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EQUILIBRIUM PRICE DYNAMICS
FOR AN EXPERIENCE GOOD

by

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Abstract

Equilibrium price dynamics for an experience good monopolist depends on the evolution of the ratio of informed to uninformed consumers. Consumers use prices and possibly firm age as signals of quality. Under a variety of informational assumptions, the price of a high-quality good initially lies above the full information monopoly price, never increases, and eventually falls (perhaps gradually) to the full information monopoly price. Kreps' criterion characterizes unique equilibria for a large class of environments.

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I. Introduction

We analyze the dynamics of equilibrium pricing in a new market with repeat purchases in which a monopolist knows whether the quality of its product is high or low, but consumers do not initially. Since consumers do learn about the quality of the product by purchasing it, the product is what Nelson (1970) terms an experience good. Moreover, consumers might also learn about product quality from price signals, or other market information, such as the age of the firm.

We focus on the price dynamics associated with a gradual diffusion of information about a new product. Specifically, we assume that consumers only gradually learn about the existence of the product, that "new consumers" who have only just entered the market do not directly observe its quality, and that "old consumers" obtain perfect word-of-mouth information about quality after one period. Thus, we consider price signalling in a market environment in which informed consumers and uninformed consumers coexist, and in which the ratio of informed to uninformed increases over time.

Under a variety of informational assumptions, we conclude that the price of a high-quality good will be initially above the full information monopoly price, will never increase, and will be lowered eventually (perhaps gradually) to the full information monopoly price. The basic intuition is simple. Assuming that costs rise with quality, the high-quality firm prefers to signal its quality with high (rather

than low) prices. As time passes, information about the product diffuses; consequently, quality becomes easier to signal, while the opportunity cost of supramonopoly prices increases. These forces exert a downward pressure on the high-quality price.

Throughout the paper, we maintain the assumption that new consumers do not directly observe quality, and adopt Kreps' (1984) refinement of sequential equilibrium as our equilibrium concept. In Section II, we develop the "ignorant consumer model" in which new consumers observe current price and the age of the firm, but do not observe the past history of prices. We find that a unique separating equilibrium typically exists and characterize its properties. In this equilibrium, high-quality products are introduced at a price above the full information monopoly price. The high-quality price then gradually declines over time, as information about the product diffuses. Eventually, when the stock of informed consumers is sufficiently large, the high-quality price rests permanently at the full information monopoly price. Price declines more quickly the faster the rate of information diffusion. Pooling can never occur in the first period, and, for a large set of parameter values, pooling cannot occur in any period.

In Section III, we discuss the "hindsight consumer model," in which new consumers observe the current price, the age of the firm, and all past prices. Although new consumers now condition their beliefs on a greater number of signals, a unique separating equilibrium again exists, and it is the same equilibrium as derived in the ignorant

consumer model. We also discuss a recursive argument whereby Kreps' refinement is applied to a dynamic environment with history-dependent strategies.

Section IV contains the "confused consumer model" in which new consumers know only the current price. Thus, new consumers do not know past prices, or even the age of the firm. If the horizon is sufficiently long, then there exists no fully separating equilibrium. The argument is intuitive. As information about the product diffuses, the high-quality firm will eventually desire to charge its full information monopoly price and maximize profits from the large stock of informed consumers. But this creates an incentive for the low-quality firm to introduce its product at the high-quality monopoly price, thus pretending to be a mature high-quality firm. A kind of "intertemporal pooling" is therefore inevitable. There do often exist unique equilibria in "simple step strategies" of the following form. The high-quality firm introduces its product at a high separating price for a fixed number of periods, and then permanently lowers its price to the high-quality monopoly price. On the other hand, the low-quality firm initially mimics the mature high-quality firm by charging the high-quality monopoly price for a fixed number of periods, and then permanently lowers its price to the low-quality monopoly price. While there might also exist equilibria in more complicated step strategies, we do not find them plausible.

In a final section, we draw conclusions and briefly discuss a few extensions. Before proceeding, however, we relate our analysis to previous literature.

The optimal pricing of an experience good by a monopolist has been studied previously by Shapiro (1983) under the assumptions that consumers' initial point expectations about quality are exogenous and common knowledge, that consumers do not draw any inference about quality from observed prices or any other market variables, and that each consumer learns perfectly about quality upon buying the good once. Shapiro considered both the "optimistic case," in which consumers initially overestimate quality, and the converse "pessimistic case." In the optimistic case, the optimal policy for the firm is to temporarily "milk its reputation" by gradually lowering its price during introductory periods and then to permanently raise its price to the full information monopoly price. In the pessimistic case, the firm charges a low introductory price for one period and then permanently charges a price above the full information monopoly price.

A restrictive feature of Shapiro's analysis is the assumption that consumers do not draw any inferences about quality from the observed price path. Ignoring the signalling role of prices enables Shapiro to focus purely on the consequences for price dynamics of bundling information about product quality with the product itself. We focus instead on price signalling. Towards this end, we develop an equilibrium model of price dynamics in which consumers draw rational inferences about product quality from all available information,

including observed prices and possibly the age of the firm. We focus most of our attention on "separating equilibria," in which equilibria prices perfectly signal information about product quality. Such equilibria obviously lack the property that information about quality is bundled with the product.

Milgrom and Roberts (1986) also analyzed an experience good monopoly, focusing on conditions under which introductory price and advertising might jointly signal product quality.^{1/} They also briefly considered price dynamics, concluding that introductory price must be lower than the subsequent regular price. This rising price path follows from their assumptions that the product might be either "satisfactory" or "unsatisfactory" for each individual consumer, that product "quality" determines the probability of the product being satisfactory for a random consumer, that consumers have different reservation prices for a satisfactory product and a zero reservation price for an unsatisfactory product, that each consumer learned whether the product was satisfactory only after making an initial purchase, and that consumers who do not make an initial purchase leave the market. In a separating equilibrium all consumers correctly infer the true quality by observing introductory price and advertising, consumers with sufficiently high expected reservation prices make initial purchases, and the monopolist raises its second period price to extract all surplus from the marginal consumer who found the product satisfactory. Somewhat analogously to Shapiro's pessimistic case, it is the fact that information about product satisfactoriness is bundled with the product that leads to the rising

price path. While our analysis is similar in spirit to Milgrom and Roberts's, we ignore advertising, as well as consumer-specific uncertainty about tastes (satisfactoriness), and instead focus purely on the dynamics of price signalling arising from information diffusion.

Our distinction between informed and uninformed consumers is reminiscent of a related literature on product selection in which by assumption some consumers directly observe quality while others do not. In this context, Chan and Leland (1982), Cooper and Ross (1984), and Farrell (1980) analyzed competitive markets, while Cooper and Ross (1985) analyzed monopoly. This literature argued that informed consumers create a market externality by attracting high-quality products.

We are concerned instead with pure adverse selection. Thus, the market's provision of quality is determined by chance rather than by technology decisions or entry decisions.^{2/} Nevertheless, some issues are similar to those in the product selection literature. In particular, informed consumers might create a market externality by making it "easier" for a high-quality firm to distinguish itself by price signalling.

II. The Ignorant Consumer Model

We consider a new product which is either high quality or low quality. Letting q index quality, we represent these possibilities as $q = H$ and $q = L$. "Nature" determines the firm's quality of product, with $r \in (0,1)$ being the commonly known probability that $q = H$.

There is a continuum of consumers, with mass M , and each consumer has an inelastic demand for one unit of the product. If the product is known (or believed) to be low quality, then consumers are assumed to have a common reservation price P^L , where $P^L > 0$. If, instead, the product is known (or believed) to be high quality, then consumer preferences are heterogeneous. In this latter case, consumers are distributed uniformly on $[P^L, 1+P^L]$, where $v \in [P^L, 1+P^L]$ corresponds to a reservation price. The uniform case is convenient, as it generates a linear demand. For a price P , valuation v , and quality q , a consumer's indirect utility function is $U(P,v,q)$, where $U(P,v,H) = v - P$ and $U(P,v,L) = P^L - P$.

High-quality production involves a marginal cost of c , where $c > 0$. Fixed costs are ignored with no essential loss of generality. For simplicity, we take low-quality production to be costless.

A single stock of potential consumers lives throughout the T -period game. T can be finite or infinite, although we will typically think of T as "large". Consumers gradually learn about the existence of the new product. Let Z_t be defined as the number of consumers unaware at the beginning of date t of the product's existence. We assume that $Z_{t+1} = (1-\alpha) \cdot Z_t$, where $\alpha \in (0,1)$. In words, each period a fraction α of uninformed consumers (with uniformly distributed valuations) learn of the new product. Using the initial condition $Z_1=M$, one can show that $Z_t = (1-\alpha)^{t-1} \cdot M$. Notice that Z_t is strictly decreasing over time and that Z_t approaches zero in the limit.

In any period t , new consumers (that is, consumers entering the market in period t) do not know product quality, past prices, or past purchase decisions. They only know t (the firm's age) and P_t (the firm's current price). While new consumers are incompletely informed, they are rational and therefore use P_t and t in forming beliefs about q . As new consumers are ignorant of past prices, we call the model developed in this section "the ignorant consumer model."

At any date, consumers who have previously entered the market are referred to as "old." Old consumers are assumed to know product quality. Such knowledge could come from direct experience with the product or from communication with users of the good. Since new consumers always learn quality before their second period in the market, optimal consumer strategies will be of a static nature; the interesting issue of "experimental" buying does not arise in our model.^{3/}

Summarizing, at time t the monopolist faces a market of $M-Z_t$ old consumers and $\alpha \cdot Z_t$ new consumers. The old know quality, and the new try to infer quality.

