Discussion Paper No. 740

THE CORE OF RESALE-PROOF INFORMATION TRADES

by

Mikio Nakayama* and Luis Quintas**

April 1988

Abstract

This paper considers the core of a game describing resale-proof trades of an information good. The concept of resale-proofness has been introduced by Nakayama and Quintas [6] in order to formulate a trade which is self-binding in the sense that no agent has an incentive to sell the information after acquiring it when the resale is freely allowed. We characterize the core, and derive a necessary and sufficient condition for the core to consist of a single outcome, the monopolistic imputation. These results show that the sole seller cannot in general enjoy the monopolistic position in the resale proof trade.

* Faculty of Economics, Toyama University, 3190 Gofuku, Toyama 930, Japan.

** IMASL Universidad Nacional de San Luis, Chacabuco y Pedernera, 5700 San Luis, Argentina.

The authors are currently visiting the Department of Managerial Economics and Decision Sciences, Northwestern University, Evanston, Illinois 60208.

1. Introduction.

This paper considers the core of a game describing a resale-proof trade of an information good. The concept of "resale-proofness" has been introduced by Nakayama and Quintas [6] in an attempt to formulate a self-binding trade such that no agent has an incentive to sell the information after acquiring it when the resale is freely allowed. Such a notion of self-binding trade is inevitable especially when, as in the case of a technical know-how, it is impossible or difficult to provide patent protection for it.

One of the important properties obtained by Nakayama and Quintas [6] is that a minimal resale-proof set of agents attains the maximal total profit over all resale-proof sets. That is, if the information is shared by the agents in a minimal resale-proof set, the total profit is maximal in all resale-proof sets, and there remains no possibility for the information to be sold further. Since the sole seller can choose any buyers, it might be argued that the seller may have a monopolistic power in determining the allocation of profits as an outcome of the resale-proof trade. In fact, if the profit to any holder of the information is same, which is an assumption made in Nakayama and Quintas [6] for simplicity, it is easy to see that a typical outcome of the trade is the monopolistic allocation: the seller obtains all the profits. This is because any buyer claiming a positive net profit in the trade can be expelled from the trade whenever the minimal resale-proof set is a proper subset of all agents.

One of our results indicates, however, that in the non-symmetric case the sole seller can no longer enjoy the monopolistic position; namely, the core can include an imputation with the seller obtaining only the minimum level of profit. A necessary and sufficient condition for the monopolistic imputation to be the only outcome in the core is that the seller is the only agent who is in all profitable resale-proof sets with the maximal total profit over all resale-proof sets. This result is an extension of the symmetric case.

In the literature, several cooperative games describing trades of information have appeared (see, e.g., Muto, Potters and Tijs [5], Muto Nakayama [3] or Muto, Nakayama, Potters and Tijs [4]). All of these games are defined under the assumption that any coalition can make a binding agreement not to resell the acquired information. This is a strong assumption when there exists a prospect to gain by reselling the information. However, the game to be considered here can be defined without any such assumption, because the sole seller can choose any resale-proof set which is, by definition, self-binding in the sense already stated. This is a conceptual novelty in our construction of a cooperative game.

The crucial assumption in defining the concept of resale-proofness is that the profit to any holder of the information may become smaller as the set of the holders becomes larger. But, this is rather a typical observation in an oligopolistic situation of information trading (see, e.g. Kamien and Tauman [1] or Muto [2]).

The paper is organized as follows. In the next section, we restate the definition of resale-proofness, and prove a preliminary result that a smaller resale-proof set attains a higher total profit in the trade, which is an extension of the result obtained in the symmetric case. In section 3, we define the game and derive several results characterizing the core. It will turn out that the agents in the intersection of all minimal resale-proof sets play a crucial role in characterizing the imputations in the core. Finally, in the last section, we conclude with a few remarks.

2. Resale-Proof Trades.

Let $N=\{1,...,n\}$ be the set of all agents, where 1 is the sole seller of the information and the others are potential buyers. By a holder of the information, we mean the seller or the buyer who is assumed to have acquired the information. For each $i\in N$ and $S\subseteq N$, let $E^i(S)$ be the monetary profit to agent i when S is the set of all holders of the information.

