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The Dissipation of Profits by Brand Name Investment and Entry When Price Guarantees Quality*

by

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1. Introduction

The theory of "quality guaranteeing prices" as first described by Klein and Leffler [1981] has proven to be a very useful concept in understanding the role that prices may play as signals of quality in markets where consumers cannot easily evaluate quality prior to purchase.[1] The basic idea is that firms' only incentive to provide high quality goods in such a market is the desire to induce repeat sales. However, the prospect of inducing repeat sales is only attractive if price exceeds average salvageable costs of production. Therefore price must exceed average salvageable production costs in order for high quality to be produced.

In markets where sunk costs are high this constraint will not bind; in the competitive zero profits equilibrium, price will be well above average salvageable costs in order that sunk costs be covered. However, in markets where sunk costs are not so large the constraint that price must exceed average salvageable costs by some given amount will bind -- i.e. -- firms producing at efficient size will earn profits by charging the quality guaranteeing price. Given that price cannot be competed lower, (consumers correctly predict that firms charging lower prices will be "fly-by-night" operators) an interesting question arises concerning how these profits will be dissipated. Two theories have been suggested.

Klein and Leffler [1981] assume that firms will operate at efficient scale in equilibrium - i.e. - at the bottom of their U-shaped average cost curves. They then predict that excess profits will be dissipated by investment in non salvageable selling costs[2] such as elaborate firm logos, deluxe storefronts and selling areas, promotional activity which familiarizes large numbers of people with the product or advertising which creates long-term demand for the product. They call these investments brand name capital investments.

Consumers directly value brand name investments. This is why firms are able to compete for customers by investing in brand name capital.[3] However, the equilibrium has the property that firms overinvest in brand name capital - i.e. - consumers would prefer that firms lower both brand name investments and price. However this cannot occur because of the role that price plays in guaranteeing quality. Shapiro [1983] has constructed an elegant version of this first theory where the sunk investment cost consist of producing high quality at low prices for an initial period until consumers become aware of the firm. He also assumes that firms operate at efficient scale.

Allen [1984], on the other hand, has correctly pointed out that these profits could equally well be dissipated by entry of more firms. Under the assumption that brand name capital investment as modelled by Klein and Leffler is not possible, Allen constructs an equilibrium where entry occurs until profits generated by the quality guaranteeing price are just sufficient to cover the productive sunk costs associated with capital equipment.

The equilibria generated by these two approaches are quite different. In the Klein and Leffler approach the market appears competitive in the sense that firms operate at the bottom of their U-shaped average cost curves and are on their marginal cost curve -- i.e. -- they do not wish to sell any more at the market price. However, there is excessive investment in non-salvageable selling costs. In the Allen approach the market appears monopolistically competitive in the sense that firms operate at less than efficient scale and are off their marginal cost curve - i.e. - they all wish to sell more at the market price. However, there is no excessive investment in non-salvageable selling costs.

By assuming that only one of the two possible types of profit dissipation is possible, the above-mentioned authors were able to construct elegant clear models describing the nature of this dissipation. However, in most markets both types of dissipation are a priori possible. This raises the question as to what form we would expect rent dissipation to take in such markets and whether measurable characteristics can be identified which influence the nature of this dissipation. Will markets where quality is difficult to verify prior to purchase generally be characterized by excess capacity and "normal" levels of brand name capital investment or by full capacity and excessive levels of brand name capital investment or by some combination of both and why?

This paper addresses the above question by using a variant of the models developed to investigate quality competition among firms where price and entry are regulated.[4] The basic difference between this paper's model and those developed in the regulation context is that in this paper's model only price is fixed, the "no entry" condition of the regulation models is replaced by a zero profits condition. This paper also proves more complete existence theorems which are applicable to the above cited work as well.[5]

Stigler [1968] has analyzed a similar problem to that considered by the above cited regulation models in the context of a monopolistically competitive market. He investigates whether non-price competition will dissipate profits when price and the number of firms are fixed.[6] Once again, the basic difference between Stigler's analysis and that of this paper, is that in this paper only price is fixed and entry is allowed. Then the question becomes whether entry or non-price competition will dissipate profits given that they will be dissipated in equilibrium.

Firms are modelled as competing for customers by investing in brand name capital. For any given number of firms the level of brand name investment is determined by a symmetric Nash equilibrium. Entry or exit of firms occurs so that profits are zero. In general, profit dissipation will occur both through entry and brand name investment -- i.e. -- firms will operate at less than full capacity and also engage in excessive brand name investment. A key determinant of the extent to which the solution is closer to that suggested by Klein and Leffler or by Allen is the elasticity of demand with respect to investments in brand name capital. As this elasticity grows larger, Nash equilibrium levels of brand name investment rise for any number of firms and this results in more profit dissipation occurring through brand name investments.

Therefore, in markets where customer loyalties are not very firm and small increases in brand name investment can yield large shifts in market share, we would expect profit dissipation to take the form primarily of brand name investment. However, when market share is less responsive to brand name investment, profit dissipation will occur more through excess capacity. The predictions of Klein and Leffler and Allen can thus be viewed as describing the two extreme cases for the continuum of equilibria which can actually occur.

2. The Model

All firms are identical. They can produce x units of the good according to the cost function.

(2.1) $cx + S$

up to a capacity of $\bar{x}$. Assume that c and S are positive; S represents an annualized sunk or non-salvageable cost. Firms can also choose a level of brand name capital investment. Let B represent the annualized value of this; B must be non-negative.

The standard argument for the existence of a quality guaranteeing price will not be repeated here. Rather, it will simply be assumed that there is some premium, $m > 0$, such that consumers (correctly) expect firms to become fly-by-night operators if a premium of less than m is charged.[7] It will also be assumed that firms would earn positive profits at this price if they operated at full capacity and invested in no brand name capital -- i.e. --

$$(2.2) \qquad \bar{x}m > S$$

Finally recall from the introduction that Klein and Leffler [1981] argue that firms will invest in brand name capital that is of some value to consumers but that at the margin the cost of capital will exceed its value to consumers. The simplest manner to formally investigate this is to assume that all brand name capital costs more to produce than consumers are willing to pay for it. (Alternatively, investments in brand name capital which consumers value at more than cost can be viewed as being already included in the sunk cost term, S). The implication of this, and assumption (2.2), is that price will equal c + m in equilibrium. At prices above c + m firms will earn positive profits unless they invest in brand name capital. However, if a firm changes a price greater than c + m and invests in brand name capital, it can increase its profits by reducing both price and brand name investment since consumers are willing to pay less than cost for brand name advertising.

This argument is formalized in Appendix A. It is shown that if brand name capital costs more to produce than consumers are willing to pay for it, the only symmetric Nash equilibrium in price and brand name investment involves every firm charging $c + m$. This argument reduces the model to one where firms choose levels of brand name investment and price is fixed at $c + m$.

In order to analyze this model the manner in which demand is determined as a function of brand name investment (given price is $c + m$) must be specified. In Section 3 a fairly specific functional form is analyzed. It is assumed that

$$x_i = s_i(B_1, \ldots, B_n)\, Q \tag{2.3}$$

where

$$s_i(B_1, \ldots, B_n) = \frac{B_i^e}{\sum_{j=1}^{n} B_j^e}\,. \tag{2.4}$$

where subscript i denotes firm i and e and Q are positive constants. Aggregate demand, Q, remains constant and firm i's share, s_i, increases as it invests more but decreases as other firms invest more.

