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QUANTITY DISCOUNT SCHEMES FOR CHANNEL COORDINATION

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ABSTRACT

This paper analyzes the implications of the quantity discount scheme suggested by Jeuland and Shugan (1983) for channel coordination when non-price variables are included in the model. Using an alternative formulation of the extended Jeuland-Shugan model it is shown that there exists at least one case in which the discount schedule parameter which determines the allocation of profits among channel members is unique. Using the same alternative formulation it is also shown that the Jeuland-Shugan quantity discount scheme does not insure that non-price variables are set at levels which maximize joint channel profits. Finally, an explanation is provided for these divergent results.

I. Introduction

Jeuland and Shugan (1983) have demonstrated the need for channel coordination. Using a simple model consisting of two channel members, a manufacturer and a retailer, they found that when the manufacturer and the retailer set their margins independently, the sum of these margins will not equal the margin which maximizes joint channel profits. Moreover, they demonstrated that coordination among channel members 'can lead to results where all channel members receive larger profits (p. 266).' To induce channel coordination, Jeuland and Shugan proposed a quantity discount scheme and derived the optimal quantity discount schedule. Moorthy (1986) discusses the posssibility of implementing the discount schedule as a two-part tariff. Both the discount schedule and the two-part tariff include a side payment which affects the allocation of channel profits. In the two-part tariff case, once the value of the side payment is determined, there is a single allocation of channel profits which is consistent with the maximization of channel profits. In the quantity discount scheme, the allocation of channel profits remain indeterminate even after the size of the side payment is fixed.

Jeuland and Shugan extend their model to include two non-price decision variables, one for each channel member. The second control variable for the manufacturer is q, the quality of the product, while for the retailer it is s, the shelf space allocated to the product. Jeuland and Shugan found that the inclusion of these non-price variables did not alter the margin coordination problem. Without coordination, both the manufacturer and the retailer have the incentive to raise their margin above the level at which joint channel profits are maximized. In a brief discussion of the extended model (expression (21) p. 255) Jeuland and Shugan note that their quantity discount schedule also provides proper incentives for margin coordination in the presence of non-price variables.

In this note we utilize a double moral hazard model developed by Cooper and Ross (1984) to analyze the implications of the non-price variables for the quantity discount scheme. Under the assumptions of our model, we show that there exists at least one case in which the allocation of channel profits is unique. Further, we show that under the Jeuland-Shugan quantity discount scheme, at least one channel member will find it optimal to deviate from the level of q and/or s which maximizes joint channel profits.

This note is organized as follows. In section II, we discuss the extended Jeuland-Shugan model. Section III deals with the determination of the discount schedule parameter. In Section IV, we analyze the coordination of q and s. Section V contains the conclusion.

II. The Extended Jeuland-Shugan Model

We retain the notation and major assumptions of Jeuland and Shugan (p. 242). Their quantity discount schedule t(Q) has the following form:

(1) $t(Q) = k_1[p(D) - c - C] + (k_2/D) + C,$

where p(D) is the inverse demand function; C and c are the variable costs associated with quality and shelf-space; and k_1 and k_2 are constants determined through negotiations between the two channel members. Since k_2 does not affect the analysis which follows, we will assume without loss of generality that $k_2 = 0$. To simplify the notation, we will denote k_1 simply by k.

Recall that in the basic Jeuland-Shugan model the manufacturer and retailer determine, respectively, the wholesale and retail margins. In the extended model the manufacturer also determines the level of control variable q, quality of the product, and the retailer determines the level of control variable s, shelfspace.

We modify the Jeuland-Shugan model in two ways. First, let the manufacturer's fixed costs F(q) be an increasing function of q, and the retailer's fixed costs f(s) an increasing function of s. Conventionally, fixed costs refer to expenses associated with plant and equipment. In the long run, of course, these variables are not fixed; however, in the short run, the prohibitive costs associated with altering the quantities of these factors of production lead the firm to treat these factors as if they were fixed. It is these types of expenses which are most likely to be omitted in the construction of a quantity discount schedule. Our model explores the consequences of these omissions. Mathematically, we assume that $F'(q) > 0$, $F''(q) > 0$, $f'(s) > 0$ and $f''(s) > 0$.

