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THE LIMITS OF MONOPOLIZATION THROUGH ACQUISITION

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Abstract

We address the question of whether competitive acquisition of firms by their rivals can result in a complete or partial monopolization of a homogeneous product industry. This question is modelled in terms of a four-stage noncooperative game in pure strategies. Analysis of subgame perfect Nash equilibria of this game shows that, under general weak assumptions, monopolization of an industry through acquisition is limited to industries with relatively few firms. For industries with a large number of firms, complete monopolization is impossible while partial monopolization is insignificant in scope and can be completely eliminated by prohibiting any owner from acquiring over 50 percent of the industry. Moreover, if the industry consists of four or more firms, there is always an equilibrium outcome in which the industry is not even partially monopolized and the original oligopolistic structure is retained.

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I. Introduction

A conventional view of how an industry is monopolized is through the acquisition by one firm of all its rivals. This view appears to underlie the antitrust authorities efforts to inhibit such behavior through the issuance of merger guidelines. A firm that violates them risks an attempt by the Justice Department or the Federal Trade Commission to block the merger through the Federal Courts. A tacit assumption in the conventional view is that the firms being acquired react passively, perhaps out of fear that they will be victimized by predatory acts if they fail to sell out, or because they are unaware of what their buyer is attempting to accomplish. Selten (1978) has called the credibility of predatory pricing into question while McGee (1980) has questioned its actual role in the acquisition of rivals on favorable terms. The supposition that firms are unaware of what a rival seeking to acquire them is attempting to accomplish is belied in reality by their common appeal to the antitrust laws to ward off takeover.

Our purpose is to determine the limits of monopolization through acquisition in the absence of any legal barriers to such activity but in the presence of firms fully aware of the consequences of acquiring or being acquired by rivals, not susceptible to incredible threats, and behaving strategically with respect to this activity. In order to focus attention solely on this issue, we assume that the industry is composed of n identical firms with regard to the product they sell and their costs of production, which are assumed to be linearly increasing in the quantity produced, and that entry into the industry is difficult. The functional form of the industry's inverse demand function is assumed to be arbitrary but downward sloping with a uniformly concave revenue function. The interaction among owners vis-a-vis their firms is supposed to be describable as a Cournot-Nash oligopoly. It is also assumed that all the relevant variables and strategies available to all the firms are common knowledge. Under these mild assumptions we model the strategic behavior of the firms' owners, in the formation of coalitions via acquisition of some firms by other owners, as a four-stage noncooperative game in pure strategies. In this game we allow owners to profit both from selling and buying firms and from operating those that they own. We characterize possible and impossible subgame perfect Nash equilibria of this game, and show (Theorem 5) that for large industries there are substantial limits to the extent of industry monopolization obtained via acquisitions.

The effects and desirability of horizontal mergers have been addressed by Salant, Switzer and Reynolds (1983) in the context of a Cournot oligopoly with a homogeneous product and linear demand and cost functions. They conclude that any coalition of firms, behaving as a merged firm, that consists of fewer than eighty percent of the industry's members will be disadvantageous. That is, members of the coalition would be better off abandoning it than staying in.[1] However, they assume (and this is a critical assumption in their model) that every coalition of firms behaves as if it were a single firm. The approach we adopt here allows an owner of

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[1] Deneckere and Davidson (1985) on the other hand find merger advantageous when the firms produce differentiated products and engage in price competition.

several firms to operate some of them in competition with each other. This emanates from an owner of several firms following a bi-level decision process, where at the first level he decides how many firms to operate and at the second level, knowing how many firms are operated by all other owners, decides the optimal output level of every firm he chose to operate. Hence, in deciding how many of his firms to operate, he takes into account the resulting second level production decisions of all other owners. In other words, an owner of several firms strategically chooses how many to operate in anticipation of the resulting Cournot equilibria at the actual production stage.

As already mentioned, the strategic behavior of the firms' owners is posed in terms of a four-stage noncooperative game in pure strategies. In its first stage each owner of a firm chooses an asking price at which he would sell out. We assume that a firm is owned by one and only one owner initially and that it can be sold only in its entirety. In the second stage each owner announces which firms he wants to purchase at the asking prices. If several buyers seek to acquire the same firm, the one with the lower index (they are numbered 1 through n) is assigned the purchase. In the third stage each owner decides how many of the firms he acquired will be active, i.e., operated at a positive level. He does this assuming that the managers of the active firms seek their individual maximum profits even when several of them belong to a single owner. Finally, in the last stage each manager decides the output level of his active firm. These output decisions, as those in the other stages of the game, are made under the usual Cournot-Nash assumptions. The final profit realized by each owner includes the net second stage ownership trading profits plus the last stage production profits from all the active firms he owns. We employ the subgame perfect Nash equilibrium as the solution concept for this game and therefore develop it by working backwards from the Nash equilibrium of the last stage to that of its first stage.

A few remarks regarding the intuition of our analysis and results are in order as its formal phase involves a considerable amount of algebraic computations. Let us begin with a perhaps counter-intuitive result, namely that an owner of several firms might optimally choose to operate more than one of them at a positive level. This may appear especially surprising in the presence of constant marginal costs. And in fact when one owner does purchase all his rivals he only operates one firm, as intuition suggests. However, if he does not own all the firms, then operating more than one of those he controls at a positive level, in response to competition from the others, may be optimal. For while he does compete against himself by doing so, the effect of this internal competition is diluted by the presence of active rivals. That is, by competing against himself he captures some sales from his rivals and thereby gains market share. Thus, while total industry profit declines his share of it enlarges enough to increase his total profit.

Relying on the above considerations we show in Theorem 1 that in any merged subgame perfect Nash equilibrium of the game, where by merged we mean that the number of firms operated by all owners is less than the initial n, there is one and only owner operating fewer firms than he owns. Furthermore, this owner must possess over 50 percent of the industry's firms. Hence, in order to prevent monopolization via acquisition, a prohibition against any single owner acquiring more than 50 percent of the industry's firms will suffice. In addition, we show in Theorem 3 that for numerically large industries when owners behave strategically, even this prohibition may be unnecessary.

To explain these results, note first that it is intuitively clear that no owner will sell out at a price below the profit level he can realize at the Cournot equilibrium of the original n-firm oligopoly. Second, and less obviously, under complete monopolization of the industry by one owner acquiring all his rivals, the buyer will be ready to pay the n - 1 sellers altogether the difference between the monopoly and the single firm profit in an n-firm oligopoly. Consequently, in a complete monopoly equilibrium the buyer ends up paying the above difference and netting a single firm's profit in an n-firm oligopoly (Proposition 6). The implication is that a seller is usually strictly better off from the sale of his firm while a buyer is never better off. Naturally, the buyer would be better off becoming a seller. However, he cannot achieve this by backing off from his request to buy all the other firms, since at the given asking prices, if no other owner seeks to buy firms, it is his best response to become the buyer. It turns out, however, that if the number of firms in the industry is sufficiently large, then by lowering his own asking price he can persuade another owner to purchase his firm in a way that benefits both. Hence complete monopolization (Corollary 3) and, using similar arguments, any significant partial monopolization (Theorem 3) become impossible equilibria as the number of firms in the industry becomes sufficiently large. Instead two possible equilibrium market structures can emerge. The first is an unmerged equilibrium in which each owner asks a sufficiently high price for his firm, and no firm is purchased by another, resulting in retention of the original oligopoly. This equilibrium is shown in Theorem 2 to hold whenever there are four or more firms in the industry. In the second possible equilibrium, one owner possesses slightly more than one-half, say K, of the industry's firms (including his own) and operates fewer, say $r < K$. However, this buyer's net profit only equals his initial single firm profit in an n-firm Cournot equilibrium, while each of the K - 1 sellers realizes more, and those that have not sold out realize a higher profit still. The latter phenomenon occurs because the nonsellers benefit from the reduction in the number of actively competing firms. The K - 1 sellers realize altogether the difference between the Cournot equilibrium profit of the r firms being operated by the buyer in the merged industry and the single firm profit in an n-firm oligopoly. However, because these profits are to be shared among all the K - 1 sellers and this number equals or exceeds the number r of firms operated by their buyer, it follows that a seller cannot make more than a nonseller in equilibrium. Moreover, the number K, in equilibrium, is such that it is unprofitable for any one of the K - 1 sellers to become a nonseller. As it turns out, for large industries in such an equilibrium, the fraction of the industry held by the buying owner cannot substantially exceed one-half (Corollary 4) and the number of firms he decides not to operate is relatively small (Corollary 5). Hence, the extent of industry monopolization approaches zero as the number of its firms increases (Theorem 5).

