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PRODUCT DURABILITY UNDER MONOPOLY AND COMPETITION

by

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Introduction

The profit maximizing output rate attained under monopoly and that achieved under competition have been thoroughly analyzed and compared. Less is known about the impact of these polar market structures upon the qualitative characteristics of a product. Recently E. Kleiman and T. Ophir [6], David Levhari and T. N. Srinivasan [7], and R. L. Schmalensee [11] have attempted to demonstrate that the organization of an industry can affect the durability of its product. In particular, they sought to show that with cost an increasing function of durability, a profit-maximizing monopolist tends to select a lower durability, or more rapid decay, for the product than a perfectly competitive firm facing the same cost conditions. The demonstrations employed comparative statics analyses of stationary states. Peter L. Swan [12, 13] disputed the conclusion that durability depends on market structure and argued that this result appears because the approach to the stationary state is ignored.

Many of the authors cited posit constant returns to scale. Swan's results rest on a very strong form of this hypothesis however. He assumes there is no short run; the same unit cost is realized no matter how rapidly or widely the production rate is varied. Thus the steady state is achieved instantaneously, with the firm producing in the first moment of time a volume several times as large as the constant production rate which will prevail thereafter [13, p. 351]. It is the effect of this initial lump of output (and its cost and profit) that enables Swan to reverse the conclusions previously drawn. While Swan's extreme interpretation of "constant returns to scale" may be appropriate in some circumstance, we think it worthwhile to view the approach to the stationary state under the more usual convention that returns to scale is a long run concept, with diminishing returns to varying proportions in the short run.

We reconsider the question of the relation between market structure and the durability of consumer goods. The full optimization problem is studied, from the time of planning to produce the good forward. The scale of plant and durability of product must be selected once-for-all but the rate of output is freely variable. We assume an n-firm industry, with all firms facing identical cost conditions, having perfect foresight, and considering the output of the $n-1$ other firms to be independent of its own actions. Specializing to $n=1$ readily yields monopoly results, while letting n grow without bound gives the competitive solution (under appropriate conditions [10]).

The model, with modest restriction on the revenue and cost functions, is developed in the next section. The optimal production plan involves a temporally falling production rate and rising stock of the durable, regardless of the size, n, of the industry. To compare durabilities chosen by a monopolist and competitor, we specialize to quadratic revenue and cost functions and generate explicit solutions. The short run average production cost is U-shaped while long run average cost is constant. Long run unit cost is an increasing function of product durability.

While the algebra of the quadratic example is tedious, the outline of calculations and findings can be summarized as follows. We first find the explicit optimal paths of output and stock of the durable for given capital stock and product durability. The steady state is approached only as $t \to \infty$. The speed of approach is more rapid with greater durability, lower discount rate, larger demand or larger number of firms. Conditions for positive production to be economic are also found in the form of explicit bounds on the ratio of maximum demand price to minimum unit cost.

Next, optimal plant size is found for given durability and then optimal durability is derived. The pure competitor's durability choice in our model coincides with that of earlier investigators, as do the limiting price and stock of the durable. The monopolist's choice of durability is lower than the competitor's, in qualitative agreement with earlier findings based on analysis of the steady state only. The quantitative results differ however, reflecting the impact of the period of approach to the steady state. We also show that the larger the market for service of the durable, the lower the monopolist's chosen durability.

Finally we look at the effect on price and output of regulating the monopolist's choice of durability toward the competitive benchmark. Since price and stock change through time, we need a summary measure or average price. Certainly the steady state price is inappropriate since it is a bound, never attained and approached only as $t \to \infty$. The summary measure must take the entire price path into account, with weight given to consumers of the near and intermediate term as well as to those of the far distant future. We suppose the societal discount rate coincides with the firm's discount rate to find the weighted average. We find that, contrary to prior studies, durability regulation leads to higher average price.

General Revenue and Cost

We posit a consumer durable with infinite life. The rate of service provided by a unit of the good decays exponentially at rate b, although the quality of service rendered is unaltered. It is convenient to regard the durable as evaporating at rate b, and to deal with the effective stock of the good.

Suppose the durable is produced by an industry composed of $n \geq 1$ firms, each facing the same cost conditions. Each firm considers the output of the $n-1$ other firms to be independent of its own actions. We shall let $n=1$ and $n \to \infty$ to study the polar cases of monopoly and pure competition respectively. For ease in discussion, we focus on a single "representative" firm in the industry.

