Discussion Paper No. 695

REPUTATION IN BARGAINING AND DURABLE GOODS MONOPOLY

by

Lawrence M. Ausubel*
and
Raymond J. Deneckere*

June 1986

*Department of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60201.

The authors' work was supported in part by National Science Foundation Grants SES-85-69701 and IST-86-09129, by the Kellogg School's Paget Research Fund, and by National Science Foundation Grant SES-83-20464 at the Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford California. We would like to thank Varadarajan Chari, Kenneth Judd, and Roger Myerson for helpful suggestions and stimulating discussions. The usual disclaimer applies.

REPUTATION IN BARGAINING AND DURABLE GOODS MONOPOLY

by

Lawrence M. Ausubel* and Raymond J. Deneckere*

This paper analyzes durable goods monopoly in an infinite-horizon, discrete-time game. We prove that, as the time interval between successive offers approaches zero, all seller payoffs between zero and static monopoly profits are supported by subgame perfect equilibria. This reverses a well-known conjecture of Coase. Alternatively, one can interpret the model as a bargaining game with one-sided incomplete information in which the uninformed party makes all the offers. Under that interpretation, our "Folk Theorem" applies to the set of sequential equilibria.

Key Words: Durable goods monopoly, bargaining, Coase Conjecture, reputation.

---

*Department of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, 2001 Sheridan Road, Evanston, Illinois 60201.

1. Introduction

Assume that a single firm controls the supply of an infinitely durable good. In a classic [1972] paper, Ronald Coase asked what sales plan this monopolist would adopt to maximize her profits. Coase provided a partial, negative answer by observing that the naive policy of forever offering the good at a static monopoly price is not credible. To paraphrase Martin Hellwig [1975], the monopolist who announces such a policy cannot "keep a straight face"--she has an irresistable temptation to cut the price at future dates, to generate additional sales and profits. Coase supplemented his answer by conjecturing that, with rational consumer expectations, "the competitive outcome may be achieved even if there is but a single supplier." Several subsequent authors have supported this conjecture by producing models with subgame perfect equilibria in which the introductory price approaches marginal cost.

Nevertheless, Coase's original puzzle concerning the optimal monopoly pricing rule remains essentially unsolved. In this paper, we propose a solution: the firm introduces the durable good at approximately the static monopoly price. She then follows the slowest rate of price descent that enables her to maintain her credibility. As the time interval between successive periods of the game approaches zero, the rate of descent can be made arbitrarily small while preserving subgame perfection. This enables the supplier to earn nearly static monopoly profits. Thus we argue that, even in a durable goods market, a monopoly _is_ a monopoly.

The identical reasoning carries over to bargaining games with incomplete information, since essentially the same mathematical model may depict either a continuum of actual consumers with different valuations or a single buyer with a continuum of possible valuations. Thus, in an infinite-horizon bargaining game where the seller makes repeated offers to a buyer whose valuation she does not know, we prove that there exist sequential equilibria where the seller essentially extracts monopoly surplus. The Coase Conjecture, in contrast, would predict that the buyer obtains almost all the gains from trade.

The intuition behind Coase's Conjecture is as follows: once an initial quantity of the good has been sold, the monopolist will find it tempting to sell some additional output as long as her accumulated sales remain below the competitive output. If there is virtually no restraint to the rate at which the monopolist can sell additional units (if the time interval between successive offerings is very small), the market may be saturated with the competitive output "in the twinkling of an eye" (Coase [1972]).

Bulow [1982] analyzed Coase's reasoning in a finite-horizon model. In the last period, the monopolist who lacks commitment power charges the static monopoly rental price for the residual demand curve. By backward induction, Bulow calculates the monopolist's best action in each previous period, and shows that it is always unambiguously lower than the static monopoly price. Stokey [1981] formalized the Coase Conjecture for infinite-horizon models. Under an assumption of self-fulfilling expectations (which depend continuously on the stock of goods), she proved that the unique equilibrium of a continous-time model is for the monopolist to price at marginal cost. She also demonstrated that the backward induction equilibrium of an infinite-horizon discrete-time model satisfies the Coase Conjecture. Gul, Sonnenschein and Wilson [1986] discovered a continuum of additional subgame perfect equilibria in this game. However, for a general class of demand curves, these (weak-Markov) equilibria behave qualitatively like the backward induction equilibrium--they satisfy the Coase Conjecture.

Kahn [1986] considered the durable goods monopolist with increasing marginal cost. Bond and Samuelson [1984] examined monopoly sale of a durable good which is subject to depreciation. These authors show that, as long as the interval between successive periods is positive, the inability to commit does not undermine monopoly power as much as in the standard model. Nevertheless, as the time interval shrinks, output still approaches the competitive level.

The noncooperative bargaining literature developed in parallel to the study of durable goods monopoly.[1] To escape from the typical complete information result (e.g., Rubinstein [1982]) that all bargaining is concluded in the first round, this literature introduced incomplete information into the bargaining process. This often added a continuum of sequential equilibria (e.g., Fudenberg and Tirole [1983]). One modeling technique, however, offered apparent promise for yielding results with greater predictive value: restricting the game to one-sided bargaining with one-sided uncertainty. When the uninformed party makes all the offers (the informed party only responding with "yes" or "no"), the complications of strategic communication largely disappear. The widespread perception in the literature was that the multiplicity of equilibrium outcomes vanished as well.

The first paper to explore this approach was Sobel and Takahashi [1983]. They study the bargaining game which is the direct analogue to durable goods monopoly. Mirroring Bulow [1982] and Stokey [1981], they prove the existence of a unique sequential equilibrium in the finite-horizon model and of a backward induction equilibrium in the infinite-horizon model; both satisfy the Coase Conjecture.

Fudenberg, Levine and Tirole [1985] analyze two distinct cases in the infinite-horizon bargaining game. In the first case, where the buyer's valuation is known to strictly exceed the seller's, Fudenberg-Levine-Tirole prove that the model generically has a unique sequential equilibrium. Moreover, negotiations always end after a finite number of rounds, and so the Coase Conjecture applies. In the second, and more reasonable case, where there is no gap between the lowest buyer valuation and the seller's valuation, sales occur over infinite time. The three authors proved, under reasonably general assumptions (which we extend in this paper), the existence of a backward induction equilibrium. This equilibrium is necessarily weak-Markov, and therefore satisfies the Coase Conjecture.

The main result of our paper is a Folk Theorem for the "no gap" case.[2] As the time interval between successive periods approaches zero in durable goods monopoly (bargaining), the set of monopolist (seller) payoffs associated with subgame perfect (sequential) equilibria expands to the entire interval from zero to static monopoly profits.

Let us provide an interpretation of the equilibria constructed in this paper. Initially, consumers believe they are facing a "strong" monopolist who will continue to adhere to the main price path specified in the equilibrium. However, the moment a deviation from the main price path occurs, consumers decide they are dealing with a "weak" monopolist who has read the [1972] Coase paper (and believes its message). Since the Coase price path yields profits barely above the competitive level, the prospect of such a reversal of expectations deters the monopolist from ever deviating. We refer to these equilibria as "reputational equilibria."

Our paper has important implications for the bargaining literature. (We develop some of these in a sequel, Ausubel and Deneckere [1986].) Much of the impetus for "refinements"[3] of the sequential equilibrium concept stems from the observation that it allows excessive freedom in specifying players' beliefs (about the type of opponent they are facing) afer a zero probability event occurs. This freedom in updating beliefs sneaks "incredible threats" in through the back door.[4] Observe, however, that the vast multiplicity of sequential equilibria in the current bargaining game is not susceptible to the usual refinements. "Reputation," in our equilibria, does not involve the seller's type[5]--buyers have no beliefs to be updated when off-equilibrium behavior is observed. Meanwhile, the buyer's language is so condensed that there is no room for the seller to make alternative inferences about the buyer's type. Hence, a restriction on updating rules has no effect on the set of equilibrium payoffs. Because the multiplicity of equilibria displayed here does not rely upon incompleteness of information, the criticism cannot be advanced that with the right kind of incomplete information, any outcome can be supported as a sequential equilibrium (Fudenberg and Maskin [1986]).

Gul, Sonnenschein and Wilson [1986] limited the set of outcomes by restricting attention to weak-Markov equilibria. We reject this approach for three reasons. First, the restriction is not natural. If a restriction were natural, it would be to stationary strategies--but these three authors, as well as Fudenberg, Levine and Tirole [1985], demonstrated that such strong-Markov equilibria often do not exist. Furthermore, weak-Markov equilibria are as bizarre as many others one might exclude--off the equilibrium path, they often require the monopolist to correct her own mistakes.[6] Second, while stationarity may at once seem appealing and possess cutting power, we should be warned how this restriction operates in other infinite-horizon games. In the supergame version of the prisoner's dilemma, stationarity selects a unique subgame perfect equilibrium: the infinite repetition of the static Nash outcome. Third, casual empiricism suggests that history does matter.

The ultimate irony of this paper is that we extend and use two types of results (which seemingly endorse the Coase Conjecture) to prove the Folk Theorem (which reverses the Coase Conjecture). After describing the model (Section 2) and presenting a linear example (Section 3), we first prove a general existence result on weak-Markov equilibria (Section 4 and Appendix A). We then show that price paths associated with weak-Markov equilibria are uniformly low compared to the demand curve (Section 5 and Appendix B). We proceed to establish the main result of our paper: the Folk Theorem under very general conditions (Section 6). We conclude with Section 7.

2. The Model

We consider a market for a good which is infinitely durable, and which is demanded only in quantity zero or one. There is a continuum of infinitely lived consumers, indexed by $q \in I = [0,1]$. The preferences of these consumers are completely specified by a monotone nonincreasing function $f: [0,1] \to \mathbb{R}_+$, where $f(q)$ denotes the reservation value of customer q, and by a common discount rate r. More precisely, if individual q purchases the good at time t for the price p_t, he derives a net surplus of:

$$e^{-rt}[f(q) - p_t]$$

Consumers seek to maximize their net surplus. The monopolist, meanwhile, faces a constant marginal cost of production, which we assume (without loss of generality) to equal zero. Her objective is to maximize the net present value of profits, using the same discount rate as consumers.

The monopolist offers the durable good for sale at discrete moments in time, spaced equally apart. The symbol z will denote the time interval between successive offers, and so sales occur at times $t = 0, z, 2z, \ldots, nz, \ldots$. We will sometimes refer to the "period" n rather than to the "time" t ($= nz$). Within each period, the timing of moves is as follows: first, the monopolist names a price; then, consumers who have not previously purchased decide whether or not to buy. After a time interval z elapses, play repeats.

A strategy for the monopolist specifies the price she will charge in each period, as a function of the history of prices charged in previous periods and the history of purchases by consumers. A strategy for a consumer specifies, in each period, whether or not to buy in that period, given the current price charged and the history of past prices and purchases. Formally, let $G(z,r)$ denote the above game when the time interval between successive sales is z and payoffs are discounted at the rate r. Let σ be a <u>pure</u> strategy for the monopolist. σ is a sequence of functions $\{\sigma(n)\}_{n=0}^{\infty}$. The function $\sigma(n)$ at date nz determines the monopolist's price in period n as a function of the prices she charged in previous periods, and the actions chosen by consumers in the past as summarized by the set $Q_n = \{q$: consumer q did not buy at any time $t < nz\}$. We impose measurability restrictions on joint consumer strategies below which imply that the set Q_n will be a measurable set, i.e., $Q_n \in \Omega$, where Ω is the Borel σ-algebra on I. Then $\sigma(n)\colon Y^n \times \Omega \to Y$, with $Y = [0, f(0)]$ and Y^n the n-fold Cartesian product of Y. A strategy combination for consumers is a sequence of functions $\{\tau^n\}_{n=0}^{\infty}$ where $\tau^n\colon Y^{n+1} \times \Omega \times I \to \{0,1\}$ is such that for each $y^{n+1} \in Y^{n+1}$ and each $B \in \Omega$, $\tau^n(y^{n+1}, B, \cdot)$ is measurable. Decision "0" is to be interpreted as a decision not to buy in the current period; decision "1" indicates that a sale takes place in the current period.

We will assume that consumers are anonymous, i.e., that neither the monopolist nor fellow consumers can identify a particular consumer's reservation price. Furthermore, we will assume that deviations by sets of measure zero of consumers can change neither the actions of remaining consumers nor those of the monopolist.[7] The above assumptions render all histories that have identical prices and that lead to the same measure of consumer acceptances observationally equivalent. Thus, τ^n and $\sigma(n)$ are constant on histories that have identical prices and that result in the same measure, $m(Q_n)$, of consumer acceptances (where $m(\cdot)$ denotes Lebesgue measure).