The number e is a measure of the responsiveness of a firm's market share (and thus its demand) to changes in its brand name investment. To see this, consider $\partial s_i/\partial B_i$ evaluated where $B_j = B$ for every j; this will be the expression of interest in the subsequent analysis because symmetric equilibria will be calculated.

(2.5) $$\frac{\partial s_i}{\partial B_i}(B, \ldots, B) = \frac{e(n-1)}{B n^2}$$

This increases with e. Note also that e is closely related to the elasticity of demand with respect to brand name investment.

(2.6) $$\frac{\partial x_i}{\partial B_i}\frac{B_i}{x_i} = \frac{n-1}{n} e$$

The major role played by the assumption that demand satisfies the particular functional form specified in (2.3) and (2.4) is to guarantee the existence of a unique equilibrium and to guarantee that the first order conditions for firms actually are sufficient for a global maximum. However the major point of this paper is not to provide a careful existence proof but to argue that the responsiveness of demand to brand name investment determines the extent to which profits are dissipated by brand anme investment _vs._ entry. Section 4 generalizes this result to a large class of demand curves where e is a parameter such that the derivative of a firm's demand with respect to its brand name investment increases in e. (As shown by (2.5), the demand curves defined by (2.3) and (2.4) obey this property.)

The most obvious case in which to apply this theory is that of advertising. It is well-documented that advertising often has the required properties of a brand-name investment; current advertising can generate sales increases in a number of subsequent periods but has no salvage value if the firm goes out of business.[8] To apply this theory it is necessary to be able to tell if a given market has a high value of e or not. Of course if the data exists the elasticity of demand with respect to advertising can simply be directly estimated. A large literature exists where this is done. See Aaker and Carmen [1982] for a very complete review.[9] Unfortunately (publically available) data often does not exist, and a number of estimation difficulties also arise.[10] Other studies have attempted to finesse this issue by simply identifying more easily observable characteristics of industries which are correlated with high levels of advertising. After identifying a positive correlation, if plausible reasons exist why the characteristic might make the elasticity of demand with respect to advertising larger (which in turn would cause larger amounts of advertising) and no plausible reasons can be found for the causality to be reversed (i.e. -- for advertising to generate the characteristic), then a conclusion that the characteristic causes a higher elasticity of demand seems warranted.[11] Borden's [1942] treatise on advertising represents an early attempt to do this through detailed qualitative analysis of a number of industries. Farris and Albion [1981] provide a very complete review of a large number of more modern econometric studies. An excellent study not review by Farris and Albion is by Porter [1978]. Most modern textbooks on marketing and advertising also describe characteristics of industries which result in high elasticities of demand with respect to advertising. See Buzzel, et. al. [1972] for example. Finally, Telser [1978] outlines an information-based theory of advertising which explains why many of the characteristics identified in the above studies might cause the elasticity of demand with respect to advertising to be higher, although it contains no empirical work, itself.

Based on the above references the following market characteristics seem to cause higher values of e. First, a number of factors could result in consumers having less information about products. For example, if the market is growing rapidly this implies that large numbers of new (and less informed) customers are entering the market. If there is a high rate of innovation or products are redesigned frequently consumers will be less informed about current offerings. Finally, if there is a rapid turnover of customers (as is the case for baby-products, for example) there will be a constant inflow of new, uninformed consumers. The return to advertising information about products should be higher when there are more uninformed consumers. Second, if products in the market are highly differentiated informative advertising may be more efficacious in attracting customers because it can stress these differences. Third, the marginal cost of reaching consumers may be lower in some markets. This will increase the responsiveness of demand per dollar of advertising expenditures. For example if a product is widely used by most consumers on a regular basis general mass media provides an effective method of contacting potential customers. If only a small fraction of the population are regular customers the issue becomes whether specialized media exist which allow advertising to be targeted. The fourth fact differs from the preceding three in that it is based not on the idea that advertising conveys information but that advertising can change the nature of the product in peoples minds. It seems clear that some products are more suitable for image advertising than others. Scherer [1980] refers to this as "status considerastions"[12] while Borden terms it "emotional buying motives."[13] The problem with this explanation is that it appears to be difficult to identify objectively measurable product characteristics which determine the susceptibility of consumers to image advertising. Thus it may be difficult to predict which types of markets will exhibit large values of e due to image advertising. There appears to be less consensus on the effects of other industry and product characteristics on the value of e, such as frequency of purchase, number of brands, market size, durability of goods or the share of a consumer's income spent on the product.

Having described the structure of the market's cost and demand curves, an equilibrium can now be defined. In order to do this some further notation will be useful. For any firm size, x, the number of firms required to satisfy demand is given by Q/x. Since firms' maximum capacity is $\bar{x}$, the smallest number of firms possible is given by

$$(2.7) \qquad \bar{n} = \frac{Q}{\bar{x}} .$$

It will be assumed that $\bar{n} \geqslant 2$ - i.e. - demand is large enough to require at least two firms in the market. The largest number of firms for which it is possible for profits to be non-negative is

$$(2.8) \qquad \bar{\bar{n}} = \frac{Qm}{S} .$$

If there are $\bar{\bar{n}}$ firms, they are just large enough to cover their sunk costs S; thus zero profits are earned if B = 0. For any number of firms in the interval $[\bar{n}, \bar{\bar{n}}]$, the amount of brand name investment per firm which disspiates all profits is given by

$$(2.9) \qquad B = \frac{Q}{n} m - S$$

Let $\phi(n)$ denote the function in (2.9).

Only symmetric equilibra will be considered. Therefore an equilibrium is an ordered pair (B,n) such that (i) each firm is operating at less than or equal to capacity by producing an equal share of demand - i.e. - $n \geqslant \bar{n}$, (ii) there is an n firm symmetric Nash equilibrium in brand name investment when all firms invest at a level B, and (iii) all firms earn zero profits.

To formalize this, it is convenient to ignore the restriction that n must be an integer. To do this the demand functions in (2.3) and (2.4) will be redefined. Let $r(B, \hat{B}, n)$ denote a particular firm's demand when it invests in B, all other firms invest in $\hat{B}$ and there are n firms. This is given by

$$(2.10) \qquad r(B, \hat{B}, n) = \frac{B^e}{B^e + (n-1)\hat{B}^e} Q$$

Let $R(B,\hat{B},n)$ denote the minimum of the firm's demand and $\bar{x}$. This is the actual amount that the firm will sell.

$$(2.11) \qquad R(B,\hat{B}\ n) = \min \{\bar{x},\ r(B,\hat{B},n)\}$$

An equilibrium can now be defined.

<u>Definition</u>: An equilibrium is a pair of numbers (B,n) such that

$$(2.12) \qquad n \geq \bar{n}$$

$$(2.13) \qquad B \in \underset{B^* \geq 0}{\operatorname{argmax}}\ R(B^*,B,n)\ m - B^*$$

and

$$(2.14) \qquad \frac{Qm}{n} - B - S = 0$$

3. Equilibrium

Theorem 1 describes the unique equilibrium in this model.

Theorem 1:

Fix the value $e > 0$. Then there is a unique equilibrium, $(n^*(e), B^*(e))$. Let $\hat{n}(e)$ denote the unique positive solution to

$$(3.3) \qquad Qm\left[\frac{n(1-e) + e}{n^2}\right] - S = 0$$

The value $\hat{n}(e)$ is in the interval $(0,\bar{\bar{n}})$. Then the equilibrium is determined by[14]

$$(3.4) \qquad n^*(e) = \max\,(\hat{n}(e), \bar{n})$$

$$(3.5) \qquad B^*(e) = \phi(n^*(e))$$

proof:

See Appendix B.

