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Preemptive Bidding and the Role of the Medium of Exchange in Acquisitions

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Abstract

In an asymmetric information setting, consisting of a market for corporate control in which an initial bidder's offer brings forth potential competition, the role of the medium of exchange in acquisitions is analyzed. A securities offer is shown to have a relative advantage in that it can induce target management to make an efficient, given its information, accept or reject decision on the offer. A cash offer is shown to have a relative advantage in that it is a "cheaper" medium with which to "preempt" potential competition. An equilibrium is developed in which both types of offers are observed, and implications are derived.

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1. Introduction

In structuring its offer to acquire a firm, an acquirer must, among other things, make a determination as to what the medium of exchange in the offer will be. That is, in addition to determining how high an offer to make, an acquirer must determine the form which the payment would take, whether it be cash, debt, equity, or some combination. In a world of homogeneous information, no transactions costs, and no taxes, the medium of exchange would be irrelevant. Such is not the case though if these assumptions are relaxed. Then, the medium of exchange will become a non-trivial decision variable. There have been previous studies which have examined the role of the medium of exchange from an accounting/tax perspective. Among the relevant issues are whether the acquisition will be accounted for as a purchase or a pooling, and whether the transaction will be taxable or will qualify as tax free (see for instance Carleton, Gulkey, Harris, and Stewart [3]).¹ This study takes a different approach and analyzes the problem in a setting of asymmetric information.

A key economic difference between a cash offer and an offer of securities is that the value of a cash offer is independent of the future profitability of the acquired target, while the value of a securities offer is not. With homogeneous information, this would not matter as any package of securities would be valued the same by the bidder and target, and in turn, any two packages with the same value would be perfect substitutes. This is not the case with asymmetric information. The willingness to offer and/or accept a given package of cash and securities may indicate something about

¹
the information of the bidder and/or the target. Thus the determination of
the value of a particular offer becomes more complex.

The particular focus of this study is on the role of the medium of
exchange as a bidder's strategic tool in "preempting" potential competition.
This, where preemption refers to the making of an offer which deters
potential competitors. Consider the problem facing a potential acquirer. It
may expend resources to study the profitability of acquiring another firm.
If it should then go ahead with an offer, other potential acquirers will
observe the offer, and thus learn of a potentially profitable acquisition. A
preemptive bid may be a way to cope with this unwanted competition. Suppose
the expected profit to becoming a competing bidder were decreasing in the
initial bidder's valuation for the target's assets. When bidding against a
high valuing initial bidder, a competitor may face a lower probability of
winning the bidding, and a lower expected profit given that it does win.
Given this scenario, if the initial bidder could credibly signal to its
potential competitors that it had a sufficiently high valuation, it could
deter the potential competitors. How then could an initial bidder credibly
signal such information? As Fishman [6] and P'ng [20] have demonstrated, a
high initial bid can be a credible signal of a high valuation. Both studies
model the takeover bidding process along the lines outlined above, and the
resulting equilibria are ones in which high valuing initial bidders may make
high initial bids in order to preempt their competition. These models
however, deal only with cash offers. Since only bidders observe private
information, only bidders would have private information on the value of an
equity or risky debt offer. Therefore any such offer would be perceived by
the target as being of low value, and would thus be dominated by cash.
The model here extends this analysis by allowing the target, in addition to the bidders, to observe private information regarding the profitability of an acquisition. This is an important possibility to consider. Target management would probably be expected to have the best information on the current condition of the target's physical assets, and the best information on the target's existing contractual commitments. It would thus seem likely that target management might possess private information on the profitability of the target assets under alternative control.

To see that securities offers would then be an important possibility, recognize that a bidder would prefer to make its offer contingent upon the information of the target. For instance, the bidder could specify a high payment if the target's information indicated a profitable acquisition, and a low payment otherwise. This could induce the target to make an efficient, given its information, accept or reject decision. That is, the target can be induced to accept the offer if it has observed "good news," and to reject the offer otherwise. If however, the target's information cannot be verified, it will not be possible to make such a contingent offer. An alternative though, is a securities offer. Rather than making the value of the offer contingent upon the target's information about future cash flows, the value of a securities offer is contingent upon the future cash flows themselves. If structured properly, a securities offer can also induce an efficient accept/reject decision on the part of the target. Contrast this with a cash offer. Since the value of a cash offer is independent of the profitability of the acquisition, a target can make its accept or reject decision independent of its information. Thus a cash offer cannot induce an efficient accept/reject decision.
A unique equilibrium is developed for a model of the bidding problem described above. In equilibrium, both securities offers and cash offers will be observed. The advantage of a securities offer is its ability to induce the target to make an efficient, given its information, accept/reject decision on the offer. A cash offer cannot do this. A cash offer, however, has the advantage that it is a "cheaper" medium with which to preempt a potentially competing bidder. In equilibrium, lower valuing bidders will make securities offers which do not preempt the potential competition, and higher valuing bidders will make preemptive cash offers.

Hansen [12] also studies the role of the medium of exchange in an asymmetric information setting, and an equilibrium is developed in which bidders make both cash and securities offers. As is the case here, the motivation behind a securities offer lies in its capacity to allow for a state-contingent payment to be offered to the target. The motivation behind cash offers, however, differs between the two models. In Hansen [12], cash offers are made in order to circumvent a problem of adverse selection. Bidders are assumed to possess private information or their own pre-merger values. In equilibrium, if a bidder makes a securities offer, it will be perceived as having a low value, and bidders with high pre-merger values will offer cash. Here, cash offers are made in an attempt to preempt a potentially competing bidder. Preemption is cheaper with cash offers.

The paper is laid out as follows. Section 2 sets out the basic model. Section 3 discusses the bidders' problems for the case in which bidders make either cash or risky debt offers, and Section 4 develops the equilibrium for the problem. Section 5 discusses the possibility of advance disclosure of the target's private information. It is demonstrated that subject to a
constraint that bidders are not deterred from studying the target altogether, the target's expected profit would be higher under a policy of advance disclosure. Section 6 discusses the bidding problem for the case in which bidders make either cash or combination cash and equity offers. With the use of simplifying assumptions, equilibrium outcomes will be unchanged. Section 7 discusses some implications of the model and Section 8 concludes the paper.

2. The Model

The model is an extension of that analyzed in Fishman [6]. It is presumed that the reason for an acquisition is to gain control over the target's operating strategy in order to implement a more profitable strategy than is currently being followed. Questions concerning what this change of strategy actually involves are not addressed. It is simply taken as given that the possibility of implementing a more profitable strategy may exist. Also, the possibility that bidders are attempting to profit on private information regarding the value of the target's assets as is, is not studied here. It is assumed that there exists no private information on the value of the target's assets as is.

For a given target, it is assumed that there are two potential bidders, termed 1 and 2, and it is assumed that the target and both bidders are all-equity firms with risk-neutral shareholders. Also, the managements of the target and both bidders are assumed to be acting to maximize the expected wealth of their respective shareholders, and it is assumed that there is no collusion among any of the three parties.

Bargaining between bidders and the target is not considered. It is assumed that the offer which has the highest value will be accepted given
that it is at or above a known minimum acquisition price. Though the analysis will go through with any minimum acquisition price, it is assumed that the minimum acquisition price is equal to the target’s prebid market value, denoted \( v_0 \).

It is assumed that there exists private information on the value of the target’s assets under each bidder’s control. Specifically, at a known cost \( k_1 > 0 \), bidder \( 1 \) can privately observe a signal, \( s_1 \), which conveys information on its own valuation but is independent of the other bidder’s valuation. The cost, \( k_1 \), is taken to include any direct costs of studying the target, for instance investment banker’s fees, and also any opportunity costs involved. The bidder could, for example, be working on other projects. In addition, the target is assumed able to costlessly observe a private signal, \( s_0 \), which conveys information on both bidders’ valuations. It is assumed that \( s_0 \) is independent of \( s_i \), for \( i = 1, 2 \). Also, the signals \( s_0 \), \( s_1 \), and \( s_2 \) cannot be physically verified. Thus contracts explicitly contingent on these signals cannot be enforced.

For \( i = 1, 2 \), let \( \{ f_i(s_i) \} \) be the probability density function of \( s_i \), where \( f_i(s_i) \) is strictly positive on the interval \([i, h]\) and zero elsewhere. It is assumed that \( s_0 \) has the following two-point distribution:

\[
\tilde{s}_0 = \begin{cases} 
\alpha & \text{with probability } 1 - \gamma \\
\beta & \text{with probability } \gamma 
\end{cases}
\]

where \( 0 < \gamma \leq 1 \). These distributions are common knowledge. The signals \( s_i \) and \( s_0 \) both convey information on bidder \( i \)'s valuation from controlling the target’s assets. Given realizations of \( s_1 \) and \( s_0 \), bidder \( i \) could then compute its valuation. Let \( v_i \) (for \( i = 1, 2 \)) denote bidder \( i \)'s valuation, where for a given realization of \( s_1 \), \( v_i = \sqrt{s_i} \), for \( \tilde{s}_0 = \alpha \), and \( v_i = \sqrt{s_i} \).
for \( z_0 = \beta \). It is assumed that \( v^0(s_1) \) and \( v^0(s_2) \) are both increasing in \( s_1 \)
and that \( v^0(s_1) > v^0(s_2) \geq 0 \), for all \( s_1 \). Moreover, it is assumed that
\( v^0(s_1) < v_0 \) for all \( s_1 \). Therefore, since \( v_0 \) is the minimum acquisition
price, neither bidder can profitably acquire the target if \( z_0 = \alpha \). Finally,
it is assumed that \( Ev(\bar{z}_1, \beta) < v_0 \) Bidders' expected valuations are below
the minimum acquisition price. The significance of this assumption is that
it implies that bidders would not find it profitable to bid without first
incurring the cost to become informed of their private signal.