Let Σ be the pure strategy space for the monopolist, and T be the set of strategy combinations for consumers. The strategy profile $\{\sigma,\tau\}$, with $\tau = \{\tau_n\}_{n=0}^{\infty}$, generates a path of prices and sales which can be computed recursively. The pattern of prices and sales over time in turn determines the payoff to the players. Let $\pi(\sigma,\tau)$ be the discounted present value of profits generated by the strategy profile $\{\sigma,\tau\}$, and let $u^q(\sigma,\tau)$ be the discounted net surplus derived by consumer q. The profile $\{\sigma,\tau\}$ is a Nash equilibrium if and only if

$$\pi(\sigma,\tau) \geqslant \pi(\sigma',\tau),\ \forall\ \sigma' \in \Sigma$$

and

$$u^q(\sigma,\tau_q,\tau_{-q}) \geqslant u^q(\sigma,\tau_q',\tau_{-q}),\ \forall\ \tau_q' \in T_q,\ q - \text{a.e.}$$

where τ_q is the projection of τ onto the q-th component (and similarly for T_q). An n-period history of the game is a sequence of prices in periods $0,\ldots,(n-1)$ and a specification of the set of consumers who did not buy prior to n. We denote such a history by the symbol H_n. Thus, $H_n \in Y^n \times \Omega$. The symbol H_n' refers to H_n followed by a price announced by the monopolist in period n. Thus, $H_n' \in Y^{n+1} \times \Omega$. The strategy profile (σ,τ) induces strategy profiles $(\sigma|_{H_n},\tau|_{H_n})$ and $(\sigma|_{H_n'},\tau|_{H_n'})$, after the histories H_n and H_n', respectively. (σ,τ) is a <u>subgame perfect</u> equilibrium if and only if (σ,τ) is a Nash equilibrium and $(\sigma|_{H_n},\tau|_{H_n})$ is a Nash equilibrium in the game remaining after the history H_n, for all H_n and all n, and similarly after any history H_n'. In order to ensure the existence of an equilibrium, we will have to allow the monopolist to use mixed strategies. $\hat{\Sigma}$ will denote the mixed extension of Σ. It should be clear to the reader how to modify the above definitions when mixed strategies are employed.

Given any price history $\{p_k\}_{k=0}^{n-1}$, let $q_n = m(Q_n)$ be the proportion of customers who have purchased. Observe that the single number q_n summarizes, along the equilibrium path, the actions chosen by consumers prior to period n. Indeed, as the next lemma shows, consumer maximization requires that in any equilibrium in which consumer q has bought prior to period n, all consumers with valuations $v > f(q)$ should also have bought.

<u>Lemma 2.1</u>: In any equilibrium, in every period n, and after any history H_n, a buyer accepts a price offered by the monopolist if and only if his valuation exceeds some cutoff valuation $\beta(p,H_n)$.

<u>Proof</u>: See Fudenberg, Levine and Tirole [1985, Lemma 1].

When the indifference valuation $\beta(p,H_n)$ is independent of the prior history H_n, buyer strategies are referred to as <u>stationary</u>. Stationary consumer strategies are important because they lie at the heart of the Coase Conjecture. Gul, Sonnenschein and Wilson [1986] show that any subgame perfect equilibrium of the above game in which consumers use stationary strategies must satisfy the Coase Conjecture. We will refer to this set of equilibria as the set of <u>weak-Markov equilibria</u>, and denote it by the symbol $E^{wm}(f,z)$.

In weak-Markov equilibria, the monopolist conditions her strategy on the state of the game (the measure of consumers sold to) and the previous price.

In a strong-Markov equilibrium, the monopolist would condition her strategy on the payoff-relevant part of the history, namely q_n, only. Unfortunately, strong-Markov equilibria do not, in general, exist (see Fudenberg, Levine and Tirole [1985]). When f is strictly monotone, the state of the system can be equivalently described by $v = f(q)$. Letting $\phi(p_{-1},v)$ be the monopolist's (random) pricing rule for the subgame starting at (p_{-1},q), we see that a weak-Markov equilibrium must satisfy:

$$\beta(p) - p = \delta[\beta(p) - \bar{\phi}(p,\beta(p))] \tag{2.1}$$

i.e., the buyer with valuation $\beta(p)$ is indifferent between accepting the price p in the current period and waiting for next period's expected price $\bar{\phi}(p,\beta(p))$. Also, the monopolist's pricing rule is optimal given consumer behavior, i.e., $\bar{\phi}(p_{-1},v) \in \hat{\phi}(v)$. Here $\hat{\phi}(v)$ is the convex hull of $\phi(v)$, the set of maximizers in:

$$\pi(v) = \max_p \{p[f^{-1}(\beta(p)) - f^{-1}(v)] + \delta\pi(\beta(p))\} \tag{2.2}$$

$\pi(v)$ denotes the monopolist's present discounted value of profits when the state is v, and $\delta = e^{-rz}$ is the discount factor. The above notation is convenient when f(q) is strictly monotone; we will use it in Section 3, where we present the example of a linear demand curve. Furthermore, the notation is necessary for our existence proof in Section 4, which relies crucially on the fact that a certain family of functions $\beta(p)$ is equicontinuous.

Nevertheless, this elegant notation fails when f(q) has flat sections. In this case, it is easier to think of the monopolist as choosing quantities rather than prices. The strategies of monopolist and consumers can then always be converted back into price strategies, using the trick in Gul, Sonnenschein and Wilson [1986, Theorem 1]. Since $\beta(p)$ may be taken to be monotone, consumer strategies in a weak-Markov equilibrium can be equivalently described by a monotone acceptance function $P(q)$, where $P(q)$ satisfies $\beta(P(q)) = f(q)$. Thus, consumer q accepts a price p if and only if $p \leq P(q)$. If the monopolist expects buyers to use the stationary strategy $P(\cdot)$, then along the equilibrium path her present discounted value of profits when all buyers in the interval $[0,q]$ have bought, $R(q)$, must satisfy the dynamic programming equation:

$$R(q) = \max_{y \geq q} \{P(y)(y - q) + \delta R(q)\}$$

Furthermore, consumer optimality requires:

$$f(q) - P(q) = \delta[f(q) - S(q)]$$

where $S(q) = P(t(q))$ and $t(q)$ is the monopolist's optimal choice when q buyers have bought.

One final remark: our model above can equally be interpreted as a bilateral bargaining problem in which a seller with a known valuation offers an object for sale to a buyer with privately known valuation. In each round of this infinite-horizon bargaining game, the seller makes a price offer which the buyer can either accept or reject. The bargaining continues until acceptance occurs. If $F(v)$ is the (common knowledge) distribution function of buyer valuations, then $F(v) = 1 - y$, where $y = \inf\{q: f(q) = v\}$. For expositional ease, all of our subsequent definitions, theorems and proofs will be phrased in the language of durable goods monopoly. However, all of our results also hold for the bargaining game, provided one substitutes "sequential equilibrium" whenever the phrase "subgame perfect equilibrium" appears. It should also be understood that $m(Q_n)$ corresponds to the seller's truncation, after history H_n, of his prior on the buyer's valuation.

3. A Linear Example

In this section we analyze a linear demand example with unit slope and unit intercept, and characterize the set of all equilibrium payoffs attainable by the seller in the game $G(z,r)$ when z is small. We will do this by proving the existence of reputational equilibria in which the monopolist refrains from rapid price cutting out of fear of inducing expectations (on the part of consumers) of even more rapid future price reductions. A particular stationary equilibrium (which we will refer to as the Coase path), will play a central role in the construction of these reputational equilibria. It is the limit of finite horizon equilibria, as first described by Stokey [1981], and later, in the bargaining context, by Sobel and Takahashi [1983]. This equilibrium is the unique one in which consumer strategies are analytic (Gul, Sonnenschein and Wilson [1986]). Furthermore, it is strong-Markov, i.e., it requires no randomization off the equilibrium path.

In this equilibrium, the monopolist's pricing rule for the subgame starting at the node v (i.e., after any history in which a measure of $(1 - v)$ customers have bought), is given by $\phi(v) = \alpha v$, yielding a present discounted value of profits of $\pi(v) = (\alpha/2)v^2$. Consumer strategies are described by their indifference valuation $\beta(p) = p/\sqrt{1 - \delta}$. An explicit expression for α is given by $\alpha = 1 - \delta^{-1} + \delta^{-1}\sqrt{1 - \delta}$. The reader can easily verify that the above strategies indeed form an equilibrium.

<u>Definition 3.1</u>: A <u>simple reputational strategy</u> is a triple (p,ω,μ), where p

denotes the initial price to be charged, ω refers to the price rule $p(v) = \omega v$ the monopolist follows if she set an initial price of p and no deviation from this price rule occurred in the past, and μ refers to the price rule $p(v) = \mu v$ to be followed otherwise.

We may now state our main result:

Theorem 3.2: For every $r > 0$ and every $\theta > 0$, there exists a $\bar{z} > 0$ such that for every $z \leqslant \bar{z}$ there exist subgame perfect equilibria yielding the monopolist all payoffs in $[\theta, \pi^* - \theta]$, where π^* is the static monopoly profit.

Proof: We will construct equilibria in which the seller follows a simple reputational strategy (p,ω,α). Let ε be the fraction, implicit in the pricing rule $p(v) = \omega v$, by which the monopolist lowers prices along the equilibrium path. The corresponding price sequence $p_n = \varepsilon^n p$ implies a pattern of historic variables v_n of:

$$v_n = \frac{p_{n-1} - \delta p_n}{(1 - \delta)} = \frac{(1 - \varepsilon\delta)}{(1 - \delta)} \varepsilon^{n-1} p \equiv \gamma p_{n-1}, \quad n \geqslant 1$$

Since v_n is strictly decreasing in n as long as $\varepsilon < 1$, the monopolist will have positive sales along the equilibrium path provided $p < \gamma^{-1}$. Sales in period n $(n \geqslant 1)$ can then be calculated to be:

$$v_n - v_{n+1} = \gamma(p_{n-1} - p_n) = \gamma(1 - \varepsilon)p_{n-1}$$

implying a pattern of profits of

$$\pi_n = \sum_{k=0}^{\infty} \delta^k (v_{n+k} - v_{n+k+1}) p_{n+k} = \gamma(1 - \varepsilon) \sum_{k=0}^{\infty} \delta^k p_{n+k-1} p_{n+k}$$

$$= \frac{\gamma\varepsilon(1-\varepsilon)}{1-\delta\varepsilon^2}\, p_{n-1}^2 = \frac{(1-\delta)\varepsilon(1-\varepsilon)}{(1-\varepsilon\delta)(1-\varepsilon^2\delta)}\, v_n^2$$

for all $n \geq 1$. Observe that since the Coase path (represented by the pricing rule $p(v) = \alpha v$) is subgame perfect starting at any node v, it will suffice to show that the seller does not have any incentive to deviate from the path $p_n = \varepsilon^n p$ when such deviations induce expectations that the future pricing rule will be $p(v) = \alpha v$. Since <u>optimal</u> deviation from p_n in period n will yield Coase conjecture profits of $(\alpha/2)v_n^2$, the seller will not deviate in period n ($n \geq 1$) if and only if:

$$\frac{(1-\delta)\varepsilon(1-\varepsilon)}{(1-\varepsilon\delta)(1-\varepsilon^2\delta)} \geq \frac{\alpha}{2}$$

Choosing $\varepsilon = e^{-sz}$, where $s = r^{3/2}\, z^{1/2}$, and recalling the definition of α, we see that the above condition can be rewritten as:

$$R(z) = \frac{2\, e^{-(r+s)z}(1-e^{-rz})(1-e^{-sz})}{(1-e^{-(r+s)z})(1-e^{-(r+2s)z})(e^{-rz}-1+\sqrt{1-e^{-rz}})} \geq 1$$

Observe that $\lim_{z \to 0} R(z) = 2$. Hence, there exists $\bar{z}_1 > 0$ such that for all z $(0 < z < \bar{z}_1)$, $R(z) > 1$.