QED.

Theorem 2 describes the function $\hat{n}(e)$. Its behavior determines the comparative statics of the equilibrium with respect to e.

Theorem 2:

(i) $\hat{n}'(e) < 0$

(ii) $\hat{n}(0) = \bar{\bar{n}}$

(iii) There exists a value $\bar{e}$ such that $\hat{n}(\bar{e}) = \bar{n}$.

proof:

Straightforward calculus applied to (3.3) yields the results.

QED.

Therefore n^* falls from $\bar{\bar{n}}$ to $\bar{n}$ over the interval $[0,\bar{e}]$ and then equals $\bar{n}$ for larger values of e. Similarly B^* rises from 0 to $Q\bar{x} - S$ over the interval $[0,\bar{e}]$ and then equals $Q\bar{x} - S$ for larger values of e.

Recall that e is a measure of the responsiveness of demand to brand name investment. The comparative statics follow intuitively from this. As demand grows more responsive to investment the equilibrium amount of brand name investment per firm grows. Thus the firm size which dissipates profits is larger and there are fewer firms. As e gets close to zero, equilibrium brand name investment converges to zero as well. Because of the demand structure assumed in (2.3) and (2.4) the marginal return to investment for any given firm is infinite if all the other firms choose zero levels of investment. Thus the symmetric equilibrium always involves a positive amount of investment for $e > 0$. As e grows larger, the equilibrium amount of brand name investment eventually grows large enough to dissipate all profits even if firms operate at full capacity.

Klein and Leffler predict that firms will operate at full capacity and brand name investment will dissipate profits. Thus they predict that the equilibrium will be $(\bar{n}, \varphi(\bar{n}))$. Allen predicts that entry will dissipate all profits. This is given by $(\bar{\bar{n}}, \varphi(\bar{\bar{n}}))$. The above analysis shows that Allen's solution occurs for $e = 0$. As e grows, profits begin to be dissipated by brand name investment, yielding a solution intermediate between that of Allen and Klein and Leffler. For values of $e \geq \bar{e}$, the Klein and Leffler equilibrium occurs.

The fact that the Allen solution occurs only in the limit as e converges to zero while the Klein and Leffler solution occurs over a large interval of values for e should not be treated as significant. This is a byproduct of the particular functional forms used in the analysis. For example, if brand name investment by any firm must exceed some threshold level before it has any effect (as is often thought to be the case for advertising) then the Allen solution could occur for an interval of values for e. Similarly, if cost curves were U-shaped, the Klein-Leffler solution will generally only occur in the limit as e converges to ∞.[15]

The robust point of this analysis is that brand-name investments will not simply grow automatically to dissipate profits in a world where price must exceed non-salvageable costs in order to guarantee quality. Levels of brand name investment are determined as the outcome of strategic interaction between firms as they compete for customers. As the elasticity of demand with respect to investments in brand name capital grows larger, equilibrium levels of investment will be dissipated to a greater extent by brand name investments.

4. Generalization and Demand Functions

The major role played by the assumption that demand satisfies the particular functional form specified in (2.3) and (2.4) is to guarantee the existence of a unique equilibrium and to guarantee that the first-order conditions for firms' actually are sufficient for a global maximum. Given the existence of a stable equilibrium, the comparative statics proposition that a greater responsiveness of demand for brand-name investment will cause larger levels of brand name investment and fewer firms in equilibrium is actually quite general. To see this let

$$(4.1) \qquad q(B,\hat{B},n,e)$$

denote a firm's demand if it chooses a level of brand name investment, B, all other firms choose a level, $\hat{B}$, there are n firms, and e is a parameter measuring the responsiveness of demand to brand-name investment. As before, a symmetric equilibrium is an ordered pair (B,n) such that B is a symmetric Nash-equilibrium brand name investment level with n firms and each firm earns zero profits. In what follows it will be assumed that there exists an e^* such that a unique equilibrium exists for every e in some neighborhood of e^* and that this equilibrium is uniquely determined by the first-order conditions. Let (B(e), n(e)) denote the equilibrium given e and let B^* and n^* denote, respectively, $B(e^*)$ and $n(e^*)$. Then B(e), n(e) are determined by [16]

$$(4.2) \qquad q_1(B,B,n,e)\, m - 1 = 0$$

$$(4.3) \qquad q(B,B,n,e)\, m - S - B = 0$$

Equation (4.2) is the first-order condition for a symmetric Nash-equilibrium and (4.3) is the zero profits condition. The goal is to evaluate $B'(e^*)$ and $n'(e^*)$.

Two types of assumptions are required to do this. First, a number of natural structural assumptions concerning q need to be made. Assume that the derivatives of q evaluated at (B^*,B^*,n^*,e^*) have the following signs:

(i) $q_1 > 0$, $q_2 < 0$, $q_3 < 0$

(ii) $q_{11} < 0$

(iii) $q_{12} \leqslant 0$

(iv) $q_{13} \leqslant 0$

(v) $q_{14} > 0$

The assumptions have straightforward, natural interpretations as described below.

(i) If a firm invests more in brand name capital its demand increases. However if other firms invest more or if more firms enter and each invest the same amount, the firm's demand drops.

(ii) This is a necessary condition for each firm's choice of brand-name investment to be optimal.

(iii) As other firms spend more on brand name investment the marginal effect of a given firm's expenditure is less.

(iv) As there are more firms the marginal impact of a firm's expenditure on advertising diminishes. This is related to (iii). As n increases but expenditures per firm, $\hat{B}$, stay constant the aggregate expenditures of other firms increase. This reduces the marginal effectiveness of a given firm's brand name investment. This assumption is not required for the following proof. In fact much the reverse is true. The fact that q_{13} is not positive generates an extra difficulty in the proof. However, since this is a property one would expect to be true it is included in order that the manner in which this difficulty is addressed is made more clear.[17]

(v) As e increases, the marginal effectiveness of brand name investment also increases. This simply defines the sense in which e is a measure of the responsiveness of demand to brand name investment.

The second type of assumption which must be made concerns the stability of the equilibrium -- i.e. -- Samuelson's correspondence principal will be employed. The assumption that the equilibrium is locally stable for some natural adjustment process will be used to derive comparative statics results. First consider adjustment of the number of firms, n. It is reasonable to suppose that n increases when profits are positive and decreases when profits are negative. Thus the adjustment process is defined by

$$(4.4) \qquad \dot{n} = f_n(q(B,B,n,e)\, m - S - B)$$

where $\dot{n}$ denotes the time derivative of n and f_n is any function with the properties that, $f_n(0) = 0$, and f_n is increasing. Now consider adjustment of B. If all other firms are choosing B, a given firm will choose a higher (lower) value of brand name investment if its marginal profits from choosing B are positive (negative). Since this is true for all firms, the value of B will rise (fall) if the marginal value of firms' profits are positive (negative). Thus the adjustment process is defined by

$$(4.5) \qquad \dot{B} = f_B(q_1(B,B,n,e)\, m - 1)$$

where $\dot{B}$ denotes the time derivative of B and f_B is any function with the properties that $f_B(0) = 0$ and f_B is increasing.