We proceed now to define a sequential equilibrium [Kreps and Wilson (1982)] for the game. In a sequential equilibrium, each player's strategy must be a best response to the strategies of other players at every information set. Of course, for a player to determine his best response at an information set, he must have some beliefs about the type of players against whom he is playing. A new consumer, for example, cannot determine a best response to a given price unless he has some belief about product quality. On the equilibrium path, beliefs are

required to agree with Bayes' Rule. However, off the equilibrium path, information sets are reached with probability zero and Bayes' Rule is inapplicable. This freedom in specifying off-equilibrium path beliefs generally leads to a multiplicity of sequential equilibria.

Consider first an old consumer at time t . He knows the current price (P_t), product quality (q), and his own valuation (v). He may also know some (or all) of the history of trade. However, since the consumer is "small" relative to the market, it is unreasonable to focus on strategies in which he responds to past play in hopes of influencing future play. The "small" consumer simply maximizes utility in each period. Thus, his purchase decision depends only on P_t , q , and v , for these three variables alone determine his utility.^{4/} We represent this strategy as $s^o(P_t, v, q)$. $s^o(\cdot)$ maps into the set $\{0, 1\}$, where 1 (0) indicates (don't) buy. Thus equilibrium requires that:

$$(N1) \quad s^o(P_t, v, q) = \begin{cases} 1, & \text{if } q = H \text{ and } P_t \leq v ; \\ 1, & \text{if } q = L \text{ and } P_t \leq P^L ; \\ 0, & \text{otherwise .} \end{cases}$$

Consider next a new consumer at time t . By assumption, he knows only P_t , t , and v . After observing P_t , he forms a belief about the quality of the firm's product. Let $b_t(P_t)$ be this belief function, where $b_t(P_t)$ gives the probability with which new consumers at time t believe the firm to be high quality, given the observation of P_t . We can represent the new consumer's strategy as $s_t^N(P_t, v)$. Unlike old

consumers, the strategies of new consumers are time dependent. This is because t determines the number of new and old consumers, and it therefore affects the firm's incentives. Thus, a given price may signal something quite different to a new consumer in period t as compared to a new consumer in some later period. Equilibrium behavior for a new consumer requires:

$$(N2) \quad s_t^N(P_t, v) = \begin{cases} 1, & \text{if } b_t(P_t) \cdot v + (1 - b_t(P_t)) \cdot P^L \geq P_t ; \\ 0, & \text{otherwise .} \end{cases}$$

At time t , the firm knows q , t , and the history of prices and purchase choices. However, for reasons analogous to those discussed above, only q and t affect the firm's incentives in an interesting way. Thus, we represent the firm's strategy at time t as $P_t(q)$. The function P_t maps into $[0, 1 + P^L]$.

Since past prices do not affect consumer strategies, an optimizing firm will choose, at every t , $P_t(q)$ to maximize profit in period t . Let $\pi_t(q, b_t(P_t), P_t)$ be the t -th period profit to a firm of quality type q , charging price P_t , and facing new consumers with beliefs $b_t(P_t)$. ($s^O(\cdot)$ and $s_t^N(\cdot)$ are implicit in $\pi_t(\cdot)$.) Then, equilibrium behavior for the firm necessitates:

$$(N3) \quad \text{for } q = H \text{ or } L, P_t(q) \in \underset{P_t}{\operatorname{argmax}} \pi_t(q, b_t(P_t), P_t) .$$

Finally, we must say something about beliefs. As discussed above, they must be consistent with Bayes' Rule on the equilibrium path:

- (N4) (i) if $P_t(H) = P_t(L)$, then $b_t(P_t(H)) = r$;
- (ii) if $P_t(H) \neq P_t(L)$, then $b_t(P_t(H)) = 1$
- and $b_t(P_t(L)) = 0$.

Off the equilibrium path--that is, for $P_t \notin \{P_t(H), P_t(L)\}$ --sequential equilibrium imposes no restrictions on beliefs.

We now have the following definition:

Definition: A sequential equilibrium is a coupling of strategies and beliefs satisfying (N1)-(N4) for every t .

As mentioned above, the freedom which the sequential equilibrium concept affords in the specification of disequilibrium beliefs generally leads to multiple equilibria. One can ask, however, whether the beliefs supporting a particular equilibrium are plausible. The following refinement, due to Kreps, discards equilibria which are not stable against implicit credible speeches that a high-quality firm might make to consumers.^{5/}

Definition: Consider a sequential equilibrium E . Let $\{\pi_t^*(H), \pi_t^*(L)\}_{t=1}^T$ be the profile of single period equilibrium firm profits. E satisfies Kreps' criterion if, for all t , there does not exist a disequilibrium price P_t' satisfying the following conditions:

Condition 1: $\pi_t(H, 1, P_t') > \pi_t^*(H)$

Condition 2: $\pi_t(L, 1, P'_t) < \pi_t^*(L)$.

Kreps' criterion is a restriction on out-of-equilibrium beliefs. To understand this refinement, imagine an equilibrium E for which there exists t and P'_t satisfying Conditions 1 and 2. Now, suppose P'_t is announced. Which firm type should new consumers believe to be behind this deviation? Condition 2 indicates that the low-quality firm has no incentive to undertake such a deviation--it makes more profit with the equilibrium price $P_t(L)$ than with the deviant price P'_t even if new consumers believe it is high quality after seeing P'_t .^{6/} By contrast, Condition 1 says that the high-quality firm could increase its profits with a deviation to P'_t , if in charging P'_t it were believed to be the high-quality firm. Thus, the high-quality firm has an incentive to charge P'_t and (implicitly) explain to new consumers that they should set $b_t(P'_t) = 1$, since such beliefs are the only beliefs consistent with the observation of P'_t . E then fails Kreps' criterion, because it is not stable against implicit communication.

Our specification of Kreps' criterion is static; it compares only the current period profits of high- and low-quality firms. Yet our model is dynamic. However, in the ignorant consumer model, deviations from equilibrium prices can only affect the purchasing behavior of current new consumers through their belief function. Consequently, only current period profits can be affected by the deviation, and our static specification of Kreps' criterion is reasonable.

We adopt the following equilibrium concept:

Definition: An equilibrium is a sequential equilibrium which satisfies Kreps' criterion.

An equilibrium is said to be separating in period t if $P_t(H) \neq P_t(L)$. Pooling in period t is said to occur in the converse case. A separating equilibrium is separating in every period. Any other equilibrium is a pooling equilibrium, although we shall sometimes distinguish between full pooling and partial pooling. Note that, by definition, we only consider pure strategy equilibria.

Before examining the set of equilibria, it is useful to characterize $\pi_t(L, b_t(P_t), P_t)$ and $\pi_t(H, b_t(P_t), P_t)$. The following equations, discussed below, can be shown to hold:

$$\pi_t(L, b_t(P_t), P_t) = \begin{cases} P_t \cdot [M - (1-\alpha)Z_t] , & \text{if } P_t \leq P^L ; \\ 0 , & \text{if } P_t \geq P^L + b_t(P_t) \text{ and } P_t > P^L ; \\ P_t \cdot [1 - (P_t - P^L)/b_t(P_t)] \cdot \alpha Z_t , & \text{otherwise .} \end{cases}$$

$$\pi_t(H, b_t(P_t), P_t) = \begin{cases} (P_t - c) \cdot [M - (1-\alpha)Z_t] , & \text{if } P_t \leq P^L ; \\ (P_t - c) \cdot (1 + P^L - P_t) \cdot (M - Z_t) , & \text{if } P_t \geq P^L + b_t(P_t) \text{ and } P_t > P^L \\ (P_t - c) \cdot \{ (1 + P^L - P_t) \cdot (M - Z_t) + [1 - (P_t - P^L)/b_t(P_t)] \alpha Z_t \} , & \text{otherwise.} \end{cases}$$

Recall that there are $M - Z_t$ old consumers and αZ_t new consumers in period t . The demand curves for different types of consumers are

illustrated in Figure 1. Old consumers buy when $P_t \leq P^L$, if $q = L$, and buy when $v \geq P_t$ if $q = H$. Thus, for prices between $(1+P^L)$ and P^L , a fraction $1+P^L - P_t$ of the old consumers buy when $q = H$. The corresponding demand curves are given by D_0^L and D_0^H , respectively.

A new consumer with valuation v does not know quality and buys if $b_t(P_t) \cdot (v - P_t) + (1 - b_t(P_t)) \cdot (P^L - P_t) \geq 0$. Thus, for prices between $b_t(P_t) + P^L$ and P^L , a fraction $(1 - (P_t - P^L)/b_t(P_t))$ of new consumers buy, the corresponding demand curve being given by D_n^b .

Insert Figure 1 here

$\pi_t(q, b_t(P_t), P_t)$ is then simply the profit to a firm of type q charging P_t and facing beliefs $b_t(P_t)$, when old and new consumers make their respective buying decisions as described above.

We now state our first two lemmas.

Lemma 1: In any equilibrium, $P_t(H) \geq P^L$ and $P_t(L) \geq P^L$.

Proof: Notice that $\pi_t(L, b_t(P_t), P_t)$ and $\pi_t(H, b_t(P_t), P_t)$ are strictly increasing in P_t , for all functions $b_t(\cdot)$, when $P_t < P^L$. Thus, were $P_t(H)$ less than P^L , the high quality firm could increase its price slightly and increase profit. The same is true for the low-quality firm.

Q.E.D.

Lemma 2: In any equilibrium, if there exists t such that $P_t(H) \neq P_t(L)$, then $P_t(H) > P^L$ and $P_t(L) = P^L$.

Proof: Suppose instead that $P_t(L) > P^L$. Then, since the equilibrium is separating, the low-quality firm makes zero profit in period t . This is contradictory, since, against any beliefs, this firm could profit with $P_t = P^L$. Thus, by Lemma 1, we have that $P_t(L) = P^L$. Using Lemma 1 once more gives $P_t(H) > P^L$.

