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PRODUCT DIFFERENTIATION

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Abstract

In a simple oligopoly with quantity setting firms, we analyze the conditions under which more product differentiation makes tacit collusion easier. If the ease of collusion is measured by the maximum ratio of interest at which a given output can be maintained or by the minimum feasible output, then the net affect can go either way. On the other hand, both the rate of collusive output to cartel output and the ratio of collusive profits to cartel profits go monotonically to one as product differentiation increases to zero cross elasticity.

1. Introduction

The relationship between product differentiation and tacit collusion has been widely discussed since the seminal work of Chamberlin (1936). Tacit collusion can be analyzed in the context of repeated games such that an agreement is supported by various punishment schemes according to which a deviation is penalized by lower profits in the following periods. An agreement is self enforcing if the short run gains from violating it are smaller than the long run losses from the ensuing punishment. When this is applied to product differentiation, the typical intution offered is that two things happen as products become less close substitutes. First, the potential gains from cheating on an agreement supposedly goes down and secondly, the severity with which a deviator can be punished is conjectured to go down as well. The net result would then depend on the balance between these two forces.

Earlier analyses of this issue have been either informal or performed in the context of inoptimal punishment schemes such that the choice of equilibrium have been open to question. Very recently, Abreu (1983) has, however, derived the optimal punishment schemes for a wide range of repeated games. The purpose of the present paper is to analyze the collusive effects of product differentiation in the context of optimal punishments.

2. Model

We consider a market in which n + 1 oligopolists compete on quantity and face inverse demand curves of the form

$$(1) \qquad p_i = 1 - \beta q_i - (1 - \beta) \sum_{j \neq i}^{n+1} q_j, \quad i,j = 1,2,\ldots,n+1$$

where q_i is the quantity of firm i and increasing values of $\beta \in [1/2, 1]$

measure increasing product differentiation.

If we conceive of the oligopolists as repeating this game infinitely while discounting their profits according to a "small"[1] identical rate r, we can model tacit collusion as supported by the optimal punishments derived by Abreu (1986, Sec. 4).[2] According to his results, the maximum extent of collusion can be characterized by two numbers: y_2, which is the output per firm in periods not immediately preceded by violations, and y_1, which is the output per firm in periods immediately preceded by violations (of y_2 or y_1). Denote the single period profit per firm by $\pi(y)$ if all firms produce y, and $\pi^*(y)$ if the firm maximizes single period profit given that all others produce y. Abreu's result is that (y_1,y_2) is globally optimal if

$$\pi(y_1) = -\frac{1}{r}\pi(y_2) \tag{2}$$

$$\pi^*(y_1) = 0 \tag{3}$$

$$\pi^*(y_2) - \pi(y_2) \leq \frac{1}{r}\pi(y_2) \tag{4}$$

The first condition, (2), characterizes the optimal punishment as one which inflicts losses equal to the net present value of all future profits. This is clearly the most severe punishment under which firms will stay in the market. For this to be credible it must furthermore be the case that firms find it (weakly) unattractive to cheat on the punishment. This is captured in (3). By (4) it is finally required that the one period gains from cheating (which will entail zero net present value thereafter) be weakly dominated by the long term benefits from the agreement. As long as y_2 is greater than the cartel output, (4) will hold with equality.

If r is sufficiently large such that future penalties count for little (2)-(4) cannot hold for any y_1, y_2 and a simple two-phase punishment scheme will not be optimal. It is, however, important to note that r is a measure of the reaction lag (rather than an annual discount rate) such that very small values of r may be realistic in practice.

In the absence of production costs,[3] the analogs of (2)-(4) for our model are

$$(2') \qquad y_1 - y_1^2[\beta + (1 - \beta)n] = -\frac{1}{r}(y_2 - y_2^2[\beta + (1 - \beta)n])$$

$$(3') \qquad y_1 \geqslant [(1 - \beta)n]^{-1}$$

$$(4') \qquad \frac{1}{4\beta}[1 - (1 - \beta)ny_2]^2 \leqslant \frac{1 + r}{r}(y_2 - y_2^2[\beta + (1 - \beta)n])$$