Thus equation (4.4) states that n adjusts so that the zero profits equilibrium condition, (4.3), is satisfied. Equation (4.5) states that B adjusts so that the Nash equilibrium condition, (4.2) is satisfied. A phase diagram analysis of this process is the clearest method of illustrating the necessary conditions for this process to be stable.[18] Let

$$(4.6) \qquad B = \xi(n)$$

denote the values of B and n which satisfy (4.2). By assumptions (ii) - (iv) $\xi'(n) < 0$. Similarly, let

$$(4.7) \qquad B = \psi(n)$$

denote the solution to (4.3). Once again, by (i) and by (4.2), ψ' is positive when evaluated at n^*.

Figure 1: $|\psi'| > |\xi'|$

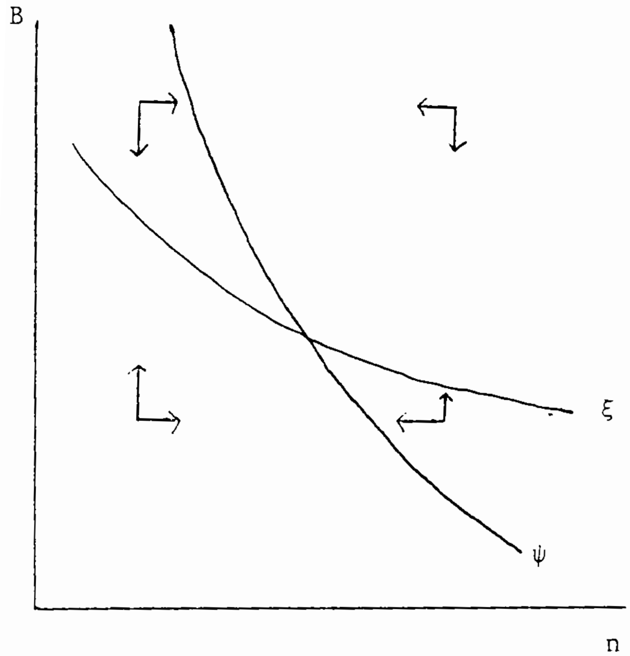
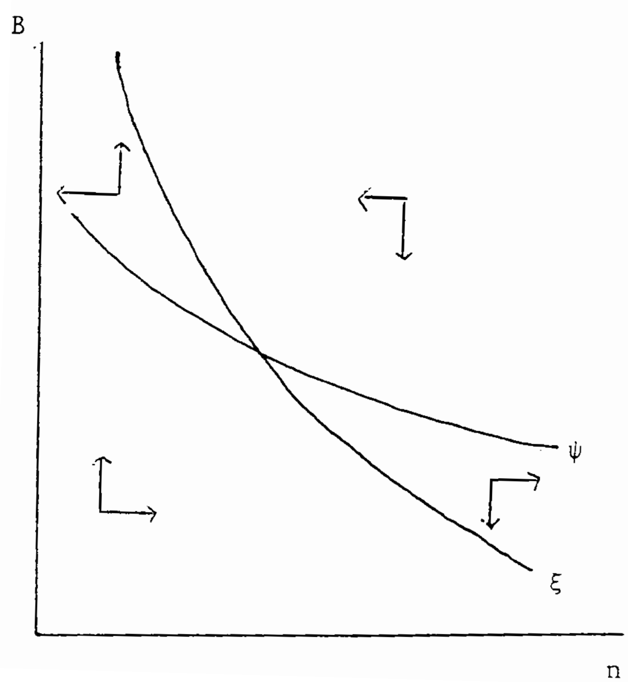


Figure 2: $|\psi'| < |\xi'|$



There are two possible cases, determined by whether ψ or ξ is steeper at their intersection point. As the phase diagrams in Figures 1 and 2 make clear, the equilibrium is stable if and only if ψ is steeper -- i.e. -- $|\psi'| > |\xi'|$.

The desired comparative statics result is now straightforward.

Theorem 3

Suppose that the above assumptions are satisfied. Then an increase in the responsiveness of demand to brand name investment causes the equilibrium level of brand name investment per firm to rise and the equilibrium number of firms to fall -- i.e. --

$$B'(e^*) > 0 \tag{4.8}$$

$$n'(e^*) < 0 \tag{4.9}$$

Proof:

From Figure 1 it is sufficient to show that ξ shifts up when e increases. (From (4.3), ψ does not depend on e.) Totally differentiating (4.2) yields

$$\frac{\partial B}{\partial e} = \frac{-q_{14}}{q_{11} + q_{12}} \tag{4.10}$$

where all the derivatives are evaluated at (B^*, B^*, n^*, e^*). By assumptions (ii) and (iii) the denominator is negative; by assumption (v) the numerator is negative. Therefore, $\partial B/\partial e$ is positive -- i.e. -- ξ shifts up as e grows.

Q.E.D.

Appendix A

This appendix formalizes the verbal argument in the body of the paper that when all brand name capital costs more to produce than consumers are willing to pay for it, all firms will change a price of c + m in equilibrium. Suppose there are n firms charging prices $(p_1, \ldots, p_n)$ and choosing levels of brand investment $(B_1, \ldots, B_n)$. Let $q_i^n(p_n \ldots, p_n, B_1, \ldots, B_n)$ denote the demand for firm i's product. Four assumptions about these demand curves will be made. An explanation follows the statement of each assumption.

(i) If $p_i < c + m$, then $q_i^n = 0$.

Consumers (correctly) believe that a firm charging less than c + m will not produce a high quality product. Thus no consumer will purchase a product for less than c + m.

(ii) There exists an $\bar{\bar{x}} > \bar{x}$ such that for any two firms, i and j, if

(A.1) $$B_i > B_j$$

(A.2) $$p_i \geq p_j + \frac{B_i - B_j}{\bar{\bar{x}}}$$

and

(A.3) $$p_j \geq c + m,$$

then all consumers prefer (p_j, B_j) to (p_i, B_i).

Assumption (ii) can be interpreted by considering the following thought experiment. Suppose a firm is serving x customers by offering (p_j, B_j) (where x must be less than or equal to the capacity of $\bar{x}$.) Now suppose the firm raised its level of brand name investment to B_i. If it continues to serve the same x customers it must raise price by $(B_i - B_j)/x$ to cover its costs. As x grows larger the required price rise is of course smaller. Assumption (ii) states that consumers will never prefer more brand investment with a correspondingly higher price. (Even if the firm was of size $\bar{\bar{x}}$ which is bigger than capacity, the required price rise would be too large.) This is thus the formal statement of the idea that brand name investment costs more to produce than consumers are willing to pay for it.

(iii) If $p_i < p_j$ and $B_i \geq B_j$ then all consumers prefer (p_i, B_i) to (p_j, B_j).

This simply states that a strictly lower priced product with at least as much brand name investment is strictly preferred by all consumers.

A symmetric equilibrium[19] is a number of firms, price and brand name investment level, (n,p,B) with $n \geq 2$ such that the following three conditions are satisfied.

(A.4) $\quad \bar{x} \geq q_i^n(p,\ldots,p,\ B,\ldots,B)$

(A.5) $\quad q_i^n\ (p,\ldots,p,B,\ldots,B)\ (p-c) - S - B = 0$

(A.6) $\quad$ (p,B) is a Nash-equilibrium among the existing n firms

Theorem A.1 now states the desired result. All the equilibria involve $p = c + m$ and to check that condition (A.6) holds, it is sufficient to check that B is a Nash equilibrium in levels of brand name investment among the existing firms given all firms set price equal to $c + m$.