Q.E.D.

Lemmas 1 and 2 tell us that prices are never below P^L and that, when separation occurs, the low-quality firm charges P^L and the high-quality firm charges a higher price.^{7/}

We first consider separating equilibria. If separation occurs in some period t , then it must be that $\pi_t(L,0,P^L) \geq \pi_t(L,1,P_t(H))$. Otherwise, the low-quality firm would mimic the high-quality firm, thereby destroying the putative equilibrium. We are thus led to examine the set $\{P_t | \pi_t(L,0,P^L) = \pi_t(L,1,P_t)\}$. This equation has an upper and a lower root, \bar{P}_t and \underline{P}_t , respectively, where

$$\bar{P}_t = \frac{1+P^L}{2} + \left[\frac{(1+P^L)^2}{4} - P^L \cdot (1+X_t) \right]^{1/2},$$

$$\underline{P}_t = \frac{1+P^L}{2} - \left[\frac{(1+P^L)^2}{4} - P^L \cdot (1+X_t) \right]^{1/2}, \text{ and}$$

$$X_t = (M-Z_t)/\alpha \cdot Z_t.$$

Notice that X_t is the ratio of old consumers to the number of new consumers. It is strictly increasing in t , ranging from 0 (at

$t=1$) to infinite (as $t \rightarrow \infty$). We assume that $1 > P^L$ and represent \bar{P}_t and \underline{P}_t in Figure 2 as functions of X_t .^{8/} The upper boundary of the parabola characterizes the path of \bar{P}_t , while the lower boundary characterizes the path of \underline{P}_t . The critical point \bar{x} satisfies $\pi_t(L, 0, P^L) \leq \pi_t(L, 0, (1+P^L)/2)$ for $X_t \leq \bar{x}$, and conversely for $X_t \geq \bar{x}$. Since X_t is strictly increasing in t , there exists a critical date \bar{t} such that $X_t < \bar{x}$ for all $t < \bar{t}$ and $X_t \geq \bar{x}$ for all $t \geq \bar{t}$. For any $t < \bar{t}$ and price inside the parabola--that is, for any $P_t \in (\underline{P}_t, \bar{P}_t)$ --we have $\pi_t(L, 0, P^L) < \pi_t(L, 1, P_t)$. Thus, the low-quality firm will mimic any high-quality price $P_t \in (\underline{P}_t, \bar{P}_t)$. This leads immediately to the following lemma.

Insert Figure 2 here

Lemma 3: In any equilibrium, if there exists $t < \bar{t}$ such that $P_t(H) \neq P_t(L)$, then either $P_t(H) \geq \bar{P}_t$ or $P_t(H) \leq \underline{P}_t$.

It is important to understand the parabola in Figure 2. By following the high-quality firm to a price $P_t(H) > P^L$, the low-quality firm both gains and loses. It gains because it fools new consumers into buying at a high price. But, it loses because its old consumers, who know better, refuse to buy at the high price. Over time, the number of new consumers diminishes relative to the number of old consumers. Consequently, the low-quality firm becomes less and less willing to mimic the high-quality firm. For this reason, the parabola narrows about $(1+P^L)/2$, the maximizer of $\pi_t(L, 1, P_t)$, as time passes.

Eventually, a time \bar{t} is reached such that $X_t \geq \bar{x}$, and after which the low-quality firm even refuses to mimic $(1+P^L)/2$. Thus, for $t > \bar{t}$, mimicry is always unattractive and \bar{P}_t and P_t are undefined.

$P^H \equiv (1+P^L + c)/2$ is the maximizer of $\pi_t(H,1,P_t)$. That is, P^H is the high-quality firm's full information monopoly price. If $P^H > \bar{P}_1 = 1$, then the low-quality firm would be unwilling to mimic the high-quality firm's favorite price. In this case, the natural separating equilibrium has $P_t(H) = P^H$ and $P_t(L) = P^L$. The more interesting problem emerges when $P^H < 1$ or, equivalently, $P^L + c < 1$. In this latter case, separation is costly to the high-quality firm. For the remainder of the paper, we assume $P^L + c < 1$.

Define \tilde{t} as the lowest value of t such that $\pi_t(L,0,P^L) \geq \pi_t(L,1,P^H)$. Since $P^L + c < 1$, we know that $\tilde{t} > 1$. Notice that the low-quality firm has no desire to mimic P^H when $t \geq \tilde{t}$. Thus, at the time \tilde{t} the high-quality product enters into a "mature" phase--information about the product has become sufficiently diffuse that the high-quality firm can separate with its monopoly price, P^H .^{9/}

The following theorem establishes that, if a separating equilibrium exists, then the corresponding price paths for high- and low-quality firms are unique.

Theorem 1: In any equilibrium, if there exists t such that $P_t(H) \neq P_t(L)$, then $P_t(H) = \text{Max}(\bar{P}_t, P^H)$ and $P_t(L) = P^L$.

Proof: Assume the existence of an equilibrium in which, for some t , $P_t(H) \neq P_t(L)$. By Lemma 2, $P_t(L) = P^L$ and $P_t(H) > P^L$. Suppose to the contrary that $P_t(H) \neq \text{Max}(\bar{P}_t, P^H)$. Consider Figure 3.

Suppose first that $t > \tilde{t}$, so that $x_t \geq \tilde{x}$ and $P^H > \bar{P}_t$. Then $P_t(H) \neq P^H$ and $\pi_t(H, 1, P^H) > \pi_t(H, 1, P_t(H)) \equiv \pi_t^*(H)$. Further, since $t > \tilde{t}$, $\pi_t(L, 1, P^H) < \pi_t(L, 0, P^L) \equiv \pi_t^*(L)$. This contradicts Kreps' criterion.

Suppose next that $t \leq \tilde{t}$, so that $x_t \leq \tilde{x}$ and $P^H \leq \bar{P}_t$. Our contrary hypothesis is now that $P_t(H) \neq \bar{P}_t$. Thus, using Lemma 3, $P_t(H) > \bar{P}_t$ or $P_t(H) \leq \underline{P}_t$. Consider first $P_t(H) > \bar{P}_t$. Is this possible? To see that it is not, pick some $P'_t \in (\bar{P}_t, P_t(H))$. Since $\bar{P}_t \geq P^H$, $\pi_t(H, 1, P'_t) > \pi_t(H, 1, P_t(H)) \equiv \pi_t^*(H)$. Further, since $P'_t > \bar{P}_t$, $\pi_t(L, 1, P'_t) < \pi_t(L, 0, P^L) \equiv \pi_t^*(L)$. Thus, $P_t(H) > \bar{P}_t$ contradicts Kreps' criterion. A similar argument shows $P_t(H) < \underline{P}_t$ to be contradictory also.

The remaining possibility is $P_t(H) = \underline{P}_t$. It is straightforward to show that $\pi_t(H, 1, \bar{P}_t) > \pi_t(H, 1, \underline{P}_t)$ if and only if $c > 0$. Thus, for small $\epsilon > 0$, $\pi_t(H, 1, \bar{P}_t + \epsilon) > \pi_t(H, 1, \underline{P}_t) \equiv \pi_t^*(H)$. But $\pi_t(L, 1, \bar{P}_t + \epsilon) < \pi_t(L, 0, P^L) \equiv \pi_t^*(L)$. Again, we have a contradiction.

Q.E.D.

Insert Figure 3 here

Figure 3 illustrates the prices which must be charged when separation occurs. (The parabola is the same as in Figure 2.) When the market is young, the number of old consumers is small. The low-quality

firm then finds mimicry profitable for a large range of prices. Consequently, new consumers believe that quality is high only if a very high price is charged. As time passes, the number of old consumers increases and the low-quality firm finds mimicry more costly, as it entails a "burning up" old consumers. New consumers at later dates therefore require a less high price to be convinced that quality is high. Finally, as the firm matures and the number of new consumers gets very small, each firm type charges its full information monopoly price. Thus, the price of a high-quality product starts high and descends to its full information level.

It may seem surprising that the young high-quality firm separates with the high price \bar{P}_t instead of the low price \underline{P}_t . However, a simple intuition underlies this result. Since high-quality production is more costly, P^H is closer to \bar{P}_t than to \underline{P}_t . Thus, the high-quality firm prefers to separate with high prices. Put differently, high prices signal high quality because high quality costs more than low quality.^{10/}

So far, we have derived some necessary conditions for a separating equilibrium. We turn next to a complete characterization of necessary and sufficient conditions.

Theorem 1 identifies the best price path for the high-quality firm subject to the condition that the low-quality firm is unwilling to mimic any prices along the path. We now address whether the high-quality firm is willing to follow this path.

Separation can occur only if the high-quality firm chooses not to monopolize its old consumers at the loss of its new consumers.

Specifically, for the high-quality firm not to defect to P^H , we need $\pi_t(H, 1, P_t(H)) \geq \pi_t(H, 0, P^H)$. For $t \geq \tilde{t}$, this inequality is clearly met. Setting $\pi_t(H, 1, P_t) = \pi_t(H, 0, P^H)$ when $t < \tilde{t}$ defines a "no-defect" root:^{11/}

$$\bar{P}_t^0 = P^H + 1/2 \cdot (1 + P^L - c)(1 + X_t)^{-(1/2)} .$$

As illustrated in Figure 4, this curve asymptotically approaches P^H from above. The high-quality firm will defect from the price $P_t(H)$ if $P_t(H) > \bar{P}_t^0$. Clearly, for large t , $P_t(H)$ must be close to P^H .

