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INFORMATIONAL EXTERNALITIES AND THE SCOPE OF EFFICIENT DOMINANT STRATEGY MECHANISMS*

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ABSTRACT

This paper concerns a group choice model in which efficient choice requires the revelation of at least some of the group members' private information. A mechanism is a game that determines the group choice and transfer payments from the headquarters to each member as functions of the members' messages to headquarters. Each member's utility is the sum of a direct payoff from the group choice plus the transfer payment from the headquarters. A mechanism is dominant if it implements efficient choice in dominant strategies. Informational externalities exist when the direct payoff to one member depends upon information that is privately held by other members. This paper examines the effect of informational externalities upon the existence of dominant mechanisms. In particular, the impact of informational externalities upon the Groves mechanisms is studied.

1. Introduction

Each agent in a group typically has some private information. When a collective action is considered, the agents may be asked to reveal some of their private information to a central agent (or headquarters) in order to permit an efficient choice. An agent may find it in his best interest, however, to lie; this can lead to an inefficient choice for the group.

This paper concerns the problem of devising transfer payments (positive or negative) from the headquarters to the agents that will induce them to reveal their private information. To be more precise, we regard the process whereby the agents communicate with headquarters as a game, where each agent's strategy set is his range of possible messages. A *mechanism* is a game that determines the group choice and a set of transfer payments from the headquarters to the agents as a function of the agents' messages. A mechanism is *dominant* if (i) each agent has a dominant strategy, and (ii) the headquarters makes an efficient choice when the agents choose these strategies. We examine conditions under which dominant mechanisms exist.

As a starting point of our analysis, we present a model of group decision-making. In our model, each agent's utility is the sum of the transfer payment from headquarters plus a *direct payoff* that results from the group choice itself. Our model differs from most of the models in the mechanism design literature in one important respect: in this paper, an agent's direct payoff may depend upon information that is observed *only* by the other agents. We refer to this phenomenon as an *informational externality*. It is usually assumed that an agent's direct payoff depends only upon the choice itself, information that is common knowledge, and his own private observations (e.g., his personal characteristics), but not upon the private observations of the other agents. Informational externalities do not preclude the existence of dominant mechanisms, but they do place limits on their existence. The purpose of this paper is to explore these limits.

Informational externalities are illustrated in the internal transfer problem, which concerns the transfer of an item within a firm. One division (the manufacturer) can produce an item for another (the marketing division). The headquarters must decide if this should be done. When the item is transferred, the direct payoff to the manufacturer is the cost of the item, and the direct payoff to the marketing division is the price that it expects to receive when it later sells the item outside the firm. Informational externalities exist if the marketing division would alter its estimate of this price if it somehow acquired the manufacturer's information about the cost of production. This might occur if this cost and the cost of the item to competing firms were dependent random variables. Similarly, if the manufacturing division itself had the option of marketing the item, then this opportunity cost must be considered in determining the manufacturer's direct payoff. Informational externalities exist if the manufacturer's estimate of this cost would change if it acquired the marketing division's knowledge of the marketplace. This example is more formally presented in Section 2.

A <u>revelation mechanism</u> is a mechanism in which each agent can completely reveal his private information. A natural starting point for our analysis is a particular family of revelation mechanisms - the <u>Groves mechanisms</u>. These mechanisms originated in the work of Clarke [1], Groves [7], and Vickrey [12]. In several models, it has been shown that a Groves mechanism solves the misrepresentation problem by making truth-telling into a dominant strategy for each agent. We reexamine these results in light of the problems that can be caused by informational externalities.

Three independence properties (I-III) are formulated that limit the dependence of any agent's direct payoff upon the other agents' information. Property I states that no informational externalities exist; Properties II and III are less restrictive. Property I implies that II and III hold, while II does not imply III, nor does III imply II.

When Property I is satisfied, any Groves mechanism makes truth-telling into a dominant strategy for each agent. This is the "classical" result about Groves mechanisms. Property III is a <u>necessary</u> condition for the existence of a Groves mechanism that makes truth-telling into a dominant strategy for each agent. We then formulate a class of <u>quasi</u>-Groves mechanisms for the set of direct payoff functions that satisfy Property II. These mechanisms are identical to the Groves mechanisms when Property I holds, but they form a completely separate class when informational externalities exist. For all direct payoff functions in the set defined by Property II, truth-telling is a dominant strategy for each agent under any of the quasi-Groves mechanisms. Property II is therefore a <u>sufficient</u> condition for the existence of dominant mechanisms. We also prove in this paper that none of these three properties is necessary for the existence of a dominant mechanism.

These results are meaningful, because it is easy to imagine that these properties would not be satisfied in many of the real-world situations where Groves mechanisms have been proposed as solutions to the misrepresentation problem. For instance, the Groves mechanisms have been considered as a way to insure efficient choice in the internal transfer problem (see [9], [11]). Our work shows that informational externalities may make the Groves mechanisms ineffective in such situations.