Theorem A.1

Given the cost curves described by (2.1) and (2.2), and given assumptions (i) - (iii), (n,B,p) is a symmetric equilibrium if and only if

(A.7) (A.4) and (A.5) are satisfied

(A.8) $p = c + m$

(A.9) B is a Nash equilibrium in brand name investment levels.

proof:

Equilibrium Implies (A.7) (A.8) and (A.9):

Suppose $(\hat{n},\hat{p},\hat{B})$ is an equilibrium. Obviously it satisfies (A.7) and (A.9). A graphical interpretation of assumption (ii) is the clearest way of showing that (A.8) is satisfied.

Hold firm size fixed at some level, x. Then iso-profit lines in (p,B) space are given by

$$B = (p-c)x - S - \pi \tag{A.10}$$

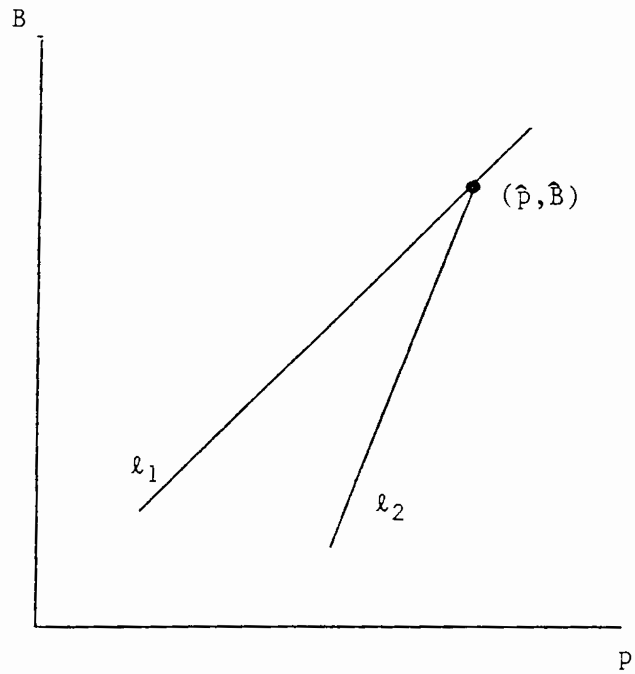
where π is the profit level. In particular note that they are straight lines with slope x. Now suppose a firm is offering $(\hat{p},\hat{B})$. It knows that it can retain all its customers (and even possibly attract new ones) if when it lowers price to p it also lowers B by no more than determined by (A.2).[20] In particular, for $p < \hat{p}$, the values of (p,B) which satisfy (A.2) with equality are given by

$$(A.11) \quad B = \hat{B} - (\hat{p}-p)\bar{\bar{x}}$$

This is a straight line through $(\hat{p},\hat{B})$ with slope of $\bar{\bar{x}}$. By assumption x must be less than or equal to $\bar{x}$ which is strictly less than $\bar{\bar{x}}$. Figure A.1 illustrates this. Line ℓ_1 is the iso-profits line. Line ℓ_2 represents deviations from $(\hat{p},\hat{B})$ determined by (A.11). In particular note that if the firm lowered price and correspondingly adjusted B to satisfy (A.11), it would increase its profits even if its demand stayed constant. But, by construction, movements along ℓ_2 increase the firms attractiveness to all customers and thus the firm's demand will not diminish. Thus it is profitable for the firm to both lower its price and brand name investment if it can. It will not be able to do this if $\hat{p} = c + m$ or if $\hat{B} = 0$. Thus in any Nash equilibrium it must be that price equals c + m or brand name investment is zero.

It suffices to show that no equilibrium exists with $\hat{B} = 0$ and $\hat{p} > c + m$. If $\hat{B} = 0$, then by (2.2) zero profits implies that firm size is less than $\bar{x}$. By assumption (iii) a firm could generate a discrete increase in demand by an arbitrarily small price cut. Since each firm is operating at less than capacity it would wish to do so.

Figure A.1



(A.7), (A.8) and (A.9) imply equilibrium:

Suppose that $(\hat{n},\hat{p},\hat{B})$ satisfies (A.7) - (A.9). Obviously it satisfies (A.4) and (A.5). Therefore it must be shown that (A.6) is saitsfied. It suffices to prove the following lemma.

Lemma A.1:

Let $\hat{\pi}$ denotes a firm's profits when it offers $(\hat{p},\hat{B})$. Suppose a firm contemplates a deviation to (p^*,B^*) generating profits of $\pi^* > \hat{\pi}$. Then there exists a B^{**} such that if the firm offered $(\hat{p},B^{**})$ generating profits of π^{**}, that $\pi^{**} > \pi^*$.

First it will be explained why this lemma implies (A.6). Then the lemma will be proven. Because firms are in a Nash equilibrium with respect to investment levels (i.e. -- (A.9 is true), $\pi^{**} \leqslant \hat{\pi}$. Therefore no contract (p^*,B^*) with $\pi^* > \hat{\pi}$ can exist if Lemma A.1 is true. This means that (A.6) is true.

It remains to prove Lemma A.1. Consider the same graphical analysis as Figure A.1 where (p^*,B^*) replaces $(\hat{p},\hat{B})$. Two cases are possible. These are drawn in Figures A.2 and A.3. Recall that ℓ_1 is in the iso-profit line through (p^*,B^*) and ℓ_2 is a set of price investment pairs which are more attractive to all consumers than (p^*,B^*). Let $\tilde{p}$ be the price where ℓ_2 intersects the horizontal axis. Two cases exist.

Case #1 $\tilde{p} \leqslant c + m$ (Figure A.2):

In this case, the firm can lower its price to c + m and the level of brand name investment determined by ℓ_2, is non-negative. Thus offering a price of c + m and an investment level of B^{**} produces more profits than offering (p^*,B^*).

Case #2 $\tilde{p} > c + m$ (Figure A.3):

In this case, lowering price to c + m would require a negative brand name investment level so the argument in Case #1 cannot be used. It will be shown that (A.7)-(A.9) imply that this case cannot occur. Consider offering the price $\tilde{p}$ and choosing zero brand name investment. Let $\tilde{\pi}$ denote the profit level. By construction $\tilde{\pi} > \pi^*$. By assumption $\pi^* > \hat{\pi}$. Therefore $\tilde{\pi} > \hat{\pi}$. Since $\hat{\pi}$, this implies that $\tilde{\pi} > 0$. This implies that $(\tilde{p},0)$ must generate a positive number of customers. However by assumption (iii) this cannot be since $\tilde{p} > \hat{p}$ and $\tilde{B} = 0 \leq \hat{B}$ and other firms are still offering $(\hat{p},\hat{B})$.

QED.