We will concentrate on the interesting case in which y_2 is above the cartel output, so we assume:

$$y_2 > \frac{1}{2}\,[\beta + (1 - \beta)n]^{-1}$$

this enables us to solve (4') to find y_2 as

$$(5) \qquad y_2 = [(1 - \beta)^2n^2r + 4(1 + r)\beta^2 + 4(1 + r)\beta(1 - \beta)n]^{-1}$$

$$\cdot\; [(1 - \beta)nr + 2\beta(1 + r) - 2\beta(1 + r)^{1/2}]$$

From this, (2') can be solved for y_1, and optimality of the scheme can be checked by seeing if y_1 satisfies (3') which always will be the case if r is not too large. So we will conduct our analysis in the intersection of the two sets

$$S_1 \equiv \{(\beta,r,n) | \text{" the r.h.s. of (5)"} > \tfrac{1}{2}[\beta + (1-\beta)n]^{-1}\}$$

$$S_2 \equiv \{(\beta,r,n) | \text{" the } y_1 \text{ solving (2')"} \geqslant [(1-\beta)n]^{-1}\}$$

where S_1 is the area in which it is meaningful to talk about more or less collusion while S_2 is the domain of optimality for Abreu's scheme. Roughly speaking, S_1 rules "too small" values of r, n out, while S_2 does not allow r to be "too big." As an example, $(1/2, 1, 10) \in S_1 \cap S_2$.

3. Analysis

In order to look at the effect of product differentiation on the maximum degree of tacit collusion, we can vary β in

$$(4'') \qquad \frac{1}{4\beta}[1 - (1-\beta)ny_2]^2 - \frac{1+r}{r}(y_2 - y_2^2[\beta + (1-\beta)n])$$

$$= \pi^*(y_2,\beta,r,n) - \frac{1+r}{r}\pi(y_2,\beta,r,n) \equiv F(y_2,\beta,r,n) = 0$$

under the assumption that $(\beta,r,n) \in S_1 \cap S_2$.

To measure the implications of increasing product differentiation, we will differentiate $F(\cdot)$ with respect to β. To this end, we first find

$$(6) \qquad \frac{\partial\pi^*}{\partial\beta} = \frac{1}{4\beta^2}[1 - (1-\beta)ny_2][ny_2(1+\beta) - 1]$$

$$= \frac{1}{4\beta^2}[-n^2(1-\beta^2)y_2^2 + 2ny_2 - 1]$$

This is positive if $(1 - \beta)/[(n(1 - \beta^2)] < y_2 < (1 + \beta)/[(n(1 - \beta^2)]$. We know that $y_2 < [2\beta + (1 - \beta)n]^{-1}$, the Nash output, and that $y_2 > \frac{1}{2}[\beta + (1 - \beta)n]^{-1}$, the cartel output. Since $(1 + \beta)/[(n(1 - \beta^2)] \geqslant [2\beta + (1 - \beta)n]^{-1}$ for all elements of $S_1 \cap S_2$ and $(1 - \beta)/[(n(1 - \beta^2)] \leqslant 1/2[\beta + (1 - \beta)n]^{-1}$ except when $n = 1$ and $\beta < 1$, we have that $\partial\pi^*/\partial\beta \geqslant 0$ unless $n = 1$ and $\beta < 1$. In this latter case, however, $y_2(n = 1)$ from (5) is below the cartel output if $r < \frac{4\beta}{1 - \beta}(1 + r)^{1/2}$. If this is not the case, y_1 from (2') is below $(1 - \beta)^{-1}$ such that (3') is violated. So $\partial\pi^*/\partial\beta > 0$. Intuitively, increased differentiation leaves the firm with a bigger residual demand $[1 - (1 - \beta)ny_2]$ which dominates the increased slope (β) of its inverse demand. So, contrary to common intuition, <u>increasing product differentiation offer firms greater temptations to cheat</u> in this model.

We next find

$$\frac{\partial\pi}{\partial\beta} = y_2^2(n - 1) \geqslant 0 \tag{7}$$

Intitively, increased differentiation leads, ceteris paribus, to larger profits, such that the optimal punishment $(1/r)\pi(y_2)$ can be more severe. So, again contrary to common intuition, <u>increasing product differentiation makes it possible to penalize a cheater harder</u> in this model.

For any given element of $S_1 \cap S_2$, we can find y_2 from (5) and insert (6) and (7) into

$$\frac{\partial F}{\partial\beta} = \frac{\partial\pi^*}{\partial\beta} - \frac{1 + r}{r}\frac{\partial\pi}{\partial\beta} \tag{8}$$

to see how these two effects net out. In general, the penalty effect will be relatively stronger for smaller values of r (because of $(1 + r)/r$), higher values of β, and smaller values of n (since y_2 is of the order $1/n$). Intuitively, penalties count more if r is small, the steeper inverse demand hurts the temptation more as β goes up, and the ability to penalize goes down as n increases (and π decreases). Conversely, the temptation effect will be relatively stronger for larger r, smaller β, and larger n. Again, the sign of $\frac{\partial F}{\partial \beta}$ at any particular point in $S_1 \cap S_2$, can be evaluated from (5)-(7).

