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Learning-by-Doing and the Introduction of New Goods

by

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Abstract

A dynamic general equilibrium model is developed in which goods are valued according to the characteristics they contain, the set of goods produced in any period is endogenously determined, and learning-by-doing is the force behind sustained growth. It is shown that the set of produced goods changes in a systematic way over time, with goods of higher quality entering each period, and those of lower quality dropping out. The model is then used to study the effect of introducing a "traditional" sector in which there is no learning.

1. Introduction

Perhaps the most remarkable feature of economic growth in the developed countries, especially in the period beginning with the industrial revolution, is the extent to which the production of goods and services has not merely grown, but changed drastically in composition. Candles gave way to whale oil lamps, which in turn gave way to gas lights and then to incandescent bulbs. The latter have, in their turn, been partially displaced by fluorescent, neon, mercury vapor and sodium vapor lights. Casual empiricism suggests that this example is typical rather than exceptional: many of the goods and services produced today were unknown three hundred years ago, and many produced then are--except through books and museums--unknown now.

By contrast, most of the aggregative models of growth and development that economists have developed to date (the work of Ramsey [1928], Solow [1956], Cass [1965], Koopmans [1965], and their many followers), concentrate almost wholly on increases in the _quantities_ of goods produced. The introduction of new goods is notable by its absence.[1] Technical change, when it appears at all, takes the form of process rather than product innovation, so that "growth" means producing more of the same good(s). Moreover, it has proved difficult to construct models giving rise to _sustained_ growth, even defined in this narrow sense. Exogenous technical change is one "engine" for sustained growth in these models (as in Solow [1959], Diamond [1965], Shell 1967], and many others); positive externalities in production are another (as in Arrow [1962], Romer [1983, 1986], and Lucas [1985]).

In this paper a simple dynamic general equilibrium model is developed in which competitive equilibrium paths feature sustained growth, and in which the introduction of new and better products is an integral part of that growth. Specifically, main features of the model are that there is a continuum of potentially producible goods; in each period only a limited subset of the goods are actually produced; over time the set of produced goods changes, with higher quality goods entering the produced set and those of lower quality dropping out; and in the long run growth continues without bound. The accumulation of knowledge, through economy-wide learning-by-doing, is the sole force behind the growth; there is no physical capital. Other features of the model are standard: labor is inelastically supplied, within each period all goods are produced with constant returns to scale technologies, and all markets are perfectly competitive.

Thus, the model is similar in several respects to those in the papers by Arrow, Romer and Lucas mentioned above: there is endogenously generated, sustained growth in per capita output; growth is driven by the accumulation of knowledge; and there is an externality in the accumulation of knowledge. It is also like the model of Arrow in that the accumulation of knowledge is the result of experience in production rather than a separate activity (although many of the arguments here would also apply to models based on R & D or education). The main differences are the absence of physical capital and the specification of the commodity space and preferences.

The absence of physical capital may at first seem startling. However, as noted above, "growth" models built around the accumulation of physical capital alone do <u>not</u> give rise to sustained growth. The models that do are those built around the (endogenously determined) accumulation of knowledge or around (exogenously given) technological change. The work presented here focuses entirely on the accumulation of knowledge, and dispenses with physical capital altogether. The benefits of this, in terms of simplicity, will be apparent; the costs will be discussed in the conclusions.

The description of the commodity space and preferences are also unusual for a model of economic growth. Since they are central to the results, they need some justification.

Why is it that (most) people in the industrialized countries no longer eat gruel, read by candlelight, or sleep in log cabins? The obvious answer is because they can afford to buy steak dinners, electric lights, and houses with central heating instead. They can afford these goods because real incomes have gone up, i.e., the real cost of producing almost all goods has gone down. Still, why doesn't the consumer eat <u>some</u> gruel as well as some steak, as convexity of preferences suggests he should? The answer to this seems clear. Gruel is cheap and provides calories, but otherwise doesn't have much to recommend it. Steak dinners provide a variety of vitamins, minerals and protein, in addition to calories, and are much tastier as well. In this sense they are strictly "better" foods. Moreover, it is impossible to get the protein, good taste, etc., without getting plenty of calories. Thus, the one thing that gruel provides is supplied in sufficient quantity by the "better" foods, and gruel is redundant. A little reflection suggests that similar arguments can be made in many other instances: a new good often replaces an old one because it does or provides everything the old one did, and more as well.

This suggests that a Lancasterian [1966] characteristics model of commodities and preferences may be a useful framework for the problem at hand. The rest of the paper shows that this is indeed the case, and is organized as follows. In section 2, specific assumptions are developed under which the dynamics of product introduction are as described above. It is also shown that such an economy will display sustained growth in the sense that GNP, as conventionally measured, will increase every period. The consequences of adding a "traditional" sector--one without learning--are explored in section 3, and the conclusions are discussed in section 4.

