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ADVERTISING AS A SIGNAL
WHEN PRICE GUARANTEES QUALITY[*]

by

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Abstract

This paper shows that in markets for experience goods where price guarantees quality that conspicuous expenditures by a new entrant on items that have no direct value to consumers can serve as a signal that the entrant will supply high quality. The reason for this is that higher initial expenditures signal that a firm is more efficient which in turn implies that the firm has a lower quality guaranteeing price. This is a different theory than Nelson's which requires that firms make a once and for all quality choice.

1. Introduction

In a thought provoking paper Nelson [1974] considered the role of
advertising in a market with the following two characteristics. First, the
good being sold was what Nelson termed an experience good; consumers could
only verify the quality of the product after purchasing it. Second, firms
made a once and for all quality choice; firms could not produce high quality
in one period and then switch to low quality in subsequent periods.
Advertising cannot directly communicate information about product quality in a
market for an experience good because claims of high quality can be copied by
low quality producers. Nelson conjectured, however, that advertising might
still serve as a signal of high quality in the class of markets he
considered. The reason is that a high quality firm would attract new
permanent customers through advertising whereas a low quality firm which
advertised would attract customers for only one period until they discovered
it was low quality. The differential reward to advertising provides the basis
for a signalling equilibrium.

This has been formally modelled in the context of a competitive market[1]
by Schmalensee [1978] and Kihlstrom and Riordin [1984]. This formal analysis
has shown that the circumstances under which such a signalling equilibrium
will exist are much more limited than the informal arguments in Nelson's paper
suggest. Two major conditions are required for a signalling equilibrium to be
likely to exist.[2] First, it must be impossible for consumers who do not
purchase a firm's product to learn about its quality. To understand the
reason for this, suppose that all consumers can learn the quality of a firm's
production in the previous period. Then a high quality firm can simply
produce a small amount of output in the first period (and sell at the low

quality price) in order to prove that it is a high quality firm. Advertising equilibra are very unlikely when firms have this simple alternative option available. Second, it must be that the non-sunk costs of high quality firms are no larger than the non-sunk costs of low quality firms. The reason for this is that the return to advertising depends not only on the number of sales but on the price-cost margin on each sale. If high quality firms have considerably higher costs they may have a lower return to advertising.

Both of these conditions are restrictive. Consumers often can learn about the quality of a firm's output in previous periods through published reports in magazines or newspapers and discussions with friends. The condition on costs still allows higher total costs for high quality firms (because of higher sunk costs) but is obviously very restrictive.

The purpose of this article is to formally show that a qualitatively different theory of conspicuous expenditure as a signal of product quality in markets for experience goods can be constructed for markets where firms do not make a once and for all quality choice but rather can switch their quality level from period to period if they desire. When there are large sunk costs which are quality specific, Nelson's assumption that firms will not change their quality level is appropriate. For example, it may be that a high quality firm is simply one which has a better idea generated by higher sunk costs in an R&D phase. Once the firm has the idea there is no incentive not to use it. In many other examples, however, higher quality is produced by purchasing more expensive inputs or spending more money on quality control. In such cases, the possibility that a firm with a reputation for high quality might choose to "milk" its reputation must be considered. This article will show that when an equilibrium must also provide firms with an incentive to maintain quality, a different reason for conspicuous expenditure to signal high quality will exist than that described by Nelson [1974].

The seminal article on the behavior of such markets was by Klein and Leffler [1981].[3] In it they described the key property of such markets -- the incentive role of price. The basic idea is that firms' only incentive to provide high quality goods is the desire to induce repeat sales. However, the prospect of inducing repeat sales is only attractive if price exceeds average salvageable costs of production. Therefore price must exceed average salvageable production costs by a certain premium in order for high quality to be produced. If sunk costs are low this constraint will bind -- i.e. -- firms producing at efficient size will earn positive profits by charging the quality guaranteeing price. Given that price cannot be competed lower (consumers correctly predict that firms charging lower prices will be "fly-by-night" operators) an interesting question arises concerning how these profits will be dissipated.

Zero-profits equilibria of a sort can be constructed where firms "burn money" prior to entering and then earn discounted profits exactly equal to the amount of money burned. "Burning money" could be interpreted as advertising, spending money in other conspicuous ways such as on overly elaborate logos or storefronts, or as producing high quality products but selling them at well below cost for an initial period. Shapiro [1983] has constructed a model of the latter sort. The problem with this "solution" to the problem of how profits will be dissipated is that the consumer beliefs required to support it are not particularly rational. Consumers must believe that only firms which burn money will produce high quality. However, this is irrational when all firms have the same cost function. A firm will produce high quality if and only if price exceeds the quality guaranteeing price independent of the amount of money it has burned. A related problem is that such a weak notion of equilibrium admits continuums of possible solutions. Beliefs exist to support

any price above the quality guaranteeing price (together with a level of advertising calculated so that profits are zero) as an equilibrium.

The idea of this paper is that adding a small amount of realism to the model allows zero profits equilibria where firms burn money to exist and to be supported by fully rational beliefs on the part of consumers. Specifically, it is assumed that marginally less efficient types of firms than the most efficient type exist -- i.e. -- a small amount of incomplete information of a variety which surely must exist is being injected into the model.

The following example will illustrate the effects of this assumption on equilibrium. Suppose that the quality guaranteeing price for the low cost firms is $5 and a low cost firm charging this earns $25 of discounted profits. First consider the case of complete information -- i.e. -- all firms are known to be low cost. Consumers know that any firm which charges $5 will produce high quality. Thus it is not rational to believe that a firm must burn $25 before it will produce high quality. Now consider the case of incomplete information -- i.e. -- there exists a potentially infinite number of marginally less efficient firms as well as a potentially infinite number of low cost firms. It will be shown in the body of the paper that higher cost firms also have a higher quality guaranteeing price. For now, simply assume this is true. Now suppose that a new firm enters and charges $5. Consumers know that the firm will produce high quality if and only if it is a low-cost firm. How can the firm prove that it is low cost? The answer is by burning $25 since a higher cost firm would make less profit than $25 and would thus be unwilling to burn $25.[4] Thus the idea of this paper is that conspicuous expenditures by a firm signal that it has low costs of production. This is important to consumers because lower-cost firms have a lower quality guaranteeing price and thus can credibly promise to supply high quality at lower prices.

In the formal analysis of this paper the beliefs which consumers hold (regarding the relationship between conspicuous expenditures, price and quality) are explicitly described as part of the equilibrium. Three notions of equilibrium are defined which require different degrees of rationality on the part of consumer beliefs. A weak equilibrium allows beliefs of any sort; no rationality requirements are imposed. At the other extreme a fully rational equilibrium requires that consumers know the structure of costs and directly calculate firms' incentives. An intermediate level of rationality is captured by the notion of a locally rational equilibrium, where consumers beliefs are required to be fully rational in an arbitrarily small neighborhood of the equilibrium. This small amount of rationality can be interpreted as the by-product of consumer experimentation in a neighborhood of equilibrium.

It is shown that although weak equilibria exist in the complete information world (in fact continuums of them exist), no locally or fully rational equilibria exist. However under incomplete information both fully and locally rational equilibria always exist. Furthermore in the case of constant marginal costs, the sets of fully and locally rational equilibria are identical and they equal the subset of weak equilibria which Shapiro [1981] selects. Thus the analysis of this paper can be viewed as a formal justification for Shapiro's equilibrium selection.

The basic idea of this paper -- that advertising can signal firm efficiency which in turn determines a firm's incentives to supply high quality -- is not original to this paper. Section III - C of Klein and Leffler [1981] is devoted to a verbal description of this idea. Farrell [1984] also makes use of this idea in a model where there is a single firm of each type and positive profits are allowed to exist in equilibrium.

Section 2 outlines a model with constant marginal costs which is considered for the bulk of the analysis. Section 3 briefly describes the incentive role of price. Section 4 describes equilibrium in a complete information model -- i.e. --where all firms are known to be low cost. Section 5 describes equilibrium in the incomplete information model -- i.e. -- where marginally less efficient firms than the lowest cost type also exist. Finally Section 6 describes how the analysis generalizes to the case of U-shaped cost curves.

2. The Model

A. Firms

There is a continuum of firm types indexed by $t \in [0,1]$. A potentially infinite number of any type of firm are available to enter the industry. Firms can produce high or low quality products. In any given period a firm can produce only one quality level but a firm can switch quality levels from period to period if it desires.