While it is clear that increasing values of β correspond to increasing product differentiation, it is not at all clear how one measures "more" collusion. We will therefore look at several different measures of collusion.

Following some tradition in the literature (Deneckere, 1983; Mookerhjee and Ray, 1985) we will first look at the critical interest rate implied by $(4'')$. That is, we will ask if increasing β's allows a given $y_2(n)$ to be maintained for even larger values of r. Using the implicit function theorem on $(4'')$, we find that

$$\frac{\partial F}{\partial r} = \frac{1}{r^2}(y_2 - y_2^2[\beta + (1 - \beta)n]) > 0.$$

Accordingly, $dr/d\beta > 0$, such that <u>more product differentiation allows a given output to be maintained with higher discount rate</u>, if $\frac{\partial \pi^*}{\partial \beta} < \frac{1 + r}{r} \frac{\partial \pi}{\partial \beta}$, that is, <u>if the penalty effect dominates</u>. Conversely, if $\frac{\partial \pi^*}{\partial \beta} > \frac{1 + r}{r} \frac{\partial \pi}{\partial \beta}$ such that the temptation effect dominates, then more differentiation requires lower discount rates.

If the firms use the inoptimal Cournot-Nash punishments suggested by Friedman (1971), the analog of $(4'')$ is

$$(4''') \qquad \pi^*(y_2) + \frac{1}{r}\pi^0 - \frac{1 + r}{r}\pi(y_2) = 0$$

where the Nash profits $\pi^0 = \beta[n(1 - \beta) + 2\beta]^{-2}$ are increasing in β such that the analog of $\frac{\partial F}{\partial \beta}$ is bigger. Since these punishments become less severe as β increases, this would tend to make collusion relatively harder in the sense that $dr/d\beta$ is smaller.[4]

An alternative measure of collusion is given by y_2 in the sense that smaller y_2's mean that output is more restrained. To look at this, we find

$$\frac{\partial \pi^*}{\partial y_2} = - \frac{n(1 - \beta)}{2\beta}[1 - n(1 - \beta)y_2]$$

which is negative since $y_2 < [n(1 - \beta)]^{-1}$ (the opposite would entail negative profits). Intuitively, cheating profits are smaller if output is higher. Similarly,

$$\frac{\partial \pi}{\partial y_2} = 1 - 2y_2[\beta + (1 - \beta)n]$$

which is nonpositive since $y_2 \geqslant (1/2)[\beta + (1 - \beta)n]^{-1}$, the cartel output. In the relevant range, profits and thus penalties are decreasing in output. After inserting y_2 from (5) we do, however, get a negative net effect:

$$\frac{\partial F}{\partial y} = - [n^2(1 - \beta)^2 + n4\beta(1 - \beta) \frac{1 + r}{r} + 4\beta^2 \frac{1 + r}{r}](1 + r)^{1/2}$$

$$\cdot [(1 - \beta)^2 n^2 r + 4(1 + r)\beta^2 + 4(1 + r)\beta(1 - \beta)n]^{-1} < 0.$$

So if we measure increasing collusion by decreasing y_2's, we get the same results if we look at increasing r's. <u>More differentiation allows lower output</u> if $\partial F/\partial \beta < 0$, that is, <u>if the penalty effect dominates</u>. Conversely, if $\partial F/\partial\beta > 0$, more differentiation forces firms to increase their outputs.

The use of y_2 as a measure of collusion suggests a problem since the cartel output, $y_c \equiv \frac{1}{2}[\beta + (1 - \beta)n]^{-1}$, is increasing in β such that we are comparing y_2 to a moving target. Another way of seeing this is that the only possible equilibrium at $\beta = 1$ gives the cartel output. So it seems that "degree of collusion" should depend upon the relationship to the cartel solution. To adjust for this, we therefore look at the effect of β on $y_2/y_c = 2y_2[\beta + (1 - \beta)n]$. Differentiation with respect to β, followed by use of (5), gives

$$(9) \qquad \frac{d(y_2/y_c)}{d\beta} = 2\left(\frac{dy_2}{d\beta}[\beta + (1 - \beta)n] - (n - 1)y_2\right)$$

$$= -2D^2 n[n^2(1 - \beta)^2 r(2 + r + 2(1 + r)^{1/2})$$

$$+ n(1 - \beta)\beta 4(1 + r + (1 + r)^{1/2}) + 4\beta^2(1 + r)r] < 0.$$

where D is the denominator from (5).