In a recent paper, Gal-Or (1987) also considered the question addressed here, employing different assumptions. In her two-stage game, owners are allowed to purchase and sell fractions of firms, the initial owner of a firm controls its output even when he becomes a minority shareholder, owners do not profit from buying and selling firms, and only interior symmetric solutions are considered at the Cournot production stage. Thus, collusive equilibria in which each owner purchases equal shares in every firm, making the industry a "monopoly in disguise" (all firms are operated but the monopoly price can prevail) are possible. However, Gal-Or considered only symmetric ownership level outcomes. Thus, the possibility, discussed in this paper, of merger by acquisition is not allowed in her model. Gal-Or established that a symmetric equilibrium in which each owner possesses 1/n of every firm, while industry output and prices are those of a monopoly, is possible only if there are two firms in the industry.

This paper is organized as follows. In section II we illustrate our main results by means of simple linear demand examples and industries consisting of three and four initial owners and firms. The general model is presented in section III and its analysis in section IV. The possibility and impossibility of some equilibria is established in section V. Section VI is devoted to a short summary. Proofs of results appear in an appendix.

II. Examples

To crystalize our results we offer the following example. Suppose the inverse demand function is $P = 20 - Q$, where Q refers to total industry production, and that the average cost of production is $C = 0$. It is not difficult to establish that a monopolist's profit in this case would be 100, a duopolist's profit would be 44.44, that each firm in a three-firm oligopoly would realize a profit of 25 at the Cournot equilibrium, and 16 if there were four firms. Let us begin with the supposition that the industry initially consists of three firms and illustrate the two equilibria that can obtain in the four-stage game, i.e., one in which the industry remains a three firm oligopoly and the other in which the industry is monopolized by a single owner.

The fourth stage of the game is characterized by the per firm Cournot equilibrium payoffs indicated above when there are one, two, or three active firms. The third stage of the game is analyzed by observing that if all three firms were owned by a single owner he would operate only one and realize a payoff of 100; if only two firms were owned by a single owner he would operate both of them as his payoff would be 50, which exceeds 44.44, the payoff from operating one firm only. Last, it is obvious that each individually owned firm will be operated.

Given the results of the third and fourth stages, each owner's decision regarding how many firms to purchase in the second stage depends on each owner's asking price for his firm in the first stage. It is immediately apparent that no owner will sell his firm for less than 25, the payoff he can realize in the original three firm oligopoly, and therefore, together with the results of stages three and four, regardless of whether each firm remains individually owned or two of them are owned by a single owner, the industry will remain an oligopoly with three active firms. Thus, we focus on the two equilibria in which no owner purchases any other firm and the one in which one owner purchases the other two firms.

We begin by noting that no two firms will be purchased by the third owner, if the combined asking price for their two firms exceeds 75, as this would yield him a net payoff of less than 25. With this observation we can construct an equilibrium in which each owner purchases no other firm. Suppose that each owner's asking price for his firm is 50. This might occur if each owner seeks to capture the full gain in profits when another owner is asking the lowest possible price of 25 and the industry is transformed into a monopoly from a three firm oligopoly. It is obvious that at these asking prices no owner has an incentive to purchase any other firm. However, to show that this is a subgame perfect Nash equilibrium we must establish that no owner has an incentive to deviate from his asking price, given the asking prices of the two others. An owner would have an incentive to change his asking price, if he could profit from being purchased by one of the others. Let us look at the situation from the perspective of the owner of the first firm. He certainly has no incentive to raise his asking price above 50 as he will not be purchased by anyone at the higher price, either. Might he have an incentive to lower his asking price so as to be purchased by one of the others? The answer is again no if we note that a potential buyer, say the owner of the third firm, will not ask to purchase the first and second (at price 50) firms together unless the sum of their asking prices is below 75, that is, unless the first owner's asking price is below 25. Moreover, if the third owner contemplates buying the first firm only, then he will have to operate both of his firms, making altogether 50 at the production stage, as compared to 25 he could realize if he did not make the purchase. Hence the third owner refrains from buying firm 1 alone or firms 1 and 2 together unless the first owner's asking price is below 25. But if the first owner asks less than 25, he will be worse off than by maintaining the asking price of 50. Thus, an asking price of 50 or more by each owner of a firm is a subgame perfect Nash equilibrium in which the original three firm oligopoly is maintained.

To demonstrate a subgame perfect Nash equilibrium in which one owner possesses all three firms and the industry is completely monopolized, we posit asking prices of V_1, 37.5, 37.5, for the first, second, and third firms, respectively,[2] where $V_1 \geq 25$. We show that these prices result in an equilibrium in which the owner of the first firm purchases the other two and realizes a payoff of 25. To demonstrate this we must show that at these prices it is a best response for the first owner to purchase the other two and that no owner has an incentive to deviate from his asking price. Certainly, at these prices the first owner is not better off by purchasing either none or only one of the other two firms. Moreover, given that the first owner seeks to purchase both other firms, each of their owners, say the third, can only seek to purchase the first owner's firm. But then, the first owner will end up owning only two firms and hence operating both, implying a profit of $37.5 + 25 - V_1$ for the third owner, as compared to 37.5 if he did not make the purchase. It follows that the third owner cannot become better off by purchasing the first firm at the asking price of V_1.

To show that no owner has an incentive to deviate from his asking price, let us look at the owner of the first firm. He may wish to escape from this situation for he profits the least among the three owners. However, he will be bought by one of the others only if he lowers his asking price below 25. But this is disadvantageous to him. Certainly, if he attempts to raise his asking price, he will still not be bought. Thus, the owner of the first firm has no incentive to change his asking price from V_1, as his payoff will not be increased. Moreover, neither of the other owners

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[2]The fact that the prices constitute an equilibrium can be demonstrated by showing that V_1, $37.5 - \varepsilon$, $37.5 - \varepsilon$, is an equilibrium as $\varepsilon \rightarrow 0$. In the limit, the single buyer of the entire industry is in fact indifferent between purchasing the two other firms and purchasing neither firm.

has an incentive to change his asking price. For example, the owner of the third firm has no incentive to lower his asking price because he will then reduce his payoff. If he raises his asking price he will not be bought, nor will the second firm, and his payoff will decline to 25. He might, however, raise his asking price and contemplate buying the first two firms. But if he were to do this, his payoff would fall below 37.5 to the extent that V_1 is greater than 25. Thus, neither of the owners of the second or third firms have an incentive to deviate from their asking prices. And the posited asking prices of V_1, 37.5, 37.5, constitute a subgame perfect Nash equilibrium at which the industry is completely monopolized.

Note, however, that the final owner of all three firms realizes the same payoff as he would in the original equilibrium of the three firm oligopoly while the two owners who sold out realize the entire gain from monopolization. Note also that this equilibrium is not unique in the sense that it would obtain for other asking prices by the owners of the second and third firms that are 25 or above but add up to 75. By employing the mode of analysis for establishing the above two equilibria, it is not difficult to show that asking prices of (100/3, 100/3, 100/3), an even division of the monopoly profit, cannot be a Nash equilibrium.

Let us now turn to the case of a four firm oligopoly and indicate the type of Nash equilibria that can exist. In particular, we will demonstrate that there is an equilibrium in which the original oligopoly structure is maintained, and that there cannot be a monopoly equilibrium in this case. Indeed, there exists another equilibrium in which one owner possesses three firms and actively operates two. In the latter equilibrium, therefore, the industry is reduced into a three firm oligopoly.

