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A TWO-PERSON REPEATED BARGAINING GAME WITH LONG-TERM CONTRACTS

by

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Abstract

Do players always attain a Pareto optimal outcome in an equilibrium point of a supergame if they are allowed to negotiate for their actions in every period ? In order to answer this , we consider a two-person repeated bargaining game in which players can make long-term contracts on their actions , if they want. We show that the stationary outcomes of subgame perfect equilibria in our repeated bargaining game are both Pareto optimal and individually rational if the equilibrium strategies for both players have " limited " memory in a sense that they do not punish each other over periods. We also point out that these outcomes are not necessarily Pareto optimal if the equilibrium strategies have unlimited memory.

1. Introduction

The theory of repeated games is one of the most important fields of game theory. The purpose of it is to investigate dynamic interactions among players in repeated situations. It helps us to understand phenomena such as cooperation, coordination, betrayal, revenge, etc., in terms of noncooperative utility-maximizing behavior of players. Among many types of repeated games, a supergame, in which the identical component game is repeated infinitely many times, has been investigated by many authors (For example, see Aumann [1], Luce and Raiffa [6], and Rubinstein [8] etc.).

The " Folk Theorem " states that the average payoffs of Nash equilibrium points in a supergame are the feasible individually rational payoffs in the component game. It shows that any Pareto optimal and individually rational payoffs in the component game can be attained by a Nash equilibrium point in the supergame. This result is significant because it implies that cooperation among players can be explained by their noncooperative behavior which does not require any outside enforcement mechanism. Since no explicit negotiations are allowed in the supergame, we can say that the Folk Theorem gives light on the so-called " tacit coordination " among players in repeated situations.

On the other hand, the Folk Theorem contains an embarrassing result that the set of all Nash equilibrium payoffs is very large in the supergame (see Kaneko [5] on this point). It has been commonly recognized since Selten [10, 11]'s pioneering work on perfect equilibrium points that the concept of a Nash equilibrium point is inadequate as a noncooperative solution concept in repeated games. One may hope that the concept of perfectness could narrow down the set of equilibrium outcomes in a supergame. Rubinstein [8], however, proved that the Folk Theorem remains true even if we adopt a subgame perfect equilibrium point as the noncooperative solution concept in the supergame.

The Folk Theorem depends crucially on the rule of a supergame that at each period of the game every player can obtain perfect information on history. Under this rule of the game , a player can employ a strategy with unlimited memory if his capability of memory is unlimited. That is , he is free to take actions dependent on history of the game as he wishes. As an extreme case, he may employ a strategy with zero-memory , i.e. , he takes his action independent of history. If all players employ strategies with zero-memory, the stationary outcome of an equilibrium point in the supergame must be just a Nash equilibrium point in the component game because each player can not be punished by the other players for his past behavior.

In the present paper, we will consider a two-person repeated bargaining game based on the supergame model. In our repeated game, two players can negotiate for their actions in every period before they choose their actions independently. They are allowed to reach a binding agreement not only about which action pair they should choose but also about how long they should play it in future. That is, the players can make a long-term contract on their actions. In the context of such a repeated bargaining, we will investigate the following questions :

(1) Can the possibility of negotiations and binding agreements reduce the multiplicity of equilibrium outcomes in a supergame in the way that non-Pareto-optimal outcomes are eliminated ?

(2) Are punishments over many periods useful for attaining cooperation i.e. Pareto optimal outcomes in repeated bargaining situations ?

The two questions above are closely interdependent. Because the equilibrium outcomes of the game depend crucially on to what extent each player punishes the other for past behavior, or in other words, how large memory the strategies players employ have. In order to answer those questions, we will introduce the concept of memory for a strategy, and will characterize the stationary outcomes of Nash and subgame perfect equilibria in our repeated bargaining game under several conditions on memory for both players' equilibrium strategies.

The bargaining between two players proceeds as follows. At the beginning of every period of the game, players 1 and 2 in turn propose their preferred action pairs and terms of contract. Player 1 first makes a proposal to player 2. Then, player 2 decides whether he accepts it or not. If player 2 accepts it, they will play the agreed action pair as many times as they agreed. If player 2 rejects it, he can make a counter-proposal to player 1. Player 1 also decides whether he accepts it or not. If he rejects it, their negotiations at the present period break down and they must decide their own strategies independently. Afterwards, the game proceeds to the next period and the same process will be repeated infinitely many times. We remark that each player can create the same situation as the supergame, if he wishes, by breaking down negotiations at every period. We employ the limit of average payoffs as the players' preferences in our repeated game.

We first show that the set of all stationary outcomes of Nash equilibrium points in our repeated bargaining game coincides with the set of all individually rational action pairs in the component game, regardless of the memory of equilibrium strategies. Therefore, as long as we adopt the Nash equilibrium point as a noncooperative solution concept, the possibility of negotiations and binding agreements does not help the two players to improve their payoffs in the supergame.

Secondly, we show that the set of all stationary outcomes of subgame perfect equilibria in our repeated bargaining game coincides with the set of all Pareto optimal and individually rational action pairs in the component game if the memory of both players' equilibrium strategies is limited in the following way : There exists at least one period t^* such that the players' behavior in any succeeding period t ($> t^*$) does not depend on the history of the game until period t^*. This means that the players do not punish each other for their past behavior until period t^*. We also point out that the set of all stationary outcomes of subgame perfect equilibria in our repeated bargaining game contains many non-Pareto-optimal action pairs if both players' equilibrium strategies have unlimited memory.

The paper is organized as follows. In Section 2, the component game of our repeated bargaining game is introduced. In Section 3, our repeated bargaining game sketched above is formalized. The concept of memory for a strategy is defined. In Section 4, the main theorems are presented. The proofs of all theorems are given in Section 5. Section 6 has the conclusion.

2. The Component Game

Let $G = (S_1, S_2; f_1, f_2)$ be a two-person game in normal form, where S_i is the set of player i's strategies and f_i his payoff function. For i = 1, 2, we assume that S_i is a compact set and f_i is a real-valued continuous function on $S = S_1 \times S_2$. A strategy pair $s = (s_1, s_2) \in S$ is also called an outcome in G. Throughout the paper, we will restrict attention to pure strategies only.

