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CAPITAL GAINS TAXATION BY REALIZATION IN DYNAMIC GENERAL EQUILIBRIUM*

by

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Abstract

One of the more controversial aspects of the tax structure is capital gains taxation. While its impact on trading strategies, volume of trade, and security prices has been studied, there has been little work examining the macroeconomic and allocative effects of capital gains taxation. In this paper we examine how capital gains taxation may differ from capital income taxation in its effects on investment and its efficiency as a source of revenue.

We jointly examine capital gains taxation and capital income taxation in a perfect foresight model of general equilibrium. We first characterize equilibrium and develop a cost of capital formula. Whereas most studies of the cost of capital use accrual "equivalents" as proxies for capital gains taxation by realization, we find that the cost of capital depends on capital gains tax parameters in fashions substantially different from its dependence on accrual taxes.

Then we analyze the effects of permanent and unanticipated changes in capital gains taxation. We find in this model that the short-run impact on capital stock due to a capital gains tax increase is possibly much greater than the long-run impact. This indicates that it may be severely misleading to extrapolate the short-run results of the empirical literature to the long run.

Next, we compare the excess burden of an increase in capital gains taxation to that of the corporate income tax, finding the capital gains tax to be less distortionary at the margin than capital income taxation if the basis value of equity is small relative to the price of the equity at the time of the tax increase or if tax rates are low. On the other hand, if the corporate income tax is high, then an increase in corporate taxation is preferable at the margin.

1. Introduction

One of the more controversial aspects of the tax structure is capital gains taxation. Its impact on trading strategies and security prices has been well studied, as in Kovenock and Rothschild, Constantinides, Constantinides and Scholes, and Stiglitz. There has been substantial empirical work (e.g., Feldstein, Slemrod, and Yitzhaki) showing that capital gains taxation affects the volume of transactions, as predicted by theory. However, there has been little work examining the effects of capital gains taxation by realization on investment incentives and on the general equilibrium movement of the capital stock. Since most productive assets are directly or indirectly subject to capital gains taxation, such a perspective is appropriate. In this paper we examine how capital gains taxation may differ from capital income taxation in its impact on the cost of capital, its impact on investment paths, and its efficiency as a source of revenue.

We jointly examine capital gains taxation and capital income taxation at the corporate level in a representative agent perfect foresight model of deterministic general equilibrium. In order to focus on the differences between an accrual tax on capital icome and a realization tax, we assume that personal taxation is such that firms want to distribute all earnings through capital gains. This is assumed possible through repurchase of stock or cross-purchasing, that is, firms purchasing each other's equity. In either case shareholders sell their stock which is rising in value at a rate sufficient for such sales to generate a continuous cash flow for shareholders. This formulation focusses on intertemporal decisions of when to liquidate assets to finance consumption and abstracts from transactions which are motivated by portfolio management considerations in the presence of uncertainty. Since the primary motive for savings is presumably a desire to shift current resources into consumption in the future by either one's heirs or self, this is an appropriate first attempt to analyze the intertemporal allocative effects of capital gains taxation.

Since this is a representative agent model, we are abstracting from the trading of assets among agents, concentrating entirely on asset sales which finance consumption. This focus is appropriate since we do not have well-developed theories of transactions *per se* and their volume. While it is true that much of the discussion of capital gains taxation has been focused on transactions-related issues such as the so-called "lock-in effect," any serious attempt to examine such issues in a dynamic model of endogenous investment will be sensitive to the assumptions concerning transactions costs and the structure of markets. This study reflects a belief that we should first deal with capital gains taxation from a purely price-theoretic view, abstracting from the transactions cost nature of capital gains taxation. A representative agent model is ideal for this purpose.

We assume that there is a tax on capital gains which is assessed at the time an asset is liquidated. In order to abstract from complex portfolio management problems (see Balcer and Judd (1985a) for a development of these issues), we also assume that all shares have the same basis value. Finally, we assume that all firms are mature in the sense that they are capable of satisfying capital needs out of earnings. This will unfortunately ignore the impact of capital gains taxation on the formation of new firms. This simplification will, however, allow us to address general equilibrium issues in a direct and intuitive fashion.