2. Learning-by-Doing and New Goods

Assume that the economy has many identical consumers and many identical firms, and all markets are perfectly competitive. All consumers and firms are infinitely long-lived, and there is no uncertainty. There is no capital; contemporaneous labor is the only factor of production; and all produced goods are perishable. All goods (including labor) are traded on spot markets in each period, and these are the only markets available. The consumer has a constant endowment of $y > 0$ units of labor each period, and his preferences are additively separable over time.

In each period there is a continuum of potentially producible goods indexed by $s \in R_+$, and a continuum of characteristics indexed by $z \in R_+$. A goods allocation in period t is represented by a piecewise continuous density, $x_t(s)$, $s \geq 0$. Good s provides one unit of each of the characteristics $z \in [0,s]$, so that the goods allocation x_t contains the allocation of characteristics q_t given by

$$(1) \qquad q_t(z) = \int_z^\infty x_t(s)ds , \qquad z \geq 0.$$

Thus, higher index goods are "better" in the sense that they provide more characteristics, and the notion of "better" or "higher quality" is not linked to any particular specification of preferences. For any preferences that are increasing in all characteristics, additional units of higher index goods are always preferred, at the margin, to units of lower index goods, regardless of the initial allocation. Define

$$X = \{x\colon R_+ \to R_+ \mid x \text{ is piecewise continuous, and for some } B \geq 0,\ x(s) = 0,\ s \geq B\}.$$

$$Q = \{q\colon R_+ \to R_+ \mid q \text{ is nonincreasing and piecewise continuously differentiable, and for some } B \geq 0,\ q(z) = 0,\ z \geq B.\}$$

Then $x_t \in X$ and $q_t \in Q$, all t, and (1) defines a one-to-one mapping between X and Q.

For simplicity, temporarily drop the subscript "t". Assume that within each period, the consumer's preferences over allocations of characteristics $q \in Q$ are additively separable and symmetric: $U(q) = \int_0^\infty u[q(z)]dz$. These preferences are tractable, yet given the link between goods and characteristics in (1) imply strong income effects. In particular, any good is inferior at high enough levels of income. The function u will be restricted as follows.

<u>Assumption 1</u>: u is strictly increasing, strictly concave, and twice continuously differentiable, with $u(0) = 0$ and $u'(0) < \infty$.

It is important that $u'(0)$ is finite, as the equilibria will involve zero consumption of many characteristics.

All goods are produced in competitive industries, with constant returns to scale technologies, and with contemporaneous labor as the only input. The links between periods come from the fact that production is subject to economy-wide learning-by-doing: the unit labor requirement for production of <u>any</u> good by <u>any</u> firm in any period depends on the <u>entire economy's</u> cumulative experience in production of <u>all</u> goods in all previous periods. That is, learning displays complete spillovers among firms, and in addition may display spillovers among goods.

Let experience in any period be described by the state variable k, an index of "knowledge capital," taking values in the set K. The variable k may be a finite-dimensional vector, $k = (k_1,\ldots,k_n)$; an infinite-dimensional vector, $k = (k_1,k_2,\ldots)$; or a real-valued function, $k(\xi)$, $\xi \geq 0$.[2] In particular it may be the function describing cumulative experience, $k_t(s) = \Sigma_0^t x_\tau(s)$, $s \geq 0$. The law of motion for k will be discussed below.

Within each period the technology displays constant returns to scale. Specifically, given $k \in K$, the total labor required to produce any goods allocation $x \in X$ is $\int_0^\infty p(s,k)x(s)ds$. The function p will be restricted as follows.

<u>Assumption 2</u>: For each $k \in K$,

(i) $p(\cdot,k)$ is twice continuously differentiable and strictly increasing, with $p(0,k) = 0$;

(ii) $p(\cdot,k)$ is weakly concave on $[0,m)$ and strictly convex on (m,∞), for some $0 \leq m < \infty$; and

(iii) $\lim_{z\to\infty} p_1(z,k) = +\infty$.

Part (i) of this assumption says that within any period the unit cost of production increases smoothly with the quality of the good, with the worthless ($s = 0$) good costless to produce. Since $p(\cdot,k)$ and $q(\cdot)$ are both differentiable, with $x = -q'$, it then follows from an integration by parts that for any allocation x containing the characteristics q,

$$\int_0^\infty p(s,k)x(s)ds = \int_0^\infty p_1(s,k)q(s)ds.$$

Hence $p_1(\cdot,k)$ can be interpreted as the unit cost function for characteristics, in the sense that the cost of producing any goods allocation is simply the cost of producing the characteristics it contains. Part (ii) of the assumption then says that for fixed knowledge k, the unit cost curve for goods is either strictly convex or weakly concave-strictly convex. Hence the unit cost curve for characteristics is either strictly increasing or "single-troughed" (where the "trough" may be a "flat"). Part (iii) says that the unit cost curve for characteristics increases without bound as $z \to \infty$.

