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EFFICIENT INVESTMENT AND TRADE WITH ASYMMETRIC INFORMATION*

by

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Abstract

This paper considers the standard model of collective decision making with private information which has often been interpreted as a public-goods problem and can also be interpreted as a trading problem with private information. However, one additional complication is added. Each individual must choose an investment level prior to learning his type and the investment choice is not publically observable. It is shown that contracts which implement the efficient outcome exist.

1. Introduction

This paper considers the standard model of collective decision making with private information which has often been interpreted as a public-goods problem. However, one additional complication is added. It is assumed that the n individuals must each choose an investment level prior to learning their type. The investment level will affect their private benefits from the public good. It is assumed that the investment level undertaken by an individual is not publically observable and thus cannot be directly contracted upon.[1] This complicates the contract design problem. An efficient contract must now not only guarantee that an efficient collective choice is made *ex-post* of the individuals learning their types but must also provide incentives for individuals to choose efficient investment levels *ex-ante*.

For the case of $n = 2$ this problem can be interpreted as a contracting problem between a risk neutral monopsony buyer and risk neutral monopoly seller where:

(i) The buyer and seller will possess private information about, respectively, the value of consumption and cost of production on the day of exchange.

(ii) At the time of contracting they do not yet possess this private information.

(iii) After the time of contracting but before the time when the parties receive their private information the seller (buyer) has an opportunity to make an investment which will lower his production costs (raise his value of consumption).

(iv) The level of investment is private information and cannot be explicitly contracted upon.[2]

(v) Before they learn their private information about production cost and value of consumption and before they must choose their investment levels, the buyer and seller can sign a contract governing the manner in which the terms of exchange will be determined. However the contract must take into account the fact that the parties have private information.[3]

In the absence of a formal contract the buyer and seller face Williamson's familiar "hold-up" problem.[4] The seller correctly fears that after he invests in sunk expenses to lower production costs that he may be unable to recover these in the bargaining over price. Thus he is unwilling to invest at the efficient level in advance. The buyer has a similar fear and will also be unwilling to invest.

Rogerson [1984] and Teresawa [1984] have built simple formal models which demonstrate this result for particular bargaining games. Tirole [1986] has generalized this to show that it will be true for broad classes of bargaining games. Tirole [1986] also investigates whether some particularly simple types of contracts such as cancellation penalties might help alleviate the problem and finds that they may not. Thus his analysis is somewhat pessimistic about the ability of incentive compatible contracts to significantly improve efficiency. A number of other papers[5] have investigated the incentive effects of some particularly simple types of contracts. As in Tirole's analysis, with the simple contracts considered, efficiency cannot generally be attained.

However none of the above papers have addressed what is surely one of the more fundamental questions about this model. Namely, what is the nature of the optimal incentive-compatible contract between the buyer and seller? It may be in some cases that the optimal contract is very complicated and thus transactions costs would prevent its use or in some cases that legal institutions prevent one or both parties from committing in advance to any binding relationship. It is this sort of idea which motivates the investigations which are described above. However, it is clearly an important step in the formal analysis of this relationship to analyze the nature of the optimal contract.

A recent very interesting paper by Konakayama, Mitsui, and Watanabe [1986] has begun investigating this question by considering a very simple problem where the buyer and seller must decide whether to exchange one unit of a good with fixed characteristics. By investing in advance the seller can stochastically reduce his cost of production. The buyer makes no investment decision. In this environment they prove the rather surprising result that the buyer and seller can sign a contract which produces the first-best outcome! (Under the optimal contract trade occurs if and only if the buyer's value exceeds the seller's cost and the seller chooses a level of investment which maximizes the expected gains from trade.) Although the analysis of the paper is elegant and clear it seems to rely heavily on the special structure of the particularly simple exchange problem being assumed. Thus it is not clear whether the authors have identified a special case of an important general idea or have simply identified a very special case with an anomalous sort of solution.