We begin by recalling that in a four firm oligopoly each firm realizes a profit of 16 at the Cournot equilibrium. Hence, an owner of two firms will actively operate both of them as 32 exceeds 25, the profit he would realize by operating only one firm. In addition, an owner of three firms will operate only two actively as he will then realize a payoff of 50, which exceeds 48 and 44.44, the payoffs from operating all three and one only, respectively. It follows that an owner of one firm will purchase two additional firms if and only if their combined asking price does not exceed 34 or more. Finally, we note also that when one owner possesses three firms the remaining firm's owner will realize a payoff of 25.

To demonstrate the possibility of an equilibrium in which the original four firm oligopoly structure is maintained, we posit asking prices for each firm of 34 or more. At these prices no owner has an incentive to purchase any other firm, or to lower his asking price so as to be purchased, alone or together with other firms. For the only price at which he would be purchased would be below 16.

Let us now demonstrate the impossibility of a complete monopoly equilibrium in which one owner, say the first, purchases all the other firms and operates only one. Note that for such an equilibrium to exist, the first owner will have to pay every other owner at least 25,[3] to every pair of owners at least 34,[4] and to the three of them altogether 84, the difference between the monopoly profit of 100 and the single firm profit of

[3]Otherwise he is better off purchasing that firm whose asking price is less than 25 and no other firms.

[4]Otherwise he is better off purchasing the two firms whose combined asking price is less than 34 and no other firm.

16 in a four-firm oligopoly. It follows that the first owner will end up making 16. Furthermore, in order to support this outcome his asking price V_1 has to be sufficiently high, for otherwise his firm will be purchased. To show why this cannot be an equilibrium we show that the first owner, by deviating and lowering his asking price close to but above 16, can guarantee that in any resulting equilibria he will be bought and be better off.

To further simplify the presentation, while still capturing the main idea of how the assumed buyer can escape his predicament, suppose that the asking prices of each of the three other owners are equal to 28. If the first owner asks V_1 for his firm, where V_1 is greater than 16, then given that he seeks to purchase the other firms, it is a best response for one of the other owners, say the second one, to purchase the first firm at some V_1. The reason for this is that the second owner knows that when he makes this purchase, the first owner will retain only three firms and hence operate only two of them. Thus, the second owner's profit[5] from the firm he just bought will be 25 while paying V_1 for it. So if the first owner sets his asking price V_1 at $16 < V_1 < 25$, then an equilibrium in which he owns all the firms is impossible since someone else will purchase his firm. In fact, at the new asking prices the only possible equilibria are those where either the second, third, or fourth owner purchases all the other firms. But whoever the buyer is, he will have an incentive to lower the asking price for his firm, and hence there cannot be a complete monopoly equilibrium in this case.

Note that what enables the first owner to escape his predicament is the fact that when his firm is bought, he will not operate all his remaining

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[5]This is in fact his extra profit. His net profit will be $28 + 25 - V_1$.

firms and hence, guarantee his purchaser an extra profit that exceeds the 16 that he (the first owner) makes now. Thus, by, say, splitting the difference both he and his potential buyer can become better off. Note also that in a complete monopoly equilibrium in a three firm industry, this cannot happen since if the first owner, making 25 in this equilibrium, sells his firm he operates his two remaining firms. Hence, he cannot guarantee a potential buyer an extra profit higher than 25. In general, it is the monopolist buyer's anticipation that if his firm is bought he will not operate all his remaining firms, thereby guaranteeing his firm's purchaser some extra profit at an asking price higher than his profit as a monopolist, that makes complete or substantial partial monopolization an impossible equilibrium in large industries. The only possible merger equilibria, that is, ones in which fewer than the original n firms are operational, are those wherein the buyer operates all his remaining firms if he sells his own firm. Such equilibria are possible only if the buyer owns slightly over one-half of the industry's original firms and operates almost all of them. This results in an insignificant degree of monopolization whenever the number of firms in the industry is large (see Theorems 4-5). As an example, it can be shown that asking prices of (V_1,V_2,V_3,V_4) satisfying $V_1 \geq 41$, $V_2 \geq 16$, $V_3 \geq 16$, $V_2 + V_3 = 34$, and $V_4 \geq 50$ result in an equilibrium in which the first owner purchases the second and third firms. In this equilibrium only three out of the initial four firms will be active.

III. The Model

We now turn to the formal description of our four-stage game. We posit an industry consisting of n identical firms producing a single good whose overall quantity is denoted by Q, and facing an inverse demand function $P(Q)$. Every firm has the same constant marginal cost technology and there is no fixed cost, i.e., if firm i produces a quantity q_i, then $C(q_i) = Cq_i$. We assume that the following properties hold:

(I) P is twice continuously differentiable, $P(0)$ and $P'(0)$ are finite, and $P'(Q) < 0$ for all $Q \geq 0$.

(II) $P(0) > C$ and for some $\hat{Q} > 0$, $P(\hat{Q}) < C$ holds.

(III) The industry revenue function $QP(Q)$ possesses a negative second derivative which is bounded from below, i.e., there exists a real number $\beta > 0$ such that $(QP(Q))'' \geq -\beta$ for all $Q \geq 0$. Note that this assumption implies strict concavity of the industry revenue function.

We now proceed to describe the "acquisition game," G. Initially, each firm is owned and controlled by a single owner. These owners are the players of the game. In the acquisition game, each owner can purchase other firms or sell his firm. Naturally, if a firm is sold it becomes controlled by its buyer. Let $N = \{1,2,\ldots,n\}$.

<u>Stage 1</u>: Each owner $i \in N$ announces a selling (or asking) price V_i for (the whole of) his firm. Let $V = (V_1,V_2,\ldots,V_n)$.

<u>Stage 2</u>: Given V, owner $j \in N$ decides which firms $I_j(V)$ he wants to purchase at the announced prices where $I_j(V) \subset N - \{j\}$. Let

$$m(i) = \begin{cases} \min \{j \in N: i \in I_j(V)\} & \text{if } i \in \cap_{j \in N} I_j(V), \\ i, & \text{otherwise.} \end{cases}$$

The ownership index of owner j in firm i is denoted by ε_{ij} and is determined as follows. Let

$$\varepsilon_{ij} = \begin{cases} 1, \text{ if } m(i) = j, \\ 0, \text{ otherwise.} \end{cases}$$

It follows that owner j gets to own firm i if and only if he either already possesses it and no other owner asks to purchase it or he asks to purchase it and no owner $\ell \neq i$, $\ell = 1,\ldots,j - 1$ seeks to purchase i.

Denote $I = (I_1, I_2, \ldots, I_n) = I(V)$ and let ε be the $n \times n$ matrix whose (i,j) entry is ε_{ij}. Note that $\varepsilon = \varepsilon(I,V)$. For $j \in N$ let

$$K_j = \Sigma_{i \in N} \varepsilon_{ij}$$

denote the number of firms owned (and controlled) by j. Let $K = (K_1, K_2, \ldots, K_n) = K(I,V)$. Since all firms are identical only knowledge of K is required in the next stage.

<u>Stage 3</u>: Given K, owner $j \in N$ decides how many of his firms, $0 \leq r_j \leq K_j$, will be active (competing with each other). Let

$$r = (r_1, r_2, \ldots, r_n) = r(I,V)$$

and denote by

$$m = \Sigma_{i \in N} r_i$$

the number of active firms, and by $M \subseteq N$ the set of active firms.

<u>Stage 4</u>: Given r, owner $j \in N$ chooses the production level of each of his r_j active firms, i.e., by the manager of each active firm independently

seeking to maximize its profits. Letting $q_i = 0 \ \forall \ i \notin M$, we denote by

$$q = (q_1, q_2, \ldots, q_n) = q(r, I, V),$$

the vector of quantities produced by all firms in N as a result of this decision and let $Q = \Sigma_{i \in N} q_i$.

The payoff to each player (initial owner) is the sum of the stage 4 profits of all the firms he controls plus the net trade cash flow in stage 2.

We are concerned with characterizing properties of subgame perfect equilibria (hereafter SPNE) of the game G.

