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A STOCHASTIC MODEL OF HOUSEHOLD BRAND SWITCHING BEHAVIOR

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ABSTRACT

Kahn, Morrison and Wright (1986) recently studied the effect of aggregating individual members' purchasing behavior to the household level. Their results show that for exponentially distributed interpurchase times (of individuals), the household choice process approaches a zero-order process as the number of individuals in the household increases, even though the purchasing behavior of each individual is a first order process. The purpose of this paper is to show that their results hold for any arbitrary interpurchase-time distribution that has a density over some interval. Therefore additional support is provided for their conclusion that the empirically observed zero-order choice behavior at the household level may not convey much information about individuals' choice behavior.

1. Introduction

Brand choice is an important aspect of the purchasing behavior of a household. Generally, a buyer has to choose one of several brands existing in the market place on successive purchase occasions. On each occasion, the brand choice may be either completely random or based on choices made on previous occasions. Examination of households' successive purchases of specific products reveals that few, if any, households are completely brand loyal (Ehrenberg, 1972; Bass, 1974; Blattberg and Sen, 1976) and that frequent brand switching is typical for frequently-purchased, low priced products.

Panel data, a longitudinal history of household purchases, has been used extensively by marketing researchers (Frank, 1962; Massy, 1966; Jeuland, Bass, Wright, 1980; and Bass et al. 1984) to study the multibrand buying behavior of households. A general finding of these studies is that the purchase behavior on a given occasion appears to be independent of past purchases. Such independence of choice probability is often termed as constituting a zero-order process of multibrand buying.

An important fact concerning the use of panel data in studying multibrand buying is that the unit of observation is a <u>household</u> and not a particular individual. In other words, the panel data is an aggregation of all family members' purchase histories. Thus, an interesting question in connection with the use of panel data is: "To what extent does household purchasing behavior depend on that of individual family members?" This question can be stated differently as: "What is the effect of aggregating individual family members' purchasing behavior to the household level?" Kahn, Morrison and Wright, 1986, (herein after referred as KMW) formulated a model to answer this question. They consider a household consisting of individuals who have first order (Markovian) purchasing behavior and whose interpurchase times are independent and identically distributed (i.i.d.) exponential random variables. Their results show that for exponentially distributed interpurchase times, the extent of dependence between successive purchases decreases as the number of individuals in the household increases.

An intuitive explanation of KMW's result is the following: As the number of individuals in a household increases, the likelihood for two successive purchases made by the household to be from the same individual decreases and therefore, since the brand choice processes for different individuals are assumed to be independent, the degree of dependence between successive <u>household</u> purchases must also decrease. In fact, one might conjecture that the above explanation should be valid for any "well-behaved" interpurchase-time distribution and not necessarily only for exponential interpurchase-time distribution. The purpose of this paper is to show that the above conjecture is indeed true and therefore our results provide additional support for KMW's conclusion.

Our analyses rely on renewal theoretic arguments. We provide an interesting probabilistic interpretation of the quantity D (proposed by KMW) which measures the extent to which the household brand choice process is a zero-order process. This interpretation also suggests an alternative statistical test for zero-order hypothesis which might have greater statistical power than the commonly used tests. Moreover, we bound the speed at which the aggregated household brand choice process approaches a zero-order process for the class of NBUE (New Better than Used in Expectation) interpurchase-time distributions. This class includes the Erlang family of distributions. In fact, we show analytically that the speed of convergence of D to zero in the general case is no worse than the speed of convergence when the interpurchase-time is exponentially distributed.

The rest of the paper is organized as follows: Section 2 contains a detailed description of the model formulation. Section 3 presents the main results. Section 4 establishes a bound on D for a general class of interpurchase-time distributions. Finally, Section 5 contains the conclusions.

2. The Model

Consider a household consisting of n individuals indexed by $i=1,2,\ldots,n$. We assume that the purchase behaviors for different individuals in the household are independent and that each individual has i.i.d. interpurchase times with distribution function F and finite mean λ. In addition, to avoid pathological situations, we assume that F has a density over some interval.