The basic problem unfolds as follows. A bidder, call it bidder 1,
exogenously learns of a potentially profitable acquisition. Assume that
there is a large enough number of firms which would not be potentially
profitable acquisitions, so that studying random firms is not a profitable
strategy. Once bidder 1 learns of the target, it can incur the cost \( k_1 \)
to observe its private information, and then perhaps make an offer for the
target. Should bidder 1 not make an offer, the problem will end with the
knowledge of the target remaining known only to bidder 1.\(^6\) Suppose, though,
that bidder 1 does make an offer for the target. The offer will identify the
target as a potentially profitable acquisition. A second bidder, call it
bidder 2, will observe bidder 1's offer and will determine whether or not it
will compete for the target. After bidder 2 makes this determination, the
target is assumed to costlessly observe the signal \( z_0 \).\(^7\)

If bidder 2 chooses to compete, it will incur the cost \( k_2 \) to observe its
private signal. Then a competitive open auction (i.e., the value of the high
offer continues rising from the value of bidder 1's initial offer until only
one bidder remains) for the target will follow. If bidder 2 chooses not to
compete, bidder 1's initial offer will stand as the high offer. In either
case, the target will then either accept or reject the high offer. Specifically, it will accept this offer if its value equals or exceeds $V_0$, and it will reject it otherwise. It is important to note that the target will be able to observe any and all offers before having to accept or reject one. In particular, bidder 1's initial offer does not have to be rejected in order to observe an offer (should one materialize) from bidder 2. Finally, if an offer is accepted, control of the target's assets changes, and the acquirer privately observes its valuation.

A second bidder will observe a first bidder's initial offer before having to determine whether to compete. Therefore, if a first bidder's offer contains any information (about the first bidder's valuation), a second bidder will be able to utilize this information in making its decision. This in turn implies that a first bidder can, through its initial offer, influence a second bidder's action. In particular, a first bidder will have an incentive to bid so as to deter a potential second bidder, thus limiting competition. The problem is first analyzed for cash and risky debt offers.

3. The Bidders' Problems with Cash Offers and Risky Debt Offers

A cash offer consists of a price at which the bidder will purchase 100% of the shares of the target. With a cash offer $p$, the profits from an acquisition are as follows. Bidder $i$ will receive $V_i - p$, and the target will receive $p - V_0$. The debt offer considered here is an offer of pure discount debt, maturing immediately after the acquisition is consummated, and written against the assets of the target. For a debt offer with face value $p$, the profits from an acquisition are as follows. Bidder $i$ will receive $\max(V_i - p, 0)$, and the target will receive $\min(V_i, p)$. If the target assets under bidder 1's control are worth more than $p$, the debt liability will be
satisfied. If the target assets under bidder 1’s control are worth less than \( p \), the shareholders (the bidder) will default on the bonds and turn over the firm to the debtholders (the target). 8

Since \( v_0 \) is the minimum acquisition price, a bidder making a debt offer must set the face value of the offer \( p \geq v_0 \), for otherwise the offer is certain to be unsuccessful. Also, a bidder would set \( p < \varphi(s_1) \), for otherwise the bidder’s profit would be zero with certainty. Therefore, a bidder will set \( v_0 \leq p < \varphi(s_1) \). Consider now the target’s response to such an offer. If the target has observed \( \hat{s}_0 = \beta \), then the target’s profit from the offer would be equal to

\[
\min(\varphi(\hat{s}_1), p) - v_0 = p - v_0 \geq 0,
\]

and thus the target would accept the offer. Suppose though, that the target has observed \( \hat{s}_0 = \alpha \). In this case, the target’s profit from the offer would be equal to

\[
\min(\varphi(\hat{s}_1), p) - v_0 - \varphi(\hat{s}_1) - v_0 < 0,
\]

and thus the target would reject the offer. The debt offer induces the target to accept the offer when it has observed “good news” concerning the profitability of a change in control (i.e., \( \hat{s}_0 = \beta \)), and to reject the offer when it has observed “bad news” (i.e., \( \hat{s}_0 = \alpha \)). This is an efficient decision rule. 9,10 Contrast this with the cash offer. The target’s profit on a cash offer, \( p \), is equal to \( p - v_0 \), which is independent of the realization of \( \hat{s}_0 \). So in determining whether to accept or reject a cash offer, a target would not need to consult its private information. Let \( \sigma = (p, \theta) \) denote an offer, where \( p \geq v_0 \) is the face value of the offer and \( \theta \in (C, D) \) denotes the type, cash or debt respectively, of offer.

Consider now the problem facing a first bidder. Once it has exogenously
learned of a potentially profitable target for acquisition, it must determine whether or not to incur the cost \( k_1 \) to observe \( \bar{s}_1 \). To determine this, the optimal strategy given that bidder 1 has observed \( \bar{s}_1 \) must be determined first. Therefore, the problem which arises once the bidder has observed \( \bar{s}_1 \) is analyzed first.

Consider a first bidder which has incurred the (now sunk) cost \( k_1 \), and has observed \( \bar{s}_1 \). Recall that if \( \bar{z}_0 = \alpha \), bidder 1’s valuation is equal to \( v_0(\bar{s}_1) \), if \( \bar{z}_0 = \beta \) bidder 1’s valuation is equal to \( v_0(\bar{s}_1) \), and that \( v_0(\bar{s}_1) > v_0(\bar{s}_1) \) for all \( \bar{s}_1 \). Therefore, since the minimum acquisition price is \( v_0 \), if \( v_0(\bar{s}_1) < v_0 \), bidder 1 will not make an offer. Define \( \ell_0 \) such that \( v_0(\ell_0) = v_0 \) (the discussion implicitly assumes that \( \ell_0 \geq \ell \), though this is not important). Now suppose a first bidder has observed \( \bar{s}_1 \geq \ell_0 \). Since \( v_0(\bar{s}_1) \) is increasing in \( \bar{s}_1 \), there is some possibility that the bidder and target can profit from an acquisition, and thus some offer, either cash or debt, should be considered.

First consider the case in which bidder 1 makes an initial offer of debt with face value \( p \) such that \( v_0(\bar{s}_1) > p \geq v_0 \). Bidder 2 will observe the offer and will then determine whether to compete for the target. If bidder 2 does not compete, bidder 1’s initial offer will stand as the high offer. If bidder 2 does compete, it will incur the cost \( k_2 \), and observe \( \bar{s}_2 \). Then a competitive open auction will follow. The value of the high offer will rise until only one bidder remains. To see what form (i.e., cash or debt) the ultimately high offer in the auction will take, consider the incentives of the higher valuing bidder. Ideally, it would want to structure the offer so as to (i) induce the target to accept the offer if and only if \( \bar{z}_0 = \beta \), and (ii) acquire the target (if \( \bar{z}_0 = \beta \)) at the minimum necessary cost, which is
equal to the second highest valuation. As discussed above, the debt offer can have just this structure. Thus the ultimately high-valued offer in an auction will be debt with a face value equal to \( \min(\max(p, \beta(s_2)), \beta(s_1)) \).

That is, if the realization of \( s_2 \) is such that \( \beta(s_2) \neq p \), bidder 2 cannot profitably top the initial bid, and the initial bid will stand as the high bid. If \( s_2 \) is such that \( \beta(s_2) > p \), the high bid will be bid up to the second highest valuation. So the high bid will be a debt offer with face value \( p \), bidder 1's initial bid, if bidder 2 does not compete, and it will be a debt offer with face value \( \min(\max(p, \beta(s_2)), \beta(s_1)) \) if bidder 2 does compete. Whichever it turns out to be, the target will then accept this high offer if it has observed \( s_0 = \beta \), and it will reject it if it has observed \( s_0 = \alpha \).

Bidder 1’s expected profit from an initial offer of debt can now be computed. Let \( e \) denote the decision of bidder 2, where \( e = 1 \) refers to bidder 2 competing and \( e = 0 \) refers to bidder 2 not competing. Let

\[
\Pi_1(s_1, p, \theta, e) = \gamma(\beta(s_1) - p)
\]

\[
\Pi_1(s_1, p, D, 0) = \gamma(\beta(s_1) - p) - E \min(\max(p, \beta(s_2)), \beta(s_1))
\]

for \( e = 0 \) and \( e = 1 \).

Now consider the case in which bidder 1 makes an initial cash offer. After bidder 2 observes the offer, it will determine whether or not to compete. If bidder 2 does not compete, bidder 1’s initial offer will stand as the high offer. Since this is an offer of cash, the target will then accept the offer irrespective of the realization of \( s_0 \). If bidder 2 does
compete, it will incur the cost $k_2$ to observe $\tilde{r}_2$. Then, as in the case of an initial debt offer, a competitive open auction would follow. In fact, if $\tilde{r}_0 = \delta$, the payoffs will be the same as with an initial debt offer. The higher valuing bidder would acquire the target with an offer valued at $\min(\max(p, \psi^\delta(\tilde{s}_2)), \psi^\delta(s_2))$. Suppose, though, that the target observes $\tilde{r}_0 = a$. In this case, the target has no incentive to await the outcome of an auction. As discussed above, if the target would receive any offer higher than the initial cash offer $p$, in an auction, it would be a debt offer. So if the target has observed $\tilde{r}_0 = a$, it knows that it would not be willing to accept a debt offer, and it would simply accept the initial cash offer.

Bidder 1's expected profit from an initial offer of cash can now be computed. For a realization of $\tilde{s}_1 \geq \tilde{r}_0$ and an initial cash offer $p$, such that $\psi^\delta(\tilde{s}_1) > p \geq \psi_0$,

$$\begin{align*}
\Pi_1(s_1, p, C, 0) &= (1-\gamma)[\psi^\delta(s_1)-p] + \gamma[\psi^\delta(s_1)-p] \\
\Pi_1(s_1, p, C, 1) &= (1-\gamma)[\psi^\delta(s_1)-p] \\
&\quad + \gamma[\psi^\delta(s_1) - \mathbb{E}\min(\max(p, \psi^\delta(\tilde{s}_2)), \psi^\delta(s_1))]
\end{align*}
$$

for $e = 0$ and $e = 1$.