The purpose of this paper is to show that the result of Konakayama Mitsui and Watanabe [1986] is remarkably general in three respects. First, it generalizes to arbitrarily complex exchange problems where the seller and buyer must potentially agree on the price and characteristics of a large number of different goods. Second, it generalizes to situations involving more than two parties. Third, it generalizes to situations where all parties must make an investment in advance of exchange. Furthermore the generalization also provides a much clearer explanation of why the result is true. Finally the generalization also gives a clearer indication of the limits of the result. In particular it will not generalize in an obvious way to cases where one party's investment directly affects another party's utility. (For example, it will not generalize to a case where the seller invests money in innovation to increase the buyer's value of consumption.) Thus it suggests that investment problems where investment affects other parties' values may be fundamentally different from problems where investment only directly affects the investor's utility.

The idea behind the proof is quite simple. Consider the standard public goods with private information problem (with no investment.) As shown by D'Aspermont and Gerard-Varet [1979a], a sufficient condition for a mechanism to be Bayesian Incentive Compatible is for the transfer rules to exhibit a property which they term being "subjectively discretionary." Using this insight they constructed the first balanced mechanism which implements the efficient outcome as a Bayesian equilibrium. Cremer and Riordan [1985] have constructed an "improved" mechanism which accomplishes the same thing. It is "improved" in that n-1 of the n players follow dominant strategies instead of merely Bayesian-Nash optimal strategies. However it is easy to verify that their transfer functions are also all subjectively discretionary.

This paper proves that if one adds an investment choice to the standard model, that the investment choices will also be efficient under any mechanism with subjectively discretionary transfer functions. Thus both D'Aspermont and Gerard-Varet's [1979] and Cremer and Riordan's [1985] mechanisms will yield efficient investment choices. It is straightforward to verify that the contract identified by Konakayama Mitsui and Watanabe [1986] is simply the Cremer-Riordan mechanism in their special environment.

Another large literature on investment efficiency and contracts exists which makes a different basic assumption than this paper and the related papers described above. This difference will be briefly described in order to distinguish the two approaches. This paper assumes that both firms' investment levels and their type are private information on the day of exchange. Thus information is <u>asymmetric</u> on the day of exchange.[6] The other literature assumes that these variables are publically observable but not verifiable by a court. That is, parties are <u>symmetrically</u> informed on the day of exchange. Under this approach, bargaining is assumed to resolve any <u>ex-post</u> inefficiencies. However investment inefficiencies may still occur and contracts can affect investment levels by affecting the threat point of the <u>ex</u>-<u>post</u> bargaining game. Early papers in this literature applied this idea to problems in law and economics [Rogerson 1984, Shavell 1984], labor [Grout 1984] and vertical integration [Grossman and Hart 1986].

Recently a literature considering more abstract models has begun. See, for example, Hart and Moore [1985], Huberman and Kahn [1986] and Green and Laffont [1987].

2. The Model

Since the standard model is so well known it will be described as briefly as possible. See Green and Laffont [1977] and D'Aspermont and Gerard-Varet [1979a] for more detailed discussions.

There are n individuals indexed by $i \in \{1, \ldots, n\}$. Individual i is of some type a_i chosen from the set A_i. The collective choice to be made is to choose an element x from the set X. If Mr. i is of type a_i, the collective choice is x, and he receives I dollars, then Mr. i's utility is given by

$$(2.1) \qquad v_i(x,a_i) + I.$$

Finally individual i also makes an investment choice y_i from the set Y_i. Individual i's type is then determined by the distribution function $F(a_i/y_i)$[7]. It will be assumed that the a_i's are distributed independently of one another.[8]

Some mathematical notation will be useful. Let A denote $\overset{n}{\underset{i=1}{\pi}} A_i$ and let a denote an element of A. Let a_{-i} denote the vector $(a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ and let $(a_{-i}\ a_i^*)$ denote the vector $(a_1, \ldots, a_{i-1}, a_i^*, a_{i+1}, \ldots, a_n)$. Define Y, y, y_{-i}, and (y_{-i}, y_i^*) similarly. Finally let $E\{\ /y\}$ denote the expectation operator over a given the investment vector y and let $E_{-i}\{\ /y_{-i}\}$ denote the expectation operator over a_{-i} taking y_{-i} as given.