The market is viewed as a two brand market, indexed by $j=0,1$. The brand under consideration is denoted by 1 and the other brand(s) by 0. We further assume that each individual makes purchases independently according to a first-order brand choice process, i.e., a two-state Markov chain $\{X_k^i,\ k \geq 1\}$, where X_k^i denotes the brand bought by individual i on the k^{th} purchase occasion, i=1,2,...n. Let the i^{th} individual's transition probability matrix P_i be of the form

$$P_i = \begin{bmatrix} P_i(1|1) & P_i(0|1) \\ P_i(1|0) & P_i(0|0) \end{bmatrix}$$

where $P_i(\ell|j)$ is the conditional probability that the i^{th} individual chooses brand ℓ on his $(k+1)^{th}$ purchase occasion given that he choose brand j on the k^{th} occasion, ℓ=0,1 and j=0,1. The steady state probabilities corresponding to P_i will be denoted by π_{ji}, j=0,1.

Similarly, at the household level, let $\{Z_k,\ k \geq 1\}$ denote the household's brand choice process where

$$Z_k = \begin{cases} 1 & \text{if brand 1 is bought at the } k^{th} \text{ household purchase occasion} \\ 0 & \text{otherwise.} \end{cases}$$

Note that the process $\{Z_k,\ k \geq 1\}$ is an aggregation of individual brand choice processes $\{X_k^i,\ k \geq 1\}$, i=1,2,...,n, and thus, it is in general not a Markov chain. However for this process there exists a "transition rate" matrix P, where

$$P = \begin{bmatrix} P(1|1) & P(0|1) \\ P(1|0) & P(0|0) \end{bmatrix}$$

with $P(\ell|j)$ = transition rate from state j to state ℓ for j=0,1 and ℓ=0,1. The transition rate $P(\ell|j)$ is defined as the proportion of times the process next enters state ℓ after leaving state j.

Now, if the household brand switching process were zero-order, i.e., successive brand choices were i.i.d., then the rows of P would be identical, implying that $P(1|1) = P(1|0)$. Since $\{Z_k, k \geq 1\}$ is in general not a zero-order process, we shall, following KMW, study a quantity D defined by

$$D = P(1|1) - P(1|0),$$

which measures the degree of dependence between two consecutive purchases at the household level. We shall examine the limiting behavior of D as the number of individuals in the household approaches infinity. For this purpose, we need to relate the elements of P to the elements of P_i for $i=1,2,\ldots,n$ _and_ the interpurchase-time distribution F.

First, we observe that at any purchase occasion _in steady state_, the distribution of the random variable Y_e representing the time to next purchase for all individuals _other than_ the one making the current purchase is given by the equilibrium distribution F_e (the assumption of F having a density over some interval is needed here), defined for $t \geq 0$ by

$$F(t) = \frac{1}{\lambda} \int_o^t [1 - F(u)]\, du$$

(see, e.g., Ross, 1983, p. 76).

To facilitate understanding we begin by considering the case of two individuals, say, A and B. If a person makes two consecutive purchases, then his/her interpurchase time Y is less than the other person's equilibrium interpurchase time Y_e. On the other hand, if the person making the purchase on the second occasion is different from the one making the first, then Y is greater than Y_e.

Now viewing $P(1|1)$ as a conditional probability, we have

$$P(1|1) = P(Z_{k+1} = 1|Z_k = 1)$$

$$= P(Z_{k+1} = 1, Z_k = 1) / P(Z_k = 1).$$

To calculate the numerator, consider four mutually exclusive events as follows:

Event	Probability
A makes two consecutive purchases	$(1/2)\ \pi_{1A}\ P(Y<Y_e)\ P_A\ (1|1)$
B makes two consecutive purchases	$(1/2)\ \pi_{1B}\ P(Y<Y_e)\ P_B\ (1|1)$
A buys first and B buys next	$(1/2)\ \pi_{1A}\ P(Y>Y_e)\ \pi_{1B}$
B buys first and A buys next	$(1/2)\ \pi_{1B}\ P(Y>Y_e)\ \pi_{1A}$

For example, the probability of the first event is obtained as follows: The factor 1/2 is due to the fact that A purchases half of the time; π_{1A} is the steady state probability of A buying brand 1; $P(Y < Y_e)$ is the probability that A purchases brand 1 again. The probabilities for the other events are obtained similarly.