We follow Swan's suggestion of allowing each firm to retain ownership of its output and to collect rental fees. (This assumption circumvents difficulties potential purchasers would have in assessing future price movements, necessary to rational decision on their part.) Let $y(t)$ be the representative firm's output rate at t so that its effective stock $q(t)$ obeys the differential equation and initial condition

$$(1)\quad q'(t) = y(t) - bq(t) \qquad q(0) = 0$$

Let $X(t)$ be the total effective stock of durable held by the remaining $n-1$ firms of the industry. Then, with $Q(t)$ the total industry stock,

$$(2)\quad Q(t) = X(t) + q(t)$$

where $Q(t)$ is also the rate of service available from the stock extant at time t. The industry demand function for the durable's service is assumed stationary and representable in inverse form as

$$(3)\quad p = p(Q)$$

where p is the rental price for use of Q.

The firm's cost function $C(y;k,b)$, likewise assumed stationary, has as arguments the production rate y, the scale of plant k, and durability b. While the influence of durability on cost was considered by the earlier writers [6,7,11,12,13], the impact of scale on production cost is discussed more extensively in [1,4].

The representative firm is to select a production plan $y(t)$ for $t \geq 0$, plant scale k, and durability b to maximize the present value of rentals less production costs:

$$(4)\quad \underset{y(t);k,b}{\text{maximize}} \int_0^\infty e^{-rt}[q(t)p(q(t) + X(t)) - C(y(t);k,b)]dt$$

where r is the discount rate and $X(t)$ is exogenous to the firm. Problem (4) is one of optimal control, with state variable q and control y, for given k and b. We proceed stepwise by finding the optimal production plan $y(t)$ for $t \geq 0$, for any k and b, then finding the optimal scale k and durability b.

We assume the firm's revenue function is bounded and strictly concave in q:

$$(5)\quad R(q) \equiv qp(q+X),\ R'(q) = p(Q) + qp'(Q),\ R''(q) < 0$$

while the cost function $C(y;k,b)$ is strictly convex in $y, C_{yy} > 0$. The subscripts indicate partial differentiation. We assume positive production is worthwhile. Let $\mu(t)$ be the multiplier associated with (1) and form the Hamiltonian

$$H = e^{-rt}[R(q(t)) - C(y(t);k,b)] + \mu(t)[y(t) - bq(t)]$$

Since $R-C$ is concave in q and y, conditions necessary and sufficient for a production plan to be optimal [5,8] are that $y(t)$ and the multiplier function $\mu(t)$ obey (1) and

$$(6) \quad \partial H/\partial y = - e^{-rt} C_y(y(t);k,b) + \mu(t) = 0$$

with

$$(7) \quad \mu'(t) = - \partial H/\partial q = - e^{-rt} R'(q(t)) + b\mu(t)$$

Equation (7) may be solved with terminal condition

$$\lim_{t\to\infty} \mu(t) = 0$$

to yield

$$(8) \quad \mu(t) = \int_t^{\infty} e^{-rs} e^{-b(s-t)} R'(q(s))ds$$

Thus $\mu(t)$ is the value at time 0 of the increment to rentals attributable to a marginal unit produced at time t. Combining (6) and (8) gives the "marginal revenue equals marginal cost" condition that must hold at each instant t:

$$(9) \quad \int_t^{\infty} e^{-(r+b)(s-t)} R'(q(s))ds = C_y(y(t);k,b)$$

Since this marginal equality must hold for all t, it may be differentiated with respect to time to give

$$(10) \quad y'(t) = [(r+b)C_y(y(t)) - R'(q(t))]/C_{yy}(y(t))$$

Equations (1) and (10) constitute the first order differential equations satisfied by an optimal production plan $y(t)$ and stock $q(t)$ of the individual firm, for given k, b.