Insert Figure 4 here

Like \bar{P}_t and P_t , \bar{P}_t^0 depends on time only in so far as it depends on X_t . That is, the fundamental dynamic variable is the ratio of new and old consumers. Define

$$\Delta(X_t) \equiv \begin{cases} \bar{P}_t^0 - \bar{P}_t & , \quad \text{for } t < \tilde{t} , \\ 1 & , \quad \text{if } t \geq \tilde{t} . \end{cases}$$

We have the following theorem.

Theorem 2: A separating equilibrium exists if and only if $\Delta(X_t) \geq 0$ for all t . By Theorem 1, $P_t(H) = \text{Max}(\bar{P}_t, P^H)$ and $P_t(L) = P^L$ in any separating equilibrium.

Proof: Suppose first that a separating equilibrium exists.

Assume to the contrary that there exists t such that $\Delta(X_t) < 0$. Thus, for some $t < \tilde{t}$, $\bar{P}_t^0 < \bar{P}_t$. But then the high-quality firm would defect from \bar{P}_t to P^H , which is a contradiction.

Going the other way, suppose that $\Delta(X_t) \geq 0$, for all t . We will verify a sequential equilibrium with $P_t(H) = \text{Max}(\bar{P}_t, P^H)$ and $P_t(L) = P^L$, and beliefs $b_t(P_t) = 0$ for all $P_t \notin \{P_t(H), P_t(L)\}$. Any deviation to $P_t > P^L$ causes the high-quality firm to lose new customers. Suppose first that $t < \tilde{t}$. Given the described beliefs, the high-quality firm has two deviation candidates. First, a deviation to P^H might be considered. But, since $\Delta(X_t) \geq 0$ for all t , this deviation doesn't increase profit. Second, a deviation to P^L attracts all consumers and might seem profitable. It is not:

$$\begin{aligned} \pi_t(H, 1, \bar{P}_t) - \pi_t(H, 0, P^L) &\geq \pi_t(H, 1, \bar{P}_1) - \pi_t(H, 0, P^L) \\ &= c \cdot (1 - P^L) \cdot [M - (1 - \alpha) \cdot Z_t] > 0. \end{aligned}$$

Suppose next that $t \geq \tilde{t}$. The only candidate for deviation now is P^L . Arguing as above, one can show such a deviation to be unprofitable. Thus, for any t , $P_t(H)$ is a best response. It is straightforward to verify that the low-quality firm has no profitable deviation. Similarly, consumers are behaving optimally, given their beliefs. Thus, a sequential equilibrium exists.

It also satisfies Kreps' criterion. To see this, observe that $\pi_t(H, 1, P_t^!) > \pi_t(H, 1, P_t(H))$ implies $\pi_t(L, 1, P_t^!) > \pi_t(L, 0, P^L)$.

Q.E.D.

In Figure 5 we illustrate parameter values for which $\Delta(X_t) \geq 0$ for all t . (The only parameters of $\Delta(1)$ are P^L and c) As the

figure illustrates, $\Delta(X_t) \geq 0$ for all t is true for a very large set of environments.^{12/} It is possible, however, that t exist such that $\Delta(X_t) < 0$ if P^L and c are sufficiently low. In that case no separating equilibrium exists.

Insert Figure 5 here

Thus, if $\Delta(X_t) < 0$ for some t , then an equilibrium exists only if $P_t(H) = P_t(L)$. That is, negative Δ 's must go with pooling equilibria. Lemma 4 shows that Kreps' criterion also rules out pooling in period one.

Lemma 4: In any equilibrium, pooling cannot occur in period one.

Proof: We show that pooling cannot exist at $t=1$. Suppose to the contrary that pooling occurred at price P_1 . Then $b_1(P_1) = r$. Define $Q(P,b) \equiv [1-(P-P^L)/b] \cdot \alpha M$. Since there are no old customers, both high- and low-quality firms sell a quantity $Q_1 = Q(P_1,r)$.^{13/} The high-quality firm earns profits $\pi^H \equiv (P_1 - c)Q_1$ and the low-quality firm earns profits $\pi^L \equiv P_1 Q_1$. Clearly $1 \geq P_1 \geq \max\{P^L, c\}$; otherwise one or the other firm would defect. Consider $P' > P_1$ such that $P' \cdot Q(P',1) = \pi^L$. Note that $P \cdot Q(P,1)$ is concave in P and is maximized at $(1/2)(1+P^L)$. Thus, since $0 < \pi^L < P_1 \cdot Q(P_1,1)$, P' clearly exists. Furthermore, $P \cdot Q(P,1)$ must be decreasing in P for $P \geq P'$. If $b(P') = 1$, then profits of the low-quality firm at price P' are equal to π^L , while the profits of the high-quality firm are

equal to $\pi^H + c[Q_1 - Q(P', 1)] > \pi^H$. Moreover, profits for the low-quality firm are decreasing in P , for $P \geq P'$, even if consumers continue to believe that the firm is high quality. Therefore, there exists a price slightly above P' that fails Kreps' criterion. It follows that pooling cannot occur in period 1.

Q.E.D.

Intuitively, pooling equilibria fail to exist because (by Kreps' criterion) there always exists a high price at which a high-quality firm can profitably distinguish itself. This deviant price keeps high-quality revenue--and hence low-quality profit--constant. Since demand is less at the high price, the deviation increases high-quality profit. This line of argument may fail at later dates when a stock of old consumers exists: By moving to the high deviant price, the high-quality firm may move further from P^H , causing a loss of profit on old consumers. Nevertheless, when the prior on high-quality is small, pooling can never occur. This point is established in the following theorem:

Theorem 3: If $r \leq \max(c - P^L, P^L \cdot (3 + 2\sqrt{2}))$, then, in any equilibrium, pooling can never occur.

Proof: We show first that pooling cannot occur when $r \leq P^L \cdot (3 + 2\sqrt{2})$. Suppose to the contrary that pooling occurs at $\bar{P} > P^L$ at some t . By Theorem 3, $t \geq 2$. $\pi_t(L, 0, P^L) \leq \pi_t(L, r, \bar{P})$ is clearly necessary. Equivalently, pooling implies that $0 \leq \bar{P} \cdot [r + P^L - \bar{P}] - r \cdot P^L \cdot [X_t + 1] \equiv g(\bar{P}, t)$. Notice g is strictly

decreasing in t . Therefore, $g(\bar{P},1) > g(\bar{P},2) \geq 0$ is necessary.

Since $X_1 = 0$ and $X_2 > 1$, it follows that

$f_1(\bar{P}) \equiv \bar{P} \cdot [r + P^L - \bar{P}] - r \cdot P^L > 0$ and $f_2(\bar{P}) \equiv \bar{P} \cdot [r + P^L - \bar{P}] - 2 \cdot r \cdot P^L > 0$. f_1 and f_2 are strictly concave and maximized at $(r + P^L)/2$.

Thus, $f_2((r + P^L)/2) > 0$, which in turn implies $r \in (0, P^L(3 - 2\sqrt{2}))$ or $r \in (P^L(3 + 2\sqrt{2}), 1)$. The latter possibility contradicts our

hypothesis. Suppose then that $0 < r < P^L(3 - 2\sqrt{2})$. Then $0 < r < P^L$.

Notice that $f_1(P) = 0$ if and only if $P = (r + P^L)/2 \pm 1/2 \cdot |P^L - r|$.

Thus, $f_1(\bar{P}) > 0$ implies $\bar{P} \in (r, P^L)$. This contradicts $\bar{P} > P^L$.

Obviously, pooling can never occur at $\bar{P} < P^L$. By Kreps' criterion, $\bar{P} = P^L$ is also impossible. (Recall from the proof of Theorem 2 that $\pi_t(H, 1, \bar{P}_t) > \pi_t(H, r, P^L)$.) Thus, if $r < P^L(3 + 2\sqrt{2})$, then pooling cannot occur.

Suppose next that $r \leq c - P^L$. If pooling occurs in any period t at some price \bar{P} , then $\bar{P} \geq c$ and $\bar{P} < r + P^L$. (If the latter inequality did not hold, the low-quality firm would deviate to P^L .) Thus, pooling implies $r > c - P^L$, contradicting our hypothesis.

Q.E.D.

Intuitively, if a good product is hard to find (i.e., if r is small), then the price which can be commanded in a putative pooling equilibrium is low and, consequently, one firm or the other will refuse to pool. Note, moreover, that if P^L is not too small, then pooling equilibria fail to exist for any $r \in (0, 1)$. In particular, $P^L \geq [1/(3 + 2\sqrt{2})]$ is sufficient for pooling equilibria not to exist.

We now summarize our results. Separating equilibrium exists if and only if P^L and c are sufficiently large (Figure 5). If it does exist, then high- and low-quality prices are uniquely determined at each date by the ratio of old to new consumers (Figure 3). High-quality prices decline over time, eventually resting at the (high-quality) full information monopoly price. Low-quality prices are constant at the (low-quality) full information monopoly price. A higher rate of information diffusion (α) decreases the ratio of old to new customers at all dates except the first, resulting in a faster decline in high-quality prices. Pooling can not occur in the introductory period. If r is sufficiently small (or P^L is not too small) pooling can't occur.

In characterizing the unique separating equilibrium, we have relied on the informational assumption that new consumers are "ignorant" of the past; they know only the current price and the firm's age. The next two sections respectively modify this assumption in two directions. The "hindsight consumer model" grants new consumers more information (viz., past prices are known), while the "confused consumer model" gives new consumers less information (viz., firm age is no longer known). As the basic methodology of our analysis should be clear by now, we switch to a less formal exposition.

III. The Hindsight Consumer Model

In this section, we drop the assumption that consumers are ignorant of the history of trade. We assume now that consumers have (twenty-twenty) hindsight--a new consumer at time t knows t , P_t , and

the history of prices. Using all of this information, he attempts to infer quality. All other assumptions from the previous section are maintained.