Competitive equilibrium prices and quantities are then determined as follows. At the beginning of period t, knowledge k_t is given. The assumptions of perfect competition and constant returns to scale then imply that all goods are priced at cost. (Since learning spills over completely and with no lag to other firms, it is not in the interest of any producer to suffer current losses in order to accelerate learning.) That is, with the price of labor normalized to unity, the function $p(\cdot,k_t)$ describes competitive equilibrium goods prices. Equilibrium quantities are then determined by the preferences of the representative consumer. The (as yet unspecified) law of motion for knowledge then determines knowledge in the subsequent period, k_{t+1}, as a function of k_t and x_t. Therefore, given initial knowledge k_0 in period 0, the equilibrium paths for knowledge, prices and output can be determined. The goal here is to find assumptions under which only a limited set of goods is produced in each period, and over time lower quality goods drop out of the produced set and higher quality goods enter. In the context of this model, the latter will be interpreted to mean that equilibrium quantities $\{x_t\}_{t=0}^{\infty}$ have the following features: in each period t the set of goods actually produced is an interval $[A_t,B_t]$, and that both $\{A_t\}$ and $\{B_t\}$ are increasing sequences.

First consider the determination of equilibrium quantities within any period. That is, consider a consumer with the preferences above and an endowment of labor $y > 0$, facing the prices $p(\cdot,k)$. His problem is:

$$(2) \qquad \max_{x \in X} \int_0^{\infty} u\left(\int_z^{\infty} x(s)ds\right)dz$$

$$\text{s.t.} \qquad \int_0^{\infty} p(s,k)x(s)ds - y \leq 0,$$

$$x(s) \geq 0, \qquad \text{all } s.$$

The solution to this problem is characterized in the following lemma.

<u>Lemma 1</u>: Let u and p satisfy Assumptions 1 and 2 respectively. Then for any $k \in K$, the solution x to (2) is unique, and has the following form:

$$x(s) \begin{cases} = 0 & s \in [0,\ A) \\ > 0 & s \in [A,B] \\ = 0 & s \in (B,\ \infty), \end{cases} \tag{3}$$

where

$$A = \max\ \{s \geq 0 |\ p(s,k) - sp_1(s,k) = 0\}, \tag{4}$$

and $B > A$. Moreover, x is continuous on $[A,B]$.

<u>Proof</u>: The problem in (2) is equivalent to:

$$\underset{q \epsilon Q}{\text{Max}} \int_0^{\infty} u(q(z))dz \tag{5}$$

$$\text{s.t.} \quad \int_0^{\infty} p_1(z,k)q(z)dz - y \leq 0, \tag{6}$$

$$q'(z) \leq 0, \quad \text{all } z. \tag{7}$$

The feasible set for this problem is convex, and under Assumption 1 the objective function is strictly concave. Hence the solution--if one exists--is unique, and satisfies the first order condition

$$\int_0^{s} u'[q(z)]dz - \lambda p(s,k) \leq 0, \quad \text{with equality if } q'(s) < 0, \quad \text{all } s. \tag{8}$$

First it will be shown that for any $\lambda > 0$, there is a unique function $\psi(z,\lambda)$ satisfying (7) - (8); and then that for an appropriate choice of λ, (6) also holds.

Define $A \geq 0$ by (4). If $p(\cdot,k)$ is strictly convex, then $A = 0$. If $p(\cdot,k)$ is concave-convex, then $A > 0$ is as shown in Figure 1. Note that in either case $p_1(\cdot,k)$ is strictly increasing on $[A,\infty)$, and $p(s,k) \geq sp_1(A,k)$, all s.

Fix $\lambda > 0$. If $u'(0) < \lambda p_1(A,k)$, let $\psi(z,\lambda) = 0$, all z. Clearly (7) and (8) hold. If $u'(0) \geq \lambda p_1(A,k)$, define $B \geq A$ by $u'(0) = \lambda p_1(B,k)$; it follows from parts (ii) and (iii) of Assumption 2 that B is well-defined. Then define $\psi(\cdot,\lambda)$ by

(9a) $$u'[\psi(z,\lambda)] = \lambda p_1(z,k), \qquad z \in [A,B];$$

(9b) $$\psi(z,\lambda) = \psi(A,\lambda), \qquad z \in [0,A);$$

(9c) $$\psi(z,\lambda) = 0, \qquad z \in (B,\infty);$$

as shown in Figure 2. Note that $\psi_1(z,\lambda) = 0$, for $z \in [0,A) \cup (B,\infty)$. Moreover, since both u' and $p_1(\cdot,k)$ are continuously differentiable, it follows from (9a) that $\psi(\cdot,\lambda)$ is continuously differentiable on (A,B), with $\psi_1(z,\lambda) = \lambda p_{11}(z,k)/u''[\psi(z,\lambda)]$. Since u is strictly concave and $p(\cdot,\lambda)$ is--on this region--strictly convex, it follows that $\psi_1(z,\lambda) < 0$, so that $\psi(\cdot,\lambda)$ satisfies (7).