Our second modification of the Jeuland-Shugan model involves the demand function. Let demand vary directly with q and s, and inversely with p. Further, assume D(p,q,s) is weakly separable in retail price, and quality and service:

$$(2) \quad D(p,q,s) = h(p)l(q,s).$$

We assume that $h'(p) < 0$, $l_q(q,s) > 0$, $l_{qq}(q,s) < 0$. $l_s(q,s) > 0$, and

$l_{ss}(q,s) < 0$, where $l_q(q,s)$ and $l_s(q,s)$ are the partial derivatives of $l(q,s)$ with respect to q and s, and $l_{qq}(q,s)$ and $l_{ss}(q,s)$ are the corresponding second derivatives. Intuitively, (2) imposes the restriction that the price elasticity of demand is independent of the levels of q and s. The optimal price (and therefore the optimal margins) is independent of the levels of quality and shelf-space. Although this demand function is a special case of that analyzed by Jeuland and Shugan, we find it attractive in this instance since the margin coordination problem is identical.

Let m equal the optimal sum of the manufacturer margin G and the retailer margin g. Suppose that the manufacturer adopts the quantity discount schedule suggested by Jeuland and Shugan in order to induce channel coordination in the determination of manufacturer and retailer margins. Under these assumptions, we now discuss the determination of q and s in a Nash equilibrium context.

Consider first the coordinated solution, where q and s are set by the single owner of both channel members. This first best solution will maximize $[mD(q,p,s)] - F(q) - f(s)$, where $m = G + g$, and the coordinated solution (q^*,s^*) satisfies

(3) $h(p).l_q(q,s).m = F'(q)$ and

(4) $h(p).l_s(q,s).m = f'(s)$.

Denote the solution to (3) as $q^*(s)$, and the solution to (4) as $s^*(q)$. These are the reaction functions for the single owner, and the intersection of these reaction functions determines the optimal values of q and s.

Now consider the situation where the choice of q and s are not coordinated. For given s, the manufacturer chooses q to maximize his profits. Assuming that the Jeuland-Shugan quantity discount scheme is in effect, the manufacturer and retailer profit functions are given by

(5) $k.m.h(p).l(q,s) - F(q)$, and

(6) $(1-k).m.h(p).l(q,s) - f(s)$.

The solution to the manufacturer's problem, $\hat{q}(s;k)$, will satisfy

(5a) $h(p).l_q(q,s).m.k = F'(q)$.

Similary, the retailer choice of $\hat{s}(q;k)$, will satisfy

(6a) $h(p).l_s(q,s).m.(1-k) = f'(s)$.

We assume that the manufacturer and retailer always supply positive levels of quality and service. Then $\hat{q}(0;k)$ and $\hat{s}(0;k)$ are both positive for $0 < k < 1$. It follows that the reaction functions have positive intercepts. However, the slopes of these reaction functions will depend upon the sign of $l_{qs}(q,s)$, since the slope of $\hat{q}(s;k)$ can be found from (5a) to be

(5b) $dq/ds = [-k.m.h(p).l_{qs}(q,s)]/$
$\{[k.m.h(p).l_{qq}(q,s)] - F''(q)\}$.

Similarly, (6) implies that the slope of $\hat{s}(q;k)$ is

(6b) $ds/dq = [-(1-k).m.h(p)\ l_{qs}(q,s)]/$
$\{[1-k).m.h(p).l_{ss}(q,s)] - f''(s)\}$.