IV. Analysis of the Model

Analysis of Stage 4: For $i \in M$, q_i is determined via the manager seeking to

$$\text{(1)} \qquad \max_{q_i \geq 0} \pi(q, m) = q_i [P(\Sigma_{i \in M} q_i) - C].$$

The first order necessary conditions for this problem are

$$\text{(2.1)} \qquad \partial\pi/\partial q_i = P(Q) - C + q_i P'(Q) = 0, \text{ if } i \in M, \ q_i > 0,$$

$$\text{(2.2)} \qquad \partial\pi/\partial q_i = P(Q) - C + q_i P'(Q) \leq 0, \text{ if } i \in M, \ q_i = 0.$$

First note that whenever (2) holds then $P(Q) > C$ is satisfied, since otherwise, if $P(Q) \leq C$ the assumption that $P' < 0$ will imply through (2) that $Q = 0$ and hence $P(0) \leq C$ contradicting Assumption II. This however implies that $q_i = 0$ for some $i \in M$ cannot hold, since (2.2) will imply then $P(Q) \leq C$. It follows that at a stage 4 equilibrium of G, $q_i > 0 \ \forall \ i \in M$. But then (2.1) implies that $q_i = q_j = q \ \forall \ i,j \in M$. Or,

$$(3) \qquad q = -(P - C)/P',$$

where for convenience we use the notation $P = P(Q)$, $P' = P'(Q)$. Consequently, $Q = mq = -m(P - C)/P'$, or

$$(4) \qquad m[P(Q) - C] + QP'(Q) = 0.$$

Lemma 1: For every $1 \leq m \leq n$ the game G has a unique stage 4 equilibrium. In this equilibrium each active firm produces a positive quantity given by (3).

For proofs of all our results, see the Appendix.

Let $Q(m)$ denote the unique, by Lemma 1, solution to (4). Equation (4) describes the relationship between the total output, $Q(m)$, and the number of active firms, m, operated by all owners collectively.

If we now substitute $q_i = q = Q/m$ in (1) we obtain the individual firm's stage 4 profit:

$$(5) \qquad \pi(m) = (1/m)Q[P(Q) - C],$$

where $Q = Q(m)$ solves (4). An interesting property of $Q(m)$ is obtained by differentiating (4) with respect to m,

$$(6) \qquad P - C + [mP' + P' + QP'']Q'(m) = 0$$

or,

$$(7) \qquad Q'(m) = - \frac{P - C}{(m + 1)P' + QP''},$$

where the denominator of (7) cannot vanish since this would imply by (6) that $P(Q) = C$. However, (4) would then imply $Q = 0$. Hence, $P(0) = C$ which is impossible by Assumption II. Since

$$(8) \qquad [QP(Q)]'' = 2P' + QP''$$

we obtain from (7)

$$(9) \qquad Q'(m) = - \frac{P - C}{(m - 1)P' + (QP)''}.$$

Assumptions (I) and (III) and (9) immediately imply that aggregate industry production increases with the number of active firms:

<u>Proposition 1</u>: For $m \geq 1$, $Q'(m) > 0$ holds.

Our next proposition shows that aggregate industry profits decrease as the total number of active firms operated increases. Let

$$\Pi(m) = m\pi(m) \tag{10}$$

denote aggregate industry profit. By (5)

$$\Pi(m) = Q[P(Q) - C]. \tag{11}$$

Thus

$$\Pi'(m) = [P(Q) - C + QP'(Q)]Q' = \tag{12}$$

$$[P(Q) - C + qP'(Q)]Q' + (m - 1)qP'(Q)Q' = (m - 1)qP'(Q)Q',$$

where the second equality in (12) follows from $Q = mq$ and the last one follows (2.1). By Lemma 1, $q > 0$ and since $P' < 0$ holds, it follows that $\Pi'(m) < 0$ for $m > 1$. Thus:

<u>Proposition 2</u>: Aggregate profits are a declining function of the total number of active firms.

It follows as an immediate corollary to Proposition 2 that an individual firm's profit increases as the number of active firms declines.

<u>Corollary 1</u>: $\pi(m)$ is a decreasing function of m.

<u>Analysis of Stage 3</u>: In this stage, owner $j \in N$ solves

$$\max_{0 \le r_j \le K_j} r_j \pi(\Sigma_{i \in N} r_i), \tag{13}$$

where π is given by (5). Note that the above objective function depends on r_j and on the sum of r_i, $i \neq j$. Hence we let $t_j = \Sigma_{i \neq j} r_i$ be the number of firms operated by all other owners except j and denote by

$$(14) \qquad T(r,t) = r\pi(r + t) = [r/(r + t)]Q(r + t)(P[Q(r + t)] - C).$$

Thus, problem (13) becomes

$$(15) \qquad \max_{0 \leq r_j \leq K_j} T(r_j, t_j).$$

The next two lemmas present the intuitive properties of the stage 3 profit function T.

<u>Lemma 2</u>: An owner possessing at least one firm ($K_j \geq 1$) will operate at least one ($r_j^* \geq 1$).

<u>Lemma 3</u>: An owner possessing $K_j = n$ firms will operate only one.

Let us now derive an expression for the derivative of T with respect to r, $T'(r;t)$. From (14) and by definition of $m = r + t$,

$$(16) \qquad T(r,t) = r\pi(m)$$

and therefore

(17) $$T'(r,t) = \pi(m) + r\pi'(m).$$

Differentiating (10) and substituting into (17) we obtain

(18) $$T'(r,t) = \pi(m) + (r/m)[\Pi'(m) - \pi(m)] = (\pi/m)[t + r\Pi'/\pi].$$

Substituting (5) and (12) into (18) we obtain

$$T'(r,t) = (\pi/m)[t + r(m - 1)P'Q'/(P - C)],$$

and by substituting (9) and then rearranging

(19) $$T'(r;t) = \frac{\pi}{m}\left[\frac{t(QP)'' + (m - 1)(m - 2r)P'}{(QP)'' + (m - 1)P'}\right].$$

It follows immediately that for $r \leq m/2$, $T' > 0$. Moreover, since $m = r + t$, we have that $r \leq m/2$ is equivalent to $r \leq t$. Thus,

<u>Lemma 4</u>: If $t \geq r$, then $T'(r,t) > 0$.

The implications of Lemma 4 are straightforward. Assuming r_j to be a continuous variable, if the solution to (15) is obtained at r_j^* satisfying $r_j^* < K_j$ then $T'(r_j^*;t_j) \leq 0$ must hold, implying, in view of Lemma 4, that $t_j < r_j$ holds. In other words:

<u>Proposition 3</u>: If an owner finds it optimal to operate fewer firms than he owns (i.e., $T'(r_j,t_j) \leq 0$), then it must be that the number of firms he operates is greater than the number of firms operated by all other owners

combined.

We mentioned already that when an owner of several firms decides how many more firms to actively operate, he evaluates the trade-off between the cost of increasing competition with himself and the profit he gains by taking sales from his rivals. Naturally the cost side will dominate when he owns many firms and the others operate a few, while the profit side will dominate when he owns a few firms and the others operate many. Hence he will tend to operate fewer firms of those he owns in the former case and more of them in the latter. The striking implication of Proposition 3 is that if he owns fewer firms than the others operate, he will choose to operate all of them. An immediate consequence of this is that in any stage 3 equilibrium outcome, there cannot be more than one owner operating fewer firms than he owns.

<u>Proposition 4</u>: In any stage 3 subgame perfect Nash equilibrium of G (and hence any SPNE of G) there can be at most one owner for which $r_j^* < K_j^*$.

<u>Definition</u>: A SPNE to the game G is said to be <u>merged</u> if the number m of firms operated by all owners is fewer than the initial number, n. If m = n, in an SPNE, then this equilibrium will be <u>unmerged</u>. If m = 1 in a SPNE then we have a <u>complete monopoly equilibrium</u>.

In the sequel we will characterize possible SPNE's of the game G. In particular, we will identify the instances in which an unmerged equilibrium exists. In cases where there are possibilities for merged equilibria we will be interested in the <u>degree of industry monopolization</u>, namely, $(n - m + 1)/n$, and with possible regulatory rules that will prevent merged equilibria. Such a rule will be given as a corollary to our next theorem, which is a direct consequence of Proposition 4.