Assumption 2.1 For each $i\in N$,

$$E^i(S) \geq E^i(T) > 0 \quad \text{if } \{1,i\}\subseteq S\subseteq T\subseteq N,$$

$$E^i(S) = 0 \quad \text{if } \{1,i\}\not\subseteq S.$$

This assumption simply says that the profit to any holder may decrease as the set of the holders becomes large, and that the profit to any non-holder is zero. We assume that this fact is common knowledge. For each $T\subseteq N$ with $1\in T$, define the function $E(\bullet|T):2^T-\{\emptyset\} \rightarrow R_+$ by

$$E(S|T) = \Sigma_{i\in S}E^i(T) \text{ for each } S\in 2^T-\{\emptyset\}.$$

We write $E(S|S)=E(S)$ for all $S\subseteq N$ with $1\in S$. $E(S|T)$ is the total profit of the holders in the subset S of T when T is the set of the holders. We are now ready to define a profitable resale of the information.

Definition 2.1. Let $1\in H\subseteq N$, and assume that H is the set of the holders. A resale from $S\subseteq H$ to $T\subseteq N-H$ is said to be profitable iff $S\neq\Phi$, $T\neq\Phi$

and $E(S \cup T|H \cup T) > E(S|H)$. The resale is called a <u>sale</u> iff $S=\{1\}=H$.

Notice that the profitability is only <u>tentative</u>, because it presupposes that no further resale is carried out after the resale. The problem is, then, how to make sure the profitability when every holder is fully allowed to resell. In Nakayama and Quintas [6], we formalized the idea of an enforceable resale as follows.

<u>Definition 2.2</u>. Let $1 \in H \subseteq N$, and assume that H is the set of the holders.

(i) For $|N-H|=1$, we say $S \subseteq H$ has an <u>enforceable</u> resale to N-H iff the resale is profitable.

(ii) Suppose that the definition is completed for $|N-H| = 1, \ldots, k < n-1$. Then, for $|N-H|=k+1$, we say $S \subseteq H$ has an <u>enforceable</u> resale to $T \subseteq N-H$ iff the resale is profitable <u>and</u> there is no $T' \subseteq H \cup T$ which has an enforceable resale to some $P \subseteq N-(H \cup T)$.

Thus, a resale is enforceable if it is profitable and there exists no further enforceable resale after the resale.

<u>Definition 2.3</u>. Let $1 \in M \subseteq N$. We say M is <u>resale-proof</u> iff there does not exist $S \subseteq M$ which has an enforceable resale to some $T \subseteq N-M$.

A resale-proof set always exists: N is resale-proof because $N-N=\Phi$. We say M is a <u>profitable</u> resale-proof set iff M is resale-proof and $E(\{1\})<E(M)$. Notice that if M is a profitable resale-proof set, then $|M| \geq 2$.

A number of results may be obtained which extend the symmetric case considered in Nakayama and Quintas [6]. Among them, only the following theorem is necessary as a basis of the construction of our game.

Let J be the set of all profitable resale-proof sets. Hereafter, $\subset$ stands for the proper set inclusion.

Theorem 2.1. Let $M, M' \in J$ satisfy $M \subset M'$. Then, $E(M) > E(M')$.

Proof. Suppose that $E^i(M) < E(\{i\} \cup (M'-M) | M')$ for some $i \in M$. Then, this i has a profitable resale to $M'-M$. But, since M' is resale-proof by assumption, this resale must be enforceable. This contradicts the assumption that M is resale-proof. Hence, we must have

$$E^i(M) \geq E(\{i\} \cup (M'-M) | M') \quad \text{for all } i \in M.$$

Then, summing over all $i \in M$ in both sides, we have

$$\begin{aligned} E(M) &\geq E(M|M') + |M| E(M'-M|M') \\ &= E(M') + (|M|-1) E(M'-M|M') \\ &> E(M'), \end{aligned}$$

because $|M|>1$, $M'-M \neq \Phi$ and $E(M'-M|M')>0$ by Assumption 2.1. QED

3. The Core of the Game.

For each $S \subseteq N$, define the function $v: 2^N \rightarrow R_+$ by

$$\begin{aligned} v(S) &= \max\{E(M) | M \subseteq S, \text{ and } M \in J \cup \{\{1\}\}\} && \text{if } 1 \in S, \\ &= 0 && \text{if } 1 \in N-S. \end{aligned}$$

The pair (N,v) is called a game in characteristic function form. The scenario behind this definition of v is that for each given set S of agents including the seller, the seller may choose the minimal profitable resale-proof set if it is included in S; and, if otherwise, he does not trade with any buyer in S.