The sequence of decisions and distribution of information is as follows. The relationship can be thought of as evolving over 4 periods. In period 1 the individuals' types are not yet drawn and investment levels have not yet been chosen. The distribution functions, F_i, and utility functions, v_i, are public information. The individuals can sign a contract[9] in period 1. In period 2 individuals choose an investment level. Their choice is private information. In period 3 the individuals' types are drawn according to the distributions F_i. Their types are private information. Finally in period 4 the contract is executed.

It will be assumed that unique maxima to various maximization problems exist. Let $\phi^*(a)$ be the unique maximum over the set X to

$$(2.2) \qquad \sum_{i=1}^{n} v_i(x, a_i) \; .$$

Let y^* be the unique maximum over the set Y to

$$(2.3) \qquad E\{\sum_{i=1}^{n} v_i(\phi^*(a),\; a_i)/y\} \; .$$

An outcome is defined to be a vector $y = (y_1, \ldots, y_n)$ and a function ϕ: $A \rightarrow X$ where y gives the investment choices of individuals and ϕ denotes the rule for determining the collective choice given the types. An outcome will be said to be exchange-efficient if $\phi = \phi^*$, investment efficient if $y = y^*$, and efficient if it is both exchange and investment efficient. The efficient outcome of course maximizes the expected sum of utilities and would be the rule the individuals would choose if y and a were public information and could be contracted upon.

Following the familiar revelation mechanism approach, a contract is a function (d,t) from A to X x R^n where d is called the decision rule and t is called the transfer rule. It is interpreted as follows. In period 1 the individuals agree on the contract. Then in period 4 the individuals announce their types. If $\hat{a}$ is the vector of announcements, then $d(\hat{a})$ is the collective choice and $(t_1(\hat{a}), \ldots, t_n(\hat{a}))$ is the vector of payments received by the individuals - i.e. - Mr. i receives $t_i(\hat{a})$ dollars. If $t_i(\hat{a})$ is negative Mr. i pays money. As part of the definition of a contract it will be assumed that the transfer rule must balance. That is, it will be assumed that the contract must satisfy

$$(2.4) \qquad \sum_{i=1}^{n} t_i(a) = 0$$

for every $a \in A$. This means that side payments between the individuals must sum to zero.

A strategy for Mr. i is a vector $(y_i, \alpha_i(a_i))$ where y_i is an investment level and $\alpha_i(a_i)$ is Mr. i's announced type given his real type is a_i. Let (y,α) denote a vector of strategies for all n individuals. The vector (y,α) will be called an equilibrium strategy under the contract (d,t) if (y,α) is a Bayesian-Nash equilibrium given the contract (d,t).

Definition:

(y,α) is an equilibrium strategy given the contract (d,t) if (2.5) and (2.6), below, hold.[10]

$$(2.5) \qquad \alpha(a_i) \in \operatorname*{argmax}_{\hat{a}_i} E_{-1} \left[v_i(d(\alpha_{-i}, \hat{a}_i), a_i) + t_i(\alpha_{-i}, \hat{a}_i)/y_{-i} \right]$$

$$(2.6) \qquad y_i \;\varepsilon\; \operatorname*{argmax}_{\hat{y}_i} E\left[v_i(d(\alpha),\, a_i) + t_i(\alpha)/(y_{-i},\, \hat{y}_i)\right]$$

A contract will be said to implement an outcome if equilibrium strategies will yield that outcome.

Definition:

The contract (d,t) implements the outcome (y,ϕ) if there exists an equilibrium strategy $(\hat{y},\hat{\alpha})$ to (d,t) such that

$$(2.7) \qquad d(\hat{\alpha}(a)) = \phi(a) \quad \text{for every } a \;\varepsilon\; A$$

and

$$(2.8) \qquad y = \hat{y} \;.$$

A contract will be called exchange efficient, investment efficient or efficient if, respectively, it implements an exchange efficient, an investment efficient or the efficient outcome. The goal of the paper is to investigate whether an efficient contract exists.