Noting that $P(Z_k=1)=(\pi_{1A} + \pi_{1B})/2$, we therefore have

$$P(1|1) = \{[\pi_{1A}P_A(1|1) + \pi_{1B}P_B(1|1)]P(Y<Y_e) + 2\pi_{1A}\pi_{1B}P(Y>Y_e)\}/(\pi_{1A}+\pi_{1B}). \tag{2.1}$$

Similarly,

$$P(1|0) = \{[\pi_{0A}P_A(1|0) + \pi_{0B}P_B(1|0]P(Y<Y_e) + [\pi_{0A}\pi_{1B} + \pi_{0B}\pi_{1A}]P(Y>Y_e)\}/(\pi_{0A}+\pi_{0B}). \tag{2.2}$$

Now consider the general case of n individuals. In order for the same individual to make two consecutive purchases, his interpurchase time Y must be less than $Y_{e,n-1}$ where $Y_{e,n-1}$ = minimum of (n-1) independent random variables each distributed as Y_e. Denote the probability of this event by α_{n-1}, i.e., $\alpha_{n-1} = P(Y<Y_{e,n-1})$.

Similarly, if two different persons make the purchases then Y must be greater than $Y_{e,n-1}$. Consequently, the probability of this event will be $1-\alpha_{n-1}$. Moreover, given that the individual who makes the first purchase does not make the next purchase, it is equally likely for the next purchase to be made by any of the remaining (n-1) individuals. Following this argument we get

$$(2.3)\qquad P(1|1) = \frac{\alpha_{n-1}\left[\sum_{i=1}^{n} \pi_{1i} P_i(1|1)\right] + (1/(n-1))(1-\alpha_{n-1})\left[\sum_{i\neq j} \pi_{1i}\pi_{1j}\right]}{\sum_{i=1}^{n} \pi_{1i}}$$

and

$$(2.4)\qquad P(1|0) = \frac{\alpha_{n-1}\left[\sum_{i=1}^{n} \pi_{0i} P_i(1|0)\right] + (1/(n-1))(1-\alpha_{n-1})\left[\sum_{i\neq j} \pi_{0i}\pi_{1j}\right]}{\sum_{i=1}^{n} \pi_{0i}}.$$

To evaluate P(1|1) and P(1|0) further, we must now make specific assumptions on the interpurchase-time distribution F. We consider the following examples.

<u>Example 1</u>. Exponential interpurchase times: $F(t) = 1-\exp(-t/\lambda), \lambda>0$.

For F exponential, it is easy to show that $Y_{e,n-1}$ is exponentially distributed with parameter $1/[\lambda(n-1)]$. Hence

$$(2.5)\qquad \alpha_{n-1} = \lambda/[\lambda + \lambda(n-1)] = 1/n.$$

Substituting (2.5) into (2.3) and (2.4), we have

$$(2.6)\quad P(1|1) = (1/n)\ [\sum_{i=1}^{n} \pi_{1i}\ P_i\ (1|1) + \sum_{i\neq j} \pi_{1i}\ \pi_{1j}]/\sum_{i=1}^{n} \pi_{1i}$$

and

$$(2.7)\quad P(1|0) = (1/n)\ [\sum_{i=1}^{n} \pi_{0i}\ P_i\ (1|0) + \sum_{i\neq j} \pi_{0i}\ \pi_{ij}]/\sum_{i=1}^{n} \pi_{0i}.$$

The expressions (2.6) and (2.7) agree with equations (4) and (5) in KMW (p. 267).

<u>Example 2</u>. Erlang interpurchase times: $Y \sim$ Erlang $(m, 1/(\lambda m))$, $m \geq 1$.

Consider the case of two individuals. Since Y can be viewed as a sum of m.i.i.d. exponentially distributed "phases", it is not difficult to see (e.g., Ross, 1985, p. 209) that

$$P(Y>Y_e) = \sum_{j=1}^{m} P(Y>Y_e | Y_e \text{ is in phase } j)\ P(Y_e \text{ is in phase } j)$$

$$= \sum_{j=1}^{m} \sum_{i=j}^{m+j-1} \binom{m+j-1}{i} (1/2)^{m+j-1} (1/m).$$

For example, if $m=2$, then $P(Y > Y_e) = 5/8$ and $P(Y < Y_e) = 3/8$. The above can then be substituted into (2.1) and (2.2) to find D.

3. Main Results

In this section, we present our main results in the form of two theorems. Theorem 1 reveals an interesting probabilistic interpretation for D (defined in the previous section). Theorem 2 deals with the asymptotic behavior of D as the number of individuals in the household approaches infinity.