The final section of this paper concerns the more general problem of the existence of dominant mechanisms. In a special case of our model, we formulate Property D (for "dependence"), which expresses the presence of informational externalities. Property D implies that II does not hold. In this specialized setting, we prove that D is sufficient to insure that no dominant mechanism exists; moreover, we show that D is satisfied by "almost all" determinations of our model in this restricted setting. This in some sense mitigates our criticism of Groves mechanisms, for it suggests that the dominant strategy solution concept is simply too strong a requirement for the decision problems that can be represented by our model.

We conclude this introduction by briefly describing the relationship of our work to several other papers. In a special case of our model, Green and Laffont [5] showed that the Groves mechanisms are the only revelation mechanisms for which (i) truth-telling is a dominant strategy for each agent, and (ii) headquarters makes an efficient choice when the agents tell the truth. This strong characterization assumes that informational externalities are not present, and that each agent's direct payoff function assumes a "dense" range of values. Since the focus of our paper is the effect of informational externalities upon dominant strategy implementation of efficiency, the Green and Laffont characterization does not apply to our model. Our analysis in fact shows that Green and Laffont's characterization does not generalize to include all cases where informational externalities may exist: the Groves mechanisms are not always dominant in these cases, and other mechanisms (e.g., the quasi-Groves mechanisms) may be dominant. The reader should also note that though we do not make any assumptions about the range of the agents' direct payoff functions, such assumptions would not alter these conclusions.

Our model does assume that an agent's utility is additively <u>separable</u> (i.e., it is the sum of his direct payoff plus the transfer payment). We also do not impose the <u>budget constraint</u>, which requires the transfer payments from the headquarters to the agents to sum to zero. Green and Laffont [6, Ch. 5.2] showed that while separability is not necessary for the existence of dominant mechanisms, such mechanisms cannot exist when the class of non-separable utility functions is sufficiently rich. Green and Laffont [5] contains an example where nonseparability precludes the existence of dominant mechanisms. Similarly, in some cases dominant mechanisms exist that satisfy the budget constraint (e.g., see Groves and Loeb [8, Sec. 2.6]). The Green and Laffont characterization that is discussed above, however, can be used to show that such examples require strong assumptions about the agents' utility functions (see [6, Ch. 5.3] for further discussion). These results reveal some of the limits of dominant mechanisms as solutions to the misrepresentation problem. We assume separability and we do not impose the budget constraint in this paper in order to properly isolate a third limitation on the usefulness of dominant mechanisms — informational externalities. The reader should also note that the budget constraint may be inappropriate in the internal transfer problem (for instance), for it may not be important that the budget of the headquarters balances as long as the organization as a whole balances its budget. See Radner [11] for further discussion.

Using a Bayesian model of collective choice, d'Aspremont and Gérard-Varet [3,4] studied how correlation between the agents' private observations affects the existence of revelation mechanisms for which: (i) truth-telling is a Bayesian-Nash equilibrium; (ii) an efficient choice is made when the agents tell the truth; (iii) the transfers sum to zero no matter what the agents choose to report. As discussed above, our paper is focused on the dominant solution concept, and we do not attempt to balance the budget; the budget constraint is a prime interest of d'Aspremont and Gérard-Varet [3,4]. The most important distinction between our model and theirs is that we consider a larger family of direct payoff functions. As noted earlier, this is what distinguishes our paper from most work on mechanism design. In d'Aspremont and Gérard-Varet [3,4], an agent's direct payoff depends only upon the agent's own observation and the group choice itself; the agent is concerned with the other agents' observations only because they may affect the messages to headquarters, and hence also the group choice. In our model, an agent's direct payoff may in principle depend upon any feature of the state of nature; an agent is interested in the observations of others not only because they may indirectly affect the group choice, but also because they could directly affect his own preferences over the set of possible choices. Because we consider a more general class of direct payoff functions, correlation between the agents' observations is not the true source of the problems that we consider (as it is in d'Aspremont and Gérard-Varet [3,4]), but only an incidental effect; informational externalities can exist even when the agents' observations are independent. This will be clarified by a formal discussion of our model.

2. Model

There are n agents, and an institution that is separate from each of the agents - the headquarters. Headquarters must choose an element of a set A (the set of alternatives) for the group of agents. Each agent is risk-neutral.

We assume that there is a probability space (Ω,B,μ), which we refer to as the space of environments. Every element of Ω specifies a value for each of the economic parameters that is relevant to the decision problem at hand, Ω describes the range of possible environments, and μ expresses the likelihood that a particular set of circumstances would occur. We assume that (Ω, B, μ) is common knowledge. In a moment, we shall discuss what the agents and headquarters actually know about the environment. Note that an element of Ω would determine the characteristics of the agents themselves, if these characteristics could vary. An example of Ω will be given shortly.

We assume that agent i's utility function $U_i(\cdot)$ has the form

$$U_i(a,\omega,t_i) = W_i(a,\omega) + t_i.$$

Here, a is in A, ω is in Ω, $W_i(a,\omega)$ is the ith agent's <u>direct payoff</u>, and t_i is a <u>transfer payment</u> from headquarters to the ith agent. This is the linear form that was mentioned in the Introduction. We assume that headquarters knows each of the functions $W_1(\cdot),\ldots,W_n(\cdot)$.