For the bulk of the paper cost functions will be assumed to have a particularly simply form to keep the analysis as simple and clear as possible. It will be assumed that marginal costs of production are constant (up to a fixed capacity) and there are no sunk costs. In Section 6 it will be shown that the results apply in essentially unchanged form to the case of U-shaped average cost curves. It is also straightforward to generalize the results to the case where sunk costs are positive but are not so large that a first-best equilibrium results. This generalization will not be presented.

Formally let $c_h(t)$ and $c_\ell(t)$ be the marginal cost of producing a unit of high or low quality output for a firm of type t. Let $\bar{x}$ be the maximum possible output for any firm. The interest rate is denoted by the positive number r.

The following assumptions are made about $c_h(t)$ and $c_\ell(t)$. A discussion of these assumptions follows their formal statement

(A.1) $\qquad 0 \leqslant c_\ell(t) < c_h(t).$

(A.2) $\qquad c_h$ is strictly increasing in t.

(A.3) $\qquad c_\ell$ is weakly increasing in t.

(A.4) $\qquad c_h - c_\ell$ is weakly increasing in t.

(A.5) $\qquad c_h$ and c_ℓ are continuous in t.

Assumption (A.1) simply states that output is costly to produce and high quality costs more to produce than low quality. Assumptions (A.2) - (A.4) describe the nature of firm diversity. Assumptions (A.2) and (A.3) state that type can be thought of as a measure of efficiency, with higher types being less efficient. Note that only the cost of producing high quality need be strictly increasing in t. This allows the case where c_ℓ is constant in t. This could be interpreted as a situation where producing low quality involves a simple technology known to all firms. Thus firms are only differentiated by their efficiency in producing high quality. The most extreme case would be where "low quality" means producing nothing. Then c_ℓ would be zero for every t.

Assumption (A.4) states that the cost differential to producing high vs. low quality is weakly increasing in type. If less efficient firms simply have proportionately higher costs of both forms then the cost differential is strictly increasing. If both c_h and c_ℓ are shifted up by the same amount as

firms grow less efficient, then $c_h - c_\ell$ is constant and thus also satisfies

(A.4). Both of these examples are special cases of the specification

(2.1) $c_h(t) = a_h(t) c_h(0) + b_h(t)$

(2.2) $c_\ell(t) = a_\ell(t) c_\ell(0) + b_\ell(t)$

where a_h, b_h, and a_ℓ and b_ℓ functions of t such that

      (i) $a_h - a_\ell$ is weakly increasing in t.

(2.3)   (ii) $b_h - b_\ell$ is weakly increasing in t.

      (iii) $a_h(0) = a_\ell(0) = b_h(0) = b_\ell(0) = 0.$

This satisfies (A.4)

Finally, assumption (A.5) states that firms almost as efficient as the lowest-cost firms exist. Note that the assumption only requires that marginally less efficient firms exist. Thus it is consistent with the assumptions of the model that low cost firms and firms very nearly as efficient as them are the only types of firms which exist; whether firms which are much less efficient than the lowest cost firms exist is irrelevant to the results of the paper.

The effect of introducing firm diversity of the type described by (A.2) - (A.5) is to demand that the equilibrium chosen be robust to the possibility that slightly less efficient firms exist. This seems a minimal requirement to impose on any equilibrium.

The results of this paper can be derived under a different formulation of firm diversity than that of (A.2) - (A.5). It can instead be assumed that all types have the same cost function but that higher types discount the future more heavily.[5] This alternative assumption generates the same two properties as (A.2) - (A.5) which drive the analysis. Namely

      (i)   higher types earn lower discounted expected profits

      (ii) higher types have a higher quality guaranteeing price.[6]

This formulation will not be formally treated in this paper since it is a straightforward variation of the analysis which is presented.

B. Conspicuous Expenditures

It is assumed that, upon entering the market, firms can, if they wish, engage in an activity in some amount A, where A is a non-negative real number. The activity costs a type t firm $a(t)$ dollars per unit. This activity has three properties:

      (i)   Consumers can observe a firm's choice of A.

      (ii)   The activity does not directly affect consumers' preferences.

      (iii) $a(t)$ is positive, non-decreasing in t and continuous in t.

As argued in the introduction, expenditures on non-informative advertising offer a natural interpretation of A and the choice of notation is meant to reflect this interpretation. To simplify the exposition, conspicuous expenditures will often simply be called advertising, but it should be kept in mind that other interpretations are possible. For example, other conspicuous sunk expenditures include elaborate store-fronts and sales areas, or sponsorship of highly visible (and highly expensive) sporting or artistic events. Section III - C of Klein and Leffler [1981] and the introduction of Milgrom and Roberts [1986] offer very convincing and eloquent arguments that such expenditures often seem to be designed to simply maximize the amount of money which can be proven to have been spent (which is of course consistent with a signalling interpretation of these expenditures). In these cases it is natural to assume that the price of the activity, $a(t)$, is the same for all types.

A somewhat different interpretation of A is for it to be sales of the product at the market price for low quality products labelled as such, $c_\ell(0)$. If consumers can observe the number of sales of a firm at the end of the introductory period, (i) is satisfied. Since the product is homogeneous consumers learn nothing about it in the trial period and (ii) is satisfied. Finally, $a(t)$ is $c_h(t) - c_\ell(0)$ which is strictly increasing in t, positive, and continuous so (iii) is satisfied.

C. Consumer Information

The most efficient method of describing the informational environment of consumers is to simply consider a list of relevant economic variables and describe the extent to which consumers are assumed to be able to observe each of them.

First, with respect to product quality consumers are assumed to be unable to directly determine the quality of a particular product without consuming it. However, through word-of-mouth communication and consumer publications consumers are assumed to be able to learn the quality of any particular firm's output in previous periods.

Second, consumers are unable to observe a firm's type.

Third, consumers must be able to observe firms' choices of A. If A is interpreted as introductory sales then consumers must be able to observe past sales levels of firms.

Fourth, consumers are unable to observe the output of a firm during the period they are purchasing from it. This assumption seems so natural that the reader might wonder why it is explicitly stated. How can a consumer purchasing a car from General Motors in March of 1987 possibly know what GM's sales are for all of 1987? Klein and Leffler, for example, make this assumption without explicitly discussing it. However, subsequent to Klein and Leffler, Allen [1984] has written a paper where a substantial amount of the analysis considers the case where current output is assumed to be observable.[7] Thus it seems worth remarking that Klein and Leffler's original assumption is maintained in this analysis.

Fifth, for beliefs to be fully rational it must be that consumers know the structure of costs and can directly calculate firms' incentives. The weaker criteria of local rationality can be interpreted as the outcome of consumer experimentation for values close to equilibrium and thus does not require this knowledge.

3. The Incentive Effects of Price

The purpose of this section is to briefly describe the incentive effects of price originally described by Klein and Leffler [1981].[8] A price-output pair, (p,x), will be said to be incentive compatible for a firm of type t if a firm of type t would prefer to produce high quality and stay in the market over an infinite horizon rather than to produce low quality at some point and then exit.

Suppose a firm produces high quality forever. Its discounted profits are

$$(3.1) \qquad \frac{[p - c_h(t)] \; x}{r} \; .$$

If it produces low quality and exits its profits are

$$(3.2) \qquad \frac{[p - c_\ell(t)] \; x}{(1+r)} \; .$$

The firm has an incentive to produce high quality if and only if (3.1) is greater than or equal to (3.2).[9][10] This is true if and only if

$$(3.3) \qquad p \geqslant c_h(t) + r[c_h(t) - c_\ell(t)] \; .$$

Let $\phi(t)$ denote the RHS of (3.3).

The intuition underlying (3.3) is straightforward. According to (3.3) price must exceed $c_h(t)$ by a certain premium, $r[c_h(t) - c_\ell(t)]$, in order for the firm to have an incentive to produce high quality. The advantage to producing high quality is repeat business; however, repeat business is only valuable insofar as price exceeds $c_h(t)$. Thus the firm has an incentive to

supply high quality if price exceeds $c_h(t)$ by a certain premium where the
premium is related to the profits to be made by producing low quality and
exiting.

Note that whether (p,x) is incentive compatible does not depend on x.
This is an artifact of the particular cost functions assumed (and simplifies
the analysis) but is not a general property as will be seen in Section 6. Klein
and Leffler call $\phi(t)$ the "quality guaranteeing price" for a firm type t.

4. Equilibrium Under Complete Information

This section defines and analyzes equilibrium when there are only low
cost firms -- i.e. -- all firms are of type 0. This is necessary in order to
clearly understand the role that the existence of less efficient types plays
in determining the nature of equilibrium.