Figure A.2

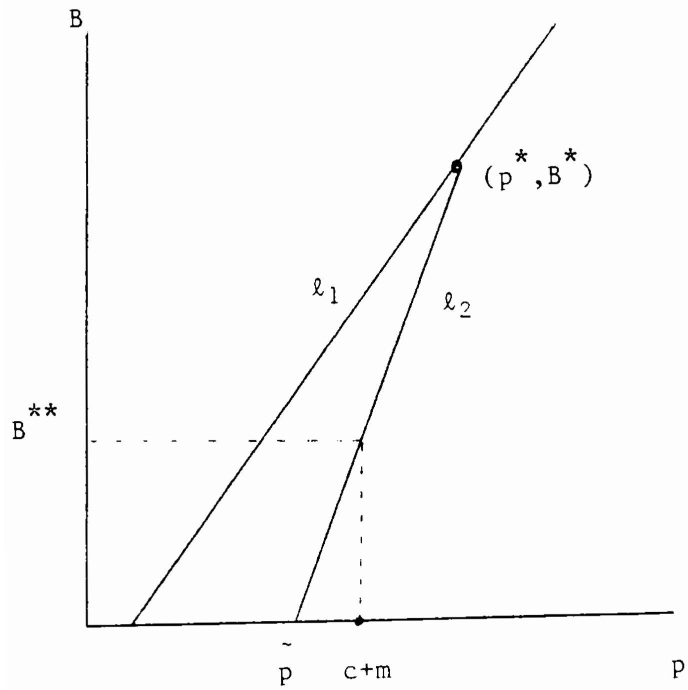
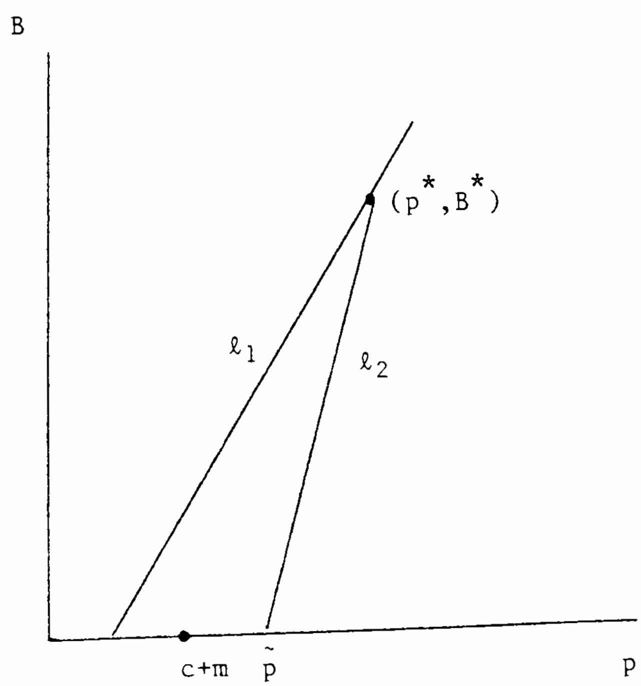


Figure A.3



Appendix B -- Proof of Theorem 1

A firm's profit given it chooses B, other firms choose $\hat{B}$, there are n firms, and the capacity constraint is ignored, is given by

$$\pi(B,\hat{B},n) = r(B,\hat{B},n)\ m - S - B \tag{B.1}$$

where r is defined in (2.10). The first and second derivatives are given by

$$\frac{\partial\pi}{\partial B}(B,\hat{B},n) = \frac{(n-1)e\ B^{e-1}\ \hat{B}^e}{(B^e + (n-1)\ \hat{B}^e)^2}\ Qm - 1 \tag{B.2}$$

and

$$\frac{\partial^2\pi}{\partial B^2}(B,\hat{B},n) = \frac{(n-1)\ e\ \hat{B}^e\ B^{e-2}}{(B^e + (n-1)\ \hat{B}^e)^3}\ Qm\ \lfloor (e-1)(n-1)\ \hat{B}^e - (e+1)\ B^e \rfloor. \tag{B.3}$$

Recall that an equilibrium is defined to be an ordered pair (B,n) which satisfies (2.12) - (2.14). The first order conditions corresponding to (2.13) can be calculated using (B.2). These are

$$B = \frac{n-1}{n^2}\ Qme, \quad \text{for } n > \bar{n} \tag{B.4}$$

$$B \leq \frac{n-1}{n^2}\ Qme, \quad \text{for } n = \bar{n} \tag{B.5}$$

The proof is now in two steps. The first step is to show that for every $e > 0$, there exists a unique (B,n) satisfying (2.12), (2.14), (B.4) and (B.5). The second step is to show that this solution also satisfies (2.13).

Step #1:

By substituting (B.4) and (B.5) into (2.14), equations (2.12), (2.14), (B.4), and (B.5) can be written as

$$\text{(B.6)} \qquad Qm \left\lfloor \frac{n(1-e) + e}{n^2} \right\rfloor - S = 0$$

for $n > \bar{n}$

$$\text{(B.7)} \qquad B = \frac{n-1}{n^2} \quad Qme$$

or

$$\text{(B.8)} \qquad Qm \left\lfloor \frac{n\,(1-e) + e}{n^2} \right\rfloor - S < 0$$

and

for $n = \bar{n}$

$$\text{(B.9)} \qquad B = \frac{Qm}{n} - S$$

Straightforward calculus shows that a unique $\hat{n}(e) > 0$ exists such that the LHS of (B.6) and (B.8) equals zero; furthermore it is positive to the left and negative to the right.[21] Therefore if $\hat{n}(e) \geq \bar{n}$ the unique solution to (B.6) - (B.9) is given by $\hat{n}(e)$ and the B determined by (B.7). If $\hat{n}(e) < \bar{n}$, the unique solution to (B.6) - (B.9) is given by $\bar{n}$ and the B determined by (B.9).

Step #2:

By (B.3) if $e \leq 1$, $\pi(B,\hat{B},n)$ is globally concave in B. Therefore the first order conditions determine a global maximum.

Now suppose that $e > 1$. By (B.3) for any $\hat{B}$ and n there exists some value $\bar{B}$ such that

(B.10) $\frac{\partial^2\pi}{\partial B^2}(B,\hat{B},n) \gtreqless 0 \leftrightarrow B \lesseqgtr \bar{B}$.

(B.11) $\pi(0,\hat{B},n) = - S$

(B.12) $\lim_{B\to\infty} \pi(B,\hat{B},n) = - \infty$.

and

(B.13) $\frac{\partial\pi}{\partial B}(0,\hat{B},n) = -1$

Therefore π as a function of B must be of one of two possible types. It may exhibit a local interior minimum followed on a local interior maximum. This is illustrated in Figure B.1. The second possibility is that it may have no stationary points. This is exhibited in Figure B.2.

It can now be shown that the solution identified in Step #1 is a global maximum for each firm. First suppose that the solution (B^*,n^*) satisfies (B.6) and (B.7). Then $\frac{\partial\pi}{\partial B}(B^*,B^*,n) = 0$. By (A.3) and (B.6) $\frac{\partial^2\pi}{\partial B^2}(B^*,B^*,n) < 0$. Therefore π must be as in Figure B.1 with the local interior maximum occurring at B^*. This is a global maximum if and only if profits exceed $- S$ at B^*. However, profits equal zero by construction.