A few more definitions regarding the structure of contracts will be useful. Consider the contract (d,t). It will be called naively exchange efficient (NEE) if it selects the optimal collective choice given the announcements - i.e. - if $d = \phi^*$. The transfer rule for Mr. i can always be written in the form

$$(2.9) \qquad t_i(a) = \sum_{j \neq i} v_j(d(a),a_j) + g(a)$$

for some function $g(a)$. The transfer rule t_i will be called discretionary if $g(a)$ does not depend on a_i - i.e. - if

$$(2.10) \qquad g(a_{-i},\hat{a}_i) = g(a_{-i},\hat{\hat{a}}_i)$$

for every $a_{-i} \varepsilon A_{-i}$ and $\hat{a}_i$, $\hat{\hat{a}}_i \varepsilon A_i$.

It will be called subjectively discretionary given y_{-i} if Mr. i's expectation of g (calculated when he knows his type) does not depend on a_i - i.e. - if

$$(2.11) \qquad E_{-i}\left[g(a_{-i},\hat{a}_i)/y_{-i}\right] = E_{-i}\left[g(a_{-i},\hat{\hat{a}}_i)/y_{-i}\right]$$

for every $\hat{a}_i$, $\hat{\hat{a}}_i \varepsilon A_i$. Obviously, the first property implies the second. That is, if t_i is discretionary then it is subjectively discretionary for every $y_{-i} \varepsilon Y_{-i}$.

3. Analysis

A. A Fixed Investment Level

In Section A it will be assumed that there is only a trivial investment choice. Each Y_i is the one element set $\{y_i\}$. This produces the standard collective choice model considered by the public goods literature. The results of these papers necessary for this paper's analysis will be reported in this section. Then Section B will show how these results generalize to cases where the investment choice is non-trivial.

Early work on dominant strategy mechanisms relied on the observation that if a contract (d,t) is NEE and discretionary for Mr. i, then it is a dominant strategy for Mr. i to truthfully reveal his type. In particular, then, if (d,t) is NEE and discretionary for every i then it implements the efficient outcome by an equilibrium strategy where it is a dominant strategy for each individual to truthfully reveal his type. Unfortunately, except for very special cases, it is not possible to create such contracts which balance.[11]

D'Aspermont and Gerard-Varet [1979a] observed that the weaker property of being subjectively discretionary is sufficient to induce a Bayesian-Nash equilibrium where each individual truthfully reveals his type. This is stated as Proposition 1, below.

<u>Proposition 1</u>: (D'Aspermont and Gerard-Varet [1979a], Theorem 5)

Suppose $Y_i = \{y_i\}$ for every i. Suppose that (d,t) is NEE and that t_i is subjectively discretionary given y_{-i} for every i. Then:

(i) It is an equilibrium strategy for each individual to truthfully reveal his type.

(ii) Therefore (d,t) implements the efficient outcome.

proof:

See D'Aspermont and Gerard-Varet [1979a].

QED

They then constructed a (balanced) contract which was NEE and subjectively discretionary for every i. This will be described in Section 4. More recently Cremer and Riordan [1985] have constructed another efficient contract. Like the original mechanism of D'Aspermont and Gerard-Varet, the Cremer-Riordan mechanism is NEE and subjectively discretionary for every i and this is why it is efficient. However, it is "improved" in the sense that (n-1) of the n transfer rules are discretionary (not merely subjectively discretionary). Thus n-1 of the n individuals have a dominant strategy and only the n^{th} individual has merely a Bayes-Nash optimal strategy. This will also be described in Section 4.

B. Variable Investment Levels

In Section B it will be assumed that the sets Y_i may have more than one element. Proposition 2 shows that Proposition 1 generalizes to this environment. Recall that (y^*,ϕ^*) is the efficient outcome.