An outcome will be defined to be an ordered triple (A,p,x) where A is the
amount firms advertise prior to entering and p and x are the price and
quantity firms sell in all subsequent periods. Note that attention is
restricted to stationary outcomes. A set of beliefs, B, is a set of ordered
pairs, (A,p), interpreted as follows. Consumers believe that a firm which
advertises at a level A and charges a price p will produce high quality if and
only if (A,p) is an element of B and the firm has never produced low quality
in a previous period.

The conditions which (A,p,x) and B must satisfy in order for (A,p,x) to
be an equilibrium supported by B will now be described. First, firms must
have an incentive to produce high quality -- i.e. -- (p,x) must be incentive
compatible as defined in Section 3.

(4.1) $p \geqslant \phi(0)$

Second, firms must be earning zero profits. If profits are positive,
entry will drive firm size down; if profits are negative firms will exit.

(4.2) $-A\ a(0) + \dfrac{[p - c_h(0)]\ x}{r} = 0$

Third, consumers beliefs must be rational at the equilibrium (A,p).

(4.3) $(A,p) \in B$

Fourth, it must not be possible for firms to reduce their advertising at
the given price (or else they would because this would increase their
profits).

(4.4) $A = \inf\{\hat{A}: (\hat{A},p) \in B\}$

Fifth, it must not be possible for an entrant to offer a lower price than
existing firms and earn positive profits. This is formalized in (4.5).
Condition (i) states that the entrant offers a lower price. Condition (ii)
states that consumers will accept the entrant's offer. Condition (iii) states
that the entrant can make positive profits either by producing high quality
over the long run or by producing low quality once and exiting.[11]

(4.5)   There does not exist an $(\hat{A},\hat{p})$ such that

          (i) $\hat{p} < p$

(ii) $(\hat{A}, \hat{p}) \in B$

(iii) $\max \left\{ \begin{array}{c} - \hat{A}\ a(0) + \dfrac{[\hat{p} - c_h(0)]\bar{x}}{r} \\[4mm] -\hat{A}\ a(0) + \dfrac{[\hat{p} - c_\ell(0)]\bar{x}}{1+r} \end{array} \right\} > 0$

The above conditions determine what will be called a weak equilibrium.

Definition: (A,p,x) is a weak equilibrium under complete information if

(i) $A \geqslant 0,\ p \geqslant 0,\ x \in (0,\bar{x}]$

(ii)   There exists a B such that (A,p,x) and B satisfy (4.1) – (4.5). (It will be said that B supports (A,p,x).)

The adjective "weak" is used in the above definition because of the very weak rationality requirements imposed on consumer beliefs. It is only required that consumers beliefs regarding the equilibrium values of A and p be confirmed. No restrictions on the rationality of their beliefs regarding other values of A and p are imposed. This results in the existence of continuums of equilibria. Theorem 1 demonstrates this.

Theorem 1

For any $p \geqslant \phi(0)$ and $x \in (0,\overline{x}]$ there exists on A such that (A,p,x) is a weak equilibrium under complete information. A is determined by

(4.6) $$A = \frac{[p - c_h(0)]x}{r\, a(0)}$$

A set of beliefs that supports (A,p,x) is

(4.7) $$B = \{(\hat{A},\hat{p}): \hat{p} \geqslant p \text{ and } \hat{A} \geqslant A\}.$$

proof:

It is straightforward to verify that conditions $(4.1) - (4.5)$ are satisfied.

                                                    QED.

Theorem 1 illustrates the reason for multiplicity of equilibria. If consumers believe that some level of advertising is required for production of high quality they will never purchase from firms advertising less and thus their beliefs will never be disconfirmed. Similarly, if consumers believe a certain price level is necessary for high quality production no sales will occur at lower prices and these beliefs will never be disconfirmed.

The requirement that beliefs of consumers need only be rational precisely at the equilibrium values of A and p may be unreasonably weak. Beliefs of consumers in actual markets are likely to be influenced by a certain amount of natural experimentation. Firms will occasionally offer slightly lower prices or slightly lower levels of advertising and some consumers will occasionally try them. If the firms prove to offer high quality consumer beliefs will be revised accordingly.

A reasonable requirement to impose on consumer beliefs may therefore be that they are locally rational -- i.e. -- rational in some neighborhood of the equilibrium A and p. In order to formally define this term it is useful to first define fully rational beliefs. These are the beliefs that consumers would have if they knew firms' cost functions and formed their expectations by directly calculating whether a given (A,p) is compatible with a stationary equilibrium where firms choose to produce high quality.

Definition: The fully rational belief set under complete information denoted by B_{FC}, is given by

(4.8) $\qquad B_{FC} = \{(A,p): A \geq 0 \text{ and } p \geq \phi(0)\}$

Beliefs are then locally rational with respect to some (A,p) if there is some neighborhood of (A,p) where beliefs are fully rational.

Definition: B is locally rational under complete information with respect to (A,p) if there exists an open neighborhood of (A,p), denoted by N, such that

(4.9) $\qquad B \cap N = B_{FC} \cap N$

The original work of Klein and Leffler and most subsequent papers have assumed that consumers' beliefs are fully rational. The weaker requirement of local rationality is more plausible because it can be interpreted as the result of local experimentation. Furthermore it yields the same non-existence result as the stronger requirement as will now be shown. First, locally rational and fully rational equilibria will be formally defined.

Definition: (A,p,x) is a locally rational equilibrium under complete information, if it is a weak equilibrium and there exists a B which supports it which is locally rational with respect to (A,p).

Definition: (A,p,x) is a fully rational equilibrium under complete information if it is a weak equilibrium and B_{FC} supports it.

Theorem 2 now states that no locally rational or fully rational equilibria exist.

Theorem 2:

There are no locally rational or fully rational equilibria.

proof:

It is sufficient to show that no locally rational equilibria exist (since a fully rational equilibrium is locally rational.) Suppose that (A,p,x) is a weak equilibrium and is supported by B which is locally rational with respect to (A,p). It is straightforward that A must equal zero. (If $A > 0$, firms would prefer to lower A if they could and by local rationality consumers will still believe high quality is forthcoming for small decreases in A.) However, $A = 0$ together with (4.1) imply that profits are positive, contradicting (4.2).

QED.

The intuition for this result is straightforward. Price must exceed $c_h(0)$ by a positive premium in order for firms to have an incentive to produce high quality. This can only be consistent with zero profits if firms must incur some sunk costs upon entering. However, if consumers beliefs are rational then no advertising can occur in equilibrium.

The results of this section apply to Shapiro's [1983] model of reputation building since he assumes that all firms have the same cost function. In Shapiro's model the activity A consists of selling A units of high quality products at a price of $c_\ell(0)$ for one period. Thus

$$(4.10) \qquad a(0) = c_h(0) - c_\ell(0).$$

Shapiro selects a particular set of weak equilibra. He chooses one equilibria for every $x \in (0, \bar{x}]$. Price is set precisely at the quality guaranteeing price

$$(4.11) \qquad p = \phi(0)$$

and A is chosen so that profits are zero. For any given x, substitution of (4.10) and (4.11) into the zero profits condition, (4.2), shows that the amount of output which must be sold at the low initial price is

$$(4.12) \qquad A = x.$$

That is, the firm must sell the same amount of output in the initial period as it will in subsequent periods in order that profits be zero. Finally, a set of beliefs that support his equilibrium is

(4.13) $B = \{ (\hat{A}, \hat{p}): \; \hat{A} \geqslant x \text{ and } \hat{p} \geqslant p \}$

Thus the outcomes selected by Shapiro are weak equilibria. However, there do not appear to be grounds within the model for differentiating them from the continuum of other weak equilibria which also exist. Furthermore, the particular weak equilibria chosen suffer from the problem of all weak equilibria -- the beliefs that consumers must have in order to support them are not locally rational. For the particular equilibria that Shapiro chooses, beliefs regarding price are locally rational. The problem is with the beliefs concerning A. Consumers must believe that a firm which initially sold even one less unit than x at a price of $c_{\chi}(0)$ would produce low quality in all subsequent periods. These beliefs are irrational since the magnitude of a firm's sunk costs upon entering is irrelevant to determining its incentives once it has entered. Even if consumers could not reason this argument through, local experimentation could be expected to reveal that slightly smaller introductory offers than x still resulted in high quality in later periods.

It is conceivable in some markets that consumers do not know the structure of costs and that local experimentation works extremely slowly to update beliefs. In this case Shapiro's equilibria are possible. However, continuums of other equilibria are then also equally possible and essentially no predictions can be made about the nature of equilibrium.