<u>Theorem 1</u>: If there is a merged SPNE to the game G, then in it there is only one owner, say j, for which $K_j \geq (n + 1)/2$. This owner operates fewer firms than he owns and all other firms are operated by their owners regardless of their ownership.

Hence, if one seeks a regulatory constraint that will prevent merged outcomes to the game G, then the following corollary applies.

<u>Corollary 2</u>: Under the restriction (regulatory constraint) that no owner can possess $(n + 1)/2$ or more firms, the game G cannot have a merged SPNE.

Let r(K) be an optimal solution to

$$(20) \qquad \max_{0 \leq r \leq K} T(r, n - K).$$

In view of Proposition 4 and Theorem 1, the following function is of interest:

$$(21) \qquad T(K) = T(r(K), n - K).$$

T(K) is the stage 4 income of an owner possessing K firms when all other owners operate all the firms they own, and when he chooses the optimal number of firms to operate, knowing that all other firms are fully operated by their owners. Next we show that this income function increases at a nondecreasing marginal rate. Note that this does not necessarily imply total profits increase since stage 2 costs go up too as additional firms are purchased.

Proposition 5: $T(K)$ satisfies

$$(22) \qquad T(K + 1) > T(K)$$

and

$$(23) \qquad T(K + 1) - T(K) \geq T(K) - T(K - 1).$$

Let $r^* = r^*(K) = (r_1^*, r_2^*, \ldots, r_n^*)$ be a stage 3 SPNE, i.e., for all $j \in N$ r_j^* solves (15) when $t_j = \Sigma_{i \neq j}\ r_i^*$. Also, let

$$R_j(K) = T(r_j^*, \Sigma_{i \neq j}\ r_i^*), \ \forall\ j \in N.$$

Analysis of Stage 2: Owner $j \in N$ solves for a given V

$$\begin{aligned} &\max_{\varepsilon_{ij}(i \neq j)} R_j(e^T\varepsilon) - \Sigma_{i \neq j}\ V_i \varepsilon_{ij} + V_j(1 - \varepsilon_{jj}) \\ &\text{s.t. } \varepsilon_{ij} = 0 \text{ if } \varepsilon_{i\ell} = 1 \text{ for some } \ell < j, \ \forall\ i \neq j \\ &\qquad \varepsilon_{jj} = 1 - \Sigma_{i \neq j}\ \varepsilon_{ji} \\ &\qquad \varepsilon_{ij} = \{0,1\}, \ \forall\ i \in N, \end{aligned}$$

where $e = (1,1,\ldots,1)^T$. Let $\varepsilon^* = \varepsilon^*(V)$ be a stage 2 SPNE and let

$$W_j(V) = R_j(e^T\varepsilon^*) - \Sigma_{i\neq j} V_i\varepsilon^*_{ij} + V_j(1 - \varepsilon^*_{jj}), \ \forall\ j \in N.$$

Analysis of Stage 1: Owner $j \in N$ solves

$$\max_{V_j} W_j(V).$$

We denote by $V^* = (V^*_1, V^*_2, \ldots, V^*_n)$ a SPNE of G.

V. Equilibria Analysis

We will now discuss some possible and impossible equilibrium outcomes of the game G. We begin with the question of the existence of unmerged equilibria to G. Indeed, if each component of V^* is sufficiently high, then no owner will be ready to buy any other firm(s). Consequently $r^*_j = K^*_j = 1$, $\forall\ j \in N$, will hold in the resulting stage 2 SPNE of the game. The question is whether this can be an overall equilibrium, namely, whether no owner can be made better off by lowering his asking price. Consider, for example, owner $j \in N$ asking V^*_j. Currently he makes

$$W_j(V^*) = T(1,\ n - 1),$$

that is, the single firm profit in an n-firm oligopoly. If he lowers his asking price to V_j and is bought, he will make

$$W_j(V^*|V_j) = V_j,$$

where $V^*|V_j$ denotes the vector V^* except that the j-th component is replaced by V_j. Hence, if there is a value V_j, satisfying

(24) $$V_j > T(1, n - 1),$$

at which it is profitable for another owner, say $i \in N$, $i \neq j$, to purchase j, then V^* cannot be an SPNE of G. It turns out, however, the whenever $n \geq 4$ this is impossible since the potential buyer i will own, when he purchases j, two firms, while the number of firms operated by all other owners will be two or more. In view of Proposition 2 the buyer will operate the two firms he owns, thus making

(25) $$W_i(V^*|V_j) = T(2, n - 2) - V_j = 2T(1, n - 1) - V_j,$$

where previously his profit was

(26) $$W_i(V^*) = T(1, n - 1).$$

From (25) and (26) it follows that i will be willing to purchase j only if

$$V_j < T(1, n - 1),$$

contradicting (24). We have proved:

<u>Theorem 2</u>: If $n \geq 4$ then there is an unmerged equilibrium to G.

We now turn to discuss the possibilities and impossibilities of existence of merged SPNE in G. If such an equilibrium exists then by Theorem 1 there is one and only one owner, say the first, owning K_1^* firms where $K_1^* \geq (n + 1)/2$ and operating $1 \leq r_1^* < K_1^*$ firms. Without loss of generality we assume that the $K_1^* - 1$ <u>seller</u> firms are $2,\ldots,K_1^*$ and the remaining firms, $K_1^* + 1,\ldots,n$, are <u>nonsellers</u>. The first owner will be called the <u>buyer</u>. V^* will be the price vector supporting an equilibrium.

<u>Proposition 6</u>: In a merged SPNE of G the buyer's profit is $T(1, n - 1) = T(1)$, that is, the profit of a single firm in an n- firm oligopoly. A nonseller makes $T(r(K_1^*), n - K_1^*)/r(K_1^*)$, and the profit V_j^* $j = 2,\ldots,K_1^*$ of a seller can neither exceed that of a nonseller nor be below that of the buyer's.

We now turn to cases where a collusive SPNE to G is impossible. Suppose V^* is a collusive SPNE of G. Proposition 6 implies that a seller is not worse off than the buyer. In fact, its proof implies that $T(K_1^*) - T(1)$ is the overall profit of the $K_1^* - 1$ sellers. Letting

(27) $$V = [T(K_1^*) - T(1)]/(K_1^* - 1),$$

then there is at least one seller profiting V or more. It is easily verified that

(28) $$V > T(1) = W_1(V^*)$$

holds and, hence, there is at least one seller whose profit exceeds the buyer's. Naturally, the first owner will prefer becoming a seller instead of being a buyer. He can achieve this by announcing that he will not buy any firms, expecting that under this announcement someone else will become the owner of K_1^* firms. However, this announcement is not a credible threat since at V* (in fact at $V^* - \varepsilon$ for any $\varepsilon \in R^n$, $\varepsilon > 0$) if all other owners announce they do not wish to purchase any firm, it is a best response for him to purchase $2,\ldots,K_1^*$. In fact, the only way he may achieve the goal of becoming a seller instead of being a buyer is by lowering his own price. Note that if V* is an SPNE for G, then whenever $V_1 \le V_\rho^*$ holds where $2 \le \rho \le K_1^*$ and $V_\rho^* = \max_{i=2,\ldots,K_1^*} \{V_i^*\}$, the asking price vector $V^*|V_1$ supports at least one stage 2 SPNE in which owner ρ buys the firms originally owned by $1,2,\ldots,\rho-1,\rho+1,\ldots,K_1^*$. Also, in view of (28) $V_\rho^* \ge V$, and hence the first owner can set his asking price V_1 to satisfy

$$(29) \qquad W_1(V^*) < V_1 \le V \le V_\rho^*.$$

Thus, at the new stage 2 equilibrium his profit will increase. However, to establish that V* cannot be an SPNE to G it is necessary to show that the first owner can set his price at a level that will make him better off in <u>any</u> resulting stage 2 equilibrium. We will establish this by showing that the first owner can set his price, V_1, to satisfy (29) and:

(i) At $V^*|V_1$, the first owner cannot own $1,2,\ldots,K_1^*$ in a stage 2 SPNE, and

(ii) At any possible stage 2 SPNE, the first owner will make at least V_1.