The definition of a sequential equilibrium for the new game is analogous to that given above. There are, however, some important differences. Let ${}^tP \equiv \{P_\tau\}_{\tau=1}^t$ be the history of prices at time t . New consumer beliefs and strategies now depend on tP as opposed to P_t . Consumer strategies are still of a static nature--in each period, consumers (new and old) choose the purchase action which maximizes expected utility.

Bayesian consistency now takes on a new form:

- (N4)' (i) If, for all $\tau \leq t$, $P_\tau(H) = P_\tau(L)$, then $b_t({}^tP(H)) = r$
- (ii) If there exists $\tau \leq t$ such that $P_\tau(H) \neq P_\tau(L)$, then $b_t({}^tP(H)) = 1$ and $b_t({}^tP(L)) = 0$.

Thus, if separation occurs in period τ , then the firm types are forever thereafter separated. Similarly, a true pooling state is reached if and only if pooling has occurred in every preceding period.

With this modification, and if T is sufficiently large, then all of our arguments and results of the previous section still apply with one exception--our argument justifying the application of the Kreps' criterion. Since Kreps' criterion is crucial for our uniqueness results, it is important to devise a new justification for it.

Recall that Kreps' criterion is based on a comparison of current period profits. This is reasonable for the ignorant consumer model

because any price deviation could only affect current profits. However, in the hindsight consumer model, future new consumers can observe any past price deviation and condition their beliefs upon it. Consequently, a deviation in period t might affect future profits, suggesting that the Kreps' criterion relies on implausibly myopic consumers.

However, this is not the case. Assume that $T \geq \bar{t}$. Recall that for $t \geq \bar{t}$, it is a dominant strategy for a low-quality firm to charge P^L , and that P^H is the full-information high-quality monopoly price. Thus new consumers should believe that $b_t(P_1, \dots, P_{t-1}, P^H) = 1$ for $t \geq \bar{t}$.¹⁴ That is, if new consumers observe price P^H at a date beyond t , then they should believe that the firm is high quality, no matter what the past sequence of prices. Given this restriction on beliefs, it is always optimal for a high-quality firm to set $P_t(H) = P^H$ and for the low-quality firm to set $P_t(L) = P^L$ for all $t \geq \bar{t}$.

Maintaining the above restriction of beliefs, we now consider period $\bar{t}-1$. Since both types of firms have fixed equilibrium strategies in all future periods, and future new consumers have fixed beliefs, any deviation in period $\bar{t}-1$ can affect only current profits. Hence, the application of Kreps' criterion is entirely sensible in this period. Moreover, by arguments analogous to those in Section II, for any subgame beginning at $\bar{t}-1$, Kreps' criterion implies the unique equilibrium price path beginning at $\bar{t}-1$ is $P_t(H) = \text{Max}\{\bar{P}_t, P^H\}$ for high quality and $P_t(L) = P^L$ for low quality.

The important point is that these equilibrium prices (and corresponding equilibrium beliefs) for the subgame beginning at $\bar{t}-1$

are independent of past prices. Thus moving back another period, and requiring that new consumers beliefs about the future correspond to the subgame equilibrium of the next period, Kreps' criterion is again sensibly applied. By a recursive argument, we drive the unique equilibrium price paths characterized in Theorem 1 and illustrated in Figure 4.

Thus application of Kreps' criterion does not imply consumer myopia. To the contrary, if consumers believe that a firm will never play a strictly dominated strategy, and also believe that Kreps' criterion will restrict the beliefs of all future generations of new consumers, then consumers must believe that the future evolution of the market is independent of the current market price. Consequently, they must believe that any current deviation from an equilibrium price will only influence current profits, and Kreps' criterion is a reasonable restriction on their beliefs.

We conclude that price dynamics in the hindsight consumer model are the same as in the ignorant consumer model.

IV. The Confused Consumer Model

This section modifies the ignorant consumer model by assuming that new consumers know neither the past history of prices nor firm age. A new consumer simply learns of the existence of the product and its current price, but has no direct information on the stock of old consumers. Therefore, at any point in time, the firm possesses two relevant pieces of private information: product quality and the size of

the stock of old consumers. New consumers might draw an inference about either or both from the observed current price. An sequential equilibrium is defined as before, except that the strategies and beliefs of new consumers must be stationary, i.e., depend only on the current price and not on time.

We modify Kreps' criterion by requiring that there does not exist a disequilibrium price, $P \notin \{(P_t(H), P_t(L))\}$, $t=1, \dots, T$, such that $\pi_t(H, 1, P) > \pi_t^*(H)$ for some t and $\pi_t(L, 1, P) < \pi_t^*(L)$ for all t . That is, if there exists an out-of-equilibrium price, which is profitable for a high-quality firm at some date if new consumers believe the product is high quality, but which is never profitable for a low-quality firm no matter what new consumers believe, then new consumers must assign probability one to high quality upon observing that price. We refer to this modified model as "the confused consumer model."

We begin by pointing out that there does not exist a separating equilibrium in the confused consumer model if the horizon T is sufficiently long and if $(P^L + c) < 1$. The reasoning is straightforward. In a separating equilibrium, the low-quality price must be P^L at every date. By Kreps' criterion, any separating high-quality price must be above P^H . Moreover, since new consumers' beliefs are time invariant, there can be at most one high-quality price observed in a separating equilibrium, for if there were more than one the high-quality firm would always defect to the one closer to P^H . However, as the market evolves and the ratio of old to new consumers gets large, the high-quality firm eventually must find it irresistible to set price

equal to P^H and to maximize profits from sales to old customers, even if it is taken to be low quality by new consumers. Thus, the separating price must be equal to P^H . However, $1 > P^L + c$ implies that the low-quality firm will profitably mimic a high-quality firm at price P^H at date 1.^{15/}

There might exist separating equilibria with a constant high-quality price above P^H if T is small. In this case, the stock of old customers never gets large enough to induce the firm to defect to P^H ; the high-quality firm always remains concerned about attracting new customers. We are mainly interested in the large T case.

With large T there very often do exist equilibria in which firms follow strategies illustrated in Figure 6. The high-quality product is introduced at price P^S for t_H periods, and then is permanently lowered to the long-run monopoly price, P^H . A low-quality product is introduced at price P^H for t_L periods, and then is lowered to the long-run monopoly price, P^L . While the figure is drawn so that t_H is greater than t_L , the converse might also be true. We will refer to these strategies as "simple step strategies."^{16/}

Insert Figure 6 here

Note that if firms did pursue step strategies then a new consumer would be able to infer high quality from price P^S and low quality from price P^L , but could make no clear inference about quality from observing price P^H . In fact, with prior beliefs that quality is high

with probability r , Bayesian consumers having observed price P^H would form posterior beliefs that quality is high with probability

$$\rho = \frac{r \sum_{\tau=t_H+1}^T Z_{\tau}}{r \sum_{\tau=t_H+1}^T Z_{\tau} + (1-r) \sum_{\tau=1}^{t_L} Z_{\tau}} \equiv \rho(t_H, t_L)$$

For the case $T = \infty$, we have

$$\rho(t_H, t_L) = \frac{r(1-\alpha)^{t_H}}{r(1-\alpha)^{t_H} + (1-r)[1-(1-\alpha)^{t_L}]}$$

That is, when new consumers observe a price P^H , they don't know whether it is an old high-quality firm or a new low-quality firm, and so they form posterior beliefs based on the relative likelihoods of these possibilities. Essentially, a type of intertemporal pooling happens.

We now develop the conditions under which the simple step strategies of Figure 6 form an equilibrium for appropriate values of P^S , t_L , and t_H and for appropriate consumer beliefs. In particular, consider the following beliefs of new consumers:

$$b^*(P) = \begin{cases} 1 & \text{for } P = P^S ; \\ \rho & \text{for } P = P^H ; \\ 0 & \text{otherwise .} \end{cases}$$

For the moment, P^S and ρ are specified arbitrarily, except that $P^S > P^H$ and $1 > \rho > P^H - P^L$. The latter inequality ensures that new consumers will buy at the pooling price, P^H .

Consider the incentives of a low-quality firm, given that new consumers have these beliefs. Clearly, charging a price other than P^S , P^H or P^L is dominated by charging P^L . Moreover, it must be that the firm prefers P^H to P^L when $t \leq t_L$, and prefers P^L to P^H when $t > t_L$. It can be shown that this is true if $(P^H/P^L)[1-(P^H-P^L)/\rho] \geq (X_t+1)$ holds for $t \leq t_L$ and the converse holds for $t > t_L$. Since X_t is strictly increasing in t , for any value of ρ , there is a unique $t_L = t_L(\rho)$ satisfying this condition. t_L is increasing in ρ .

We must also show that the low-quality firm never finds it desirable to charge P^S . This will be the case if and only if P^S is sufficiently high. Specifically, it must be that

$$P^S \geq 1/2 \cdot (1+P^L) + [1/4 \cdot (1+P^L)^2 - P^H(1-(P^H-P^L)/\rho)]^{(1/2)} \equiv \underline{P}(\rho) \quad . \quad 17/$$

The inequality assures that, for $t \leq t_L(\rho)$, the firm has no incentive to deviate from P^H to P^S . It follows a fortiori that, for $t > t_L(\rho)$, the firm has no incentive to deviate from P^L to P^S . Note that the lower bound on P^S is a function of ρ , $\underline{P}(\rho)$. Also note that $1+P^L > \underline{P}(\rho) > P^H$.