Figure B.1

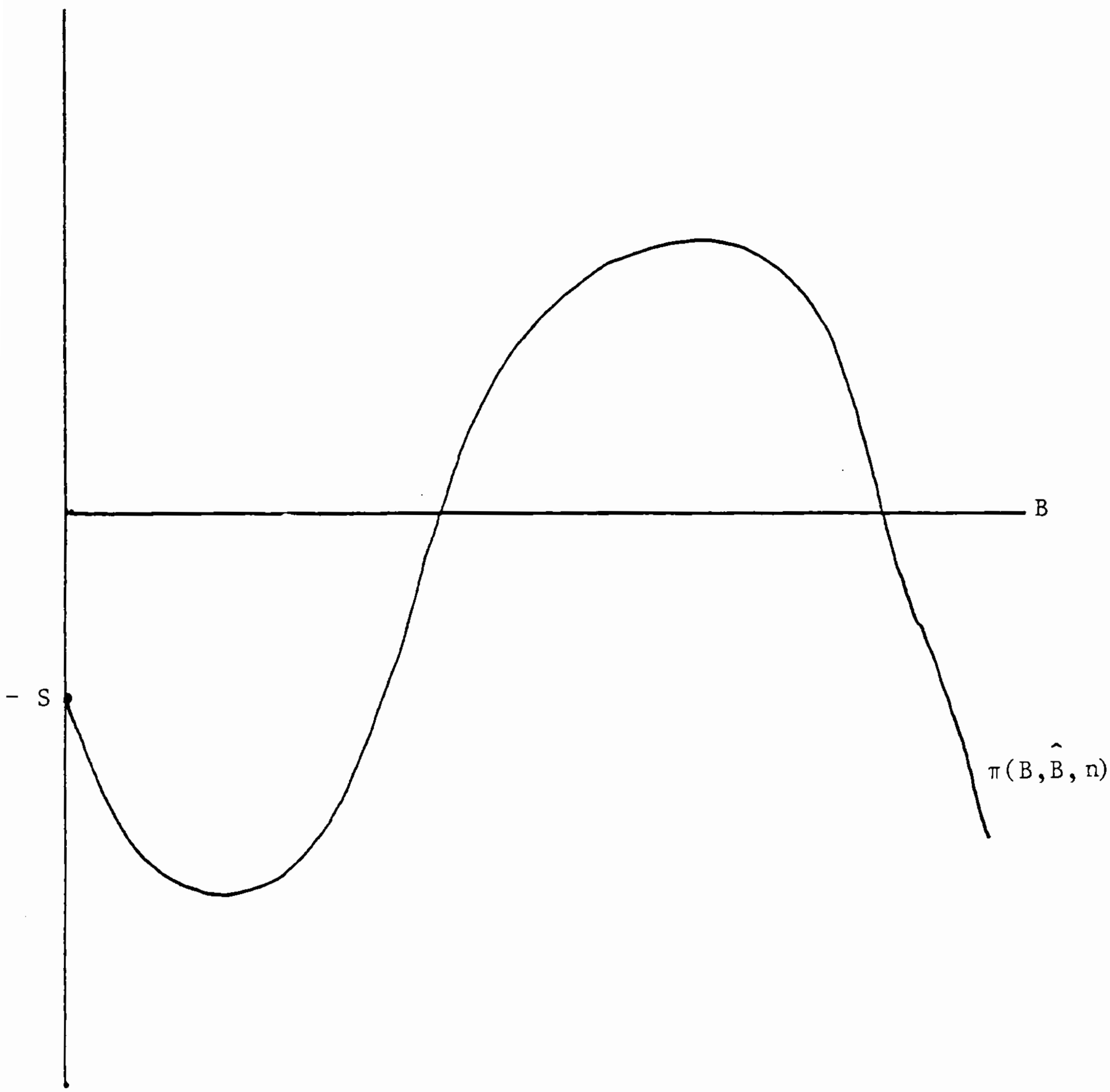
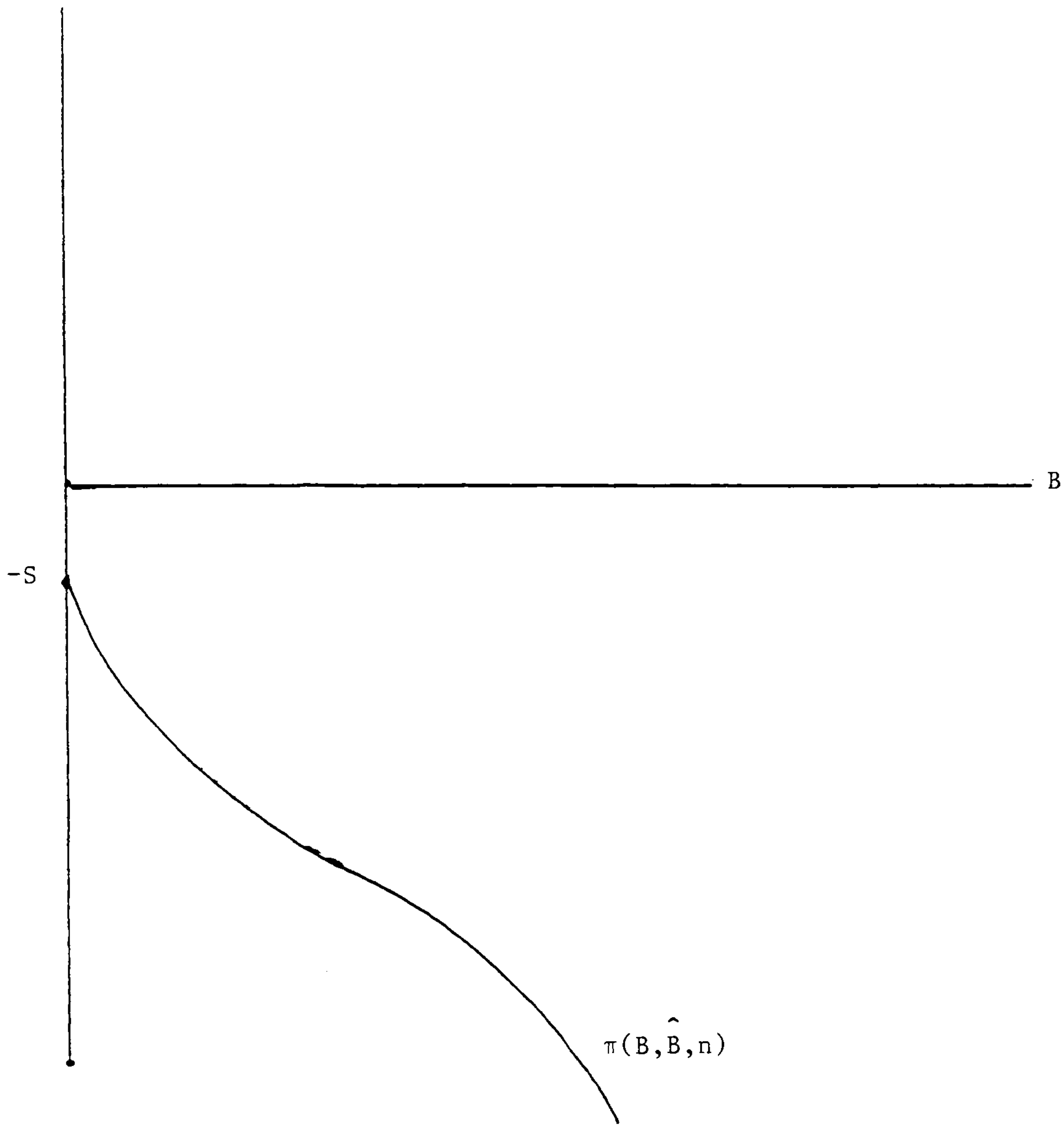


Figure B.2



Now suppose that (B^*,n^*) satisfies (A.8) and (A.(9).
Then $\frac{\partial\pi}{\partial B}(B^*,B^*,n) > 0$. Therefore π must be as in Figure B.1 and B^* is somewhere on the positively sloped portion of π. It suffices to show that $\pi(B^*,B^*,n^*) \geqslant (B,B^*,n^*)$ for every $B \leqslant B^*$. (Recall that the firms are producing at full capacity in this case). From Figure 1, this is true if profits exceed - S. However, by construction profits equal zero.