An examination of (1) and (2) makes clear several important points. First, notice that $\Pi_1(s_1, p, \delta, 0) \geq \Pi_1(s_1, p, \theta, 1)$. Holding bidder 1's initial offer fixed, bidder 1's expected profit is higher if bidder 2 does not compete. Competition from bidder 2 may be costly. Bidder 2 may raise the acquisition price, or it may outbid bidder 1 altogether. This is why bidder 1 has an incentive to attempt to deter bidder 2. Second, notice that $\Pi_1(s_1, p, \delta, e)$ is decreasing in $p$. Holding bidder 2's entry decision constant, bidder 1's expected profit is decreasing in the face value of its offer (for either type of offer). Third, notice that for $\gamma < 1$, 12
\Pi_1(s_1, p, D, e) \geq \Pi_2(s_1, p, C, e).\) Holding \(p\) and \(e\) constant, bidder 1's expected profit is higher if its offer is composed of debt rather than cash. This is because of the efficient target incentives induced by the debt offer. Despite this inequality, it will be demonstrated that bidder 1 may still have an incentive to make a cash offer. In equilibrium, it will be cheaper (i.e., require a lower \(p\)) to signal a high valuation with a cash offer. This to deter bidder 2 (i.e., to induce it to choose \(e = 0\)). Notice though, if 
\(\gamma = 1, \Pi_2(s_1, p, D, e) = \Pi_1(s_1, p, C, e).\) For this case, the target effectively possesses no private information, and cash and debt offers are equivalent.

Consider bidder 2's profit. Let \(\pi_2(s_1, s_2, s_0, p, \theta, e)\) denote bidder 2's profit conditional on \(s_1, s_2, s_0, \sigma, \) and \(e.\) For an initial offer \(\sigma = (p, \theta),\) such that \(\psi^\beta(s_0) \geq p \geq \psi_0,\)

\[
\pi_2(s_1, s_2, s_0, p, \theta, 0) = 0,
\]

\[
\pi_2(s_1, s_2, s_0, p, \theta, 0) \begin{cases} 
-k_2 & \text{if } s_0 = \alpha \\
\psi^\beta(s_0) - \min(\psi^\beta(s_1), \psi^\beta(s_2)) - k_2 & \text{if } s_0 = \beta
\end{cases}
\]

for \(e = 0\) and \(e = 1.\) Notice that for a given \(e,\) bidder 2's profit does not depend on \(\sigma = (p, \theta),\) bidder 1's initial offer. If bidder 2 does not enter the competition \((e = 0),\) bidder 2's profit is zero regardless of bidder 1's offer. If bidder 2 does enter \((e = 1),\) and chooses to bid, it would (as discussed above) bid with debt. Thus if \(s_0 = \alpha,\) bidder 2 would not acquire the target, and bidder 1's initial offer would be irrelevant. For \(s_0 = \beta,\) bidder 2 would have to offer \(\psi^\beta(s_0),\) bidder 1's valuation, in order to outbid bidder 1. Since bidder 1 would always set \(p < \psi^\beta(s_0),\) bidder 1's initial offer is again, irrelevant.
These results are however, for a given choice of \( e \), in determining its choice of \( e \), bidder 2 will in general find bidder 1’s initial offer to be an important variable. This is because bidder 1’s initial offer may contain information regarding the realization of \( \tilde{e}_1 \). More precisely, let \( \sigma(e_1) \) denote bidder 1’s bidding strategy as a function of its private information.

With bidder 1 following a pure strategy (which is the case studied here), the information which will be conveyed by bidder 1’s initial offer will be that \( \tilde{e}_1 \in S \), where \( S \) is some nonempty subset of \([e_0, h]\) (since \( \tilde{v}_1 < v_0 \) for realizations of \( \tilde{e}_1 < e_0 \), such realizations are ignored as they result in no bid). For an initial offer \( \sigma = (p, \theta) \), let \( S(\sigma) \) denote this subset. Then, given \( S(\sigma) \), bidder 2 can use Bayes’ formula to compute an updated probability density function for \( \tilde{e}_1 \), and also compute its expected profit from competing. Let \( \Pi_2(p, \theta, e) \) denote bidder 2’s expected profit conditional on bidder 1’s initial offer and on its own entry decision. Using (3), for an initial offer \( \sigma = (p, \theta) \), such that \( v^\hat{\beta}(\tilde{e}_1) > p \geq v_0 \),

\[
\begin{align*}
\Pi_2(p, \theta, 0) &= 0 \\
\Pi_2(p, \theta, 1) &= \gamma E[v^\hat{\beta}(\tilde{e}_2) \mid \tilde{e}_1 \in S(\sigma)] - \min(v^\hat{\beta}(\tilde{e}_1), v^\hat{\beta}(\tilde{e}_2)) - k_2.
\end{align*}
\]

Thus the importance of bidder 1’s initial offer (as regards bidder 2) lies solely in its effect on bidder 2’s beliefs concerning bidder 1’s private information. Through bidder 2’s beliefs, is the only way for bidder 1’s initial offer to affect bidder 2’s entry decision.

In equilibrium, a first bidder’s initial offer may affect the beliefs of a second bidder, and thus affect its actions. In particular, a first bidder may have an incentive to make a “high” initial offer to signal that it has a high valuation. This in an attempt to deter bidder 2 from competing. Bidder
2 may be deterred by such a signal because the higher it believes bidder 1's valuation to be, the lower is its expected profit from competing. There is a lower probability of winning the bidding, and given that it wins, the higher is the likely price which it will pay. This bidding problem is similar to that analyzed in Fishman [6] and P'ng [20], however there is an important difference. There only cash offers were relevant (cash offers dominated risky debt and equity offers), and the bidder simply had to determine the value of the offer. Here, in addition to choosing the value of an offer, a bidder must also choose the medium of exchange. So in equilibrium, not only will a bidder be signaling through the value of its offer; it will be signaling through its choice of offer type. We will now proceed to the development of a unique equilibrium.

4. Equilibrium

A first bidder, in making its initial offer, must determine what type of offer to make (i.e., cash or debt), and also how valuable an offer to make. In making this determination, the first bidder will take into account the effect of its choice on the beliefs of its potential competitor. For once bidder 1 has made its offer, bidder 2 will be alerted to the existence and identity of a potential profit opportunity. How bidder 2 then proceeds, i.e., whether it will compete for the target or not, will depend upon its beliefs concerning bidder 1's valuation. If it believes bidder 1 has a sufficiently high valuation, it will not compete, as the competition from bidder 1 would be too strong. So, other things equal, bidder 1 would prefer to make an offer which would signal a high valuation, and deter bidder 2. This is the strategic interaction which an equilibrium must characterize.
As is common in many signaling models, the restrictions of various equilibrium concepts will not be sufficient to lead to the existence of a unique equilibrium for the model studied here. For instance, by arbitrarily specifying the beliefs for out-of-equilibrium offers, the restrictions of a Sequential Equilibrium (see Kreps and Wilson [16]) will allow for the existence of a continuum of equilibria. Therefore a stronger equilibrium concept is required. The equilibrium concept applied here is that of a Perfect Sequential Equilibrium (see Grossman and Perry [11]). A Perfect Sequential Equilibrium combines the requirements of "sequential rationality" with a requirement of "credible" beliefs, and will be shown to lead to the existence of a unique equilibrium.

Bidder 1's bidding strategy, as a function of its private information, is denoted \( \sigma(s_1) = (p(s_1), \theta(s_1)) \). Bidder 2's updating rule, given that it has observed the initial offer \( \sigma = (p, \theta) \), is denoted \( S(\sigma) \). Given that it has observed the initial offer \( \sigma = (p, \theta) \), bidder 2's beliefs are that \( \tilde{\sigma}_1 \in S(\sigma) \). Bidder 2's strategy, as a function of its beliefs, is denoted \( \sigma(S) \). This specification takes into account that bidder 2's expected profit from competing is a function of bidder 1's initial offer only insofar as it affects its beliefs. A requirement that the bidders' strategies be sequentially rational is to be imposed.

The pair of strategies \( (\sigma', \sigma'(\cdot)) \) is defined to be sequentially rational with respect to the updating rule \( S'(\cdot) \), if and only if

(1) \( \sigma': [\mathcal{A}_0, h] \rightarrow [\mathcal{V}_0, \infty] \times \{C, D\} \), and

\[
\Pi_1(s_1, p'(s_1), \theta'(s_1), \sigma'(S'(\sigma'(s_1)))) \geq \Pi_1(s_1, p, \theta, \sigma'(S'(\sigma(\cdot))))
\]

for all \( \sigma \in [\mathcal{V}_0, \infty] \times \{C, D\} \), and for all \( s_1 \in [\mathcal{A}_0, h] \),

and

(5)
(11) \( e': S \to (0, 1) \), and \( \Pi_2(p, \theta, e'(S'(\sigma))) \geq \Pi_2(p, \theta, 0) \) for all \( e \in (0, 1) \), and for all \( \sigma \in \{V_0, =\} \times (C, D) \), where \( \Sigma \) is the set of all nonempty subsets of \([a_0, b] \).

This definition of sequential rationality corresponds to that of Kreps and Wilson [16]. Taking the updating rule as fixed, bidders must, from each point forward, be following optimal strategies.

For any bidder \( l \) strategy, \( \sigma'(*) \), define \( \hat{S}(\sigma; \sigma') = \{s_1 | \sigma'(s_1) = \sigma\} \). For a given offer \( \sigma \), \( \hat{S}(\sigma; \sigma') \) denotes the set of realizations of \( s_1 \) for which \( \sigma' \), would make the offer \( \sigma \). Note that if \( \sigma'(s_1) \neq \sigma \) for all \( s_1 \), then \( \hat{S}(\sigma; \sigma') \) is empty. A credible updating rule will now be defined. This notion of credibility is similar to that discussed by Kreps [15].