5. Equilibrium Under Incomplete Information

This section considers what might superficially appear to be a slight perturbation of the complete information model of Section 4. Namely, marginally less efficient types of firms are assumed to exist as described by (A.2) - (A.5).

The definition of a weak equilibrium does not change from Section 4. This is because higher types of firms are less efficient. Since type 0 firms earn zero profits in equilibrium, higher types cannot earn positive profits and do not enter.

Definition: (A,p,x) is a weak equilibrium under incomplete information if and only if it is a weak equilibrium under complete information.

Since the definition of a weak equilibrium does not depend on whether information is complete or not the terms "under complete or incomplete information" will no longer be appended to this term.

The difference between Sections 4 and 5 concerns the notion of fully rational beliefs. Suppose a new firm enters, advertises at a level of $\hat{A}$ and offers a price of $\hat{p}$. If $\hat{p}$ is greater than or equal to $\phi(1)$, consumers can be sure that the firm will produce high quality because price is greater than or equal to every type's quality guaranteeing price. If $\hat{p}$ is strictly less than $\phi(0)$ consumers can be sure that the firm will supply low quality because price is less than every type's quality guaranteeing price. However for values of $\hat{p}$ in the interval $[\phi(0), \phi(1))$ whether a firm prefers to offer high or low quality depends on its type. With reference to Figure 1, if the firm

Figure 1
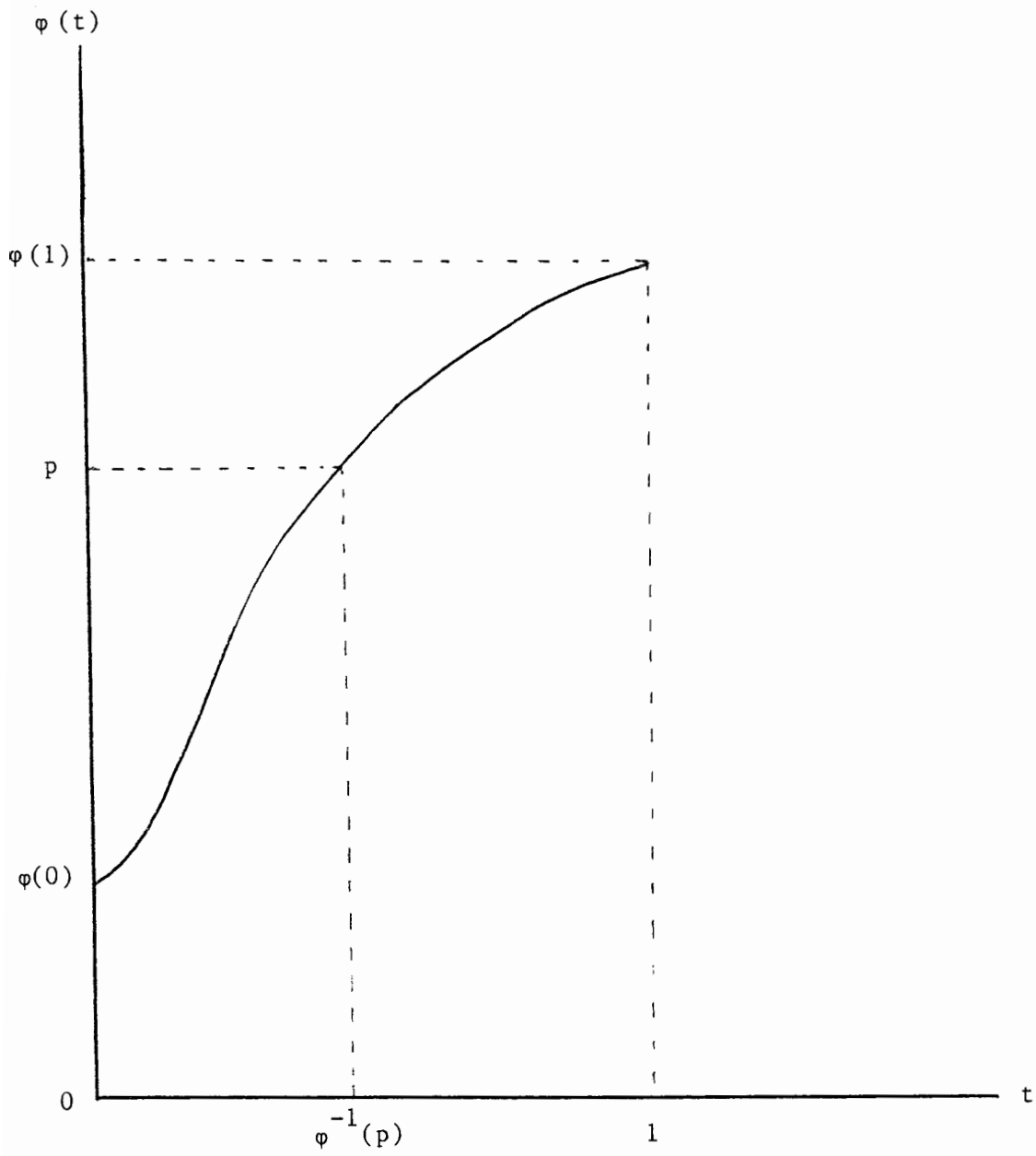
is efficient enough $\left(t \in \left[0, \phi^{-1}(p) \right] \right)$ it will prefer to produce high quality, but if it is inefficient enough $\left(t \in \left(\phi^{-1}(p), 1 \right] \right)$, it will prefer to produce low quality.

Consumers cannot directly observe a firm's type. However, the key fact determining the nature of equilibrium is that consumers can use the value of $\hat{A}$ to infer some information about the firm's type. The reason for this is that higher types are less efficient. If $\hat{A}$ is so high that firms of type a greater than some type t^* cannot possibly make non-negative profits (either by being honest or by being dishonest) then a consumer can infer that the firm's type must be less than or equal to t^*. This will now be formalized.

As a first step in this formalization consider the following problem. Suppose a firm enters, advertises at a level $\hat{A}$ and charges a $\hat{p} \in [\phi(0), \phi(1))$. Furthermore suppose that its size is known to be $\hat{x}$. Consumers can be sure that the firm will produce high quality if they can infer that it is of type $\phi^{-1}(\hat{p})$ or less. Lemma 1 characterizes the set of triples $\left(\hat{A}, \hat{p}, \hat{x} \right)$ such that types greater than $\phi^{-1}(\hat{p})$ cannot make positive profits by entering (and thus the consumer can believe high quality will be forthcoming.)

Lemma 1:

Consider any $\left(\hat{A}, \hat{p}, \hat{x} \right)$ with $\hat{p} \in [\phi(0), \phi(1))$. Suppose a firm can enter, advertise at $\hat{A}$, and sell $\hat{x}$ units at a price of $\hat{p}$ so long as it has never produced low quality in a previous period. (If it ever produces low quality it must then exit.) Then firms of type greater than $\phi^{-1}(\hat{p})$ cannot make positive profits by entering if and only if type $\phi^{-1}(p)$ earns non positive profits by entering -- i.e. --

$$(5.1) \qquad -A \; a(\hat{t}) + \frac{[\hat{p} - c_h(\hat{t})] \; \hat{x}}{r} \leqslant 0$$

where $\hat{t}$ is defined by

$$(5.2) \qquad \hat{t} = \phi^{-1}(\hat{p}).$$

proof:

First it will be shown that (5.1) is sufficient for firms of type greater than $\hat{t}$ to earn non-positive profits. Equation (5.1) states that type (5.1) earns non-positive profits by entering and producing high quality for an infinite number of periods. However by definition $\hat{t}$ is indifferent between producing high or low quality. Thus type $\hat{t}$ also makes non-positive profits from entering and producing low quality. Types higher than $\hat{t}$ are less efficient and pay at least as much for advertising. Thus they earn lower profits.

Now it will be shown that (5.1) is necessary. The contrapositive will be proven. Suppose (5.1) is false. Then type $\hat{t}$ earns positive profits by entering and either producing high quality forever or producing low quality once. (Recall type $\hat{t}$ is, by definition, indifferent between these strategies). Therefore, since profits are continuous in type, slightly higher types will also earn positive profits by entering.

QED.

The nature of fully rational beliefs can now be described. Suppose an entrant to the market advertises at level $\hat{A}$ and chooses a price $\hat{p} \in [\phi(0), \phi(1))$ such that $\hat{p}$ is less than or equal to the price charged by existing firms. Can consumers predict whether high quality will be

forthcoming? The problem with applying Lemma 1 is that consumers cannot directly observe the entrant's size. Fortunately they can infer what the entrant's size will be. Two cases must be considered.