The minimax payoff for player i in G is defined to be

$$v_i = \min_{s_j} \max_{s_i} f_i(s_i, s_j), \quad j \neq i. \qquad (2.1)$$

A strategy for player j attaining v_i is called a minimax strategy for player j against player i, and is denoted by s^{*i}_j. That is,

$$v_i = \max_{s_i} f_i(s_i, s^{*i}_j).$$

A strategy pair $s = (s_1, s_2) \in S$ is said to be individually rational if

$$f_i(s) \geq v_i, \qquad i = 1, 2.$$

For $s_j \in S_j$, a strategy for player i is said to be a best response to s_j, denoted by $b_i(s_j)$, if

$$f_i(b_i(s_j), s_j) = \max_{s_i} f_i(s_i, s_j).$$

A strategy pair $s = (s_1, s_2) \in S$ is said to be (weakly) Pareto optimal if there exists no $\tilde{s} \in S$ such that

$$f_i(\tilde{s}) > f_i(s), \quad i = 1,2.$$

Let E be the set of all strategy pairs $s \in S$ which are both individually rational and Pareto optimal. Then we define

$$w_i = \min_{s \in E} f_i(s) \quad , \quad i = 1, 2. \tag{2.2}$$

w_i is the least payoff for player i when all individually rational and Pareto optimal strategy pairs are selected. For i = 1, 2, we also define a strategy pair $m^i = (m_1{}^i , m_2{}^i) \in E$ which solves the maximization problem :

$$\max_{s \in E} f_j(s) \qquad \text{subject to} \qquad f_i (s) = w_i \ . \tag{2.3}$$

The strategy pair m^i will be used in order for player j to punish player i for deviating from a (subgame perfect) equilibrim play in the repeated bargaining game described in the next section. It can be easily proved that

$$f_j(s) \leq f_j(m^i) \quad , \quad i, j = 1, 2, \ \ i \neq j,$$

for all individually rational strategy pairs s.

3. The Repeated Bargaining Game Γ

We consider a repeated bargaining game in which the component game G is played infinitely many times. In our model, unlike the usual one of a supergame of G, at the beginning of every period two players can negotiate not only about which strategy pair they should choose but also about how long they should choose it from now on. Once they have reached an agreement, they must keep it.

The game proceeds as follows. At the beginning of every period, player 1 first proposes to player 2 a strategy pair $s^1 = (s_1^1, s_2^1)$ and a number T^1 of periods for which s^1 should be played successively. Then, player 2 decides whether he accepts player 1's proposal or not. If player 2 accepts it, they reach the binding agreement that they will play s^1 for T^1 periods from now on. If player 2 rejects the proposal, then he must counter-propose to player 1 a strategy pair $s^2 = (s_1^2, s_2^2)$ and a number T^2 of periods. If player 1 accepts player 2's proposal, they reach the agreement that they will play s^2 for T^2 periods. If player 1 rejects the proposal, their negotiations in the present period break down and they must decide their own strategies s_1 and s_2 independently. Afterwards, the same process will be repeated infinitely many times.

The repeated bargaining game sketched above can be formulated as $\Gamma = \{G^t\}_{t=1}^{\infty}$ where G^t , the game in period t, consists of the following moves :

(1) Player 1 proposes a pair $(s^1 , T^1) \in S \times N^*$ to player 2, where $N^* = \{1, 2, \ldots \} \cup \{\infty\}$.

(2) Player 2 chooses " 1 " (yes) or " 0 " (no). If he chooses 1, he accepts player 1's proposal and they will play s^1 for T^1 periods. Afterwards, the game G^{t+T^1} is played. In the case of $T^1 = \infty$, s1 will be played infinitely many times. If player 2 chooses 0, he rejects player 1's proposal and the game proceeds to the next move.

(3) Player 2 proposes a pair (s^2 , T^2) $\in S \times N^*$ to player 1 in his turn.

(4) Player 1 chooses 1 or 0 similarly to (2). If he chooses 1, he accepts player 2's proposal and they will play s2 for T^2 periods. Afterwards, the game G^{t+T^2} is played. If player 1 chooses 0, he rejects player 2's proposal and their negotiations in period t break down.

(5) After negotiations break down, players 1 and 2 choose their own strategies s_1 and s_2 independently, and then the game G^t ends. At the end of the period, each player is informed of all moves in this period and the game G^{t+1} is played under the same rule as of G^t in the next period t+1.

Formally the repeated bargaining game Γ can be described as an extensive game of infinite length, and with perfect information except that two players choose their strategies in G independently in every period after negotiations break down. The path from the origin to each move in the extensive form of Γ is called the history of the move. We can define a (pure) strategy for each player in Γ in the usual way as in an extensive game.

A strategy for player i (i = 1, 2) in Γ is represented as $\sigma_i = \{ \sigma_i^t \}_{t=1}^{\infty}$ where σ_i^t (t = 1, 2, ...) consists of the following elements :

$$\sigma_i^t = (x_i^t , y_i^t , z_i^t) . \qquad (3.1)$$

x_i^t , y_i^t and z_i^t are the functions of the history h^{t-1} of the first move in period t and also the proposals (s^1 , T^1), (s^2 , T^2) in $S \times N^*$ satisfying

$$\begin{aligned} & x_1^t(h^{t-1}) \in S \times N^* \\ & y_1^t(h^{t-1} , (s^1 , T^1), (s^2 , T^2)) \in \{ 0 , 1 \} \\ & z_1^t(h^{t-1} , (s^1 , T^1), (s^2 , T^2)) \in S_1 \end{aligned} \qquad (3.2)$$

for i = 1, and

$$x_2^t(h^{t-1} , (s^1 , T^1)) \in S \times N^*$$

$$y_2^t(h^{t-1} , (s^1 , T^1)) \in \{ 0 , 1 \} \qquad (3.3)$$

$$z_2^t(h^{t-1} , (s^1 , T^1), (s^2 , T^2)) \in S_2$$

for i = 2.

$x_1^t(h^{t-1})$ is the proposal which player 1, informed of the history h^{t-1}, makes at the first move of period t. $y_1^t(h^{t-1} , (s^1 , T^1), (s^2 , T^2))$ is the response of player 1 to player 2's proposal (s^2 , T^2) after his proposal (s^1 , T^1) is rejected. $z_1^t(h^{t-1} , (s^1 , T^1), (s^2 , T^2))$ is the strategy in S_1 which player 1 chooses after the proposals (s^i , T^i) of player i, i = 1, 2, are rejected and thus negotiations in period t break down. x_2^t, y_2^t , z_2^t can be interpreted in the similar way. The set of all strategies σ_i (i = 1, 2) for player i in Γ is denoted by Σ_i . We put $\Sigma = \Sigma_1 \times \Sigma_2$. To avoid a confusion, a strategy s_i for player i in G will be called his action in what follows.