Let $\hat{s}_k$ and $\hat{q}_k$ denote the partial derivatives with respect to **k** of $\hat{s}(q;\mathbf{k})$ and $\hat{q}(s;k)$. Then (5a) and (6a) imply

$$(7) \quad \hat{q}_k = -h(p).1_q(q,s).m/\{[h(p)\ 1_{qq}(q,s).k.m] - F''\} > 0$$

$$(8) \quad \hat{s}_k = [h(p).1_s(q,s).m]/ \{[h(p).1_{ss}(q,s).(1-k).m] - f''\} < 0.$$

The signs of (7) and (8) are intuitively appealing. An increase in **k** increases the manufacturer's share of channel profits, providing incentives for the manufacturer to increase q. On the other hand, an increase in k decreases the retailer's share, providing incentives for the retailer to decrease s.

To illustrate the determination of q and s, suppose that $1_{qs}(q,s) > 0$. Then (5a) and (6a) imply that the manufacturer and retailer reaction functions have positive slopes. Let $\tilde{q}(k)$ and $\tilde{s}(k)$ denote the Nash equilibrium values of q and s. Then $\tilde{q}(k)$ and $\tilde{s}(k)$ are the values of q and s which simultaneously solve (5a) and (6a). Graphically, $\tilde{q}(k)$ and $\tilde{s}(k)$ are given by the intersection of the reaction functions $\hat{q}(s;k)$ and $\hat{s}(q;k)$.

III. The Determination of k

Jeuland and Shugan argued that it is possible to find an interval within which k must lie; however, it is not possible to determine a unique value of k. The reason is that all values of k in this interval maximize channel profits and are Pareto optimal relative to the profits determined in a Nash equilbirium. In our formulation, there exists at least one case in which the value of k is determinate. Totally differentiating 5(a) and 6(a) with respect to k, we have

$$(9a)\quad \hat{q}_k = h(p).m.[k.l_{qs}(q,s).s_k + l_q(q,s)]/[F''(q)-h(p).m.k.l_{qq}(qs)]$$

$$(9b)\quad \hat{s}_k = h(p).m.[-l_s(q,s)+(1-k).l_{qs}(q,s)q_k]/[f''(s)-h(p).m.(1-k).l_s(q,s)]$$

In general, solving these equations simultaneously will not yield a unique value of k. However, in the special case of $l_{qs}(q,s) = 0$, the value of k is unique. In other words, if the effect of changes in quality upon demand is independent of the level of shelf space, then there will be a unique value of k which maximizes channel profits. While the nature of this interaction is an empirical question, it seems reasonable to assume that $l_{qs}(q,s) = 0$ as long as shelf space is greater than some threshold value. The mathematical details of an example illustrating this point are available in Cooper and Ross (1984).

IV. The coordination of q and s

In this section we present four propositions concerning the relationship between the Nash equilibrium values $\tilde{q}(k)$ and $\tilde{s}(k)$ and the q^* and s^* which maximize joint channel profits. Proofs of these propositions are available in Cooper and Ross, or may be obtained from the authors upon request.

Proposition I: For any value of $k, (\tilde{q}, \tilde{s})$ will deviate from the coordinated optimal solution (q^*,s^*).

To characterize the nature of these deviations from the optimal solution, we must distinguish three cases. These cases depend upon the sign of $l_{qs}(q,s)$.

Case I. $l_{qs}(q,s) > 0$.

Intuitively, this means that q and s are complements. Under this assumption, the reaction functions $\hat{q}(s;k)$ and $\hat{s}(q;k)$ are positively sloped. Now for $0 < k < 1$, equations (7) and (8) imply that $q^*(s) = \hat{q}(s;1) > \hat{q}(s;k)$ and $s^*(q) = \hat{s}(q;0) > \hat{s}(q;k)$. Graphically, this means that $\hat{q}(s;k)$ and $\hat{s}(q;k)$ lie below, respectively, $q^*(s)$ and $s^*(q)$.

Insert Figure 1 here

It is easy to see from Figure 1 that $l_{qs}(q,s) > 0$ implies that both $\tilde{q}(k)$ and $\tilde{s}(k)$ will be less than the coordinated solutions q^* and s^*. We state this formally in proposition 2.