First suppose that the entrant offers a price strictly less than the existing firms. If consumers believe the entrant will produce high quality, the entrant will be strictly more attractive to consumers than existing firms and will thus be able to operate at capacity -- i.e. -- it will be of size $\bar{x}$. Applying Lemma 1, consumers should believe that the entrant will produce high quality if and only if

$$(5.3) \qquad \hat{A} \geqslant \frac{[\hat{p} - c_h(\hat{t})] \; \bar{x}}{r \; a(\hat{t})}$$

where $\hat{t}$ is defined by (5.2)

Now suppose that the entrant offers a price equal to existing firms. Then if consumers believe the entrant will produce high quality, the entrant will be equally attractive to consumers as existing firms and will thus receive an equal number of customers. If existing firms are rationed below capacity at some level, x, so will the entrant. Applying Lemma 1, consumers should believe that the entrant will produce high quality if and only if

$$(5.4) \qquad \hat{A} \geqslant \frac{[\hat{p} - c_h(\hat{t})] \; x}{r \; a(\hat{t})}$$

where $\hat{t}$ is defined by (5.2) and x is the size of existing firms.

The above discussion leads to the following definition of fully rational beliefs.

<u>Definition</u>:   The fully rational beliefs under incomplete information given

the market outcome (A,p,x), denoted by $B_{FI}(A,p,x)$, is defined

as follows. The pair $(\hat{A},\hat{p})$ is in $B_{FI}(A,p,x)$ if and only if it

satisfies at least one of the following conditions.

    (i) $\hat{p} \geqslant \phi(1)$

(5.5)   (ii) $\hat{p} = p$, $\hat{p} \in [\phi(0), \phi(1))$,
    and (5.4) is true.

    (iii) $\hat{p} < p$, $\hat{p} \in [\phi(0), \phi(1))$,
    and (5.3) is true.

Two points should be noted. First, unlike the case of complete information,

the definition of fully rational beliefs depends on the current market

outcome. Second, beliefs for prices above the current market price are not

carefully analyzed in the above discussion. However this is not important

because in equilibrium consumers will be receiving high quality at current

prices and thus they will not consider offers at higher prices.

    As in Section 4, a locally rational set of beliefs with respect to a

given pair (A,p) is a set of beliefs which is rational in a neighborhood of

(A,p).

<u>Definition</u>:   A set of beliefs, B, is locally rational under incomplete

information with respect to (A,p,x) if there exists an open

neighborhood, N, of (A,p) such that

(5.6) $N \cap B_{FI}(A,p,x) = N \cap B$

Theorem 3 now characterizes the set of fully and locally rational equilibra.

Theorem 3:

For any $x \in (0,\bar{x}]$ there exists a unique (A,p) such that (A,p,x) is a fully or locally rational equilibrium under incomplete information. It is defined by

(5.7) $p = \phi(0)$

and

(5.8) $A = \dfrac{\left[p - c_h(0)\right] x}{r\, a(0)}$.

proof:

It is straightforward to verify that the outcomes defined by (5.7) and (5.8) together with fully rational beliefs satisfy (4.1) – (4.5).

To complete the proof it is sufficient to show that no other outcomes can be locally rational equilibria. It is clear that (5.8) must be satisfied since this is the zero profits condition.

To complete the proof it will be shown that if (A,p,x) satisfies (5.8) but does not satisfy (5.7) that it cannot be a locally rational equilibrium. There are two cases. If $p < \phi(0)$, (A,p,x) does not satisfy incentive compatibility so cannot be an equilibrium of any sort. Now suppose $p > \phi(0)$. By (5.8), A is chosen so type 0 firms make zero profits. According to (5.3) fully rational consumers would believe a firm charging p will produce

high quality so long as A is large enough so firms of type $\phi^{-1}(p)$ make zero profits. Let $\hat{A}$ denote this level of advertising. Clearly $\hat{A} < A$ since $\phi^{-1}(p) > 0$ and costs of producing high quality are strictly increasing in type. Therefore if beliefs are locally rational with respect to (A,p) a firm could reduce its advertising and consumers would still believe it would produce high quality. Thus (4.4) is violated and (A,p,x) is not a weak equilibrium.

<div align="right">QED.</div>

A number of points should be noted about this result. First, and most importantly, fully and locally rational equilibria exist under incomplete information for the reasons described in the introduction. Advertising expenditures signal a firm's efficiency which in turn determines its incentives. Thus even fully rational consumers will demand that a firm advertise before purchasing from it. Second, the locally and fully rational equilibria are identical. Thus the assumption that consumers actually know the structure of costs and directly calculate incentives is not necessary; an equally strong refinement is obtained by assuming that consumers experiment in a neighborhood of equilibrium and thus have locally rational beliefs.

Third, the set of equilibria identified in Theorem 3 are precisely those selected by Shapiro when A is interpreted as introductory sales. This is clear from the analysis at the end of Section 4. Thus this paper can be viewed as a formal justification for Shapiro's equilibrium.

6. Generalization to U-Shaped Average Cost Curves

This section describes how the results of this paper generalize the case of U-shaped average cost curves. All the important results generalize. The only result which does not generalize is that the sets of locally and fully rational equilibria under incomplete information are not necessarily identical. (It may be that some outcomes exist which are locally rational equilibria but not fully rational equilibria). To the extent that the explanation of assumptions and results is similar to that of the previous sections, it will not be repeated.

A. The Model

The structure of the model is the same as previously stated, only the cost of producing x units of high and low quality for a firm of type t is now denoted by $f_h(x,t)$ and $f_\ell(x,t)$. The following assumptions are made about f_h and f_ℓ. A discussion follows their formal statement.

(B.1) $f_h(x,t)$ and $f_\ell(x,t)$ are defined over $[0,\infty) \times [0,1]$

(B.2) $f_h(0,t) = f_\ell(0,t) = 0$

$f_h(x,t) > f_\ell(x,t) \geqslant 0$ for $x > 0$

(B.3) $f_h(x,t)$ and $f_\ell(x,t)$ are continuously differentiable in x for $x \in (0,\infty)$.

(B.4) $f_h(x,t)/x$ and $f_\ell(x,t)/x$ are strictly concave in x for

x ∈ (0,∞) and reach a local minimum in x over (0,∞).

(B.5) $f_h(x,t)$ is strictly increasing in t for x > 0.

(B.6) $f_\ell(x,t)$ is weakly increasing in t for x > 0.

((B.7) $f_h(x,t) - f_\ell(x,t)$ is weakly increasing in t for x > 0.

(B.8) $f_h(x,t)$ and $f_\ell(x,t)$ are continuous in t.

Let $MC_h(x,t)$, $MC_\ell(x,t)$, $AC_h(x,t)$ and $AC_\ell(x,t)$ denote marginal and average cost functions for the various qualities and firm types.

Assumptions (B.1) - B.4) simply formalize the statement that cost curves are smooth and U-shaped and that high quality costs more to produce than low quality. More regularity has been assumed than would be absolutely necessary to prove the results for the sake of expositional convenience. Assumptions (B.5) -(B.8) describe the nature of firm diversity and are analogous to (A.2) - (A.5). The discussions of (A.2) - (A.5) are therefore relevant here too.

B. The Incentive Effects of Price

Similar analysis to Section 3 shows that a given (p,x) is incentive compatible for a firm of type t if

(6.1) $$p \geq \frac{f_h(x,t) + r\left[f_h(x,t) - f_\ell(x,t)\right]}{x}$$

Let $\Phi(x,t)$ denote the RHS of (6.1). Notice that $\Phi(x,t)$ is always strictly greater than $AC_h(x,t)$.

