Discussion Paper No. 729 ADVERTISING AND LIMIT PRICING

by

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Abstract

We enrich the Milgrom-Roberts (1982) limit pricing model to allow an incumbent to signal his costs with price and advertisements. When advertising is a purely dissipative signal, it is not employed; however, if advertising has a positive affect on demand, then the signaling game creates an upward distortion in advertising, relative to the full information level of advertising. In either case, the signaling game results in a downward distortion in pre-entry pricing. Recent refinements of the sequential equilibrium concept are featured.

1. Introduction

When an incumbent firm has private information about the profitability of entry, a potential entrant has reason to use the incumbent's actions as signals of this information. In particular, as argued by Milgrom and Roberts (1982), an incumbent's price may be used as a signal of his costs. Their fundamental finding is that the entrant's effort to infer cost information creates a downward distortion in pre-entry pricing. 1

In this paper, we enrich the Milgrom-Roberts model by allowing the incumbent to signal his costs with price and advertisements. Specifically, we consider a two-period model in which the incumbent is privately informed as to whether his costs are high or low. In the first period, the incumbent chooses a price and a level of advertising, and these choices determine his first period profit. A single entrant observes the price-advertising selection and attempts to infer the incumbent's costs. We assume that entry can occur in the second period only and that entry is profitable if and only if the incumbent has high costs.

Clearly, in such a setting, a low-cost incumbent has incentive to separate from his high-cost counterpart. That is, a low-cost incumbent will tend to choose price-advertising pairs which would be especially unattractive, were the incumbent to have high costs. It then follows that the low-cost incumbent's choice will typically differ from the choice he would make in a complete information environment. The intriguing question concerns the direction of the distortion.

Initially, we assume that advertising does not affect consumer demand.²

Advertising is then regarded as a purely dissipative signal. A low-cost incumbent can nevertheless signal his strength with a large advertising expenditure. Indeed, a continuum of sequential equilibria (Kreps and Wilson

(1982)) always exists in which advertising is used as a signal. Since the incumbent would not advertise if the entrant were fully informed, these equilibria characterize an upward distortion in advertising (the direction of the price distortion varies).

Equilibria with positive dissipative advertising are not especially plausible, however. Since advertising is on the margin just as costly for a low-cost as for a high-cost incumbent, it is rather expensive for the former to signal his costs with an advertising outlay. Conversely, a price cut increases demand and on the margin "hurts" a high-cost incumbent more than a low-cost incumbent. The most efficient way for the low-cost incumbent to separate, then, is with a low price and no advertising.

The sequential equilibrium concept is not sensitive to the relative efficiencies of different signals. However, two recent refinements of the concept are responsive to efficiency differences. The first refinement involves the elimination of dominated strategies (Milgrom and Roberts (1986), Moulin (1981)). When dominated strategies are removed, a single separating equilibrium exists, and in this equilibrium the low-cost incumbent separates with a low price and no advertising. Thus, after dominated strategies are eliminated, the unique separating equilibrium is in fact the equilibrium initially proposed by Milgrom and Roberts.

There may also exist pooling equilibria in which both incumbent types choose the same price-advertising pair. Even after eliminating dominated strategies, some of these equilibria involve positive advertising levels. However, when a second refinement (Kreps (1984)) is employed, we are again able to remove all equilibria in which advertising occurs. In the remaining pooling equilibria, price is never higher than the full information, low-cost monopoly price.

In both cases, the distortion created by efficient signaling involves the incumbent behaving as if there were complete information but his costs were lower. This is because the difference in costs which determines the relative efficiency of signals exerts the same effect on signaling as it does on complete-information optimal choices: lower costs create an incentive to reduce prices, but still choose zero advertising. Any signaling equilibrium with positive advertising is necessarily inefficient with respect to these cost-generated incentives. Thus, if advertising is a purely dissipative signal of incumbent cost, then price will be distorted downward and the advertising signal will not be employed nor, therefore, distorted.

We next consider the possibility that advertising has a positive affect on consumer demand. The intuition developed above is easily extended. A low-cost incumbent is "hurt" less by an increase in demand than is a high-cost incumbent. Since low prices and large advertising outlays lead to high demand, the most efficient way for a low-cost incumbent to separate is with a low price and a large amount of advertising. Indeed, we prove the following: If advertising has a positive affect on consumer demand, then, once dominated strategies are removed, a unique separating equilibrium exists and this equilibrium is characterized by a downward distortion in price and an upward distortion in advertising. Moreover, the price-advertising pair selected by the low-cost incumbent is in fact the complete-information selection for an incumbent with even lower costs. Efficient signaling again creates a "cost-reducing" distortion, in that the low-cost incumbent separates most efficiently by imitating the complete-information behavior of an incumbent with lower costs. A similar result occurs with pooling equilibria: application of the second refinement removes all pooling strategies which do not give complete-information optimal choices for some

cost level lower than that of the low-cost incumbent. The remaining pooling equilibria then entail a downward distortion in price and an upward distortion in advertising.

Our ideas relate to a large volume of previous research. Bain (1956), Salop (1979), and Schmalensee (1983), for example, have all explored the sense in which advertising can deter entry. They do not, however, investigate the signaling role of advertising. Nelson (1970, 1974), Kihlstrom and Riordan (1984), and Milgrom and Roberts (1986) have each argued that dissipative advertising can signal product quality in models without entry. The recent Milgrom-Roberts paper is particularly noteworthy, as they also feature sequential equilibrium refinements in a multiple signal model. To our knowledge, the present paper is the first to analyze the signaling role of advertising in a model of entry.

The paper is organized in seven sections. The basic model is described in Section 2. In Sections 3 and 4, respectively, we characterize the set of separating equilibria and the unique undominated separating sequential equilibrium. Pooling equilibria are discussed in Section 5. Advertising is modelled as a purely dissipative signal in these sections. Then, in Section 6, we assume that advertising has a positive affect on consumer demand and characterize the corresponding unique undominated separating equilibrium, as well as the set of refined pooling equilibria. Concluding thoughts are offered in Section 7.

2. Model

Consider the following situation. There are two firms, an incumbent and a potential entrant, which interact for two periods in a market for a homogenous good. In the initial period the incumbent monopolizes the market, while at the start of the second period the entrant may choose to enter the

market. If entry occurs, the firms earn Cournot duopoly profits for the second period, while if entry does not occur the incumbent remains a monopolist. The entrant makes his choice without having complete knowledge as to the incumbent's production costs, though he might be able to infer cost information through observing the incumbent's first-period price and advertising decisions. This process of inference might in turn distort the incumbent's incentives for choosing these variables.

Consumer behavior is summarized with a market demand function X(P), where $P \geqslant 0$ denotes the market price. X(P) is assumed to be bounded, continuous, strictly decreasing for $P < \hat{P}$, where $\hat{P} > 0$, and X(P) = 0 for $P \geqslant \hat{P}$. Production costs are linear in quantity and fixed costs are zero. While the entrant does not directly observe the incumbent's unit cost, he does know that it is one of two possible levels, C^L and C^H , with $0 < C^L < C^H$. Let $\rho \in (0,1)$ give the entrant's prior probability assessment on the event that the incumbent's unit cost is C^H .