Equations (1) and (10) cannot be solved, however, unless the plans of the other firms, summarized in $X(t)$, are known. To make analysis of the multifirm industry and of the monopolist comparable, we suppose the representative firm has perfect foresight. Thus the plans of other firms are known to it and believed independent of its own actions. Finally, since all firms are assumed identical, we seek a symmetric solution in which they all have the same plans. This will yield a dynamic equilibrium with perfect foresight and no incentive to change. The modeling of firm expectations in terms of a generalized Cournot behavior is employed by Ruff [9, esp. p. 404] and related to the analysis in Henderson and Quandt [3]. The assumption of dynamic equilibrium or perfect foresight yields

$$(11)\quad X(t) = (n-1)q(t) \text{ or } Q(t) = nq(t)$$

We emphasize that this condition does not mean other firms will deliberately follow the behavior of the representative firm. (The latter supposition would amount to joint maximization of profits, with the industry acting as a cartel.) Each firm chooses its output rate independently, taking the output rate and effective stock of durable of other firms as fixed. With perfect foresight, the result of this action will be that all firms behave identically, (11).

Combining the market equilibrium condition (11) with (5) gives

$$(12)\quad R'(q) = p(nq) + qp'(nq) \quad \text{in dynamic equilibrium}$$

The stationary solution y^*, q^* to (1), (10), and (12) satisfies

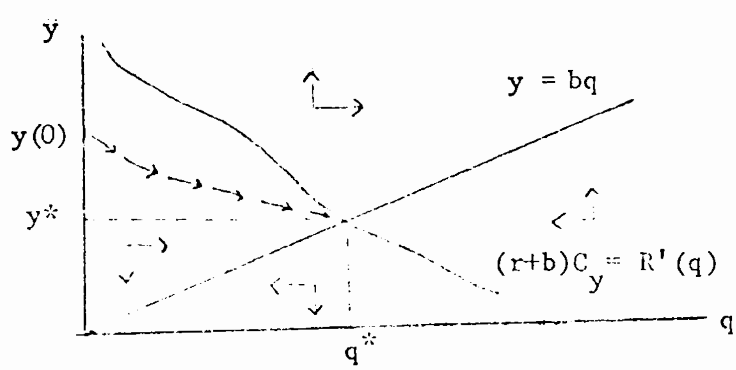
(13) $y = bq$

(14) $(r+b)C_y(y) = p(nq) + qp'(nq)$

Conditions (13) and (14) correspond to requirements agreed upon by previous investigators for monopoly $(n=1)$ with regard to steady state output and stock. (The dispute involves durability only.)

In view of the initial condition $q(0) = 0$, the stationary state is not realized at the outset and as a consequence of rising marginal production costs, it is not attained immediately. Insight into the nature of the path followed can be gained from a diagram. According to (13) the locus of points (q,y) for which q is stationary is a straight line through the origin in the q-y plane. For points above the locus, $y > bq$, (1) implies that q is increasing. Similarly, $q' < 0$ for points below the locus (13).

Likewise points (q,y) at which y is stationary satisfy (14). Since the left side of (14) is increasing in y while the right side is decreasing in q, the locus of points satisfying (14) has negative slope. Because $C_{yy} > 0$, and the numerator of the right side of (10) is increasing in y and q, y will grow above the locus of points satisfying (14) and shrink below it.



Approach to Steady State

The figure reveals that if there is an optimal solution to (4) tending to q^*, y^* it must begin at some $y(0) > y^*$ and follow a path of temporally decreasing output rate $y(t)$, which exceeds y^* until the steady state is reached. Furthermore, the stock $q(t)$ increases over time, remaining below q^* prior to the stationary state.

To find conditions governing optimal k and b, use (1) to eliminate y from (4) and differentiate with respect to these variables:

$$(15) \quad \int_0^\infty e^{-rt} C_k(y(t);k,b)dt = 0$$

$$(16) \quad \int_0^\infty e^{-rt}[q(t)C_y(y(t);k,b) + C_b(y(t);k,b)]dt = 0$$

where $q(t)$, $y(t)$ satisfy the conditions displayed earlier. The sufficient conditions for k,b are assumed to hold so that (15), (16) implicitly define positive, finite, optimal k,b. According to (15), the scale of plant minimizes average discounted production costs. Further, (16), the cost savings of making a less durable product are just offset by the cost of added production to maintain the stock of durable good.

The form of the decision rules and output path for the representative firm is the same regardless of n. However we are as yet unable to compare durabilities chosen by a monopolist and a pure competitor. Our study is facilitated by considering henceforth second order approximations to the revenue and cost functions, so we may compute solutions explicitly.

