Discussion Paper No. 682

THE MACROECONOMIC EFFECTS OF UNCERTAIN FISCAL POLICY

by

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December, 1984
Revised, February, 1986

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*The author gratefully acknowledges the comments of Yves Balcer, Jerry Green and participants of the October, 1984 NBER Tax Group meeting, and the financial support of the National Science Foundation and the Kellogg Graduate School of Managment.

Abstract

This paper examines the impact of possibly uncertain fiscal policy on resource allocation in a representative agent model of dynamic general equilibrium. Unlike most macroeconomic analysis, we assume that the government has only distortionary tax instruments. We first demonstrate existence in a constructive fashion. Second, we analytically determine the marginal impact of uncertain policy near a steady state. In particular, we find that the first-order effect of uncertain fiscal policy is arises solely from uncertainty in the timing of future policy changes, implying that conventional risk aversion considerations are of lesser importance. Furthermore, such uncertainty induces a magnification effect, in that a mean-preserving spread in the uncertainty as to when a policy will be enacted will preserve the direction but increases the magnitude of the policy's immediate effects. Third, in the case of information arrivals at Poisson times, we are able to move beyond first-order effects. We show that the anticipation of a future tax of fixed size will lead to immediate decumulation. A future tax of size sufficient to balance a growing deficit may initially stimulate investment and output, but eventually the prospect of a tax increase will drag down investment and output. In general we find that the uncertainty acts to change the rate at which returns are discounted, the change being interpretable as partially a risk premium.

1. Introduction

The direction and timing of future fiscal policy is surely an important in the determination of short-run output and its allocation between consumption and investment. Furthermore, the formation and execution of those government spending and tax policies is an uncertain process. Government spending policy is subject to uncertain and random forces, depending on changes in perceived needs. Tax changes also have much uncertainty surrounding their durability, and promised tax changes are possibly never carried out. One need only recall U.S. tax legislation in the recent past to realize that supposedly permanent tax cuts enacted one year may be cancelled the next. Sometimes tax changes are not intentional, as is the case when monetary policy interacts with a nominal tax structure, implying that uncertainty about monetary policy will cause uncertainty about taxes. Some argue that the current high deficits will force higher future taxes. However, even those with such beliefs are uncertain as to the date and magnitude of the future changes.

In this paper we adapt an often-used general equilibrium model to examine the impacts of uncertain future tax and spending changes. We examine an infinitely-lived representative agent general equilibrium model of capital accumulation with wage and capital income taxation, an investment tax credit, and government consumption. In contrast with most macroeconomic analysis (Brock and Turnovsky being a notable and early exception), we assume that the government has only distortionary tax instruments. While technology and tastes are deterministic, we examine the effects of a change in future taxes, allowing these changes and their timing to be uncertain. The first objective of this paper is to build a model combining these elements and prove the existence of an equilibrium. The proof of existence we provide is not only a logical necessity to further analysis, but also will be useful to future simulation work. Current simulation analyses of fiscal policy assume deterministic technology and policy, assumptions which should be relaxed. The constructive nature of our existence proof essentially provides an algorithm for simulation of equilibrium with uncertain policy and technology.

We next examine various macroeconomic issues using the linear models which arise when we take linearizations of the general nonlinear model around a steady state. We find several interesting relations. First, we show that the linear rational expectations solutions examined by Buiter (1983) have a substantially greater range of applicability than indicated there. Second, we demonstrate that a marginal increase of a policy parameter at some uncertain time in the future will decrease current output or the current supply of a factor if and only if a deterministically anticipated future increase in the tax parameter decreases output or the factor's supply, respectively. Moreover, we demonstrate a magnification effect in that mean-preserving spreads in the timing of a future marginal increase in a policy parameter increases the magnitude of the anticipated policy change's immediate effects. When we examine particular policies and use parameter estimates from the existing empirical literature we find a strong bias towards future increases in capital taxation reducing current labor and capital supply, and uncertainty in its timing increasing this contraction. Also, future investment tax credit increases at uncertain times will likely reduce investment. On the other hand, news of increases in future wage taxation or government consumption at uncertain future times will likely increase current labor supply and investment.

Third, we examine the problem of balanced-budget changes where agents know that the longer a necessary tax change is delayed, the greater the magnitude of the eventual tax change. Complex revenue interactions make a general analysis difficult, but we do show that the earlier results continue to hold if the initial tax rates are low.

Fourth, we show that these results in general describe future responses to new information, yielding intuitive relationships between news and its effects. For example, new information indicating an increase in the uncertainty in the time when a future wage tax will be imposed will likely increase current factor supply.

Fifth, we examine the case of information arrivals at Poisson times. In this case we can move beyond the local linear analysis and prove that policy uncertainty in that changes the rate at which returns are discounted, with the change being interpretable as a risk premium. This provides an intuitive formulation of the idea that uncertain policy introduces such a risk premium.

Section 2 describes the basic model and proves the crucial existence theorem. Section 3 derives formulae for the marginal impact of uncertain future policies and Section 4 discusses their implications. Section 5 examines the budget constraint and its implications for policy impacts. Section 6 examines the case where information arrives accoring to a Poisson process in continuous time and examines the dynamic effects of some simple examples of uncertain future policies. Section 7 concludes the paper and points to future possible developments.

2. The Model

We assume that all agents have an intertemporal utility function over consumption paths, $c(t)$, and labor paths, $\ell(t)$:

$$U = \int_0^\infty e^{-\rho t} u(c,\ell)\,dt$$

where u is a concave in consumption and labor with $u_1 > 0 > u_2$. We assume that the net production, $F(K,\ell)$, is concave in capital and labor and displays constant returns to scale. Agents may hold two perfectly substitutable assets, capital stock and government bonds. For our purposes it is sufficient to assume that either the initial stock of bonds is zero or bonds are continuously rolled over. In both cases we can ignore the bond market in examining the real evolution of the economy (see Judd (1985) or Brock and Turnovsky (1982)). Taxes are assessed on capital income at the rate of τ_K (we assume true economic depreciation) and on labor income at τ_L. An investment credit θ on gross capital investment and lump-sum transfers of Tr are made each period to each agent. The gross-of-tax returns on labor and capital are w and r, respectively.

The representative agent's problem is

$$\max_{c,\ell} \int_0^\infty e^{-\rho t} u(c,\ell)dt,$$

$$\dot{K} = (1 - \tau_K)rK + (1 - \tau_L)w\ell - c + \theta(\delta K + \dot{K}) + Tr$$

We assume that $\tau_K(t)$, $\tau_L(t)$, $\theta(t)$, and $Tr(t)$ follow stochastic processes. At times T_k, $k = 1,\ldots,n$, agents receive information, represented by $\tilde{i}_k \in \{i_1^k, i_2^k, \ldots, i_{m_k}^k\}$, concerning the paths of taxes and expenditures, where m_k is the number of possible messages at T_k. It will also be convenient to define $T_0 = 0$. Let $\pi_{I_k}^{i_\ell}$ be the probability of receiving message i_ℓ conditional on the information set at time $t \in [T_k, T_{k+1})$, denoted by $I_k \equiv \{i_1,\ldots,i_k\}$. $I_0 = \emptyset$ will denote the information at $t = 0$ that policies in the future are uncertain and follow the specified stochastic processes, and I_t will generally represent the information set as of time t. If $\ell < k$, then $\pi_{I_k}^{i_\ell}$ is 1 if $i_\ell \in I_k$ and zero otherwise. By Bayes' rule, beliefs follow a martingale, i.e., $\pi_{I_k}^{i_\ell} = E\{\pi_{I_{k+m}}^{i_\ell} | I_k\}$ for all positive k, ℓ, m. Since the realizations of $\tilde{i}_k$, k=1,..,n, are to represent information about the policy instruments, τ_K, τ_L, θ, Tr, and g must be measurable with respect to the information process, i.e., at all times t, they are functions only of calendar time and the information revealed up to time t. Therefore, let $\tau_{K,i_1,...,i_k}(t)$ be the capital income tax rate rate at $t \in [T_k, T_{k+1})$ if the cumulative information at t is $\{i_1,...,i_k\}$. $\tau_L(t)$, $\theta(t)$, Tr, and g(t) are similarly represented. Since all information is revealed in this fashion, $\tau_K(t)$, $\tau_L(t)$, $\theta(t)$, g(t) are known with no uncertainty conditional in $\{i_1,...,i_k\}$ for $t \in (T_k, T_{k+1})$, where T_{n+1} is infinity. We also assume that after T_n all policies are independent of calendar time. This is only a simplifying assumption without any apparent economic substance since T_n can be at an arbitrarily distant point in the future. The same is true of the asumption that information arrives only at a finite number of times since that number is arbitrary. In fact, we will later allow n to be infinite and T_k - T_{k-1} to be infinitesimal, thereby approximating continuous time. Finally, note that we make no stationarity assumptions concerning the evolution of information.

Before continuing some justification should be made for this admittedly cumbersome approach. If we were modelling competitive equilibrium with uncertain shocks to, say, the production function, then one could use a standard model with either discrete time or with information arriving continuously. Such a problem would reduce to a dynamic optimization problem by the efficiency of equilibrium, and be expressible in the standard dynamic programming fashion. This study instead examines a market equilibrium with distorting taxes. No such equivalence is known for such problems. One must therefore deal directly with an equilibrium formulation. As we will see, this formulation has a substantial advantage in that the messages occur at a finite number of moments, during which there are no intertemporal allocation problems to analyze, and all intertemporal allocation issues arise between the message arrival times, an interval during which the economy is deterministic. This decomposition drastically simplifies the analysis relative to a model where information and intertemporal problems arise simultaneously but does not do untoward damage to the reasonableness of the analysis since the interval between message times is arbitrarily small.

We next characterize the individual agent's solution to his intertemporal optimization problem. Since no information is revealed for $t \in (T_k, T_{k+1})$, $\lambda_{i_1,..,i_k}(t)$, the private shadow price of capital for $t \in [T_k, T_{k+1})$ if $\{i_1,..,i_k\}$ has been realized, and $K_{i_1,..,i_k}(t)$, the capital stock similarly defined, obey the deterministic equations,

(1a) $$\dot{\lambda} = \lambda\left(\rho - ((1 - \tau_K) - \delta\theta)/(1 - \theta)\right)$$

(1b) $$\dot{K} = r(1 - \tau_K)k + w(1 - \tau_L)\ell - c + \theta(\delta K + \dot{K}) + Tr$$

(1b) is derived by the standard deterministic Hamiltonian methods since the problem is deterministic between message times. We will drop the subscripts and arguments of λ and K when no confusion arises.