An updating rule, \( S'(*) \), is defined to be credible with respect to the pair of strategies \( (\sigma'(*), \sigma'(*)) \) if and only if, for all \( \sigma \in \{V_0, =\} \times (C, D) \),

1. If \( \hat{S}(\sigma; \sigma') \) is nonempty then \( S'(\sigma) = \hat{S}(\sigma; \sigma') \).

2. If \( \hat{S}(\sigma; \sigma') \) is empty then
   a) \( S'(*) = S \times S \) \( (6) \)
   b) If there exists an \( S \in S \) such that \( \Pi_1(s_1, p, \theta, e(S)) \geq \Pi_1(s_1, p'(s_1), \theta'(s_1), e(\hat{S}(\sigma'(s_1); \sigma'))) \) for all \( s_1 \in S \), and strict inequality for some \( s_1 \in S \), and
      \( \Pi_1(s_1, p, \theta, e(S)) \leq \Pi_1(s_1, p'(s_1), \theta'(s_1), e(\hat{S}(\sigma'(s_1); \sigma'))) \) for all \( s_1 \notin S \), then \( S'(\sigma) = S \). If there exists more than such \( S \), then \( S'(\sigma) \) can be set equal to any one of them.

This definition of credibility can be understood as follows.

Condition (61) is a requirement that for all offers which would be made
in the proposed equilibrium, bidder 2's beliefs must be consistent with bidder 1's strategy. Condition (6.11) is a requirement that for all offers which would not be made in the proposed equilibrium, bidder 2's beliefs must be, if possible, self-fulfilling. Suppose that for an offer, \( \sigma \), which would not be made by any bidder in the proposed equilibrium, there is a set \( S \) which satisfies (6.11b). Then, first bidders for which \( z_1 \in S \), and only those first bidders, would prefer, if it would induce bidder 2 to believe \( \bar{z}_1 \in S \), to deviate from the proposed equilibrium strategy and offer \( \sigma \). Believing \( \bar{z}_1 \in S \) would thus be self-fulfilling, and credibility requires \( S'(\sigma) = S \). A Perfect Sequential Equilibrium (PSE) can now be defined.

A pair of strategies, \( (\sigma'(\cdot), e'(\cdot)) \), combined with an updating rule \( S'(\cdot) \), constitute a PSE if and only if

(i) \( (\sigma'(\cdot), e'(\cdot)) \) is sequentially rational with respect to \( S'(\cdot) \), and

(ii) \( S'(\cdot) \) is credible with respect to \( (\sigma'(\cdot), e'(\cdot)) \).

A unique PSE will be developed. First, bidders' strategies which are sequentially rational with respect to a fixed updating rule will be characterized. Second, an updating rule which is credible with respect to a sequentially rational pair of strategies will be characterized. Combining these results will yield the unique PSE.

Some additional notation is useful. For \( z_1 \in [z_0, h] \), define

\[
P_D(z_1) = E \min(\max(\nu_0, \nu^\theta(z_2)), \nu^\theta(z_1)).
\]

\[
P_C(z_1) = \gamma E \min(\max(\nu_0, \nu^\theta(z_2)), \nu^\theta(z_1)) + (1 - \gamma) \nu^\theta(z_1).
\]

Using (1), it can be verified that for a given \( z_1 \), bidder 1 is indifferent between making a debt offer with face value \( p_D(z_1) \) given that it would deter
bidder 2, and making a debt offer with face value \( v_0 \) given that it would not deter bidder 2. Using (1) and (2), it can be verified that bidder 1 is indifferent between making a cash offer \( p_C(s_1) \) given that it would deter bidder 2, and making a debt offer with face value \( v_0 \) given that it would not deter bidder 2. Thus \( p_D(s_1) \) and \( p_C(s_1) \) represent the maximum debt and cash offers respectively, which a first bidder would be willing to offer to deter a second bidder. Comparing against an alternative of a debt offer with face value \( v_0 \) anticipates the equilibrium. If bidder 1 were not going to bid to deter bidder 2, it would make a debt offer with a zero premium. That is, it would make an offer which would only be accepted if \( \beta = \beta_0 \), and the minimum acceptable offer at that.

Since \( p_D(s_1) \) and \( p_C(s_1) \) are both increasing in \( s_1 \), the following inverse functions can be defined. For \( p^D \in [v_0, p_D(h)] \) and \( p^C \in [v_0, p_C(h)] \), define \( s_D(p) \) such that \( s_D(p_D(s_1)) = s_1 \), and \( s_C(p) \) such that \( s_C(p_C(s_1)) = s_1 \). The function \( s_D(p) \) gives the minimum value of \( s_1 \) for which bidder 1 would be willing to make a debt offer with face value \( p \), to preempt bidder 2, and the function \( s_C(p) \) gives the minimum value of \( s_1 \) for which bidder 1 would be willing to make a cash offer \( p \), to preempt bidder 2. Finally, letting

\[
g(s, p^D, p^C) = \gamma p^D + (1 - \gamma) v^D(s) - p^C,
\]

define \( s_{DC}(p^D, p^C) \) as follows. If there exists an \( s \) such that \( g(s, p^D, p^C) = 0 \), then \( s_{DC}(p^D, p^C) = s \). Otherwise, if \( g(s_0, p^D, p^C) > 0 \), then \( s_{DC}(p^D, p^C) = s_0 \). or if \( g(h, p^D, p^C) < 0 \), then \( s_{DC}(p^D, p^C) = h \).

Suppose that a debt offer with face value \( p^D \) and a cash offer equal to \( p^C \) would both preempt a second bidder. First bidders for which \( s_1 < h \) would prefer to make the preemptive debt (cash) offer. Bidder strategies which are sequentially rational with respect to a fixed
Lemma 1: For an updating rule, $S'(\cdot)$, suppose there exists a minimum $p$ for which $p \leq p_D(h)$, and $\Pi_2(p, D, 1) \leq 0$, and denote it $p_D$. Also suppose there exists a minimum $p$ for which $p \leq p_C(h)$, and $\Pi_2(p, C, 1) \leq 0$, and denote it $p_C$. Then, the unique pair of strategies which is sequentially rational with respect to $S'(\cdot)$ is given by

$$
\sigma'(s_1) = \begin{cases} 
(v_0, D) & \text{if } J_0 \leq s_1 < \min(s_D(p_D), s_C(p_C)) \\
(p_D, D) & \text{if } s_D(p_D) \leq s_1 < s_{DC}(p_D, p_C) \\
(p_C, C) & \text{if } s_1 \geq \max(s_C(p_C), s_{DC}(p_D, p_C)) 
\end{cases} \quad (7a)
$$

$$
\sigma'(S'(o)) = \begin{cases} 
1 & \text{if } \Pi_2(p, \theta, 1) > 0 \\
0 & \text{if } \Pi_2(p, \theta, 1) \leq 0. 
\end{cases} \quad (7b)
$$

Proof: See Appendix.

Bidder 1 will either make a debt or cash offer with a value equal to the minimum which would deter bidder 2's entry, given the type of offer made, or it will make a debt offer with face value equal to $v_0$, the minimum acquisition price. All other offers are dominated. Taking the updating rule as fixed, bidder 2's optimal strategy is straightforward. If its expected profit from competing is positive, it will do so, otherwise it will not.

Notice that in equilibrium, both bidders' strategies can be fully characterized by $p_D$ and $p_C$, which are determined by the updating rule. Since no first bidder would make an initial debt offer with face value greater than $p_D(h)$, we will represent the case of $\Pi_2(p, D, 1) > 0$ for all $p$ as $p_D = p_D(h)$. Similarly, we will represent the case of $\Pi_2(p, C, 1) > 0$ for all $p$ as $p_C = p_C(h)$. The bidder strategies given by (7) are the unique strategies which are
sequentially rational with respect to the fixed updating rule $S'(\sigma)$. An updating rule which is credible with respect to such strategies will now be characterized. For $I_0 \leq a \leq b \leq h$, define

$$w(a, b) = \gamma E[\nu^\beta(\tilde{x}_2) - \min\{\nu^\beta(\tilde{x}_1), \nu^\beta(\tilde{x}_2)\}] \hspace{1cm} a \leq \tilde{x}_1 \leq b - k_2.$$  

If bidder 2 believed that $a \leq \tilde{x}_1 \leq b$, then $w(a, b)$ would be its expected profit from competing. It is easily verified that $w(a, b)$ is decreasing in both $a$ and $b$. The higher bidder 2 believes $\tilde{x}_1$ to be, the lower is its expected profit from competing. Define $r$ such that $w(r, h) = 0$. The value $r$ is the critical value for which, if bidder 2 believed that $\tilde{x}_1 \geq r$, its expected profit from competing would be zero.

In what follows, it will be assumed that

$$w(I_0, h) > 0 \quad (8a)$$

and

$$p_D(r) > v_0. \quad (8b)$$

If bidder 1 would make an initial bid, it must be the case that $\tilde{x}_1 \geq I_0$. Therefore, this is the minimum which bidder 2 would learn from bidder 1’s initial bid. Assumption (8a) posits that this information would not be sufficient to deter bidder 2. Also, (8a) implies that $r$, as defined above, exists, and that $I_0 < r < h$. Assumption (8b) ensures that a cash offer equal to $v_0$ is not sufficient to deter bidder 2 from competing. Deterring bidder 2 will require a positive premium bid. The major result concerning a credible updating rule can now be stated.

**Lemma 2:** Suppose $S'(\cdot)$ is credible with respect to $(\sigma'(\cdot), e'(\cdot))$, and $(\sigma'(\cdot), e'(\cdot))$ is sequentially rational with respect to $S'(\cdot)$. If $p^D$ is the minimum value of $p$ for which $\Pi_2(p, D, l) \leq 0$, and $p^C$ is the minimum value of $p$ for which $\Pi_2(p, C, l) \leq 0$, then (i) $p^D > p_D(r)$, and (ii) $p^C = p_C(r)$.