Now observe that, for all $p \leq \gamma^{-1}$,

$$\pi_0 = [1-\gamma p]p + \delta\, \frac{\gamma\varepsilon(1-\varepsilon)}{1-\delta\varepsilon^2}\, p^2$$

Note that π_0 is a function of p and z, as $\delta = e^{-rz}$ and $\varepsilon = e^{-s(z)z}$. The simple reputational strategy $(p, \varepsilon\gamma^{-1}, \alpha)$ will induce a subgame perfect equilibrium for $z \leq \bar{z}_1$, iff

$$\pi_0(p,z) \geq \frac{\alpha(z)}{2}$$

Let $\Gamma(z) = \{p: \pi_0(p,z) \geq \frac{\alpha(z)}{2}\}$. Since $\lim_{z\to 0} \pi_0(p,z) = p(1 - p)$ and $\lim_{z\to 0} \alpha(z) = 0$, $\Gamma(z)$ grows and approaches the unit interval as $z \to 0$. In particular, the set of equilibrium payoffs supported by reputational strategies, i.e., the image of Γ under π_0, approaches the interval $[0, \pi^*]$ as $z \to 0$.[8] []

While other authors (Gul, Sonnenschein and Wilson [1986]) have noted the presence of multiple equilibria in this type of game, all previously discovered equilibria yield the seller essentially zero surplus as $z \to 0$. We showed above that when one considers the set of all subgame perfect equilibria (rather than just the set of weak-Markov equilibria) exactly the opposite result obtains.

4. Existence of Weak-Markov Equilibria

In the previous section, we constructed reputational equilibria for the linear demand example, and demonstrated that essentially all outcomes were supported by subgame perfect equilibria as the time interval between periods approached zero. Our method was to specify two price paths. The main path was defined by an initial price, p, and a slow but positive (real-time) rate of descent. This induced most customers who valued the good at greater than p to purchase in the initial period; only a trickle of customers bought in each subsequent period.

A deviation from the main path by the monopolist triggered a change in consumer expectations about future prices--consumers would expect the secondary path. By the Coase Conjecture, profits along the secondary path could be made arbitrarily small compared to the number of remaining customers by shrinking the time interval between periods. In contrast, profits along the continuation of the main path, while low, were kept bounded away from zero by a constant related to the rate of descent of prices. Hence, for any rate of descent, there was a sufficiently small time interval between periods such that the monopolist was deterred from deviating off the main path.

In order to extend this reasoning to general demand curves, we need to demonstrate two facts which we proved by formula for the linear case. We lay this groundwork here and in the next section. First, we show the existence of weak-Markov equilibria for the general demand curve (see also Appendix A). This gives us well-defined secondary paths. Then, in Section 5, we will demonstrate that these secondary paths become uniformly low as z approaches zero, enabling them to deter deviation from the main path.

We begin by defining general demand curves. Let ϕ be a real-valued correspondence on the real numbers. We will say that ϕ is monotone nonincreasing if:

$$(4.1) \qquad p_1 \in \phi(v_1) \text{ and } p_2 \in \phi(v_2) \Rightarrow (p_2 - p_1)(v_2 - v_1) \leq 0$$

and monotone nondecreasing if the inequality in (4.1) is reversed. We will call ϕ upper semicontinuous (u.s.c.) if:

$$(4.2) \qquad p^n \in \phi(v^n),\ p^n \to p \text{ and } v^n \to v \Rightarrow p \in \phi(v).$$

<u>Definition 4.1</u>: A <u>demand curve</u> f is a nonnegative-valued, monotone nonincreasing correspondence on [0,1]. Without loss of generality, we also assume:

(a) $f(q) > 0$ whenever $0 \leqslant q < 1$.[9]

(b) $f(0) = 1$.[10]

(c) f is left-continuous.[11]

(d) f^{-1} denotes the inverse correspondence of $\hat{f}$, where $\hat{f}$ is the monotone nonincreasing, u.s.c., convex-valued correspondence which agrees with f except on its points of discontinuity.[12]

In Appendix A, we prove the following theorem:

Theorem 4.2 (Existence of Weak-Markov Equilibria): Let f be any demand curve which is monotone decreasing in an interval $[\bar{q},1]$, for some $0 \leqslant \bar{q} < 1$. Then for any $r > 0$ and any $z > 0$, there exists a weak-Markov equilibrium.

This theorem strengthens results by Fudenberg-Levine-Tirole [1985], who prove existence for differentiable demand functions with derivative bounded below and above, and Gul-Sonnenschein-Wilson [1986], who prove existence for demand curves with $f(1) > 0$ that satisfy a Lipschitz condition at 1. For example, Theorem 4.2 extends existence to nondifferentiable and possibly discontinuous demand curves with $f(1) = 0$. It also contains the case where $f(1) > 0$ but $f'(1)$ is infinite. Observe that we do not prove any existence result for demand curves which have "flat sections" in all neighborhoods of 1. We also establish:

Corollary 4.3: Under the hypothesis of Theorem 4.2, there exists a weak-Markov equilibrium in which, along the equilibrium path, the monopolist plays only pure strategies.

Proof: See Appendix A.

5. The Uniform Coase Conjecture

In this section, we strengthen the "Coase Conjecture" by presenting a theorem that guarantees uniformly low prices for all weak-Markov equilibria of families of demand curves.[13] The proof of our theorem is relegated to Appendix B.

While the Uniform Coase Conjecture is of independent interest, we require it here as an intermediate step for use in the main result of the paper: the Folk Theorem of Section 6. It should be observed that there is a straightforward reason why we did not need to examine families of demand curves to treat the linear case in Section 3: given any linear demand curve, every derived residual demand curve is linear as well.[14] For generic demand curves, however, the residual demand curves are no longer rescaled versions of the original one. Thus, considerations of subgame perfection lead us naturally to study families of demand curves. We will demonstrate, for all residual demand curves arising from a demand curve f, that all price paths derived from weak-Markov equilibria are uniformly low compared to the highest remaining consumer valuation. This establishes that weak-Markov price paths may be used to deter deviation from the main price paths of reputational equilibria.

Define $\mathcal{F}_{L,M}$ to be the family of demand curves which are bounded above and below by straight lines with negative slope. We will sometimes refer to this condition as "Lipschitz above and below at 1." To be precise:

Definition 5.1: For $0 < M \leq 1 \leq L < \infty$, $\mathcal{F}_{L,M}$ is the set of all $f: [0,1] \to [0,1]$ such that:

(a) f is monotone nonincreasing, $f(0) = 1$, and $f(1) = 0$,

and

(b) $M(1 - x) \leq f(x) \leq L(1 - x)$, for all $x \in [0,1]$.

Let us also define a rescaled residual demand curve as a normalized version of the demand that remains after any proportion of customers have purchased:

Definition 5.2: Let f be any demand curve. We define f_q to be the rescaled residual demand curve of f at q ($0 \leq q < 1$) by:

$$f_q(x) = \frac{f[q + (1 - q)x]}{f(q)}, \text{ for all } x \in [0,1]$$

Lemma 5.3: If $f \in \mathcal{F}_{L,M}$, then for every q ($0 \leq q < 1$), $f_q \in \mathcal{F}_{L/M,M/L}$.

Proof: Observe that $f(q + (1 - q)x) = f(1 - (1 - q)(1 - x)) \leq L(1 - q)(1 - x)$ and $f(q + (1 - q)x) \geq M(1 - q)(1 - x)$. Meanwhile, $M(1 - q) \leq f(q) \leq L(1 - q)$, so

$$\frac{M}{L}(1 - x) \leq \frac{f[q + (1 - q)x]}{f(q)} \leq \frac{L}{M}(1 - x)$$

proving the desired result. □

Let $L = L'/M'$ and $M = M'/L'$. Lemma 5.3 demonstrates that if $f \in \mathcal{F}_{L',M'}$, then all residual demand curves arising from f are elements of $\mathcal{F}_{L,M}$. Hence, if we can show that the initial price is uniformly low for all demand curves in the family $\mathcal{F}_{L,M}$, then we will also have established that all price paths arising from weak-Markov equilibria are uniformly low (compared to residual demand). We prove this fact in the following theorem:

Theorem 5.4 (The Uniform Coase Conjecture): For every $L \geq 1$, M ($0 < M \leq 1$) and $\varepsilon > 0$, there exists $\bar{z}(L,M,\varepsilon)$ such that for every $f \in \mathcal{F}_{L,M}$, for every z satisfying $0 < z < \bar{z}(L,M,\varepsilon)$, and for every weak-Markov equilibrium $(S,P) \in E^{wm}(f,z)$, the monopolist charges an initial price less than or equal to ε (and earns profits less than ε).

Our notation will be $S(0) \leq \varepsilon$.

Proof: See Appendix B.

Theorem 5.4, as stated here and as used in establishing the existence of reputational equlibria, assumes $f(1) = 0$. But the Uniform Coase Conjecture still holds when $f(1) > 0$. Define $\mathcal{F}^c_{L,M}$ to be the family of demand curves which satisfy $f(1) \leq c$ and are "Lipschitz above and below at 1," i.e.,

$$M(1 - x) \leq f(x) - f(1) \leq L(1 - x), \text{ for all } x \in [0,1]$$

Then the following can also be proved:

Corollary 5.5: For every $L \geq 1$, $M \leq 1$, $\varepsilon > 0$ and $c < 1$, there exists $\bar{z}(L,M,\varepsilon,c)$ such that for every $f \in \mathcal{F}^c_{L,M}$, for every z satisfying $0 < z < \bar{z}(L,M,\varepsilon,c)$, and for every weak-Markov equilibrium $(S,P) \in E^{wm}(f,z)$,

$$S(0) \leq f(1) + \varepsilon$$

Finally, let us define $\bar{\mathcal{F}}_{L,M}$ to be the set of demand curves which jointly satisfy the hypotheses of Theorems 4.2 and 5.4.

Definition 5.6: $\bar{\mathcal{F}}_{L,M}$ is the set of all f such that:

(a) $f \in \mathcal{F}_{L,M}$, and

(b) f is monotone decreasing in an interval $[\bar{q},1]$, for some $0 \leq \bar{q} < 1$.

6. The Folk Theorem for Bargaining and Durable Goods Monopoly

In this section, we prove the main result of the paper.

Theorem 6.1 (The Folk Theorem): Let f be any demand curve which has the Uniform Coase Property, and let π^* denote the static monopoly profits. Then for every real interest rate r and for every $\varepsilon > 0$, there exists a $\bar{z} > 0$ such that whenever the time interval between successive offers satisfies $0 < z < \bar{z}$:

$$(6.1) \qquad [\varepsilon, \pi^* - \varepsilon] \subset SE(f,r,z)$$

In particular, the above result holds for all $f \in \bar{\mathcal{F}}_{L,M}$, and for $f(q) = (1 - q)^n$, where $0 < n < \infty$.

SE(f,r,z) denotes, for the durable goods monopoly model, the set of all monopolist payoffs arising from subgame perfect equilibria when the demand curve is f, the interest rate is r, and the time interval between periods is z. For the bargaining game with one-sided incomplete information, it denotes the set of all seller payoffs arising from sequential equilibria. Theorem 6.1 proves that SE(f,r,z) expands to the entire interval from zero to static monopoly profits, as the time interval z approaches zero. The theorem only requires that f have the Uniform Coase Property, which we now define:

Definition 6.2: We will say that f has the Uniform Coase Property if, for some $z_1 > 0$:

(6.2a) There exists a subgame perfect equilibrium (σ_z, τ_z) for all games with time interval z between periods, where $0 < z < z_1$, and, (6.2b) For every $\varepsilon > 0$, there exists $\bar{z}(\varepsilon)$ $(0 < \bar{z}(\varepsilon) < z_1)$ such that

$$S_z(q)/f(q) < \varepsilon, \text{ for all } z\ (0 < z < \bar{z}(\varepsilon))$$
$$\text{for all } q\ (0 \leq q < 1)$$

where $S_z(q)$ denotes the supremum of all prices that the monopolist charges using strategy σ_z when the current state equals q (the supremum is taken over all possible price histories).

The next lemma proves that the hypothesis of Theorem 6.1 is far from vacuous. If the demand curve has $f(1) = 0$, is monotone decreasing in a neighborhood of 1, and is "Lipschitz above and below" at 1, then Theorem 6.1 applies. Also, if f belongs to the family of demand curves studied by Sobel-Takahashi [1983], the Folk Theorem holds.

Lemma 6.3: If $f \in \bar{\mathcal{F}}_{L,M}$, or if $f(q) = (1 - q)^n$ with $0 < n < \infty$, then f has the Uniform Coase Property.

Proof: Suppose $f \in \bar{\mathcal{F}}_{L,M}$. Then f is monotone decreasing in a neighborhood of 1 (by Definition 5.6), and so there exists $(\sigma_z,\tau_z) \in E^{wm}(f,z)$ for all $z > 0$ (by Theorem 4.2). We wish to show that $\{\sigma_z,\tau_z\}_{z>0}$ satisfies (6.2b). Observe, for any z, that (σ_z,τ_z) induces a weak-Markov equilibrium for any residual demand curve arising from f. Define $(\sigma_{z,q},\tau_{z,q})$ by multiplying all prices in (σ_z,τ_z) by f(q); observe that $(\sigma_{z,q},\tau_{z,q})$ is a weak-Markov equilibrium for the rescaled residual demand curve f_q, for all $0 \leq q < 1$. (See Definition 5.2.)