We must determine how the agent reacts to information. The crucial fact is that λ cannot have any anticipated jump in expectation at $t = T_k$,

(2) $$\lambda(T_k^-) = E\{\lambda(T_k^+) \mid I_{k-1}\}$$

(T^- represents the left limit at T and T^+ represents the right limit.) This follows from the arbitrage condition

$$u_c(c,\ell)/(1 - \theta) = \lambda = E_t\{\int_t^\infty u_c(c,\ell)(r(1 - \tau_K) + \delta\theta)e^{-\rho(s-t)}dt\}$$

which states that the marginal utility of one unit of current consumption must equal the marginal utility of the future net consumption which could be achieved by investing that unit today, consuming the resulting investment tax credit, and not altering future net investment plans. At all times t, individual maximization implies that c and ℓ are chosen to satisfy

(3a) $$0 = u_c(c,\ell)w(1 - \tau_L) + u_\ell(c,\ell)$$

(3b) $$0 = u_c(c,\ell) - \lambda/(1 - \theta)$$

where at $t = T_k$, $k=1,..,n$, λ is assigned $\lambda(T_k^+)$, implicitly assuming that information arrival at k precedes choice of c and ℓ. In equilibrium, both factors must receive their marginal product:

(4a) $$F_K = r = f'(k)$$

(4b) $$F_\ell = w = f - kf'(k)$$

where $k \equiv K/\ell$ is the capital-labor ratio and $f(k) \equiv f(K,\ell)/\ell$ is output per unit of labor input.

To complete our general equilibrium system, we must describe government consumption. We will assume that the government also consumes the single produced good and that its consumption does not alter the marginal rates of substitution among private goods. This would occur if the public consumption had no utility value or entered private utility in an additively separable fashion. In order to avoid nonsensical policies, we assume that government consumption, $g(K, I_t, t)$, is a function of the capital stock as well as the state of the world and calendar time. This is done in order to impose the assumption that it is always feasible to finance government consumption without decumulating capital. We also assume that g_K exists and is continuous to avoid technical problems. Note that we allow g to depend on the information set at time t, I_t, thereby allowing it to also be stochastic.

The general equilibrium equations then become

(5a) $$\dot{\lambda} = \lambda[\rho - (f'(k)(1 - \tau_K) + \delta\theta)/(1-\theta))], \quad t \neq T_1, T_2, \ldots, T_k$$

(5b) $$\dot{K} = F(K, L(\lambda, K, \theta, \tau_L)) - C(\lambda, K, \theta, \tau_L) - g(K(t), I_t, t)$$

(5c) $$\lambda(T_k^-) = E\{\lambda(T_k^+) | I_k\}$$

(5d) $$0 < \lim_{t\to\infty} \lambda(t),\ K(t) < \infty$$

where k=K/L and $L(\lambda,K,\theta,\tau_L)$ and $C(\lambda,K,\theta,\tau_L)$ are the solutions to

(6a) $$u_1(C,L) = \lambda/(1 - \theta)$$

(6b) $$-\frac{u_2(C,L)}{u_1(C,L)} = F_\ell(K,L)(1 - \tau_L)$$

for fixed K, λ, τ_L, and θ. We impose the stability condition (5d), which can be justified by global conditions derived from the representative agent's maximization problem (see Brock and Turnovsky (1981)). The crucial assumption that we will make is that all of the possible final autonomous equilibrium systems after T_n are saddlepoint stable. This is represented by assuming that for all possible histories up to T_n there is a manifold such that for all k there is a λ on that manifold such that from that (k,λ) point the system will converge to a steady state.

Theorem 1 Assume that for each possible information set I_n at T_n^+ there is a manifold, $M^+_{I_n}(K,\lambda)$ such that for every K the system (5) will remain bounded after T_n if and only if it is at (K,λ) at T_n^+ for some λ such that $M^+_{I_n}(K,\lambda) = 0$. Then there exists an equilibrium set of random variables, K_{I_t} and λ_{I_t}, which solve (5) everywhere.

Proof: For all $t > T_n$, the system is deterministic conditional on $\{i_1,i_2,\ldots,i_n\}$. By assumption, there is a manifold, $M^+_{i_1,\ldots,i_n}(K,\lambda)$, such that the system is stable for $t \geqslant T_n$ if and only if $M^+_{i_1,\ldots,i_n}(K(T_n), \lambda(T_n^+)) = 0$. For any K, $\lambda(T_n^-)$ is determined by the condition that $\lambda(T_n^-) = E\{\lambda(T_n^+)|I_{n-1}\}$. This yields a manifold $M^-_{i_1,\ldots,i_{n-1}}(K,\lambda)$ such that $M^-_{i_1,\ldots,i_{n-1}}(K(T_n), \lambda(T_n^-)) = 0$ must hold if λ is to respond to each possible i_n at T_n in a fashion which ensures stability for $t \geqslant T_n$. For $t \in [T_{n-1},T_n)$, the system follows a deterministic path conditional on $\{i_1,\ldots,i_{n-1}\}$, implying that there is a unique manifold $M^+_{i_1,\ldots,i_{n-1}}(K,\lambda)$, such that $0 = M_{i_1,\ldots,i_{n-1}}(K(T_{n-1}), \lambda(T^+_{n-1}))$ must hold if the system is stable for $t > T_{n-1}$. We can continue inductively to compute $\lambda(T^-_{n-1})$, the manifold $M^+_{i_1,\ldots,i_{n-1}}$, $\lambda(T^+_{n-2})$, etc. In the end, we find a $M_\emptyset(k,\lambda)$ such that we have stability for all realizations if and only if $0 = M_\emptyset(K(0), \lambda(0))$. Q.E.D.

The proof of Theorem 1 is somewhat technical and misses the basic economic intuition. For purposes of clarity, a simple version of the proof of Theorem 1 applied to a standard macroeconomic problem is illustrated in Figure 1. Suppose labor is inelastically supplied and there is no investment tax credit. Then our equilibrium system reduces to

$$\dot{c} = u'(c)\big(\rho - (1 - \tau_K)f'(k)\big)/u''(c)$$
$$\dot{k} = f(k) - c - g$$

which is represented in the phase diagram in Figure 1. It is straightforward to show that in this special case all the manifolds of Theorem 1 are invertible, that is, $M_{\lambda k}$ is never zero, and hence that equilibrium is unique. For any fixed τ_K and g, the phase diagram is of the saddlepoint variety, with motion being northwesterly in the northwest quadrant, and southeasterly in the northeast quadrant, northeastly in the southwest quadrant, and southwestly in the northeast quadrant, where the quadrants are defined by the $\dot{c} = 0$ and $\dot{k} = 0$ loci. Suppose that τ_K is such that the steady-state capital stock is k^{ss} and the economy is at A. If τ_K is cut at t=0, then the $\dot{c} = 0$ locus shifts right, but the $\dot{k} = 0$ locus is unaffected, and the new steady state is B. However, at some time, T_1, it is reported that τ_K will be increased sufficiently to balance the budget at $\tilde{T}$, either T_2 or T_3, $T_2 < T_3$. If it is T_2 then the long-run system has steady state at C, but if it is T_3, the long-run steady state is D. D is left of C as long as delaying the tax increase implies a larger eventual tax rate, a valid assumption if the present value of tax revenue is increasing in the tax rate.

By stability of the equilibrium and continuity of the consumption path if the tax increase occurs at T_2, then the $t < T_2$ system must put the economy on the long-run stable manifold, $\overrightarrow{EC}$, at $t = T_2$. In that case, we can find where the economy must be at T_1 immediately after the information is revealed by shifting $\overrightarrow{EC}$ backwards in time, using the equations of motion during $t \in (T_1,T_2)$, yielding $\overrightarrow{C'D'}$. Otherwise, if T_3 is the tax increase time, the economy must be on $\overrightarrow{FD}$ at $t = T_3$ and on $\overrightarrow{F'D'}$ at T_1 after the information revelation. To find where the economy must be at T_1^-, for each k, we find the expectation of consumption at T_1^+, yielding another manifold, $\overrightarrow{GH}$. Now, to find the impact at $t = 0$ of this program, we find a $c(0)$ such that if the economy is at $(k^{ss}, c(0))$ at $t = 0^+$, it would be on $\overrightarrow{GH}$ at T_1^-. In our previous notation, $i_1 \in \{T_2, T_3\}$, $M_\emptyset^- = \overline{GH}$, $M_{T_2}^+ = \overrightarrow{E'C'}$, and $M_{T_3}^+ = \overrightarrow{F'D'}$.

In Figure 1, the equilibrium path is first a fall to J, move to K, where $\tilde{T}$ is revealed. If $\tilde{T} = T_1$, we jump up to N, then proceed to P at T_1 and converge to C along $\overrightarrow{EC}$ for $t > T_1$. Otherwise, we jump down to L at T_0, proceed to M at T_2 and move along $\overrightarrow{FD}$ for $t > T_2$. I have chosen to draw $\overline{C'E'}$ and $\overline{D'F'}$ so that output rises after T_1 if the tax increase is found to be in the distant future and the output falls after T_2 if the tax increase is found to occur soon. We shall see that this is quite plausible.

The basic assumption made in Theorem 1 is not restrictive. For example, if there are no taxes after T_n, then the hypothesis of Theorem 1 holds since the equilibrium thereafter reduces to an optimal growth path by the Pareto efficiency of equilibrium in the absence of taxes. By continuity of the equilibrium system's equations, the hypothesized stable manifold exists when taxes are small. If labor supply is inelastic, then it has been shown elsewhere (e.g., Judd (1985)) that the dynamic system (5) has a stable manifold for any combination of taxes and spending, and, again by continuity, the necessary stable manifolds exist when labor supply elasticities are small. Furthermore, if utility is separable, then a taxed equilibrium does solve some optimal growth problem (albeit the wrong one from society's point of view) for families of utility functions commonly used in empirical analyses. This equivalence to some optimal growth problem implies the existence of a stable manifold. Therefore, we see that the hypothesis of Theorem 1 is weak considering the class of models which we are examining and common beliefs about preferences. Finally, for local analysis around a steady state, we only need local existence of a stable manifold, an issue settled by examination of the steady state's eigenvalues.