For each $1 \leq i \leq n$, we assume that there exists a measurable function $X_i(\cdot)$ from (Ω, B) into some measurable space E_i. Each of these functions is common knowledge. For a specific environment $\tilde{\omega}$, however, agent i alone observes the value $\tilde{e}_i \equiv X_i(\tilde{\omega})$. This value represents agent i's private information about the true state of the environment $\tilde{\omega}$. Note that the values of $X_i(\tilde{\omega}),\ldots,X_n(\tilde{\omega})$ might not completely determine $\tilde{\omega}$; given all the agents' observations, there still might be some uncertainty about the actual environment.

By assumption, headquarters wants to choose an alternative a^* that is <u>efficient</u> in the following sense: a^* maximizes

$$E_\mu\Big(\sum_{i=1}^{n} W_i(a,\omega) \,\Big|\, X_j(\omega) = X_j(\tilde{\omega}),\ \ 1 \leq j \leq n\Big), \tag{1}$$

where $\tilde{\omega}$ is the actual environment. Headquarters, of course, does not observe

$X_1(\tilde{\omega}),\ldots,X_n(\tilde{\omega})$. In order to attain its objective, headquarters must acquire information about the observations $X_i(\tilde{\omega})$ from the respective agents. Headquarters' problem is to design transfers so that the agents will have an incentive to provide it with enough information to make an efficient choice.

Before proceeding, we first illustrate our model by formalizing the internal transfer problem.

Example: The two agents are divisions of the same firm. One division (the manufacturer) can produce a good for the other division (the marketing division). Headquarters must decide whether or not the manufacturer should supply the good to the marketing division.

An element ω of Ω determines all of the information that might be useful to headquarters or to the divisions if it were made available to them; ω might specify the cost to the manufacturer of producing the good, a complete description of the market that the marketing division would face if it acquired the good, etc. Each division makes an observation about the actual environment $\tilde{\omega}$. For instance, the manufacturer might have some information about its cost of production, and the marketing division might know something about the price it would face in the market if it tried to sell the good. When the decision is actually made, however, the manufacturing cost and the market price might not be perfectly known by the respective divisions.

Headquarters wishes to act in the best interest of the firm as a whole. Given all that is known about the true state of the environment, it should order the manufacturer to produce the item for the marketing division if and only if the expected benefit to the marketing division exceeds the expected cost to the manufacturer. If $W_1(\cdot)$ represents the benefit to the marketing division, $W_2(\cdot)$ the cost to the manufacturer, $X_1(\tilde{\omega})$ and $X_2(\tilde{\omega})$ their respective observations, and if A is the set consisting of the two options that are available to headquarters, then headquarters should choose an option that maximizes (1).

We return to the general case. Since decisions can only be based upon what is actually known, we can eliminate (Ω,B,μ) and the functions $X_1(\cdot),\ldots,X_n(\cdot)$ from the problem. This will reduce our model to a form that is similar to the more standard models. Let E denote the Cartesian product $\Pi_{i=1}^n E_i$, and let e denote the n-tuple $(e_1,\ldots,e_n)$. For $1 \leq j \leq n$, define a function $w_i(\cdot)$ from $A \times E$ into R by the following formula:

$$\text{for } a \in A \text{ and } e \in E,$$
$$w_j(a,e) \equiv E_\mu\left[W_j(a,\omega)|X_i(\omega) = e_i, \quad 1 \leq i \leq n\right].$$

If agent j knew all n components of e, his evaluation of the alternative a and the transfer t_j would be $w_j(a,e) + t_j$. Agent j, however only knows $e_j = X_j(\omega)$. He will therefore use the function $\tilde{w}_j(\cdot)$ to evaluate alternatives, where

$$\tilde{w}_j(a,e_j) \equiv E_\mu\left[W_j(a,\omega)|X_j(\omega) = e_j\right].$$

Finally, the objective for headquarters is to choose an alternative $\alpha(e)$ such that

$$\alpha(e) \in \underset{a \in A}{\operatorname{argmax}} \sum_{i=1}^n w_i(a,e).$$

Our model now differs from the more standard approach in only one respect: agent j's evaluation of the alternatives might change if he somehow acquired the information that is known only by the other agents.

<u>Informational externalitites</u> are present when $w_j(\cdot)$ and $\tilde{w}_j(\cdot)$ are distinct functions on $A \times E$ for some j. Because an agent's direct payoff may depend upon any feature of the environment, informational externalities can exist even when $X_1(\cdot),\ldots,X_n(\cdot)$ are independent; correlation between the observations is not necessary for their existence. Note that informational externalities do not exist in the special case where the agents' observations are perfectly correlated.

We now state several properties that limit the presence of informational externalities. The first property completely precludes their existence:

> Independence Property I: for $1 \leq j \leq n$, $w_j(\cdot)$ does not depend upon e_i for $i \neq j$.