Using the notation in Definition 6.2, observe that:

$$S_z(q) = f(q) \cdot S_{z,q}(0)$$

Further observe, by Lemma 5.3, that $f_q \in \mathcal{F}_{L',M'}$, where $L' \equiv L/M$ and $M' \equiv M/L$. By the Uniform Coase Conjecture (Theorem 5.4), for every $\varepsilon > 0$, there exists $\bar{z}(\varepsilon)$ such that the initial price in any weak-Markov equilibrium is less than ε, for any z $(0 < z < \bar{z})$ and any demand curve in $\mathcal{F}_{L',M'}$. We conclude:

$$S_z(q) \leqslant f(q)\varepsilon, \text{ for all } q \ (0 \leqslant q < 1)$$

Suppose $f(q) = (1 - q)^n$, where $0 < n < \infty$. Then for every $z > 0$, a weak-Markov equilibrium can explicitly be calculated which also satisfies (6.2) (see Sobel-Takahashi [1983]). []

Let p^i denote the price actually charged in period i and let q^i denote the actual proportion of customers who have purchased before period i. Let σ denote a pure (or mixed) strategy for the monopolist, which gives price(s) as a function of previous prices $(p^0,\ldots,p^{n-1})$ and the state (q^n).

Definition 6.4: For any $\vec{p} = \{p_n\}_{n=0}^{\infty}$, any $\vec{q} = \{q_n\}_{n=1}^{\infty}$, and any monopolist strategy σ, define the reputational price strategy $(\vec{p},\vec{q},\sigma)$ by:

$$p^n = \begin{cases} p_n, & \text{if } p^i = p_i \text{ for all } i \ (0 \leqslant i \leqslant n-1) \text{ and if } q^n = q_n \\ \sigma\ , & \text{otherwise} \end{cases}$$

We will further call $(\vec{p},\vec{q},\sigma)$ a reputational equilibrium if this reputational price strategy, in conjunction with optimal consumer behavior, forms a subgame perfect equilibrium.

Observe that Definition 6.4 requires that strategy σ, by itself, be associated with a subgame perfect equilibrium.

A reputational equilibrium is defined analogously for our bargaining game. $(\tilde{p},\sigma)$ is required to give a sequential equilibrium. $\tilde{q}$ can be omitted from the definition of the reputational price strategy because it is never actually observed by the seller. Rather, the seller only observes a decision to purchase, which concludes the game. We now prove Theorem 6.1, in the language of durable goods monopoly.

Proof of Theorem 6.1: Let $\{\sigma_z,\tau_z\}_{0<z<z_1}$ be the family of subgame perfect equilibria guaranteed by (6.2a), and let $\{S_z\}_{0<z<z_1}$ be defined as in Definition 6.2. Define the function g by:

$$(6.3) \qquad g(z) = \sup \left\{\frac{S_x(q)}{f(q)} : 0 < x \leqslant z \text{ and } 0 \leqslant q < 1\right\}$$

Observe that $g(z)$ is well defined for $0 < z < z_1$, since S_z is defined and $S_z(q)/f(q)$ is uniformly bounded above by 1. Further observe, by (6.2b), that $\lim_{z\to 0} g(z) = 0$.

By definition, $q_0 = 0$. Choose arbitrary sales q_1 in period zero $(0 \leqslant q_1 < 1)$. Let us define a sequence of subsequent quantities, in a fashion analogous to our treatment of the linear case in Section 3.

$$(6.4) \qquad 1 - q_{n+1} = e^{-naz(rz+g(z))}(1 - q_1), \text{ for any } a > 0 \text{ and all } n \geqslant 0$$

Equation (6.4) states that there is a constant c $(0 < c < 1)$ such that, in all periods after period zero, the monopolist sells to proportion c of all remaining customers.

Our first step is to construct a price sequence $\{p_n\}_{n=0}^{\infty}$ which yields sales of $(q_{n+1} - q_n)$ in period n (all $n \geqslant 0$). Observe that if $0 < q_1 < 1$, then (6.4) implies sales in all periods, so consumer q_{n+1} is indifferent between purchasing at price p_n in period n and at price p_{n+1} one period later, for all $n \geq 0$. Hence:

$$(6.5) \qquad f(q_{n+1}) - p_n = \delta[f(q_{n+1}) - p_{n+1}], \text{ for all } n \geq 0$$

where $\delta = e^{-rz}$. Therefore:

$$p_n = (1 - \delta)f(q_{n+1}) + \delta p_{n+1} = (1 - \delta)f(q_{n+1}) + \delta[(1 - \delta)f(q_{n+2}) + \delta p_{n+2}]$$

Telescoping this summation gives:

$$(6.6) \qquad p_n = (1 - \delta) \sum_{k=0}^{\infty} \delta^k f(q_{n+1+k}), \; n \geq 0$$

Furthermore, the price sequence $\vec{p} \equiv \{p_n\}_{n=0}^{\infty}$ implied by (6.4) and (6.6) satisfies $f(q_{n+1}) \geq p_n$ (for all $n \geq 0$) and satisfies equation (6.5), proving that consumers optimize along the sales path $\vec{q} \equiv \{q_n\}_{n=1}^{\infty}$.

Claim 1: For any q_1 $(0 \leq q_1 < 1)$, there exists $a > 0$ and $\bar{z} > 0$ such that $(\vec{p},\vec{q},\sigma_z)$ defined by (6.4) and (6.6) is a reputational equilibrium for all z satisfying $0 < z < \bar{z} < z_1$.

Proof of Claim 1: Let π_n denote profits starting from period n if the price path $\vec{p}$ is followed in all periods. Define m to be the least integer greater than $1/z$. Certainly:

$$\pi_n \geq \delta^{m-1}[q_{n+m} - q_n]p_{n+m}$$

Observe that $\delta^{m-1} \equiv e^{-(m-1)rz} \geq e^{-r}$ and, by (6.4), $q_{n+m} - q_n =$

$(1 - q_n) - (1 - q_{n+m}) \geq (1 - q_n)(1 - e^{-a(rz+g(z))})$, for all $n \geq 1$. Meanwhile, by (6.6), $p_{n+m} \geq (1 - \delta) \sum_{k=0}^{m-1} \delta^k f(q_{n+m+1+k}) \geq f(q_{n+2m})(1 - \delta) \sum_{k=0}^{m-1} \delta^k \geq (1 - e^{-r})f(q_{n+2m})$. Hence:

$$(6.7) \qquad \pi_n \geq e^{-r}(1 - q_n)(1 - e^{-a(rz+g(z))})(1 - e^{-r})f(q_{n+2m}),\ n \geq 1$$

and by similar reasoning,

$$(6.8) \qquad \pi_0 \geq q_1 p_0 \geq q_1(1 - e^{-r})f(q_m)$$

Meanwhile, let π_n^z denote profits starting from period n if (σ_z,τ_z) is followed. Let q $(0 \leq q < 1)$ denote a customer and let p_q denote the price at which customer q purchases, according to (σ_z,τ_z). Let p_q' denote the next (expected) price charged after p_q, following σ_z. Observe by the definition of g(z) that $p_q' \leq g(z)f(q)$. By consumer optimality, $f(q) - p_q \geq \delta[f(q) - p_q']$. Together these inequalities imply:

$$p_q \leq [1 - \delta + \delta g(z)]f(q), \text{ for all } q \ (0 \leq q < 1)$$

and so:

$$(6.9) \qquad \pi_n^z \leq [1 - \delta + \delta g(z)] \int_{q_n}^1 f(q)dq, \text{ for all } n \geq 0$$

Observe that the bound of (6.9) is a consequence of the Uniform Coase Property, but does not follow from the ordinary Coase Conjecture.

Let $a = 8[e^{-r}(1 - e^{-r})]^{-1}$. To prove subgame perfection, we must show that $\pi_n \geq \pi_n^z$, for all $n \geq 0$. Observe that there exists z_2 such that for all z

$(0 < z < z_2)$:

$$1 - e^{-a[g(z)+rz]} > \frac{a}{2} [g(z) + rz]$$

Hence, (6.7) implies that $\pi_n > 4(g(z) + rz)(1 - q_n)f(q_{n+2m})$ for all $n \geq 1$ and $0 < z < z_2$. Since $q_n < q_{n+2m} < 1$ and f is monotone nonincreasing:

$$\int_{q_n}^{1} f(q)dq \leq (q_{n+2m} - q_n)f(q_n) + (1 - q_{n+2m})f(q_{n+2m})$$

Furthermore, $q_{n+2m} - q_n \leq q_{1+2m} - q_1$ for all $n \geq 1$, and $\lim_{z \to 0} q_{1+2m} = q_1$, so there exists $z_3 > 0$ such that $\int_{q_n}^{1} f(q)dq \leq 2(1 - q_n)f(q_{n+2m})$ for all $n \geq 1$ and for all z $(0 < z < z_3)$. Finally, there exists $z_4 > 0$ such that $[1 - \delta + \delta g(z)] \leq 2[g(z) + rz]$ for all z $(0 < z < z_4)$. Hence, for all z satisfying $0 < z < \min\{z_1, z_2, z_3, z_4\}$ and for all $n \geq 1$, we have by (6.9) that

$$\pi_n \geq \pi_n^z, \; n \geq 1$$

It can easily be shown that we may set $\bar{z}$ so that $\pi_0 \geq \pi_0^z$ for all z $(0 < z < \bar{z})$ as well.

Claim 2: For any q $(0 < q < 1)$ and any $\lambda < 1$, there exists $\bar{z} > 0$ such that for every z $(0 < z < \bar{z})$, there is a reputational equilibrium with profits at least $\lambda q f(q)$.

Proof of Claim 2: Set $q_1 = \sqrt{\lambda}\, q$. Define m to be the least integer greater than $-\log(1 - \sqrt{\lambda})/rz$. (Observe that $e^{-rmz} \approx 1 - \sqrt{\lambda}$.) Now define $\{q_n\}_{n=2}^{\infty}$ by (6.4). Then for arbitrary $a > 0$, there exists z_5 such that for every z $(0 < z < z_5)$, $q_m < q$. By (6.6), p_0 (which induces sales of q_1) satisfies:

$$p_0 > (1 - \delta) \sum_{k=0}^{m-1} \delta^k f(q_{k+1}) \geqslant (1 - \delta^m) f(q_m) \geqslant$$

$$\geqslant [1 - (1 - \sqrt{\lambda})] f(q) = \sqrt{\lambda}\, f(q)$$

whenever $0 < z < z_5$. Hence, $\pi_0 \geqslant p_0 q_1 \geqslant \lambda q f(q)$. Using Claim 1, there exists $\bar{z} > 0$ such that $(\vec{p}, \vec{q}, \sigma_z)$ defined using $q_1 = \sqrt{\lambda}\, q$, (6.4) and (6.6) is a reputational equilibrium, for all z $(0 < z < \bar{z})$, proving Claim 2.

<u>Remainder of Proof of Theorem 6.1</u>: Given any q_1, let the quantity path $\vec{q}$ be defined by (6.4), let the price path $\vec{p}$ be defined by (6.6), and let $\pi(q_1, z)$ denote the profits associated with $\vec{q}$ and $\vec{p}$. Then:

$$\text{(6.10)} \qquad \pi(q_1, z) = \sum_{k=0}^{\infty} e^{-krz}(q_{k+1} - q_k) p_k = q_1 p_0 + \sum_{k=1}^{\infty} \delta^k (q_{k+1} - q_k) p_k$$

Suppose $q_1' > q_1$, and define $\vec{q}'$ and $\vec{p}'$ analogously. Observe, by (6.4), that $q_{k+1}' - q_k' < q_{k+1} - q_k$ for all $k \geqslant 1$, and by (6.6), that $p_k' \leqslant p_k$ for all $k \geqslant 0$. Hence, using (6.10):

$$\text{(6.11)} \qquad \pi(q_1', z) \leqslant \pi(q_1, z) + |q_1' - q_1|$$

Define $\tilde{\pi}(q_1, z)$ by:

$$\tilde{\pi}(q_1, z) = \sup_{0 \leqslant q \leqslant q_1} \pi(q, z)$$

Observe that $\tilde{\pi}$ is monotone nondecreasing in q_1 and also satisfies (6.11). Hence, $\tilde{\pi}$ is continuous in q_1, and so the image of any interval of q_1's is an interval, for any $z > 0$.