Proposition 2:

Consider the contract (d,t). Suppose that d is NEE and t_i is subjectively discretionary for y^*_{-i} for every i. Then (d,t) implements (y^*,ϕ^*).

proof:

Consider Mr. i's choice of a strategy. Suppose that for every $j \neq i$, that Mr. j chooses y^*_j and truthfully reveals his type. It will be shown that it is optimal for Mr. i to do the same. Suppose Mr. i chooses $\hat{y}_i$. By Proposition 1 Mr. i will also find it optimal to report his type truthfully. Therefore if Mr. i chooses $\hat{y}_i$ his expected utility will be

$$(3.1) \qquad E\left\{v_i(d(a),a_i) + t_i(a)/(y^*_{-i},\hat{y}_i)\right\}.$$

Substitute (2.9) into (3.1) to yield

$$(3.2) \qquad E\left\{\sum_{j=1}^{n} v_j(d(a),\ a_j)/(y^*_{-i},\ \hat{y}_i)\right\}$$

$$+ E\left\{g(a)/(y^*_{-i},\ \hat{y}_i)\right\}$$

It is obviously sufficient to show that the second term of (3.2) does not depend on $\hat{y}_i$. Rewrite it as

$$(3.3) \qquad E_i\left\{E_{-i}\ \{g(a)/y^*_{-i}\}/\hat{y}_i\right\}$$

where $E_i\{\ /\hat{y}_i\}$ denotes the expectation over a_i given $\hat{y}_i$. Since t_i is subjectively discretionary given y^*_{-i} as defined by (2.11), the inner expectation does not depend on a_i. Therefore (3.3) can be rewritten as

$$(3.4) \qquad E_i \{h(y^*_{-i})/\hat{y}_i\}$$

where h is a function of y^*_{-i}. Expression (3.4) equals $h(y^*_{-i})$ which does not depend on $\hat{y}_i$.

QED

An immediate Corollary of this is that the D'Aspermont-Gerard-Varet and Cremer-Riordan mechanisms will induce the efficient outcome. Thus efficient contracts exist.

An interesting subtlety of the proof is that it requires firms' investment choices to be unobservable. In the proof, Mr. i could assume that all other individuals would continue to tell the truth no matter what his choice of investment, $\hat{y}_i$. This was because they could not observe $\hat{y}_i$ and were assuming that Mr. i chose y^*_i. It is straightforward to show that if investment is observable but non-verifiable (and thus noncontractible)[12] that the argument will still work for Mr. i if the transfer rules for all other individuals are discretionary. Of course a balanced contract cannot be created when all n transfer rules are discretionary. The Cremer-Riordan [1985] mechanism allows one to choose n-1 of the transfer rules to be discretionary. Thus if investment is observable but non-verifiable the Cremer-Riordan [1985] mechanism will implement the efficient outcome if only one of the n individuals makes an investment choice. One simply chooses the

Cremer-Riordan mechanism which gives the other n-1 individuals the discretionary transfer rule. This result is summarized in Proposition 3.

Proposition 3:

Suppose that investment is observable but non-verifiable and that only individual 1 makes a non-trivial investment choice. (i.e. - Y_i is a singleton set for every $i \neq 1$). Then the Cremer-Riordan mechanism which gives individuals 2 through n dominant strategies implements the efficient outcome.

proof:

As above.

QED

In case of an exchange between a buyer and seller where only one of the two parties must invest in advance, Proposition 3 suggests that the Cremer-Riordan mechanism which gives the non-investing party a dominant strategy is the best mechanism to use. This mechanism is efficient whether investment is observable or not.[13]

4. The Efficient Contracts

The purpose of this section is to explicitly describe the efficient contracts which can be constructed using the mechanisms of D'Aspermont and Gerard-Varet [1979a] and Cremer and Riordan [1985]. Given Proposition 2, the efficiency of these contracts is established by showing that they are NEE and their transfer rules are subjectively discretionary. Both contracts are NEE by construction. D'Aspermont and Gerard-Varet [1979a] prove that their mechanism has subjectively discretionary transfer rules in Theorem 6 of their paper and this result will not be reproven here. Cremer and Riordan [1985] do not explicitly prove (or even claim) that their transfer rules are subjectively discretionary.[14] Therefore, although the proof is simple, it will be given here.[15]