(Note that the existence of at least one stage 2 SPNE for $V^*|V_1$ was already established above.)

Recall first that since the presumed equilibrium is merged then $r^*(K_1^*) < K_1^*$ and by Theorem 1, $K_1^* \geq (n + 1)/2$ must hold. In this equilibrium a seller j currently makes

$$(30) \qquad W_j(V^*) = V_j^*.$$

Suppose the first owner sets his asking price at V_1, and maintains purchasing $2,\ldots,K_1^*$ in stage 2 but that owner j deviates in this stage purchasing the first owner's firm, realizing that when he does this, in the new stage 3 equilibrium the first owner will operate $r(K_1^* - 1)$ firms. Also note that owner j will operate the first owner's firm he just bought and make

$$(31) \qquad \hat{W}_j = T(r(K_1^* - 1),\ n - K_1^* + 1)/r(K_1^* - 1) + V_j^* - V_1.$$

Hence, j will have the incentive to deviate if $\hat{W}_j > W_j(V^*)$, that is, by (30) and (31), if

$$V_1 < T(r(K_1^* - 1),\ n - K_1^* + 1)/r(K_1^* - 1),$$

or equivalently if

$$(32) \qquad V_1 < T(1,\ n - K^* + r(K_1^* - 1)).$$

Consider now the first owner's profit. Currently, in view of Proposition 6, he makes

$$W_1(V*) = T(1, n - 1). \tag{33}$$

We will now be concerned with the possibility of the first owner setting an asking price V_1 satisfying both (32) and (29). This would, of course, be possible in view of (33) if we establish

$$T(1, n - 1) < T(1, n - K_1^* + r(K_1^* - 1)). \tag{34}$$

In view of Corollary 1, (34) holds if

$$n - 1 > n - K_1^* + r(K_1^* - 1)$$

or if

$$K_1^* - 1 > r(K_1^* - 1). \tag{35}$$

Hence, an equilibrium in which the first owner owns K_1^* firms and operates fewer, is impossible if he will not operate all of them whenever he owns $K_1^* - 1$ firms. Before we establish conditions under which (35) holds, we prove a lemma that will be needed in the sequel.

<u>Lemma 5</u>: There exists an $\alpha > 0$ such that the inequality (35) holds whenever

$$(36) \qquad K_1^* > \frac{n+3}{2}\left[\frac{(n-2)\alpha + (n+1)\beta/(n+3)}{(n-2)\alpha + \beta/2}\right].$$

Note that the lower bound in (36) is of the order of n/2. Consider now a value of K_1^* satisfying (36). To show that a merged SPNE at which the first owner possesses K_1^* firms cannot exist, it is left to establish that if he sets an asking price V_1 satisfying (29) and (32), he expects to make at least V_1 in any resulting stage 2 SPNE. Hence, in view of (29), he will be better off lowering his asking price. As we pointed out, there is at least one such equilibrium at which the first owner's firm is bought by another owner, say ρ, who is currently a seller, together with the rest of the current sellers' firms. It is obvious that in each such equilibrium the first owner will make V_1. The question is whether there are other equilibria in which the first owner's firm is bought but he finds it advantageous to own some of the firms originally owned by the current sellers, i.e., $2,\ldots,K_1^*$. In such a stage 2 SPNE if he makes less profit than V_1 then, given that his firm is bought, it is a better stage 2 response for him not to buy any firms and thus to make V_1. We have just proved:

<u>Theorem 3</u>: For every n there exist no merged SPNE of G at which (36) holds.

The significance of Theorem 3 is manifested by the following immediate corollaries.

<u>Corollary 3</u>: For $n > (5 + \sqrt{1 + \beta/\alpha})/2$, a complete monopoly equilibrium is impossible.

It can be shown that if the industry-wide inverse demand function is linear, then $\beta/\alpha = 2$, and hence a complete monopoly equilibrium is impossible whenever there are four or more firms in the industry. This complements the example of Section II.

<u>Corollary 4</u>: For any given proportion θ, $1/2 < \theta \leq 1$, for sufficiently large n it is impossible to have a merged SPNE in which the proportion of the industry's firms owned by a single owner is θ or above.

Theorem 3 implies that the number of firms a single owner can possess in a merged SPNE of G is bounded from above by

$$K(n) = \frac{n + 3}{2}\left[\frac{(n - 2)\alpha + (n + 1)\beta/(n + 3)}{(n - 2)\alpha + \beta/2}\right],$$

which is of the order of n/2. However, this does not tell us the extent of industry monopolization in a merged SPNE. To that end we have:

<u>Theorem 4</u>: In a merged SPNE of G the active number m of firms operated by all owners satisfies

$$m \geq (n - 3)\left[\frac{(n - 2)\alpha - \beta/(n - 3)}{(n - 2)\alpha + \beta/2}\right] = m(n).$$

Consequently, we have

Corollary 5: In a merged SPNE of G, the number of firms ℓ which are not operated satisfies

$$\ell \leq \frac{3(n - 2)\alpha + (n/2 + 1)\beta}{(n - 2)\alpha + \beta/2} = \ell(n).$$

Note that $\ell(n) = 0(3 + \beta/2\alpha)$. Hence, if $\beta >> \alpha$, a large number of firms can be left nonoperational, but this number does not asymptotically depend on n, and the proportion of the industry which is left nonoperative goes to zero as $n \to \infty$. In fact, since the degree of industry monopolization $(n - m + 1)/n$ is given also by $(\ell + 1)/n$, we have

Theorem 5: The degree of industry monopolization approaches zero as $n \to \infty$.

VI. Summary

We have shown that monopolization of an industry through acquisition by one owner of his rivals is limited. Indeed, complete monopolization of the industry is possible only if it is initially small. For large industries, only unmerged equilibria or inconsequential partial monopolization equilibria are possible. Moreover, since any partial monopolization equilibrium is characterized by one owner possessing over one-half of the industry's firms, a prohibition of this possibility would eliminate even this reduction in competition.

References

Beja, A. and I. Zang (1986), "Internal Pricing and Cost Allocation for Efficient Decentralized Control," Working Paper 906/86, The Israel Institute of Business Research, Faculty of Management, Tel Aviv University.

Deneckere, R. and C. Davidson (1985), "Incentives to Form Coalitions with Bertrand Competition," Rand Journal of Economics, 16, pp. 473-486.

Gal-Or, E. (1987), "Endogenous Joint Ventures," mimeo, Graduate School of Business, University of Pittsburgh.

McGee, J. S. (1980), "Predatory Pricing Revisited," The Journal of Law and Economics, 23, pp. 289-330.

Salant, S. W., S. Switzer and R. J. Reynolds (1983), "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium," Quarterly Journal of Economics, 93, pp. 185-199.

Selten, R. (1978), "The Chain Store Paradox," Theory and Decision, 9, pp. 127-159.

Appendix

Proof of Lemma 1: First we show that the profit function faced by each producer is strictly concave. To that end all we have to show is that the revenue function faced by this producer is strictly concave. Let

$$q_i^C = \Sigma_{\substack{j \in m \\ j \neq i}} q_j .$$

Hence i's revenue function is $g(q_i) = q_i P(q_i + q_i^C)$. Now, suppressing the index i,

$$g''(q) = 2P'(q + q^C) + qP''(q + q^C).$$

By Assumption I, $g''(0) < 0$ and $g''(q) < 0 \ \forall \ q > 0$ iff $(q + q^C)g''(q)/q < 0$, $\forall \ q > 0$. That is, iff

$$\begin{aligned} &2(q + q^C)P'(q + q^C)/q + (q + q^C)P''(q + q^C) \\ &= [2(q + q^C)/q - 2]P'(q + q^C) + 2P'(q + q^C) + (q + q^C)P''(q + q^C) \\ &= (2[q + q^C]/q)P'(q + q^C) + \partial^2[QP(Q)]/\partial Q^2 \Big|_{Q=q+q^C} < 0. \end{aligned}$$

The last inequality indeed holds by Assumptions I and III.