Proposition II. For $0 < k < 1$, and $l_{qs}(q,s) > 0$, let (q^*,s^*) solve (3) and (4). Then there exists a $\tilde{q} < q^*$ and a $\tilde{s} < s^*$ such that $(\tilde{q},\tilde{s})$ solves (5a) and (6a).

Case II. $l_{qs}(q,s) = 0$

In this case, the marginal revenue curves of the manufacturer and retailer do not shift when the other channel member alters its non-price decision variable. Thus it is intuitively reasonable that this case will yield the same results as the preceding case.

Proposition III. For $0 < k < 1$, and $l_{qs}(q,s) = 0$, let (q^*,s^*) solve (3) and (4). Then there exists a $\tilde{q} < q^*$ and a $\tilde{s} < s^*$ such that $(\tilde{q},\tilde{s})$ solves (5a) and (6a).

Case III. $l_{qs}(q,s) < 0$.

In the preceding two cases, we have seen that both non-price variables $\tilde{q}(k)$ and $\tilde{s}(k)$ will be less than the values which maximize channel profits. This may be true in the case where $l_{qs}(q,s) < 0$, but it is not necessarily true. It is possible to have either $\tilde{q}(k) > q^*$ or $\tilde{s}(k) > s^*$. The reason for the ambiguity lies in the fact that q and s are substitutes. Thus the manufacturer may try to compensate for decreases in s by setting $q > q^*$. Similarly, the retailer may try to compensate for decreases in q by setting $s > s^*$. Despite this ambiguity, it is possible to show that $\tilde{q}(k) > q^*$ implies $\tilde{s}(k) < s^*$, and $\tilde{s}(k) > s^*$ implies $\tilde{q}(k) < q^*$.

Insert Figures 2(a) & 2(b) here

Proposition IV. For $0 < k < 1$, and $l_{qs}(q,s) < 0$, let (q^*,s^*) solve (3) and (4). Then the $(\tilde{q},\tilde{s})$ solving (5a) and (6a) cannot satisfy $\tilde{q} > q^*$ and $\tilde{s} > s^*$ simultaneously.

Conclusions

We have examined the implications of the Jeuland-Shugan quantity discount scheme with the inclusion of non-price variables q and s using a variant of a model suggested by Jeuland and Shugan. Conceptually, the crucial difference between the Jeuland-Shugan model and our model is that in their formulation, all costs associated with quality and shelf space are paid prior to the division of channel profits. The construction of the discount schedule insures this result. In contrast, our assumptions concerning fixed costs result in the division of channel profits prior to payment of a portion of the costs associated with quality and shelf space. Mathematically, it is possible to obtain an optimal solution in the current case by reconstructing the quantity discount schedule so that the fixed costs of quality and shelf space are paid prior to the division of channel profits. In this case, the quantity discount schedule would have the form

$$t(Q) = k_1[p(D) - c - C - f(s) - F(q)] + (k_2/D) + C + F(q).$$

However, this mathematical solution obscures the managerial insight we wish to highlight. An optimal solution requires that both channel members agree to pay all relevant costs incurred by either member prior to the division of channel profits. Our model illustrates that when costs affecting quality or shelf space are omitted from the quantity discount schedule, then the chosen values of quality and shelf space will deviate from the values which maximize channel profits.