<u>Theorem 1</u>. D is equal to the correlation coefficient between Z_k and Z_{k+1}. In other words,

$$(3.1) \qquad D = \rho(Z_k, Z_{k+1}) = \frac{\mathrm{Cov}\ (Z_k,\ Z_{k+1})}{\mathrm{Var}\ (Z_k)}\ .$$

<u>Proof</u>: For any $k \geq 1$, let $P(j) = P(Z_k = j)$ for $j=0,1$ and $P(i,j) = P(Z_{k+1} = i,\ Z_k = j)$ for $i=0,1, j=0,1$.

By definition,

$$\begin{aligned} E(Z_k) &= 1.P(Z_k=1) + 0.P(Z_k=0) \\ &= P(Z_k=1) \\ &= P(1) \end{aligned}$$

and

$$\begin{aligned} E(Z_k Z_{k+1}) &= 1.P(Z_k=1, Z_{k+1}=1) + 0.[P(Z_k=1, Z_{k+1}=0) \\ &\quad + P(Z_k=0, Z_{k+1}=1) + P(Z_k=0, Z_{k+1}=0)] \\ &= P(Z_k=1, Z_{k+1}=1) \\ &=P(1,1). \end{aligned}$$

The variance of Z_k, by definition, is

$$\begin{aligned} \mathrm{Var}(Z_k) &= E(Z_k^2) - [E(Z_k)]^2 \\ &= E(Z_k) - [E(Z_k)]^2 \\ &= E(Z_k)[1-E(Z_k)] \\ &= P(1)[1-P(1)] \\ &= P(1)P(0). \end{aligned}$$

Also, by stationarity, $E(Z_{k+1}) = E(Z_k)$ and $\mathrm{Var}(Z_{K+1}) = \mathrm{Var}(Z_k)$. The quantity D can then be expressed as

$$\begin{aligned} D &= P(1|1) - P(1|0) \\ &= \{P(0)P(1,1) - P(1)P(1,0)\}/P(0)P(1) \end{aligned}$$

$$= \{[1-P(1)]P(1,1) - P(1)P(1,0)\}/P(0)P(1)$$

$$= \{P(1,1) - P(1)[P(1,1) + P(1,0)]\}/P(0)P(1)$$

$$= \{P(1,1) - P(1)P(1)\}/P(0)P(1)$$

$$= \frac{P(Z_k=1, Z_{k+1}=1) - P(Z_k=1)\ P(Z_{k+1}=1)}{[P(Z_k=1)\ P(Z_k=0)]^{1/2}\ [P(Z_{k+1}=1)P\ (Z_{k+1}=0)]^{1/2}}$$

$$= \frac{E(Z_k Z_{k+1}) - E(Z_k)E(Z_{k+1})}{[E(Z_k^2)-(E(Z_k))^2]^{1/2}[E(Z_{k+1}^2)-(E(Z_{k+1}))^2]^{1/2}}$$

$$= \frac{\mathrm{Cov}\ (Z_k, Z_{k+1})}{(\mathrm{Var}(Z_k\ ))^{1/2}\ (\mathrm{Var}(Z_{k+1}\ ))^{1/2}}$$

$$= \rho(Z_k,\ Z_{k+1}),$$

completing the proof.

This is a very general result; it depends neither on the interpurchase-time distribution nor on the number of individuals in the household.

<u>Theorem 2</u>. When the number of individuals in a household increases, D approaches zero and hence the aggregated process $\{Z_k,\ k\geq 1\}$ looks like a zero-order process, irrespective of the interpurchase-time distribution and of the brand switching behaviors of individual family members.

<u>Proof</u>. We begin by evaluating (3.1). The covariance term $\mathrm{Cov}\ (Z_k, Z_{k+1})$ is calculated by conditioning on the pair of individuals who make the k^{th} and $(k+1)^{th}$ purchases which we denote by (I_k, I_{k+1}), where I_k (I_{k+1}) is equal to i if the i^{th} individual makes the k^{th} ($(k+1)^{th}$) purchase, i=1,2,...n. For the case of 2 individuals, say A and B, the pairs would be (A,A), (A,B), (B,A), and (B,B). In general, there would be n^2 possible pairs of $(I_k,\ I_{k+1})$.