Let $x \in R^n_+$ be a payoff vector. We write $x(R)=\Sigma_{i \in R} x_i$ if $R \subseteq N$ and $R \neq \emptyset$. The core of a game (N,v) is the set C(v) of payoff vectors x given by

$$C(v)=\{x \in R^n_+ \mid x(N)=v(N), \text{ and } x(S) \geq v(S) \text{ for all nonempty } S \subseteq N\}.$$

In order to characterize the core, we need some notations. Let us write $v(S)=E(M^S)$ for all $S \subseteq N$ with $1 \in S$. Note that M^S may not be unique. Denote by J* the set of all minimal profitable resale-proof set, i.e.,

$$J^* = \{M \in J \mid \text{there exists no } M' \in J \text{ such that } M' \subset M\}.$$

Define $J^0 \subseteq J^*$ by $J^0 = \{M \in J^* \mid E(M)=E(M^N)\}$. Finally, let $M^0 = \cap\{M \mid M \in J^0\}$. Note that $1 \in M^0$. M^0 is the set of agents who are indispensable in forming any profitable resale-proof set with the maximal total profit $E(M^N)$.

Theorem 3.1. Let $H(M^0)$ be the set defined by

$$H(M^0) = \{x \in R^n_+ \mid x_1 \geq E(\{1\}), x_i=0 \text{ for all } i \notin M^0, \ x(M^0)=E(M^N) \text{ and } x(M) \geq E(M) \text{ for all } M \in J^*\}.$$

Then, $C(v)=H(M^0)$.

Proof. Assume that $x \in C(v)$ and $i \notin M^0$. Then,

$$i \notin M \text{ for some } M \in J^0.$$

Hence, for this M we have $M \subseteq N-\{i\}$. This implies that $v(N)=E(M)=v(N-\{i\})$. Since $x \in C(v)$, we have

$$0 \leq x_i \leq v(N) - v(N-\{i\}) = 0.$$

Hence $x_i=0$ for all $i \notin M^0$. Then, $x(M^0)=x(N)=v(N)=E(M^N)$.

Next, take any $M \in J^*$ and let S=M. Then, $x(M)=x(S) \geq v(S)=E(M)$.

That $x_1 \geq E(\{1\})$ is clear. Hence, $C(v) \subseteq H(M^0)$.

Conversely, let $x \in H(M^0)$. Assume, first, that $1 \in S$. If $M' \subseteq S$ for some $M' \in J^*$, then there exists an $M \in J^*$ such that $M \subseteq S$ and $v(S)=E(M)$. Hence, $x(S) \geq x(M) \geq E(M)=v(S)$. If $M' \not\subseteq S$ for all $M' \in J^*$, then we must have

$$v(S) = E(M^S) = E(\{1\}).$$

This follows from the definition of v and the fact that S does not include any profitable resale-proof set. Hence,

$$x(S) \geq x_1 \geq E(\{1\}) = v(S).$$

If $1 \notin S$, then $x(S) \geq 0 = v(S)$. Finally, $x(N)=x(M^0)=E(M^N)=v(N)$.
Hence, $H(M^0) \subseteq C(v)$. QED

Theorem 3.1 indicates that only the members in M^0 need to be counted in considering the core.

Let $M^* = \cap\{M | M \in J^*\}$. M^* is the set of agents who are indispensable in forming any minimal profitable resale-proof set. Note that $1 \in M^* \subseteq M^0$. Let

$$I(M^*) = \{x \in R^n_+ |\ x(M^*)=E(M^N),\ x_1 \geq E(\{1\}), \text{ and } x_j=0 \text{ for all } j \notin M^*\}.$$

$I(M^*)$ is the set of payoff vectors in which the set M^* of indispensable agents as a whole obtains all the profits. When $M^*=\{1\}$, the unique payoff vector $x^m=(E(M^N),0,\ldots,0)$ in $I(\{1\})$ is called a <u>monopolistic</u> imputation. We can now refine Theorem 3.1 as follows.

<u>Theorem 3.2</u>. (i) $I(M^*)=C(v)$ iff $M^*=M^0$.