In the first period, the incumbent observes his unit cost, either \textbf{C}^L or \textbf{C}^H , and chooses the first-period price and advertising levels. Advertising is here interpreted simply as observable wasteful expenditure; the case in which advertising directly affects market demand is considered in section 6. First-period profits may be written:

$$\Pi^{i}(P,A) = (P - C^{i})X(P) - A, i = L,H$$

Let P^i give the maximizer of $\Pi^i(P,A)$, and suppose $0 < P^L < P^H < \hat{P}$. Suppose, moreover, that $\Pi^i(P,A)$ strictly increases in P for $P < P^i$ and strictly decreases for $P^i < P < \hat{P}$.

Let Π_{C}^{i} , i = L,H, denote the incumbent's Cournot profits in the second

period if entry occurs, and assume $\Pi^{\mathbf{i}}(P^{\mathbf{i}},0) > \Pi_{C}^{\mathbf{i}}$ and $\Pi_{C}^{L} > \Pi_{C}^{H} > 0$. Let F > 0 represent the start-up cost for the entrant, and denote by $\Pi_{E}^{\mathbf{i}}$ the entrant's Cournot profits when the incumbent's unit costs are $C^{\mathbf{i}}$. Assume the entrant would desire to enter if the incumbent's unit costs were high but not if they were low, which means $\Pi_{E}^{H} > F$ and $\Pi_{E}^{L} < F$.

While the entrant cannot observe unit cost prior to making his entry choice, he can observe the incumbent's first-period price and advertising decisions. Let $\rho(P,A) \in [0,1]$ give the entrant's posterior belief that unit cost is high when P and A are observed.

As a formal matter, we model this situation as an extensive-form game having four stages. First, "Nature" chooses the incumbent's unit costs, with ρ being the probability that c^H is chosen. Next, the incumbent observes c^i and chooses P and A; these must be nonnegative real numbers. The entrant then observes P and A, but not c^i , and chooses either to enter or not enter. Finally, the firms play Cournot strategies in the second period, if entry has occurred, and otherwise the incumbent receives monopoly profits. We will consider only pure-strategy sequential equilibria of this game. Let the entrant's strategy be denoted by $\hat{R}(P,A) \in \{0,1\}$, where $\hat{R} = 1$ indicates entry. The collection $\{(\hat{P}^i,\hat{A}^i)_{i=L,H},\hat{R}(P,A),\hat{\rho}(P,A)\}$ gives an equilibrium if the following three conditions are satisfied.

(A) Optimality for incumbent: For i = L,H:

$$(\hat{P}^{i}, \hat{A}^{i}) \in \underset{(P,A)}{\operatorname{arg,max}} \{\Pi^{i}(P,A) + \delta[\hat{R}(P,A)\Pi_{c}^{i} + (1 - \hat{R}(P,A))\Pi^{i}(P^{i},0)]\}$$

where $\delta \in (0,1)$ is the incumbent's discount factor.

(B) Optimality for entrant: For all (P,A), R(P,A) = 1 if and only if:

$$\hat{\rho}(P,A)\Pi_{E}^{H} + (1 - \hat{\rho}(P,A))\Pi_{E}^{L} > F$$

(C) Bayes-consistency of beliefs: If $(\hat{P}^L, \hat{A}^L) \neq (\hat{P}^H, \hat{A}^H)$:

$$\hat{\rho}(\hat{P}^L, \hat{A}^L) = 0, \quad \hat{\rho}(\hat{P}^H, \hat{A}^H) = 1$$

If
$$(\hat{P}^L, \hat{A}^L) = (\hat{P}, \hat{A}^H)$$
:

$$\hat{\rho}(\hat{P}^L, \hat{A}^L) = \rho$$

In other words, (C) requires the entrant's posterior beliefs as to C^1 to be obtained from his prior beliefs by using Bayes' rule together with the incumbent's equilibrium strategies. For (P,A) which the incumbent does not choose under either cost level, Bayes' rule cannot be used and thus $\hat{\rho}(P,A)$ may take any value; it is here that arbitrary off-equilibrium-path beliefs are allowed.

Separating Equilibria

If $(\hat{P}^L, \hat{A}^L) \neq (\hat{P}^H, \hat{A}^H)$, then observing price and advertising allows the entrant to become fully informed of the incumbent's unit cost before making his entry decision; this is called a <u>separating equilibrium</u>. In this section we will characterize the set of separating equilibria. A large class of possible price and advertising levels can arise in separating equilibria, in which the incumbent's incentives might be distorted in any direction.

Clearly, in any separating equilibrium, the incumbent will make his optimal one-period monopoly choice $(P^H,0)$ if $C^i=C^H$, since any $(P^H,A^H) \neq (P^H,0)$ would imply:

$$\Pi^{H}(\hat{P}^{H}, \hat{A}^{H}) + \delta\Pi_{C}^{H} < \Pi^{H}(\hat{P}^{H}, 0) + \delta[\hat{R}(\hat{P}^{H}, 0)\Pi_{C}^{H} + (1 - \hat{R}(\hat{P}^{H}, 0))\Pi^{H}(\hat{P}^{H}, 0)]$$

which violates (A). Let us exploit the arbitrariness of off-equilibrium-path beliefs by setting $\hat{\rho}(P,A)=1$ for all $(P,A)\neq (P^L,A^L)$, meaning that the entrant always enters unless he observes the equilibrium choices of the low-cost incumbent. In this case, we can be sure (A) is satisfied for i=H as long as:

$$\Pi^{H}(\hat{P}^{L}, \hat{A}^{L}) + \delta \Pi^{H}(P^{H}, 0) \leq \Pi^{H}(P^{H}, 0) + \delta \Pi^{H}_{C}$$

This may be expressed:

(1)
$$\Pi^{H}(\hat{P}^{L}, \hat{A}^{L}) \leq (1-\delta)\Pi^{H}(\hat{P}^{H}, 0) + \delta\Pi_{C}^{H} = \Pi^{H}$$

Thus, (\hat{P}^L, \hat{A}^L) must give the incumbent sufficiently low first-period profits under $c^i = c^H$ to discourage him from choosing it and deterring entry. Figure la depicts the isoprofit curve $\Pi^H(P,A) = \overline{\Pi}^H$, so that the shaded region H then gives the set of possible (\hat{P}^L, \hat{A}^L) which satisfy (1).

Moreover, if the incumbent does not prefer (\hat{P}^L, \hat{A}^L) under $C^i = C^L$, then his optimal choice given the beliefs we have specified will clearly be $(P^L, 0)$. This means (A) will be satisfied for i = L if:

(2)
$$\Pi^{L}(\hat{P}^{L}, \hat{A}^{L}) \geq (1-\delta)\Pi^{L}(\hat{P}^{L}, 0) + \delta\Pi_{C}^{L} = \underline{\Pi}^{L}$$

The set of (\hat{P}^L, \hat{A}^L) satisfying (2) is depicted in Figure 1b as the shaded region L. Now, as long as $(\hat{P}^L, \hat{A}^L) \in H \cap L$, it is certain that (A) is

satisfied under our specification of beliefs. Further, if either $(\hat{P}^L, \hat{A}^L) \notin H$ or $(\hat{P}^L, \hat{A}^L) \notin L$, then (A) will be violated for either i = H or i = L, respectively, for <u>any</u> specification of beliefs. (\hat{P}^L, \hat{A}^L) cannot then give a separating equilibrium. This completes the proof of:

Theorem 1: The set of price-advertising strategies which support separating equilibria is given by:

$$\{(\hat{P}^{i}, \hat{A}^{i})_{i=L,H} | (\hat{P}^{L}, \hat{A}^{L}) \in H \cap L, (\hat{P}^{H}, \hat{A}^{H}) = (P^{H}, 0)\}$$

This implies, of course, that separating equilibria do not exist if H \cap L is empty. We have existence if and only if the left-hand intercept of the isoprofit curve $\Pi^H(P,A) = \overline{\Pi}^H$, shown as \underline{P}^H in Figure 2, lies to the right of the intercept \underline{P}^L of $\Pi^L(P,A) = \underline{\Pi}^L$. Because the marginal rate of substitution of A for P is strictly increasing in unit costs, no isoprofit curve of \underline{C}^L can cut an isoprofit curve of \underline{C}^H from beneath. Thus, the region L would lie entirely beneath the curve $\overline{\Pi}^H$ if $\underline{P}^L > \underline{P}^H$, and H \cap L would be empty. As a sufficient condition for existence, lets assume that Cournot competition compresses the profit differential between cost levels:

(3)
$$\Pi_C^L - \Pi_C^H \le \Pi^L(P^L, 0) - \Pi^H(P^H, 0)$$

Corollary 1: If (3) holds, then H ∩ L is nonempty.