First, we find the joint distribution of (I_k, I_{k+1}). Using the argument presented in the previous section for deriving (2.3) and (2.4), we have

$$(3.2)\quad P(I_{k+1}=i,\ I_k=i) = P(I_k=i)\ P(I_{k+1}=i \mid I_k=i) = (1/n)\ \alpha_{n-1}$$

and

$$(3.3)\quad P(I_{k+1}=j,\ I_k=i) = P(I_k=i)\ P(I_{k+1}\neq i \mid I_k=i)\ P(I_{k+1}=j \mid I_k=i,\ I_{k+1}\neq i)$$
$$= (1/n)(1-\alpha_{n-1})[1/(n-1)] \quad \text{for } i\neq j.$$

It then follows from a well-known conditional covariance formula (Barlow and Proschan, 1975, p. 30) that

$$(3.4)\qquad \mathrm{Cov}(Z_k,\ Z_{k+1}) = E[\mathrm{Cov}(Z_k,\ Z_{k+1} \mid I_k,\ I_{k+1})] +$$
$$\mathrm{Cov}[E(Z_k \mid I_k,\ I_{k+1}),\ E\ (Z_{k+1} \mid I_k,\ I_{k+1})].$$

Noting that the two processes $\{X_k^i, k \geq 1\}$ and $\{X_k^i, k \geq 1\}$ are independent when $i \neq j$, the first term above can be evaluated as

$$E[\mathrm{Cov}(Z_k,\ Z_{k+1} \mid I_k,\ I_{k+1})]$$
$$= \sum_{i=1}^{n} \mathrm{Cov}(X_k^i,\ X_{k+1}^i)\ P(I_k=i,\ I_{k+1}=i)$$
$$+ \sum_{i\neq j} \mathrm{Cov}(X_k^i,\ X_{k+1}^j)\ P(I_k=i,\ I_{k+1}=j)$$
$$= \sum_{i=1}^{n} \mathrm{Cov}(X_k^i,\ X_{k+1}^i)\ P(I_k=i,\ I_{k+1}=i).$$

Substituting (3.2) into the above expression, we get

$$(3.5)\qquad E[\mathrm{Cov}(Z_k,\ Z_{k+1} \mid I_k,\ I_{k+1})] = (1/n)\alpha_{n-1}\ \left[\sum_{i=1}^{n} \mathrm{Cov}\ (X_k^i,\ X_{k+1}^i)\right].$$

Next, by definition of covariance and properties of conditional expectations, the second term in (3.4) is equal to

$$E[E(Z_k|I_k)E(Z_{k+1}|I_{k+1})] - E(Z_k)\ E(Z_{k+1}), \tag{3.6}$$

where

$$E(Z_{k+1}) = E(Z_k) = P(Z_k=1)$$

$$= \sum_{i=1}^{n} P(Z_k=1|I_k=i)\ P(I_k=i) \tag{3.7}$$

$$= [\sum_{i=1}^{n} \pi_{1i}]\ (1/n).$$

Again, the first term in (3.6) is evaluated by conditioning on possible pairs of (I_k,I_{k+1}) and using (3.2) and (3.3); it reduces, after some algebra, to

$$(1/n)\alpha_{n-1}\ [\sum_{i=1}^{n} \pi_{1i}^2] + \frac{1}{n(n-1)}\ (1-\alpha_{n-1})[\sum_{i\neq j} \pi_{1i}\ \pi_{1j}]. \tag{3.8}$$

Using (3.5), (3.7) and (3.8), (3.4) can now be written as

$$Cov(Z_k,\ Z_{k+1}) = (1/n)\alpha_{n-1}\ [\sum_{i=1}^{n} Cov(X_k^i,\ X_{k+1}^i)$$

$$+ \sum_{i=1}^{n} \pi_{1i}^2] + \frac{1}{n(n-1)}\ (1-\alpha_{n-1})[\sum_{i\neq j} \pi_{1i}\ \pi_{1j}]. \tag{3.9}$$

$$- (1/n^2)\ (\sum_{i=1}^{n} \pi_{1i})^2.$$

Finally, Var (Z_k) is given by

$$Var(Z_k) = P(Z_k=1)\ P(Z_k=0)$$

$$= [(1/n) \sum_{i=1}^{n} \pi_{1i}]\ [(1/n) \sum_{i=1}^{n} \pi_{0i}] \tag{3.10}$$

thus completing the evaluation of (3.1).