This property is assumed in most analyses of the misrepresentation problem. We shall also use the following hypotheses in our analysis of Groves mechanisms:

> Independence Property II: for $1 \leq j \leq n$,
>
> $w_j(a,e) = v_j(a,e_j) + y_j(e)$;

> Independence Property III: for $1 \leq j \leq n$, if
>
> $\alpha(e_j',e_{-j}) = \alpha(e_j'',e_{-j}) \equiv a$, then
>
> $\sum_{i \neq j} w_i(a,e_j',e_{-j}) = \sum_{i \neq j} w_i(a,e_j'',e_{-j})$.

Note that II and III follow from I, but II does not imply III, and III does not imply II. Properties II and III are motivated by their use in the next section.

There is no mathematical reason to expect that any of these independence properties would hold. They may or may not be true in a particular real-world problem. Our discussion of the internal transfer problem illustrates, however, that they need not hold.

3. The Problem

As mentioned in the Introduction, we regard the process whereby the agents communicate with headquarters as a game, or mechanism. To define a mechanism, we must specify a strategy set S_i for each agent, and a mapping $g(\cdot)$ from $S \equiv \Pi_{i=1}^{n} S_i$ into $A \times R^n$. The set S_i contains all of the messages that are available to the ith agent. The mapping $g(\cdot)$ is the rule with which headquarters chooses an alternative and a set of transfers, based upon the messages it receives from the agents. We let $g \equiv (\alpha, t_1, \ldots, t_n)$, where $\alpha(\cdot)$ is an element of A and $t_j(\cdot)$ is the transfer to the jth agent. For the moment, we do not assume that $\alpha(\cdot)$ has any special properties.

Let (S,g) represent the mechanism with strategy set S and outcome $g(\cdot)$. Also, let S_{-j} denote the Cartesian product of the n-1 sets $S_1, \ldots, S_{j-1}, S_{j+1}, \ldots, S_n$, let s_{-j} denote any element of S_{-j}, and let (s_j^*, s_{-j}) denote the element of S that is obtained by starting at $(s_1, \ldots, s_j, \ldots, s_n)$ and changing the jth component from s_j to s_j^*.

A <u>strategy</u> for agent j is a mapping ϕ_j: $E_j \to S_j$ that selects his messages on the basis of his observations. Let τ denote the probability measure on E that is defined using μ and mapping $(X_1, \ldots, X_n)$: $\Omega \to E$. For $i \neq j$, let ϕ_i: $E_i \to S_i$ be any strategy for the ith agent, and let ϕ_{-j}: $E_{-j} \to S_{-j}$ denote the mapping $(\phi_1, \ldots, \phi_{j-1}, \phi_{j+1}, \ldots, \phi_n)$. Given that the jth agent observes $\tilde{e}_j$ and believes that the other agents will use the strategies given by $\phi_{-j}(\cdot)$, he should choose his message s_j^* to maximize the conditional expected value of the sum of his direct payoff and his transfer. This is formally given by the function u_j:

for ϕ_{-j}: $E_{-j} \to S_{-j}$, $\tilde{e}_j \in E_j$, and $s_j \in S_j$,

$$u_j(\tilde{e}_j, s_j, \phi_{-j}) \equiv E_\tau\big[w_j\big(\alpha(s_j, \phi_{-j}(e_{-j})), e_j, e_{-j}\big) + t_j\big(s_j, \phi_{-j}(e_{-j})\big) \,|\, e_j = \tilde{e}_j\big].$$

The goal of this paper is to determine when <u>dominant</u> mechanisms exist. A mechanism is dominant if: (i) each agent i has a dominant strategy γ_i: $E_i \to S_i$; (ii) when the agents use these strategies, headquarters chooses an alternative that is efficient relative to what the agents have observed. As long as we are interested only in the existence of dominant mechanisms, the revelation principle allows us to restrict our attention to revelation mechanisms in which truth-telling is a dominant strategy for each agent. A <u>revelation mechanism</u> is a mechanism (S,g) in which each S_i equals E_i. The revelation principle states that if there exists a dominant mechanism, then there also exists a dominant revelation mechanism (E,h) that satisfies the following conditions:

i) truth-telling is a dominant strategy for each agent:

$$u_j(e_j, e_j, \phi_{-j}) \geq u_j(e_j, e_j^*, \phi_{-j}) \tag{2}$$

for all $1 \leq j \leq n$, e_j, $e_j^* \in E_j$, and $\phi_{-j} \equiv (\phi_1, \ldots, \phi_{j-1}, \phi_{j+1}, \ldots, \phi_n)$, where ϕ_i: $E_i \to E_i$;

ii) for any $e \in E$,

$$\alpha(e) \in \underset{a \in A}{\operatorname{argmax}} \sum_{i=1}^{n} w_i(a,e). \tag{3}$$

Further discussion of this revelation principle can be found in Dasgupta et.al. [2, p. 194].