Proof: Given in the appendix.

By rearranging (3), it is easily seen that our sufficiency condition

amounts to assuming the incumbent gains more from deterring entry when his costs are low than when costs are high. Thus, the low-cost incumbent can credibly separate from his high-cost counterpart, since he has more benefits to be dissipated via first-period pricing and advertising than would be recoverable under high costs.

Theorem 1 shows that a great many separating equilibria are possible, involving nearly unrestricted predictions as to pricing and advertising distortions. Figure 3 illustrates one possibility. The incumbent may choose $(\underline{P}^H,0)$ when his costs are low, which gives a separating equilibrium, but (P^L,\overline{A}) also gives an equilibrium; either price or advertising alone may be distorted, and certainly there are equilibria in which both are distorted. Moreover, the effect on pricing may be in either direction: $(\widetilde{P},\widetilde{A})$ gives an equilibrium in which the incumbent raises price above the one-period monopoly level, in contrast to the pricing implications of Milgrom and Roberts' original analysis.

4. Undominated Separating Equilibria

It is, of course, the arbitrariness of off-equilibrium-path beliefs which gives rise to so many separating equilibria. The low-cost incumbent can be induced to choose any (\hat{P}^L, \hat{A}^L) in H \cap L by the threat of certain entry following a deviation, based on very optimistic off-equilibrium-path beliefs by the entrant. In this section we will argue that such beliefs represent excessively unsophisticated behavior on the part of the entrant. If the entrant's inferences are required to be a bit more sophisticated, there will exist only one separating equilibrium, in which the incumbent never chooses positive advertising.

In particular, the entrant should take some account of the incumbent's

incentives to price and advertise when he draws inferences, even if the observed (P,A) could not have arisen under the equilibrium strategies. At the very least, the entrant should not believe the incumbent has played a strategy which he could never have had an incentive to play. This means the entrant rules out all possibility that the incumbent plays strictly dominated strategies. We will proceed by refining the set of separating equilibria through elimination of dominated strategies, as proposed by Moulin (1981) and Milgrom and Roberts (1986).

To this end, the pair (P,A) will be called dominated for C^{1} if:

$$\Pi^{i}(P,A) + \delta\Pi^{i}(P^{i},0) < \Pi^{i}(P^{i},0) + \delta\Pi^{i}_{C}$$

Thus, (P,A) is dominated for C^1 if it yields less profit under the best entry conditions than does the one-period monopoly optimum under the worst conditions. An equilibrium will be called <u>undominated</u> if $\rho(P,A) = 0$ whenever (P,A) is dominated for C^H but not for C^L , and similarly for the converse.

To study undominated equilibria, it is helpful to extend the definition of the profit function to arbitrary cost levels. Thus, for <u>any</u> real number C, put:

$$\Pi(P,A|C) = (P-C)X(P) - A$$

Let $\Psi(C) = (P(C), A(C))$ give the price and advertising levels which maximize $\Pi(P,A|C)$, for every C. Note that A(C) = 0 for all C and $P(\widetilde{C}) = 0$ for some \widetilde{C} . Assume that P(C) is continuous and strictly decreasing for P(C) > 0.6

Clearly, any $(P,A) \in H$ with $\Pi^H(P,A) < \overline{\Pi}^H$ is dominated for C^H , but as long as $(P,A) \in L$ it is not dominated for C^L . Thus, reasonable beliefs for the

entrant entail $\hat{\rho}(P,A) = 0$ at all points of $H \cap L$ for which $\Pi^H(P,A) < \overline{\Pi}^H$. If (A) is satisfied for i = L, it follows that (\hat{P}^L, \hat{A}^L) must maximize the low-cost incumbent's pre-entry profit on H, since he may obtain the most favorable entry conditions for points arbitrarily close to the maximizer. This is shown in the appendix as:

<u>Lemma</u>: In any undominated separating equilibrium: $(\hat{P}^L, \hat{A}^L) \in \underset{(P,A) \in H}{\text{argmax}} \Pi^L(P,A)$

Thus, it remains to identify this maximizer. If $P^L \leq \underline{P}^H$, then clearly it is $(P^L,0)$. Suppose $P^L > \underline{P}^H > \underline{P}^L$. Our assumptions assure that $P(C^0) = \underline{P}^H$ for some $C^0 < C^L$. In fact, it is precisely the point $(\underline{P}^H,0)$, which is optimal for C^0 , that maximizes $\Pi^L(P,A)$ on H. This is demonstrated in the appendix, and gives:

Theorem 2: There exists at most one undominated separating equilibrium, in which $(\hat{P}^L, \hat{A}^L) = (\min\{P^L, \underline{P}^H\}, 0)$.

If (3) holds, of course, then we have existence of this equilibrium.

In view of Theorem 2, there can be only one separating equilibrium in which the entrant has reasonable beliefs, involving the low-cost incumbent reducing price by the minimum amount necessary for credible signaling, and choosing zero advertising. This represents the most efficient way for the low-cost incumbent to credibly separate from his high-cost counterpart. Efficiency is always attained by imitating complete-information optimal behavior under some still-lower cost level, since it is the difference in costs which determines the relative efficiency of the signals. In particular, price reductions are more harmful to the incumbent when costs are high than

when costs are low, while increases in advertising are equally harmful under either cost level. This notion of relative harmfulness translates directly into incentives to choose price and advertising under complete information. Thus, we may view efficient signaling as entailing a "cost-reducing distortion," whereby incomplete cost information distorts incentives in a manner identical to a reduction in costs. What makes efficient signaling possible is that the entrant does not induce a countervailing incentive through unreasonable inferences.

5. Pooling Equilibria

Thus far we have considered equilibria in which the entrant becomes fully informed, but there can also exist equilibria in which the entrant learns nothing at all from observing price and advertising. These pooling equilibria are characterized by $(P^L, A^L) = (P^H, A^H)$. It is immediate, though, that pooling equilibria cannot exist when entry would be favorable under the entrant's prior beliefs. For suppose we have:

(4)
$$\rho \Pi_{E}^{H} + (1-\rho)\Pi_{E}^{L} > F$$

Then $\hat{R}(\hat{P}^L, \hat{A}^L) = 1$ in a pooling equilibrium, and the incumbent could always do better under at least one cost level by deviating to his one-period optimum, for any specification of off-equilibrium-path beliefs. Under (4), then, the entrant will always gain information in equilibrium.