Summarizing, the final expression for D is given by

$$D = \{(1/n)\alpha_{n-1}\,[\sum_{i=1}^{n} \mathrm{Cov}(X_k^i, X_{k+1}^i) + \sum_{i=1}^{n} \pi_{1i}^2\,] \tag{3.11}$$

$$+ \frac{1}{n(n-1)} \quad (1-\alpha_{n-1})[\sum_{i\neq j} \pi_{1i}\pi_{1j}] - (1/n^2)\ [\sum_{i=1}^{n} \pi_{1i}]^2\}/\mathrm{Var}(Z_k).$$

For example, if F is exponential then $\alpha_{n-1} = 1/n$ and using this value of α_{n-1} in (3.8) and (3.11), we get after some algebra the following interesting results: for exponential interpurchase times,

$$\mathrm{Cov}(Z_k, Z_{k+1}) = (1/n^2) \sum_{i=1}^{n} \mathrm{Cov}(X_k^i, X_{k+1}^i)$$

and

$$D = [\sum_{i=1}^{n} \mathrm{Cov}(X_k^i, X_{k+1}^i)] \,/\, [(\sum_{i=1}^{n} \pi_{1i})(\sum_{i=1}^{n} \pi_{0i})].$$

Note that, $P(Y < Y_e) < 1$ and hence $\alpha_{n-1} \to 0$ as $n\to\infty$. Now, assuming that $\lim\inf \mathrm{Var}(Z_k) = c>0$, (3.11) implies that

$$\lim\sup |D| \le \frac{1}{c} \lim\sup \left| \frac{1}{n(n-1)} \quad [\sum_{i\neq j}^{m} \pi_{1i}\pi_{1j}] - (1/n^2)(\sum_{i=1}^{m} \pi_{1i}^2) \right|$$

$$= \frac{1}{c} \lim\sup \left| - (1/n^2) \sum_{i=1}^{n} \pi_{1i}^2 \right| \qquad \text{(after replacing n-1 by n)}$$

$$= 0.$$

This completes the proof of the theorem.

An important implication of this theorem is that zero-order brand switching behavior at the household level is a somewhat expected phenomenon, especially when the number of effective individuals in the household in large; and thus it may not convey much information about individuals' choice behavior.

4. Bounds on D

The asymptotic result (Theorem 2) in the previous section is applicable only when the family size is "large". For moderate or small family sizes and suitable interpurchase-time distributions, one must compute the values of D explicitly in order to get a good feel as to how "close" a particular household brand choice process is to zero-order process. KMW show that for exponential interpurchase times,

$$| D | \leq 1/n.$$

The questions that can be raised are the following:

a) Is exponential distribution a realistic assumption for the interpurchase-time distribution?

b) Is it possible to analytically characterize the behavior of D when the interpurchase-time distribution is not exponential?

As a response to the first question, Chatfield and Goodhart (1975) have argued that the assumption of exponentially distributed interpurchase times is perhaps not very realistic because individuals are unlikely to make another purchase immediately following a purchase. They therefore suggested and provided some empirical support for the use of Erlang distribution as a better approximation of interpurchase-time distributions. Recently Gupta (1987) while analyzing scanner panel data for regular ground coffee, provides empirical evidence that the interpurchase-time distribution is Erlang-2.

Here we provide an answer to the second question and show that it is possible to provide a bound for D for the very general situation of NBUE (New Better than Used in Expectation) interpurchase times. It is important to note that Erlang and hence also exponential random variables are NBUE. The NBUE assumption can be interpreted as : the mean time to next purchase of the person making the current purchase is greater than or equal to that of any other person; i.e., Y is an NBUE random variable if and only if

$$E(Y-t|Y>t) \le E(Y) \text{ for all } t \ge 0.$$

It is well known (Ross 1983, p. 273) that Y is NBUE if and only if Y is stochastically larger than Y_e, i.e.,

$$Y \sim \text{NBUE} \iff P(Y > t) \ge P(Y_e > t) \text{ for all } t \ge 0,$$

and hence $P(Y < Y_{e,n-1}) \le P(Y_e < Y_{e,n-1}) = 1/n$. This fact, together with (3.11), implies that