**Proof:** See Appendix.

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Part (i) of the lemma states that in equilibrium, no debt offer below $p_0(r)$ could deter bidder 2. Part (ii) of the lemma states that the minimum cash offer which would deter bidder 2 is equal to $p_C(r)$. Notice that if $\gamma = 1$, then $p_0(r) = p_C(r)$. If $\gamma < 1$, then $\bar{s}_0 = \beta$ with certainty, and cash offers are equivalent to debt offers. Thus the minimum preemptive offers would also be equivalent. If $\gamma > 1$, however, then $p_0(r) > p_C(r)$. The cash offer which would be required to deter bidder 2 is lower than the face value of the debt offer which would be required to do the same. This can be alternatively put as follows. To deter bidder 2, bidder 1's initial offer must signal to bidder 2, at least that $\bar{s}_1 \geq r$. That $p_0(r) > p_C(r)$ means that the value of the cash offer which is required to credibly signal $\bar{s}_1 \geq r$ is lower than the face value of the debt offer which would do likewise.

This is a key result. Preemption is cheaper with cash. The intuition can be explained as follows. A debt offer is an offer which will only be accepted if $\bar{s}_0 = \beta$, that is, only when the bidder's valuation is high. A cash offer, on the other hand, will be accepted if $\bar{s}_0 = \beta$ or $\bar{s}_0 = \alpha$. Thus a cash offer is effectively a commitment to acquire the target even if $\bar{s}_0 = \alpha$, i.e., even when its valuation is low. The expected cost of this commitment is equal to $(1 - \gamma)(p - \nu^*(s_1))$. Since this expected cost is decreasing in $s_1$, it takes a higher value of $s_1$ (holding $p$ constant) to profitably make a cash offer as compared to a debt offer. Therefore, the price necessary to signal that $s_1$ is at or above any given level is lower in a cash offer as compared to a debt offer. Combining the characterization of sequentially rational strategies with the characterization of a credible updating rule yields the unique PSE.
Proposition 1: If the pair of strategies \((s^*(*), e^*(*))\), combined with the updating rule \(S^*(*))\), constitute a PSE, then

\[
\begin{align*}
\sigma^*(s_1) &= \begin{cases} 
(v_0, D) & \text{if } l_0 \leq s_1 < r \\
(p_C(r), C) & \text{if } s_1 \geq r
\end{cases} 
\end{align*}
\] (9a)

\[
\begin{align*}
e^*(S^*()) &= \begin{cases} 
1 & \text{if } \sigma = (p, D) \text{ with } p < p_D(r) \\
1 & \text{if } \sigma = (p, C) \text{ with } p < p_C(r) \\
0 & \text{if } \sigma = (p_C(r), C).
\end{cases}
\end{align*}
\] (9b)

Proof: See Appendix.

The unique PSE is a signaling equilibrium. Low valuing (i.e., \(l_0 \leq s_1 < r\)) bidders will make debt offers with a face value equal to \(v_0\). High valuing (i.e., \(s_1 \geq r\)) bidders will make cash offers \(p_C(r)\). The low valuation signaled by the debt offer will be such that bidder 2 will compete. The high valuation signaled by the cash offer will be such that bidder 2 will be deterred.\(^{13}\)

The equilibrium strategies of Proposition 1 can be substituted into the expected profit functions of the bidders and the target to determine the equilibrium expected profits of each. Bidder 1’s equilibrium expected profit from studying the target is given by

\[
\begin{align*}
\text{EL}_1(\tilde{s}_1, p^*(\tilde{s}_1), \sigma^*(\tilde{s}_1), e^*(S^*(\sigma^*(\tilde{s}_1)))) &= k_1 \\
&= E[v^D(\tilde{s}_1) \cdot p_D(\tilde{s}_1)] \cdot I_0 \leq \tilde{s}_1 < r \cdot \Pr(I_0 \leq \tilde{s}_1 < r) \\
&\quad + E[v^D(\tilde{s}_1) + (1 - \gamma)v^C(\tilde{s}_1) \cdot p_C(r)] \cdot \tilde{s}_1 \geq r \cdot \Pr(\tilde{s}_1 \geq r) \\
&\quad + \gamma E[v^D(\tilde{s}_1) - p_D(\min(\tilde{s}_1, r))] \cdot \tilde{s}_1 \geq r \cdot \Pr(\tilde{s}_1 \geq r) \\
&\quad + (1 - \gamma)E[v^D(s_1) - v^D(r)] \cdot \tilde{s}_1 \geq r \cdot \Pr(\tilde{s}_1 \geq r) \\
&\quad \cdot k_1.
\end{align*}
\] (10)

If (10) is positive, then bidder 1 will find it profitable to initially study the target. Bidder 2’s equilibrium expected profit is given by

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Finally, the target's equilibrium expected profit is given by

\[
E[\Pi] = E[\gamma p_0(\tilde{s}_1) - \nu_0 | \tilde{x}_0 \leq \tilde{s}_1 < r]Pr(\tilde{x}_0 \leq \tilde{s}_1 < r) \\
+ (p_0(r) - \nu_0)Pr(\tilde{s}_1 \geq r) \\
- \gamma E[p_0(\min(\tilde{s}_1, r)) - \nu_0 | \tilde{x}_0 \geq \tilde{s}_1 \geq \tilde{x}_0]Pr(\tilde{s}_1 \geq \tilde{x}_0) \\
- (1 - \gamma)(\nu_0 - \nu^* (r))Pr(\tilde{s}_1 \geq r).
\]

(12)

Comparative statics exercises with respect to \( k_2 \), bidder 2's cost of studying the target, can be considered. Consider a decrease in \( k_2 \). For any realization of \( \tilde{s}_1 \), bidder 2's expected profit from competing is higher, and thus its equilibrium expected profit is higher. Also, it would require a higher valuing first bidder to deter bidder 2, that is, \( r \) is decreasing in \( k_2 \). In order to credibly signal this required higher valuation, it would require a higher initial bid, that is, \( p_0(r) \) is increasing in \( r \). So it is costlier for a first bidder to deter a second bidder, and thus the first bidder's equilibrium expected profit must be lower. Consider now the target. A higher cost of deterring bidder 2 implies that there will be fewer cases in which bidder 2 is preempted. Also, for those cases in which bidder 2 is preempted, the preemptive bid will be higher. Both work to the benefit of the target, and thus the target's equilibrium expected profit will be higher. For an increase in \( k_2 \), the reverse analyses apply.

5. Advance Disclosure of the Target's Information

The structure of the problem is such that the target does not observe
its private information until after a first bidder has bid, and a second bidder has made its decision as to whether or not to study the target.

Suppose, though, that the target could observe and (truthfully) disclose its information, perhaps at some cost, prior to a first bidder's initial offer (but, in order to preserve the sequentiality of the bidding problem, after the first bidder has studied the target). This section discusses the target's incentives regarding such a disclosure policy. In particular, it will be demonstrated that subject to the constraint that an initial bidder is not deterred, a target's expected profit would be increased if such a disclosure policy were feasible.

Suppose the target observes and (truthfully) discloses the realization of $\tilde{s}_0$ before the first bidder has made an initial offer. How will the bidding contest proceed? If the target discloses $\tilde{s}_0 = \alpha$, there will be no bidding. Bidders would know that the target would not be a profitable acquisition. Now consider the case in which the target discloses $\tilde{s}_0 = \beta$.

What is the nature of the bidding problem now? The problem is the same as before with the exception that it is already known that $\tilde{s}_0 = \beta$. Thus the bidders and the target are effectively facing the same problem as before with the specification that $\gamma = 1$. Note that $r$ is defined by $w(r, h) = 0$, and thus $r$ is a function of $\gamma$. So define $r'$ such that $w(r', h) = 0$ for $\gamma = 1$.

Also note that $r' > r$. Since bidder 2's expected profit from competing is increasing in $\gamma$ (see (4)), if there is advance disclosure of $\tilde{s}_0 = \beta$, bidder 2 is more difficult to deter (as compared to the original scenario). So for the advance disclosure scenario, only first bidders for which $\tilde{s}_1 \geq r'$ would make preemptive bids.

So with probability $1 - \gamma$, $\tilde{s}_0 = \alpha$ is disclosed, in which case there is
no bidding and the profits of all are zero. With probability \( \gamma \), \( \bar{s}_0 = \beta \) is
disclosed, and expected profits are equal to the expected profits of the
original scenario, (10), (11), and (12), with the specification that \( \gamma = 1 \).
Letting \( \pi^d_1 \) denote bidder 1's profit for the advance disclosure scenario,

\[
E[\pi^d_1] - k_1 = \gamma E[\nu^\beta(\bar{s}_1)] - p_0(\min(\bar{s}_1, r')] | \bar{s}_1 \geq f_0] \Pr(\bar{s}_1 \geq f_0) \cdot k_1. \tag{13}
\]

It is clear that (10) exceeds (13). Bidder 1's expected profit is lower in
the advance disclosure scenario. Advance disclosure has an adverse effect on
bidder 1's profit for two reasons. First, as discussed above, for
realizations of \( \bar{s}_0 = \beta \), bidder 2's expected profit from competing is higher
for all \( \bar{s}_1 \). Therefore bidder 1 must signal a higher valuation to deter it.
Second, as demonstrated in the previous section, preemption is cheaper with
cash. This is because a cash offer commits the bidder to acquiring the
target irrespective of the realization of \( \bar{s}_0 \). With advance disclosure, the
bidder is not uninformed of the realization of \( \bar{s}_0 \), and thus a cash offer can
no longer serve this purpose. Cash would be equivalent to debt. Thus
advance disclosure limits the actions available to an initial bidder.
Without advance disclosure, the bidder had the option of making a debt offer
which is effectively an offer contingent upon a realization of \( \bar{s}_0 = \beta \).
Alternatively, it could make a non-contingent cash offer. With advance
disclosure, the possibility of making the non-contingent offer has been
eliminated. So not only must an initial bidder signal a higher valuation, it
must do so with a higher cost mechanism. For these reasons, an initial
bidder's expected profit would be reduced. Note that if its expected profit
is reduced enough (so that (13) is negative), it would not find it profitable
to study the target initially, and get the bidding contest going.