Section 2 develops the model, solving both the consumers' and firms' problems, leading to a cost of capital formula for the firm. Previous work, such as in King and Fullerton, has assumed that capital gains taxation is assessed in a fashion which makes it indistinguishable from a corporate income tax. In contrast with the usual presumption, we find that the instantaneous cost of capital responds in interesting and sometimes unexpected ways to changes in capital gains taxation. For example, a rise in the current capital gains tax rate will reduce the cost of capital if the present value of the basis is declining slowly and if the capital gains tax rate is falling. The key feature which makes capital gains taxation by realization different from accrual taxation is the role of the basis value, resulting in an inverse relation between the average tax rate (i.e., the tax expressed as a fraction of the asset's value) and the marginal tax rate, the latter being relevant for allocative decisions whereas the former represents revenues. These insights have relevance for policy issues that are repeatedly debated. For example, indexation of capital gains will have adverse allocative impacts since it keeps the basis too high for too long. In particular, if inflation is infinite, then indexation will turn a lump-sum tax into a distortionary capital gains tax. While it is well-known that a capital gains tax where assets have a zero basis is a lump-sum tax on wealth, this paper provides a general analysis with arbitrary paths for the asset's basis value.

Section 3 then characterizes general equilibrium for an economy where earnings are sufficient to finance a firm's desired investment spending. This equilibrium characterization is used in Section 4 where we analyze the effects of permanent and unanticipated changes in capital gains taxation. We find in this model that the short-run impact on capital stock due to a capital gains tax increase is possibly much greater than the long-run impact. This indicates that it may be severely misleading to extrapolate any short-run results of the empirical literature to the long run.

Section 5 then compares the excess burden of an increase in capital gains taxation to that of the corporate income tax. If tax rates are low, we find that the capital gains tax is less distortionary at the margin. This also holds if the basis value of equity is small relative to the price of the equity at the time of the tax increase, a condition which transforms a capital gains tax to a levy on current wealth. Since equity prices grow exponentially here, this also implies that the long-run distortion of capital gains taxation as formulated here is less than the short-run.

The model is clearly an initial study of capital gains taxation by realization which abstracts from many elements. However, the results do indicate that it is important to explicitly take into account the realization nature of capital gains taxation and discontinue the use of accrual "equivalents" in discussions of capital gains taxation. Also, the framework constructed is flexible, making possible many generalizations avoiding the unrealistic elements of the analysis below.

2. The Model

We examine a representative agent perfect foresight model of general equilibrium. Our agent will have a utility function

$$U = \int_0^\infty e^{-\rho t} u(c)\, dt$$

where ρ is the pure rate of time preference. $u(c)$ is a concave increasing function of the rate of consumption, c, and $\beta \equiv cu''(c)/u'(c)$ is the elasticity of marginal utility. For the sake of simplicity we assume that labor supply per unit of time is inelastic and normalized to be unity. Since we are comparing two forms of capital taxation this is an acceptable simplification to focus on the basic points.

Production will use both capital, k, and labor, ℓ, resulting in a gross output rate equal to $F(k,\ell)$, a constant returns to scale production function. Capital depreciates at a constant rate δ and $f(k)$ will represent net output per worker, $(F - \delta k)/\ell$, in terms of capital-labor ratio, also denoted by k since labor supply is inelastic and $\ell = 1$.

We follow Brock and Turnovsky in modelling firms' financial structure. Firms will be value-maximizers, paying taxes at a rate τ_K on corporate income net of wages and depreciation, and returning their income to investors via repurchase of shares[1]. Let q be the price of a share in a representative firm in consumption units and E be the number of shares outstanding. Therefore $V = qE$ is the value of a firm. Individuals pay taxes on realized capital gains at rate τ_c. To focus on capital gains, we are implicitly assuming that personal tax rates are so high that repurchase of shares dominates both debt and dividends as a method of financing capital formation and distributing earnings. Taxation of interest and dividend income would just add two more accrual taxes and would necessitate the use of ad hoc arguments on the cost of alternative financial structures to avoid corner solutions to the firms' problem. Our tax structure is an appropriate simplification since it allows one accrual tax and one realization tax on capital income, the dichotomy which