Let $\pi^* = \sup_{0 \leq q \leq 1} qf(q)$ and let $\pi^* = q^* f(q^*)$. Given ε $(0 < \varepsilon < \pi^*)$, define $\lambda = [\pi^* - \varepsilon]/\pi^*$. By Claim 2, there exists $z_6 > 0$ such that there exists a reputational equilibrium with profits at least $\lambda\pi^* = \pi^* - \varepsilon$ whenever $0 < z < z_6$. Also observe that $\lim_{z \to 0} \pi(0,z) = 0$, and so there exists $z_7 > 0$ such that $\pi(0,z) < \varepsilon$ whenever $0 < z < z_7$. Finally, by Claim 1, there exists $z_8 > 0$ such that $(\vec{p},\vec{q},\sigma_z)$ defined from $q_1 = 0$ is a reputational equilibrium whenever $0 < z < z_8$.

Define $\bar{z} = \min\{z_6, z_7, z_8\}$. Then for any z satisfying $0 < z < \bar{z}$, $\pi(0,z) < \varepsilon$ and $\pi(\lambda q^*, z) > \pi^* - \varepsilon$. Furthermore, we have already shown that $\pi_n \geq \pi_n^z$ for $0 < z < \bar{z}$ and $n \geq 1$, so $(\vec{p},\vec{q},\sigma_z)$ is a reputational equilibrium for all q_1 that yield $\pi_0 \geq \pi(0,z)$. Finally, we have shown that the image of $\pi(q_1,z)$ when $q_1 \in [0, \lambda q^*]$ is an interval. Since ε and $\pi^* - \varepsilon$ are both contained in that interval, we have proved (6.1). $\square$

7. Conclusion

Consider the outcome of durable goods monopoly (or bargaining) when the time interval between successive periods approaches <u>infinity</u>. In this situation, the monopolist (seller) has close to unlimited commitment power, and thus her maximum equilibrium payoff approaches static monopoly profits. Meanwhile, as we demonstrated in the Folk Theorem, the same outcome is attainable in the limit as the time interval approaches zero. We conclude that the graph of the time interval between periods versus maximum attainable profits is U-shaped.[15] Hence, the graph attains a minimum at some intermediate level--let us call this the <u>time interval of least commitment</u>.

We explain this phenomenon as the result of two countervailing forces. When the time interval between periods is short, reputational effects may be devastatingly effective in preserving monopoly power. When the time interval between periods is long, reputational effects are superfluous. The most adverse circumstance for the monopolist may be when the time interval is just long enough to preclude reputational equilibria (but still sufficiently short that the inability to commit is a problem).

We would also like to draw a comparison between the present monopoly model and the analogous oligopoly model. In "One is Almost Enough for Monopoly" (Ausubel and Deneckere [1985]), we prove Folk Theorems for durable goods oligopoly and for monopoly with potential entry. As the time interval between successive periods approaches zero, all joint payoffs between zero and static monopoly profits are possible. (Gul [1985] simultaneously and independently established a similar result for the case of oligopoly.) Recall our observation in the Introduction that there are actually two distinct cases to consider: those where there is a gap between seller and buyer valuations and those where there is none. Surprisingly, in the case of the "gap," durable goods oligopolists potentially earn much greater profits than the monopolist. The (generically) unique monopoly equilibrium has price rapidly dropping to the lowest consumer valuation, while for a sufficiently short time interval between periods, the oligopolists can support a price near the static monopoly price. However, when the time interval between periods becomes sufficiently long, the monopolist's profits outstrip the oligopolists', as the monopolist gains commitment power while the oligopolists collapse to the Bertrand outcome.

In the more reasonable[16] case of "no gap," there is a Folk Theorem for any number of firms, so the limiting sets of joint payoffs coincide. For short but nonzero time intervals between periods, the oligopolists could earn somewhat greater joint profits than does the monopolist, since the punishment following deviation can be more severe. Oligopolists may expect a zero price next period, while the consequence for the monopolist is a Coase Conjecture price, which is always greater than zero. For sufficiently long time intervals between periods, the monopolist does better, by the same reasoning as in the case of the "gap."

The dichotomy between the two cases, while surprising, is supported by the following intuition. Fudenberg, Levine and Tirole [1985] demonstrated that in the case of the "gap," sales occur in finitely many periods. In contrast, sales necessarily occur over infinite time in the case of "no gap." Hence, the paradox presented here is precisely the dichotomy between the finitely- and infinitely-repeated prisoner's dilemma.

While we have only explicitly examined bargaining models where one party makes all the offers, our analysis has implications for alternating-offer models. Following Fudenberg, Levine and Tirole [1985], we have embedded our reputational equilibria in the analogous alternating-offer model. In these equilibria, buyers make only nonserious counteroffers, which the seller rejects. Thus, the embedded equilibria are payoff equivalent to those of a one-sided bargaining model in which the seller makes all the offers and in which the time interval between offers is twice as long. This establishes a Folk Theorem for bargaining games with more general extensive forms.

Appendix A

Existence of Weak-Markov Equilibria

The theorems in both appendices make substantial use of the ideas and analysis contained in Fudenberg, Levine and Tirole [1985] and in Gul, Sonnenschein and Wilson [1986]. We learned a great deal from these authors, and we owe them a substantial intellectual debt.

Definition A.1: Let ϕ be a correspondence. If a solution exists to:

$$(A.1) \qquad p \in (1 - \delta)\beta(p) + \delta\phi(\beta(p))$$

we will call this solution the "β implied by ϕ."

Lemma A.2: Let ϕ_q be a monotone decreasing upper semicontinuous correspondence defined on domain $[f(1),f(q)]$, where $q \leq 1$. Then there exists a "β implied by the convex hull of ϕ_q" on the domain:

$$(A.2) \qquad I_q \equiv [f(1), (1 - \delta)f(q) + \delta \sup \phi_q(f(q))]$$

Furthermore, β is uniquely defined, and it is a monotone nondecreasing, Lipschitz-continuous function on I_q (with Lipschitz constant $1/(1 - \delta)$).

Proof: Without loss of generality, let ϕ be convex valued (as its convex hull is uniquely defined). Since ϕ is monotone and u.s.c., and $\delta < 1$, $(1 - \delta)v + \delta\phi(v)$ is a monotone decreasing, u.s.c. and convex-valued correspondence of v. Hence, its inverse is a uniquely-defined, monotone nondecreasing function $\beta(p)$ which satisfies relationship (A.1) on I_q.

Suppose $p_2 > p_1$. Then by (A.1):

$$p_2 - p_1 \in (1 - \delta)[\beta(p_2) - \beta(p_1)] + \delta[\phi(\beta(p_2)) - \phi(\beta(p_1))]$$

The second term of the right side is nonnegative, implying

$$p_2 - p_1 \geq (1 - \delta)[\beta(p_2) - \beta(p_1)] \geq 0$$

proving Lipschitz continuity. $\square$

Notation A.3: Suppose ϕ_q satisfies the hypothesis of Lemma A.2. Let β_q denote the "β implied by the convex hull of ϕ_q." Now let $\beta_q'(p,v)$ denote a function defined on $[0,1] \times [f(1), 1]$ by:

$$\text{(A.3)} \qquad \beta_q'(p,v) = \begin{cases} \beta_q(p), & \text{if } p \in I_q \text{ and } \beta_q(p) < v \\ v & \text{, otherwise} \end{cases}$$

$\beta_q'(p,v)$ represents the new state which arises if the historic state is v, the monopolist charges a price p, and consumers use the acceptance function β_q. If $p \in I_q$ (the domain of β_q), we choose the new state to be the minimum of $\beta_q(p)$ and v, since the monopolist cannot induce negative sales by charging a high price. Meanwhile, if $p \notin I_q$, we adopt the convention that the new state is v.

Definition A.4: Let f be monotone decreasing on $[q,1]$, where $q < 1$. Let ϕ_q be a correspondence defined on the domain $[f(1), f(q)]$, and let π_q be a function defined on the same domain. We will say that (ϕ_q,π_q) supports a weak-Markov equilibrium on $[q,1]$ if:

(a) ϕ_q satisfies (4.1), i.e., ϕ_q is monotone nondecreasing;

(b) ϕ_q satisfies (4.2), i.e., ϕ_q is u.s.c.;

(c) For every $v \in [f(1),f(q)]$:

$$\text{(A.4)} \qquad \pi_q(v) = \max_{0 \leq p \leq 1} \{[f^{-1}(\beta_q'(p,v)) - f^{-1}(v)]p + \delta\pi_q(\beta_q'(p,v))\}$$

where β_q, I_q, and β_q' are defined by (A.1), (A.2), and (A.3), respectively; and

(d) For every $v \in [f(1),f(q)]$, inf $\phi_q(v)$ and sup $\phi_q(v)$ are both contained in the argmax correspondence of (A.4).

Lemma A.5:[17] Suppose $\bar{q} < q < 1$, and let f be a monotone decreasing demand curve on $[\bar{q},1]$. Furthermore, let (ϕ_q,π_q) support a weak-Markov equilibrium on $[q,1]$. Then there exist ϕ and π such that $\phi(v) = \phi_q(v)$ and $\pi(v) = \pi_q(v)$ whenever $f(1) \leq v < f(q)$ and (ϕ,π) supports a weak-Markov equilibrium on $[\bar{q},1]$.

Proof: We proceed constructively. Suppose that $0 < q < 1$. Let $q' = \max\{0,\ q - \frac{1}{2}(1 - \delta)\pi_q(f(q))\}$. Observe that $q < 1$ implies $\pi_q(q) > 0$ (since we are in a subgame perfect equilibrium), so $q' < q$. Let us extend (ϕ_q,π_q) to $(\phi_{q'},\pi_{q'})$ defined on $[q',1]$ by:

$$\text{(A.5)} \qquad \pi_{q'}(v) = \max_{0 \leq p \leq 1} \{[f^{-1}(\beta_q'(p,v)) - f^{-1}(v)]p + \delta\pi_q(\beta_q'(p,v))\}$$

for all v satisfying $f(1) \leq v \leq f(q')$. Let $\mu_{q'}(v)$ be the convex hull of the argmax correspondence of (A.5), and define:

$$\text{(A.6)} \qquad \phi_{q'}(v) = \begin{cases} \phi_q(v) & \text{, if } f(1) \leq v < f(q) \\ \{p \in \mu_{q'}(v)\colon p \geq \sup\limits_{v'<v} (\mu_{q'}(v'))\}, & \text{if } f(q) \leq v \leq f(q') \end{cases}$$

Observe that $\phi_{q'}(v) = \phi_q(v)$ whenever $f(1) \leq v < f(q)$, by the definition of $\phi_{q'}$, and $\pi_{q'}(v) = \pi_q(v)$ whenever $f(1) \leq v \leq f(q)$, by part (c) of Definition A.4. We will now show that $(\phi_{q'},\pi_{q'})$ supports a weak-Markov equilibrium on $[q',1]$. This will effectively complete the proof, since we may define $q'' = \max\{0, q' - \frac{1}{2}(1 - \delta)\pi_q(f(q))\}$, $q''' = \max\{0, q'' - \frac{1}{2}(1 - \delta)\pi_q(f(q))\}$, etc. With a finite number of repetitions of this argument, the extension to the interval $[\bar{q},1]$ will be complete.