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For bidder 2,

\[ E[H^d_2] = \gamma E[H^d_2 | \tilde{z}_0 - v] + (1 - \gamma)E[H^d_2 | \tilde{z}_0 = v] \]

\[ = \gamma E[\phi(\tilde{z}_2) \cdot \min(\nu^B(\tilde{z}_1), \nu^N(\tilde{z}_2)) \cdot k_2 | \tilde{z}_1 < r']Pr(\tilde{z}_0 = \tilde{z}_1 < r'). \]

It can be shown that (14) exceeds (11). Bidder 2’s expected profit is higher in the advance disclosure scenario. This is because advance disclosure eliminates the cases in which bidder 2 would study the target, only to later learn that \( \tilde{z}_0 = v \).

Letting \( H^d_0 \) denote the target’s profit in the advance disclosure scenario,

\[ E[H^d_0] = \gamma E[p_0(\min(\tilde{z}_1, r')) | \tilde{z}_1 \geq \tilde{z}_0]Pr(\tilde{z}_1 \geq \tilde{z}_0). \]

(15)

It is clear that (15) exceeds (12). For the same reasons that bidder 1’s expected profit is lower, the target’s expected profit is higher with advance disclosure. There will be fewer preemptive bids, and those preemptive bids which do occur, will be at higher premiums. Note that there is a cost of adopting an advance disclosure policy. Consider the set of joint realizations of \( \tilde{z}_0 = v \) and \( \tilde{z}_1 \geq r \). In the original scenario, a first bidder would have made a preemptive bid, and the target’s profit would be equal to \( p_C(r) - v \). With advance disclosure, there would have been no bid made. Disclosure that \( \tilde{z}_0 = v \) would deter all bidding. So some profitable bids (for the target) which would have been forthcoming will be eliminated. However, subject to the constraint that an initial bidder is not deterred, the benefits exceed the costs, and a target would do better with an advance disclosure policy.

A bidder which is incompletely informed about its valuation for a
target's assets is making a bid for these assets. It has been demonstrated
that advance disclosure of the information which the bidder lacks, and which
the target will possess (i.e., the realization of $E_0$), will raise the
expected selling price for the target. It might appear that this target gain
from advance disclosure is the result of the elimination of an adverse
selection problem facing the bidder. This is not the case though. Recall
that the possibility of making a debt offer effectively allows a bidder to
make an offer which is contingent on $E_0$. Thus there is no adverse selection
problem facing a bidder. Contrast this with a model such as that of Leland
and Pyle [17]. There, the capital market is less informed about the value of
a project than is the entrepreneur which is attempting to finance the
project. Since fully contingent contracts cannot be written, an adverse
selection problem exists, and the effect of advance disclosure would be to
eliminate it, thus benefitting the entrepreneur. If contingent contracts
could be written, advance disclosure would be irrelevant. Here, even in the
presence of effectively contingent contracts, advance disclosure is still
important. This is due to the existence of one bidder's incentive to preempt
another. Advance disclosure serves to raise the cost of preemption, and the
target is a beneficiary of this cost increase.

6. Cash Offers and Combination Cash/Equity Offers

The previous sections analyzed the problem in which a bidder made either
a cash offer or a debt offer. With debt, a bidder could effectively make a
state-contingent offer. In practice, however, a common type of securities
offer observed is one which includes equity of the would-be merged entity.
This raises the question of whether the problem studied here can accommodate
such offers. With the use of simplifying assumptions, it will be demonstrated that a combination cash/equity offer can serve the same purpose as did the debt offer. Further, it can be shown that the resulting equilibrium outcomes will be unchanged.

Let \( y_1 \) denote the value of any bidder 1 assets prior to an acquisition. So if bidder 1 acquires the target, the merged entity would have a value of \( y_1 + v_1 \), where \( v_1 \) equals either \( v^\delta(s_1) \) or \( v^\theta(s_1) \). Assume that the value of \( y_1 \) is common knowledge. Also assume that \( y_1 \geq v_0 \), for \( i = 1, 2 \). That bidders are assumed larger than the target will be seen to simplify the discussion by ensuring that the merger offer will not bankrupt a bidder. If after bidder 1's initial offer, bidder 2 decides to compete, it will incur the cost to observe its signal. Assume that once it does, both \( \tilde{s}_1 \) and \( \tilde{s}_2 \) (but not \( \tilde{s}_0 \)) will be freely observable by both bidders and the target. This assumption simplifies the determination of the outcome of a competitive open auction between the two bidders. The assumption would have been irrelevant with the debt offers studied above. For there, neither the target nor the other bidder needed to know a bidder's private information to value its offer. The value of the debt offer was independent of \( \tilde{s}_2 \) (for \( i = 1 \) or 2).

With equity offers, this is no longer the case.

Consider now, a combination cash (or riskless debt) and equity offer \((p, n)\), where \( p \leq y_1 \) is a cash payment and \( 0 \leq n < 1 \) is an equity share of the merged entity. The value of such an offer (made by bidder 1) to the target is

\[
p + n(y_1 + \tilde{v}_1 - p).
\]

Notice that the assumption that \( \tilde{v}_1 \geq 0 \) and the specification that \( p \leq y_1 \) ensures that the merged entity will not be bankrupted by the offer.

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The advantage of the debt offer studied above was that it could induce the target to make an efficient accept/reject decision on the offer. That is, the target would be induced to accept the offer if $z_0 = \beta$, and to reject it if $z_0 = \alpha$. A combination cash/equity offer can be structured to do the same. For bidder 1 to induce the target to reject its offer if $z_0 = \alpha$, it must be that the offer would be worth less than $v_0$. That is,
\begin{equation}
p + n(y_1 + \varphi(s_1) - p) < v_0. \tag{17}
\end{equation}
An immediate implication of (17) is that the bidder must set $p < v_0$. Thus with the assumption that $v_0 \leq y_1$, presuming that $p \leq y_1$ is consistent with the analysis. For bidder 1 to induce the target to accept its offer if $z_0 = \beta$, it must be that the offer would be worth at least $v_0$. So consider an offer valued at $x \geq v_0$, if $s_0 = \beta$. That is,
\begin{equation}
p + n(y_1 + \varphi(s_1) - p) = x. \tag{18}
\end{equation}
Solving for $p$, (18) is equivalent to
\begin{equation}
p = \frac{n(y_1 + \varphi(s_1))}{\left(1 - n\right)}. \tag{19}
\end{equation}
Now, substituting for $p$, using (19), into (17) and rearranging yields
\begin{equation}
n > \frac{x - v_0}{\varphi(s_1) - \varphi^2(s_1)}. \tag{20}
\end{equation}
Thus a cash/equity offer $(p, n)$, where $p$ is given by (19), $n$ satisfies (20), and $x \geq v_0$, is an offer which will be accepted by the target if $s_0 = \beta$, and rejected otherwise.\textsuperscript{15}

By choosing $(p, n)$ and $x$ appropriately, any offer which could be made with debt, can be duplicated with cash and equity. Further, given the assumptions, it can be shown that the equilibrium outcomes will be unchanged. If $l_0 \leq z_1 < r$, bidder 1 will make a cash/equity offer with $p$ given by (19).
n satisfying (20), and $x = v_0$. Recall that $x = v_0$ implies that if $\bar{a}_0 = \beta$, the offer will be worth $v_0$, the minimum acquisition price. For this initial offer bidder 2 will infer that $I_0 < \bar{a}_1 < x$, and it will choose to compete. If $\bar{a}_1 \geq x$, bidder 1 will make a cash offer equal to $p_C(r)$. For this initial offer bidder 2 will infer that $\bar{a}_1 \geq x$, and it will be deterred from competing. We will now proceed to a discussion of the implications of the model.

7. Implications of the Model

In the analysis here, the key role of a securities offer was as a means of making a state-contingent offer and inducing an efficient accept/reject decision on the part of the target. The target is induced to accept the offer when its information indicates that the acquisition is profitable, and to reject it otherwise. A cash offer, which has a value which is independent of the future profitability of the acquired target, cannot induce such a target response. A cash offer ($p \geq v_0$) will always be acceptable, regardless of the target's information. Thus the model predicts that the target is more likely to reject a securities offer as compared to a cash offer. Note that for this prediction, rejection refers to the rejection of an offer in the absence of a higher offer. It does not refer to the rejection of one offer in favor of a higher one. To my knowledge, there is no existing empirical evidence on this question. There is an interesting point to make though. Dodd [4] reports that for a sample of merger offers, if the target's management rejected the offer, the price of the target's equity dropped. One conclusion which might be drawn from this finding is that this is evidence of a firm's management acting contrary to the interests of its shareholders. The theory here suggests another possibility. The rejection of a securities
offer indicates that the proposed acquisition would not be profitable. Thus the offer's rejection should result in a drop in the price of the target's equity (the price will have risen upon the initial announcement of the offer), as this is bad news for target shareholders. It would have been worse for target shareholders, though, had the offer been accepted. So the argument that management's interests diverged from those of shareholders may be less compelling for such cases. It would be interesting to examine rejected offers classified on the basis of the type of offer made.

Despite the inferior incentives induced on the part of the target, some bidders will still choose to make cash offers. This is because it is cheaper to preempt a potentially competing bidder with cash as compared to securities. The model predicts that competing bidders are more likely to be observed following an initial securities offer as compared to an initial cash offer. To my knowledge, there is no existing empirical evidence on this question either.