While the analysis is presented as a general equilibrium analysis with the true distribution of future tax parameters known, our short-run analysis is really just an examination of how different beliefs concerning those policies affect the current allocation of resources. This weaker interpretation is appropriate since if agents have common beliefs and these beliefs together with the structure of the economy are common knowledge, then the Nash equilibrium beliefs about future prices and the Nash equilibrium allocations at $t = 0$ are given by solutions to our equilibrium analysis.

Therefore, we are not just examining the impact of accurate information about future policies on current resource allocation, but we are also examining how changes in beliefs, rational or irrational, informed or uninformed, affect the current levels of output and factor supply. This more general interpretation substantially increases the usefulness of this analysis.

The equilibrium analysis of this section serves two purposes. First, it serves the usual purpose of putting our analysis on a sound footing. Moreover, since the proof is constructive it points to an algorithm for computing equilibria. Standard two-point boundary value problem methods can be used to solve for the terminal M_{I_n} manifolds. The intermediate manifolds are computed by running the system backwards from the latter ones and using condition (5c) at times of possible jumps in λ, just as we did in Theorem 1. We do not pursue this feature of our analysis in this essay, moving next to an analytical examination of this model for small changes around a steady state.

3. Marginal Analysis

In order to analyze changes in taxes, we need to know how C, L, and w are affected by changes in λ and K. We need compute only the first order properties of C and L. Using the notation $\hat{x} = dx/x$, we may express the relative changes of C, w, r, and L in terms of relative changes of K, $w/(1 - \tau_L)$ and $\lambda/(1 - \theta)$:

(7a) $$\hat{C} = \alpha(\hat{w} - \frac{d\tau_L}{1 - \tau_L}) + \beta(\hat{\lambda} + \frac{d\theta}{1 - \theta})$$

(7b) $$\hat{L} = \eta(\hat{w} - \frac{d\tau_L}{1 - \tau_L}) + \nu(\hat{\lambda} + \frac{d\theta}{1 - \theta})$$

(7c) $$\hat{K} - \hat{L} = \sigma(\hat{w} - \hat{r})$$

(7d) $$\hat{w} = \frac{\theta_K}{\sigma}(\hat{K} - \hat{L})$$

where β is defined to be the intertemporal elasticity of consumption demand, σ the elasticity of substitution between capital and labor, η the compensated elasticity of labor supply, ν the intertemporal elasticity of labor supply, α the elasticity of consumption with respect to the contemporaneous wage, and θ_K and θ_L are the capital and labor shares of income, respectively. θ_C will be the fraction of net output going to consumption in the steady state. (7c) is the definition of σ and (7d) follows from the demand curve for labor.

Solving for $\hat{L}$, $\hat{w}$, $\hat{C}$, and $\hat{r}$, in (7), we have

(8a) $$\hat{L} = [\nu(\hat{\lambda} + \frac{d\theta}{1 - \theta}) - \eta(\frac{d\tau_L}{1 - \tau_L} - \frac{\theta_K}{\sigma}\hat{K})](1 + \eta\theta_K/\sigma)^{-1}.$$

(8b) $$\hat{w} = -\frac{\theta_K}{\sigma}[\nu(\hat{\lambda} + \frac{d\theta}{1 - \theta}) - \eta\frac{d\tau_L}{1 - \tau_L} - \hat{K}](1 + \eta\theta_K/\sigma)^{-1}$$

(8c) $$\hat{r} = -\frac{\theta_L}{\theta_K}\hat{w}$$

(8d) $$\hat{C} = \alpha(\hat{w} - \frac{d\tau_L}{1-\tau_L}) + \beta(\hat{\lambda} + \frac{d\theta}{1-\theta})$$

To simplify notation we will define equilibrium values of the substitution and income effects and express $\hat{L}$ in the terms of them:

$$\nu^e \equiv \frac{\nu}{1 + \eta\theta_K/\sigma}$$

$$\eta^e \equiv \frac{\eta}{1 + \eta\theta_K/\sigma}$$

$$\hat{L} = \nu^e(\hat{\lambda} + \frac{d\theta}{1-\theta}) + \eta^e(\frac{\theta_K}{\sigma}\hat{K} - \frac{d\tau_L}{1-\tau_L})$$

that is, η^e and ν^e represent the net response of labor supply to changes in the price of consumption and the wage tax rate after one takes into account the change in the wage induced by the change in labor supply.

Suppose that the economy is in the steady state corresponding to constant policy parameters τ_K, τ_L, g, and θ, when, at t = 0, it is announced that for $t \geqslant 0$,

$$\tau_K^*(t) = \tau_K + \varepsilon\tilde{h}_K(t)$$
$$\tau_L^*(t) = \tau_L + \varepsilon\tilde{h}_L(t)$$
$$\theta^*(t) = \theta + \varepsilon\tilde{z}(t)$$
$$g^*(t) = g + \varepsilon\tilde{g}(t)$$

where $\tilde{h}_K$, $\tilde{h}_L$, $\tilde{z}$, and $\tilde{g}$ are stochastic processes measurable with respect to the information process $i_k(T_k)$. For any ε, there is a solution to our equilibrium system, (5), for these new policies, denoted $\lambda(t,\varepsilon)$ and $K(t,\varepsilon)$. Since the system is smooth in ε, so are the $M^+_{I_k}$, implying that for small ε there are unique solutions $\lambda(t,\varepsilon)$, $K(t,\varepsilon)$, which are smooth in ε. We will want to know how these small policy changes will affect the economy. This is best done by examining the impact on the paths of λ and K. Define the perturbations of λ and K caused by the policy change as follows:

$$\lambda_\varepsilon \equiv \frac{\partial}{\partial \varepsilon} \lambda(t,0) \qquad K_\varepsilon \equiv \frac{\partial}{\partial \varepsilon} K(t,0)$$

$$\dot{\lambda}_\varepsilon \equiv \frac{\partial}{\partial \varepsilon}\frac{\partial}{\partial t} \lambda(t,0) \qquad \dot{K}_\varepsilon \equiv \frac{\partial}{\partial \varepsilon}\frac{\partial}{\partial t} K(t,0)$$

where $\lambda(t,\varepsilon)$ and $K(t,\varepsilon)$ are the solutions to (5).

To determine the response of the economy to the uncertainty, we differentiate the system (5) with respect to ε, evaluating the result at $\varepsilon = 0$. The result of this linearization is

$$(9) \qquad \begin{pmatrix} \frac{\dot{\lambda}_\varepsilon}{\lambda} \\ \\ \frac{\dot{K}_\varepsilon}{F} \end{pmatrix} = J \begin{pmatrix} \frac{\lambda_\varepsilon}{\lambda} \\ \\ \frac{K_\varepsilon}{F} \end{pmatrix} + \vec{v}(t)$$

for $t \in (T_k, T_{k+1})$, $k = 0,\ldots,n-1$, where

$$J \equiv \begin{pmatrix} -\frac{1-\tau_K}{1-\theta}\frac{\theta_L}{\sigma}\nu^e f' & \frac{1-\tau_K}{1-\theta}\frac{\theta_L}{\sigma\theta_K}\left(1-\frac{\theta_K}{\sigma}\eta^e\right)f'f' \\ \\ -\beta\theta_c + \theta_L\nu^e + \alpha\nu^e\frac{\theta_K}{\sigma}\theta_c & f'\left(1+\frac{\theta_L}{\sigma}\eta^e - \alpha\frac{\theta_c}{\sigma}(1+\eta\theta_K/\sigma)^{-1}\right) \end{pmatrix}$$

and

$$\begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} f' \frac{1-\tau_K}{1-\theta}\frac{\theta_L}{\sigma}\left(\eta^{e}\frac{\tilde{h}_L(t)}{1-\tau_L} - \nu^{e}\frac{\tilde{z}(t)}{1-\theta}\right) - (\rho+\delta)\ \frac{\tilde{z}(t)}{1-\theta} + f'\frac{\tilde{h}_K(t)}{1-\theta} \\ \theta_L\left(\nu\frac{e^{\tilde{z}(t)}}{1-\theta} - \eta\frac{e^{\tilde{h}_L(t)}}{1-\tau_L}\right) + \alpha\theta_c\left\{\frac{\tilde{h}_L(t)}{1-\tau_L} + \frac{\theta_K}{\sigma}\left(\nu\frac{e^{z(0)}}{1-\theta} - \eta\frac{e^{\tilde{h}_L(t)}}{1-\tau_L}\right)\right\} - \beta\theta_c\frac{\tilde{z}(t)}{1-\theta} - \frac{\tilde{g}(t)}{F} \end{pmatrix}$$

with all parameters evaluated at the initial steady state. The signs of J_{11} and J_{21} will be important in our analysis. If leisure is normal, then $\nu > 0$ and $J_{11} < 0$. Also, J_{21} is most plausibly positive assuming normality of leisure.

At times T_k information is revealed and the shadow value of capital may jump. Define

$$j_{I_k}(T_k) \equiv \frac{\partial}{\partial \varepsilon}(\lambda_{I_k}(T_k^+,0) - \lambda_{I_{k-1}}(T_k^-,0)), \quad k = 0,1,\ldots,n$$

and set j(t) equal to zero for $t \notin \{T_0,\ldots,T_k\}$.