Notice that whether (p,x) is incentive compatible now depends on x as well as p (unlike the case of constant marginal costs.) This makes it useful to introduce some extra notation. Define a firm to be price-taking if it believes it can sell as much output as it desires at the current price. It can sell for an infinite number of periods if it produces high quality but must exit at the end of the period if it ever produces low quality. Let $\pi^h(p,t)$ and $\pi^\ell(p,t)$ denote the profits a price taking firm of type t would earn at price p by, respectively, producing high quality forever or producing low quality and exiting.

$$(6.2) \qquad \pi^h(p,t) = \max_x \frac{px - f_h(x,t)}{r}$$

$$(6.3) \qquad \pi^\ell(p,t) = \max_x \frac{px - f_\ell(x,t)}{1+r}$$

Let $H(t)$ denote the prices such that a price taking firm would choose to produce high quality (as opposed to producing low quality or not producing anything).

$$(6.4) \qquad H(t) = \left\{ p: p \geq \min_{x \in (0,\infty)} AC_h(x,t) \text{ and } \pi^h(p,t) \geq \pi^\ell(p,t) \right\}$$

Note that for the case of constant marginal costs $H(t)$ has a particularly simple form -- i.e. --

(6.5) $H(t) = \left[p^*(t), \infty \right)$

where $p^*(t)$ is $\phi(t)$. Klein and Leffler introduced the term "quality guaranteeing price" to describe $p^*(t)$ when $H(t)$ has the form in (6.5). However, $H(t)$ will not always be of this form even if cost curves are regular and U-shaped as described in (B.1) – (B.4). It may be that $H(t)$ is the empty set or is a set of disjoint intervals. No particular assumption about the nature of $H(t)$ will be made for this analysis.

C. Complete Information

Now the case of complete information, where all firms are assumed to be low cost, will be considered. For (A,p,x) to be a weak equilibrium supported by B, (p,x) must be incentive compatible,

(6.6) $p \geqslant \Phi(x,0)$,

firms must be earning zero profits,

(6.7) $-A\, a(0) + \dfrac{px - f_h(x,0)}{r} = 0$,

consumers' beliefs must be rational at the equilibrium,

(6.8) $(A,p) \in B$,

it must not be possible for consumers to reduce advertising (this is still defined by (4.4)), and finally it must not be possible for an entrant to offer a lower price than existing firms and earn non-negative profits. This last condition is formally defined by (6.9).

(6.9)   There does not exist on $(\hat{A}, \hat{p})$ such that

(i) $\hat{p} < p$

(ii) $(\hat{A}, \hat{p}) \in B$

(iii) max $\left\{ \begin{array}{c} -\dot{A}\ a(0) + \pi^h(\hat{p}, 0) \\ -Aa(0) + \pi^{\ell}(\hat{p}, 0) \end{array} \right\} \geqslant 0$

This yields the following definition.

Definition: (A, p, x) is a weak equilibrium under complete information if

(i) $A \geqslant 0$, $p \geqslant 0$, and $p \geqslant MC(x, 0)$.

(ii) There exists a B such that (A, p, x) and B satisfy (6.6) – (6.9) and (4.4).

A large number of weak equilibria exist for the same reasons as discussed for the case of constant marginal costs. Theorem 4 shows this.

Theorem 4:

Suppose that (p,x) satisfies

$$(6.10) \qquad p \geqslant \Psi(x,0)$$

and

$$(6.11) \qquad p \geqslant MC_h(x,0).$$

Then (A,p,x) is a weak equilibrium where A is defined by

$$A = \frac{px - f_h(x,0)}{r\ a(0)}$$

A set of beliefs which supports (A,p,x) is given by (4.7).

proof:

It is straighforward to verify that the conditions are satisfied.

QED.

Fully rational beliefs have the same interpretation as before. They are defined below.

Definition: The set of beliefs which is fully rational under complete information with respect to an outcome (A,p,x) denoted by $B_{FC}(A,p,x)$, is defined as follows. The pair $(\hat{A},\hat{p})$ is an element of $B_{FC}(A,p,x)$ if and only if it satisfies one of the following two conditions.

(i) $\hat{p} = p$ and $\hat{p} \geqslant \Psi(x,0)$

(ii) $\hat{p} < p$ and $\hat{p} \in H(0)$.

Locally rational beliefs are defined precisely as before and this definition will not be repeated. The definition of locally and fully rational equilibria are also the same as in Section 4. Theorem 5 now states the analogue to Theorem 2. Namely, no fully or locally rational equilibria exist.

Theorem 5:   There are no locally rational or fully rational equilibria.

proof:

The proof is very similar to that of Theorem 2.

QED.

D. Incomplete Information

For the same reasons as discussed in the case of constant marginal cost, the definition of a weak equilibrium is the same under both complete and incomplete information. Once again the nature of fully rational beliefs change, however, to reflect the fact that consumers must attempt to infer what type a firm is.

Suppose that the equilibrium is (A,p,x) and an entrant offers $(\hat{A},\hat{p})$ with $\hat{p} \leqslant p$. Can fully rational consumers predict that high quality will be forthcoming? As in Section 5 two cases must be considered. First suppose $\hat{p} = p$. In this case if consumers believe high quality is forthcoming the entrant will receive an equal number of customers as all other firms, x.

If

(6.12) $\hat{p} \geqslant \Phi(x,1)$,

the firm will definitely supply high quality. However, if $\hat{p} \in [\Phi(x,0), \Phi(x,1))$, the firm will only supply high quality if it is of type $\hat{t}$ or less where $\hat{t}$ is defined by

(6.13) $\hat{p} = \Phi\left(x,\hat{t}\right)$.

As in Section 5, to prove it is of type $\hat{t}$ or less, a firm must advertise enough so that a firm of type $\hat{t}$ would earn non-positive profits. That is, $\hat{A}$ must satisfy

(6.14) $- a\left(\hat{t}\right)\hat{A} + \dfrac{\hat{p}x - f_h\left(x,\hat{t}\right)}{1+r} \leqslant 0.$

If $\hat{p} < p$ and consumers believe a firm will produce high quality it can sell all it desires. Therefore a firm of type t will produce high quality if and only if p is an element of H(t). If

(6.15) $p \in H(t)$ for every $t \in [0,1]$

the firm will definitely produce high quality. Let $\hat{t}$ be defined by

(6.16) $\hat{t} = \sup\{ t : p \in H(t')$ for every $t' \leqslant t\}.$

It may be that $\hat{t}$ doesn't exist (because $p \in H(0)$). However if it does a firm will produce high quality if it is of type $\hat{t}$ or less. It can prove this by advertising so that a firm of type $\hat{t}$ earns non-positive profits. That is, $\hat{A}$ must satisfy

$$(6.17) \qquad - a(\hat{t})\, \hat{A} + \pi^h(\hat{p},\hat{t}) \leqslant 0.$$

The following definition formally summarizes these conditions.

Definition: The fully rational set of beliefs under incomplete information given (A,p,x), denoted by $B_{FI}(A,p,x)$, is defined as follows. The ordered pair $(\hat{A},\hat{p})$ is an element of $B_{FI}(A,p,x)$ if and only if one of the following is true.

(i) $\hat{p} = p$ and (6.12) is satisfied

(6.18)   (ii) $\hat{p} = p$ and (6.13) - (6.14) are satisfied

(iii) $\hat{p} < p$ and (6.15) is satisfied

(iv) $\hat{p} < p$ and (6.16) - (6.17) are satisfied.

Locally rational beliefs and locally and fully rational equilibria are then defined as in Section 5. Theorem 5 now considerably narrows down the set of possible locally rational (and thus fully rational) equilibria.

Theorem 5.

Suppose (A,p,x) is a locally rational equilibrium. Then it must satisfy

(6.19) $p = \Phi(x,0)$

(6.20) $p \geq MC_h(x,0)$

(6.21) $A = \dfrac{px - f_h(x,0)}{r\ a(0)}$

proof:

Conditions (6.20) and (6.21) are necessary by definition. It will now be shown that (6.19) is also necessary. By (6.6) p cannot be less than $\Phi(x,0)$. Suppose, for contradiction that $p > \Phi(x,0)$. Then according to (6.18) (i) and (ii), an entrant offering a price of p can reduce advertising below A and still be believed to produce high quality if beliefs are fully rational. Thus this must also be true for locally rational beliefs for small decreases in A. Thus (A,p,x) cannot be a locally rational equilibrium because it violates (4.4).

QED.