When (4) does not hold, however, there can exist pooling equilibria in which positive advertising is chosen, and such equilibria can even be undominated. To see this, suppose (3) is satisfied and that $\underline{P}^H < \underline{P}^L$. Choose \underline{A}^0 to satisfy:

(5)
$$0 < A^0 \le \Pi^L(P^L, 0) - \Pi^L(P^H, 0)$$

It then follows that (P^L,A^0) gives strategies for an undominated pooling equilibrium. First, (B) will be satisfied if $\widehat{R}(P^L,A^0)=0$. For all (P,A) such that $\Pi^H(P,A)<\overline{\Pi}^H$, we may set $\widehat{\rho}(P,A)=0$ in an undominated equilibrium, and $\widehat{\rho}(P,A)=1$ may be specified for all the remaining $(P,A)\neq (P^L,A^0)$.

Now, under low costs the incumbent prefers $(\underline{p}^H,0)$ with no entry to $(P^L,0)$ followed by entry, since we have assumed (3) and $\underline{p}^H < P^L$. Thus, by (5) he will certainly prefer (P^L,A^0) to $(P^L,0)$. In other words, (P^L,A^0) is in region L, and (A) will be satisfied for i=L. It is true, moreover, that isoprofit curve $\Pi^L(P,A) = \Pi^L(\underline{p}^H,0)$ lies strictly below $\overline{\Pi}^H$ for all $P > \underline{p}^H$, as shown in Figure 4. This guarantees (P^L,A^0) is not in H, and under high costs the incumbent will prefer (P^L,A^0) with no entry to his next best choice, which is $(P^H,0)$ followed by entry. Thus, (A) is satisfied for i=H, and we have an undominated pooling equilibrium with positive advertising.

The equilibrium derived above does not require the entrant to believe that the incumbent takes actions which could never possibly be in his interest. Still, the entrant is not very sophisticated in taking account of the incumbent's incentives. Suppose the entrant anticipates an equilibrium in which the incumbent will choose (P^L,A^0) under either cost level, but then unexpectedly observes the values (\tilde{P},\tilde{A}) shown in Figure 4. What should the entrant then believe? With high costs, the incumbent would have preferred the equilibrium choice (P^L,A^0) followed by no entry to (\tilde{P},\tilde{A}) , no matter what entry decision the latter elicited. This fact could easily be deduced by the entrant, so that it does not seem reasonable for him to entertain the possibility that the high-cost incumbent had deviated to (\tilde{P},\tilde{A}) . Thus,

 $\rho(\widetilde{P},\widetilde{A})=0$ gives the appropriate inference. But in this case the incumbent would prefer $(\widetilde{P},\widetilde{A})$ under low costs, and the equilibrium would collapse.

To eliminate unreasonable beliefs of this sort, we must refine the equilibrium concept still further. Following Kreps (1984), we call a set of equilibrium beliefs unintuitive if there exists $(\widetilde{P}, \widetilde{A}) \neq (\widehat{P}^L, \widehat{A}^L)$, $(\widehat{P}^H, \widehat{A}^H)$ such that:

(6)
$$\Pi^{\mathrm{H}}(\widetilde{P},\widetilde{A}) + \delta\Pi^{\mathrm{H}}(P^{\mathrm{H}},0)$$

$$<\boldsymbol{\Pi}^{H}(\boldsymbol{\hat{P}}^{H},\boldsymbol{\hat{A}}^{H}) + \boldsymbol{\delta}\big[\boldsymbol{\hat{R}}(\boldsymbol{\hat{P}}^{H},\boldsymbol{\hat{A}}^{H})\boldsymbol{\Pi}_{\boldsymbol{C}}^{H} + \big(\boldsymbol{1} - \boldsymbol{\hat{R}}(\boldsymbol{\hat{P}}^{H},\boldsymbol{\hat{A}}^{H})\big)\boldsymbol{\Pi}^{H}(\boldsymbol{P}^{H},\boldsymbol{0})\big]$$

(7)
$$\Pi^{L}(\widetilde{P}, \widetilde{A}) + \delta \Pi^{L}(P^{L}, 0)$$

$$> \Pi^{\mathrm{L}}(\hat{\mathbf{P}}^{\mathrm{L}}, \hat{\mathbf{A}}^{\mathrm{L}}) + \delta \left[\hat{\mathbf{R}}(\hat{\mathbf{P}}^{\mathrm{L}}, \hat{\mathbf{A}}^{\mathrm{L}}) \Pi_{\mathrm{C}}^{\mathrm{L}} + \left(1 - \hat{\mathbf{R}}(\hat{\mathbf{P}}^{\mathrm{L}}, \hat{\mathbf{A}}^{\mathrm{L}}) \right) \Pi^{\mathrm{L}}(\mathbf{P}^{\mathrm{L}}, 0) \right]$$

The right-hand sides of (6) and (7) give the equilibrium profits of the incumbent under high and low costs, respectively. By (6), the high-cost incumbent prefers his equilibrium choice to $(\widetilde{P},\widetilde{A})$, even if the latter leads to the most favorable entry situation. (7) indicates that the low-cost incumbent would prefer $(\widetilde{P},\widetilde{A})$ to the equilibrium, as long as choosing $(\widetilde{P},\widetilde{A})$ convinced the entrant as to the true cost level. Thus, the equilibrium could be supported only by an "unintuitive" inference of $\widehat{\rho}(\widetilde{P},\widetilde{A}) > 0$. We call an equilibrium intuitive if it can be supported by beliefs which are not unintuitive.

Certainly, the pooling equilibrium derived above is unintuitive. In fact, any pooling equilibrium with positive advertising must be unintuitive. For if we let (\hat{P}, \hat{A}) with $\hat{A} > 0$ give pooling equilibrium strategies, then there will exist $\hat{P} < \hat{P}$ with $\Pi^H(\hat{P}, 0) = \Pi^H(\hat{P}, \hat{A})$. But this implies $\Pi^L(\hat{P}, 0) > \Pi^L(\hat{P}, \hat{A})$,

meaning that the low-cost incumbent would prefer some price slightly below $\underline{\underline{P}}$, with zero advertising, if this convinces the entrant who he is, while the high-cost incumbent would prefer $(\underline{P}, \underline{A})$ no matter what reaction the other choice elicited. The details are outlined in the appendix, and a similar argument shows that any pooling price must satisfy $\underline{P} \leqslant \underline{P}^L$. This gives:

Theorem 3: In any intuitive pooling equilibrium with strategies (P,A), $\hat{P} \in [\underline{P}^H, P^L]$ and $\hat{A} = 0.8$

Once again, the affect of cost on the relative efficiency of the two signals determines the set of reasonable outcomes. Inefficient pooling strategies imply existence of other strategies which increase the low-cost incumbent's pre-entry profits without harming his high-cost conterpart, so that equilibria are unintuitive precisely when they are inefficient in this sense. Since the same incentives operate under complete information, the incumbent is led by efficient signaling to behave as if information were complete but his costs were lower. This intuition holds for both separating and pooling equilibria, so that we may summarize with:

Corollary 2: In any undominated and intuitive equilibrium, $\hat{P}^L \leqslant P^L$, $\hat{P}^H \leqslant P^H$, and $\hat{A}^L = \hat{A}^H = 0$.

Therefore, signaling of cost information will not distort the incumbent's advertising decision, but it can give rise to a downward distortion in pricing.