A strategy pair $\sigma = (\sigma_1 , \sigma_2)$ for the two players in Γ uniquely determines the sequence of action pairs in G,

$$a(\sigma) = \{ s^t \}_{t=1}^{\infty} , \qquad s^t \in S , \quad t = 1, 2, \dots ,$$

where each s^t is played in period t. In this paper, we assume that players evaluate the sequence $a(\sigma)$ of action pairs according to the limit of the means of the payoffs in all periods. That is, the payoff function for player i in Γ is defined by

$$F_i(\sigma) = \underline{\lim}_{T \to \infty} (1/T) \sum_{t=1}^{T} f_i(s^t) \qquad (3.4)$$

where $a(\sigma) = \{ s^t \}_{t=1}^{\infty}$.

Definition 3.1. A strategy pair $\sigma^* = (\sigma_1^* , \sigma_2^*) \in \Sigma$ is said to be a (Nash) equilibrium point if

$$F_i(\sigma_i^* , \sigma_j^*) \geq F_i(\sigma_i , \sigma_j^*) \quad , \forall \sigma_i \in \Sigma_i , \quad i = 1, 2.$$

An equilibrium point σ^* in Γ is said to be stationary if $a(\sigma^*) = (s, s, \ldots)$ for some $s \in S$.

Γ , regarded as an extensive game, includes the following five types of subgames starting in each period t : (1) Γ^t, which begins with the first move of period t ; (2) $\Gamma_2^t(s^1 , T^1)$, which begins after player 1 has proposed $(s^1 , T^1) \in S \times N^*$; (3) Γ_2^t, which begins with a proposal of player 2 ; (4) $\Gamma_1^t(s^2 , T^2)$, which begins after player 2 has proposed (s^2 , T^2) ; (5) Γ_{12}^t, which begins after negotiations have broken down.

Let $\sigma = (\sigma_1 , \sigma_2)$, $\sigma_i = \{ \sigma_i^t \}_{t=1}^{\infty}$ (i = 1, 2), be a strategy pair in Γ , and let $\sigma^t = (\sigma_1^t , \sigma_2^t)$ for each t. Given a subgame Γ' of Γ , σ induces a unique strategy pair on Γ', denoted by $\sigma|_{\Gamma'}$. We also denote by $\sigma^t|_{\Gamma'}$ the behavior for players which σ (or σ^t) induces on the period t-game of Γ'. In the same manner as (3.4), we can define the payoff of player i in the subgame Γ' when $\sigma|_{\Gamma'}$ is employed. This payoff is denoted by $F_i(\sigma)|_{\Gamma'}$.

Definition 3.2. An equilibrium point $\sigma^* = (\sigma_1^* , \sigma_2^*)$ in Γ is said to be subgame perfect if, for any subgame Γ' of Γ, $\sigma^*|_{\Gamma'}$ is an equilibrium point in Γ'.

Subgame perfectness, introduced by Selten [10], excludes " irrational " moves of players in subgames off the equilibrium path. Especially it excludes incredible threats by players such as the permanent play of the minimax strategies.

As we have mentioned in the Introduction, the equilibrium outcomes of the repeated game depends crucially on what kind of punishments two players incorporate into their equilibrium strategies. In order to investigate the role of punishments in our repeated bargaining game Γ , we introduce the concept of memory for strategies in the following.

Definition 3.3. Let $\sigma = (\sigma_1, \sigma_2)$, $\sigma_i = \{\sigma_i^t\}_{t=1}^{\infty}$ (i = 1, 2), be a strategy pair in Γ , and let $\mathcal{M}$ be defined by

$$\mathcal{M} = \{ M^t \mid M^t \subset \{1, \ldots, t-1\}, \; t = 2, 3, \ldots \}.$$

σ is said to have memory $\mathcal{M}$ if the following condition holds for every t = 2, 3, ... : Let Γ^t and $\tilde{\Gamma}^t$ be any two subgames of Γ starting with the first move in period t such that the histories of both subgames are identical in all periods $k \in M^t$. Then, $\sigma^t|_{\Gamma^t} = \sigma^t|_{\tilde{\Gamma}^t}$.

Definition 3.3 says that, when two players employ strategies with memory $\mathcal{M}$, their behavior in every period t depend on history only in previous periods $k \in M^t$. In this case, they do not punish each other for their past behavior in all other periods $l \notin M^t$. The set of all strategy pairs with memory $\mathcal{M}$ in Γ is denoted by $\Sigma^{\mathcal{M}}$.

Let $\mathcal{M} = \{ M^t \mid t = 2, 3, \ldots \}$ and $\mathcal{M}' = \{ M'^t \mid t = 2, 3, \ldots \}$. We say that memory $\mathcal{M}$ is larger than memory $\mathcal{M}'$ if $M^t \supset M'^t$ for all $t \geq 2$. Then, the largest memory is $\mathcal{M}^* = \{ M^t \mid M^t = \{1, \ldots, t-1\}$ for all $t \geq 2.\}$, and the least memory $\mathcal{M}^0 = \{ M^t \mid M^t = \phi$ for all $t \geq 2.\}$. If $\mathcal{M}$ is larger than $\mathcal{M}'$, then $\Sigma^{\mathcal{M}} \supset \Sigma^{\mathcal{M}'}$. Note that $\Sigma^{\mathcal{M}^*} = \Sigma$.

Definition 3.4. A strategy pair $\sigma = (\sigma_1, \sigma_2)$ in $\Sigma^{\mathcal{M}}$ is said to have unlimited memory if $\mathcal{M} = \mathcal{M}^*$, and limited memory otherwise. In particular, σ is said to have zero-memory if $\mathcal{M} = \mathcal{M}^0$.

When players employ strategies with unlimited memory, any restriction is not imposed on their behavior. On the other hand, when they employ strategies with zero-memory, their behavior in each period is independent of history in all past periods. Therefore, strategy pairs with zero-memory prevent them from punishing each other over periods.

4. Characterizations of Equilibria with Memory $\mathcal{M}$ in Γ

We will characterize Nash and subgame perfect equilibria with memory $\mathcal{M}$ in Γ. In what follows, we consider only stationary equilibrium points. The proofs of all theorems are given in Section 5.

Theorem 4.1. There exists a Nash equilibrium point $\sigma^* = (\sigma_1^*, \sigma_2^*) \in \Sigma$ in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$ if and only if $s^* \in S$ is individually rational.