Quadratic Revenue and Cost

Let us specify

$$(17) \quad p(Q) = \alpha - \beta Q \qquad \alpha > 0,\ \beta > 0$$

$$(18) \quad C(y;k,b) = y^2 + Ay + B \qquad A > 0,\ B > 0$$

For comparability of our results with those of recent writers, the cost function's parameters are restricted to exhibit constant returns to scale and a minimum unit production cost that rises with the good's durability; in particular set

$$(19) \quad A = m(b) - 2k$$

with

$$(20) \quad m'(b) < 0 \text{ and } 2m'(b) + (r+b)m''(b) > 0$$

and

$$(21) \quad B = k^2$$

To verify that our cost function has the desired properties, note that the optimal scale for producing y (the scale that minimizes the cost of y) satisfies $C_k = -2y + 2k = 0$ or $k = y$. Hence the long run cost function is $C(y;y,b) = m(b)y$, with constant returns to scale as desired, and minimum unit cost $m(b)$.

The short run average cost function $C(y)/y = y + A + B/y$ exhibits the traditional U-shape, falling for $y < B^{\frac{1}{2}} = k$, tangent to long run average cost at $y = k$, and rising for $y > k$. Short run marginal cost $C_y(y) = 2y + A$ is positive and rising throughout.

The requirement on the second derivative of $m(b)$ in (20) was also imposed in [11] and [13]. It can be shown to assure the second order condition for a maximum with respect to durability choice will be satisfied by both the monopolist and the pure competitor of our model.

Combining (10) and (12) with assumptions (17) and (18) gives

$$(22)\quad y'(t) = (r+b)y(t) + \beta(n+1)q(t)/2 - [\alpha - (r+b)A]/2$$

Thus (1) and (22) form the pair of simultaneous linear differential equations satisfied by the output rate and stock of the firm with a quadratic cost function facing linear industry demand. The solution of this system has the general form

$$q(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + q^*$$

$$y(t) = c_3 e^{\lambda_1 t} + c_4 e^{\lambda_2 t} + y^*$$

where q^*, y^* satisfy

$$(23)\quad y^* = bq^*$$

$$(24)\quad q^* = [\alpha - (r+b)A]/\ [2(r+b)b + \beta(n+1)]$$

and with λ_1, λ_2 roots of the characteristic equation

$$\lambda^2 - r\lambda - b(r+b) - \beta(n+1)/2 = 0$$

The roots are real and of opposite sign. Since our concern is with solutions tending toward y^*, q^* we take the negative root only and call it $-w$ so

$$(25) \quad w = \{[(r+2b)^2 + 2\beta(n+1)]^{\frac{1}{2}} - r\}/2 > 0$$

Since (1) implies $q'(0) = y(0)$, we find that

$$(26) \quad q(t) = q^*(1-e^{-wt})$$

$$(27) \quad y(t) = q^*(b + (w-b)e^{-wt})$$

Also, since

$$w - b = \{[(r + 2b)^2 + 2\beta(n+1)]^{\frac{1}{2}} - (r+2b)\}/2 > 0$$

we have

$$(28) \quad w > b$$

Thus the optimal path begins at $q(0) = 0$, $y(0) = wq^*$ and declines linearly ($y'(t)/q'(t) = dy/dq = w - b$) to q^*, y^*. The speed of adjustment, or approach to the steady state, varies with w. Since

$$\partial w/\partial b = (r+2b)/(r+2w) > 0$$

$$\partial w/\partial r = -(w-b)/(r+2w) < 0$$

$$\partial w/\partial \beta = (n+1)/2(r+2w) > 0$$

$$\partial w/\partial n = \beta/2(r+2w) > 0$$

the adjustment will be slower as the product durability, the discount rate, or demand are greater. The larger the number of firms in the industry, the more rapid the convergence to the steady state.