6. Positive Advertising Effects

If advertising does influence demand, however, then signaling will in general distort both advertising and pricing decisions. It will no longer be most efficient for the low-cost incumbent to signal with price alone, but rather with a combination of both price and advertising.

Advertising might influence demand either by affecting consumer tastes directly, or by informing consumers as to the existence and function of the product. Demand will now be written X(P,A), with X strictly increasing in A on the region where demand is positive. Suppose X(P,A) is bounded and continuous. With $\Pi^{i}(P,A)$ now defined using X(P,A), let (P^{i},A^{i}) give the unique maximizer of $\Pi^{i}(P,A)$, i = L,H, with $X(P^{i},A^{i}) > 0$. For simplicity, we will not otherwise alter the earlier assumptions concerning second-period profits, so that Π^{i}_{C} and Π^{i}_{E} , i = L,H, obey the same restrictions as given above. 9 Equilibrium is defined exactly as before.

As above, for every real number C define:

$$\Pi(P,A|C) = (P-C)X(P,A) - A$$

To rule out "fat" isoprofit curves, assume that the level sets $\Pi(P,A|C) = k$ have empty interior for all C and k. Let $\Psi(C) = (P(C), A(C))$ give the unique maximizer of $\Pi(P,A|C)$, which we shall again suppose is continuous in C. It is easy to see that a reduction in C has a direct effect which tends to reduce P(C) and raise A(C), since increases in demand are more profitable when the markup is higher. But the cross effects of advertising on the profitability of price increases and <u>vice-versa</u> could counteract the direct effect, possibly leading cost reductions to have net effects on P(C) and A(C) which are in opposite directions. By assuming that the cross effects

are of sufficiently small magnitude, we can be sure that P(C) strictly increases and A(C) strictly decreases in C when P(C), A(C) > 0, at least for C \leq C . Note that Ψ (C) intersects the P = 0 axis for some cost level \widetilde{C} \leq 0, as shown in Figure 5.10

It is simple to show, as in section 3, that $(P^H, A^H) = (P^H, A^H)$ in any separating equilibrium, and that the set of possible separating equilibrium strategies for i = L is defined by:

$$\Pi^{\mathrm{H}}(\mathbf{\hat{P}^{L},\hat{A}^{L}}) \leq (1-\delta)\Pi^{\mathrm{H}}(\mathbf{P^{H},A^{H}}) + \delta\Pi^{\mathrm{H}}_{\mathbf{C}} = \Pi^{\mathrm{H}}$$

$$\boldsymbol{\Pi^{L}(\hat{\boldsymbol{P}}^{L}, \hat{\boldsymbol{A}}^{L})} \geq (1-\delta)\boldsymbol{\Pi^{L}(\boldsymbol{P}^{L}, \boldsymbol{A}^{L})} + \delta\boldsymbol{\Pi^{L}_{C}} = \boldsymbol{\underline{\Pi}^{L}}$$

The region H is once again the set of (P,A) which satisfy $\Pi^{H}(P,A) \leq \overline{\Pi}^{H}$. A pair (P,A) is called dominated for C^{i} if:

$$\Pi^{i}(P,A) + \delta\Pi^{i}(P^{i},A^{i}) < \Pi^{i}(P^{i},A^{i}) + \delta\Pi^{i}_{C}$$

and undominated equilibria are defined as before.

Now, the lemma applies here precisely as stated above, so that (\hat{P}^L, \hat{A}^L) cannot support an undominated separating equilibrium if it does not maximize $\Pi^L(P,A)$ on H. As long as $(P^L,A^L) \notin H$, the element of H which maximizes $\Pi^L(P,A)$ will again be the point of intersection of $\Psi(C)$ and Π^H , shown as $(P(C^0),A(C^0))$ in Figure 6. This is proven in the appendix, and establishes:

Theorem 4: If $(P^L, A^L) \notin H$, then there exists at most one undominated separating equilibrium, in which $(\hat{P}^L, \hat{A}^L) = (P(C^0), A(C^0))$ for some $C^0 < C^L$.

Further, we may define intuitive equilibria by replacing $(P^{i}, 0)$ with (P^{i}, A^{i}) in (6) and (7), and in the appendix we prove an analogous result:

Theorem 5: In any intuitive pooling equilibrium with strategies (P,A), $(P,A) = \Psi(C)$ for some $C \in [C^0,C^L]$.

Thus, it continues to be true that efficient signaling involves cost-reducing distortions, whether or not the equilibrium is informative. When advertising affects demand, however, reduced costs gives an incentive for greater advertising, at least when cross effects are small. It then becomes necessary to distort advertising upwards in order to equate as nearly as possible the relative efficiency of the two signals; in Figure 6, for example, this occurs at a point of tangency between $\overline{\Pi}^H$ and an isoprofit curve of the low-cost incumbent. Note further that Theorems 4 and 5 hold true whether or not $\Psi(C)$ slopes downward as C increases. From the proofs it is easily observed that the distortions must be cost-reducing and demand-increasing, whether or not they involve lower price and higher advertising in particular. We feel, however, that in most cases these give the directions of the distortions that will actually occur.

As a general matter, then, we may conclude that incomplete cost information distorts the incumbent's incentives towards those of a firm with lower costs. This leads to price reductions when advertising is purely dissipative, and both reduced price and increased advertising when there are positive advertising effects. The net effect of signaling is that demand is increased relative to complete information.

7. Conclusion

We have extended the Milgrom-Roberts (1982) model to the case in which the incumbent firm uses price and advertising to signal his costs. Our analysis indicates a downward distortion in price, whether advertising is dissipative or demand-enhancing. Limit pricing can therefore be expected to occur, even when the incumbent has the option of signaling costs with advertising. We have also argued that the signaling process does not distort the choice of a purely dissipative advertising variable; however, when advertising has a positive influence on demand, an upward distortion in advertising occurs. In general, the entrant's effort to infer cost information leads the incumbent to act as if he had lower costs in a complete information setting. Finally, these distortions will always lead demand to be greater than under complete information, so that consumer welfare improves as a byproduct of signaling.

APPENDIX

<u>Proof of Corollary 1:</u> If $P^L \leq \underline{P}^H$, then we immediately have $(P^L,0) \in H \cap L$. Suppose $P^L > \underline{P}^H$.

$$\begin{split} & \Pi^{L}(\underline{P}^{H},0) + \delta\Pi^{L}(\underline{P}^{L},0) - \Pi^{L}(\underline{P}^{L},0) - \delta\Pi_{C}^{L} \\ & = \Pi^{L}(\underline{P}^{H},0) + \delta\Pi^{L}(\underline{P}^{L},0) - \Pi^{L}(\underline{P}^{L},0) - \delta\Pi_{C}^{L} \\ & - \{\Pi^{H}(\underline{P}^{H},0) + \delta\Pi^{H}(\underline{P}^{H},0) - \Pi^{H}(\underline{P}^{H},0) - \delta\Pi_{C}^{H} \} \\ & = [\Pi^{L}(\underline{P}^{H},0) - \Pi^{H}(\underline{P}^{H},0)] - [\Pi^{L}(\underline{P}^{L},0) - \Pi^{H}(\underline{P}^{H},0)] \\ & + \delta[\Pi_{C}^{H} - \Pi_{C}^{L}] + \delta[\Pi^{L}(\underline{P}^{L},0) - \Pi^{H}(\underline{P}^{H},0)] \\ & > [\Pi^{L}(\underline{P}^{H},0) - \Pi^{H}(\underline{P}^{H},0)] - [\Pi^{L}(\underline{P}^{L},0) - \Pi^{H}(\underline{P}^{H},0)] \\ & > [\Pi^{L}(\underline{P}^{H},0) - \Pi^{H}(\underline{P}^{H},0)] - [\Pi^{L}(\underline{P}^{L},0) - \Pi^{H}(\underline{P}^{L},0)] \\ & = (\underline{c}^{H} - \underline{c}^{L})[\underline{x}(\underline{P}^{H}) - \underline{x}(\underline{P}^{L})] > 0 \end{split}$$

This shows that $(\underline{P}^H,0)$ satisfies (2), and $(\underline{P}^H,0) \in L$. Of course, $(\underline{P}^H,0) \in H$ also.