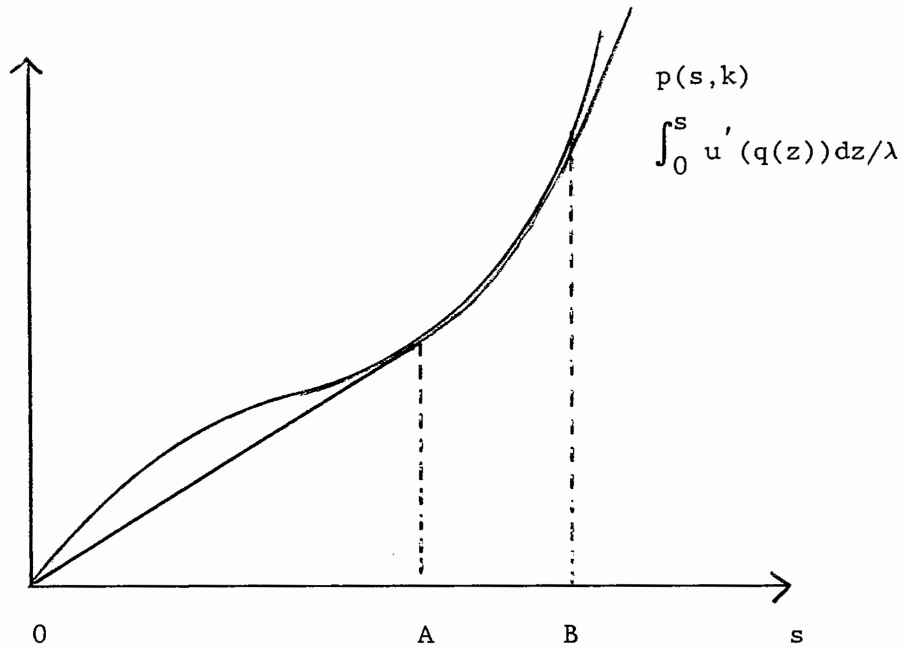

Figure 1

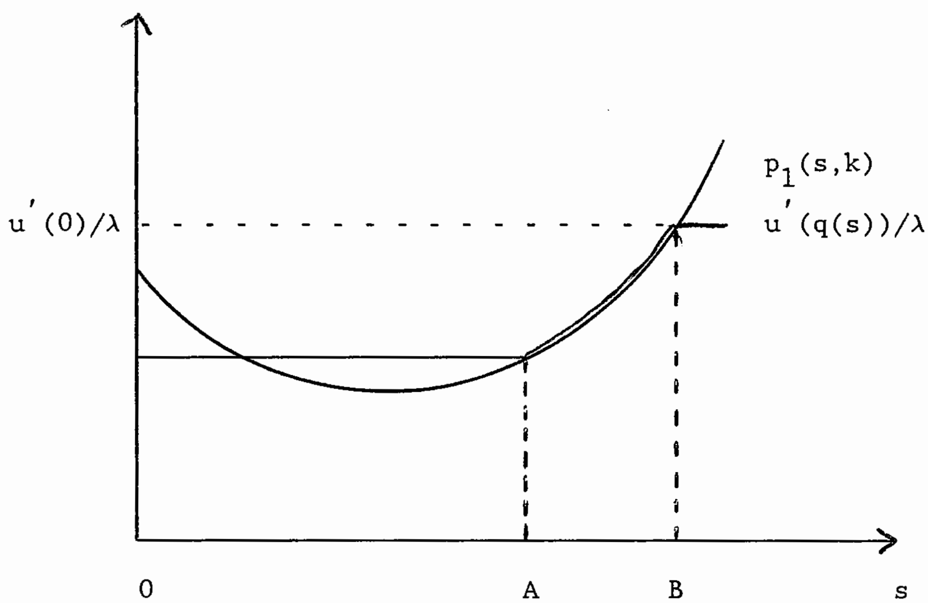

Figure 2

Next consider (8). Since $u'[\psi(A,\lambda)] = \lambda p_1(A,k)$, and $p(s,k) \geq s p_1(A,k)$, all s, it follows from (9b) that for $s \in [0,A)$,

$$\int_0^s u'[\psi(z,\lambda)]dz - \lambda p(s,k) \leq s\{u'[\psi(A,\lambda)] - \lambda p_1(A,k)\} = 0.$$