Note that our real task in defining $g(\cdot)$ is to devise transfers so that (2) is satisfied. The choice of an $\alpha(\cdot)$ that satisfies (3) is immediate, as long as there exists at least one alternative in A that maximizes $\sum_{i=1}^{n} w_i(a,e)$. We assume throughout this paper that such an alternative exists for every e in E.

4. The Scope of Groves Mechanisms

In Theorem 1, we prove that a Groves mechanism satisfies (2)-(3) if Property I holds, and only if Property III holds. A Groves mechanism may still be dominant when Property III is not satisfied, for efficiency may be implementable with non-truthful dominant strategies. Theorem 1 simply states that the Groves mechanisms do not guarantee the existence of dominant mechanisms when Property III does not hold.

Assuming that Property II holds, we then define a family of quasi-Groves mechanisms. In a corollary to Theorem 1, we prove that each of these mechanisms is dominant. Theorem 1 and its corollary together show that the Groves mechanisms are not identical to the family of dominant mechanisms when informational externalities may exist.

As a technical result, the corollary also shows that neither Property I not Property III is necessary for the existence of dominant mechanisms. In the following section, we shall show that Property II is also unnecessary for the existence of dominant mechanisms.

Definition: A Groves mechanism is any revelation mechanism (E,g) that has the following properties:

i) $g(\cdot)$ satisfies (3);

ii) for each value of j, the transfer $t_j(\cdot)$ to the jth agent has the form

$$t_j(e) = \sum_{i \neq j} w_i\big(\alpha(e),e\big) + A_j(e_{-j}),$$

where $A_j(\cdot)$ is any function on E_{-j}.

Before we state Theorem 1, we first prove the following lemma. Given the reports of the other agents, agent j may be able to influence both the headquarter's choice of an alternative and the transfer that he receives by varying his message. The lemma states that if we are considering a revelation mechanism in which truth-telling is a dominant strategy for the jth agent, then this agent can change the transfer that he receives only when he also causes headquarters to change its choice of an alternative. The transfer to the jth agent can therefore be written as a function of the reports of the other agents and the alternative that is chosen. The lemma is part of the folklore of the theory of dominant strategy implementation; a proof in a more restricted setting can be found in Green and Laffont [5, Lemma 1], for instance.

Lemma: Let (E,g) be a revelation mechanism for which truth-telling is a dominant strategy for some agent j. If $\alpha(e_j', e_{-j}^*) = \alpha(e_j'', e_{-j}^*)$ for some $e_j', e_j'' \in E_j$ and $e_{-j}^* \in E_{-j}$, then $t_j(e_j', e_{-j}^*) = t_j(e_j'', e_{-j}^*)$.

Proof of Lemma: For $i \neq j$ and $e_i \in E_i$, let $\phi_i(e_i) \equiv e_i^*$, and define $\phi_{-j}\colon E_{-j} \to E_{-j}$ by $\phi_{-j} \equiv (\phi_1, \ldots, \phi_{j-1}, \phi_{j+1}, \ldots, \phi_n)$. The proof is by contradiction. If $t_j(e_j', e_{-j}^*) < t_j(e_j'', e_{-j}^*)$, then

$$u_j(e_j', e_j', \phi_{-j}) \equiv E_\tau\left[w_j\left(\alpha(e_j', \phi_{-j}(e_{-j})), e\right) + t_j\left(e_j', \phi_{-j}(e_{-j})\right) \middle| e_j = e_j'\right]$$

$$= E_\tau\left[w_j\left(\alpha(e_j', e_{-j}^*), e\right) + t_j(e_j', e_{-j}^*) \middle| e_j = e_j'\right]$$

$$< E_\tau\left[w_j\left(\alpha(e_j'', \phi_{-j}(e_{-j})), e\right) + t_j\left(e_j'', \phi_{-j}(e_{-j})\right) \middle| e_j = e_j'\right]. \tag{4}$$

The expected value in (4) equals $u_j(e_j', e_j'', \phi_{-j})$. This contradicts (2). A similar argument can be made if $t_j(e_j'', e_{-j}^*) < t_j(e_j', e_{-j}^*)$. Q.E.D.

Theorem 1: Let (E,g) be a Groves mechanism. If Independence Property I holds, then (E,g) satisfies (2) (i.e., truth-telling is a dominant strategy for each agent). Moreover, If (E,g) satisfies (2), then Independence Property III must be satisfied.

Proof: We first assume that I holds. Let $\tilde{e}_j$ denote the observation of the jth agent, while $\phi_{-j}\colon E_{-j} \to E_{-j}$ represents the strategies of the other agents. Assuming that agent j reports e_j^*, we first write out the value of $u_j(\tilde{e}_j, e_j^*, \phi_{-j})$:

$$u_j(\tilde{e}_j, e_j^*, \phi_{-j}) \equiv E_\tau\Big[w_j\big(\alpha(e_j^*, \phi_{-j}(e_{-j})), e\big) + \sum_{i \neq j} w_i\big(\alpha(e_j^*, \phi_{-j}(e_{-j})), e_j^*, \phi_{-j}(e_{-j})\big)$$