Theorem 4.1 shows that the stationary outcomes of all Nash equilibrium points with unlimited memory in the repeated bargaining game Γ are identical to those of the supergame of G, in which no explicit negotiations are allowed between two players. This means that, as long as we adopt the Nash equilibrium point as a noncooperative solution concept for Γ, the possibility of negotiations and binding agreements does not help players to improve their average payoffs in the supergame of G. This disappointing result comes from the fact that a Nash equilibrium point in Γ may contain a kind of threat by each player, i.e., to refuse any proposal by the other if it is not equal to his proposed one, and also to choose a minimax strategy against the other whenever negotiations break down. In many Nash equilibrium points of Γ, player 1 proposes a non-Pareto-optimal action pair with a threat to reject all other proposals by player 2 even if the proposals are more profitable for both of them. Clearly, such a threat is irrational and incredible.

Theorem 4.2. For $s^* \in S$, the following conditions are equivalent.

(1) s^* is individually rational.

(2) There exists a Nash equilibrium point $\sigma^* = (\sigma_1^*, \sigma_2^*)$ with zero-memory $\mathcal{M}^0$ in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$.

(3) There exists a Nash equilibrium point $\sigma^* = (\sigma_1^* , \sigma_2^*)$ with any limited memory $\mathcal{M}$ in Γ such that $a(\sigma^*) = (s^*, s^*, \dots)$.

Theorem 4.2 shows that the memory of strategies has no influence on the set of all Nash equilibrium outcomes in Γ. This is in contrast to the fact that any Nash equilibrium point with zero-memory in the supergame results in an infinite sequence of a Nash equilibrium point in the component game G. This result, of course, comes from the special rule of our repeated bargaining game Γ. During one period of Γ, two players can negotiate with each other before they select their own actions independently. Therefore, even a strategy with zero-memory enables each player to inflict an " intra-period " punishment on the other, i.e., he can punish the other by using a minimax strategy immediately after negotiations break down.

Next, we will characterize subgame perfect equilibria with memory $\mathcal{M}$ in Γ.

<u>Theorem 4.3.</u> There exists a subgame perfect equilibrium point σ^* with zero-memory $\mathcal{M}^0$ in Γ such that $a(\sigma^*) = (s^*, s^*, \dots)$ if and only if s^* is both Pareto optimal and individually rational.

By introducing subgame perfectness and zero-memory, we can eliminate non-Pareto-optimal action pairs from the set of equilibrium outcomes of Γ. The essense of the proof is very simple. Assume that the two players agree to play a non-Pareto-optimal action pair s^* infinitely many times in a subgame perfect equilibrium point $\sigma^* = (\sigma_1^* , \sigma_2^*)$ with zero-memory $\mathcal{M}^0$. Since s^* is not Pareto optimal, there exists an action pair $\tilde{s}$ more profitable to both of them than s^*. Suppose that player 2 makes the new proposal that they should play $\tilde{s}$ infinitely many times. If player 1 accepts it, they will play $\tilde{s}$ forever and thus player 1 will obtain the average payoff $f_1(\tilde{s})$. On the other hand, even if he rejects it, the equilibrium point σ^* will lead players 1 and 2 to reach the original agreement again in the next period because σ^* has zero-memory. Hence, to accept player 2's proposal is the optimal behavior of player 1, and thus player 2 can increase his average payoff by proposing the eternal play of $\tilde{s}$. This shows that any equilibrium point with a non-Pareto-optimal stationary outcome can not be subgame perfect if it has zero-memory.

Furthermore, Theorem 4.3 shows that in our repeated bargaining game Γ a very simple strategy with zero-memory $\mathcal{M}^0$ is sufficient for attaining every Pareto optimal and individually rational outcome as the stationary play of a subgame perfect equilibrium point. A strategy with zero-memory prevents each player from inflicting a " inter-period " punishment on the ther, i.e., he can not punish the other over periods. Only the " intra-period " punishment which we have already mentioned is necessary for the players to reach a Pareto optimal and individually rational outcome.

We can weaken the condition of zero-memory in Theorem 4.3 as follows.

<u>Theorem 4.4.</u> Let $\mathcal{M} = \{ M^t \mid M^t \subset \{1, \ldots, t-1\},\ t = 2, 3, \ldots \}$ satisfy the following condition : there exists at least one period t^* (≥ 2) such that

$$M^t \subset \{ t^*, \ldots, t-1 \}, \quad \forall\ t \geq t^*. \tag{4.1}$$

Then, there exists a subgame perfect equilibrium point σ^* with memory $\mathcal{M}$ in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$ if and only if s^* is both Pareto optimal and individually rational.

(4.1) means that the behavior of both players in every period t ($\geq t^*$) does not depend on any history of the game from period 1 to period $t^* - 1$. The essential point of this condition is that, even if negotiations broke down before period t^*, both players' behavior in period t^* and in all succeeding periods does not depend on how negotiations broke down. Therefore, they do not inflict punishments on each other for their past behavior before period t^*. Theorem 4.4 shows that, if the two players do not make the " inter-period " punishment mentioned above, they can always attain a Pareto optimal and individually rational outcome by a subgame perfect equilibrium point.

Finally, we investigate what kind of action pairs will be stationary outcomes of subgame perfect equilibria if players are free to punish each other over periods.

Theorem 4.5. There exists a subgame perfect equilibrium point $\sigma^* = (\sigma_1^* , \sigma_2^*)$ with the unlimited memory $\mathcal{M}^*$ in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$ if

$$f_i(s^*) \geq w_i \quad , \quad \forall\ i = 1, 2. \qquad (4.2)$$

When no limitations are imposed on the memory of strategies in Γ, each player is free to punish the other for the behavior in previous periods. In any subgame perfect equilibrium point of Γ , such punishments themselves also must be subgame perfect equilibrium points of the relevant subgames. The essense of Theorem 4.5 is in that the players can employ as the punishments subgame perfect equilibrium points with zero-memory constructed in Theorem 4.3 of which stationary outcomes are both Pareto optimal and individually rational. The punishment level on player i (i = 1, 2) is his least payoff w_i in all Pareto optimal and individually rational payoffs. For every action pair $s^* \in S$ satisfying (4.2), we can construct a subgame perfect equilibrium point $\sigma^* = (\sigma_1^* , \sigma_2^*)$ in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$ by incorporating the punishments above. If player 1 does not propose s^*, then he will be punished by player 2 from the next period so that his average payoff will be w_1. On the other hand, if player 2 rejects player 1's proposal s^*, then he will be punished by player 1 so that his average payoff will be w_2. These types of punishments cause that the two players agree to the eternal play of a non-Pareto-optimal action pair. We remark that the punishment mentioned above is not possible if the strategies have memory $\mathcal{M}$ satisfying (4.1).

5. Proofs

In this section, the proofs of all theorems in the last section are given.

Proof of Theorem 4.1. if-part. Define a strategy $\sigma_i^* = \{\sigma_i^{*t}\}_{t=1}^{\infty} \in \Sigma_i$ for player i in Γ as follows : for each t = 1, 2, ... ,

(1) Propose (s^* , ∞) to player j ($j \neq i$).