First, let us show that $\phi_{q'}$ is monotone, nondecreasing and u.s.c. Suppose $f(1) \leq v_1 < v_2 \leq f(q')$, $p_1 \in \phi_{q'}(v_1)$, and $p_2 \in \phi_{q'}(v_2)$. Without loss of generality, assume $v_1 \geq f(q)$ (otherwise, monotonicity and u.s.c. follow from the fact that $\phi_{q'}$ coincides with ϕ_q). Then $p_1 \in \mu_{q'}(v_1)$ and $p_2 \in \mu_{q'}(v_2)$, the convex hull of the argmax correspondence of (A.5). Clearly, also, $p_1 \in I_q$ and $p_2 \in I_q$, so:

$$\pi_{q'}(v_1) = [f^{-1}(\beta_q(p_1)) - f^{-1}(v_1)]p_1 + \delta\pi_q(\beta_q(p_1)) \geq$$

$$\geq [f^{-1}(\beta_q(p_2)) - f^{-1}(v_1)]p_2 + \delta\pi_q(\beta_q(p_2))$$

and

$$[f^{-1}(\beta_q(p_1)) - f^{-1}(v_2)]p_1 + \delta\pi_q(\beta_q(p_1)) \leq$$

$$\leq [f^{-1}(\beta_q(p_2)) - f^{-1}(v_2)]p_2 + \delta\pi_q(\beta_q(p_2)) = \pi_{q'}(v_2)$$

Subtracting the first inequality from the second:

$$[f^{-1}(v_1) - f^{-1}(v_2)]p_1 \leq [f^{-1}(v_1) - f^{-1}(v_2)]p_2$$

If $f^{-1}(v_1) - f^{-1}(v_2) > 0$, we conclude $p_1 \leqslant p_2$, proving monotonicity. If $f^{-1}(v_1) - f^{-1}(v_2) = 0$, monotonicity immediately follows from (A.6), since then, $\mu_{q'}(v_1) = \mu_{q'}(v_2)$. Meanwhile, since f is monotone decreasing, f^{-1} is continuous. Since the domain of f^{-1} is compact, f^{-1} is also uniformly continuous. Consequently, if $v^n \to v$, the maximands of (A.5) corresponding to v^n uniformly converge to the maximand corresponding to v. By the Theorem of the Maximum, if $p^n \in \mu_{q'}(v^n)$ for every n and $p_n \to p$, then $p \in \mu_{q'}(v)$. By (A.6), $\phi_{q'}$ is also u.s.c.

Second, let us show that $\pi_{q'}$ and the $\beta'_{q'}$ implied by the convex hull of $\phi_{q'}$ satisfy (A.4). Observe, by Lemma A.2, that $\beta_{q'}$ is uniquely defined from $\phi_{q'}$, and coincides with β_q for all $p \in I_q$. Define $\tilde{\pi}$ by:

$$\text{(A.7)} \qquad \tilde{\pi}(v) = \max_{0 \leqslant p \leqslant 1} \left\{[f^{-1}(\beta'_{q'}(p,v)) - f^{-1}(v)]p + \delta\pi_{q'}(\beta'_{q'}(p,v))\right\}$$

We will now demonstrate that (A.7) has its argmax in I_q for all v satisfying $f(1) \leqslant v \leqslant f(q')$, and so $\tilde{\pi}(v)$ of (A.7) coincides with $\pi_{q'}(v)$ of (A.5). Suppose not. Let p' be an element of the argmax for v', such that $p' \notin I_q$. Observe that $f^{-1}(\beta'_{q'}(p',v')) \leqslant q$, $f^{-1}(v') \geqslant q'$, and $\pi_{q'}(\cdot) \leqslant \tilde{\pi}(\cdot)$. Hence:

$$\tilde{\pi}(v') = [f^{-1}(\beta'_{q'}(p',v')) - f^{-1}(v')]p' + \delta\pi_{q'}(\beta'_{q'}(p',v')) \leqslant$$

$$\leqslant [q - q']p' + \delta\pi_{q'}(v') \leqslant \tfrac{1}{2}(1 - \delta)\pi_{q'}(f(q)) + \delta\pi_{q'}(v') <$$

$$< \pi_{q'}(v') \leqslant \tilde{\pi}(v')$$

a contradiction.

We have thus that (A.7) has its argmax in I_q, and so $\tilde{\pi}$ coincides with $\pi_{q'}$

(for all v satisfying $f(1) \leq v \leq f(q')$). This proves that $\pi_{q'}$ satisfies (A.4). We conclude that $(\phi_{q'}, \pi_{q'})$ supports a weak-Markov equilibrium on $[q',1]$. $\square$

Lemma A.6: Let f be a monotone decreasing demand curve on $[q,1]$, where $q < 1$. Suppose that (ϕ,π) supports a weak-Markov equilibrium on $[q,1]$. Then π is monotone nondecreasing and continuous, satisfying:

$$0 \leq \pi(v_2) - \pi(v_1) \leq f^{-1}(v_1) - f^{-1}(v_2), \text{ if } v_1 < v_2$$

Proof: Let β be implied by ϕ. By definition, π satisfies:

$$\pi(v) = \max_p \left\{[f^{-1}(\beta(p)) - f^{-1}(v)]p + \delta\pi(\beta(p))\right\}$$

Let $v_1 < v_2$, $p_1 \in \phi(v_1)$ and $p_2 \in \phi(v_2)$. Observe that the monopolist, starting from state v_2, can charge a price of p_1 and follow her equilibrium strategy in the subgame which ensues. Hence:

$$\pi(v_2) \geq [f^{-1}(\beta(p_1)) - f^{-1}(v_2)]p_1 + \delta\pi(\beta(p_1))$$

whereas:

$$\pi(v_1) = [f^{-1}(\beta(p_1)) - f^{-1}(v_1)]p_1 + \delta\pi(\beta(p_1))$$

Since p_1 is nonnegative and $f^{-1}(v_1) \geq f^{-1}(v_2)$, we have shown $\pi(v_2) \geq \pi(v_1)$.

Meanwhile, suppose that $p^1, p^2, \ldots$ is an equilibrium sequence of prices that the monopolist charges when she starts from state v_2. Let p^k be the first price in this sequence such that $\beta(p^k) \leq v_1$. Observe that the monopolist, starting from state v_1, can charge prices $p^1,\ldots,p^k$ in the first k periods and follow her equilibrium strategy in the subgame which ensues. Hence:

$$\pi(v_1) \geq \delta^{k-1}[f^{-1}(\beta(p^k)) - f^{-1}(v_1)]p^k + \delta^k\pi(\beta(p^k))$$

whereas:

$$\pi(v_2) \leq [f^{-1}(v_1) - f^{-1}(v_2)] + \delta^{k-1}[f^{-1}(\beta(p^k)) - f^{-1}(v_1)] + \delta^k\pi(\beta(p^k))$$

Since each p^i must be less than or equal to one to generate positive sales we conclude:

$$\pi(v_2) - \pi(v_1) \leq f^{-1}(v_1) - f^{-1}(v_2)$$

as desired. $\square$

Lemma A.7: Suppose $\{\pi_n\}_{n=1}^{\infty}$ is a sequence of monotone functions, each mapping an interval I_D of the reals into a compact interval I_R of the reals. Further suppose there exists a continuous function π: $I_D \to I_R$, such that $\{\pi_n\}_{n=1}^{\infty}$ converges pointwise, at all the rationals in I_D, to π. Then $\{\pi_n\}_{n=1}^{\infty} \to \pi$ uniformly in the supremum norm.

Proof: A standard exercise in real analysis, omitted here for brevity.

We are now ready to prove:

Theorem 4.2 (Existence of Weak-Markov Equilibria): Let f be any demand curve which is monotone decreasing in an interval $[\bar{q},1]$, for some $0 \leq \bar{q} < 1$. Then for any $r > 0$ and any $z > 0$ there exists a weak-Markov equilibrium.

Proof: Consider the sequence of demand curves:

$$f_n(q) = \begin{cases} f(q) & , \text{ if } 0 \leqslant q \leqslant \frac{n-1}{n} \\ (n - nq)f(\frac{n-1}{n}) + (1 - n + nq)f(1), & \text{if } \frac{n-1}{n} < q \leqslant 1 \end{cases}$$

Observe, for future use, that:

$$\sup_{f(0) \leqslant v \leqslant 1} \left| f_n^{-1}(v) - f^{-1}(v) \right| \leqslant 1 - \frac{n-1}{n} \to 0$$

so $f_n^{-1} \to f^{-1}$ uniformly. Also observe that, for every n, f_n is linear in the interval $[(n - 1)/n, 1]$. Hence, one can readily calculate, explicitly, a linear-quadratic pair $(\tilde{\phi}_n, \tilde{\pi}_n)$ which supports a weak-Markov equlibrium on $[(n - 1)/n, 1]$ for f_n. (See Section 3.) By Lemma A.5, this pair can be extended, if necessary, to (ϕ_n, π_n) which supports a weak-Markov equilibrium on $[\bar{q}, 1]$ for f_n. Let β_n be the β implied by the convex hull of ϕ_n.

By Lemma A.2, the sequence $\{\beta_n\}_{n=1}^{\infty}$ is an equicontinuous family in the supremum norm. By the Ascoli Theorem, it has a convergent subsequence which uniformly converges to β. Now let $r_1, r_2, \ldots,$ be an enumeration of the rationals in $[f(1), f(\bar{q})]$, and take further subsequences so that $\{\pi_n(r_i)\}_{n=1}^{\infty}$ converges for every $i \geqslant 1$. For notational simplicity, let $\{\beta_n, \pi_n\}_{n=1}^{\infty}$ denote the result of this construction, and for every rational r in $[f(1), f(\bar{q})]$, let π be defined as the pointwise limit $\pi(r) = \lim_{n \to \infty} \pi_n(r)$. Observe, by Lemma A.6, that for every pair (r_1, r_2) of rationals such that $f(1) < r_1 < r_2 \leqslant f(\bar{q})$, and for every n:

$$0 \leqslant \pi_n(r_2) - \pi_n(r_1) \leqslant f_n^{-1}(r_1) - f_n^{-1}(r_2)$$

implying, since $f_n^{-1} \to f^{-1}$, uniformly, that:

$$0 \leqslant \pi(r_2) - \pi(r_1) \leqslant f^{-1}(r_1) - f^{-1}(r_2)$$

Extend π to all values in $[f(1), f(\bar{q})]$ in the unique way which gives continuity for that entire interval. Observe, by Lemma A.7, that $\{\pi_n\}_{n=1}^{\infty}$ uniformly converges to π. So we have that $\{\beta_n, \pi_n\}_{n=1}^{\infty}$ uniformly converges to (β, π).

Define β^{-1} by $\beta^{-1}(v) = \{p\colon \beta(p) = v\}$. Finally, define the ϕ correspondence ϕ by:

$$\text{(A.8)} \qquad \phi(v) = \frac{1}{\delta}[\beta^{-1}(v) - (1 - \delta)v]$$

Claim: (ϕ, π) supports a weak-Markov equilibrium on $(\bar{q}, 1]$ for the (limit) demand curve f.

Proof of Claim: First let us show that "β is implied by ϕ." Let $\beta(p) = v$. Then by (A.8), $\phi(v) = (1/\delta)[\beta^{-1}(v) - (1 - \delta)v]$. Rearranging, $\beta^{-1}(v) = (1 - \delta)v + \delta\phi(v)$. But $p \in \beta^{-1}(v)$, by assumption, so (A.1) is satisfied.

Second, we will show that ϕ, π and β jointly satisfy all the requirements of Definition A.1. Observe that β is monotone and Lipschitz-continuous (with Lipschitz constant $1/(1 - \delta)$), since it is defined as the uniform limit of the β_n, each of which is monotone and Lipschitz-continuous (by Lemma A.2). Hence, β^{-1} is monotone and u.s.c., and $\beta^{-1}(v_1) - \beta^{-1}(v_2) \geqslant (1 - \delta)(v_1 - v_2)$ if $v_1 \geqslant v_2$. So by equation (A.8), ϕ is also monotone and u.s.c., as required.

Define:

$$\beta_n'(p,v) = \min\{\beta_n(p),\ v\};\ \beta'(p,v) = \min\{\beta(p),\ v\}$$

and:

$$J_n(p,v) = [f_n^{-1}(\beta_n'(p,v)) - f_n^{-1}(v)]p + \delta\pi_n(\beta_n'(p,v))$$

$$J(p,v) = [f^{-1}(\beta'(p,v)) - f^{-1}(v)]p + \delta\pi(\beta'(p,v))$$

What remains to be shown is that:

(A.9) $$\pi(v) = \max_{0 \leq p \leq 1} J(p,v), \text{ for all } v \in [f(1), f(\bar{q})]$$

and that $\phi^{\min}(v) \equiv \inf \phi(v)$ and $\phi^{\max}(v) \equiv \sup \phi(v)$ are both contained in the argmax correspondence of (A.9). This will follow from the Theorem of the Maximum.

Each f_n and f is strictly monotone on $[\bar{q},1]$; hence f_n^{-1} and f^{-1} are well-defined and continuous functions on the compact interval $[f(1),\ f(\bar{q})]$. Thus, f^{-1} is also uniformly continuous on that interval; given $\varepsilon > 0$, there exists $\varepsilon' < \varepsilon/5$ such that:

$$\left|f^{-1}(v_1) - f^{-1}(v_2)\right| < \frac{\varepsilon}{5},$$

whenever $f(1) \leq v_1 \leq v_2 \leq f(\bar{q})$ and $\left|v_1 - v_2\right| < \varepsilon'$.