Now consider the incentives of the high-quality firm. Clearly this firm also will never consider any price other than P^S , P^H or P^L . Consequently, it must be that firm prefers P^S to P^H for $t \leq t_H$, and conversely for $t > t_H$. It can be shown that this holds if

$$(P^S-c)(1+P^L-P^S)(1+X_t) \geq (P^H-c)(1+P^L-P^H)X_t + (P^H-c)(1-(P^H-P^L)/\rho)$$

holds for $t \leq t_H$ and the converse holds for $t > t_H$. For any values of ρ and P^S , there is a unique $t_H = t_H(\rho, P^S)$ satisfying this condition. t_H is decreasing in ρ .

Similarly, for $t \leq t_H$, the high-quality firm must prefer to set P^S and be taken for high quality, then set P^L and be taken for low quality. This will be true if $(P^S - c)(1 + P^L - P^S) \geq (P^L - c)$, or, equivalently,

$$P^S \leq P^H + 1/2[(1+c)^2 + (P^L)^2]^{(1/2)} \equiv P^\dagger$$

Finally we consider beliefs. The Kreps' criterion requires that $P^S = \underline{P}(\rho)$. To see this, suppose $P^S > \underline{P}(\rho)$, and recall that by definition a low-quality firm will always prefer to set price equal to P^L or P^H , rather than to set price at $\underline{P}(\rho)$ and be taken for high. However, a young high-quality firm prefers $\underline{P}(\rho)$ to P^S , as long as new consumers continue to regard it as high quality, because $\underline{P}(\rho)$ is closer to P^H . Thus, by the Kreps' criterion, if new consumers observe a defection to $\underline{P}(\rho)$ they must believe high quality with probability one. But this creates an incentive for a young high-quality firm to defect from P^S to $\underline{P}(\rho)$. Therefore, it must be that $P^S = \underline{P}(\rho) \leq P^\dagger$.

Beliefs must also satisfy Bayes' rule along the equilibrium path. This requires that

$$(**) \quad \rho = \rho(t_H(\rho, \underline{P}(\rho)), t_L(\rho)) .$$

Since the functions $t_H(\cdot)$ and $t_L(\cdot)$ are step functions, (**) may not have a fixed point, in which case an equilibrium in simple strategies

does not exist. This is really technically, however, arising from the discreteness of time period. Equilibrium will exist over wide ranges of parameter values.^{18/}

To illustrate consider an example with $T = \infty$ and with parameters:

$$r = 0.05 \quad ;$$

$$c = 0.25 \quad ;$$

$$p^L = 0.05 \quad ;$$

$$\alpha = 0.001 \quad .$$

Then $p^H = 0.65$. Calculation establishes that an equilibrium in simple step strategies uniquely exists. The high-quality firm charges $p^S = 0.74$ for one period, and then permanently lowers price to $p^H = 0.65$. The low-quality firm introduces its product at $p^H = 0.65$ for four periods, and then permanently lowers price to $p^L = 0.05$. New consumers observing p^H believe that quality is high with probability $\rho = 0.93$. Thus, even though a separating equilibrium does not exist, prices do convey considerable information about product quality.

We conjecture that there might also exist equilibria in more complicated step strategies. For example, the low-quality firm might mimic the high-quality firm in several declining steps. This requires that the low-quality firm charge two different prices, both above p^L , at different points in time, even though the firm is always indifferent between these alternatives. The indifference arises because the firm is selling only to new new consumers at these prices, and these consumers

have stationary beliefs about quality. Since spurious price adjustments by the low-quality firm do not seem plausible to us, we did not investigate these equilibria.^{19/}

V. Conclusion

A high-quality experience good will be introduced at a high price that is lowered eventually to the full-information monopoly price. The high introductory price signals product quality. As information about the product diffuses two effects occur. First, the opportunity cost of signalling quality to new consumers rises as the high-quality firm becomes increasingly tempted to extract monopoly rents from informed consumers. This effect is particularly pronounced if new consumers do not know the firm's age, resulting in intertemporal pooling at the high-quality monopoly price. Second, if new consumers do observe firm age, it becomes easier to signal quality, and price gradually declines to the full-information price, perfectly signalling quality at each date.

Our model relies on many special features that can be relaxed. For example, linear costs and demands are not crucial for our conclusions. More substantive extensions of the model also suggest themselves. One possibility is to have overlapping generations of consumers. This would change the time path of the ratio of old to new consumers, which is the fundamental state variable in our model. All of our main conclusions would remain unaltered if the long-run ratio of uninformed to informed consumers were sufficiently low. If not, so that there were always a large proportion of uninformed consumers in the

market, then some of our conclusions might change. For example, separating equilibria might exist in confused consumer model, and our defense of the Kreps' condition in the hindsight consumer model is no longer valid. However, all our results for the ignorant consumer model would still go through.

Another extension that would help integrate our analysis with the previous literature would be to introduce an additional idiosyncratic taste variable for consumers that is learned only by experience. Since the taste variable is idiosyncratic it cannot be signalled by price, and therefore product information would be bundled with consumption, as in Shapiro's and Milgrom and Roberts' models. The interaction of price-signalling and information-bundling merits further research.

Another interesting extension would be to consider the price dynamics of new entry into an established market. What price path would be selected by an entrant into a market already inhabited by an incumbent? On the one hand, high prices may enable the high-quality entrant to separate most easily; but, on the other hand, the entrant might price itself out of the market with a high price. Bagwell (1985a) and Farrell (forthcoming) offer some preliminary results in this vein, but much work remains.

Still another extension would be to allow firms to choose quality. Will high prices be associated with high quality when there is moral hazard in quality? Riordan (1986) argues that this association does exist, if firms can commit to a price over a period of re-

purchases.^{20/} In effect, by picking a high price, a firm indicates an incentive to provide high quality in order to retain repeat purchases.

A final extension that we find intriguing would be to introduce advertising into the model. Our model assumes that information about the product's existence diffuses at an exogenous rate. An alternative assumption is that information diffusion depends on advertising expenditures. In Milgrom and Roberts's model, consumers observed both price and the entire advertising budget of the firm.^{21/} In equilibrium both jointly signalled quality. Suppose instead that consumers randomly observed advertising messages, but never observed the firm's entire advertising budget. Then advertising might (imperfectly) inform consumers about the existence of product, while price by itself might signal quality. Moreover, in equilibrium high quality firms might have a greater return to information diffusion, and so might advertise relatively more. Consequently, and contrary to Nelson's argument, a positive correlation between product quality and advertising need not imply that advertising is a signal of quality. We intend to develop these ideas more fully in further research.

Footnotes

1/ For other discussions of introductory price as a signal, see Bagwell (1985b), and of advertising as a signal see Kihlstrom and Riordan (1984) and Nelson (1974).

2/ Sharpe (1986) analyzed a multi-firm market in which quality is determined by chance and some consumers are informed. He focused on stationary pooling equilibria. In contrast, we vary the number of informed consumers in a natural way and emphasize nonstationary separating equilibria in a monopoly market.

3/ See Grossman, Kihlstrom and Mirman (1977) for a discussion of experimental buying.

4/ We restrict the domains of strategies with an intuitive notion of the strategic "size" of various players. Essentially the same restrictions emerge when a "payoff-relevant" or "Markov" refinement is imposed. See, e.g., Maskin and Tirole (1983).

5/ See Kohlberg and Mertens (forthcoming) for a more general notion of stability.

6/ Since this is the "most favorable" belief for the firm, it follows that the low-quality firm is worse off no matter what new consumers believe.

7/ Lemmas 1, 2, and 3 (see below) describe properties of sequential equilibria and do not depend on Kreps' criterion.

8/ If $P^L \geq 1$, then for ϵ such that $0 < \epsilon \leq 1$, $\pi_t(L, 1, P^{L+\epsilon}) = \pi_t(L, 0, P^L) - P^L \cdot (M - Z_t) - \epsilon \cdot \alpha \cdot Z_t \cdot (P^L - 1) - \epsilon^2 \cdot \alpha \cdot Z_t < \pi_t(L, 0, P^L)$. That is, a low-quality firm's marginal revenue, for a price increase above P^L , is negative no matter how beliefs adjust. Thus, if $P^L \geq 1$, then the low-quality firm is not tempted to mimic the high-quality firm at any price or time. The following can be shown: if $P^L \geq 1 + c > 1$, then the unique equilibrium has $P_t(H) = P_t(L) = P^L$, for all t ; if instead $1 + c \geq P^L \geq 1$, then in any equilibrium at any t , $P_t(L) = P^L$ and $P_t(H) = (1 + P^L + c)/2 \equiv P^H$.

9/ Sequential equilibria exist in which $P_t(H) \neq P^H$, for some $t \geq \tilde{t}$, but by Theorem 1 are eliminated by Kreps' criterion.

10/ The proof of Theorem 1 indicates that the Kreps' criterion is necessary to get the high price separating equilibrium uniquely. The argument that "high" prices signal high quality is implicit (but is not discussed) in Milgrom and Roberts (JPE, forthcoming). Ramey (1986) has established independently that high prices signal high quality in a model with a continuum of types; his argument does not rely on Kreps' criterion.

11/ This equation also has a lower root which is always below p^H . We have no interest in the lower root, since Theorem 1 establishes that $P_t(H) \geq P^H$ in a separating equilibrium.

12/ Our calculations can be derived as follows. First, minimize $\Delta(X_t)$ with respect to X_t . This gives a cubic first-order equation. Next, apply Cardano's formula to find the minimizing X_t . Plug this X_t into $\Delta(\cdot)$ to get a minimum value function, $\Delta^*(P^L, c)$. For a given c , find the critical P^{L*} such that $\Delta^*(P^L, c) \geq 0$, for $P^L \geq P^{L*}$. c and P^{L*} are plotted in Figure 5.