First consider the D'Aspermont-Gerard-Varet solution. Define $\gamma_i(a_i)$ by

$$(4.1) \qquad \gamma_i(a_i) = E_{-i}\left\{\sum_{j\neq i} v_j(\phi^*(a), a_j)/y^*_{-i}\right\} .$$

for every i. (Recall that (y^*, ϕ^*) is the efficient outcome). Then the D'Aspermont-Gerard-Varet contract is given by

$$(4.2) \qquad d(a) = \phi^*(a)$$

$$(4.3) \qquad t_i(a) = \gamma_i(a_i) - \frac{1}{n-1} \sum_{j\neq i} \gamma_j(a_j) + z_i$$

where z_i are any n constants satisfying

$$(4.4) \qquad \sum_{i=1}^{n} z_i = 0.$$

Now consider the Cremer-Riordan solution. Define $\delta_i(a_i)$ by

$$(4.5) \qquad \delta_i(a_i) = E_{-i}\left\{\sum_{j=1}^{n} v_j(\phi^*(a),a_j)/y^*_{-i}\right\} .$$

Recall that Cremer and Riordan construct a contract where n-1 of the n individuals have dominant strategies. The contract where all individuals except Mr. i have a dominant strategy is defined as follows. The decision rule is

$$(4.6) \qquad d(a) = \phi^*(a)$$

The transfer rules for all individuals except Mr. i are defined by

$$(4.7) \qquad t_j(a) = \sum_{k\neq j} v_k(\phi^*(a),a_k) - \delta_i(a_i) + z_j$$

where the z_j are (n-1) constants. Finally, Mr. i's transfer rule is

$$(4.8) \qquad t_i(a) = -\sum_{j\neq i} t_j(a) .$$

The transfer rules for all individuals other than Mr. i are obviously discretionary (and thus also subjectively discretionary). It will now be shown that t_i is also subjectively discretionary.

Proposition 4:

The transfer rule t_i defined by (4.8) is subjectively discretionary given y^*_{-i}.

proof:

The transfer rule t_i can be rewritten as

$$(4.9) \qquad t_i(a) = \sum_{j=2}^{n} v_j(\phi^*(a),a_j) + \Gamma(a)$$

where

$$(4.10) \qquad \Gamma(a) = (n-1)\delta_i(a_i) - (n-1)\sum_{j=1}^{n} v_j(\phi^*(a),a_j) - \sum_{j\neq i} k_i$$

By taking expectations and using the definition of $\delta_i(a_i)$ in (4.5), we have

$$(4.11) \qquad E_{-i}\left\{\Gamma(a)/y^*_{-i}\right\} = -\sum_{j\neq i} k_i$$

In particular, the expectation does not depend on a_i. Therefore t_i is subjectively discretionary given y^*_{-i}.

QED

5. The Contracting Problem of Konakayama, Mitsui, and Watanabe [1986]

Konakayama, Mitsui, and Watanabe [1986] consider a special case of the general contracting problem considered by this paper. In their example $n = 2$. Let Mr. 1 be the seller and Mr. 2 be the buyer. Let $X = \{0,1\}$ be the decision set where 0 denotes no trade and 1 denotes trade. Mr. 1's utility is

$$v_1(x, a_1) = \begin{cases} -a_1, & x = 1 \\ 0, & x = 0 \end{cases} \tag{5.1}$$

where $A_1 = [0,1]$. That is, a_1 is the seller's cost if trade occurs. If no trade occurs he does not produce the good and his cost is zero. Mr. 2's utility is

$$v_2(x, a_2) = \begin{cases} a_2, & x = 1 \\ 0, & x = 0 \end{cases} \tag{5.2}$$

where $A_2 = [0,1]$. That is, a_2 is the buyer's value if trade occurs. If no trade occurs the buyer does not consume the good and his value is zero. Finally the authors assume that only Mr. 2 (the buyer) makes a non-trivial investment decision. For Mr. 2, $Y_2 = R$ and Mr. 2's type is determined by $F_2(a_2/y_2)$. Mr. 1's type is determined by $F_1(a_1)$.

The authors construct a contract which implements the first-best outcome. Furthermore, Mr. 2 has a dominant strategy to truthfully reveal his type. It is straightforward to show that the authors' contract is the Cremer-Riordan mechanism for their special environment.

6. Conclusion

The result of this paper -- that contracts guaranteeing both efficient investment and trade exist -- is remarkably general in some respects. Arbitrarily general type spaces, investment spaces and utility functions are allowed. Furthermore, the collective decision itself can be arbitrarily complex and every individual can be allowed to have a non-trivial investment choice.