(2) When player j proposes (s, T), accept it if $s = s^*$, and reject it otherwise.

(3) When negotiations break down, choose a minimax strategy s_i^{*j} against player j.

Clearly, $a(\sigma^*) = (s^*, s^*, \ldots)$ and thus $F_i(\sigma^*) = f_i(s^*)$ for i = 1, 2.

We show that σ^* is a Nash equilibrium point of Γ. First, consider the case that player 1 deviates from σ^*. Suppose that player 1 changes his proposal to (s, T), $s \neq s^*$, at period 1. Then, player 2 rejects it and in turn proposes (s^*, ∞) to player 1. If player 1 accepts it, s^* is played forever. If he rejects it, his payoff in period 1 will be at most the minimax payoff v_1 ($\leq f_1(s^*)$) because player 2 chooses a minimax strategy s_2^{*1} against him when negotiations break down. Since player 2's strategy σ_2^{*t} in period t ($t \geq 2$) does not depend on any past history, player 1's payoff will be at most $f_1(s^*)$ in every period. Hence, player 1 can not increase his average payoff in Γ by deviating unilaterally from σ^*. The similar arguments hold for player 2.

only if-part. Let $\sigma^* = (\sigma_1^*, \sigma_2^*) \in \Sigma$ be a Nash equilibrium point of Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$. Assume that $f_i(s^*) < v_i$ for some i. Without loss of generality, we can let i = 1. We denote by $b_1(s_2) \in S_1$ a best response of player 1 to $s_2 \in S_2$. Remark that $f_1(b_1(s_2), s_2) \geq v_1$ for any s_2. Define a strategy σ_1' for player 1 as follows : for each period t = 1, 2, ... ,

(1) Propose $((b_1(s_2^*), s_2^*), \infty)$ to player 2.

(2) Reject any proposal by player 2.

(3) When negotiations break down, choose a best response $b_1(s_2)$ to the strategy s_2 for player 2 assigned by σ_2^*.

From the remark above, if the strategy pair (σ_1', σ_2^*) is played, player 1's payoff will be at least v_1 in every period. Hence,

$$F_1(\sigma_1', \sigma_2^*) \geq v_1 > f_1(s^*) = F_1(\sigma^*),$$

which contradicts that σ^* is an equilibrium point of Γ. Q.E.D.

<u>Proof of Theorem 4.2.</u> (1) $\Rightarrow$ (2) : The equilibrium point σ^* in Γ constructed in the proof of if-part of Theorem 4.1 has zero-memory. (2) $\Rightarrow$ (3) : This is clear from the definition of memory. (3) $\Rightarrow$ (1) : We can prove this from only-if part of Theorem 4.1. Q.E.D.

<u>Proof of Theorem 4.3.</u> <u>only if-part.</u> Let $\sigma^* = (\sigma_1^*, \sigma_2^*)$ be a subgame perfect equilibrium point with zero-memory in Γ such that $a(\sigma^*) = (s^*, s^*, \ldots)$. From Theorem 4.2, s^* must be individually rational. Assume that s^* is not Pareto optimal. Then, there exists some $\tilde{s} \in S$ such that $f_i(\tilde{s}) > f_i(s^*)$ for all i = 1, 2. Consider the subgame $\Gamma_1^t(s^2, T^2)$ for any t and any (s^2, T^2). By the " backward argument " we can examine the equilibrium condition of player 1's response to player 2's proposal (s^2, T^2). If player 1 rejects it, the sequence ($s^*, s^*, \ldots$) of strategy pairs will be played from the next period t + 1 by σ^* since σ^* has zero-memory. Then, his average payoff in $\Gamma_1^t(s^2, T^2)$ will be $f_1(s^*)$. If he accepts (s^2, T^2), his average payoff in $\Gamma_1^t(s^2, T^2)$ will be $f_1(s^*)$ when $T^2 < \infty$, and $f_1(s^2)$ when $T^2 = \infty$. Hence the equilibrium condition of player 1's response is as follows.

$$\begin{cases} \text{accept} & \text{if} \quad T^2 = \infty \quad \text{and} \quad f_1(s^2) > f_1(s^*) \\ \text{accept or reject} & \text{if} \quad T^2 < \infty \quad \text{or} \quad f_1(s^2) = f_1(s^*) \\ \text{reject} & \text{if} \quad T^2 = \infty \quad \text{and} \quad f_1(s^2) < f_1(s^*). \end{cases}$$

In the subgame Γ^t, if player 2 rejects player 1's proposal and counter-proposes $(\tilde{s}, \infty)$ to player 1, player 1 will accept it from the equilibrium condition above. Then, player 2's average payoff in Γ^t increases from $f_2(s^*)$ to $f_2(\tilde{s})$. This contradicts that $\sigma^*|_{\Gamma^t}$ is a Nash equilibrium point of Γ^t.

<u>if-part.</u> Define a strategy σ_i^* for player i in Γ as follows. For each period t,

(1) Propose (s^*, ∞) to player j ($\neq$ i).

(2) When player j proposes (s, T), accept it if $s = s^*$ or $f_i(s) > f_i(s^*)$ and reject it otherwise.

(3) When negotiations break down, choose a minimax strategy s_i^{*j} against player j.

Clearly, σ^* has zero-memory and $a(\sigma^*) = (s^*, s^*, \ldots)$. We will first show that for each subgame Γ^t starting with the first move in period t $\sigma^*|_{\Gamma^t}$ is a Nash equilibrium point of Γ^t. Since $a(\sigma^*|_{\Gamma^t}) = (s^*, s^*, \ldots)$, $F_i(\sigma^*)|_{\Gamma^t} = f_i(s^*)$ for all $i = 1, 2$. Assume that player i (i = 1, 2) alone deviates from σ^*. At each period t' ($\geq t$), any proposal (s, T) of player i with $f_i(s) > f_i(s^*)$ is rejected by player j since $f_j(s^*) \geq f_j(s)$ from the Pareto optimality of s^*. Moreover, when negotiations break down, player i's payoff will be at most v_i because player j chooses a minimax strategy s_j^{*i} against player i. Since $f_i(s^*) \geq v_i$ from the individual rationality of s^*, player i can not increase his payoff in period t' by deviating from σ^*. Since player j's strategy σ_j^* has zero-memory, player i can not increase his payoff in all succeeding periods by the same reason as above. Therefore, $\sigma^*|_{\Gamma^t}$ is a Nash equilibrium point of Γ^t.