13/ For pooling to occur, $Q_1 > 0$ is clearly necessary.

14/ Obviously, our justification for the Kreps' criterion is relying on the additional refinement of eliminating dominated strategies. See Milgrom and Roberts (1986). In defining a dominated strategy, we assume optimal behavior on the part of old consumers.

15/ Note that this argument does not depend on the Kreps' criterion.

16/ Simple strategies with the high-quality product being introduced at a low price are ruled out by Kreps' criterion.

17/ This is the positive root of $\pi_t(L, 1, P^S) = \pi_t(L, \rho, P^H)$. Only new consumers buy at P^S or P^H , and so the root is independent of t .

18/ Since we have not been able to show that (**) is a contraction, multiple equilibria might exist in some cases.

19/ The only equilibria with unique best responses for firms are equilibria in simple step strategies. Therefore, suppressing consumers, we have a unique strict equilibria.

20/ See also Farrell (forthcoming). Ramey has investigated the case in which quality is chosen with error. He finds that an underinvestment in quality will typically occur. This suggests that the small r case may in fact be the appropriate case.

21/ The same is true in Kihlstrom and Riordan.

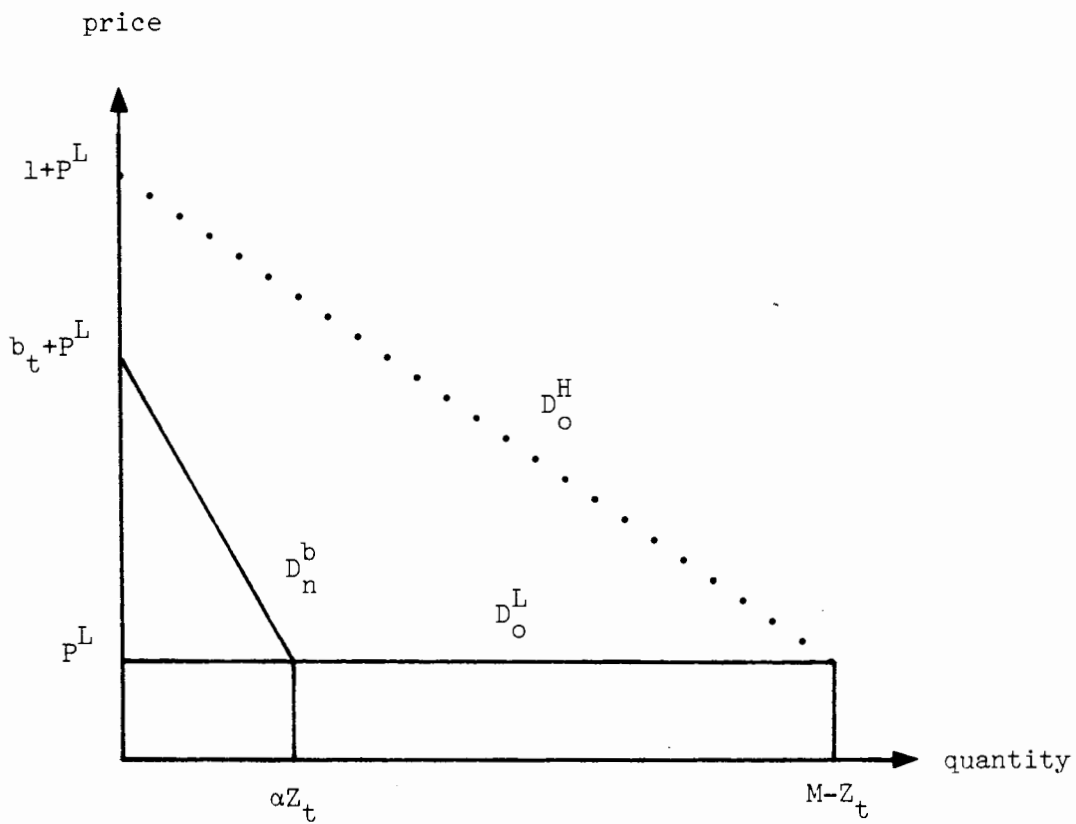
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Figure 1

Demand Curves in Period t



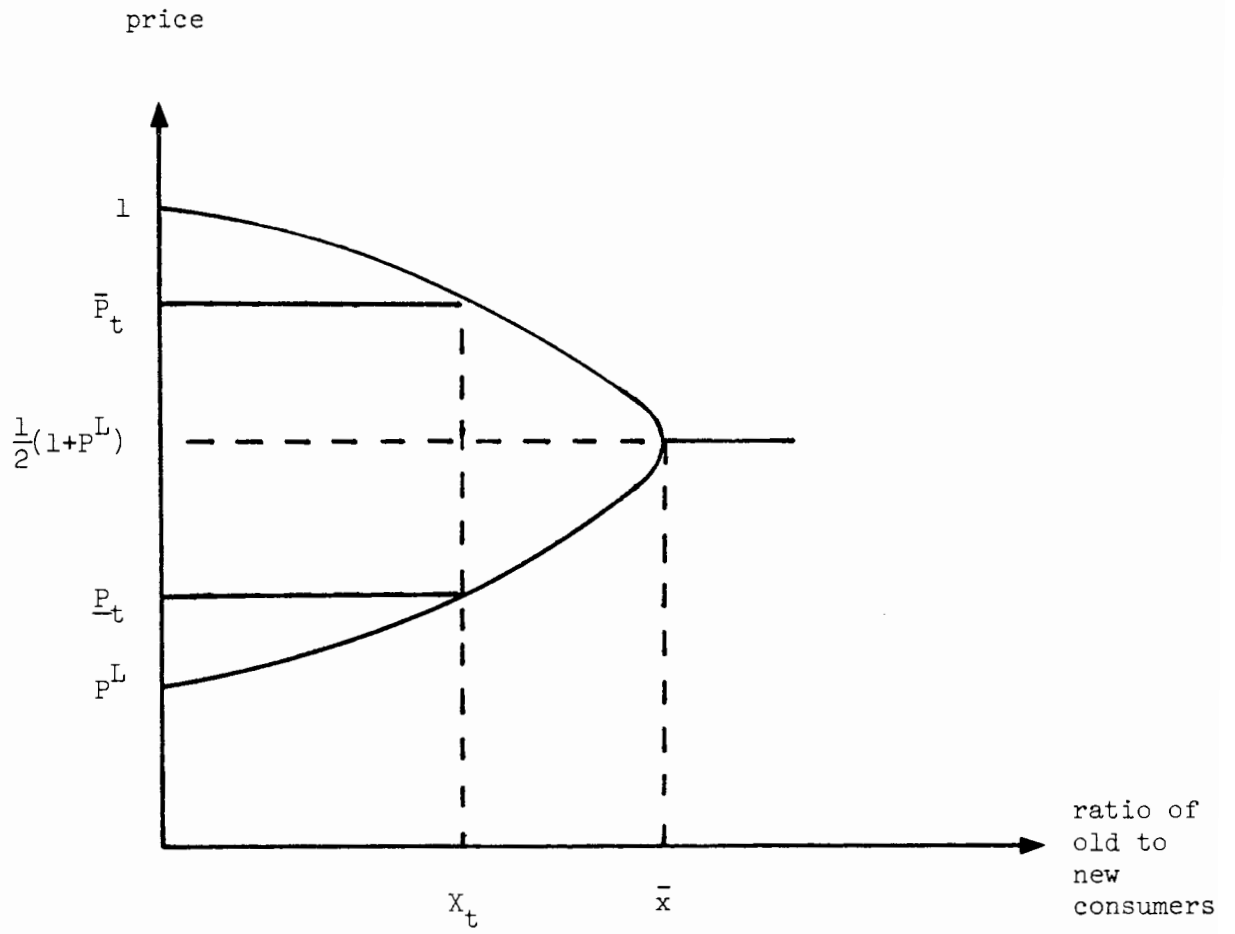
D_o^H : Demand of old consumers when $q = H$

D_o^L : Demand of old consumers when $q = L$

D_o^b : Demand of new consumers with beliefs b_t

Figure 2

Separating Prices



Prices outside of the parabola are separating.

