manchester School 55 (3), 287-304.

Dixon, H. (1987b), "Approximate Bertrand Equilibria in a Replicated Industry," Review of Economic Studies 54(1), 47-62.

Dowrick, S. (1986), "von Stackelberg and Cournot Duopoly: Choosing Roles," Rand Journal of Economics 17 (2), 251-260.

Gal-Or, E. (1985), "First Mover and Second Mover Advantages," International Economic Review 26 (3), 649-653.

Kreps, D. and J. Scheinkman (1983), "Cournot Pre-commitment and Bertrand Competition Yields Cournot Outcomes," Bell Journal of Economics 14 (2), 326-337.

Levitan, R. and M. Shubik (1972), "Price Duopoly and Capacity Constraints," International Economic Review 13, 111-123.

Markham, J.W. (1951), "The Nature and Significance of Price Leadership," American Economic Review 41, 891-905.

Maskin, E. and J. Tirole (1986), "A Theory of Dynamic Oligopoly, Parts I-III," Harvard Institute of Economic Research, Discussion Paper No. 1270.

Ono, Y. (1978), "The Equilibrium of Duopoly in a Market of Homogeneous Goods," Economica 45, 287-295.

Ono, Y. (1982), "Price Leadership: A Theoretical Analysis," Economica 49, 11-20.

Osborne, M. and C. Pitchik (1986), "Price Competition in a Capacity Constrained Duopoly," Journal of Economic Theory 38, 238-260.

Oxenfeldt, A. (1951), Industrial Pricing and Market Practices (Prentice-Hall, Inc., New York).

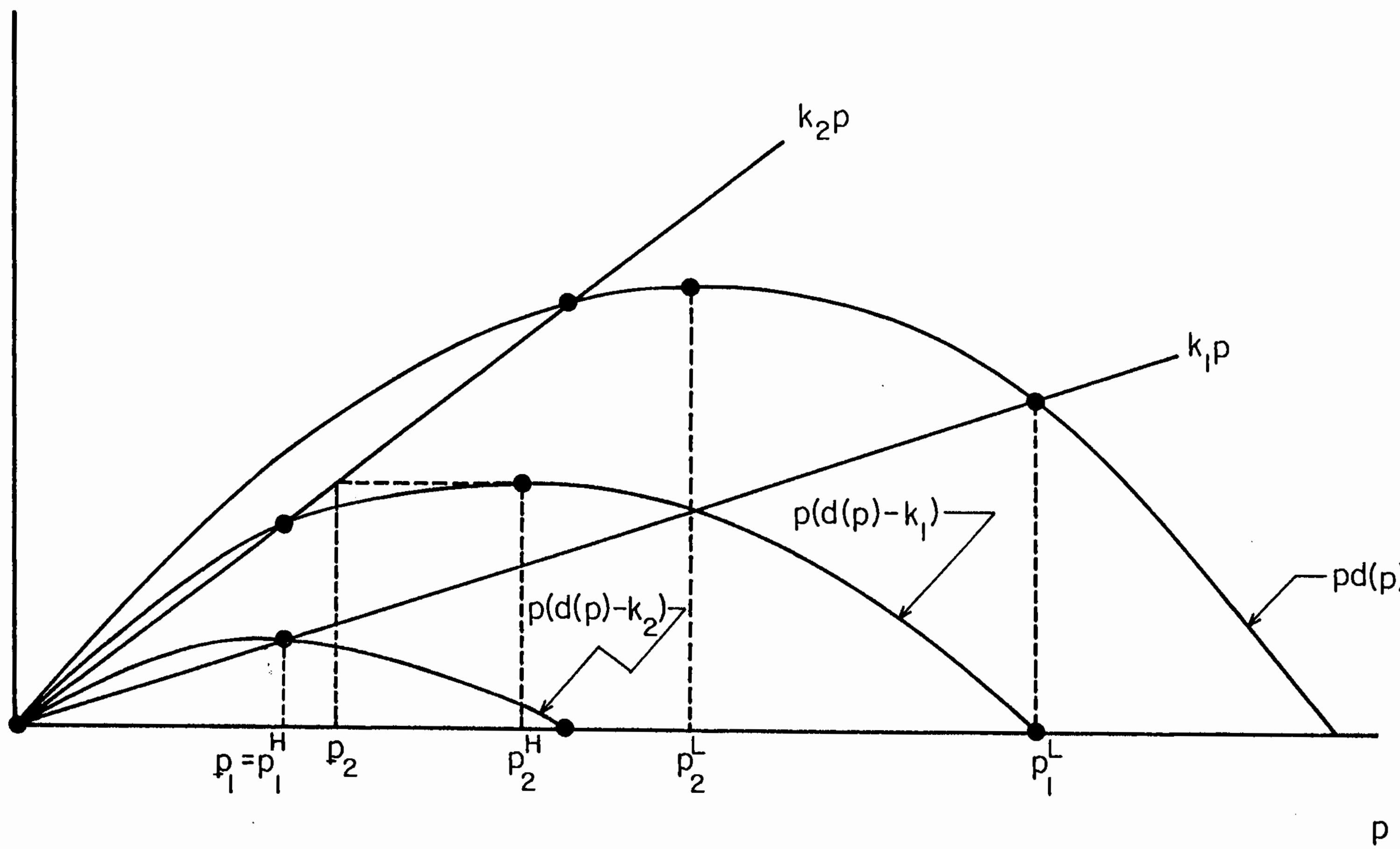
Rotemberg, J. and G. Saloner (1986), "Price Leadership," Massachusetts Institute of Technology, Department of Economics, Working Paper No. 412.

Scherer, F.M. (1970), Industrial Market Structure and Economic Performance, First Edition, (Rand McNally, Skokie, IL) p. 164.

Shubik, M. and R. Levitan (1980), Market Structure and Behavior (Harvard University Press, Cambridge, MA).

Stigler, G. (1947), "The Kinky Oligopoly Demand Curve and Rigid Prices," Journal of Political Economy 55 (5), 432-449. Reprinted in G. Stigler, The Organization of Industry, (University of Chicago Press, Chicago, 1983).

Figure 1



k_2p
k_1p
$p(d(p)-k_1)$
$pd(p)$
$p(d(p)-k_2)$
$\underline{p}_1=p_1^H$
$\underline{p}_2$
p_2^H
p_2^L
p_1^L
p