Hence (8) holds for $s \in [0,A)$. For $s \in [A,B]$,

$$\int_0^s u'[\psi(z,\lambda)]dz - \lambda p(s,k)$$

$$= Au'[\psi(A,\lambda)] + \int_A^s u'[\psi(z,\lambda)]dz - \lambda[p(A,k) + \int_A^s p_1(z,k)dz]$$

$$= A\{u'[\psi(A,\lambda)] - \lambda p_1(A,k)]\} + \int_A^s \{u'[\psi(z,\lambda)] - \lambda p_1(z,k)\}dz$$

$$= 0,$$

where the second line uses (9b), the third uses (4), and the last uses (9a). Hence (8) also holds for $s \in [A,B]$. Finally, using this result and (9c), it follows that for $s \in (B,+\infty)$,

$$\int_0^s u'[\psi(z,\lambda)]dz - \lambda p(s,k) = \int_B^s [u'(0) - \lambda p_1(z,k)]dz.$$

From the definition of B and the fact that $p(\cdot,k)$ is strictly convex in this region, it follows the integrand on the right is negative, so that (8) holds for $s \in (B,+\infty)$.

Next note that for each z, $\psi(z,\cdot)$ is monotone in λ--strictly monotone for $z \in [0,B]$--with $\lim_{\lambda\to\infty} \psi(z,\lambda) = 0$ and $\lim_{\lambda\to 0} \psi(z,\lambda) = +\infty$. Hence for a unique value λ^*, $\int_0^\infty p_1(z,k)\psi(z,\lambda^*)dz = y$, so that $q(z) = \psi(z,\lambda^*)$, all z, is a solution to (6) - (8). Moreover, it is clear that if $y > 0$, then $q \neq \theta$, so that $A < B$. Taking $q'(A)$ and $q'(B)$ to be the right and left derivatives respectively, it follows that $x = -q'$ is the unique solution to (2), and has the properties claimed. [ ]

Lemma 1 shows that within each period, the set of goods produced in competitive equilibrium is a bounded interval [A,B]. The lower boundary A of the produced set is zero if $p(\cdot,k)$ is strictly convex, and is determined by the tangency condition illustrated in Figure 1 otherwise. Thus, it is in either case determined by properties of the unit cost function $p(\cdot,k)$ alone. The upper bound B of the produced set depends, in either case, on properties of the preferences and the value of the labor endowment, as well on the cost function.

Lemma 1 also shows that any concave-convex unit cost function $p(\cdot,k)$ can be replaced with its greatest convexification (the greatest weakly convex function that is everywhere equal to or less that $p(\cdot,k)$), without changing the solution to the consumer's problem. To see this, refer again to Figure 1. Suppose that the cost function pictured there is replaced by the function (not pictured) that is equal to zero at zero; is equal to $p(\cdot,k)$ on $[A,\infty)$; and is linear on the interval $[0,A]$. Clearly, at these prices the consumer cannot do better than the allocation chosen at prices $p(\cdot,k)$.

Characterizing the evolution over time of a competitive economy's production of goods, requires characterizing the behavior of the set $[A,B]$ as knowledge increases. To do this, another assumption will be needed.

Assumption 3: For any $k, \hat{k} \in K$ with $k < \hat{k}$:

(i) for $A, \hat{A}$ defined in (4), $A < \hat{A}$; and

(ii) $p_1(z,\hat{k})/p_1(z,k)$ is not greater than unity and weakly decreasing in z, for $z \in [0,A]$; and is less than unity and strictly decreasing in z, for $z \in (A,\infty)$.

Part (i) of this assumption ensures that the lower bound of the produced set shifts to the right as knowledge increases. (Note that $p(\cdot,\hat{k})$ cannot be strictly convex on all of R_+ if $\hat{k} > k$, since this would imply $\hat{A} = 0$.) Part (ii) ensures that an increase in knowledge reduces the cost of every characteristic (and hence of every good), and has a relatively greater effect on the costs of higher-index characteristics.

The next lemma describes how the set of produced goods changes as knowledge increases.

Lemma 2: Let u satisfy Assumption 1, and let p satisfy Assumptions 2 and 3. Let $k, \hat{k} \in K$, with $\hat{k} > k$, and let (x,λ) and $(\hat{x},\hat{\lambda})$ be solutions of (2), for k and $\hat{k}$ respectively. Let $[A,B]$ and $[\hat{A},\hat{B}]$ be the intervals on which x and $\hat{x}$ respectively are positive. Then $A < \hat{A}$ and $B < \hat{B}$.

Proof: The first claim follows trivially from part (i) of Assumption 3.