QED.

Footnotes

1. See, for example, Allen [1984], Rogerson [1983], and Shapiro [1983].

2. Investment in sunk costs does not change the quality guaranteeing price.

3. Thus brand name investment does not play a signalling role in this model. This distinguishes this model from those where advertising is of no direct value to consumers but occurs in equilibrium because it signals high quality. See, for example, Milgrom and Roberts [1984], Nelson [1974], and Schmelensee [1978].

4. See Panzar [1975, 1979], Schmalensee [1976, 1977] and Vander Weide and Zalkind [1981]. This paper is most closely related to the approach in Schmalensee [1976].

5. In the symmetric equilibrium identified by Schmalensee [1976], Schmalensee shows that each firm is at a local maximum. This paper shows that in fact each firm is at a global maximum. This paper's result that there always exists a unique number of firms such that industry profits are zero is also new.

6. Stigler goes on to investigate whether price or non-price competition will be more effective in dissipating profits, given a fixed number of firms.

7. See Klein and Leffler [1981], Shapiro [1983], Allen [1984], or Rogerson [1983] for a formal treatment. Note in particular that because marginal costs are constant, the quality guaranteeing price differential, m, is independent of firm size. This considerably simplifies the formal analysis. See Allen [1984] for a detailed analysis of the behavior of the quality guaranteeing price when marginal cost is not constant.

8. This point has been theoretically modelled by Nerlove and Arrow [1962]. Empirical estimates of the long-run effects of advertising are contained in Lambin [1976], Palda [1965] and Telser [1962]. For a more complete discussion of this and further references see Aaker and Carman [1982], pages 63-64.

9. See Cowling [1972], Telser [1962] and Lambin [1976] for particularly interesting examples.

10. See Schmalensee [1972] for a thorough discussion of these difficulties.

11. The major characteristic where the issue of reverse causality arises is concentration. If, as is often argued, advertising causes barriers to entry, then advertising may cause concentration. See Comanor and Wilson [1974] for a thorough treatment and references to the voluminous literature on this topic.

12. Page 383.

13. Page 426.

14. Recall that ϕ is defined by (2.9).

15. To see this, suppose that firms have increasing marginal costs and no capacity constraint. In the Klein and Leffler solution all firms operate on their marginal cost curve and investment in brand name capital dissipates profits. However, since price equals the marginal cost of the last unit sold there is no first-order effect on revenues minus production costs if brand name investment is reduced and less is sold. However there is a first order effect on investment costs. Thus there cannot be positive equlibrium levels of investment if firms are exactly on their marginal costs curves.

16. In what follows let q_i denote the derivative of q with respect to the ith variable and let q_{ij} denote the cross-partial of q with respect to the ith and jth variables.

17. As will be seen below, the comparative statics result requires that the equilibrium be stable. If $q_{13} \geqslant 0$ the equilibrium is always stable. If $q_{13} < 0$, it is possible for the equilibrium not to be unstable. Therefore it must simply be assumed that the equilibrium is stable in order to derive the result.

18. See Quirk [1976] for a more formal treatment of this.

19. At the cost of a bit more notational complexity the same argument in this Appendix can be applied to the case of assymetric equilibra. However since only symmetric equilibra are calculated in the body of the paper, only symmetric equilibria will be considered here as well.

20. Ignore, for the moment, the constraints that $p \geqslant c + m$ and $B \geqslant 0$. This will be considered momentarily.

21. It is <u>not</u> true that the expression in square brackets in the LHS of (B.6) is strictly decreasing in n for every $n \geqslant 0$. However it can be shown that it is strictly decreasing until it equals zero and then never exceeds zero again. This is sufficient to establish the existence of a unique solution to (B.6).

References

Aaker, David and James Carman [1982], "Are You Overadvertising," Journal of Advertising Research, 22, 57-70.

Allen, Franklin [1984], "Reputation and Product Quality," Rand Journal of Economics, 15, 311-327.

Borden, Neil [1942], The Economic Effects of Advertising, Richard D. Irwin, Inc., Chicago.

Buzzel, Nourse, Mathews, Jr., Levitt [1972], Marketing: A Contemporary Analysis, 2nd edition, McGraw Hill, New York.

Comanor, W., and T. Wilson [1974], Advertising and Market Power, Harvard University Press, Cambridge.

Cowling, Keith [1972], "Optimality in Firms Advertising Policies: An Empirical Analysis," in Market Structure and Corporate Behavior: Theory and Empirical Analysis of the Firm, Keith Cowling, ed., Grey-Mills Publishing Ltd., London, 85-104.

Farris, Paul and Mark Albion [1981], "Determinants of the Advertising-to-Sales Ratio," Journal of Advertising Research, 21, 19-27.

Klein, Benjamin and Keith Leffler [1981], "The Role of Market Forces in Assuring Contractual Performance," Journal of Political Economy, 89, 615-641.

Lambin, Jean-Jaques [1976], Advertising, Competition and Market Conduct Over Time, North-Holland Publishing Co., Amsterdam.

Milgrom, Paul and John Roberts [1984], "Price and Advertising Signals of Product Quality", Stanford Graduate School of Business Research Paper 742.

Nelson, Phillip [1974], "Advertising as Information," Journal of Political Economy, 82, 729-754.

Nerlove, M. and K. Arrow [1962], "Optimal Advertising Policy Under Dynamic Conditions," Economica 29, 129-42.

Palda, Kristian [1965], "The Measurement of Cumulative Advertising Effects," Journal of Business, 38, 162-179.

Panzar, John [1975], "Regulation, Service Quality and Market Performance: A Model of Airline Rivalry," Memorandum 184, Centre for Research in Economic Growth, Stanford University.

Panzar, John [1979], Regulation, Service Quality and Market Performance: A Model of Airline Rivalry, Garland Publishing, New York.

Porter, Michael E. [1978], "Optimal Advertising: An Intra-Industry Approach," in Issues in Advertising: The Economics of Persuasion, David Tuerck, ed., American Enterprise Institute, Washington, 91-114.

Quirk, James P. [1976], Mathematical Notes for Intermediate Microeconomics, Science Research Associates, Inc., Chicago.

Rogerson, William P. [1983], "Reputation and Product Quality," Bell Journal of Economics, 14, 508-516.

Scherer, F.M. [1980], Industrial Market Structure and Economic Performance, Houghton Mifflin Co., Boston.

Schmalensee, Richard [1972], The Economics of Advertising, North-Holland Publishing Co., Amsterdam.

Schmalensee, Richard [1976], "A Model of Promotional Competition in Oligopoly" Review of Economic Studies, XLIII, 493-507.

Schmalensee, Richard [1977], "Comparative Static Properties of Regulated Airline Oligopolies" Bell Journal of Economics, 8, 565-576.

Schmalensee, Richard [1978], "A Model of Advertising and Product Quality," Journal of Political Economy, 86, 485-503.

Shapiro, Carl [1983], "Premiums for High Quality Products as Returns to Reputations," Quarterly Journal of Economics, XCVIII, 659-679.

Stigler, George J. [1968], "Price and Non-Price Competition," Journal of Political Economy, 76, 149-54.

Telser, Lester [1962], "Advertising and Cigarettes," Journal of Political Economy, 70, 471-499.

Vander Weide, James and Julie Zalkind, [1981], "Deregulation and Oligopolistic Price-Quality Rivalry," American Economic Review, 71, 144-154.