Since $\lambda_{I_{k-1}}(T_k^-) = E\{\lambda(T_k^+)|I_{k-1}\}$, we may conclude that

$$0 = E\{j(T_k)|I_\ell\}, \quad \ell < k \tag{10}$$

The set of linear equations (9) can be combined with the definition of j to yield a representation of the dynamic system valid for all nonnegative t:

$$\begin{pmatrix} \frac{\dot{\lambda}_\varepsilon}{\lambda} \\ \frac{\dot{K}_\varepsilon}{F} \end{pmatrix} = J \begin{pmatrix} \frac{\lambda_\varepsilon}{\lambda} \\ \frac{K_\varepsilon}{F} \end{pmatrix} + \vec{v} + \begin{pmatrix} \sum_{k=0}^{n} j(t)\delta_{(T_k)} \\ 0 \end{pmatrix} \tag{9'}$$

where $\delta_{(t)}$ is the Dirac delta function at t or, equivalently, the density of the measure with point mass at t. Solving (9')[1] together with the asymptotic stability conditions, implies that for any terminal information set, I_n,

$$\frac{\lambda_\varepsilon(0)}{\lambda} = \frac{J_{11} - \mu}{J_{21}} \left(E\{V_2(\mu) | I_n\} + K_\varepsilon(0) \right) - E\{V_1(\mu)\} - \sum_{k=1}^{n} j_{I_n}(T_k)\, e^{-\mu T_k}$$

where μ is the positive eigenvalue of J and $V_i(\mu)$ is the present value of $v_i(t)$ discounted at μ. While $\lambda_\varepsilon(0)$ appears to depend on I_n, it cannot since it can depend only on the information available at t = 0. Therefore, if we weight each of these equivalent representations of $\lambda_\varepsilon(0)$ by the probability of each terminal information set, the $j_{I_n}(T_k)$ terms will be eliminated by (10), yielding

$$(11) \qquad \frac{\lambda_\varepsilon(0)}{\lambda} = \frac{J_{11} - \mu}{J_{21}} \left(E\{V_2(\mu)\} + \frac{K_\varepsilon(0)}{F} \right) - E\{V_1(\mu)\}$$

One also finds that $I_\varepsilon(0)$, the change in investment at t = 0, equals

$$(12) \qquad I_\varepsilon(0) = (\beta C - f\nu^e(\theta_L + \alpha\theta_c \frac{\theta_K}{\sigma}))E\{f' H_K(\mu) - (\rho + \delta)Z(\mu) + \mu Z(\mu) - z(0)\}(1-\theta)^{-1}$$

$$+ E\{G(\mu)\}(\mu + \frac{(1 - \tau_K)f'}{(1 - \theta)} \frac{\theta_L}{\sigma} \nu^e) - g(0) - \zeta K_\varepsilon(0)$$

$$+ \beta C \frac{(1 - \tau_K)f'}{(1 - \theta)} \frac{\theta_L}{\sigma} \eta^e \frac{E\{H_L(\mu)\}}{1 - \tau_L} + \left(\eta^e \theta_L f - \alpha C(1 - \eta^e \frac{\theta_K}{\sigma})\right)\left(\frac{\mu E\{H_L(\mu)\} - h_L(0)}{1 - \tau_L}\right)$$

where ζ is the negative eigenvalue of J and $H_K(\mu)$, $H_L(\mu)$, $Z(\mu)$, and $G(\mu)$ are the (random) present values of $\tilde{h}_K(t)$, $\tilde{h}_L(t)$, $\tilde{z}(t)$, and $\tilde{g}(t)$, respectively, all discounted at μ. At t = 0, the impact on labor supply is given by[2]

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[1](9') is a system of $m_1 \times m_2 \times \ldots \times m_n$ linear equations which, together with (10) and the measurability of λ and K with respect to information, has a unique stable solution. The Laplace transform technique is the preferred solution method given our interest in present values and initial effects.

$$(13) \qquad \hat{\ell}(0) = \nu^e\left(\frac{\lambda_\varepsilon(0)}{\lambda} + \frac{z(0)}{1-\theta}\right) - \eta^e \frac{h_L(0)}{1-\tau_L}$$

For $t = T_k$, $k = 1,\ldots,n$, the jumps $j(T_k)$ are solved inductively. Suppose i_1^* is realized at T_1 and in that case the jump is $j_{i_1^*}$. Then taking the expectation of $\lambda_\varepsilon(0)$ over all I_n which contains i_1^*, we find

$$(14) \qquad \frac{\lambda_\varepsilon(0)}{\lambda} = \frac{J_{11}-\mu}{J_{21}}\left(E\{V_2(\mu)|i_1^*\} + \frac{K_\varepsilon(0)}{F}\right) - E\{V_1(\mu)|i_1^*\} - j_{i_1^*}e^{-\mu T_1}$$

By iterating this procedure for all possible I_2, I_3, etc., we can solve for all $j_{I_k}(T_k)$, yielding

$$(15) \qquad j_{I_k}(T_k) = e^{\mu T_k}(E\{\lambda_\varepsilon(0)|I_k\} - E\{\lambda_\varepsilon(0)|I_{k-1}\})$$

This expression has a natural interpretation. It says that the jump in λ is equal to difference in the expected jump at $t = 0$ due to the incremental information multiplied by $e^{\mu T_k}$. (The $e^{\mu T_k}$ term does not imply any increasing sensitivity because the conditional expectations differ to only order $e^{-\mu T_k}$, being differences in policies which are equal for $t < T_k$ and discounted at μ.) Similarly, $I_\varepsilon(T_k^+) - I_\varepsilon(T_k^-)$, the jump in investment in response to new information at T_k, is given by

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[2]Equations (11, 12, 13) are all similar to the formulae computed in Buiter (1983). However, as Buiter points out in footnote 6, his analysis was limited to deterministic problems. In particular, he assumed that agents' expectations are "held with complete subjective certainty" even though these beliefs are allowed to change over time in his model. In this model, agents are aware of the uncertainty and respond rationally to news. We also make no assumptions concerning the stochastic processes governing information revelation, whereas Buiter claims validity for his formulae only for stationary processes with independent increments and zero mean.

$$(16) \qquad I_\varepsilon(T_k^+) - I_\varepsilon(T_k^-) = e^{\mu T_k}\left(E\{I_\varepsilon(0)|I_k\} - E\{I_\varepsilon(0)|I_{k-1}\}\right)$$

We have succeeded in determining the first-order effects of uncertainty in fiscal policy variables and expressing them in an intuitive form. Of particular interest is the fact that the resolution of uncertainty is not a factor in this first-order analysis. Only the actual policy changes matter.

4. Uncertain Timing in Tax Changes

Given the machinery developed in the previous section, we are now able to determine the marginal effect of timing uncertainty on current factor supply. In general, we find that timing uncertainty about a future parameter change accentuates the anticipation effect of the deterministic change occurring at the expected time.

Theorem 2: Suppose that at $t = 0$ one of two policy changes is believed: either that $\tau_K(\tau_L,\theta,g)$ will be increased by ε at $\tilde{T}_1$ or that $\tau_K(\tau_L,\theta,g)$ will be increased at $\tilde{T}_2$. Assume $\tilde{T}_2$ is riskier than $\tilde{T}_2$. If $\lambda_\varepsilon^i(0)$ is the impact on $\lambda(0)$ of policy i, $i = 1,2$, then

$$\lambda_\varepsilon^2(0)/\lambda_\varepsilon^1(0) > 1.$$

Furthermore, this magnification effect also applies to the investment and labor supply responses at $t = 0$.

Proof: Such changes in τ_K are represented by

$$\tilde{h}_K^i(t) = \begin{cases} 0, & t < \tilde{T}_i \\ 1, & t > \tilde{T}_i \end{cases}$$

where $i = 1,2$. Then $H_K^i(\mu) = e^{-\mu\tilde{T}_i}/\mu$ is the present value of $h_K^i(t)$, $i = 1,2$

discounted at μ. Since $e^{-\mu T}/\mu$ is convex in T and positive,

$$E\{e^{-\mu\tilde{T}_2}\} > E\{e^{-\mu\tilde{T}_1}\}.$$

Since $V_2(\mu)$ and $V_1(\mu)$ are proportional to $E\{H_K(\mu)\}$ when we change τ_K alone, the magnification effect on λ of uncertainty follows. Similarly, we have the result for such changes in τ_L, θ, and g. By (12) and (13), the result is immediately extended to the impact on investment and labor supply since z(0), g(0), and $h_L(0)$ are zero for future policy changes. Q.E.D.

We next examine the direction of the effects on factor supply for the various policy parameters.

Theorem 3. Factor supplies react to policy changes according to:

(i) For both an increase in the future level of the capital income tax and a mean-preserving increase in the timing of a capital income tax increase, investment and labor supply are reduced if and only if

$$J_{21} = -\beta\theta_c + \nu^e\theta_L - \alpha\,\theta_K\sigma^{-1}\nu^e\theta_c > 0. \tag{17}$$

(ii) A future increase in θ or an increase in timing uncertainty of a θ increase will reduce investment if

$$J_{21} = -\beta\theta_c - \nu^e\theta_L - \alpha\theta_K\sigma^{-1}\nu^e\theta_c > 0 \text{ and } \mu - \rho - \delta > 0. \tag{18}$$

(ii) A future increase in g or an increase in the timing uncertainty of a future g increase will increase investment if

$$\mu + \frac{(1 - \tau_K)f'}{1 - \theta}\,\theta_L\nu^e\sigma^{-1} > 0. \tag{19}$$

and will increase labor supply if $(J_{11} - \mu)/J_{21} < 0$.

(iv) A future increase in the wage tax or an increase in the timing uncertainty of a future increase of τ_L increases investment if and only if

$$\eta^e \beta\theta_c(\rho - \delta\theta/(1 - \theta))\theta_L/\sigma + \eta^e\theta_L\mu - \alpha\theta_c(1 - \eta^e\theta_K/\sigma)\mu > 0 \tag{20}$$

Proof. The proofs of (17)-(20) follow from (12) and Theorem 2.

The economic relevance of the conditions of Theorem 3 is not immediately transparent from their statements. There are two ways to examine the economic plausibility of conditions (17) through (20). First, one could find empirical estimates of the basic structural parameters and substitute them into these conditions. Since this has been done somewhat in another context, we will here summarize the result of that effort. Judd (1984) calculates the eigenvalues and other critical parameters of several alternative parameterizations of our model. Labor supply parameters were taken from studies which represent the range of current estimates. A convenient and popular way to discuss these estimates is to give the compensated wage elasticity and the income elasticity of labor supply implied by the utility function u in a static model. Macurdy (1981) suggests compensated wage elasticities near .08 and income elasticities near .06, whereas Macurdy (1983) suggests a compensated wage elasticity of .7 and an income elasticity of -.77. Hausman (1981) argues for a wage elasticity of around .2 and an income elasticity of near -.6. Other estimates, such as those by Abbot-Ashenfelter (1976, 1979) fall into this range, as well as the utility function used in Auerbach-Kotlikoff-Skinner (1983). σ was allowed to vary from .4 to 1.0. β was taken to be between -2.0 and 0.5, a range suggested by the work of Weber (1970, 1975), Hansen-Singleton (1982), and Hall (1981). Tax parameters were allowed to vary over the range suggested by the Fullerton-King (1984) and Feldstein, et al. (1983). Since our qualitative results are insensitive to the choices made from this collection of parameterizations, we will not discuss them further. We will refer to these parameterizations to only plausibly sign crucial terms. In particular, $\mu > \rho$ and $J_{21} > 0 > J_{11}$ for all these parameterizations.

The second way to examine the plausibility of conditions (17) through (20) is to find conditions on the structure of preferences which help to sign the expressions. One such restriction which will help is separability in the utility function between consumption and labor. Also, the requirement that utility be concave will be helpful.