Next, we will show by the backward argument that σ^* also induces Nash equilibrium points in the other four types of subgames of Γ starting in period t.

(a) Γ_{12}^{t}: In σ^*, the two players choose minimax strategies against each other if negotiations break down in period t, and they will play s^* forever from period t+1. Hence, $F_i(\sigma^*)|_{\Gamma_{12}^{t}} = f_i(s^*)$. Note that $F_i(\sigma^*)|_{\Gamma_{12}^{t}} = F_i(\sigma^*)|_{\Gamma^{t+1}}$ where Γ^{t+1} is the subgame of Γ_{12}^{t} starting with the first move in period t+1. From the zero-memory of σ_j^* and the definition of F_i, we have

$$F_i(\sigma_i, \sigma_j^*)|_{\Gamma_{12}^{t}} = F_i(\sigma_i, \sigma_j^*)|_{\Gamma^{t+1}}, \quad \forall \sigma_i \in \Sigma_i .$$

On the other hand, since $\sigma^*|_{\Gamma^{t+1}}$ is a Nash equilibrium point of Γ^{t+1},

$$F_i(\sigma^*)|_{\Gamma^{t+1}} \geq F_i(\sigma_i, \sigma_j^*)|_{\Gamma^{t+1}}, \quad \forall \sigma_i \in \Sigma_i .$$

Therefore, we have

$$F_i(\sigma^*)|_{\Gamma_{12}^{t}} \geq F_i(\sigma_i, \sigma_j^*)|_{\Gamma_{12}^{t}}, \quad \forall \sigma_i \in \Sigma_i ,$$

which implies that $\sigma^*|_{\Gamma_{12}^{t}}$ is a Nash equilibrium point of Γ_{12}^{t}.

(b) $\Gamma_1^{t}(s^2, T^2)$: For simplicity, let $\tilde{\Gamma} = \Gamma_1^{t}(s^2, T^2)$. In $\tilde{\Gamma}$, player 1 first has to decide whether he accepts player 2's proposal (s^2, T^2) or not. In order to see that $\sigma^*|_{\tilde{\Gamma}}$ is a Nash equilibrium point of $\tilde{\Gamma}$, it suffices to show that the response indicated in (2) is optimal for player 1, provided that $\sigma^* = (\sigma_1^*, \sigma_2^*)$ is employed after player 1 makes any response. From the definition of σ^*, we have

$$F_1(\sigma^*)|_{\tilde{\Gamma}} = \begin{cases} f_1(s^2) & \text{if} \quad f_1(s^2) > f_1(s^*) \text{ and } T^2 = \infty \\ f_1(s^*) & \text{if} \quad f_1(s^2) \leq f_1(s^*) \text{ or } T^2 < \infty . \end{cases}$$

When $f_1(s^2) > f_1(s^*)$ or $s^2 = s^*$, player 1 accepts the proposal (s^2, T^2) of player 2 if he obeys σ_1^*. Then $F_1(\sigma^*)|_{\tilde{\Gamma}}$ is equal to $f_1(s^2)$ if $T^2 = \infty$, and to $f_1(s^*)$ if $T^2 < \infty$. If he rejects (s^2, T^2), he can obtain at most $F_1(\sigma^*)|_{\Gamma_{12}^{t}} = f_1(s^*)$ in the next subgame Γ_{12}^{t}, as we have shown in (a). Hence, to accept (s^2, T^2)

is his optimal behavior in this case. When $f_1(s^2) \leq f_1(s^*)$ and $s^2 \neq s^*$, player 1 rejects the proposal (s^2, T^2) if he obeys σ_1^*. Then, $F_1(\sigma^*)|_{\tilde{\Gamma}}$ is equal to $f_1(s^*)$. If he accepts it, he will obtain the lower payoff $f_1(s^2)$ than $f_1(s^*)$ for T^2-periods. From period $t + T^2$, he will obtain at most $f_1(s^*)$ from the zero-memory of σ^*. Hence, to reject (s^2, T^2) is his optimal behavior in this case.

(c) Γ_2^t : In σ^*, player 2 proposes (s^*, ∞) to player 1, and then player 1 accepts it. Hence, $F_2(\sigma^*)|_{\Gamma_2^t} = f_2(s^*)$. Assume that player 2 proposes (s, T) with $f_2(s) > f_2(s^*)$ in order to increase his payoff. Since $f_1(s) \leq f_1(s^*)$ from the Pareto-optimality of s^*, player 1 rejects it. Player 2 will obtain at most $f_2(s^*)$ in the next subgame $\Gamma_1^t(s, T)$. This means that $\sigma^*|_{\Gamma_2^t}$ is a Nash equilibrium point of Γ_2^t.

(d) $\Gamma_2^t(s^1, T^1)$: Similarly to (b), we can show that σ^* induces a Nash equilibrium point in $\Gamma_2^t(s^1, T^1)$ for any pair (s^1, T^1).

By the discussion above, we have shown that σ^* induces a Nash equilibrium point in every type of subgames of Γ. Q.E.D.

<u>Proof of Theorem 4.4.</u> Since the zero-memory $\mathcal{M}^0$ clearly satisfies (4.1), it is sufficient from Theorem 4.3 to prove the only-if part. Let $\sigma^* = (\sigma_1^*, \sigma_2^*)$ be a subgame perfect equilibrium point with the memory $\mathcal{M}$ satisfying (4.1) such that $a(\sigma^*) = (s^*, s^*, \ldots)$. Assume that s^* is not Pareto optimal. Then, there exists some $\tilde{s} \in S$ such that $f_i(\tilde{s}) > f_i(s^*)$ for all $i = 1, 2$. Now, suppose that player 2 unilaterally deviates from σ^* in a way that he rejects player 1's any proposal and counter-proposes $(\tilde{s}, \infty)$ from period 1 to period t^*-1. If player 1 accepts player 2's proposal in some period t ($< t^*-1$), then player 2 can increase his average payoff from $f_2(s^*)$ to $f_2(\tilde{s})$. This contradicts that σ^* is a subgame perfect equilibrium point in Γ. Therefore, it suffices us to consider a case that negotiations broke down in all periods before period t^*-1. We examine the equilibrium condition for player 1's response to $(\tilde{s}, \infty)$ in period t^*-1.