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[1]While periodic stock repurchase to replace dividends is not allowed, the constant returns to scale assumption makes random patterns of reinvestment and repurchase perfect substitutes for periodic stock repurchases. Also, what is crucial for our purposes is that firms buy shares of some corporation from private shareholders, not that an investor sells his shares of firm XYZ to firm XYZ. In light of these features, we ignore the restrictions on repurchases. Also, since one objective of this essay is to see how a true capital gains tax affects the economy, it is appropriate to assume a pure capital gains tax.

is the focus of this study.

Some of the revenue from the taxes may go to finance government expenditure at a rate of g units of the numeraire good. This expenditure is assumed to have no impact on demand for private consumption. All remaining revenues are rebated in a lump-sum fashion to consumers.

The Consumer's Problem

The first crucial element in the determination of investment decisions is the consumption-savings choice of the individual consumer-investors. We turn now to characterizing his intertemporal choices. Since individuals hold equity, an individual's holding of equity is the appropriate index of his wealth. λ be the private shadow price of equity. The representative individuals' budget constraint is

$$(1) \qquad 0 = q\dot{E} + c - w - \tau_c(q - q_b)\dot{E} - Tr$$

where q_b is the base value of equity, assumed constant across a portfolio,[2] w is the wage rate, and Tr equals transfers from the government to the agent.

We allow q_b to be a function of time. This is natural since q_b is to represent the real value of the assets' basis and inflation will cause the real basis value of equity to decrease. Also, one feature of U.S. capital gains tax law is the treatment of capital gains at death where an heir is allowed to increase the basis value of an asset without paying a capital gains tax. At such times, q_b would also rise. However, given the stochastic nature

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[2]This is a necessary simplification if we are to keep the consumer's problem finite-dimensional. If the various units of the same asset in a portfolio carried different base values due to different purchase times, a nontrivial inventory problem arises in the consumers' choice of which units of an asset to sell. We abstract from these complications to concentrate on general equilibrium effects of capital gains taxation. See Balcer and Judd (1985a) for a partial equilibrium analysis of these vintage problems.

of death, allowing a rising q_b could only be a rough approximation of this feature even under the maintained assumption that agents have altruistic intergenerational attitudes. The difficulty in accurately modelling such a feature is that different dynastic families would differ in the date of the last death, destroying our ability to aggregate individual dynastic family decisions into a low-dimensional aggregate system. Therefore, this essay will focus on the inflation interpretation of q_b changes, leaving the death pass through issue for further study.

The first-order conditions from optimal control theory imply that

$$\text{(2a)} \qquad \dot{\lambda} = \rho\lambda$$

$$\text{(2b)} \qquad p \equiv u'(c) = \lambda(q(1 - \tau_c) + \tau_c q_b)^{-1}$$

Define $\phi \equiv \rho - \frac{\dot{p}}{p}$, the consumption rate of return. Then (2) implies that

$$\text{(3)} \qquad \phi = \frac{\dot{q}(1 - \tau_c) + \tau_c \dot{q}_b - \dot{\tau}_c(q - q_b)}{q(1 - \tau_c) + \tau_c q_b}$$

Equation (3) represents the first-order condition which equates the intertemporal marginal rate of substitution at any moment with the intertemporal return to the investor. The return to holding the asset in the face of capital gains taxation is expressed intuitively by the right side of (3). The denominator is the after-tax value of equity. The numerator is the sum of returns to holding the asset. If an investor holds the investment, he earns $\dot{q}(1 - \tau_c)$ from appreciation of the equity value, $\tau_c \dot{q}_b$ from an increase in the basis value, and $-\dot{\tau}_c(q - q_b)$ if the capital gains tax rate changes at the rate $\dot{\tau}_c$.

Rearranging (3) shows that if an investor is indifferent between buying and selling a unit of an asset, its price must obey

$$(4) \qquad \dot{q} = q\left(\phi + \frac{\phi q_b \tau_c}{(1 - \tau_c)q} + \frac{\dot{\tau}_c(q - q_b) - \tau_c \dot{q}_b}{(1 - \tau_c)q}\right) \equiv q\phi^*$$

Equation (4) tells us that q must grow at rate ϕ^* in order to generate a return just sufficient to satisfy the investor after taxes. ϕ^* will be a crucial element in determining the cost of funds to a firm.