By Lemma A.6, we also have:

$$\left|\pi_n(v_1) - \pi_n(v_2)\right| \leq \left|f_n^{-1}(v_1) - f_n^{-1}(v_2)\right|$$

for all n. Since we have shown that $\pi_n \to \pi$ uniformly and $f_n^{-1} \to f^{-1}$ uniformly, this implies:

$$|\pi(v_1) - \pi(v_2)| \leq |f^{-1}(v_1) - f^{-1}(v_2)| \leq \frac{\varepsilon}{5}$$

whenever $|v_1 - v_2| < \varepsilon'$. Also $\beta_n \to \beta$ uniformly, so there exists $\bar{n}$ such that for every $n \geq \bar{n}$, we have $|f_n^{-1}(v) - f^{-1}(v)| < \varepsilon/5$, $|\pi_n(v) - \pi(v)| < \varepsilon/5$, and $|\beta_n'(p,v) - \beta'(p,v)| < \varepsilon'$. Hence for all v, $(f(1) \leq v \leq f(\bar{q}))$, for all p, $(f(1) \leq p \leq 1)$, and for all $n \geq \bar{n}$:

$$|J_n(p,v) - J(p,v)| \leq |f_n^{-1}(\beta_n'(p,v)) - f^{-1}(\beta_n'(p,v))| +$$

$$+ |f^{-1}(\beta_n'(p,v)) - f^{-1}(\beta'(p,v))| + |f_n^{-1}(v) - f^{-1}(v)| +$$

$$+ |\pi_n(\beta_n'(p,v)) - \pi(\beta_n'(p,v))| + |\pi(\beta_n'(p,v)) - \pi(\beta'(p,v))|$$

$$< \frac{\varepsilon}{5} + \frac{\varepsilon}{5} + \frac{\varepsilon}{5} + \frac{\varepsilon}{5} + \frac{\varepsilon}{5} = \varepsilon$$

So we have established that J_n converges uniformly to J in the supremum norm. First, consider any v where $\phi(v)$ is single-valued. Then for every n, select $p_n \in \beta_n^{-1}(v)$ and $p_n' \in \phi_n(v)$ such that

$$p_n = (1 - \delta)v + \delta p_n'$$

(This is possible by the definition of β_n in (A.1).) Choose a convergent subsequence of $\{p_n, p_n'\}_{n=1}^{\infty}$ and let (p,p') be the limit. Then:

$$p = (1 - \delta)v + \delta p'$$

Observe that since $p_n \to p$ and $\beta_n \to \beta$ uniformly, we have:

$$v = \beta_n(p_n) \to \beta(p)$$

implying $p \in \beta^{-1}(v)$. Then by the definition of the ϕ correspondence:

$$\phi(v) = \frac{1}{\delta}[p - (1 - \delta)v] = \frac{1}{\delta}[(1 - \delta)v + \delta p' - (1 - \delta)v] = p'$$

But $p_n' \in \text{argmax } J_n(p,v)$ for all n and $p_n' \to p'$; by the Theorem of the Maximum, $p' \in \text{argmax } J(p,v)$ and $\pi(v) = J(p',v)$, as desired.

Now consider any v where $\phi(v)$ is multiple-valued. Since ϕ is monotone, there are only countably many such points; hence, there exists $v^n \downarrow v$ such that $\phi(v^n)$ is single valued for all n. By the above argument, $\phi(v^n) \in \text{argmax } J(p,v^n)$; by the fact that ϕ is u.s.c. and monotone, $\lim \phi(v^n) = \sup \phi(v)$. By the Theorem of the Maximum, $\sup \phi(v) \in \text{argmax } J(p,v)$, as desired; and by a symmetric argument, we also obtain $\inf \phi(v) \in \text{argmax } J(p,v)$.

This completes the proof of the claim.

We have shown that there exists a weak-Markov equilibrium on $[\bar{q},1]$, the monotone decreasing part of the demand curve. We complete the proof of the theorem by observing that we can extend this equilibrium to the entire interval $[0,1]$. This involves a change in the notation to that of Appendix B, and so the details are omitted here. (We need to change notation because f is not longer necessarily strictly decreasing, so more than one q may be associated with a single v. The "state" is thus no longer uniquely described by v.)

We define $P(q)$ to be the left-continuous selection from $\beta^{-1}(f(q))$ and define $R_n(q) = \pi_n(f(q))$, for all $\bar{q} \leq q \leq 1$. The analogue to Lemma A.5 still holds in the new notation, by an almost identical proof; hence (P,R) can be extended to the entire interval [0,1], yielding the desired result. $\square$

Finally, we obtain:

<u>Corollary 4.3</u>: Under the hypothesis of Theorem 4.2, there exists a weak-Markov equilibrium in which, along the equilibrium path, the monopolist plays only pure strategies.

<u>Proof</u>: Let us construct the monopolist's strategy $\phi^*(p,v)$ for the case $\bar{q} = 0$ so that the demand curve is everywhere monotone decreasing. (A somewhat more intricate argument demonstrates the corollary when $\bar{q} > 0$.)

In the proof of Theorem 4.2, we constructed (ϕ,π) which supports a weak-Markov equilibrium on $[\bar{q},1]$. $\phi^*(p,v)$ will give the actual price (or prices) to charge if the <u>previous price</u> was p and the <u>current state</u> is v. Suppose we are in the initial period (i.e., $v = f(\bar{q})$). The monopolist sets:

$$\phi^*(\cdot,f(\bar{q})) = \sup \phi(f(\bar{q})), \text{ with probability one.}$$

Suppose we are in any later period, and p was the previous price charged. Observe, by equation (A.8), that:

$$\text{(A.10)} \qquad p' \equiv \frac{1}{\delta}[p - (1 - \delta)\beta(p)] = \theta \sup \phi(\beta(p)) + (1 - \theta) \inf \phi(\beta(p))$$

has a solution $\bar{\theta}$, where $0 \leq \bar{\theta} \leq 1$. Define:

$$\phi^{*}(p,v) = \begin{cases} \sup \phi(v), \text{ with probability } \bar{\theta} \\ \inf \phi(v), \text{ with probability } (1 - \bar{\theta}) \end{cases}$$

Observe that this gives optimal behavior for the monopolist, since $\sup \phi(v) \in \operatorname{argmax} J(p,v)$ and $\inf \phi(v) \in \operatorname{argmax} J(p,v)$. It also gives optimal behavior for consumers, since (A.10) implies:

$$\beta(p) - p = \delta[\beta(p) - p']$$

so the consumer with valuation $\beta(p)$ is indifferent between purchasing this period at price p and next period at expected price p'.

<u>Observation</u>: $\beta(p)$ contains a flat section if and only if $\phi(v)$ is multivalued.

<u>Proof of the Observation</u>: Rewriting (A.8), $\phi(v) = \frac{1}{\delta}[\beta^{-1}(v) - (1 - \delta)v]$. If $\phi(v)$ is multivalued, $\beta^{-1}(v)$ must be multivalued, implying that $\beta(p)$ has a flat section. Conversely, if $\beta(p)$ has a flat section, $\beta^{-1}(v)$ is multivalued, implying that $\phi(v)$ is multivalued.

Now suppose that, at some point in time, randomization is called for, implying $p' \equiv \frac{1}{\delta}[p - (1 - \delta)\beta(p)] < \sup \phi(\beta(p))$. Then ϕ is multivalued at $v \equiv \beta(p)$, implying β is flat at p. Moreover, $p < p_1 \equiv \sup \beta^{-1}(v)$, or else p' would equal $\sup \phi(\beta(p))$. But then, in the previous period, the monopolist charged p when she could have charged $p_1 > p$ such that $\beta(p_1) = \beta(p)$. Since along every weak-Markov equilibrium path, there are positive sales in every period, the monopolist suboptimized in the previous period.

We conclude that, if randomization is called for, we are off the equilibrium path. $\square$

Appendix B

The Uniform Coase Conjecture

Proof of Theorem 5.4: Suppose not. Then there exist real numbers $L \geq 1$, M $(0 < M \leq 1)$ and $\varepsilon > 0$, a sequence $\{f_n\}_{n=1}^{\infty} \subset \mathcal{F}_{L,M}$, a sequence of positive numbers $\{z_n\}_{n=1}^{\infty} \to 0$, and a sequence of weak-Markov equilibria $\{S_n,P_n\}_{n=1}^{\infty}$ such that $S_n(0) \geq \varepsilon$ for all $n \geq 1$. (S_n,P_n) denotes a weak-Markov equilibrium associated with demand curve f_n and interval between periods z_n.[18] S_n gives the firm's price strategy as a function of q along the equilibrium path (or, possibly a mixed strategy, depending on q and the previous price, off the equilibrium path). So $S_n(0)$ denotes the initial price the monopolist charges. Meanwhile, P_n denotes the consumer's reservation price strategy, also as a function of q--consumer q accepts any price less than or equal to $P_n(q)$ and rejects any price greater than $P_n(q)$, in the equilibrium (S_n,P_n). P_n may be assumed to be left-continuous.

Let R_n denote the monopolist's value function as a function of q, associated with (S_n,P_n).[19] Observe, by reasoning identical to that of the proof of Lemma A.6, that:

$$0 \leq R_n(q_1) - R_n(q_2) \leq q_2 - q_1$$

whenever $q_1 \leq q_2$.[2] Hence, each R_n is Lipschitz-continuous in q with a Lipschitz constant of 1. Therefore, $\{R_n\}_{n=1}^{\infty}$ is an equicontinuous family. Let $\{R_{n_i}\}_{i=1}^{\infty}$ denote a uniformly convergent subsequence in the supremum norm, and let R denote the limit.

Now let $r_1,r_2,r_3,\ldots$ be an enumeration of the rationals in [0,1]. Let $\{P_{n_{i_j}}\}_{j=1}^{\infty}$ denote a convergent subsequence of $\{P_{n_i}(r_1)\}_{i=1}^{\infty}$. Continue this construction of successive convergent subsequences for all n and r_n, and (for notational simplicity) let $\{P_n\}_{n=1}^{\infty}$ denote the result of the construction. For every rational $r \in [0,1]$, let $\bar{P}(r)$ be defined by

$$\bar{P}(r) = \lim_{n\to\infty} P_n(r)$$

and extend the function $\bar{P}$ to all reals in [0,1] by imposing left-continuity. Let y denote $\inf\{r: \bar{P}(r) < \varepsilon\}$. Observe that each P_n is left-continuous, so $\bar{P}(y) \geq \varepsilon$, but $\bar{P}(r) < \varepsilon$ for every $r > y$. Furthermore, any weak-Markov equilibrium has sales in every period; hence $S_n(0) \geq \varepsilon$ implies $P_n(0) \geq \varepsilon$ for all n. But P_n is monotone nonincreasing and (by the Lipschitz constant L at 1):

$$P_n(1 - \frac{\varepsilon}{2L}) \leq f_n(1 - \frac{\varepsilon}{2L}) \leq L[1 - (1 - \frac{\varepsilon}{2L})] = \frac{\varepsilon}{2}$$

for all $n \geq 1$, so we have that $0 \leq y < 1$.

<u>Case I</u>. Suppose $\bar{R}(y) > 0$.

Since $R_n \to R$ uniformly, there exists a rational q ($y < q < 1$) and an integer $\bar{n}_1$ such that:

$$R_n(q) > \frac{1}{2}\bar{R}(y), \text{ for all } n \geq \bar{n}_1$$

Since $q > y$, $\bar{P}(q) < \varepsilon$, so there exists $\alpha > 0$ and integer $\bar{n}_2$ such that:

$$P_n(q) < \varepsilon - \alpha, \text{ for all } n \geq \bar{n}_2.$$

We will establish a lower bound on the real time t before which the price can drop by α, in equilibrium, and hence before which consumer q purchases. In particular, consumer 0 prefers to purchase at the initial equilibrium price, which is at least ε, than to purchase at a price below $\varepsilon - \alpha$ at time t, so:

$$1 - \varepsilon \geq e^{-rt}[1 - (\varepsilon - \alpha)]$$

$$e^{-rt} \leq 1 - \frac{\alpha}{1 - \varepsilon + \alpha}$$

This gives an upper bound on the profits attainable by the monopolist:

(B.1) $$R_n(0) \leq \int_0^q P_n(x)dx + e^{-rt}R_n(q), \text{ for all } n \geq \bar{n}_2$$

But choose any integer m and let $z \leq 1/m^2$. Then for any consumer reservation price function P_n, the monopolist may charge prices $\frac{m-1}{m}$, $\frac{m-2}{m}$, ... , $\frac{1}{m}$, respectively, in the first (m - 1) periods. This earns the monopolist within 1/m of all "available surplus," within a factor $e^{-(m-2)z}$ of discounting. Hence:

$$R_n(0) \geq e^{-(m-2)z}\{\int_0^1 P_n(x)dx - \frac{1}{m}\}$$

Since $z_n \to 0$, there exists $\bar{n}_3(m)$ such that $z_n \leq 1/m^2$ for all $n \geq \bar{n}_3(m)$, and so:

(B.2) $$R_n(0) \geq e^{-1/m}\{\int_0^1 P_n(x)dx - \frac{1}{m}\}, \text{ for all } n \geq \bar{n}_3(m).$$

Since $R_n(q)$ is bounded away from zero for all $n \geq \bar{n}_1$, there exists an integer m such that (B.1) and (B.2) are contradictory for $n \geq \max\{\bar{n}_1, \bar{n}_2, \bar{n}_3(m)\}$.