Expected profits, conditional on the medium of exchange, can also be considered. In equilibrium, lower valuing first bidders (i.e., \( I_0 \leq \tilde{s}_1 < r \)) will choose to make securities offers valued at \( v_0 \). Then, a second bidder, after observing this initial offer, would choose to compete. Thus a first bidder's expected profit, conditional on its making a securities offer, is given by (using (1)),

\[
\gamma \mathbb{E}(\nu^\beta(\hat{z}_1) \cdot p^0(\hat{z}_1) \mid I_0 \leq \tilde{s}_1 < r) \cdot k_1. \tag{21}
\]

Higher valuing first bidders (i.e., \( \tilde{s}_1 \geq r \)) will choose to make cash offers equal to \( p^0(r) \). This offer would deter a potentially competing second bidder. Thus a first bidder's expected profit, conditional on its making a cash offer, is given by (using (2)),

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\[ E[\gamma \cdot (1 - \gamma) v^0(\hat{s}_1) \cdot P_C(r) | \hat{s}_1 \geq r] - k_1 \\
= \gamma E[p_D(\hat{s}_1) | \hat{s}_1 \geq r] + (1 - \gamma) E[v^0(\hat{s}_1) - v^0(r) | \hat{s}_1 \geq r] - k_1. \tag{22} \]

It is straightforward to verify that (22) exceeds (21). A first bidder which is observed making a cash offer will have a higher expected profit than one which is observed making a securities offer. Empirically, Asquith, Bruner, and Mullins [1] and Gordon and Yagil [8] study the returns to bidders in samples of merger offers, conditional on the medium of exchange. Consistent with the implication here, both studies report a significantly higher equity price revaluation for bidders which made cash only as compared to those which made combination cash and securities offers.\(^{17}\)

Whether the market would value a target's expected profit upon receiving a cash offer higher or lower than that of a securities offer is ambiguous. Since there would be competition following an initial securities offer, a target's expected profit given an initial securities offer is equal to

\[ \gamma E[p_D(\hat{s}_1) | \hat{s}_0 \leq \hat{s}_1 \leq r] - v_0. \]

Since a competing bidder would be deterred by a cash offer, a target's expected profit given an initial cash offer is equal to

\[ P_C(r) \cdot v_0. \]

Thus the target's expected profit is higher given an initial cash offer as compared to an initial securities offer if

\[ P_C(r) > \gamma E[p_D(\hat{s}_1) | \hat{s}_0 \leq \hat{s}_1 < r] + (1 - \gamma)v_0. \]

If the minimum price which would deter bidder 2's entry is sufficiently high, then the target would have a higher expected profit given an initial cash offer. Empirically, Gordon and Yagil [8], Walkling and Huang [22] and Wansley, Lane, and Yang [23] have studied the returns to targets of merger
offers, conditional on the medium of exchange. Each of the studies finds significantly higher equity price revaluations for targets of cash only offers as compared to targets of combination cash and securities offers.\footnote{18}

8. Conclusion

A key economic difference between a cash-only merger offer and one which includes securities lies in the difference in target management’s incentives induced by the two types of offers. Suppose target management possesses private information regarding the profitability of the target assets under alternative control. Since the value of a securities offer will depend upon the future profitability of the acquired target, target management will be induced to utilize this information in making its decision as to whether or not to accept the offer. Since the value of a cash offer is independent of the future profitability of the acquired target, target management will make its accept/reject decision without consulting its information. With a focus on this difference, and in a setting consisting of a market for corporate control in which an initial bidder’s offer brings forth potential competition, the role of the medium of exchange in acquisitions was analyzed.

In the equilibrium developed, both securities and cash offers would be observed. The relative advantage of a securities offer is that it can be structured so as to induce target management to make an efficient, given its information, accept/reject decision on the offer. A cash offer cannot induce such a response. The relative advantage of a cash offer is that it is a cheaper medium with which to preempt a potential competitor. That is, preemption requires the potential competitor to expect a non-positive profit from competing. In equilibrium, a lower valued cash offer (as compared to a securities offer) can credibly signal a valuation sufficient to convey such
information. The nature of the equilibrium is as follows. Lower valuing bidders would find it optimal to make non-preemptive securities offers, and higher valuing bidders would find it optimal to make preemptive cash offers. Among the implications derived from the equilibrium are:

1. The probability that target management will reject an offer is higher if the offer consists of securities.
2. The probability that competing bidders will be observed is higher following a securities offer.
3. An initial bidder’s expected profit from an offer is lower given that the bidder has been observed making a securities offer.

An interesting avenue for further work is a deeper examination of the issue of advance information disclosure by target management. It was demonstrated that target shareholders would benefit from a policy of advance disclosure. How then, in practice, could such a policy be implemented? More generally, bidders will seek to minimize the costs of preempting potential competition, and targets will seek to maximize these costs (subject to the constraint that an initial bidder is not deterred). What other phenomena in the market for corporate control can be explained (at least partially) as responses to such incentives?
Footnotes

1. See Dreyfus [5] for a strategic, tax-based analysis of a bidder’s choice of medium of exchange. Equity offers have the advantage that target shareholders can defer the tax on the transaction, whereas cash offers lead to the immediate taxation of the transaction. Cash offers however, are shown to have an advantage in that they are a lower-cost way to induce target shareholders to tender. This is because the free-rider problem (see Grossman and Hart [9]) is alleviated relative to the one faced in a tax-deferred offer.

2. The nature of the problem is similar to that of limit-pricing problems. See for instance Milgrom and Roberts [18]. See also Giammarino and Heinkel [7] and Khanna [14] for related sequential bidding models of takeovers.

3. Grossman and Hart [10] study a model in which a bidder can profit from both an increase in the productivity of the target’s assets, and private information regarding the value of the target’s assets as is.

4. No conflict of interest between management and shareholders is studied. For analyses of corporate control contests when there is a conflict of interest, see Baron [2] and Harris and Raviv [13].

5. Suppose the problem described is exactly replicated, period after period, until such time as the target is acquired. With an infinite horizon and stationary environment, the market value of the target, \( v_0 \), would be the same at the beginning of every period. Furthermore, the alternative to a merger offer in any given period is \( v_0 \). Thus \( v_0 \) would be the target’s reservation price in a take-it-or-leave-it offer.

6. There are other possible strategies which are not studied here. For instance, a bidder could invest in the target’s equity and then announce its information. This could lead to another bidder making a bid with the possibility that the initial bidder could profit from its investment.

7. The possibility that the target can observe and disclose \( v_0 \) prior to any bidding is discussed in Section 3.

8. It was assumed that the debt was issued against the value of the target assets only. The identical offer could also have been constructed if the debt was to be issued against combined bidder and target assets. Suppose \( y_b \) is the value of bidder I’s assets exclusive of an acquisition, and suppose \( y_t \) is common knowledge. If bidder I acquires the target, the merged entity will have a value of \( y_b + v \). Issue debt with face value \( p \) to the target which is subordinate to debt with face value \( y_t \) issued to the bidder. The payoffs from this offer are identical to the payoffs of the original offer.

9. That there exists a securities offer which can induce the target to make an efficient, given its information, accept/reject decision holds true for general specifications of the joint distribution of \( y_0 \), \( s_0 \), and \( v(s_1, s_2) \).

10. There is an implicit assumption that there is no trading of this debt prior to its maturity. This simplifies the discussion by eliminating questions concerning the appropriate target managerial objective function. With trading, target shareholders who will not be selling their debt have
different interests than those who will.

11. Independence of $x_1$ and $x_2$, which is assumed here, is a stronger assumption than is necessary. That $E[\Pi,(p, \theta, 1)|x_1]$ be decreasing in $x_1$ is sufficient for the analysis which follows. If this condition did not hold, the knowledge that $x_1$ were high, would not be a deterrent.

12. Other studies of "multiple variable signaling" equilibria include Milgrom and Roberts [19] and Viswanathan [22]. The former analyzes the choice of product price and advertising expenditure as a signal of product quality. The latter analyzes the choice of debt/equity and dividend levels as a signal of a firm's profitability.

13. Bidder 1's responses have only been specified for debt offers with face values less than $p_0(r)$ and for cash offers less than or equal to $p_0(r)$. Bidder 2's response to other offers cannot be uniquely specified. This nonuniqueness is not crucial though. These are offers which bidder 1 would never make, regardless of bidder 2's response. Thus varying bidder 2's response to such offers would not affect the sequence of choices which would actually be observed for any realization of $x_1$.

14. The cash payment can be financed through internal funds (i.e., out of $y_1$), or through borrowing with the lender receiving debt of the merged entity.

15. Note that there is no nonnegativity constraint on $p$. The case of $p < 0$, a cash outflow from target shareholders, has been allowed. Such an offer corresponds to an offer of warrants. Target shareholders would receive warrants for the fraction $n$ of the merged entity with an exercise price of $p$.

16. Note, again that the analysis utilizes the strong assumption that the realizations of $x_1$ and $x_2$ would be freely observable by all once bidder 2 enters. A more satisfactory analysis awaits the relaxation of this assumption.

17. The Asquith, Bruner, and Mullins [1] results were obtained via discussion with Paul Asquith.

18. Another hypothesis consistent with these findings involves the differential tax treatment of the two types of offers. Since target shareholders can defer the taxes on the gains from equity offers (until such time as the equity is sold), but cannot defer the taxes on the gains from cash offers, it requires a higher valued cash offer to be successful.

- r2 -
Appendix

Proof of Lemma 1.

That (7b) is bidder 2’s optimal strategy is immediate. Consider now bidder 1’s optimal strategy. Let \( \sigma^D = (v_0, D) \), \( \sigma^C = (p^D, C) \), and \( \sigma^C = (p^C, C) \).

For \( \sigma = (p, D) \), where \( v_0 < p < p^D \),

\[
\Pi_1(s_1, p, D, e'(S'(\sigma))) = \Pi_1(s_1, v_0, D, 1) < \Pi_1(s_1, v_0, D, 1) = \Pi_1(s_1, v_0, D, e'(S'(\sigma^D))).
\]

For \( \sigma = (p, D) \), where \( p > p^D \),

\[
\Pi_1(s_1, p, D, e'(S'(\sigma))) < \Pi_1(s_1, p^D, D, e'(S'(\sigma^D))) \leq \Pi_1(s_1, p^D, D, 0) - \Pi_1(s_1, p^D, D, e'(S'(\sigma^D))).
\]

For \( \sigma = (p, C) \), where \( v_0 \leq p < p^C \),

\[
\Pi_1(s_1, p, C, e'(S'(\sigma))) = \Pi_1(s_1, p, C, 1) \leq \Pi_1(s_1, v_0, D, 1) = \Pi_1(s_1, v_0, D, e'(S'(\sigma^D))).
\]

For \( \sigma = (p, C) \), where \( p > p^C \),

\[
\Pi_1(s_1, p, C, e'(S'(\sigma))) < \Pi_1(s_1, p^C, C, e'(S'(\sigma^D))) \leq \Pi_1(s_1, p^C, C, 0) = \Pi_1(s_1, p^C, C, e'(S'(\sigma^C))).
\]

Therefore offers other than \( \sigma^D \), \( \sigma^D \), and \( \sigma^C \) are dominated.