$$+ A_j(e_{-j}) \,|\, e_j = \tilde{e}_j\Big].$$

Applying I, this reduces to

$$u_j(\tilde{e}_j, e_j^*, \phi_{-j}) = E_\tau\Big[\sum_{i=1}^{n} w_i\big(\alpha(e_j^*, \phi_{-j}(e_{-j})), e_j, \phi_{-j}(e_{-j})\big) \,|\, e_j = \tilde{e}_j\Big] \qquad (5)$$

$$+ E_\tau\big[A_j(\phi_{-j}(e_{-j})) \,|\, e_j = \tilde{e}_j\big].$$

Agent j's choice of e_j^* affects only the first term in the sum in (5). By (3), setting e_j^* equal to $\tilde{e}_j$ maximizes $\sum_{i=1}^{n} w_i\big(\alpha(e_j^*, \phi_{-j}(e_{-j})), \tilde{e}_j, \phi_{-j}(e_{-j})\big)$ for any value of e_{-j}; hence, the sum in (5) is maximized when $e_j^* = \tilde{e}_j$. This verifies (2).

Assuming that (2) holds for a Groves mechanism, the lemma states that Property III must also be satisfied. Q.E.D.

Corollary: Suppose that Independence Property II holds. Let (E,g) be any revelation mechanism where $\alpha(\cdot)$ satisfies (3) and each $t_j(\cdot)$ has the following form:

$t_j(e) \equiv \sum_{i \neq j} v_i\big(\alpha(e), e_i\big) + A_j(e_{-j})$, where
$A_j(\cdot)$ is any real-valued function on E_{-j}.

Then under (E,g), truth-telling is a dominant strategy for each agent.

This corollary can be proven by applying the techniques that were used to prove Theorem 1.

5. The Generic Nonexistence of Dominant Mechanisms

Our goal in this section is to develop a necessary condition for the existence of dominant mechanisms. We begin with the following conjecture:

a dominant mechanism can exist only if Independence Property II holds. (6)

This conjecture is false in the class of models for which the set of alternatives A and each of the sets E_i is finite. It is easy to devise examples in this case that refute (6). One begins by constructing an example of our model such that: (i) Property II holds; (ii) there exists a mechanism (E,g) such that (3) holds and the inequality in (2) is strictly satisfied (i.e., truth-telling is the unique dominant strategy for each agent). The functions $w_1(\cdot),\ldots,w_n(\cdot)$ can then be perturbed so that they no longer satisfy Property II. If the perturbations are sufficiently small, (2)-(3) will be satisfied by the same (E,g) together with the new, perturbed version of the example.

Using this procedure, it is possible to devise counterexamples to (6) that involve sets E_1, $E_2,\ldots,E_n$ and A of arbitrary size. This is true as long as we do not require the mappings in the model to have any other properties besides measurability.

For this reason, we now both weaken the conjecture and restrict ourselves to a special case. We assume there are only two agents, and that the set of alternatives contains only two elements - $A \equiv \{a_0, a_1\}$. Each E_i is a closed interval on the real line; for convenience, we assume that $E_i = [0,1]$ for

$i = 1,2$. We assume that $w_1(\cdot)$ and $w_2(\cdot)$ are C^1 functions of the environment. Finally, we assume that the probability measure τ is absolutely continuous with respect to the Lebesgue measure on E. It is possible to generalize the following results, but our ideas are amply illustrated by this simple case.

The following notation is needed:

$$\bar{w}_j(e) \equiv w_j(a_1,e) - w_j(a_0,e);$$

$$\bar{w} \equiv \bar{w}_1 + \bar{w}_2;$$

$\sigma \equiv \tau'$, the density of the probability measure τ.

Headquarters should choose a_0 whenever $\bar{w} < 0$, and a_1 whenever $\bar{w} > 0$. The set of points where $\bar{w}(\cdot)$ changes sign is therefore especially important, for at these points headquarters must switch from one alternative to the other. Let $Z(\bar{w})$ denote this subset of $\bar{w}^{-1}(0)$:

$$Z(\bar{w}) = C\ell\{e|\bar{w}(e) > 0\} \cap C\ell\{e|\bar{w}(e) < 0\}.$$

Theorem 2 describes conditions on $\bar{w}_1(\cdot)$ and $\bar{w}_2(\cdot)$ under which dominant mechanisms cannot exist. Property D is one of the hypotheses of this theorem. Property D is satisfied at an environment $\tilde{e}$ in $Z(\bar{w})$ if the following regularity conditions hold at $\tilde{e}$ for either $j = 1$ or $j = 2$:

$$\text{i)}\quad \partial\bar{w}_j/\partial e_{-j}(\tilde{e}) \neq 0; \tag{7}$$

ii) $\partial\bar{w}/\partial e_j(\tilde{e}) \neq 0.$ (8)

iii) $\partial\bar{w}/\partial e_{-j}(\tilde{e}) \neq 0.$ (9)

Condition (7) implies that $w_j(\cdot)$ does not satisfy Independence Property II. Conditions (8) and (9) imply that for i = 1, 2, when the ith agent reports $\tilde{e}_i$, the (-i)th agent could implement either a_0 or a_1 by perturbing his message away from $\tilde{e}_{-i}$. These conditions are illustrated in Figure 1. In this specialized setting, it is clear that Property II is too strong to serve as a necessary condition for the existence of dominant mechanisms, for it can be violated far away from $Z(\bar{w})$ without affecting the incentives of the given mechanism. Property D expresses this idea, for it states that Property II is violated somewhere in E in an essential way. Theorem 2 is therefore a weaker result than (6).