Next we show that for every $1 \leq m \leq n$, (4) has a unique positive solution Q, and hence $q = Q/m$ must hold in a stage 4 equilibrium to G if an equilibrium exists. Let

$$f(Q) = m(P(Q) - C) + QP'(Q).$$

Then, by Assumptions I and II, for every $1 \le m \le n$, $f(0) > 0$ and since $P' < 0$ by Assumption I and $P(\hat{Q}) < C$ for some $\hat{Q} > 0$, by Assumption II, then $f(\hat{Q}) < 0$. By continuity of f, it follows that a positive solution to (4) must exist. To establish the uniqueness of this solution we show that f is decreasing. Indeed, by Assumptions I and III:

$$f'(Q) = (m + 1)P'(Q) + QP''(Q) = (m - 1)P'(Q) + (QP(Q))'' < 0$$

for all $Q \ge 0$ and $1 \le m \le n$. It follows that the only possible stage 4 equilibrium to G is the symmetric one. But if all $j \neq i$ set $q_j = q$, then by strict concavity of i-th profit function, setting $q_i = q$ is the unique best response possible for i. Hence a stage 4 equilibrium does exist. [ ]

<u>Proof of Corollary 1</u>: From (10),

$$\text{(A.1)} \qquad \Pi'(m) = [m\pi(m)]' = m\pi' + \pi < 0,$$

which implies $\pi' < 0$ since $\pi \ge 0$. [ ]

<u>Proof of Lemma 2</u>: Since $P(Q(m)) > C$ must hold $\forall\ m \ge 1$ (see the paragraph following (2)) then $\pi(m)$ given by (5) is positive for $m \ge 1$, so an owner can always make positive income be selecting $r_j \ge 1$ as compared to zero income when $r_j = 0$. [ ]

Proof of Lemma 3: For any choice of $1 \leq r_j \leq n$ this owner will realize the total industry profits (since $t_j = n - K_j = 0$ holds). By Proposition 2, he will maximize his profits by letting $r_j^* = 1$. []

Proof of Proposition 4: From (19) it follows immediately that $T' \leq 0$ implies that $r > m/2$. But m is the total number of active firms and there cannot be more than one owner who operates more than one-half of them. []

Proof of Theorem 1: Proposition 4 implies that only one owner, say j, will decide on $r_j < K_j$ and hence all other firms will be fully operated. It follows that $t_j = n - K_j$. By Proposition 3, $r_j > t_j$ must hold or altogether $K_j > r_j > n - K_j$ or $K_j > n - K_j$ and since K_j is an integer

$$K_j - 1 \geq n - K_j$$

must hold, implying $K_j \geq (n + 1)/2$. []

Proof of Proposition 5: To establish (22) we should show

$$T(r(K), n - K) < T(r(K + 1), n - K - 1).$$

Now,

$$T(r(K), n - K) = T(r(K), n - K - 1 + 1)$$

using the definition (14) of T

$$= [r(K)/(r(K) + 1)]T(r(K) + 1, n - K - 1) < T(r(K) + 1, n - K - 1)$$
$$\le T(r(K + 1), n - K - 1),$$

since $r(K) \le K$ and hence $r(K) + 1 \le K + 1$. To establish (23), we first show that

$$\text{(A.2)} \qquad T(K + 1) - T(K) \ge T(K)/r(K)$$

holds. Indeed, to establish (A.2) we have to show

$$\text{(A.3)} \qquad T(K + 1) \ge ([1 + r(K)]/r(K))T(K).$$

Now the left side of (A.3) is

$$\text{(A.4)} \qquad T(K + 1) = T(r(K + 1), n - K - 1),$$

while the right side is

$$\text{(A.5)} \qquad \frac{1 + r(K)}{r(K)} T(K) = \frac{1 + r(K)}{r(K)} T(r(K), n - K) = T(r(K) + 1, n - K - 1).$$

Since

$$T(r(K + 1), n - K - 1) \ge T(r(K) + 1, n - K - 1),$$

(A.3) and hence (A.2) follows (A.4) and (A.5). In view of (A.2), to establish (23) it is sufficient to show that

$$\text{(A.6)} \qquad T(K)/r(K) \geq T(K) - T(K - 1)$$

or that

$$\text{(A.7)} \qquad T(K - 1) \geq ([r(K) - 1]/r(K))T(K).$$

Now, the left side of (A.7) is

$$\text{(A.8)} \qquad T(K - 1) = T(r(K - 1), n - K + 1)$$

and the right side is

$$\text{(A.9)} \qquad ([r(K) - 1]/r(K))T(K) = [(r(K) - 1)/r(K)]T(r(K), n - K)$$
$$= T(r(K) - 1, n - K + 1).$$

Since

$$T(r(K) - 1, n - K + 1) \leq T(r(K - 1), n - K + 1),$$

(A.7) and hence (A.6) follow (A.8) and (A.9). []

<u>Proof of Proposition 6</u>: Let $D = \{2,\ldots,K_1^*\}$. The buyer's profit

$$\text{(A.10)} \qquad W_1(V^*) = T(K_1^*) - \Sigma_{i \in D} V_i^*.$$

A necessary condition for a SPNE is that at V* the buyer cannot be better off by purchasing any subset $S \subset D$, that is,

$$(A.11) \qquad T(K_1^*) - \Sigma_{i \in D} V_i^* \geq T(|S| + 1) - \Sigma_{i \in S} V_i^*$$

or

$$T(K_1^*) - T(|S| + 1) \geq \Sigma_{i \in D-S} V_i^*, \ \forall \ S \subseteq D,$$

where $S = \emptyset$ is a possible choice. Alternatively

$$(A.12) \qquad T(K_1^*) - T(K_1^* - |S|) \geq \Sigma_{i \in S} V_i^*, \ \forall \ S \subseteq D$$

should hold. The remainder of the proof follows the proof of Theorem 5 of Beja and Zang (1986) and is furnished here for the sake of completeness. First, we need some preliminary lemmas. For $A \subseteq D$ let

$$(A.13) \qquad d(A;V^*) = \Sigma_{i \in A} V_i^* - T(K_1^*) + T(K_1^* - |A|),$$

and note that $d(A;V^*) \leq 0$ in view of (A.12).

<u>Lemma A.1</u>: For every $A,B \subseteq D$

$$(A.14) \qquad d(A \cup B;V^*) + d(A \cap B; V^*) \geq d(A;V^*) + d(B;V^*).$$

<u>Proof</u>:

$$(A.15) \qquad d(A \cup B;V^*) + d(A \cap B;V^*) = \Sigma_{i \in A} V_i^* + \Sigma_{i \in B} V_i^* - 2T(K_1^*)$$

$$+ T(K_1^* - |A \cup B|) + T(K_1^* - |A \cap B|)$$

(A.16) $$d(A;V^*) + d(B;V^*) = \Sigma_{i \in A} V_1^* + \Sigma_{i \in B} V_1^* - 2T(K_1^*)$$
$$+ T(K_1^* - |A|) + T(K_1^* - |B|).$$

In view of (A.15) and (A.16), (A.14) holds iff

$$T(K_1^* - |A \cup B|) + T(K^* - |A \cap B|) \geq T(K_1^* - |A|) + T(K_1^* - |B|)$$

or iff

(A.17) $$T(K^* - |A \cap B|) - T(K_1^* - |B|) \geq T(K_1^* - |A|) - T(K_1^* - |A \cup B|).$$

Note that on each side of (A.17) the difference between the arguments of T is $|B - A|$, and that $K_1^* - |B| \geq K_1^* - |A \cup B|$. Hence, (A.17) follows from (23). [ ]

<u>Lemma A.2</u>: In a merged SPNE of G

$$d(A;V^*) = d(B;V^*) = 0 \Rightarrow d(A \cap B;V^*) = d(A \cup B;V^*) = 0.$$

<u>Proof</u>: The proof follows Lemma A.1 since $d(\cdot;V^*) \leq 0$ in an SPNE. [ ]

<u>Lemma A.3</u>: In a merged SPNE of G if $d(D;V^*) < 0$ then there exists a $j \in D$ such that

$$d(A;V^*) < 0, \; \forall \; A \subset D \text{ containing } \{j\}.$$

Proof: Suppose to the contrary that for every $i \in D$ there is a subset $A_i \subseteq D$ containing $\{i\}$ such that $D(A_i;V^*) = 0$. Clearly, $\cup_{i\in D} A_i = D$. Moreover, Lemma A.2 implies that $d(A_1 \cap A_2;\ V^*) = 0$ and applying Lemma A.2 recursively we obtain $d(D;V^*) = 0$, a contradiction. []

Lemma A.4: In a merged SPNE of G

$$T(K_1^*) - T(1) = \Sigma_{i\in D} V_i^*. \tag{A.18}$$

holds.