(17) in Theorem 3 holds in all the parameterizations described above and always if $u(c,\ell)$ is additively separable in c and ℓ. ν, the intertemporal labor supply elasticity, is positive in all the estimates cited above, validating the common belief that leisure is normal. Hence, the only way that mean-preserving increases in timing uncertainty could possibly not reduce investment and labor supply is if α is substantially negative and σ is small. Concavity of u in ℓ assures $\beta - \alpha < 0$, so our condition holds as long as $\theta_K \sigma^{-1}$ is less than unity, a condition which holds for the most favored estimates of θ_K and σ. A positive J_{21} says that at the steady state values of capital and its private shadow price, investment increases if the private shadow price increases. From these comments, we conclude that it is most likely that future random increases in τ_K reduces factor supply at $t = 0$.

We saw above that the first part of (18) is likely to hold. The second part also happens to hold for <u>all</u> of our parameterizations. Therefore, we see that future uncertain θ increases will likely reduce investment.

Since $\mu > 0$ always and $\nu > 0$ if leisure is normal, we find a strong bias in the effect of future expenditures on initial investment. Normality in leisure also implies that $J_{11} < 0 < \mu$, so $(J_{11} - \mu)/J_{21}$ is also most likely to be negative. Therefore, future g increases will most likely raise current investment and uncertainty about when they occur will further increase investment. This is true in all of our examples.

First note that (20) holds if α is zero, that is, if utility is separable in consumption and leisure. In most of the parameterizations mentioned, (20) holds, indicating a bias for τ_L timing uncertainty to increase investment. The only exception was when the static analysis of Hausman was combined with β around $-.6$ yielding a small compensated labor supply elasticity but a large positive α. This exception is a strained example since Hausman's estimates were based on a static analysis where the value of β is irrevelevant.

Our analysis of how factor supply and output reacts at $t = 0$ to uncertainty also applies to the problem of determining how they react at later times to "news." (15) and (16) show how the private value of capital and investment reacts to news at T_k, $k = 1,\ldots,n$. For example, if the news at T_k increases uncertainty about the time of a future capital income tax increase or spending cut, then investment will likely drop. We also can use (15) and (16) together with Theorem 3 to determine how the economy reacts to changes in expectations of what future policies will be. For example, if τ_K was expected to rise in the future but new information indicates that τ_K will be reduced, the impact will be an increase in investment.

Overall, we find that we can solve for the crucial conditions governing the first-order impact of uncertainty over future policy and plausibly sign the effects when this model is parameterized with current estimates of tastes and technology. In the next section we examine implications for balanced-budget policies.

5. Balanced-Budget Policies

In many situations, the uncertainty takes into account the impact of delay on revenue. For example, suppose there is a tax reduction at t=0 giving rise to a deficit, which may be expected to be financed by future taxes. In this case, longer delays lead to greater tax increases. Therefore, there will be an interaction between the resolution of uncertainty concerning the timing of budget-balancing policies and the final budget-balancing policy. In order to determine the effect of timing uncertainty under the belief that the budget will be balanced in a particular fashion along all paths, we must first develop a balanced-budget condition for our model.

Since the representative agent must be willing to hold outstanding bonds, the government must choose a policy which obeys the budget constraint

$$0 = \int_0^\infty (g + Tr - \tau_K KF_K - \tau_L \ell F_\ell + \theta(\delta K + \dot{K}))e^{-\int_0^s \phi(z)dz} ds$$

for <u>all</u> paths, where

$$\phi(t) = \rho - \dot{p}(t)/p(t), \quad p(t) \equiv u_1(c(t),\ell(t))$$

is the consumption rate of return. This follows from Brock and Turnovsky. When we perturb our economy by ε, differentiation of the budget constraint with respect to ε shows that the marginal policies must obey

$$\begin{aligned}(21) \qquad 0 = {} & Kf'(k)H_K(\rho) + \ell(f(k) - kf'(k))H_L(\rho) - \tau_L\Gamma_\varepsilon(\rho)(f(k) - kf'(k)) \\ & - \delta KZ(\rho) - G(\rho) + \tau_K f'(k)X_\varepsilon(\rho) - \theta(\rho+\delta)X_\varepsilon(\rho) \\ & + Kf''(k)(\tau_K - \tau_L)\left(\frac{X_\varepsilon(\rho)}{K} - \frac{\Gamma_\varepsilon(\rho)}{L}\right)\frac{K}{L} - T(\rho)\end{aligned}$$

where $H_K(\rho)$, $H_L(\rho)$, $Z(\rho)$, $G(\rho)$, $X_\varepsilon(\rho)$, $\Gamma_\varepsilon(\rho)$, and $T(\rho)$ are the random present values of $\tilde{h}_K(t)$, $\tilde{h}_L(t)$, $\tilde{z}(t)$, $\tilde{g}(t)$, $K_\varepsilon(t)$, $\ell_\varepsilon(t)$, and $Tr_\varepsilon(t)$, respectively, all discounted at the rate ρ, with each being also a function of I_n, the realization of all information. It is most convenient to regard transfers as the residual policy variable. Since the timing of transfers do not matter except through their present value, one could generally assume incremental transfers are set at some time after T_n to balance the budget. In examining (21), one should note that there are no risk premia in this expression. This is expected since we are examining the first-order effects of uncertainty when we take the first derivative with respect to ε whereas risk premia are related to the variances of risks, a second-order property. Taking a second derivative with respect to ε would generate such risk premia, but that is beyond the scope of this paper.

A general analysis imposing budget balance on all paths without resort to transfers is obviously extremely complex. However, if there are no distortionary taxes initially, we can determine the effect of uncertainty on balanced-budget changes in tax parameters. If all taxes and credits are zero initially, (20) implies that if τ_K is reduced by ε at $t = 0$, then to balance the budget at T, it must be raised by $\varepsilon e^{\rho T}$. In this case, (11) becomes

$$\frac{\lambda_\varepsilon(0)}{\lambda} = -\frac{f'}{1-\theta}\, E_0\{e^{(\rho-\mu)\tilde{T}}\} \tag{22}$$

Equation (22) shows that uncertainty in T will reduce λ at t=0, and also reduce investment since $\rho \neq \mu$ is most likely. We can similarly analyze balanced-budget intertemporal policy changes in any tax, yielding Theorem 7.

<u>Theorem 4</u>: If the economy is initially in a steady state with little taxation, then the results of Theorem 3 continues to hold if current tax cuts are balanced by the permanent tax increases at $\tilde{T}$ sufficient to balance the

budget.

Proof: If tax levels are all zero, this follows from (22) and the formulae yielding impacts on current investment and labor supply, just as Theorem 3 followed. Since the budget constraint is continuous in the tax parameters, these results continue to hold for small tax levels.

Another example of intertemporal budget balance is when a tax is cut permanently at $t = 0$ with the loss in revenue to be balanced by future cuts in government consumption. Judd (1985) shows that if the tax cut included a reduction in τ_K and if labor were supplied inelastically, current investment is likely reduced. We can again determine the impact of uncertainty in the timing of the spending cut if all tax parameters are initially zero. From (21), if $h_K = -1$ and g is to be reduced permanently at $\tilde{T}$ by $\gamma\varepsilon$ to balance the budget, then γ must be $e^{\rho\tilde{T}}$. Hence $H_K(\mu) = -\mu^{-1}$ and $G(\mu)$ must be $-e^{(\rho-\mu)\tilde{T}}/\mu$, implying that

$$(23) \qquad \frac{I_\varepsilon(0)}{F} = \frac{J_{21}f'}{\mu} - \left(\mu + \frac{(1-\tau_K)f'\theta_L\nu^e}{(1-\theta)\sigma}\right) E\{e^{(\rho-\mu)\tilde{T}}\}$$

Since $e^{(\rho-\mu)T}$ is convex in T we have shown Theorem 5.

Theorem 5: If an immediate cut in τ_K (τ_L) is to be balanced by a future permanent cut in government consumption and current taxes are low, then an increase in the uncertainty concerning the timing of the spending cut reduces current investment if $\rho \neq \mu$ and if future increases in τ_K (τ_L) reduce current investment.

Proof: The τ_L case is proven just as the τ_K case above. Since the balanced-budget condition is continuous in τ_K and τ_L, the results also apply to small

τ_L and τ_K.

We again find a strong bias for timing uncertainty in future spending cuts to reduce current investment even when the level of the cut depends on the time of the cut. Also, since these results hold when there are no taxes initially, they seem to be driven by price effects alone.

Recall that our short-run formulas also modeled reactions to news or changes in expectations. This interpretation of our formulas point out the complexity of interactions between government policy and the business cycle. For example, suppose that from a balanced position the government cut taxes without cutting expenditures. This would cause a deficit in the short-run. The direction of the economy's reaction to that deficit would depend crucially on the current expectations of future policies used to balance the budget. If the initial expectation is an expenditure cut then our calculations indicate that the initial reaction is a recession, that is, a fall in factor supply and output. Suppose, however, that the expected expenditure cuts do not materialize, leading agents to believe that expenditures will not change. Furthermore, suppose that they come to expect that in the future a labor tax increase and/or a decrease in investment incentives will be chosen to balance the budget. The total effect of those altered expectations is that the recessionary impact of expected future expenditure cuts would be eliminated and replaced by the expansionary effect of the expected future labor tax increase and/or the expected future loss of investment incentives. This would be consistent, for example, with the belief that the government has some commitment to keep corporate income tax rates down, but may decide to increase business tax revenues in the future by cutting out investment incentives. Furthermore, if there is great uncertainty as to when these tax changes will be made, their expansionary effects are increased.

This example is just one case showing how even the direction of the economy's movement depends crucially on the evolution of agents' expectations. In particular, this shows that it would be important to include variables which reflect the movements of such expectations in any empirical analysis of such an episode. The value of this model is the ability to precisely model the effects of changing expectations on the economy.

6. The Case of Poisson Information Arrivals

In many situations, it may be more natural to think that information arrives in lumps at any time. Formally, this can be modelled by a Poisson process in continuous time. We can take a limit of our discrete time analysis and model a continuous-time Poisson process for information arrival. In this case, we also find an intuitive and informative representation of equilibrium even in the general case.