Figure 2 illustrates the possible equilibria according to Theorem 5. The (p,x) pairs which are possible equilibria are on the darkened portion of $\Phi(x,0)$. The reason underlying this result is exactly the same as that underlying Theorem 3 where it was shown that the equilibria must all have (p,x) pairs on $\phi(0)$. (i.e. -- price must be equal to $\phi(0)$ for every x). Suppose that (p,x) is strictly above the incentive compatibility constraint. This means that types less than or equal to some $\hat{t} > 0$ will produce high

Figure 2
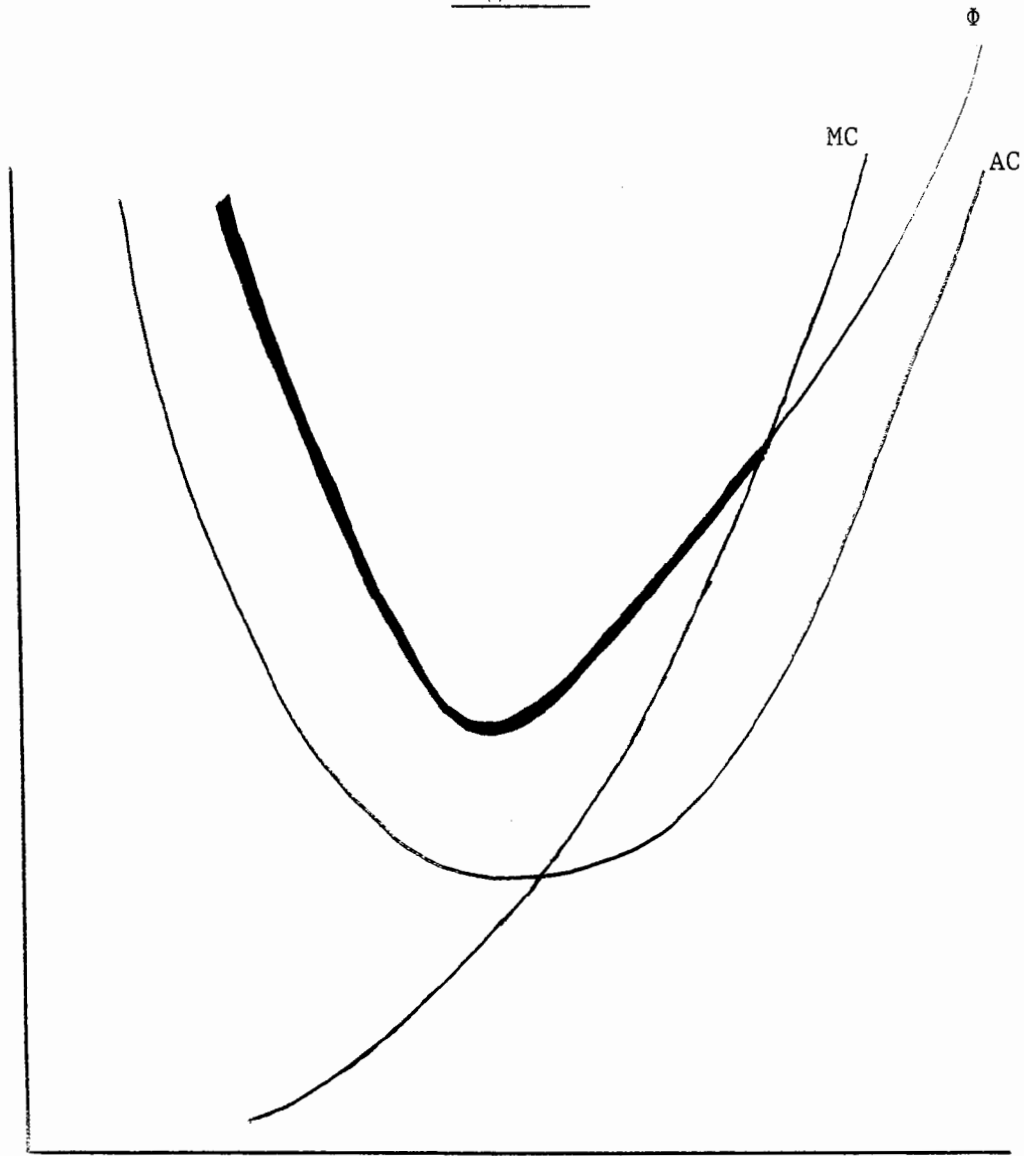
quality. Fully rational consumers will only demand that firms advertise
enough so that type $\hat{t}$ consumers earn zero profits and this allows type 0 firms
to earn positive profits. (If beliefs are locally rational type 0 firms still
can reduce advertising a small amount below their zero profit level.)

In Theorem 3 it was shown that the analagous conditions to those in
Theorem 5 were not only necessary but also sufficient for a fully or locally
rational equilibrium. This was because $\phi(t)$ is both the incentive
compatibility constraint and quality guaranteeing price for a firm of type
t. Therefore if (p,x) is on $\phi(0)$ (i.e. -- $p = \phi(0)$) it is impossible for an
entrant to offer a lower price and attract customers. However when cost
curves are U-shaped it may well be possible for a (p,x) pair to be on
$\Phi(x,0)$ (i.e. -- $p = \Phi(x,0)$) but for the quality guaranteeing price for type 0
firms to be below p.[12] If this is true a firm of type 0 could successfully
enter and disrupt the potential equilibrium. Theorem 6 shows that the
conditions of Theorem 5 together with a condition which states entry at a
lower price is impossible are necessary and sufficient for a fully rational
equilibrium. This condition is

(6.22) $p \leqslant \inf H(0)$.

If a quality guaranteeing price exists (6.22) is simply

(6.23) $p \leqslant p^*(0)$.

If (6.22) is met a firm cannot enter by offering a lower price than p. At any lower price firms of type 0 would prefer to produce low quality. Thus even if some type of firm preferred to produce high quality at some $\hat{p} < p$ it could not prove that it was not a type 0 firm.

Theorem 6:

(A,p,x) is a fully rational equilibrium if and only if it satisfies (6.19) - (6.22).

proof:

First it will be shown that (6.19) - (6.22) are necessary for equilibrium. Conditions (6.19) - (6.21) are necessary by Theorem 5. Now consider (6.22). Suppose it is not true. Then there exists a $\hat{p} < p$ such that $\hat{p} \in H(0)$. Thus a firm of type 0 could enter by charging $\hat{p}$ and choosing $\hat{A}$ large enough so it earns zero profits. This violates (6.9).

Now it will be shown that these conditions are sufficient for equilibrium. It must be shown that (A,p,x) and $B_{FI}(A,p,x)$ satisfy (6.6) - (6.9) and (4.4). It is totally straightforward to verify that (6.6) - (6.8) and (4.4) are satisfied so this will not be done here. It will be shown that (6.9) is satisfied. Suppose a firm offers $\hat{p} < p$. By (6.22) $\hat{p} \in H(0)$. Therefore it is clear from (6.18) that $(\hat{p},\hat{A})$ cannot be in $B_{FI}(A,p,x)$ for any $\hat{A}$ at which a firm of type 0 makes non-negative profits. Since type 0 firms are the most efficient, $(\hat{p},\hat{A})$ is in $B_{FI}(A,p,x)$ implies that every type of firm makes negative profits offering it. Thus no entry is possible.

QED.

It is obvious that a continuum of outcomes satisfy (6.19) – (6.21), as illustrated in Figure 2. However it is not obvious that any of these outcomes also satisfy (6.22). Theorem 7 shows that at least one outcome does and thus that a fully rational equilibrium exists.

Theorem 7:

A fully rational equilibrium exists.

proof:

For this proof attention will be restricted to prices greater than or equal to

(6.24) $\min\limits_{x \in (0,\infty)} AC_h(x,0).$

Define $x_h(p)$ and $x_\ell(p)$ by

(6.25) $x_h(p) = \underset{x}{\text{argmax}} \; px - f_h(x,0)$

and

(6.26) $x_\ell(p) = \underset{x}{\text{argmax}} \; px - f_\ell(x,0).$

The following lemma will first be proven.

(6.27)   If $MC\big(x_h(p),\, 0\big) \leqslant \Phi\big(x_h(p),\, 0\big)$, then $p \in H(0)$.

By the LHS of (6.27),

$$(6.28) \qquad \pi^h(p,0) \leqslant \frac{px_h(p) - f_\ell(x_h(p),0)}{1+r} \ .$$

Because $x_h \neq x_\ell$,

$$(6.29) \qquad \frac{px_h(p) - f_\ell(x_h(p),0)}{1+r} < \pi^\ell(p,0).$$

Combining (6.28) and (6.29) yields

$$\pi^h(p,0) < \pi^\ell(p,0)$$

which implies the RHS of (6.27).

Now the theorem can be proven. Two cases must be considered. First suppose that

$$(6.30) \qquad \Phi(x,0) > MC_h(x,0)$$

for every $x > 0$. Then by (6.27) $H(0)$ is the empty set so (6.22) is vacuous and all outcomes satisfying (6.19) - (6.21) are fully rational.