The maximum present value V(k,b) the firm can obtain with given k and b is

$$V(k,b) = \int_0^\infty e^{-rt}[p(nq(t))q(t) - C(y(t);k,b)]dt$$

$$= \int_0^\infty e^{-rt}[\alpha q(t) - \beta nq^2(t) - y^2(t) - Ay(t) - B]dt$$

where q(t), y(t) are given by (26), (27). On substituting for these functions, integrating, and performing some tedious collection of terms, one eventually obtains

$$(29) \quad rV(k,b) = q^{*2}w^2z - B$$

where

$$(30) \quad z = 1 - \beta(n-1)/(r+w)(r+2w)$$

To show that the expression z is positive, we establish an identity useful for both present and later calculations, namely

$$(31) \quad 2w(w+r) \equiv 2b(b+r) + \beta(n+1)$$

This result may be obtained by rearranging (25) to $r+2w = [(r+2b)^2 + 2\beta(n+1)]^{\frac{1}{2}}$, squaring and simplifying. Next, using (31) we find that

$$(r+w)(r+2w) = r(r+w) + 2w(r+w)$$

$$= r(r+w) + 2b(b+r) + \beta(n+1) > \beta(n-1)$$

from which it follows that

(32) $0 \leq z \leq 1$

The closed form expression for $V(k,b)$ can now be used to find the optimal scale of plant k for given durability b. Differentiating (30)

(33) $\partial rV/\partial k = 2(w^2 zq^* \, \partial q^*/\partial k - k) = 0$

In view of (24), (31) and (19), we have $\partial q^*/\partial k = (r+b)/w(w+r)$ so that solving (33) for $k = k^*$ yields

(34) $k^* = [\alpha - (r+b)m(b)](r+b)z/2[(r+w)^2 - (r+b)^2 z]$

Since

$$\partial^2 V/\partial k^2 = [(r+b)^2 z/(r+w)^2 - 1] \, 2/r < o$$

(34) does maximize V with respect to k for given b.

One can now find successively that

(35) $A^* \equiv m(b) - 2k^* = [m(b)(r+w)^2 - \alpha(r+b)z]/[(w+r)^2 - (r+b)^2 z]$

(36) $q^{**} \equiv [\alpha - (r+b)m(b) + 2k^*(r+b)]/2w(w+r)$

$= [\alpha - (r+b)m(b)](w+r)/2w[(r+w)^2 - (r+b)^2 z]$

(37) $V^*(b) \equiv V(k^*,b) = [\alpha - (r+b)m(b)]^2 z/4r[(w+r)^2 - (r+b)^2 z]$

The foregoing conclusions rest on the implicit assumption that demand is sufficiently large for production to be worthwhile. But q^* and therefore $q(t)$ will be positive and $A^* > 0$ will be satisfied if we suppose that the ratio of maximum demand price to minimum unit costs satisfies

(38) $1 < \alpha/(r+b)m(b) < (r+w)^2/(r+b)^2 z$

Positivity of q^* is assured by the first inequality. The second inequality renders $A^* > 0$. In view of (28) and (32), the upper bound in (38) always exceeds the lower bound and the restriction is nonvacuous. Note that $V^*(b) > 0$.

Taking into account (25) and (30), one may find after some computation that

$$(39)\quad V'(b) = \frac{[\alpha - (r+b)m(b)]^2}{2r[(r+w)^2/z-(r+b)^2]}\left\{-\frac{[m(b)+(r+b)m'(b)]}{\alpha-(r+b)m(b)} + [(w+r)^2/z - (r+b)^2]^{-1}\left[(r+b) + \frac{(w+r)(1-z)(3r+4w)(r+2b)}{2z^2(r+2w)^2} - \frac{(r+w)(r+2b)}{z(r+2w)}\right]\right\}$$

so that the optimal durability b^* renders

$$(40)\quad \frac{m(b)+(r+b)m'(b)}{\alpha-(r+b)m(b)} = \frac{(r+b)2z^2(r+2w)^2 + (w+r)(1-z)(3r+4w)(r+2b) - 2z(r+2w)(r+w)(r+2b)}{2z^2(r+2w)^2[(w+r)^2/z - (r+b)^2]}$$

In particular, for the monopolist $n = 1$ so that $z = 1$ and (40) reduces to

$$(41)\quad m(b) + (r+b)m'(b) = r[\alpha - (r+b)m(b)]/(r+2w)(w+b+2r) > 0$$

It can be shown that $V^{*''} < 0$ for $n = 1$ provided (41) is satisfied so (41) implicitly defines the monopolist's optimal durability.