Suppose that n, the number of time periods at which information is received, is an infinite integer. Also assume that $T_k - T_{k-1}$ is an infinitesimal dt and that ndt is a large finite number. (See Keisler (1976) for the elementary details of infinitesimal calculus used here.) This specification approximates a continuous stream of information over a long horizon. Furthermore, suppose that $I_k = \{i_k^1, i_k^2\}$ with

$$\pi_{I_k}^{i_k^1} = 1 - h(T_k)dt.$$

where h(t) is some positive function of time. This structure models a situation where there are two possible messages at each time T_k, where message i_k^1 is infinitely more likely than i_k^2 for each k. The i_k^2 messages are therefore Poisson events which occur with a hazard rate $h(\cdot)$. For many problems this is a reasonable model of "news". One example we will study is the situation where the budget is out of balance, and agents expect that at some future time there will be an immediate and permanent increase in taxes sufficient to balance the budget.

Using equations (5) we can represent the Poisson information process in an enlightening fashion. To do this, let us follow λ from T_k^+ to T_{k+1}^+. From T_k^+ to T_{k+1}^-, (5a) is obeyed, implying that

$$(24) \qquad \frac{\lambda(T_{k+1}^-) - \lambda(T_k^+)}{dt} \approx \lambda(T_k^+)[\rho - \frac{f'(k)(1-\tau_k) - \delta\theta}{1-\theta}]$$

At T_{k+1}^+, λ "jumps" to, say, λ_j if message i_k^j, j = 1, 2, is received at T_{k+1}. By (5c), λ_1 and λ_2 are related by

$$(25) \qquad (\lambda_1 - \lambda(T_{k+1}^-))(1 - h(T_{k+1})dt) = -(\lambda_2 - \lambda(T_{k+1}^-))h(T_{k+1})dt$$

If the "unlikely" message is received, λ jumps to λ_2. Combining (24) and (25) we find that as long as the likely message is received, that is, the Poisson event does not occur, then λ is a smooth function of time and

$$(26) \quad \dot{\lambda}(T_k) = \frac{\lambda(T_{k+1}^+) - \lambda(T_k^+)}{dt} \approx \lambda(T_k^+)[\rho - \frac{f'(k)(1-\tau_K) - \delta\theta}{1-\theta} - \frac{\lambda_2 - \lambda(T_k)^+}{\lambda(T_k^+)} h(T_{k+1})]$$

This expression for the rate of change in the marginal private value of capital is an informative decomposition. We see that as long as the low probability event does not occur, λ moves according to (5a) except that ρ is increased by the product of $(\lambda_2 - \lambda(T_k^+))\lambda(T_k^+)^{-1}$, the relative jump in λ if the low probability event did occur, and $h(T_{k+1})$, the hazard rate of the low probability event.

The intuitive description of (26) derives from the observation that an instantaneous drop in λ represents a capital loss since λK is a measure of the total utility value of capital. For example, an unanticipated increase in τ_K represents an unanticipated fall in income from assets. This loss has value equal to $K\Delta\lambda$ where $\Delta\lambda$ is the drop in λ which occurs when τ_K is unexpectedly increased. $\Delta\lambda/\lambda$ is the relative loss of the private value to capital. Since a possible fall in value of capital equal to $\Delta\lambda/\lambda$ of all capital occurs with chance $h(T_{k+1})dt$, (26) says that the after-tax return on capital is discounted at ρ, the pure rate of time preference, plus $-h\Delta\lambda/\lambda$, a term which one is tempted to interpret as a "risk premium". The interpretation of the difference between the effective discount rate and the pure rate of time preference as a risk premium provides some intuition about consumption patterns. Recall that if utility is separable, then λ and c move in opposite directions. Therefore, if the fear is the implementation of a policy which causes capital loss of $K\Delta\lambda$ with hazard rate h, then λ rises at the rate $\rho - \bar{r} - h\Delta\lambda/\lambda$ and consumption rises at the rate $\beta(\rho - \bar{r} - h\Delta\lambda/\lambda)$, where $\bar{r}$ is the after-tax return on investment and β is the intertemporal rate of substitution in consumption. This shows in particular that during periods of a relatively high risk of a policy which causes a capital loss we expect consumption to rise and investment to fall until either the uncertainty is resolved or the period during which this risk exists passes.

This "risk premium" term represents a response to risk, but not just a premium above expected return. The change in the private shadow price of capital may be caused by a change in the mean return to capital, in particular if the shadow price of capital falls in response to an increase in the capital income tax rate. We shall continue to use the term risk premium in this context with the understanding that we mean the total compensation the agent requires to hold the risky asset.

We complete this section with a discussion of some more precise examples. To increase the specificity of the analysis, we assume that we are near a steady state and that the uncertain policy changes are small. However, it will be clear that the qualitative features will persist for large changes.

First, suppose that the economy has arrived at the steady state corresponding to a capital income tax of τ_K, and there is a new expectation of a future increase in that capital income tax. In particular, suppose that there is a constant hazard rate, γ, of a fixed τ_K increase of ε for t between 0 and some finite but distant time. Since $T_k - T_{k-1} = dt$ for all k, then if it is determined at T_k that there be no tax increase between T_{k+1} and T_k, the resulting jump in λ at T_k will equal

$$e^{\mu T_{k-1}} \lambda(T_k) \frac{f'}{1-\theta} \left(- E\{H_K(\mu)|I_k\} + E\{H_K(\mu)|I_{k-1}\}\right) \simeq \lambda(T_k) \frac{f'}{1-\theta} \frac{\gamma dt}{(\gamma+\mu)}$$

where I_{k-1} is the information set immediately before the tax increase and I_k is the information immediately after. Therefore, until the tax increase actually occurs, (10) is approximated uniformly by the deterministic system:

$$(27) \qquad \begin{pmatrix} \dfrac{\dot{\lambda}_\varepsilon}{\lambda} \\ \\ \dfrac{\dot{K}_\varepsilon}{F} \end{pmatrix} = J \begin{pmatrix} \dfrac{\lambda_\varepsilon}{\lambda} \\ \\ \dfrac{K_\varepsilon}{F} \end{pmatrix} + v + \begin{pmatrix} \dfrac{f'}{1-\theta}\dfrac{\gamma}{\gamma+\mu} \\ \\ 0 \end{pmatrix}$$

This system is the linearization of

$$(28a) \qquad \dot{\lambda} = \lambda\left(\rho - \frac{f'(k)\left(1 - \tau_K - \varepsilon\gamma/(\gamma+\mu)\right) - \delta\theta}{1-\theta}\right)$$

(28b) $$\dot{K} = Lf(k) - C(\lambda, K, \tau_L, \theta)$$

for small ε.

As above, one way to interpret the equilibrium is that the constant hazard rate effectively increases the discount rate before the actual tax increase. This "risk premium" is further seen to be proportional to the magnitude of the ultimate tax change, ε. The constant of proportionality is related to the timing uncertainty and the structure through the hazard rate, γ, and the positive eigenvalue, μ, being their product divided by their sum.

(28a) also gives a different interpretation of the risk term. In this form, it is apparent that the anticipation of the tax increase causes the agents to act as if a fraction of the tax increase were already enacted, that fraction being $\gamma/(\mu + \gamma)$. That anticipation of a tax increase thereby raises the effective tax rate and the cost of capital to firms, even before the tax is enacted.

This interpretation leads us to the description of the dynamics of this example, which are represented in Figure 2. Again, we assume that labor supply is inelastic as in Figure 1 and examine the phase diagram in c-K space. Initially the steady state is at A with a capital stock of k_1. While agents wait for the tax increase, (28a) says that the agents act as if there had been a tax increase of $\varepsilon\gamma/(\gamma + \mu)$. This causes the $\dot{c} = 0$ locus to move left, becoming a vertical line at k_2, the steady state capital stock if τ_K were increased by $\varepsilon\gamma/(\gamma + \mu)$. Note that the $\dot{K} = 0$ locus is unaffected by τ_K changes. Therefore, the steady state under this intermediate situation is at C with a stable manifold of $\overrightarrow{BC}$. When the tax is finally increased by ε, uncertainty no longer plays a role and the steady state under the final tax is at E with an even smaller capital stock k_3 and stable manifold $\overrightarrow{DE}$.

With this system of stationary loci and auxiliary computations, we are now able to characterize the dynamic behavior of the economy before and after the tax increase. First, we know from computing (15) for this case that the economy will jump up in response to the actual tax increase. Furthermore, from the stability of the economy, we know that the jump must be to the stable manifold corresponding to the final tax rate and that after the tax increase the economy converges to the steady state at E.

More detailed examination of (15) shows that the jump at the time of the tax increase is independent of the time of the increase since the hazard rate is constant. This information tells us that the evolution of the economy prior to the tax increase must not be explosive. The only way for that to be true is for the economy to be converging to the intermediate steady state at C. Therefore, at the moment the agents come to believe that this random tax increase will occur in the future, the economy jumps to B, a point on the stable manifold around the C, implying an increase in consumption and a decrease in investment. Such an economy therefore immediately begins capital decumulation and contraction of output, approaching an intermediate capital stock and output level. This continues until the actual tax increase takes place, at which time the consumption again jumps up, say to D, and the economy completes the eventual capital decumulation and contraction by moving down the terminal stable manifold $\overrightarrow{DE}$.