Proof of Lemma: H closed and upper contour sets $\Pi^{L}(P,A) \ge k$ compact imply existence of:

$$(\widetilde{P}, \widetilde{A}) \in \underset{(P,A) \in H}{\operatorname{argmax}} \Pi^{L}(P,A)$$

Suppose $\Pi^L(\hat{P}^L,\hat{A}^L) < \Pi^L(\hat{P},\hat{A})$. For any $\epsilon > 0$, we have:

$$\Pi^{\mathrm{H}}(\widetilde{P},\widetilde{A}+\varepsilon) < \Pi^{\mathrm{H}}(\widetilde{P},\widetilde{A}) \leq \overline{\Pi}^{\mathrm{H}}$$

while for ϵ sufficiently small:

(A1)
$$\Pi^{L}(\hat{P}^{L}, \hat{A}^{L}) < \Pi^{L}(\hat{P}, \hat{A} + \varepsilon)$$

so that $(\widetilde{P}, \widetilde{A} + \varepsilon) \in L$. This implies $\widehat{\rho}(\widetilde{P}, \widetilde{A} + \varepsilon) = 0$, since the equilibrium is undominated. But then (Al) implies violation of (A) for i = L. Conclude $\Pi^L(\widehat{P}, \widehat{A}^L) = \Pi^L(\widetilde{P}, \widetilde{A}).$ Q.E.D.

<u>Proof of Theorem 2</u>: Suppose $P_L > \underline{p}^H$. If $\Pi^H(\hat{p}^L, \hat{A}^L) < \overline{\Pi}^H$, we may write:

$$(P^{H}-C^{H})X(P^{H}) > (\hat{P}^{L}-C^{H})X(\hat{P}^{L}) - \hat{A}^{L}$$

(A2)
$$(\hat{P}^L - C^L)X(\hat{P}^L) - \hat{A}^L \ge (\underline{P}^H - C^L)X(\underline{P}^H)$$

where the latter inequality uses the Lemma and $(\underline{P}^H,0)\in H$. Adding these inequalities gives:

$$(C^{H}-C^{L})(X(\underline{P}^{L}) - X(\underline{P}^{H})) > 0$$

which implies $X(\hat{P}^L) > X(\underline{P}^H)$. But using (A2), $C^0 < C^L$ implies:

$$(\hat{P}^{L}-C^{0})X(\hat{P}^{L}) - \hat{A}^{L} > (P^{H}-C^{0})X(P^{H})$$

which contradicts $P(C^0) = \underline{P}^H$. Thus, the maximizing point must satisfy $\overline{\Pi}^H(\hat{P}^L, \hat{A}^L) = \overline{\Pi}^H$.

Now, any $(P,A) \neq (\underline{P}^H,0)$ with $\Pi^H(P,A) = \overline{\Pi}^H$ must certainly have $P > \underline{P}^H$, so that $X(\underline{P}^H) > X(P)$. We then have:

$$(\underline{P}^{H}-C^{L})X(\underline{P}^{H}) - [(P-C^{L})X(p) - A]$$

$$= (\underline{P}^{H}-C^{L})X(\underline{P}^{H}) - [(P-C^{L})X(P) - A]$$

$$+ (P-C^{H})X(P) - A - [(\underline{P}^{H}-C^{H})X(\underline{P}^{H})]$$

This proves $(\underline{P}^{H},0)$ uniquely maximizes $\Pi^{L}(P,A)$ on H.

Proof of Theorem 3: Suppose (\hat{P}, \hat{A}) with $\hat{A} > 0$ gives pooling equilibrium strategies. Since $(\hat{P}, \hat{A}) \notin \text{int}(H)$, there exists $\hat{\underline{P}} \in [\underline{P}^H, \hat{P})$ with:

Q.E.D.

$$(P-C^H)X(P) - A = (P-C^H)X(P)$$

 $= (c^{H} - c^{L})(x(p^{H}) - x(p)) > 0$

But $X(\underline{P}) > X(P)$ then implies:

$$(\underline{P}-\underline{C}^{L})\underline{X}(\underline{P}) - \underline{A} < (\underline{\underline{P}}-\underline{C}^{L})\underline{X}(\underline{\underline{P}})$$

It follows that for sufficiently small $\epsilon > 0$, $\Pi^{H}(\hat{P},\hat{A}) > \Pi^{H}(\hat{P}-\epsilon,0)$ and

 $\Pi^L(\hat{P},\hat{A}) < \Pi^L(\hat{P}-\epsilon,0)$, and belief are unintuitive unless $\hat{\rho}(\hat{P}-\epsilon,0) = 0$. But then (A) is violated for i = L.

Now suppose $(\hat{P},0)$ gives pooling equilibrium strategies with $\hat{P} > P^L$. Since $\Pi^L(\hat{P},0) > \underline{\Pi}^L > 0$, there exists $\underline{\hat{P}} < P^L$ with:

$$(\hat{P}-C^L)X(\hat{P}) = (\hat{P}-C^L)X(\hat{P})$$

 $X(\underline{P}) > X(P)$ then implies:

$$(\hat{P}-C^H)X(\hat{P}) > (\hat{\underline{P}}-C^H)X(\hat{\underline{P}})$$

Thus, for some sufficiently small $\epsilon > 0$ intuitive beliefs require $\rho(\underline{P}+\epsilon,0)=0$, in which case (A) is violated for $i=L_{\bullet}$ Q.E.D.

Proof of Theorem 4: We first establish the existence of an intersection point $(P(C^0), A(C^0))$ of $\Psi(C)$ and $\Pi^H(P,A) = \overline{\Pi}^H$ with $P(C^0), A(C^0) > 0$. Since $\Pi^H(P^H,A^H) > \overline{\Pi}^H > 0 > \Pi^H(0,A)$, $\Psi(C)$ and $\Pi^H(P,A)$ are continuous, and $\Psi(C)$ intersects the P = 0 axis, it is sufficient to prove that $\Pi(\Psi(C)|C^H)$ is strictly increasing in C for $C < C^H$.