Among these three offers,

\[
\Pi_1(s_1, p^D, D, e'(S'(\sigma^D))) - \Pi_1(s_1, v_0, D, e'(S'(\sigma^D))) = \Pi_1(s_1, p^D, D, 0) - \Pi_1(s_1, v_0, D, 1) - \gamma(v^D(s_1) - p^D) - \gamma(v^\theta_1(s_1) - \text{Emin}(\max(v_0, \alpha^\theta_1), v^\theta(s_1))) - \gamma(p_0(s_1) - p^D) \geq \langle \langle \rangle \rangle 0 \text{ as } s_1 \geq \langle \langle \rangle \rangle s_0(p^D). \tag{A1}
\]

\[
\Pi_1(s_1, p^C, C, e'(S'(\sigma^C))) - \Pi_1(s_1, v_0, D, e'(S'(\sigma^D))) = \Pi_1(s_1, p^C, C, 0) - \Pi_1(s_1, v_0, D, 1) - \gamma(v^\theta(s_1) + (1 - \gamma)v^C(s_1) - p^C) - \gamma(v^\theta(s_1) - \text{Emin}(\max(v_0, \alpha^\theta_1), v^\theta(s_1))) - \gamma(p_0(s_1) - p^C) \geq \langle \langle \rangle \rangle 0 \text{ as } s_1 \geq \langle \langle \rangle \rangle s_0(p^C). \tag{A2}
\]
\[ S_{1}(s_1, p^C, C, e'(S'(e^D))) - S_{1}(s_1, p^D, D, e'(S'(e^D))) = S_{1}(s_1, p^C, C, 0) - S_{1}(s_1, p^D, D, 0) = \gamma p^D + (1 - \gamma)\nu^D(s_1) - p^C \geq 0 \] 

That (7a) is bidder 1's optimal strategy follows directly from inequalities (A2) and (A3).

**Proof of Lemma 2.**

Let \( \sigma^D = (\nu^D, D) \), \( \sigma^C = (p^C, C) \).

(1) Establish \( p^D \geq p^D(r) \).

Say \( p^D < p^D(r) \). There are two cases to consider.

(a) \( s_{DC}(p^D, p^C) \leq s_{D}(p^D) \).

This implies \( p^C < p^D(r) \), which implies \( s_C(p^C) < s_C(p^D(r)) \).

Using lemma 1, credibility requires \( S'(\sigma^C) = S(\sigma^C; \sigma^C) = [s_C(p^C), s_{DC}(p^D, p^C)], h \). Therefore \( E_2(p^C, C, 1) = w(\max(s_C(p^C), s_{DC}(p^D, p^C)), h) > w(r, h) = 0 \), which is a contradiction.

(b) \( s_{DC}(p^D, p^C) > s_{D}(p^D) \).

Using lemma 1, credibility requires \( S'(\sigma^D) = S(\sigma^D; \sigma^C) = [s_D(p^D), s_{DC}(p^D, p^C)] \). Therefore \( E_2(p^D, D, 1) = w(s_D(p^D), s_{DC}(p^D, p^C)) > w(s_D(p^D(r)), h) = w(r, h) = 0 \), which is a contradiction.

Thus \( p^D \geq p^D(r) \).
(11) Establish $p^C = p_C(r)$.

Say $p^C > p_C(r)$.

Using lemma 1, credibility requires $S'(\sigma^C) = \hat{S}(\sigma^C; \sigma')$
- $\{\max(s_C(p^C)), s_{DC}(p^D, p^C), h\}$. Therefore, using (1), $\Pi_2(p^C, C, 1)$
- $w(\max(s_C(p_C(r)), s_{DC}(p^D, p_C(r))), h) = w(\max(r, s_{DC}(p^D, p_C(r))), h)$
- $w(r, h) = 0$.

Say $p^C < p_C(r)$.

$s_C(p^C) < s_C(p_C(r)) - r$ and $s_{DC}(p^D, p^C) < s_{DC}(p^D, p_C(r)) - r$.

Using lemma 1, credibility requires $S'(\sigma^C) = \hat{S}(\sigma^C; \sigma')$
- $\{\max(s_C(p^C)), s_{DC}(p^D, p^C), h\}$.

Therefore, $\Pi_2(p^C, C, 1) = w(\max(s_C(p^C)), s_{DC}(p^D, p^C), h)$
- $w(r, h) = 0$, which is a contradiction.

Say $p^C > p_C(r)$.

Using lemma 1, for $\sigma = (p_C(r), C)$, $\hat{S}(\sigma; \sigma')$ is empty.

Therefore to determine $S'(\sigma)$ consider (611). It will be shown that there exists a unique $S \in S$ for which (611) is satisfied for $e = (p_C(r), C)$. There are two cases to consider.

(a) Say $S \in S$ and $E[\pi_2(\tilde{z}_1, \tilde{z}_2, \tilde{z}_0, p_C(r), C, 1)|\tilde{z}_1 \in S] > 0$.

For $s_1 \in [t_0, h]$
- $\Pi_2(s_1, p_C(r), C, e'(S)) = \Pi_2(s_1, p_C(r), C, 1)$
- $\Pi_2(s_1, \tilde{y}_0, D, 1) = \Pi_2(s_1, \tilde{y}_0, D, e'(S(\sigma^C; \sigma')))$
- $\Pi_1(s_1, p'(s_1), \hat{S}(s_1; \sigma')))

and there exists no such $S$ which satisfies (611b).

(b) Say $S \in S$ and $E[\pi_2(\tilde{z}_1, \tilde{z}_2, \tilde{z}_0, p_C(r), C, 1)|\tilde{z}_1 \in S] \leq 0$.

For $s_1 \in [t_0, h]$
- $\Pi_2(s_1, p_C(r), C, e'(S)) = \Pi_2(s_1, p_C(r), C, 0)$
- $\Pi_1(s_1, p^D, C, 0) = \Pi_1(s_1, p^C, C, e'(S(\sigma^C; \sigma')))$.

- A3 -
Using (A2) and the fact that $s_C(p_C(r)) = \tau$, for $s_1 \in \{r, h\}$,

\[ \Pi_1(s_1, p_C(r), C, e'(S)) = \Pi_1(s_1, p_C(r), C, 0) \]
\[ \geq \Pi_1(s_1, \gamma_0, D, 1) = \Pi_1(s_1, \gamma_0, D, e'(S(s)'; s'))) \]

Using (A3), for $s_1 \in \{s_D(p_D, p_C(r)), h\}$,

\[ \Pi_1(s_1, p_C(r), C, e'(S)) = \Pi_1(s_1, p_C(r), C, 0) \]
\[ \geq \Pi_1(s_1, p^D, D, 0) = \Pi_1(s_1, p^D, D, e'(S(s)'; s'))) \]

Therefore, since $e'(s_1)$ is equal to $s^0$, $s^D$, or $s^C$, we have

that for $s_1 \in \{s_0, h \} \cap \{r, h\} \cap \{s_D(p_D, p_C(r)), h\}$,

\[ \Pi_1(s_1, p_C(r), C, e'(S)) \not\geq \Pi_1(s_1, p'(s_1), e'(S(s_1); s'))) \]

Since $p^D \not\geq p_0(r)$ (by (1)),

\[ s_D(p^D, p_C(r)) \not\leq s_D(p_0(r), p_C(r)) = \tau \]

Therefore $[s_0, h] \cap \{r, h\} \cap \{s_D(p_D, p_C(r)), h\} = [r, h]$.

So for $s_1 \in \{r, h\}$, $\Pi_1(s_1, p_C(r), C, e'(S)) \geq \Pi_1(s_1, p'(s_1), e'(S(s_1); s')))$, with strict inequality holding for $s_1 > r$.

Using (A2) and the fact that $s_C(p_C(r)) = \tau$, for $s_1 \in \{s_0, r\}$,

\[ \Pi_1(s_1, p_C(r), C, e'(S)) = \Pi_1(s_1, p_C(r), C, 0) \]
\[ < \Pi_1(s_1, \gamma_0, D, 1) = \Pi_1(s_1, \gamma_0, D, e'(S(s)'; s'))) \]
\[ \leq \Pi_1(s_1, p'(s_1), e'(S(s_1); s'))) \]

Lastly, for $S = \{r, h\}, \Pi(s_2(\tilde{S}_1, \tilde{S}_2, \tilde{\gamma}_0, p_C(r), C, 1)|\tilde{s}_1 + S = w(r, h) = 0$, and we have that $S = \{r, h\}$ uniquely satisfies

$(67ib)$.

Therefore $s'(r) = (r, h)$ and $\Pi(p_C(r), C, 1) = w(r, h) = 0$, which is a contradiction.

Thus $p^C = p_C(r)$.

Q.E.D.

- A4 -
Proof of Proposition 1.

That $e^*(S^*(e))$ is given by (9b) follows directly from lemma 2.

Using lemma 2, $s_d(p^D) \geq s_d(p_D(r)) = r$, $s_c(p^C) = s_c(p_C(r)) = r$, and $s_D(p^D, p^C) \leq s_D(p_D(r), p_C(r)) = r$.

Therefore $\min(s_d(p^D), s_c(p^C)) = r$, $s_d(p^D) \geq s_D(p^D, p^C)$, and

$\max(s_c(p^C), s_D(p^D, p^C)) = r$.

That $\sigma^*(s_L)$ is given by (9a) then follows directly from lemma 1.

Q.E.D.
References