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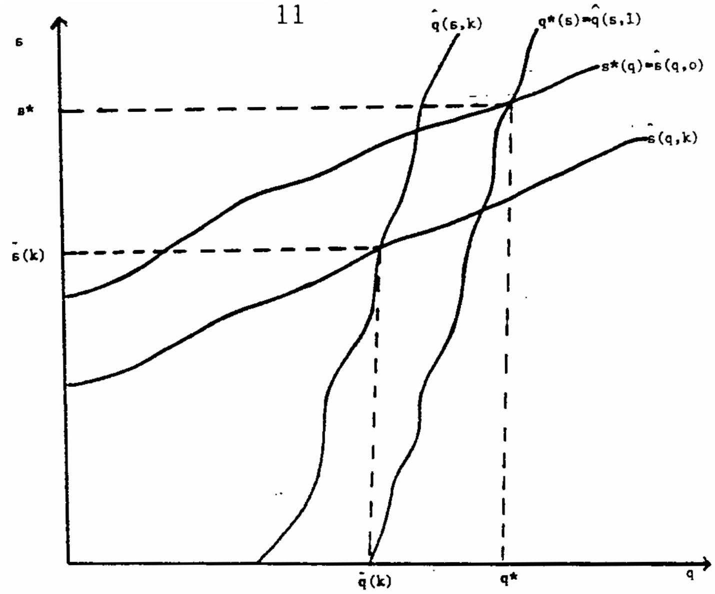

Figure 1

Case I: $l_{qs}(q,s) > o$

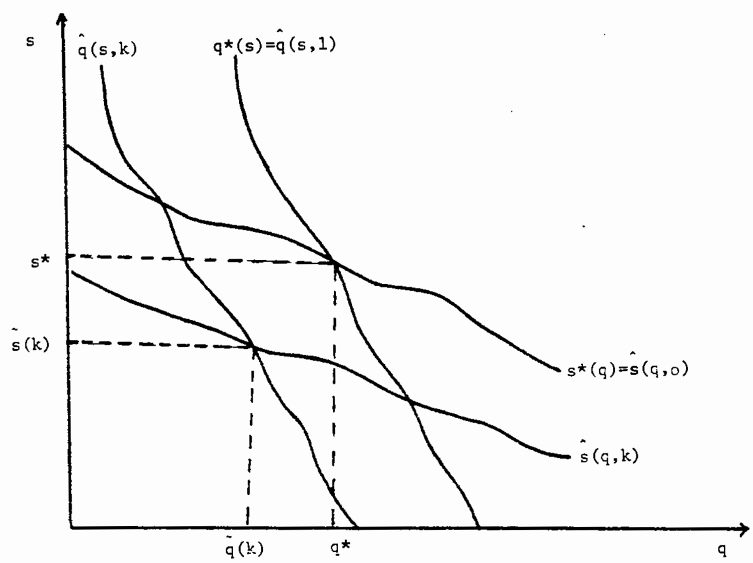

Figure 2(a)

Case III: $l_{qs}(q,s) < o$, $\tilde{q}(k) < q^*$ and $\tilde{s}(k) < s^*$

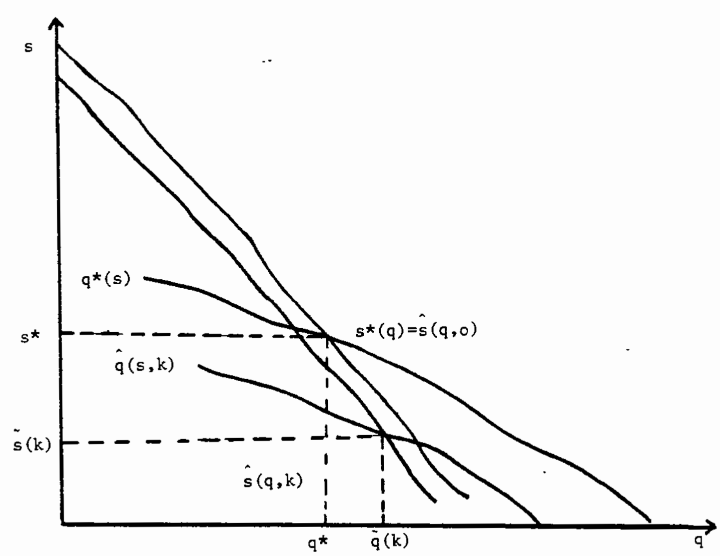

Figure 2(b)

Case III: $l_{qs}(q,s) < o$; $\tilde{q}(k) > q^*$ and $\tilde{s}(k) < s^*$