In the general case of large tax changes we do not have the simple quantitative result that individuals incorporate the expected tax increase immediately into their investment decisions. However, the qualitative behavior will be the same. Figure 3 represents the situation for the case of a large anticipated tax increase. After the tax increase, the system will move to the stable manifold corresponding to the new tax rate. Let $\lambda^S(K)$ be the value the value of λ corresponding to K along the post-tax-increase stable manifold. This shows that in (26) λ_2, the value to which λ jumps upon the imposition of the new tax, must be $\lambda^s(K)$ if the tax rate occurs when the capital stock is K. Therefore, during the wait for the tax increase, the economy is described by the following equations:

$$\text{(29a)} \qquad \dot{\lambda} = \lambda\left(\rho - \frac{f'(k)(1 - \tau_K) - \delta\theta}{1 - \theta} - \gamma\,\frac{\lambda^s(K) - \lambda}{\lambda}\right)$$

$$\text{(29b)} \qquad \dot{K} = Lf(k) - C(\lambda, K, \tau_L, \theta)$$

The intermediate system therefore has the same $\dot{K} = 0$ locus as the initial and terminal systems, but has a substantially different $\dot{\lambda} = 0$ locus, which is expressed by the equation

$$\text{(30)} \qquad \lambda = \lambda^s(K)\;\frac{\gamma}{\gamma + \rho - \left(f'(k)(1 - \tau_K) - \delta\theta\right)/(1 - \theta)}$$

Since the denominator of this expression is positive but less than γ for capital stocks less than the initial capital stock, we see that the intermediate $\dot{\lambda} = 0$ locus lies above $\lambda^s(K)$ but equals $\lambda^s(K)$ at the initial steady state capital stock, k_1. This implies that the intermediate $\dot{c} = 0$ locus in c - K space lies below the post-tax stable manifold in c - K space, $c^s(K)$, but coincides with $c^s(K)$ at the initial steady state capital stock, k_1. Figure 3 represents this relation since D and E lie on $c^s(K)$, and $\overrightarrow{FC}$ represents the $\dot{c} = 0$ locus in c - K space which corresponds to (30), the $\dot{\lambda} = 0$ locus in λ - k space. However, the intermediate dynamic system is still saddlepoint stable. Therefore, there are both a stable manifold and an unstable manifold around its steady state at C. The stability of the total system argues that when it is announced (or it comes to be believed) that there will be a tax increase at some future random time, it will jump to B on the intermediate stable manifold $\overrightarrow{BC}$ and converge towards the intermediate system's steady state at C while waiting for the tax change. At the time of the tax increase, the economy will jump from its position on the intermediate stable manifold to the position, say D, on the terminal stable manifold corresponding to the current capital stock. The position of the intermediate $\dot{c} = 0$ locus shows that the intermediate steady state capital stock, k_2, is between the original capital stock, k_1, and the eventual capital stock under the new high tax, k_3. Therefore, the economy first engages in some capital decumulation in response to the initial expectation of the future tax increase, but completes the capital decumulation only after the future tax is imposed.

This response to a fixed-size tax rate increase at some random future time is intuitive in that the anticipation of the tax increase causes the agents to partially incorporate the tax increase into their investment evaluations immediately but completely incorporate the tax increase only after the tax increase takes effect. This leads to monotonically decreasing paths for output and the capital stock. On the other hand, we find very different behavior if the future tax rate is determined by balanced-budget considerations. We next examine a simple example. Suppose that there is no taxation initially when τ_K is reduced at t = 0 by ε, thereby creating a subsidy to capital formation. The resulting deficit is financed immediately by bonds and eventually by an increase in τ_K at some future time. We again assume that there is uncertainty as to when the increase occurs, with hazard rate of the increase again equalling γ. In this case we will also assume that if the tax increase does not occur before T, some distant time, it will occur at T. By the balanced-budget condition, (21), the τ_K increase which would balance the intertemporal budget constraint must equal $\varepsilon\, e^{\rho t}$ if it occurs at t. If we started with a positive, this relation between the time of the tax increase would be much more complex and depend on the exact timing of information. Since this example is relatively simple, and continuity implies that it displays the same behavior if the initial tax rate were small, we stick with this case. (28a) then becomes

$$\text{(31)} \qquad \dot{\lambda} = \lambda\left(\rho - f'(k)(1 + \varepsilon) + \frac{f'\gamma}{\gamma + \mu}\, \varepsilon\, e^{\rho t}\right)$$

Integration of (31) and (28b) for the case of inelastic labor supply shows that the dynamic path is characterized by the phase diagram in Figure 4. (For the sake of brevity, we leave the details to the reader.) From (11) and (12) we find that

$$\frac{\lambda_{\varepsilon}(0)}{\lambda} = \frac{f'(\mu - \rho)}{\mu(\mu + \gamma - \rho)} + o(T^{-1})$$

implying that the shadow price of capital initially increases. (Since T is assumed large, we separate $o(T^{-1})$ terms from the dominant effects.) If labor is inelastically supplied, this implies that consumption falls, investment rises, and soon after output rises. Immediately, the $\dot{c} = 0$ locus moves from its original position above k_1 towards the right, becoming the vertical line above k_2. However, as time passes without a tax increase, the $\dot{c} = 0$ locus slides left since the economy at time t acts as if $\tau_K = \varepsilon(e^{\rho t}\gamma/(\gamma + \mu) - 1)$. When the tax increase occurs at $\tilde{T}$, the tax rises to $\varepsilon e^{\rho \tilde{T}}$, the new steady state is $k_{\tilde{T}}$, and the economy jumps to the stable manifold corresponding to the realized terminal tax rate. By (15), the λ jump at the time of the tax increase is downward, implying an increase in consumption. These considerations together imply that the typical path is as displayed in Figure 4: an initial fall in consumption from A to B, then a rise in capital stock, output, and consumption along BC, followed by a period of disinvestment and falling output along CD, followed by a phase of falling consumption along DE. This fall in capital, output, consumption continues until the tax increase is decided, causing consumption to jump from E, the economy's position at the time the tax increase is implemented, to F, and convergence of the economy to its final steady state at G. (An early resolution of the tax increase would eliminate some of the later stages.)

This second exercise shows that a tax cut financed by a tax increase at some future uncertain time can lead to an initial expansion, but that expectations of high taxes can dampen that expansion and cause a contraction even before the tax is increased. If the tax were a general income tax, then this description is still valid as long as the labor supply is not too elastic. The crucial aspect is that the expectation of high taxes will eventual depress capital formation and output even before the imposition of the tax, but these contractionary features will be offset in the short run by the price effect induced by the temporarily reduced taxation of investment income. This stimulus is only temporary since if the tax increase takes too long, its eventual large size is immediately internalized by investors and overwhelms the short-run effect. Therefore, the immediate stimulus is misleading since it does not reflect the long-run implications of the policy.

These examples are suggestive in showing the range of application of this analysis.

9. Conclusions

This paper has taken a general equilibrium model of growth and elastic labor supply with taxation and analyzed the impacts of uncertainty about the timing of future taxation on current factor supply. While the model is highly stylized, we do find several interesting and strong biases. Such timing uncertainty generally has a determinate impact on factor supply and accentuates the mean effect, indicating that predictions from analyses which ignore timing uncertainty will be biased towards zero. These results continue to hold if budget balance is imposed for all realizations.

Somewhat surprising is the fact that this timing uncertainty has such determinate effects since the conventional risk premia associated with the timing uncertainty do not enter into our first-order analysis. The next step in this analysis should be work to determine the impacts when we move away from first-order effects. The constructive nature of our existence proof points to a possibly useful algorithm to investigate the effects of more general problems. Furthermore, a second-order expansion of the equilibrium would analytically yield insights into the global impact of uncertain policy.

While much remains to be done in analyzing the impact of tax policy uncertainty, this analysis has given us the initial first-order analysis and also solved the general existence problem needed to move to a richer analysis.

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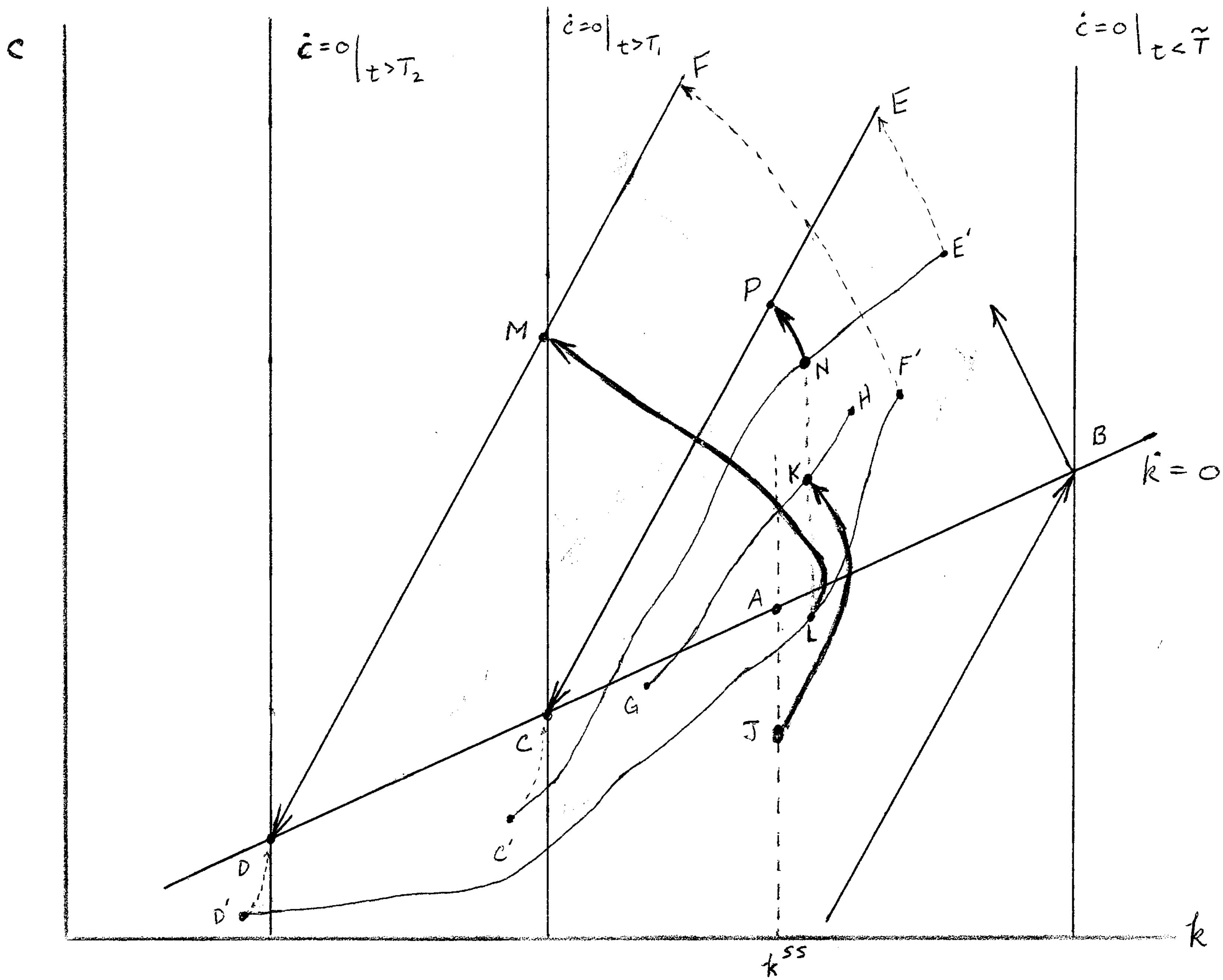