Figure 3

Equilibrium Separating Prices

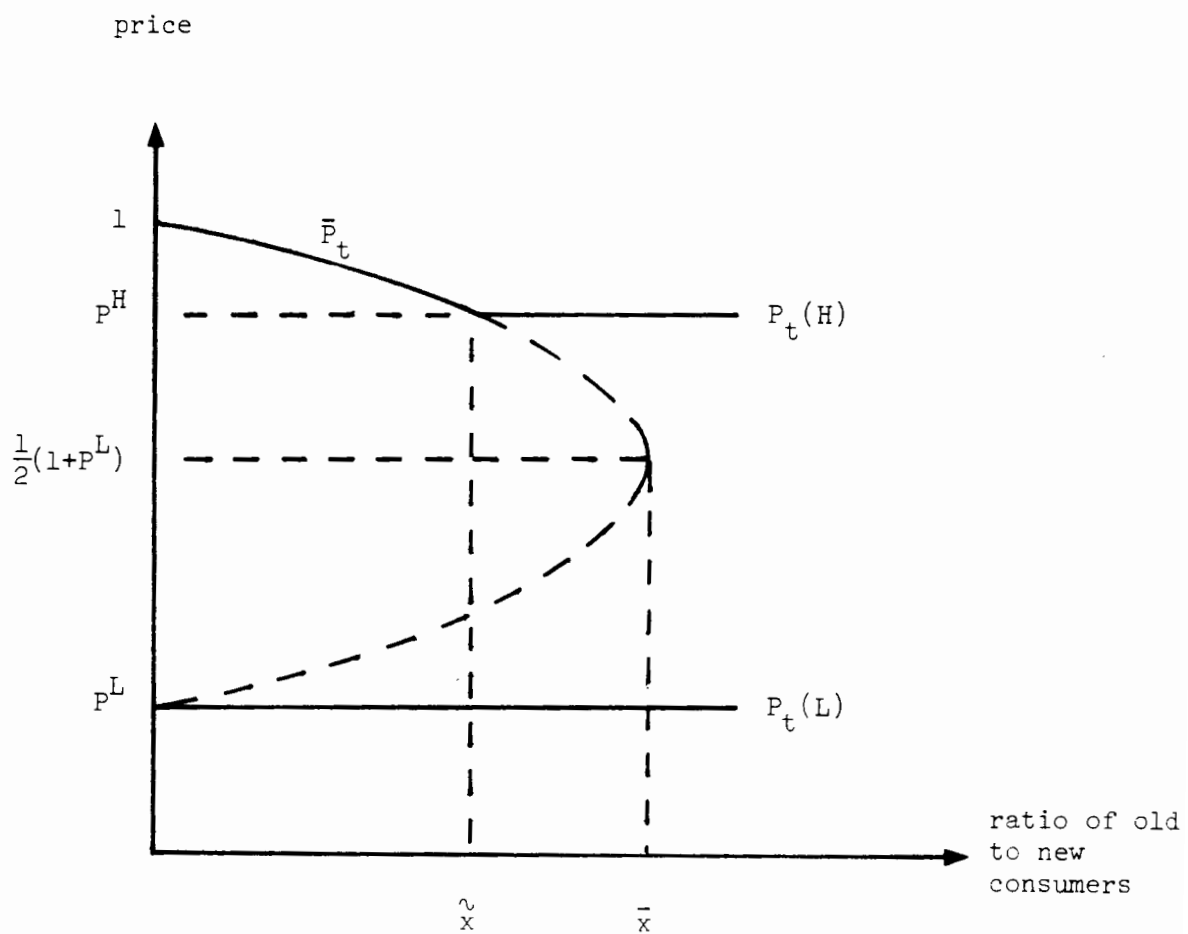
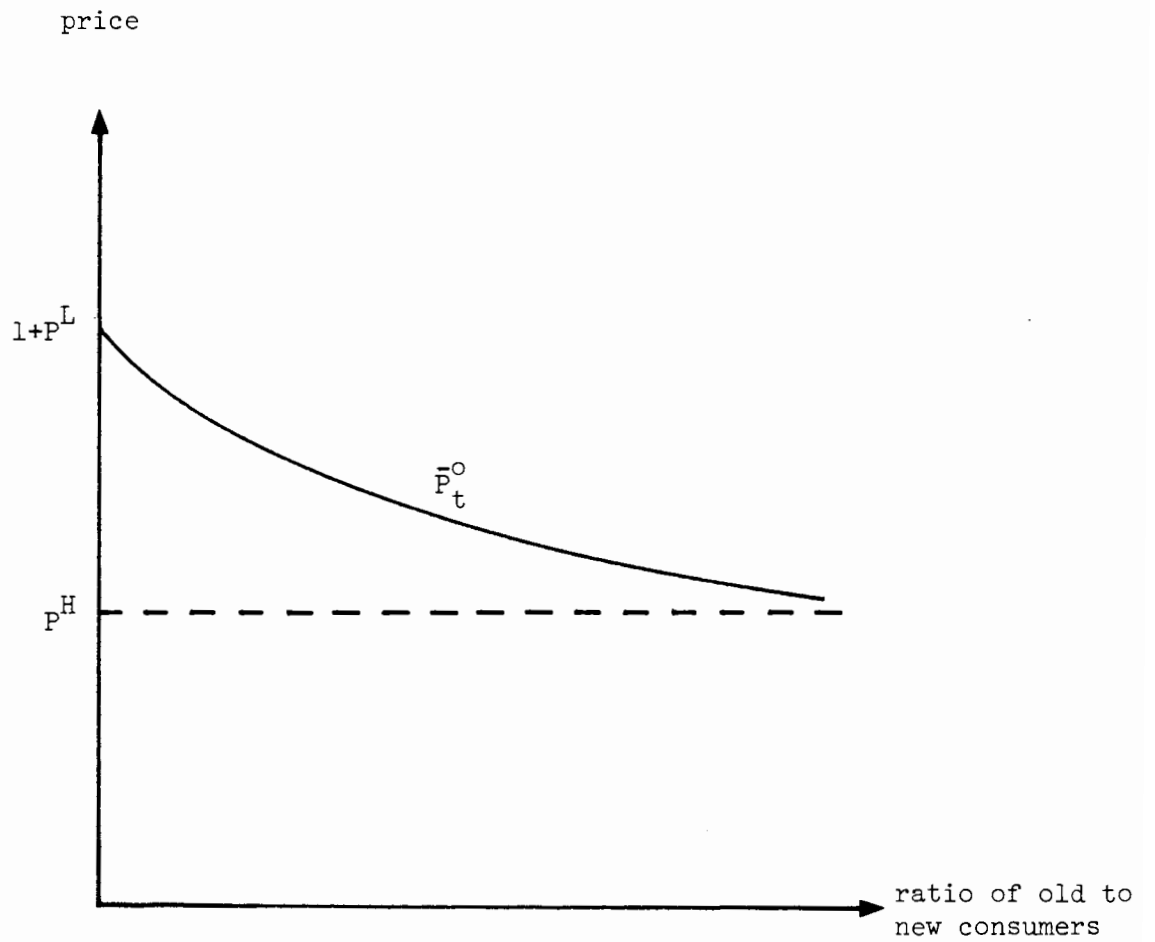


Figure 4

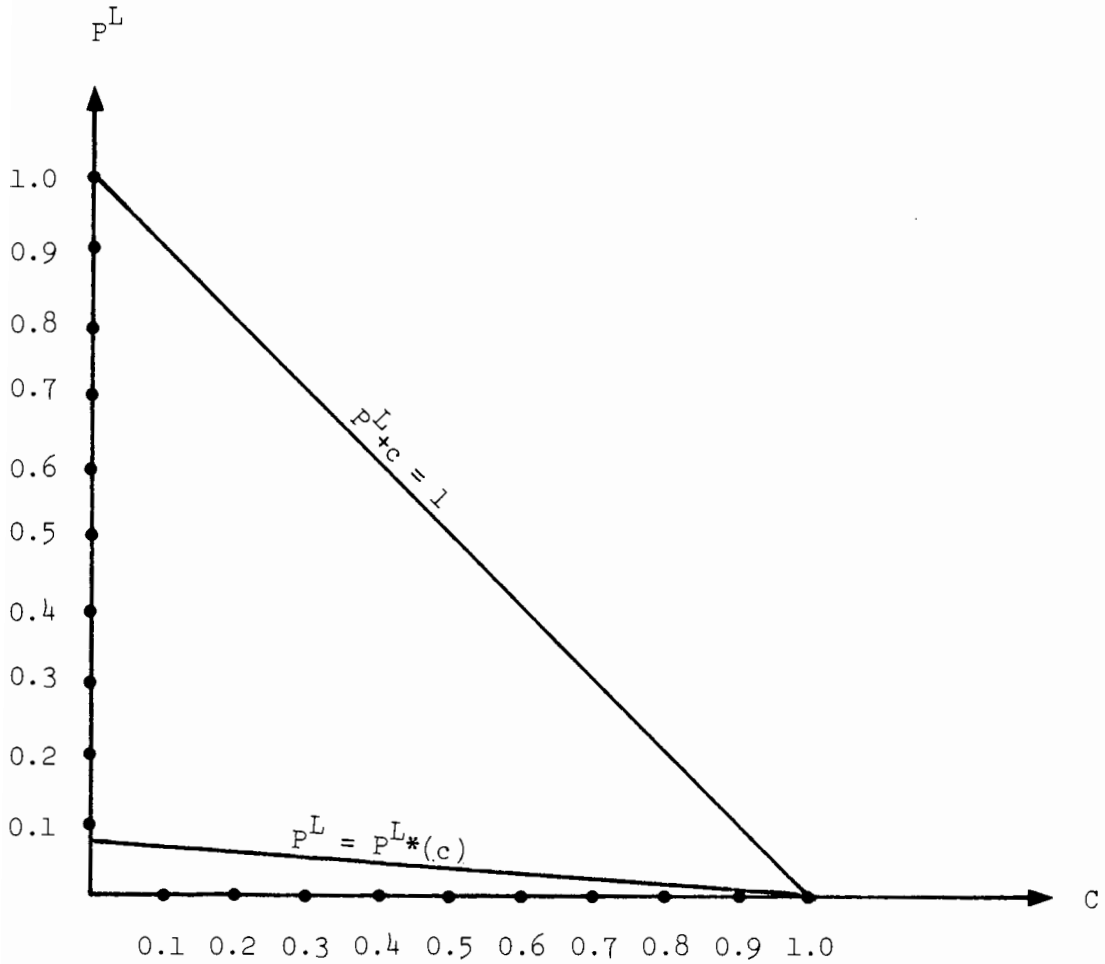
"No-Defect" Prices



Prices above \bar{P}_t^O lead to defection, and so cannot arise in equilibrium.

Figure 5

Parameter Values Supporting
Separation



For $\{P^L, c\}$ above $P^L + c = 1$, the high-quality firm can separate with its monopoly price. For $\{P^L, c\}$ below $P^L + c = 1$ and above $P^L = P^{L*}(c)$, the unique separating equilibrium (Theorem 1) exists.

For $\{P^L, c\}$ below $P^L = P^{L*}(c)$, $\Delta(X_t) < 0$ for some t and a separating equilibrium does not exist.

Figure 6

Simple Step Strategy Equilibrium