Now suppose that (6.30) is not true. Then let $\hat{p}$ be defined by

$$(6.31) \qquad \hat{p} = \inf\{p: \Phi(x_h(p), 0) \leqslant MC_h(x_h(p), 0)\} \ .$$

This exists because the functions are continuous. Furthermore, by continuity

$$(6.32) \qquad \Phi(x_h(\hat{p}), 0) = MC_h(x_h(\hat{p}), 0).$$

It will now be shown that $\hat{p}$ and $x_h(\hat{p})$ satisfy (6.19), (6.20), and (6.22) and

thus form a fully rational equilbrium when combined with an A satisfying

(6.21). Condition (6.23) implies (6.19). Condition (6.20) is satisfied by

the definition of x_h. Finally it must be shown that (6.22) is satisfied.

Suppose that

(6.33) $\hat{p} > \inf H(0)$

for contradiction. Then there exists a $p < \hat{p}$ such that $p \in H(0)$. By (6.27)

(6.34) $MC(x_h(p),0) > \Phi(x_h(p,0))$

which contradicts (6.31).

<div align="right">QED.</div>

Figure 3 illustrates the nature of equilibrium when Φ is not always above

MC_h. A case where MC_h and Φ cross only once is illustrated. Note that the

inf H(0) (which is the quality guaranteeing price if it exists) is greater

than $\hat{p}$, in accord with the analysis in Theorem 7. The part of Φ darkened by a

heavier line is the set of fully rational equilibria.

In general there may be more locally rational equilibria than fully

rational equilibria -- i.e. -- some of the outcomes which satisfy (6.19) -

(6.21) but do not satisfy (6.22) may be locally rational equilibria. An

(A,p,x) satisfying (6.19) - (6.21) but not (6.22) was eliminated as a possible

fully rational equilibria by showing that an $(\hat{A},\hat{p})$ existed with $\hat{p} < p$ such

that fully rational consumers would believe that high quality would be

forthcoming. However, the values of $(\hat{A},\hat{p})$ were not generally in a

Figure 3
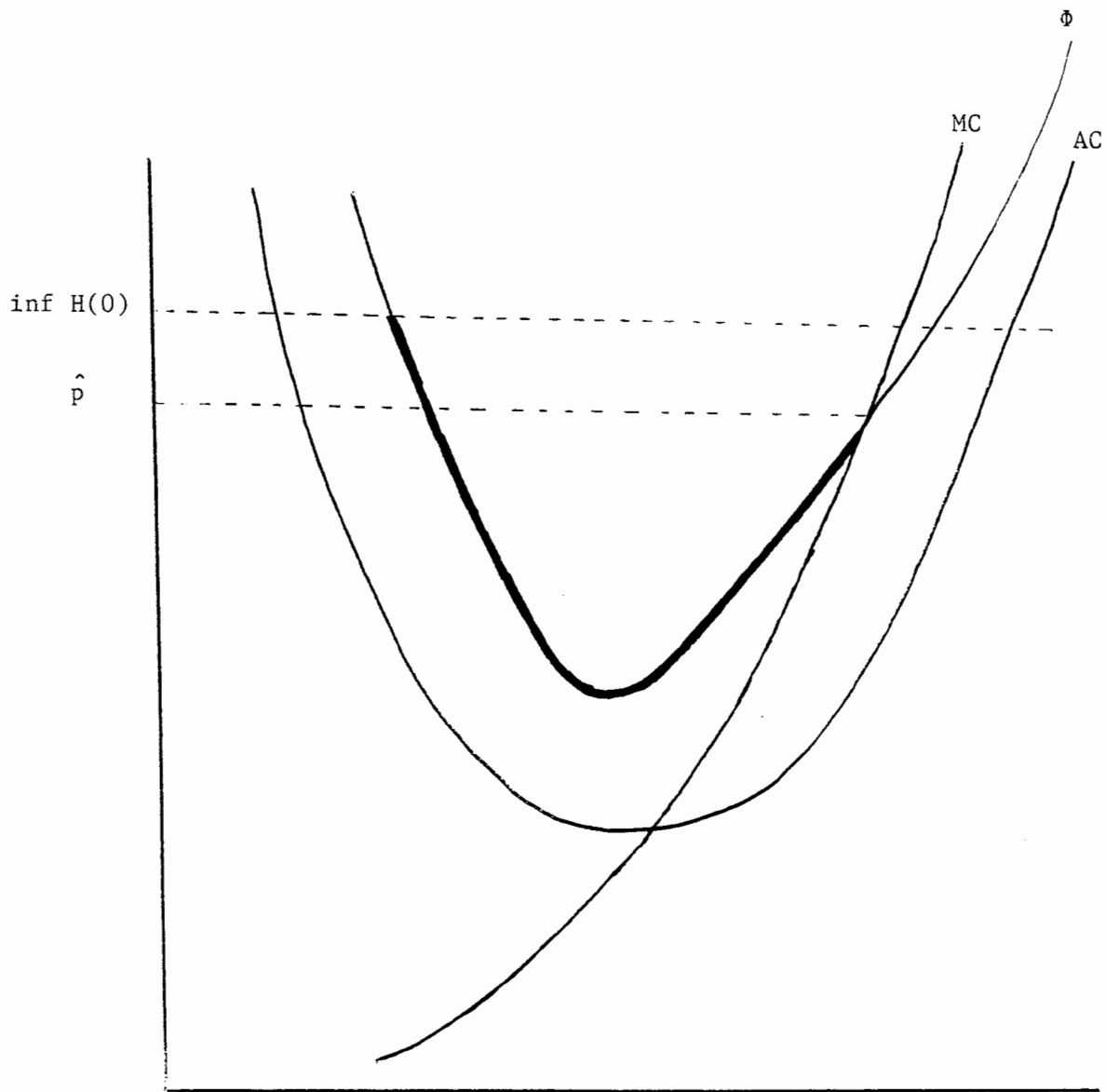
neighborhood of (A,p). Even if $\hat{p}$ could be chosen close to p, it is not in general possible to guarantee that $\hat{A}$ can be chosen close to A. It does not seem possible to provide a simple characterization of precisely which outcomes are locally rational.

7. Conclusion

It has been shown that in markets for experience goods where price guarantees quality that conspicuous expenditures by new entrants on items that have no direct value to consumers can serve as a signal that an entrant will supply high quality. The reason for this is that higher initial expenditures signal that a firm is more efficient which in turn implies that the firm has a lower quality guaranteeing price. Possible conspicuous expenditures include advertising which is not directly informative and initial sales at low prices.

This is a different theory of advertising as a signal than that proposed by Nelson [1974] and formalized by previously mentioned papers. Nelson's theory applies to markets where firms make a once-and-for-all quality choice and additionally requires a number of restrictive assumptions as explained in the introduction.

This paper also provides a theory explaining how profits accruing from the existence of quality guaranteeing prices are dissipated. Rational consumers demand that a firm "burn" an amount of money equal to the present discounted value of the profit stream a low-cost firm would earn. Only by doing this can firms prove they are low-cost and thus prove that they have the incentive to supply high quality.

Footnotes

[1]By a competitive market it is meant that a potentially infinite number of firms of any type exists so profits of any type are driven to zero. Milgrom and Roberts [1986] have recently modelled a monopoly version of this idea.

[2]See Kihlstrom and Riordan [1974] for precise descriptions of necessary and sufficient conditions for existence.

[3]See Allen [1984], Rogerson [1983, 1986] and Shapiro [1983] for subsequent analyses.

[4]Furthermore if firms which are nearly as efficient as the low-cost firms exist, a firm must burn the entire $25 since if it burned any less, a marginally less efficient firm would find it profitable to mimic the action (but then supply low quality.)

[5]The analysis can also be carried out, of course, if higher types are both more inefficient and discount the future more heavily.

[6]This term is formally defined in Section 3.

[7]Allen's analysis of this case does shed light on the mathematical structure of the model. Furthermore Allen also considers the case where current output is unobservable.

[8]See Klein and Leffler [1981] or any of the previously cited subsequent treatments for a more detailed discussion of this topic.

[9] It is straightforward to show that the strategy of producing high quality for a finite number of periods before producing low quality once and exiting is never strictly preferred to both strategies considered above.

[10]Throughout this paper it will be assumed that firms indifferent between producing high quality and low quality choose to produce high quality. Similarly, firms indifferent between producing low quality or not entering choose not to enter.

[11]Since the entrant is charging a lower price than existing firms it can sell $\bar{x}$ units.

[12]If $\lim_{x \to 0} AC_h(x,0) = \infty$ and a finite quality guaranteeing price exists this must be the case for small enough values of x. (This is because $\Phi(x,0) > AC_h(x,0)$. Therefore $\lim_{x \to \infty} \Phi(x,0) = \infty$.)

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