Figure 1: Cut τ_K at 0, raise at $\tilde{T}$

Fig. 2

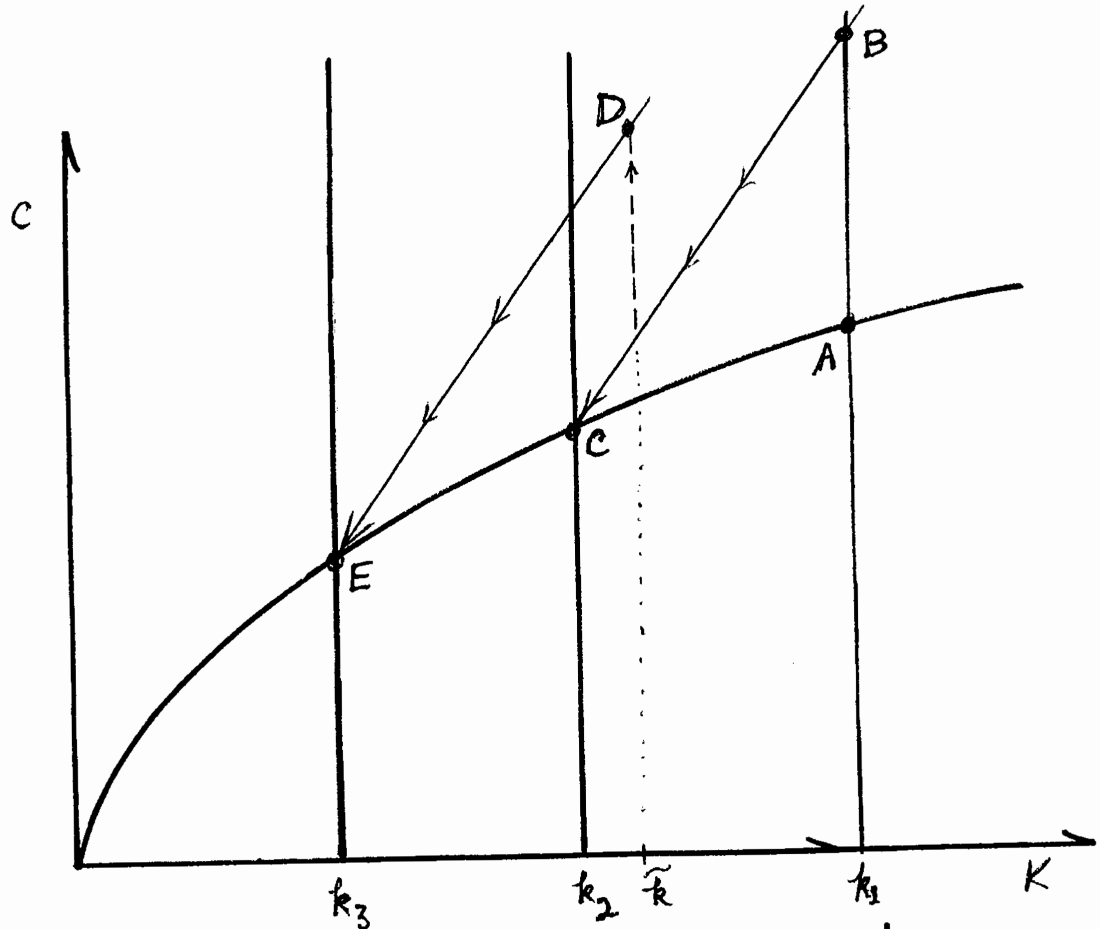
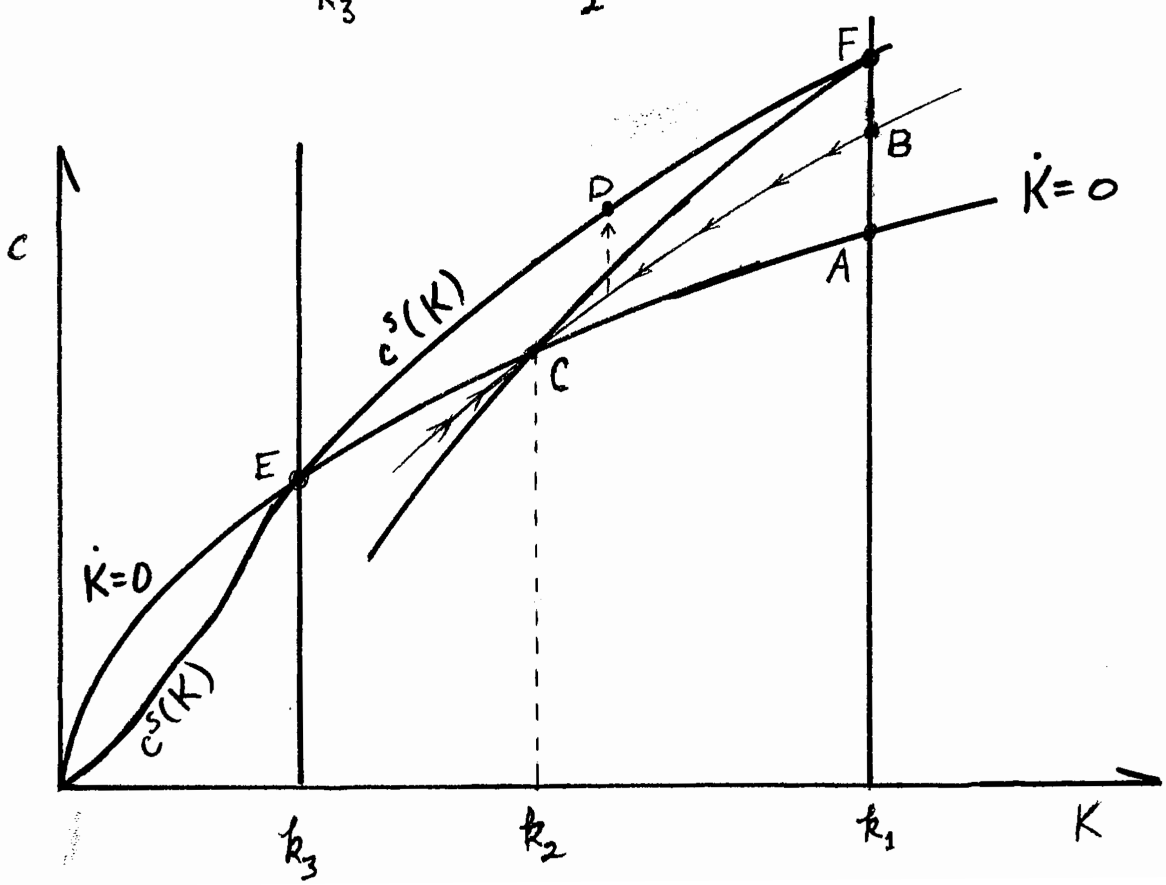


Fig. 3



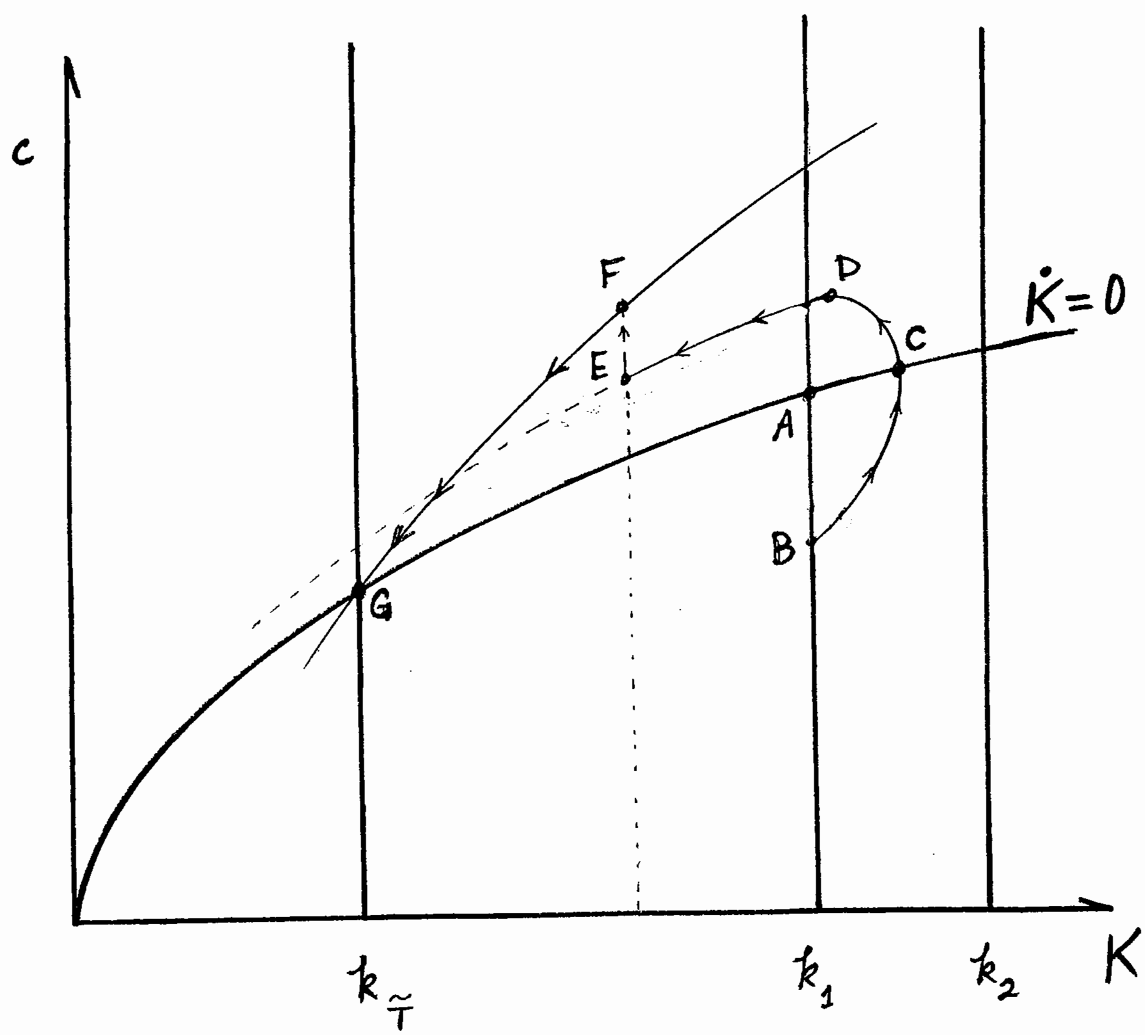


Fig. 4