If player 1 accepts it, his average payoff will be $f_1(\tilde{s})$. Otherwise, his average payoff will be $F_1(\sigma^*)|_{\Gamma^{t^*}}$, which is independent of history in all past periods since the memory $\mathcal{M}$ of σ^* satisfies (4.1). Thus, the equilibrium condition for player 1's response to $(\tilde{s}, \infty)$ is

$$\begin{cases} \text{accept} & \text{if} \quad f_1(\tilde{s}) > F_1(\sigma^*)|_{\Gamma^{t^*}} \\ \text{accept or reject} & \text{if} \quad f_1(\tilde{s}) = F_1(\sigma^*)|_{\Gamma^{t^*}} \\ \text{reject} & \text{if} \quad f_1(\tilde{s}) < F_1(\sigma^*)|_{\Gamma^{t^*}} \end{cases} .$$

First consider the case that $f_1(\tilde{s}) > F_1(\sigma^*)|_{\Gamma^{t^*}}$. From the equilibrium condition above, player 1 will accept player 2's proposal $(\tilde{s}, \infty)$ and thus player 2 will obtain the average payoff $f_2(\tilde{s})$ greater than $f_2(s^*)$. This contradicts that σ^* is a subgame perfect equilibrium point in Γ. Next, consider the case that $f_1(\tilde{s}) \leq F_1(\sigma^*)|_{\Gamma^{t^*}}$. Assume that player 1 deviates from σ^* in the same manner as player 2, i.e., from period 1 to period $t^* - 1$, he proposes $(\tilde{s}, \infty)$ and , if it is rejected by player 2, then he also rejects any proposal of player 2. By the same reason as above, it suffices us to assume that negotiations broke down in all periods before period $t^* - 1$. Then, the average payoff of player 1 in Γ will be $f_1(\tilde{s})$ if player 2 accepts $(\tilde{s}, \infty)$ in period $t^* - 1$, and $F_1(\sigma^*)|_{\Gamma^{t^*}}$ otherwise since $\mathcal{M}$ satisfies (4.1). Whichever happens, player 1 will obtain his average payoff greater than $f_1(s^*)$. This also contradicts that σ^* is a subgame perfect equilibrium point in Γ. Q.E.D.

<u>Proof of Theorem 4.5.</u> We define a strategy $\sigma_i^* = \{ \sigma_i^{*t} = (x_i^t , y_i^t , z_i^t) \}_{t=1}^{\infty}$ for player i (i = 1, 2) in Γ as follows. Let h^{t-1} be any history of the first move of player 1 in every period t = 1, 2, ...

case (i) : No breakdown of negotiations happens in the history h^{t-1} (the case that t = 1 is included).

$$x_1^t(h^{t-1}) = (s^* , \infty) , \tag{5.1}$$

$$y_1^t(h^{t-1}, (s^*, T^1), (s^2, T^2)) = \begin{cases} 1 & \text{if} \quad f_1(s^2) > f_1(m^2) \text{ and } T^2 = \infty \\ 0 & \text{otherwise,} \end{cases} \tag{5.2}$$

$$y_1^t(h^{t-1}, (s^1, T^1), (s^2, T^2)) = \begin{cases} 0 & \text{if} \quad f_1(s^2) < f_1(m^1) \text{ and } T^2 = \infty \\ 1 & \text{otherwise,} \end{cases} \tag{5.3}$$

where $s^1 \neq s^*$, and

$$z_1^t(h^{t-1}, (s^1 , T^1), (s^2 , T^2)) = s_1^{*2} . \tag{5.4}$$

We also define

$$x_2^t(h^{t-1} , (s^1 , T^1)) = \begin{cases} (s^* , \infty) & \text{if} \quad s^1 = s^* \\ (m^1 , \infty) & \text{if} \quad s^1 \neq s^*, \end{cases} \tag{5.5}$$

$$y_2^t(h^{t-1} , (s^1 , T^1)) = \begin{cases} 1 & \text{if} \quad s^1 = s^* \text{, or } f_2(s^1) > f_2(m^1) \text{ and } T^1 = \infty \\ 0 & \text{otherwise,} \end{cases} \tag{5.6}$$

$$z_2^t(h^{t-1} , (s^1 , T^1) , (s^2 , T^2)) = s_2^{*1} . \tag{5.7}$$

case (ii) : Negotiations break down in the history h^{t-1}.

Let $\tilde{t}$ be the first period of Γ in which negotiations break down in h^{t-1}. Let (s^1 , T^1) be the proposal of player 1 in period $\tilde{t}$. Then, we define

$$\sigma_i^{*t} = \begin{cases} \sigma_i^t(m^2) & \text{if} \quad s^1 = s^* \\ \sigma_i^t (m^1) & \text{if} \quad s^1 \neq s^* \end{cases} \tag{5.8}$$

where for $j = 1, 2$ $\sigma(m^j) = (\sigma_1(m^j), \sigma_2(m^j))$, $\sigma_i(m^j) = \{\sigma_i^t(m^j)\}_{t=1}^{\infty}$ $(i = 1, 2)$ is a subgame perfect equilibrium point in Γ with zero-memory satisfying $a(\sigma(m^j)) = (m^j, m^j, \dots)$. The existence of such $\sigma(m^j)$ is guaranteed by Theorem 4.3.

Clearly we have $a(\sigma^*) = (s^*, s^*, \dots)$. By the backward argument, we will prove that in every period t (t = 1, 2, ...) σ^* induces a Nash equilibrium point on every type of subgames of Γ, i.e., Γ^t, $\Gamma_2^t(s^1, T^1)$, Γ_2^t, $\Gamma_1^t(s^2, T^2)$ and Γ_{12}^t. For all subgames except Γ^t, we will prove this in the case that player 1 proposed (s^1, T^1) with $s^1 \neq s^*$. The similar proof holds in the case that $s^1 = s^*$. From the definition of σ^*, we remark that it suffices us to consider only case (i).

(a) Γ^t : Consider the deviation of player 1 from σ^*. Assume that player 1 proposes (s^1, T^1), $s^1 \neq s^*$, to player 2. By (5.6), player 2 accepts it only if $f_2(s^1) > f_2(m^1)$ and $T^1 = \infty$, and then player 1's average payoff will be $f_1(s^1)$. But, since

$$f_1(s^1) \leq f_1(m^1) = w_1 \leq f_1(s^*)$$

because of (4.2) and the Pareto optimality of m^1, player 1 can not increase his average payoff in this case. When player 2 rejects (s^1, T^1), player 2 counter-proposes (m^1, ∞) to player 1 by (5.5). If player 1 rejects it, $\sigma(m^1)$ will be played from period t + 1 as shown in (5.8). Hence, whether player 1 accepts player 2's proposal or rejects it, his average payoff will be at most $f_1(m^1) = w_1$. Therefore, the deviation from σ^* is not beneficial to player 1 in either case. The same result holds for player 2.