Theorem 2: Assume that (i) $\bar{w}_1(\cdot)$ and $\bar{w}_2(\cdot)$ are C^1, (ii) Property D holds at some environment $\tilde{e}$ in $Z(\bar{w}) \cap \text{Int } E$, and (iii) $\sigma(\cdot)$ is positive in some neighborhood of $\tilde{e}$. Then no dominant mechanisms exist.

Proof: Without loss of generality, we make the following assumptions: (i) Property D holds at $\tilde{e}$ for j = 1; (ii) when agent 2 reports $\tilde{e}_2$, agent 1 implements a_0 by reporting a value slightly less than $\tilde{e}_1$, and he implements a_1 by reporting a value slightly more than $\tilde{e}_1$. Condition (8) implies that a C^1 function $\gamma(\cdot)$ is defined in some neighborhood $[\underline{e}_2,\bar{e}_2]$ of $\tilde{e}_2$ by the equation $\bar{w}(\gamma(e_2),e_2) = 0$. By (9), this interval can be chosen so that $\gamma'(\cdot)$ is nonzero over it.

The proof is by contradiction. Let $t_1(\cdot)$, $t_2(\cdot)$ be transfers that satisfy (2)-(3). The lemma to Theorem 1 implies that $t_1(\cdot)$ has the form

$$t_1(e_1,e_2) = \begin{cases} h(a_1,e_2) & \text{if } \bar{w}(e_1,e_2) > 0, \\ h(a_0,e_2) & \text{if } \bar{w}(e_1,e_2) < 0. \end{cases}$$

For e_1^* near $\tilde{e}_1$, let e_2^* denote $\gamma^{-1}(e_1^*)$. We consider strategies $\phi(\cdot)$ for agent 2 such that: (i) $\phi(e_2) = \underline{e}_2$ if $e_2 \leqslant \underline{e}_2$; (ii) $\phi(e_2) = \bar{e}_2$ if $e_2 \geqslant \bar{e}_2$; (iii) $\phi'(e_2) > 0$ if $\underline{e}_2 < e_2 < \bar{e}_2$. Given such a strategy $\phi(\cdot)$, agent 1's expected utility when he observes $\tilde{e}_1$ and reports e_1^* is

$$\int_0^{\underline{e}_2}[w_1(a_1,\tilde{e}_1,e_2) + h(a_1,\underline{e}_2)]\sigma(\tilde{e}_1,e_2)de_2$$

$$+ \int_{\underline{e}_2}^{\phi^{-1}(e_2^*)}[w_1(a_1,\tilde{e}_1,e_2) + h\big(a_1,\phi(e_2)\big)]\sigma(\tilde{e}_1,e_2)de_2$$

$$+ \int_{\phi^{-1}(e_2^*)}^{\bar{e}_2}[w_1(a_0,\tilde{e}_1,e_2) + h\big(a_0,\phi(e_2)\big)]\sigma(\tilde{e}_1,e_2)de_2$$

$$+ \int_{\bar{e}_2}^{1} [w_1(a_0,\tilde{e}_1,e_2) + h(a_0,\bar{e}_2)]\sigma(\tilde{e}_1,e_2)de_2.$$

Note that the first and the last terms in this expression do not vary with perturbations of e_1^*. The first order condition for utility maximization at $e_1^* = \tilde{e}_1$ reduces to

$$0 = w_1\big(a_1,\tilde{e}_1\phi^{-1}(\tilde{e}_2)\big) + h(a_1,\tilde{e}_2) - w_1\big(a_0,\tilde{e}_1,\phi^{-1}(\tilde{e}_1)\big) - h(a_0,\tilde{e}_2),$$

or equivalently,

$$\bar{w}_1\big(a_1,\tilde{e}_1,\phi^{-1}(\tilde{e}_2)\big) = h(a_0,\tilde{e}_2) - h(a_1,\tilde{e}_2).$$

The right side of this expression does not depend upon the value of $\phi^{-1}(\tilde{e}_2)$; the left side does, however, because of (7). Since $\phi^{-1}(\tilde{e}_2)$ can be any value in $(\underline{e}_2,\bar{e}_2)$, we have a contradiction. Q.E.D.