Before proceeding to the many-firm case, let us examine the effect of a demand shift upon the monopolist's choice of durability. Rearrange (41) to

$$r[\alpha - (r+b)m(b)] - [m(b) + (r+b)m'(b)](w+b+2r)(r+2w) = 0$$

and differentiate implicitly with respect to b, α, β. Any shift in α or β must be accompanied by a compensating change in b to maintain the above equality. The derivative of the left side with respect to b is negative, with respect to α is $r > 0$, and with respect to β is

$$- [m(b) + (r+b)m'(b)]\ (5r+2b+4w)/(r+2w) < 0$$

Therefore an increase in demand, reflected as either an increase in the intercept α or reduction in slope β, brings about an increase in the monopolist's choice of decay rate b. To understand why greater demand will be accompanied by reduced product durability in a monopolized industry, recall that unit production cost increases with the durability of the good. The greater the output rate, the larger the total cost savings associated with a marginal reduction in product durability.

For the case of pure competition, we let the number of firms grow without bound so each firm can have negligible impact on the market. Ruffin's work [10, Theorem 1] assures that the solution will tend to the competitive solution as n tends to infinity, given the cost structure assumed here. The first order condition for the pure competitor's choice of product durability is obtained by taking the limit as $n \to \infty$ of (40). To evaluate this limit we appeal to the concept of order of a function, see [2]. Both z and w depend on n and from their definitions we have

$$z = \frac{r[(r+2b)^2 + 2\beta(n+1)]^{\frac{1}{2}} + (r+2b)^2 + 4\beta}{r[(r+2b)^2 + 2\beta(n+1)]^{\frac{1}{2}} + (r+2b)^2 + 2\beta(n+1)}$$

so that

$$\lim_{n\to\infty} n^{\frac{1}{2}} z = r/(2\beta)^{\frac{1}{2}}$$

and hence $z \sim r/2\beta n)^{\frac{1}{2}}$ as $n \to \infty$. Similarly, one can find that $\lim_{n\to\infty} wn^{-\frac{1}{2}} = (\beta/2)^{\frac{1}{2}}$ so that $w \sim (\beta n/2)^{\frac{1}{2}}$ as $n \to \infty$. Since w and z are of the same order as $n^{\frac{1}{2}}$ and $n^{-\frac{1}{2}}$ respectively for large n, the highest order term in the denominator of the right side of (40) is of the same order as $n^{3/2}$ while the numerator is of order n. Thus the ratio tends to proportionality with $n^{-\frac{1}{2}}$ as n becomes very large. Therefore as $n \to \infty$, (40) reduces to

$$[m(b) + (r+b)m'(b)]/[\alpha - (r+b)m(b)] = 0$$

or

$$(42) \quad m(b) + (r+b)m'(b) = 0$$

The result that competitive durability obey (42) was found also by [11] using analysis of the stationary state. Indeed, since $w \to \infty$ as $n \to \infty$, the stationary state is achieved immediately in the case of an indefinitely large number of perfectly foresighted firms in the industry.

The left side of (42) is assumed in (20) to be an increasing function of b; indeed that condition is necessary for $\lim_{n\to\infty} V^{*''} < 0$ when (42) holds. It follows that the decay rate selected by the monopolist to satisfy (41) must be greater than the decay rate chosen by the pure competitor to satisfy (42). Thus the monopolist chooses faster deterioration or lower durability than the purely competitive firm does. This is the qualitative conclusion reached by [6, 7, 11] with a comparative statics analysis of the problem. The conclusion is in contrast to that reached by Swan under the assumption that all factors of production are freely and instantaneously variable.

Although our focus is on durability , it is of some interest to check that the remaining conclusions about the purely competitive industry obtained by letting $n \to \infty$ are in accord with the findings of earlier contributors. As remarked above, the stationary state is achieved immediately in the polar case of pure competition. While the representative firm's scale of plant, output, and stock of durable each tend to zero, the industry stock of durable of course does not. The industry stock is

$$Q^{**} = nq^{**} = \frac{\alpha - (r+b)m(b)}{[2w(w+r)/n]\ [1 - (r+b)^2 z/(w+r)^2]}$$

The numerator is free of n and for large n the denominator is dominated by its term of highest order in n, namely

$$2w(w+r)/n = [2b(b+r) + \beta(n+1)]/n \to \beta \text{ as } n \to \infty$$

on recalling (31). It is now apparent that

$$\lim_{n\to\infty} Q^{**} = [\alpha - (r+b)m(b)]/\beta$$

and therefore the competitive price

$$\lim_{n\to\infty} p(Q^{**}) = (r+b)m(b)$$

where b is given implicitly by (42), as was to be shown. Thus the limiting industry price and stock of durable obtained from our model are the same as those of the previous literature for the competitive industry.