Consider B and $\hat{B}$. Let q and $\hat{q}$ be the allocations of characteristics corresponding to x and $\hat{x}$ respectively. First it will be shown that

$$(10) \qquad p_1(\hat{B},\hat{k})/p_1(\hat{B},k) < \lambda/\hat{\lambda}.$$

Suppose the contrary. Then it follows from part (ii) of Assumption 3 that $p_1(z,\hat{k})/p_1(z,k) \geq \lambda/\hat{\lambda}$, all $z \in [0,\hat{B}]$. Since $\lambda p_1(z,k) \geq u'(q(z))$, all $z \in [A,+\infty)$, and $\hat{A} > A$, it then follows that

$$u'[\hat{q}(z)] = \hat{\lambda} p_1(z,\hat{k}) \geq \lambda p_1(z,k) \geq u'[q(z)], \qquad \text{all } z \in [\hat{A},\hat{B}].$$

This in turn implies that $\hat{q}(z) \leq q(z)$, all $z \in [\hat{A},\hat{B}]$, and hence that $\hat{q}(z) = \hat{q}(\hat{A}) \leq q(\hat{A}) \leq q(z)$, all $z \in [0,\hat{A})$. Since $p_1(z,\hat{k}) \leq p_1(z,k)$, all z, with strict inequality on $(A,+\infty)$, it then follows that the budget constraint (6) cannot hold for both situations. Hence (10) holds, and it follows that

$$\lambda p_1(B,k) = u'(0) = \hat{\lambda} p_1(\hat{B},\hat{k}) < \lambda p_1(\hat{B},k) .$$

Since $p_1(\cdot,k)$ is strictly increasing in z for $z \in (A,+\infty)$, it then follows that $B < \hat{B}$. []

Lemma 2 shows that under Assumptions 1 - 3, the set of produced goods shifts to the right as knowledge grows. Specifically, greater knowledge implies that lower-index goods drop out of the produced set and higher-index goods enter.

Finally, to characterize the competitive equilibrium of a multiperiod economy, the dynamics of knowledge accumulation must be specified. Let $h: K \times X \to K$ be the law of motion for knowledge, $k_{t+1} = h(k_t, x_t)$. The only restriction on the function h that will be needed is the following.

<u>Assumption 4</u>: For all $k \in K$ and all $x \in X$, $h(k,x) \geq k$, with equality only if $x = \theta$.[3]

<u>Theorem 1</u>: Let u satisfy Assumption 1, let p satisfy Assumptions 2 and 3, let h satisfy Assumption 4, and let $k_0 \in K$ be given. Then the unique competitive equilibrium sequence of prices, allocations, and knowledge, $\{p(\cdot,k_t),\ x_t(\cdot),\ k_t\}_{t=0}^{\infty}$, for an economy beginning with knowledge k_0 in period 0, has the following properties. In each period $t = 0,1,\ldots,$ goods prices $p(s,k_t)$ are strictly increasing in s; only goods in a finite range $[A_t,B_t]$ are produced; and the allocation x_t is continuous on $[A_t,B_t]$. Over time, the sequence of price functions $\{p(\cdot,k_t)\}$ is strictly decreasing; and the sequences $\{A_t\}$, $\{B_t\}$ and $\{k_t\}$ are all strictly increasing.

<u>Proof</u>: All of the claims follow directly from Lemmas 1 and 2, and Assumption 4. []

Using the model just described, it is possible (easy, in fact) to measure the rate of growth in real output, even though new goods are being produced every period. The reason is that unproduced goods in any period have a well-defined price: their unit cost of production. Hence it is quite simple to compare the value output in periods t and $t + 1$, both evaluated at period t prices. Doing so gives a conventional measure of period-to-period growth in real GNP. The next theorem shows that the rate of growth, so measured, is always positive.

Theorem 2: Under the assumptions of Theorem 1,

$$\int_0^\infty p(s,k_t)x_{t+1}(s)ds \;>\; \int_0^\infty p(s,k_t)x_t(s)ds, \quad \text{all } t.$$

Proof: It follows immediately from (2) and Assumption 3 that

$$\int_0^\infty p(s,k_t)x_{t+1}(s)ds \;>\; \int_0^\infty p(s,k_{t+1})x_{t+1}(s)ds$$

$$= \; y \; = \; \int_0^\infty p(s,k_t)x_t(s)ds, \quad \text{all } t. \qquad []$$

The rate of growth may be increasing, decreasing or constant over time, or display more complicated behavior, depending on the particular assumptions made about the functions u, p and h.[4]

3. Incorporating a "Traditional" Sector

Suppose that the economy has, in addition to the "learning" sector described above, a "traditional" sector in which there is no learning. For simplicity call these sectors manufacturing and agriculture. Take preferences of the representative consumer to be

$$(11) \qquad V[a, \int_0^\infty u(q(z))dz],$$

where a is the quantity of agricultural goods consumed, and V is continuous, strictly increasing and strictly concave. Without loss of generality, assume that units of agricultural goods have been defined so that one unit of labor produces one unit of agricultural goods. Then the technology is

$$a + \int_0^\infty p_1(z,k)q(z)dz - y \leq 0. \tag{12}$$

The assumptions of perfect competition and constant returns to scale imply that, with the price of labor normalized to unity, the competitive equilibrium price of agricultural goods is unity and the prices for manufactured goods are given by $p(\cdot,k)$. Competitive equilibrium quantities are given by the solution to the consumer's problem: maximize (11) subject to (12) and the constraints $a \geq 0$, and $q'(z) \leq 0$, all z.