Case II. Suppose $\bar{R}(y) = 0$.

By hypothesis, (S_n, P_n) is a subgame perfect equilibrium for all n. Suppose that, in the initial period, the monopolist chooses to deviate by charging a price of $\varepsilon/2$. This defines a subgame. We will show that, for sufficiently large n, the posited behavior under (S_n, P_n) in this subgame cannot be optimal for both the monopolist and consumers.

Observe that any weak-Markov equilibrium has sales in every period; hence $P_n(0) \geq \varepsilon$ for all n. Customer 0 is optimizing when he purchases at price $\varepsilon/2$, so he must believe that the price will not drop rapidly thereafter. In particular, let t_n be the first (real) time in which the price will drop below $\varepsilon/4$. Since customer 0 is willing to purchase at price $\varepsilon/2$:

$$1 - \frac{\varepsilon}{2} \geq e^{-rt_n}[1 - \frac{\varepsilon}{4}]$$

Define t by $e^{-rt} = (1 - \varepsilon/2)/(1 - \varepsilon/4)$. Then for all n, $t_n \geq t$; i.e., a price less than or equal to $\varepsilon/4$ is not charged until at least time t.

Recall that y has been defined so that $\bar{P}(y) \geq \varepsilon$. Therefore, there exists a sequence of rationals $y_n \uparrow y$ such that $P_n(y_n) \geq \varepsilon/2$ for all n. (If $y = 0$, let $y_n = 0$ for all n.) For arbitrarily chosen $z > 0$, there exists $\bar{n}_1$ such that $z_n < z$ for all $n \geq \bar{n}_1$. Since $\bar{R}(y) = 0$, $R_n \to R$ uniformly, and $y_n \to y$, there also exists $\bar{n}_2$ such that $R_n(y_n) < 1/4\ \varepsilon e^{-rt}z$ for all $n \geq \bar{n}_2$. Write n for $\max\{\bar{n}_1, \bar{n}_2\}$. Meanwhile, let m be the greatest even integer less than t/z. Let $p_1, \ldots, p_m$ denote the first m prices charged by the monopolist along a subgame arising after the monopolist charges an initial price $p_0 = \varepsilon/2$. (In the event that a mixed strategy is called for in period 1, let p_1 be the largest price which the monopolist randomizes over.) Observe, by our definitions, that $p_m > \varepsilon/4$. Following Gul-Sonnenschein-Wilson [1986], define

$$a_i = \varepsilon/2 - \frac{2i}{m}[\varepsilon/2 - p_m] \quad (0 \leqslant i \leqslant m/2)$$

and define the sequence $p_0', \ldots, p_{m/2}'$ by $p_i' = a_{k_i}$ (for $0 \leqslant i \leqslant m/2$), where:

$$k_0 = 0$$

$$k_i = \begin{cases} \inf\{r \geqslant k_{i-1} + 1 : \exists\ p_j \in [a_r, a_{r-1}]\}, & \text{if over a nonempty set} \\ m/2 & \text{, otherwise} \end{cases}$$

Observe that, by following $p_0', \ldots, p_{m/2}'$, the monopolist "does not lose time" on any sale and loses at most $2(p_0 - p_m)/m$ on each sale. Furthermore, since $R_n(y_n) < 1/4\ \varepsilon e^{-rt} z$ and since each sale before time t is at a price greater than $\varepsilon/4$, the total number of customers sold to at $p_1, \ldots, p_m$ is less than z.

Let V_n denote the net present value of profits from following the equilibrium price path $p_1, p_2, p_3, \ldots$ after a price $p_0 = \varepsilon/2$ was charged. Let V_n' denote the value from following $p_1, \ldots, p_{m/2}$ in the first m/2 periods and then continuing optimally. Let V_n'' denote the value to the monopolist of playing optimally, beginning in the period after a price p_m is charged. Observe:

$$V_n' - V_n \geqslant [e^{-rt/2} - e^{-rt}]V'' - \frac{2}{m}(p_0 - p_m)z$$

We now place a lower bound on V_n''. Observe that, in the period after p_m is charged, customer $1 - \varepsilon/4L$ remains in the market since

$$P_n(1 - \frac{\varepsilon}{4L}) \leqslant f_n(1 - \frac{\varepsilon}{4L}) \leqslant L[1 - (1 - \frac{\varepsilon}{4L})] = \frac{\varepsilon}{4}$$

by the Lipschitz constant L at 1. Meanwhile, customer $1 - \varepsilon/8L$ prefers to purchase at a price of $[1 - e^{-rz}]f_n(1 - \varepsilon/8L)$ this period to purchasing at a price of zero next period. By our "Lipschitz below at 1" assumption:

$$f_n(1 - \frac{\varepsilon}{8L}) \geq M[1 - (1 - \frac{\varepsilon}{8L})] = \frac{M}{L} \cdot \frac{\varepsilon}{8}$$

Hence, a price of $M/L \cdot \varepsilon/8 \cdot (1 - e^{-rz})$ induces all customers in the interval $[1 - \varepsilon/4L,\ 1 - \varepsilon/8L]$ to purchase, so:

$$V_n'' \geq [1 - e^{-rz}]\ \frac{M}{L^2} \cdot \frac{\varepsilon^2}{64}$$

Recall that $(p_0 - p_m) \leq \varepsilon/4$ and $m \approx t/z$. Hence, for sufficiently small z (and the implied n):

$$V_n' - V_n \geq (e^{-rt/2} - e^{-rt})(1 - e^{-rz})\ \frac{M}{L^2} \cdot \frac{\varepsilon^2}{64} - \frac{\varepsilon}{3t}\ z^2$$

$$\geq (1 - e^{-rz})\left\{(e^{-rt/2} - e^{-rt})\ \frac{M}{L^2} \cdot \frac{\varepsilon^2}{64} - \frac{\varepsilon}{3t} \cdot \frac{z^2}{1 - e^{-rz}}\right\}$$

Observe that $(1 - e^{-rz})$ is always positive, and the first term in brackets is positive and independent of z. Meanwhile, $\lim_{z \to 0}(z^2/(1 - e^{-rz})) = 0$, so for sufficiently small choice of z, $V_n' - V_n > 0$. This contradicts our hypothesis that, for all n, (S_n, P_n) is a subgame perfect equilibrium. $\square$

Notes

[1]For a thorough review of this literature, see Rubinstein [1985a].

[2]For further discussion of the case with a gap between seller and buyer valuations, see Ausubel and Deneckere [1985] and Section 7.

[3]See, for example, Rubinstein [1985b], Grossman and Perry [1986], and Cho and Kreps [1985].

[4]Subgame perfection eliminates "incredible threats" efficiently in complete information games.

[5]One can also introduce reputation effects by adding consumer uncertainty about the monopolist's marginal cost (seller's valuation). See Ausubel and Deneckere [1986].

[6]Even with a linear demand curve, there exist weak-Markov equilibria which require the monopolist to randomize off the equilibrium path. See Gul, Sonnenschein and Wilson [1986].

[7]This restriction may affect the equilibrium set, as demonstrated in Gul, Sonnenschein and Wilson [1986]. Nevertheless, it seems like an eminently natural regularity requirement.

[8]A more rigorous argument can be found in the proof of Theorem 6.1.

[9] f can be normalized to satisfy (a), unless $f(q) \equiv 0$.

[10] The definition already assumes $0 < f(0) < \infty$. We normalize so that $f(0) = 1$.

[11] The definition requires the demand curve to be continuous except, possibly, at a countable set of points. Our anonymity assumption permits us to replace the actual demand curve with a left-continuous function f which agrees with it except on a set of measure zero.

[12] Reasoning analogous to the previous footnote.

[13] For a survey of related literature, see the Introduction.

[14] Identical reasoning applies to the case where $f(q) = (1 - q)^n$, considered by Sobel and Takahashi [1983]. Furthermore, this is the only demand curve that is closed under the joint operation of truncation and rescaling (see Definition 5.2).

[15] We do not mean this literally. The graph may display discontinuities and certainly contains inflection points.

[16] The case of the "gap" translates into consumer demand which is perfectly inelastic at the monopolist's marginal cost.

[17] This lemma builds on Fudenberg, Levine and Tirole [1985], Lemma 3, and Gul, Sonnenschein and Wilson [1986], Lemma 5.

[18] We adopt this notation, in lieu of the notation of Appendix A, in order to be able to fully describe equilibrium strategies in cases where f_n is <u>not</u> necessarily monotone decreasing, and so more than one q may be associated with a single v.

[19] In fact, whenever f_n is monotone decreasing, π_n is well-defined and so $R_n(q) \equiv \pi_n(f(q))$ for all $q \in [0,1]$. Hence, by Lemma A.6:

$$0 \leq R_n(q_1) - R_n(q_2) \equiv \pi_n(f(q_1)) - \pi_n(f(q_2)) \leq f^{-1}(f(q_2)) - f^{-1}(f(q_1)) = q_2 - q_1$$

References

Ausubel, L. and R. Deneckere, "One is Almost Enough for Monopoly," C.M.S.E.M.S. Discussion Paper No. 669, Northwestern University, 1985.

Ausubel, L. and R. Deneckere, "Reputation in Bargaining with Two-sided Incomplete Information," manuscript, Northwestern University, 1986.

Bond, E. and L. Samuelson, "Durable Good Monopolies with Rational Expectations and Replacement Sales," The Rand Journal of Economics, 15 (1984), 336-345.

Bulow, J., "Durable Goods Monopolists," Journal of Political Economy, 90 (1982), 314-332.

Cho, I.-K. and D. Kreps, "More Signalling Games and Stable Equilibria," mimeo, Stanford Graduate School of Business, 1985.

Coase, R., "Durability and Monopoly," Journal of Law and Economics, 15 (1972), 143-149.

Fudenberg, D. and J. Tirole, "Sequential Bargaining with Incomplete Information," Review of Economic Studies, 50 (1983), 221-247.

Fudenberg, D., D. Levine and J. Tirole, "Infinite Horizon Models of Bargaining with One-Sided Incomplete Information," in Game Theoretic Models of Bargaining, A. Roth (Ed.), Cambridge University Press, 1985.

Fudenberg, D. and E. Maskin, "The Folk Theorem in Repated Games with Discounting and with Incomplete Information," Econometrica, 54 (1986), 533-554.

Grossman, S. and M. Perry, "Sequential Bargaining under Asymmetric Information," Journal of Economic Theory, 39 (1986), 120-154.

Gul, F., "Foundations of Dynamic Oligopoly," mimeo, University of Pennsylvania, 1985.

Gul, F., H. Sonnenschein and R. Wilson, "Foundations of Dynamic Monopoly and the Coase Conjecture," Journal of Economic Theory, 39 (1986), 155-190.

Hellwig, M., "Monopolistic Discrimination Over Time: Remarks on Periodization in Economic Models," Economic Research Program, Research Memorandum No. 174, Princeton University, 1975.

Kahn, C., "The Durable Goods Monopolist and Consistency with Increasing Costs," Econometrica, 54 (1986), 275-294.

Rubinstein, A., "Perfect Equilibrium in a Bargaining Model," Econometrica, 50 (1982), 97-109.

Rubinstein, A., "A Sequential Theory of Bargaining," IMSSS Technical Report No. 478, Stanford University, 1985a.

Rubinstein, A., "A Bargaining Model with Incomplete Information," Econometrica, 53 (1985b), 1151-1172.

Sobel, J. and I. Takahashi, "A Multi-Stage Model of Bargaining," Review of Economic Studies, 50 (1983), 411-426.

Stokey, N., "Intertemporal Price Discrimination," Quarterly Journal of Economics, 93 (1979), 355-371.

Stokey, N., "Rational Expectations and Durable Goods Pricing," Bell Journal of Economics, 12 (1981), 112-128.