So for any $(\beta,r,n) \in S_1 \cap S_2$, <u>more product differentiation allows the ratio of collusive output to cartel output to decrease monotonically</u>. Intuitively, even though differentiation may increase collusive outputs this effect is dominated by the simultaneous increase in cartel output. It is difficult to judge whether this result will pertain to a more general class of models, but we suspect that it will. In particular, since a related result--that the ratio of Nash output to cartel output will go down as differentiation goes up--will hold in most models. Given that the collusive outputs are between these two extremes, one might then conjecture that the above result will be quite robust to changing assumptions.

A final measure of collusion could be the ratio of collusive profits to cartel profits, $\pi^c \equiv (1/4)[\beta + (1 - \beta)n]$. Since these will decrease as β goes up it might also here be useful to adjust for this. We therefore look at the effect of β on $\pi/\pi^c = 4y_2[\beta + (1 - \beta)n] - 4y_2^2[\beta + (1 - \beta)n]^2$. Differentiation gives

$$\frac{d(\pi/\pi^c)}{d\beta} = 4(\frac{dy_2}{d\beta}[\beta + (1 - \beta)n] - (n - 1)y_2)(1 - 2y_2[\beta + (1 - \beta)n])$$

$$= 2\,\frac{d(y_2/y_c)}{d\beta}(1 - \frac{y_2}{y_c}) > 0$$

where the first factor is negative by (9), whereas the second is negative since $y_2 > y_c$.

So for any $(\beta,r,n) \in S_1 \cap S_2$, <u>more product differentiation allows the ratio of collusive profits to cartel profits to increase monotonically</u>. Using arguments similar to those for the y_2/y_c measure, one could speculate that this result will partain in a more general class of models.

4. <u>Conclusion</u>

In a simple supergame we have analyzed the effect of product differentiation on the maximum degree of tacit collusion. <u>For absolute measures, reaction speed or output, the net effect can go either way</u>. Loosely speaking, differentiation will tend to favor collusion more when firms are fewer and react faster: conditions which tend to make collusion easy in the first place. While this is type of result one would expect, it is important to note that the net effect comes about through <u>increasing</u> temptation and penalty effects, instead of opposite. One could speculate that our--here faulty--common sense about the problem applies to price games rather than quantity games.

For relative measures of collusion, output or profits, the effect of increased differentiation is always one which facilitates collusion. Since differentiation narrows the gap between competition and cooperation, this result is perhaps not that suprising. Our standard intuition fits this result better and relative measures may be the appropriate way to measure collusion.

When interpreting the results it is important to keep in mind that they only have been established for a range of parameter values, the inverse demand (1) and a quantity game. For (β, r, n) outside S_2, Abreu's simple punishment scheme is no longer optimal and a more complicated analysis is necessary. Although such an analysis is outside the scope of this apper, the intuition behind our results seems quite robust and one would expect the same basic tradeoffs to be present, perhaps with differing net results. An identical argument applies to alternative inverse demand functions. It is possible that some functions will lead to uniform effects of differentiation in the entire feasible set, but the underlying tradeoffs should still be there. For price games, Deneckere (1983; 1984) has found an ambiguous effect of differentiation on collusion in the context of Friedman's (1971) Nash punishments. We do, however, not yet have a satisfactory theory of optimal penalty schemes for such models. Consider, for example, the case of homogeneous products. The worst punishment is given by price = (unit cost) forever and any price is sustainable if $n < r/(1 + r)$. This is troubling in two ways. First, since r measures the reaction lag it should be very small with this information structure. Second, it is not intuitively plausible that no attempt to rebuild a crumbled agreement would appear. One might conjecture that we need more complicated models to analyze collusion in price games. One possibility is to get to a more realistic cost structure (as in Mookherjee and Ray, 1985), another is to look explicitly at capacity constraints (in which case the argument of Kreps and Scheinkman, 1983, may lead us back towards quantity games).

Notes

[1]The meaning of "small" will be made precise below.

[2]As stated, Abreu's results require homogeneous products, but inspection of his proofs reveal that identical strategy spaces and symmetric profit functions in the sense $\pi_i(q_1,q_2,\ldots,q_{i-1}, q_j,\ldots,q_{j'}, q_{j'+1},\ldots,q_{n+1}) = \pi_{i'}(q_1,q_2,\ldots,q_{i-1}, q_{j'},\ldots,q_i, q_{j'+1},\ldots,q_{n+1})$, $\forall i,j'$, will suffice. The model fulfills these requirements.

[3]This entails negative prices under y_1. If firms face constant unit costs above $\beta[\beta + (1 - \beta)n]^{-1}$ we avoid this mole.

[4]In this context it is interesting to note that Deneckere (1983; 1984) finds that differentiation all in all makes collusion easier for a duopoly where inverse demand is slightly different from (1). So we have to take care when interpreting partial and net effects.

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