Our next theorem states that the hypotheses of Theorem 2 are satisfied by almost all $\bar{w}_1(\cdot)$, $\bar{w}_2(\cdot)$ for which $Z(\bar{w})$ is nonempty, together with almost all continuous density functions $\sigma(\cdot)$. This paper is not concerned with those $\bar{w}_1(\cdot)$, $\bar{w}_2(\cdot)$ for which $Z(\bar{w})$ is empty; in those cases, $\alpha(\cdot)$ is constant on E, and there is no reason for headquarters to try to overcome the misrepresentation problem. $\bar{C}^1(E,\mathbb{R}^2)$ denotes the subset of $C^1(E,\mathbb{R}^2)$ consisting of all $\bar{w}_1(\cdot)$, $\bar{w}_2(\cdot)$ for which $Z(\bar{w})$ is nonempty, and $D(E,\mathbb{R})$ is the set of all continuous density functions on E. Each of these sets is given the appropriate induced Whitney topology (see Hirsch [10] for further discussion).

Theorem 3: For all $(\bar{w}_1,\bar{w}_2,\sigma)$ in some open, dense subset of $\bar{C}^1(E,\mathbb{R}^2) \times D(E,\mathbb{R})$, there are no dominant mechanisms.

Proof: Given Theorem 2, this result is proven by simply applying the definitions of the Whitney C^1 and C^0 topologies. Q.E.D.

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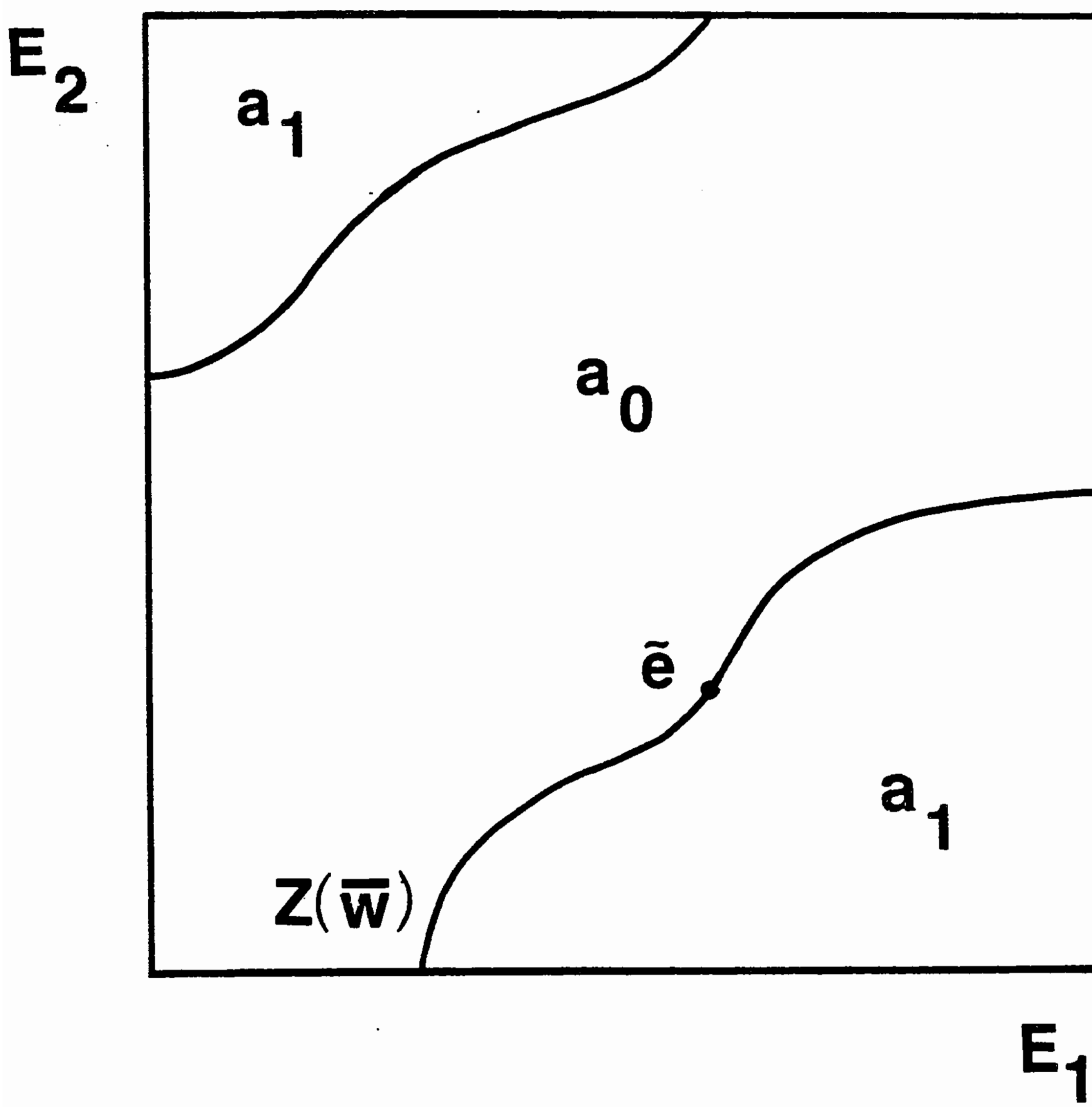
E2
a1
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ẽ
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Z(w̄)
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