Durability Regulation

Since the monopolist would select a more rapid rate of decay than the competitive industry, we ask about the consequences of prespecifying a lower decay rate than he would freely choose. In particular, we are concerned with the effect of such regulation on price and output. Since these variables change over time, we must use some measure of average price and output. Since price varies inversely with stock, we first investigate the effect of durability regulation upon average stock. We assume that only durability is prespecified, with the firm free to pick its output rate and plant scale.

As discussed at some length before, the steady state stock is an inappropriate summary measure since it exceeds the actual stock during all finite time. We need a definition of "average" that reflects the consumers of all time periods, present and future. By supposing that the social discount rate is the same as the firm's discount rate, we can easily calculate the desired measure. Define the average stock to be that level which, if maintained from time 0 forward, would give the same discounted total as does the actual discounted stock stream. Specifically, for the monopolist

$$\bar{q}/r = \int_0^\infty e^{-rt} q(t)dt$$

The stock of each time t is given weight reflecting its proximity in time to the point at which planning is done, with time discounted at rate r. On replacing q^* with q^{**} in (26) to reflect optimal plant and substituting from (36) with $n=1$ so $z=1$,

$$\bar{q} = r\int_0^\infty e^{-rt} q^{**}(1-e^{-wt})dt = wq^{**}/(w+r)$$

$$= [\alpha - m(b)(r+b)]/2\ [(r+w)^2 - (r+b)^2]$$

The manner in which the monopolist's $\bar{q}$ varies with durability b is

$$\frac{\partial \bar{q}}{\partial b} = - \frac{[m(b) + (r+b)m'(b)]}{2[(r+w)^2 - (r+b)^2]} + \frac{[\alpha - m(b)(r+b)]r(w-b)}{(r+2w)[(r+w)^2 - (r+b)^2]^2}$$

While the sign of this expression is not readily apparent, it may be evaluated at the monopolist's choice of b. Substituting from (41) gives

$$(43) \quad \partial \bar{q}/\partial b \Big|_{(41)} = [m(b) + (r+b)m'(b)]/2\,[(r+w)^2 - (r+b)^2] > 0$$

Thus, if a monopoly were required to produce a more durable good than it finds most profitable, that is, a smaller decay rate b, the average stock resulting from optimizing over its remaining variables would likewise be reduced. This means that the average price $\bar{p} = \alpha - \beta\bar{q}$ would be <u>higher</u> under regulation than under full profit maximization! The last result is the opposite of that reached in [7] and [11]. It is easy to show that requiring greater durability also results in smaller average output as well as smaller average stock and higher average price.

Conclusions and Open Questions

In our reinvestigation of the relationship between market structure and durability of consumer goods, we have raised substantive and methodological questions. While comparison of steady states often is a fruitful means of analysis, our findings suggest the results so obtained may be qualitatively misleading. In particular, we do concur in the previously established result (disputed only by Swan, as far as we know) that a monopolist tends to produce a good of lower durability than would an otherwise identical competitive firm. On the other hand, we find that regulating durability toward the competitive benchmark tends to raise the average price for a unit of service and reduce the average level of services, rather than the opposite as found by Levhari and Srinivasan and by Schmalensee via comparative statics.

Thus we tend to agree with Swan in accepting the principle that the period of approach to the steady state may be highly relevant to the analysis and results obtained. However, we do not concur with his supposition that the monopolist's adjustment period is but an instant, with plant being installed, employed, and sold without friction all in that moment of time. Nor do we agree with the tacit assumption of many previous investigators that the firm will choose the scale of plant that minimizes the cost of long run equilibrium output even though that equilibrium will not be achieved immediately. The profit maximization hypothesis suggests selection of an intermediate scale that achieves the most economical compromise between near and far term needs.

In this investigation we employed an industry composed of firms pursuing generalized Cournot behavior as the vehicle of analysis and second order approximations to obtain explicit solutions. It remains to be seen whether our findings would be altered if other behavioral assumptions were posited or more general functional forms were used. Further, we left open the consequences of permitting the firm to modify its scale of plant and/or product durability, perhaps incurring some costs of adjustment in so doing.

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