To this end, pick $\underline{C} < \overline{C} < C^H$. Let $(\overline{P}, \overline{A}) \equiv (P(\overline{C}), A(\overline{C}))$ and $(\underline{P}, \underline{A}) \equiv (P(\underline{C}), A(\underline{C}))$. Then:

(A3)
$$(\bar{P}-\bar{C})X(\bar{P},\bar{A}) - \bar{A} - (\bar{P}-\bar{C})X(\bar{P},A) + A > 0$$

$$(\underline{P}-\underline{C})X(\underline{P},\underline{A}) - \underline{A} - (\overline{P}-\underline{C})X(\overline{P},\overline{A}) + \overline{A} > 0$$

Adding these inequalities gives:

$$(\overline{C}-C)(X(P,A) - X(\overline{P},\overline{A})) > 0,$$

whence X(P,A) > X(P,A). Using (A3), we may write:

$$(\vec{P}-C^{H})X(\vec{P},\vec{A}) - \vec{A} - (\underline{P}-C^{H})X(\underline{P},\underline{A}) + \underline{A} > 0,$$

which proves monotonicity and establishes the existence of a positive intersection point. Clearly, an analogous argument establishes that $\Pi(\Psi(C)|C^H)$ is strictly decreasing in C, for $C > C^H$.

Put $(P^0, A^0) \equiv (P(C^0), A(C^0))$. Using the Lemma, one may show that $\Pi^H(\hat{P}^L, \hat{A}^L) = \bar{\Pi}^H$ by using the same argument as in Theorem 2.

Now, for all $(P,A) \neq (P^0,A^0)$ with $\Pi^H(P,A) = \overline{\Pi}^H$, we have:

$$(P^{0}-C^{0})X(P^{0},A^{0}) - A^{0} - (P-C^{0})X(P,A) + A > 0$$

(A4)
$$(P-C^H)X(P,A) - A - (P^O-C^H)X(P^O,A^O) + A^O = 0$$

Adding these yields:

$$(c^{H}-c^{0})[X(P^{0},A^{0}) - X(P,A)] > 0$$

so that $X(P^0,A^0) > X(P,A)$. Using (A4), we may write:

$$(P^{0}-C^{L})X(P^{0},A^{0}) - A^{0} - (P-C^{L})X(P,A) + A$$

=
$$(C^{H}-C^{L})[X(P^{O},A^{O}) - X(P,A)] > 0$$

This proves (P^0,A^0) uniquely maximizes $\Pi^L(P,A)$ on H.

Q.E.D.

Proof of Theorem 5: Suppose (\hat{P}, \hat{A}) with $(\hat{P}, \hat{A}) \notin \Psi(\mathbf{R})$ gives pooling equilibrium strategies. Since $\Pi^H(P^H, A^H) > \Pi^H(\hat{P}, \hat{A}) > \overline{\Pi}^H$, we can argue as above to establish the existence of $\hat{C} < C^H$ such that:

(A5)
$$(\hat{P}-C^H)X(\hat{P},\hat{A}) - \hat{A} = (P(\hat{C})-C^H)X(P(\hat{C}),A(\hat{C})) - A(\hat{C})$$

$$(P(C)-\hat{C}) \times (P(\hat{C}),A(\hat{C})) - A(\hat{C}) > (\hat{P}-\hat{C}) \times (\hat{P},\hat{A}) - \hat{A}$$

Adding these inequalities gives:

$$(c^{H}-\hat{c})(x(p(\hat{c}), A(\hat{c})) - x(\hat{p}, \hat{A})) > 0$$

so that X(P(C),A(C)) > X(P,A). Using (A5):

$$(\hat{P}-C^L)X(\hat{P},\hat{A}) - \hat{A} < (\hat{P}(\hat{C}) - C^L)X(\hat{P}(\hat{C}),\hat{A}(\hat{C})) - \hat{A}(\hat{C})$$

Thus, since $\Pi^{\hat{\mathbf{I}}}(P,A)$ is continuous and $\Pi^{\hat{\mathbf{H}}}(P,A)$ has strict monotonicity properties on $\Psi(C)$, there exists $(\widetilde{P},\widetilde{A})$ near $(P(\widehat{C}),A(\widehat{C}))$, such that $\Pi^{\hat{\mathbf{H}}}(\widehat{P},\widehat{A}) > \Pi^{\hat{\mathbf{H}}}(\widetilde{P},\widetilde{A})$ and $\Pi^{\hat{\mathbf{L}}}(\widehat{P},\widehat{A}) < \Pi^{\hat{\mathbf{L}}}(\widetilde{P},\widetilde{A})$. Intuitive beliefs then imply $\widehat{\rho}(\widetilde{P},\widetilde{A}) = 0$, and $\widehat{\mathbf{A}}$ is violated for i = L.

Now suppose $(\hat{P}, \hat{A}) = (\hat{P}(\hat{C}), \hat{A}(\hat{C}))$ for $\hat{C} > \hat{C}^L$. Under our assumptions, it is

easy to establish the existence of $C' < C^L$ such that:

(A6)
$$(P-C^{L})X(P,A) - A = (P(C') - C^{L})X(P(C'),A(C')) - A(C')$$

$$(P(C') - C')X(P(C'),A(C')) - A(C') > (P-C')X(P,A) - A$$

Adding these inequalities gives:

$$(c^{L}-c')(x(P(C'),A(C')) - x(\hat{P},\hat{A})) > 0$$

or X(P(C'),A(C')) > X(P,A). From (A6) we have:

$$(\hat{P}-C^{H})X(\hat{P},\hat{A}) - \hat{A} > (P(C') - C^{H})X(P(C'),A(C')) - A(C')$$

Thus, for some $(\widetilde{P},\widetilde{A})$ near (P(C'),A(C')), $\Pi^H(\widehat{P},\widehat{A}) > \Pi^H(\widetilde{P},\widetilde{A})$ and $\Pi^L(\widehat{P},\widehat{A}) < \Pi^L(\widetilde{P},\widetilde{A}). \quad (\widehat{P},\widehat{A}) \text{ cannot then give intuitive pooling equilibrium}$ strategies. Q.E.D.

NOTES

- 1 See Ramey (1985) for an extension of the Milgrom-Roberts model that allows for capacity choice by the entrant.
- This could be the case, for example, if consumers were perfectly informed about product quality and if price information were costless. Nelson (1970,1974), Kihlstrom and Riordan (1984), and Milgrom and Roberts (1986) have shown that advertising would be informative to consumers when product quality information is incomplete; Bagwell (forthcoming) has shown that consumers will respond to signals of production cost when price information is costly.
- In addition to the possibilities mentioned in note 2, advertising might increase demand by affecting consumer tastes directly, or by informing consumers of the existence of a product (see, e.g., Butters (1977) and Schmalensee (1983)).
- While the sequential equilibrium concept is formally defined for games with finitely many actions, the definition of equilibrium we offer is the obvious extension of this concept to our game. See Kreps and Wilson (1982) and Kreps and Ramey (forthcoming) for further discussion.
- To see this, fix $(\widetilde{P}, \widetilde{A})$, $\widetilde{P} > 0$, and choose (P,A) with $P < \widetilde{P}$ and $\Pi^H(P,A) = \Pi^H(\widetilde{P},\widetilde{A})$. Then $\Pi^L(P,A) > \Pi^L(\widetilde{P},\widetilde{A})$, meaning that the isoprofit curve of the low-cost incumbent passing through $(\widetilde{P},\widetilde{A})$ must lie above that of the high-cost incumbent for all $P < \widetilde{P}$.