First it will be shown that there may be equilibrium paths that display no growth, and that these paths are unstable in the sense that a (large enough) perturbation in the initial state sets the economy onto a path of sustained growth. For any $U > 0$ and $k > 0$, define $E(U,k)$ to be the expenditure function for manufactured goods:

$$E(U,k) = \min_{q \in Q} \int_0^\infty p_1(z,k)q(z)dz,$$

$$\text{s.t.} \quad \int_0^\infty u(q(z))dz \geq U.$$

With E so defined, the share of total manhours devoted to each sector is then given by the solution to:

$$\max_{a,U\geq 0} \; V(a,U), \qquad \text{s.t.} \quad a + E(U,k) - y \leq 0.$$

If the preferences and technology are such that $V_1(y,0) \geq V_2(y,0)/E_1(0,\theta)$, then in equilibrium an economy with no experience in manufacturing $(k_0 = \theta)$, produces no manufactured goods $(U = 0)$. Such an economy remains stagnant forever $(k_t = \theta,$ all $t)$. However, if this economy somehow acquires enough experience to reverse that inequality, it then produces manufactured goods $(U > 0)$, so that experience grows $(k_{t+1} > k_t)$. The same is then true in every subsequent period as well. Thus, there may be a dynamic competitive equilibrium that is unstable against (large enough) perturbations in the initial state.

Next consider the change over time in hours devoted to agriculture. It follows from Assumption 3 that if $\{k_t\}$ is strictly increasing, then the prices of all manufactured goods fall over time. This has two effects. The change in relative prices tends to decrease consumption of agricultural goods, but the increase in real income tends--assuming that agricultural goods are "normal"--to increase the quantity consumed. The net effect is the sum of these substitution and income effects, and either may predominate. This statement can be made precise by studying the market and compensated demand functions for agricultural goods. Since the prices of manufactured goods depend on knowledge, in this context both demand functions will have k as an argument instead of the usual vector of goods prices. For simplicity let k be a scalar.

It is useful first to define the indirect utility function $\hat{U}$ by

$$\hat{U}(e,k) = \max_{q\epsilon Q} \int_0^\infty u(q(z))dz \tag{13}$$

$$\text{s.t.} \quad \int_0^\infty p_1(z,k)q(z)dz - e \leq 0.$$

Call $\hat{U}(e,k)$ the "felicity" attainable from manufactured goods when total expenditure on those goods is e and prices are $p(\cdot,k)$. It is immediate that since u is strictly increasing and strictly concave, $\hat{U}$ is strictly increasing and strictly concave in its first argument.

With $\hat{U}$ so defined, consider the two problems

$$\max_a V[a, \hat{U}(y - a, k)], \tag{14}$$

and

$$\min_{a,y} y \quad \text{s.t.} \quad V[a, \hat{U}(y - a, k)] = \bar{v}. \tag{15}$$

Since V is strictly concave and $\hat{U}$ is strictly concave in its first argument, both have unique solutions; call them $\alpha(k,y)$ and $[\alpha^c(k,\bar{v}), y^c(k,\bar{v})]$. The functions α and α^c are the market and compensated demand functions for agricultural goods.

Assume that (14) and (15) have interior solutions, $0 < a < y$. Then α and α^c are characterized by the appropriate first order conditions, and, in the case of α^c, by the utility constraint. Differentiating these conditions one finds that

$$\frac{\partial \alpha}{\partial k} = (\hat{U}_2/\hat{U}_1)\frac{\partial \alpha}{\partial y} + \frac{\partial \alpha^c}{\partial k}. \tag{16}$$

Thus, the effect on the demand for agricultural goods of a change in knowledge (and hence a change in manufactured goods prices) can be decomposed into an income effect and a substitution effect. It is tedious but straightforward to show that, as usual, the former is of ambiguous sign and the latter is negative.

5. <u>Conclusions</u>

Several specific features of the technology and preferences are important for obtaining the results in Theorem 1. First, it is important that learning display spillovers among goods. Otherwise, learning simply reinforces existing patterns of production, which works against both the introduction of new goods and the discontinuation of old ones. Krugman [1985] has explored such a technology, with a fixed, bounded set of goods, in the context of international trade. The conclusion there is that once an international pattern of specialization is established, it persists. Because each country learns only about the goods it has produced itself, the initial pattern of comparative advantage is simply exacerbated as production occurs. Similar conclusions can be expected in a closed economy.