We can solve for q in terms of the initial price and taxes, yielding a decomposition of the price into tax and fundamental components. Integration of (4) yields a solution for q in terms of p:

$$(5) \qquad q(t) = e^{\int_0^t \phi(s)ds}\left(q(0) + \int_0^t e^{-\int_0^s \phi(z)dz} x(s)ds\right)$$

where

$$\phi = \phi + \dot{\tau}_c/(1 - \tau_c)$$

$$x = (\phi \tau_c q_b - \dot{\tau}_c q_b - \tau_c \dot{q}_b)/(1 - \tau_c)$$

(5) shows that the price at any future time is an exponential multiple of the initial value and a present value of the tax parameters. This formula for q will be valuable below in our equilibrium analysis.

<u>The Firm's Problem and its Cost of Capital</u>

We next address the firm's problem and integrate it with the individual's problem to formulate a cost of capital formula. The firm chooses net investment and labor use to maximize value with net earnings, γ, distributed in the form of share repurchase. Therefore

(6) $$\gamma(k,\ell,I) = (1 - \tau_K)(F(k,\ell) - \delta k - w\ell) - I + \theta(\delta k + I)$$

where I is net investment, $\dot{k} - \delta k$. Since $V \equiv qE$ is the market value of a firm, $\dot{V} = \dot{q}E + q\dot{E}$. However, $q\dot{E} = -\gamma$, since cash flow net of investment expenditures is sent to the equity holders via repurchase. Also $\dot{q}E = (\dot{q}/q)qE = \phi^* V$. Therefore,

$$\dot{V} + \gamma = V\phi^*$$

Integrating, assuming V does not grow exponentially at rate ϕ^*, yields[3]

(7) $$V(t) = \int_t^\infty e^{-\int_t^s \phi^*(z)dz}((1 - \tau_K)(F(k,\ell) - \delta k - w\ell) - I + \theta(\delta k + I))ds$$

We assume that the firm at t chooses net investment, I, and labor use, ℓ, to maximize value at t. We first examine the value maximizing problem at $t = 0$. This problem is

$$\max_{I,\ell} \int_0^\infty e^{-\int_0^t \phi^*} \gamma(k,\ell,I)dt$$

$$\dot{k} = I$$

$$I \leqslant (1 - \tau_K)(F(k,\ell) - w\ell)(1 - \theta)^{-1} - \delta k$$

Since we want to abstract from vintage aspects of capital gains taxation, we assume that firms' equity has the same basis value at all times. This essentially says that all firms are always identical, the usual assumption in

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[3]This follows from the transversality condition at infinity from the consumer's problem. See Brock and Turnovsky.

aggregate models. This also implies however that no new equity is issued by any firm and that there are never any new firms. Recall that we also assumed that personal taxes were so high that bond financing is prohibitively costly. All these elements are indicative of our focus on an economy of mature firms which finance all investment through retained earnings. This is formally represented in the firm's maximization problem by the upper bound on net investment. Given our focus on mature firms, we will proceed below under the assumption that this will not be binding. This is also an appropriate assumption later when we examine marginal effects since a marginal change around a steady state, where the constraint surely does not bind, cannot cause a constraint to become binding. While this focus on an economy of mature firms does lose some generality, it does describe the majority of corporations in the U.S. Also, the introduction of elements which lead to heterogeneous basis values would necessitate the use of the much more complex framework of Balcer and Judd (1985a). Therefore, the simplifying assumptions of this study are justified also by the relatively simple analysis and analytic (as opposed to numerical) results which follow.