(ii) $\{x^m\}=C(v)$ iff $\{1\}=M^0$.

<u>Proof</u>. (i) <u>sufficiency</u>. Let $M^*=M^0$. Then, by definition, $H(M^0) = H(M^*) \subseteq I(M^*)$. Let $x \in I(M^*)$. Then, for all $M \in J^*$, we have

$$x(M) = x(M \cap M^*) = x(M^*) = x(M^0) = E(M^N) \geq E(M).$$

Hence, $I(M^*) \subseteq H(M^*)=H(M^0)$, which, by Theorem 3.1, implies $I(M^*)=H(M^0)=C(v)$.

<u>necessity</u>. Assume that $M^* \neq M^0$. Then, $M^* \subset M^0$, which implies $J^0 \subset J^*$. Define a payoff vector y by

$$y_1 \geq E(\{1\}),$$

$$y(M^*) = \max\{E(M) \mid M \in J^* - J^0\}, \text{ and}$$

$$y(M^0 - M^*) = E(M^N) - y(M^*) > 0.$$

Then, we have

$$y_i = 0 \text{ for all } i \notin M^0, \quad y(M^0) = E(M^N),$$

$$y(M) = y(M^0) = E(M^N) = E(M) \text{ for all } M \in J^0, \text{ and}$$

$$y(M) \geq y(M^*) \geq E(M) \text{ for all } M \in J^* - J^0.$$

Hence, by Theorem 3.1, $y \in H(M^0) = C(v)$, but $y \notin I(M^*)$. Hence, $I(M^*) \neq C(v)$.

(ii). Letting $M^* = \{1\}$, the conclusion follows from (i). QED

In the symmetric case where the profit to any holder is same and depends only on the number of the holders; namely, $E^i(S) = f(|S|)$ for all $i \in S$ and all S with $1 \in S \subseteq N$, we can immediately state the following corollary: the core consists only of the monopolistic imputation iff every minimal profitable resale-proof set is a proper subset of N.

Theorem 3.2 also implies that if there exists a "big" buyer who is indispensable in any minimal resale-proof set, then the sole seller may no longer be able to enjoy the monopolistic position. The following theorem in fact states that this can happen.

<u>Theorem 3.3</u>. Assume that $J \neq \emptyset$. Then:

(i) $I(M^*) \subseteq C(v)$.

(ii) If $|M^*| \geq 2$, then $x_1 = E(\{1\})$ for some $x \in C(v)$.

(iii) If $|M^*| = 1$ and $|M^0| \leq 2$, then $x_1 > E(\{1\})$ for all $x \in C(v)$.

(iv) If $M^0 \cap M = M^*$ for some $M \in J^*$, then $x(M^*) \geq E(M)$ for all $x \in C(v)$.

If, moreover, $M^* = \{1\}$, then $x_1 \geq E(M)$ for all $x \in C(v)$.

Proof. (i) Let $x \in I(M^*)$. Since $M^* \subseteq M$ for all $M \in J^*$,

$$x(M) \geq x(M^*) = E(M^N) \geq E(M) \quad \text{for all } M \in J^*, \text{ and}$$

$$x_1 \geq E(\{1\}).$$

Hence, the result follows from Theorem 3.1.

(ii) Since $|M^*| \geq 2$, there exists $x \in I(M^*)$ such that $x_1 = E(\{1\})$. The conclusion then follows from (i).

(iii) Suppose there existed $x \in C(v)$ such that $x_1 = E(\{1\})$. By Theorem 3.1, it then follows that

$$x((M - \{1\}) \cap M^0) = x(M) - E(\{1\})$$

$$\geq E(M) - E(\{1\}) > 0 \quad \text{for all } M \in J^*.$$

Hence, $(M - \{1\}) \cap M^0 \neq \Phi$ for all $M \in J^*$, which implies that $M^0 \neq \{1\}$. Assume, then, that $M^0 = \{1, i\}$ for some $i \neq 1$. Then, it must be true that $i \in M - \{1\} \subseteq M$ for all $M \in J^*$, which implies $M^0 = \{1, i\} \subseteq M^*$ by definition. This contradicts the assumption that $|M^*| = 1$.