Second, it is important that "forward" spillovers be stronger than "backward" spillovers. This is the basic content of Assumption 3, which is similar in spirit to the restriction made in Wan's [1975] model of learning. An assumption of this sort is needed to ensure that new goods are introduced.

Finally, the characteristics model of preferences provides an analytically tractable framework for introducing interactions among goods. Specifically, it allows one to retain the simplicity of additive separability, without some of its drawbacks. Preferences that are additively separable over <u>goods</u> are <u>not</u> particularly well suited to obtaining the type of results in Theorem 1. The reason is that they imply a preference for diversity in the goods consumed, which is then a strong force against abandoning the production of any good. Income effects and/or changes in relative costs can offset this force, but joint restrictions on the technology and preferences are then needed to ensure that the latter are strong enough to produce the desired conclusions.

An unusual feature of the model above is the absence of physical capital. This implies, of course, that the model can say nothing about long-run rates of investment, rates of return on capital, etc. However, physical capital could be incorporated in a variey of ways. For example, one could add a capital-goods sector that produces a homogeneous output with an unchanging technology. The output of this sector would be combined with labor, and the resulting "aggregate physical input" used as a factor of production in both the consumption-goods and capital-goods sectors. One would then be able to study questions about long-run rate of investment, etc. However, it seems unlikely that the results in Theorems 1 and 2 would be changed. Thus, the omission of physical capital limits the scope of the model, but seems unlikely to change the basic conclusions.

Research and development, also absent here, provides another source for sustained growth through the introduction of new goods. However, R & D could, at least in principle, also be incorporated. The results in Theorems 1 and 2 will hold whenever preferences and unit costs satisfy Assumptions 1 and 2, and one or more factors cause the unit cost function to change over time as described in Assumption 3. The factor affecting unit costs might be R & D or firm-specific learning-by-doing, instead of or in addition to the economy-wide learning-by-doing described here. However, the imperfectly competitive markets and dynamic incentive problems that R & D or firm-specific learning entails will make the model very much harder to analyze.

Notice, too, that in some situations R & D and learning-by-doing are hard to distinguish, as in Wan [1975]. It is <u>not</u> accurate simply to view improvements in technology as attributable to R & D if they involve a cost, and to learning-by-doing if they do not. In a learning-by-doing model the relevant cost is an opportunity cost. It is therefore a little less obvious, but certainly no less real. The model above is typical in this respect. The agents there face a tradeoff each period between current utility and the benfits of future cost reduction. Current production can serve either purpose, or both. From a firm's point of view, the opportunity cost of faster learning is lower current profits. Hence for firms in competitive markets, the choice is quite simple. Since future cost reduction is a pure public good, while the costs are completely internal to the firm, the benefits of learning receive no weight in any firm's production decisions.[5]

Finally, notice that the model above might also be viewed as representing a sector--food, clothing, transportation, etc., with an entire economy then composed of several such sectors, as in Clemhout and Wan [1970]. Would such a multidimensional extension display the same qualitative properties? It is difficult to say. The one-dimensional model here has the property that goods that are close in terms of consumption are also close in terms of production requirements. A multidimensional model would make such an assumption more problematic.

Footnotes

[1] An exception to this generalization is the model of research and development introduced in Judd [1985]. However, that model is an explanation of product differentiation; it does not yield sustained growth in the long run. Schmitz [1986] looks at a modified version of Judd's model and studies optimal long-run product development. Although product development does, in this case, proceed without bound, it is not clear whether his results can in any sense be interpreted as competitive equilibrium outcomes.

[2] In general, K may be any set with a relationship "$\geq$" satisfying:

i. $k \geq k$, all $k \epsilon K$, (reflexive);

ii. $k_A \geq k_B$ and $k_B \geq k_C$ implies $k_A \geq k_C$, all k_A, k_B, $k_C \epsilon K$, (transitive).

The relationship need not be complete. That is, there may be $k, \hat{k} \epsilon K$ such that $k \ngeq \hat{k}$ and $\hat{k} \ngeq k$.

[3] It would seem reasonable to require that h be increasing in x, for each fixed k, but this assumption is not needed for Theorem 1. It would be needed to get sensible results in an analysis of <u>optimal</u> allocations, not discussed here.

[4] An example in which the economy converges asymptotically to a constant rate of growth is available upon request from the author. The key features of this example are that experience is one-dimensional, and additional restrictions are imposed on the cost function and the law of motion for

knowledge. These assumptions make costs and learning stationary when scaled to an appropriate (common) point in characteristic space.

[5] It follows, of course, that the competitive equilibrium is, in general, not Pareto-efficient. The representative agent's total discounted utility would be increased if production in each period were distorted, at least a little, toward the production of goods that resulted in more learning.

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