$$\limsup |D| \le \limsup \frac{1}{cn} \quad \alpha_{n-1} \left| \sum_{i=1}^{n} \mathrm{Cov}(X_k^i, X_{k+1}^i) + \right.$$

$$\left. \sum_{i=1}^{n} \pi_{1i}^2 - \frac{1}{n-1} \sum_{i \ne j} \pi_{1j}\pi_{1j} \right| + \frac{1}{c} \left| - \frac{1}{n} \sum_{i=1}^{n} \pi_{1i}^2 \right|$$

$$\le \limsup \{ \frac{1}{cn^2} \sum_{i=1}^{n} | \mathrm{Cov}(X_k^i, X_{k+1}^i) | + \frac{2}{cn^2} \sum_{i=1}^{n} \pi_{1i}^2$$

$$+ \frac{1}{c(n-1)n^2} \sum_{i \ne j} \pi_{1i}\, \pi_{1j} \}$$

$$= \limsup \left(\frac{1}{cn} + \frac{2}{cn} + \frac{2}{cn} \right)$$

$$= \limsup \left(\frac{5}{cn} \right).$$

Thus, the speed of convergence of D to zero is again no worse than O(1/n) for the wide class of NBUE interpurchase times.

5. Conclusion

This paper generalizes the results of KMW and shows that their results hold even when the interpurchase times have a well-behaved, arbitrary distribution. We therefore provide additional support to their main conclusion that the empirically observed zero-order brand switching behavior at the household level does not convey much information about individuals' switching behavior. Thus, one should be careful when attempting to extrapolate household brand switching behavior to that of individuals.

A useful outcome of our analysis is the possibility of a more powerful statistical test for the zero-order hypothesis. Our Theorem 1 can be used to develop such a test which could then be employed in empirical work.

Our results also suggest that more empirical work should be done on product categories where only one member of a household is a consumer of the product. Clearly, in this instance, the aggregation issue would be irrelevant, and tests of the zero-order hypothesis might then yield useful information.

References

Barlow, R.E. and F. Proschan (1975), Statistical Theory of Reliability and Life Testing: Probability Models, New York: Holt, Rinehart and Winston.

Bass, Frank M. (1974), "The Theory of Stochastic Preference and Brand Switching," Journal of Marketing Research, XI (Feb.), 1-20.

Bass, Frank M., Moshe M. Givon, Manohar U. Kalwani, David Reibstein, and Gordon P. Wright (1984), "An Investigation into the Order of the Brand Choice Processes," Marketing Science, Vol. 3, No. 4 (Fall), 267-287.

Blattberg, Robert C. and Subrata Sen (1975), "Market Segments and Stochastic Brand Choice Models," Journal of Marketing Research, 13 (Feb.), 34-45.

Chatfield, C. and G. J. Goodhart (1975), "A Consumer Purchasing Model with Erlang Inter-Purchase Time," Journal of the American Statistical Association, 68 (Dec.), 828-835.

Ehrenberg, A.S.C. (1972), Repeat-Buying: Theory and Applications, Amsterdam: North-Holland.

Frank, R.E. (1962), "Brand Choice as a Probability Process," Journal of Business, 35, 43-56.

Gupta, Sunil (1987), "Impact of Sales Promotions on When, What, and How Much to Buy," Working Paper, Graduate School of Management, UCLA.

Jeuland, Abel P., Frank M. Bass, and Gordon P. Wright (1980), "A Multibrand Stochastic Model Compounding Heterogeneous Erlang Timing and Multinomial Choice Processes," Operations Research, Vol. 28, No. 2 (March-April), 255-277.

Kahn, Barbara E., Donald G. Morrison and Gordon P. Wright (1986), "Aggregating Individual Purchases to the Household Level," Marketing Science, Vol. 5, No. 3 (Summer), 260-268.

Massy, William F. (1966), "Order and Homogeneity of Family Specific Brand Switching Processes," Journal of Marketing Research, 3 (Feb.), 48-54.

Ross, Sheldon M. (1983), Stochastic Processes, New York: John Wiley & Sons.

Ross, Sheldon M. (1985), Introduction to Probability Models, 3rd edition, New York: Academic Press.

Wierenga, B. (1974), An Investigation of Brand Choice Processes, Rotterdam: Universitaire Pers, Rotterdam.