(iv) Let $x \in C(v)$. By Theorem 3.1, $x_i = 0$ for all $i \notin M^0$. Hence, for $M \in J^*$ with $M^0 \cap M = M^*$, we have $x(M^*) = x(M \cap M^0) = x(M) \geq E(M)$. QED

The case in which $|M^*| = 1$ and $|M^0| \geq 3$ remains to be answered. The following example indicates that, in this case, there exists a game (N, v) such that $x_1 = E(\{1\})$ for some $x \in C(v)$.

Example 3.1: Let $N = \{1, 2, 3, 4\}$ and let E^i, E and v be given by the following tableau:

SETS	E^1	E^2	E^3	E^4	E	v
{1}	5				5	5
{1,2}	2	3			5	5
{1,3}	2		3		5	5
{1,4}	2			3	5	5
{1,2,3}	2	3	3		8	8
{1,2,4}	2	2		3	7	7
{1,3,4}	2		2	2	6	6
{1,2,3,4}	2	1	1	1	5	8

It is easy to see that: $J=\{\{1,2,3\},\{1,2,4\},\{1,3,4\}\}$; $M^* = \{1\}$; $M^0 = \{1,2,3\}$ and $C(v) = \{ x \in R^n_+ \mid x(N)=8 , x_4=0 , x_1 \geq 5 , x_1+x_3 \geq 6 , x_1+x_2 \geq 7 \}$. Therefore, $x=(5,2,1,0) \in C(v)$ and $x_1=E(\{1\})$.

It is not difficult to obtain a similar example when $|M^0|>3$.

4. Concluding Remarks.

We have shown that the non-symmetry in the profit functions of agents may weaken the monopolistic position of the sole seller in the resale-proof trade of an information good. The big buyers who are indispensable in forming any profitable resale-proof set have played a crucial role in enlarging the core of the game: it includes an imputation x with $x_1=E(\{1\})$ whenever there exists at least one such big buyer. Notice, here, that the external effect to the big buyers, expressed by Assumption 2.1, must nevertheless be reasonably strong enough to prevent them from reselling the information in the enforceable way. It will be clear by definition that any agent who is unaffected by the externality at all cannot be the member in any resale-proof set except N.

The core analysis presented here is not, in itself, a very novel research strategy. In fact, if complete patent protection was possible, we could carry out a similar analysis as to the core of the following game (N,v'):

$$
\begin{aligned}
v'(S) &= \max\,\{E(T) \mid 1 \in T \subseteq S\}, && \text{if } 1 \in S, \\
&= 0 && \text{if } 1 \in N-S.
\end{aligned}
$$

The game (N,v') is a straightforward extension of the one considered in Muto, Nakayama, Potters and Tijs [4] as an example of the "big boss game".

The point of departure of our analysis is that we did not make any assumption on the possibility of a binding agreement among agents when forming a coalition. The resale-proof trade is, by definition, self-binding so that the seller has only to consider a resale-proof trade in each conceivable coalition.

The resale-proof trade may not generate Pareto optimal outcomes in general. Theorem 2.1 does not assure the overall maximality of the total profit in a minimal resale-proof set. This is, however, to be expected since the resale-proofness is not a fully cooperative concept in the sense stated above. From a normative point of view, therefore, the desirability of patent protection whenever possible is obvious: the above game (N,v') has a Pareto optimal core. In this sense, the lack of Pareto optimality might be viewed as a cost due to the inability of patent protection, or of making a binding contract among agents.

<u>References</u>.

1. M.I.Kamien and Y.Tauman, The private value of a patent: a game theoretic approach, Journal of Economics, Suppl.4 (1984),93-118.

2. S.Muto, Possibility of relicensing and patent protection, European Economic Review 31 (1987),927-945.

3. _______, and M.Nakayama, A cooperative game of information trading: the core and the nucleolus, Discussion Paper No.757, Department of Managerial Economics and Decision Sciences, Northwestern University (1988).

4. _______,__________, J.Potters and S.Tijs, On big boss games, Report 8719, Department of Mathematics, Catholic University (1987), forthcoming in Economic Studies Quarterly.

5. _______, J.Potters and S.Tijs, Information market games, Report 8633, Department of Mathematics, Catholic University (1986).

6. M.Nakayama and L.Quintas, Resale-proof trades of information, Discussion Paper No.763, Department of Managerial Economics and Decision Sciences, Northwestern University (1988).