(b) Γ_{12}^t : When negotiations break down in period t, $\sigma(m^1)$ will be employed from period t + 1 independent of the outcome in period t as shown in (5.8). Since the payoffs in period t does not influence the average payoffs for players, σ^* induces a Nash equilibrium point on the subgame Γ_{12}^t.

(c) $\Gamma_1^t(s^2, T^2)$: We prove that (5.3) is the optimal response for player 1 to player 2's proposal (s^2, T^2). If player 1 rejects (s^2, T^2), then his average payoff will be $f_1(m^1) = w_1$ from (5.8). Therefore, when $f_1(s^2) < f_1(m^1)$ and $T^2 = \infty$, his optimal response is to reject (s^2, T^2). In other cases, if he accepts (s^2, T^2), then his average payoff will be $f_1(s^2)$ ($\geq f_1(m^1)$) when $T^2 = \infty$, and $f_1(s^*)$ ($\geq f_1(m^1)$) when $T^2 < \infty$. Therefore, in these cases, to accept (s^2, T^2) is the optimal response for player 1.

(d) Γ_2^t : Suppose that player 2 proposes (s^2, T^2) to player 1. We consider the three cases ; (1) $f_1(s^2) < f_1(m^1)$ and $T^2 = \infty$, (2) $f_1(s^2) \geq f_1(m^1)$ and $T^2 = \infty$, (3) $T^2 < \infty$. In the case (1), player 1 rejects (s^2, T^2) from (5.3), and player 2's average payoff will be $f_2(m^1)$. In the case (2), player 1 accepts (s^2, T^2) from (5.3). If $f_1(s^2) > f_1(m^1)$, then we have $f_2(s^2) \leq f_2(m^1)$ since m^1 is Pareto optimal. If $f_1(s^2) = f_1(m^1)$, then we have $f_2(s^2) \leq f_2(m^1)$ from (2.3), too. In the case (3), player 1 accepts (s^2, T^2) from (5.3), and player 2's average payoff will be $f_2(s^*)$ ($\leq f_2(m^1)$). Therefore (m^1, ∞) is the optimal proposal for player 2.

(e) $\Gamma_2^t(s^1, T^1)$: Similarly to (c), we can prove that (5.6) is the optimal response for player 2 to (s^1, T^1). Q.E.D.

6. Conclusion

From the results in Section 4, we can answer the questions posed in the Introduction. One may think that the introduction of negotiations and binding agreements trivially leads players to cooperation, i.e., Pareto optimal outcomes, in a supergame. However, this is not the case as shown in Theorems 4.1, 4.2 and 4.5. Theorems 4.1 and 4.2 show that, if we employ the Nash equilibrium point as a noncooperative solution concept, the multiplicity of equilibrium outcomes still remains even in our repeated bargaining game. Theorem 4.5 shows that even if we employ the subgame perfect equilibrium point, Pareto optimal outcomes are not necessarily attained if players punish each other severly over periods. Theorems 4.3 and 4.4 show that, in order to reach cooperation, it is important for players to forget each other's behavior in previous periods, or, in other words, to have a kind of " forgiveness " against each other. We can find similar observations in the results of computer tournaments of iterated Prisoner's Dilemma performed by Axelrod [2].

We conclude the paper with a few remarks. We have restricted our attention to stationary equilibrium points, and have allowed players to reach only the agreements that they should jointly choose the identical action pairs for finitely or infinitely many periods. If they are allowed to agree to any sequence of action pairs, payoffs by jointly-mixed strategies in the component game can be realized as the average payoffs of non-stationary equilibrium points in the repeated game. See Benoit and Krishna [3].

The rule of our bargaining game allows players to agree that they should play a certain action pair forever. In practice, it may be considered unnatural that they bind themselves to keep such an eternal agreement. The eternal contracts are caused by the use of the average payoff because any sequence of payoffs in finitely many periods are negligible. If we adopt the discounted payoff as the preference of a player and the discount factor is sufficiently close to 1, they could be replaced by contracts with finitely but sufficiently long terms.

Among many repeated bargaining games appeared in the literature, the multistage unanimity game introduced by Kalai and Samet [4] is closely related to our model. In order to compare both models, we interpret the multistage unanimity game as follows. The n players propose simultaneously their preferred action combinations in the component game. If their proposals coincide, they can reach the binding agreement that they will take the agreed upon action combination forever. Otherwise, they can make proposals again. When no agreement is reached after a given number of repetitions, negotiations end in failure and the players will choose their (presupposed) noncooperative actions forever. We can say that our repeated bargaining game describes in more detail how players reach a binding agreement than the unanimity game. The major merit of such detailed description of negotiations is in that a subgame perfect equilibrium point is sufficient to assure a Pareto optimal and individually rational outcome through players' noncooperative behavior while the multistage unanimity game needs a stronger solution concept like a persistent equilibrium point. On the other hand, the condition of zero-memory for strategies (corresponding to subgame symmetry in Kalai and Samet [4]) is essential in both models.

Finally, the present paper is an attempt to reduce the multiplicity of equilibrium outcomes in a supergame by introducing the possibility of negotiations. If we make more elaborate repeated bargaining game models, we may be able to narrow down further the set of equilibrium outcomes, and at last to select a unique bargaining solution such as the Nash bargaining solution. For these attempts, see Moulin [7] and Rubinstein [9].

References

[1] R.J. Aumann, Surveys of repeated games, in " Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern, " Bibliographisches, Institut Mannheim/Wien/ Zurich, 1981.

[2] R. Axelrod, " The Evolution of Cooperation, " Basic Books, Inc., New York, 1984.

[3] J.P. Benoit and V. Krishna, Finitely repeated games, Econometrica 53 (1985), 905-922.

[4] E. Kalai and D. Samet, Unanimity games and Pareto optimality, International Journal of Game Theory 14 (1985), 41-50.

[5] M. Kaneko, Some remarks on the folk theorem in game theory, Mathematical Social Sciences 3 (1982), 281-290.

[6] R.P. Luce and H. Raiffa, " Games and Decisions, " Wiley, New York, 1957.

[7] H. Moulin, Bargaining and non-cooperative implementation, mimeo., Laboratoire d'Econometrie de l'Ecole Polytechnique, Paris, 1982.

[8] A. Rubinstein, Equilibrium in supergames with the overtaking criterion, Journal of Economic Theory 21 (1979), 1-9.

[9] __________ , Perfect equilibrium in a bargaining model, Econometrica 50 (1982), 97-109.

[10] R. Selten, A simple model of imperfect competition, where 4 are few and 6 are many, International Journal of Game Theory 2 (1973), 141-201.

[11] __________ , Reexamination of the perfectness concept for equilibrium points in extensive games, Internationl Journal of Game Theory 4 (1975), 25-55.