Proof: Note that in view of (A.12) and (A.13), if (A.18) does not hold then $d(D;V^*) < 0$, implying by Lemma A.3 that for some $j \in D$ $d(A;V^*) < 0$, $\forall A \subset D$ containing $\{j\}$. It follows that V_j can be increased above V_j^* without violating the conditions in (A.12) and hence V^* cannot be an SPNE of G. []

We now continue with the proof of Proposition 6. In view of Lemma A.4 and (A.10) we get that the buyer's profit

$$W_1(V^*) = T(1) = T(1, n - 1).$$

Certainly, a nonseller j will make

$$W_j(V^*) = T(r(K_1^*), n - K_1^*)/r(K_1^*). \tag{A.19}$$

As for seller i,

$$W_i(V^*) = V_i^*,$$

and it is easy to verify that

$$W_1(V^*) \le W_i(V^*) \le W_j(V^*), \ i = 2,\ldots,K_1^*, \ j = K_1^* + 1,\ldots,n$$

holds. Indeed, if a seller's asking price is below T(1, n - 1) then by a sufficient increase in his asking price he will not be bought. In this case he will make at least T(1, n - 1). If a seller charges a price V_i^* which is higher than $W_j(V^*)$ given by (A.19), then the nonseller j will set his price at $W_j(V^*) < V_j < V_i^*$. At this price the buyer is better off purchasing j instead of i and j will then increase his profits from $W_j(V^*)$ to V_j. []

<u>Proof of Lemma 5</u>: To establish (35) it is sufficient to show that

$$\text{(A.20)} \qquad T'(r, n - K_1^* + 1) < 0$$

holds for $r \ge K_1^* - 2$. Let

$$g(r,t) = (t - r)(r + t - 1)P' + t(QP)'',$$

where P' and (QP)" are evaluated at Q(t + r) and note that since m = r + t, $\pi g(r,t)$ is the numerator of (19). In view of (19) and Assumptions I and

III, (A.20) is satisfied iff

(A.21) $$g(r,\ n - K_1^* + 1) > 0$$

holds for

(A.22) $$r \geq K_1^* - 2.$$

Now,

$$g(r,\ n - K_1^* + 1) = (n - K_1^* + 1 - r)(r + n - K_1^*)P' + (n - K_1^* + 1)(QP)'',$$

and by Assumptions I and III, (A.21) can hold only if (note that $n \geq K_1^*$)

$$n - K_1^* + 1 - r < 0$$

or

(A.23) $$r > n - K_1^* + 1.$$

Since (A.22) is assumed to hold, (A.23) will hold if

$$K_1^* - 2 > n - K_1^* + 1$$

or

(A.24) $$K_1^* > (n + 3)/2,$$

and then

(A.25) $$g(r, n - K_1^* + 1) = (r - [n - K_1^* + 1])(r + n - K_1^*)(-P') + (n - K_1^* + 1)(QP)''$$

Lemma A.5: There exists a $\alpha > 0$ such that $P'(Q(m)) \leq -\alpha$ for all $m \geq 1$.

Proof of Lemma A.5: Note that for every m, equation (4) that is

(A.26) $$m[P(Q(m)) - C] + Q(m)P'(Q(m)) = 0.$$

should hold. Suppose to the contrary that $\lim_{m\to\infty} P'(Q(m)) = 0$ for a subsequence of the natural numbers. Since $P'(Q) < 0 \ \forall \ Q \geq 0$ and P' is continuous this can only happen if $Q(m)$ is unbounded from above for the same subsequence. However, this will imply by Assumption II that $P(Q(m)) < C$ holds for sufficiently large m and since $P' < 0$ (A.26) cannot hold for these values of m. [ ]

To complete the proof of Lemma 5, note that by Lemma A.5 and Assumption III we obtain from (A.25)

(A.27) $$g(r,n - K_1^* + 1) \geq (r - [n - K_1^* + 1])(r + n - K_1^*)\alpha - (n - K_1^* + 1)\beta.$$

To find out when (A.21) holds we have to find conditions on K_1^*, satisfying (A.24), under which the right side of the last expression, (A.27), is positive for $r \le K_1^* - 2$. In this expression we denote by $h(r)$ the coefficient of α. Then

$$h(r) = r^2 + r[n - K_1^* - (n - K_1^* + 1)] - (n - K_1^*)(n - K_1^* + 1)$$
$$= r^2 - r - (n - K_1^*)(n - K_1^* + 1)$$

obtains its unconstrained minimum at $r = 1/2$ and is increasing for $r > 1/2$. In particular, since by (A.24) $K_1^* - 2 \ge 1/2$ for $n \ge 2$, then, for $r \ge K_1^* - 2$, $h(r)$ is increasing and therefore it is sufficient to establish positivity of the right side of (A.27) at $h(K_1^* - 2)$, or to show that

$$(K_1^* - 2 - [n - K_1^* + 1])(K_1^* - 2 + n - K_1^*)\alpha - (n - K_1^* + 1)\beta > 0$$

or that

$$[2K_1^* - (n + 3)](n - 2)\alpha - (n + 1)\beta + K_1^*\beta > 0$$

or that

$$\text{(A.28)} \quad K_1^* > \frac{(n + 3)(n - 2)\alpha + (n + 1)\beta}{2(n - 2)\alpha + \beta} = \frac{n + 3}{2}\left[\frac{(n - 2)\alpha + (n + 1)\beta/(n + 3)}{(n - 2)\alpha + \beta/2}\right].$$

Hence, if both (A.24) and (A.28) hold, then (A.21) and hence (A.20) will hold for $r \ge K_1^* - 2$. Note, however, that because

$$(n + 1)\beta/(n + 3) \ge \beta/2$$

as long as $n \geq 1$, then the lower bound set by (A.28) is higher than the one set in (A.24). Hence, (36) follows. [ ]

Proof of Corollary 3: A complete monopoly is impossible if (36) holds with $K_1^* = n$. That is, following some algebraic manipulations, if

$$(n - 3)(n - 2)\alpha > \beta.$$

Hence, n should be greater than the largest root of the quadratic equation

$$n^2 - 5n + 6 - \beta/\alpha = 0,$$

implying the inequality $n > (5 + \sqrt{1 + \beta/\alpha})/2$. [ ]

Proof of Corollary 4: Note that (36) can be read as

$$\theta = K_1^*/n > \frac{n + 3}{2n}\left[\frac{(n - 2)\alpha + (n + 1)\beta/(n + 3)}{(n - 2)\alpha + \beta/2}\right] = f(n),$$

and since $f(n) \to 1/2$ as $n \to \infty$, the result follows. [ ]

Proof of Theorem 4: By Proposition 1, $r^*(K_1^*) > t_1^* = n - K_1^*$. Hence, $m = r^*(K_1^*) + t_1^* > 2t_1^* = 2(n - K_1^*)$.

Since $K_1^* \leq K(n)$ must hold for a merged SPNE to exist, it follows that

$$m \geq 2(n - K(n)) =$$

$$= 2n - (n + 3)\left[\frac{(n - 2)\alpha + (n + 1)\beta/(n + 3)}{(n - 2)\alpha + \beta/2}\right] = m(n). \qquad []$$

<u>Proof of Corollary 5</u>:

$$\ell = n - m \leq n - m(n) = \ell(n). \qquad []$$