- Continuity of P(C) is plausible under the assumption that X(P) is bounded. If $\lim_{P\to 0} X(P) = +\infty$, then the revenue-maximizing price will be optimal for C = 0, while P = 0 would be optimal for C < 0; this may create a discontinuity at C = 0. For all subsequent analysis, however we need only assume that $P(C^0) = \underline{P}^H$ for some C^0 , which is a weaker assumption. Note further that boundedness of X(P) enters at no other point in our analysis.
- ⁷ Existence of such a c^0 follows from $\underline{p}^H > 0$, implied by (1), together with the assumptions on P(C). See note 6.
- As in Milgrom and Roberts (1986), application of the intuitive criterion leaves a continuum of possible pooling equilibria; see also Theorem 5.
- In particular, we rule out direct intertemporal linkages between the incumbent's first-period advertising and second-period profits, of the sort discussed by Schmalensee (1983), for example.
- Again, continuity of $\Psi(C)$ is plausible based on boundedness of X(P,A). The assumptions needed for the theorems are somewhat weaker than those given in the text, however. See note 6.
- 11 A condition analogous to (3) is sufficient for existence of this equilibrium.

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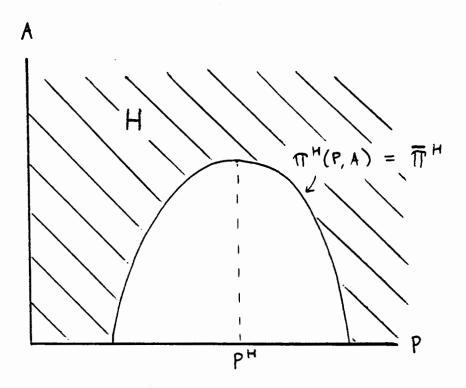


Figure 1a

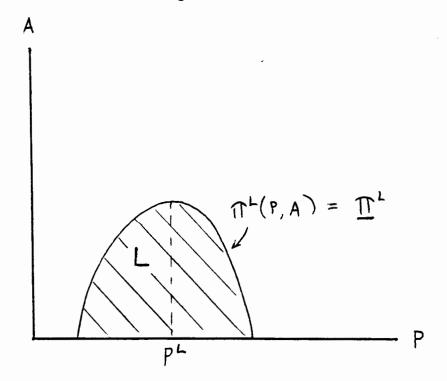


Figure 16

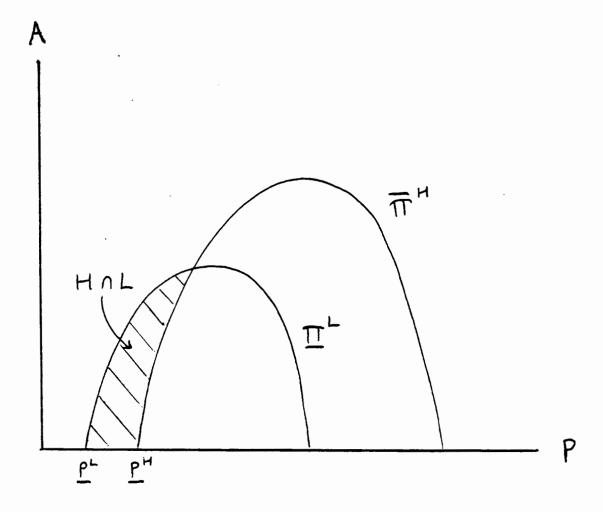


Figure 2

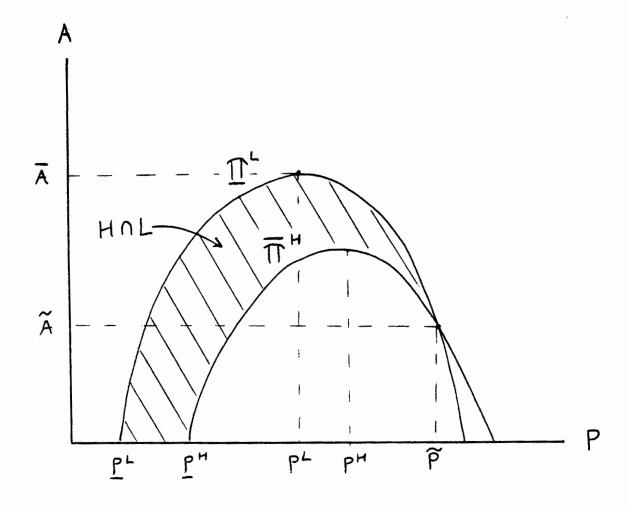


Figure 3

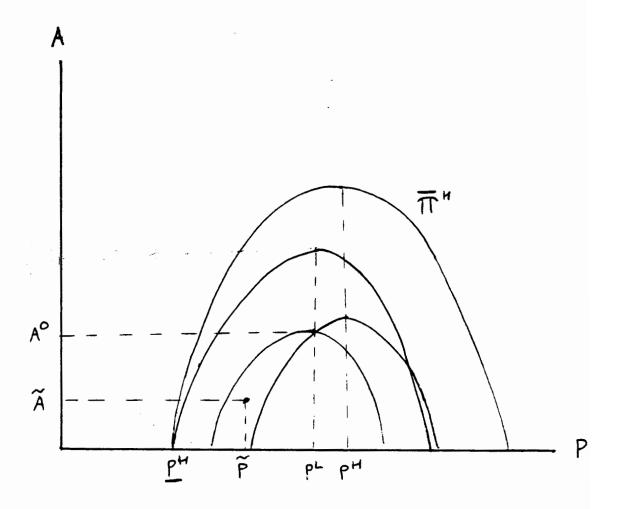


Figure 4

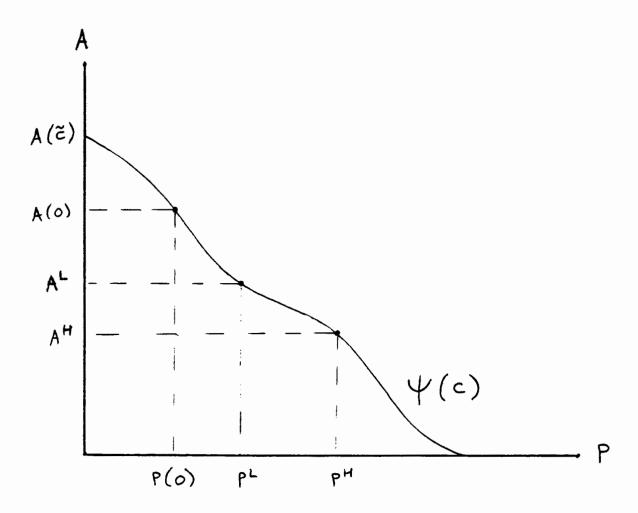


Figure 5

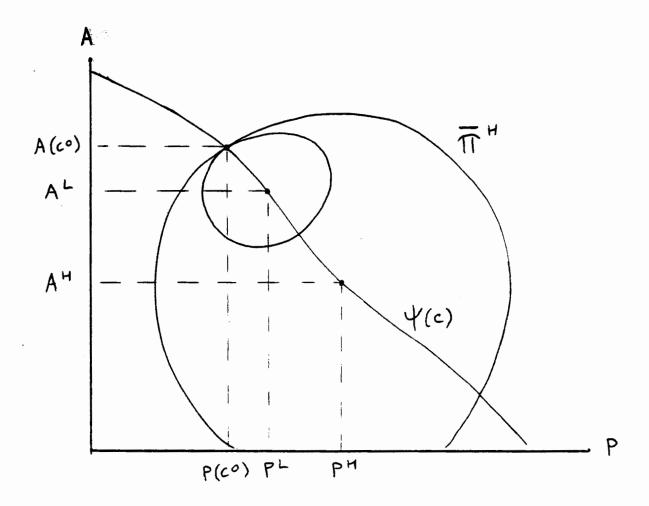


Figure 6