There appear to be three major limitations to the results. These are important to note in their own right. However they also suggest interesting directions for future research. First, the contracts of this paper may be extremely long and complex. Transactions costs and bounded rationality may make such contracts infeasible. The previously cited literature which restricts analysis to simple classes of contracts can be viewed as a response to this limitation.

Second, it may be difficult to enforce some types of contractual commitments due to bankruptcy or legal constraints. This paper has assumed an ex-ante individual rationality constraint. (At the time of contracting in period 1 individuals can commit to contracts which they may later wish to renege on.) Myerson and Satterthwaite [1983] have solved for the optimal contract in a simple two person model with interim individual rationality constraints when there is no investment problem. Even with no investment problem an efficient outcome cannot be achieved. Therefore the optimal contract with an investment problem would clearly not be efficient either. It would be interesting to see how the need to provide investment incentives affects the Myerson-Satterthwaite solution.

Third, the structure of the proof makes it fairly clear that the result of this paper will not generalize easily to the case where one individual's investment decision affects another individual's type. However this could be a fairly common situation in real trade situations. For example, a seller may invest in innovation which will increase the value of consumption to the buyer. Thus an important topic for future research is to analyze the nature of the optimal contract in this type of situation. The nature of the solution is likely to be quite different than the solution derived in this paper.

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Footnotes

1. At the end of the paper it will be shown how the results of the paper extend to the case where investment levels are publically observable but not objectively verifiable (and this not contractible). However for the bulk of the paper it will be assumed that investment levels are private information.

2. See footnote 1.

3. That is, contracts must be incentive compatible. This is formally defined in the body of the paper.

4. See Williamson [1975, 1979] and Klein, Crawford, and Alchian [1978].

5. See Shavel [1980, 1984], Rogerson [1984], and Wiggins [1988].

6. As explained in footnote 1 the case where investment levels are publically observable will also be considered by this paper. However information about individuals' types is still private on the day of exchange. Thus this is still fundamentally an asymmetric information model.

7. It is no more general to assume that the utility functions are of the form

$$u_i(x,a_i,y_i) + I$$

with type determined by $G_i(a_i/y_i)$. In this case define a "new" type space

A_i^* to be $A_i \times Y_i$. Now a distribution function $F_i(a_i^*/y_i)$ and utility function

$v_i\,(x,a_i^*) + I$ can be defined in the obvious manner.

8. Both D'Aspermont and Gerard-Varet [1979] and Cremer and Riordan [1985] show that their mechanisms will still work in some cases where the a_i's are not independent. The results of this paper would then still be true as well. For simplicity, formal analysis will be restricted to the case of independent types.

9. A contract will be defined below.

10. The vector $(\alpha_{-1},\hat{a}_i)$ denotes $(\alpha_1(a_1), \ldots, \alpha_{i-1}(a_{i-1}), \hat{a}_i, \alpha_{i+1}(a_{i+1}), \ldots, \alpha_n(a_n))$. The vector α denotes $(\alpha_1(a_1), \ldots, \alpha_n(a_n))$.

11. See Green and Laffont [1977].

12. This is Grossman and Hart's [1986] and Tirole's [1986] terminology. Verifiable means that investment levels are objectively observable by the courts and can be contractually specified. Thus "observable but non-verifiable" means investment levels can be observed by the n individuals but cannot be contractually specified.

13. Note that Konakayama, Mitsui and Watanabe [1986] chose the reverse mechanism. In their example only the buyer invests. However, they chose the Cremer-Riordan mechanism which gives the buyer a dominant strategy. See the discussion in Section 5.

14. In the Cremer-Riordan mechanism the n-1 individuals with dominant strategies clearly have discretionary transfer rules. The question is whether the transfer rule of the n^{th} individual is subjectively discretionary.

15. Cremer and Riordan directly prove that their contract is efficient in their model with no investment choice. A somewhat simpler proof would be to show that their transfer rules are subjectively discretionary. Efficiency of the contract then follows from D'Aspermont and Gerard-Varet's result reported as Proposition 1 in this paper.