If $\mu(t)$ is the present value shadow price of capital at t for the firm's value maximization problem, then the present value Hamiltonian is

$$H = e^{-\int_0^t \phi^*} \gamma(k,\ell,I) + \mu I.$$

By the Pontryagan Maximum principle

$$\text{(8a)} \qquad \dot{\mu} = -\frac{\partial H}{\partial k} = -e^{-\int_0^t \phi^*}((1 - \tau_K)(F_k - \delta) + \delta\theta)$$

$$\text{(8b)} \qquad 0 = -e^{-\int_0^t \phi^*}(1 - \theta) + \mu$$

Differentiating (8b) and substituting into (8a), we find that a necessary condition for value maximization is that

$$(9) \qquad \frac{(1 - \tau_K)f'(k) + \delta\theta - \dot{\theta}}{1 - \theta} = \phi^*, \quad f(k) - kf'(k) = w$$

Equation (9) shows that the cost of capital formula at each moment is

$$(10) \quad f' = \frac{\phi^*(1 - \theta) - \delta\theta + \dot{\theta}}{1 - \tau_K}$$

$$= \frac{\phi(1 - \theta)}{1 - \tau_K}\left(1 + \frac{\tau_c}{1 - \tau_c}\frac{q_b}{q}\right) + \frac{(1 - \theta)\dot{\tau}_c(q - q_b)}{(1 - \tau_c)(1 - \tau_K)q} - \frac{(1 - \theta)\tau_c\dot{q}_b}{(1 - \tau_c)(1 - \tau_K)q} + \frac{\dot{\theta} - \delta\theta}{1 - \tau_K}$$

We can regard ϕ as the net rate of return required by an investor since, being the consumption rate of return, it would be the net return on any short-term bond willingly held by a consumer.

This cost of capital formula yields the demand for capital as a function of the cost of investment funds. Since it is derived from the firm's value maximization problem, it can be interpreted as a firm's demand for capital in terms of the instantaneous required net return, ϕ, the basis value of a firm's equity, the tax structure, investment tax credit, and the time derivatives of these parameters. The importance of the time rates of change in policies is of interest and indicates that we should move away from the common practice of assuming constant policies.

Before examining the cost of capital formula, we should consider an example which provides the right intuition for our results. Suppose an asset is worth \$100 today and \$110 tomorrow, yielding a 10% pretax rate of return. Assume there is a 50% capital gains tax with the basis of the asset being \$50. If the asset is sold today, the net sale is \$75, whereas \$80 will be received if sold tomorrow, yielding a $6\frac{2}{3}\%$ rate of return. However, if the base were zero the choice is between \$50 today and \$55 tomorrow, a 10% return. This shows that the rate of return to holding the asset is unaffected by the capital gains tax if the base is zero, but not otherwise. In general, this shows that the average tax rate, that is, the tax payment as a fraction of the asset's value, may differ substantially from the equivalent accrual tax rate. In fact, the average tax is high precisely when the marginal disincentive to holding is least and vice-versa. This will account for some results below which are unique to capital gain taxation.

In examining the dependence of capital demand on tax and taste parameters, it is convenient to make the reasonable assumptions that $0 \leq \theta, \tau_K, \tau_c < 1$, and $q > q_b > 0$. Some of the relationships are clear. If the required rate of return, ϕ, rises then capital demand falls since f' rises. This just reflects the intuition that if investors require a higher net return, capital demand falls. Capital demand also falls if $\dot{\theta}$ rises, reflecting the fact that an increasing investment tax credit gives incentives to delay investment and receive a greater investment subsidy later.

The dependence of f' on the time rate of change in the real value of the equity's basis is also unambiguous. If q_b is rising, then investment is less than if q_b were falling. A low q_b makes the capital gains tax more like a lump-sum tax on wealth. Reduction of the basis level reduces the distortion and stimulates investment demand, and the more rapidly q_b is falling, the greater the stimulation. One way in which q_b can be more rapidly decreased is an increase in inflation. This reduction in the real value of the basis has been a nontrivial source of revenue for the U.S. during the past inflationary periods according to Feldstein, Slemrod, and Yitzhaki. While they refer to this as a distortion, they only show that investors paid more taxes than they would have if inflation had not accelerated. In this model, we find that inflation may raise investment as well as revenues. If capital is undersupplied due to a distortionary tax on capital income, then this stimulation would be beneficial and in fact counter the distortion. We must be careful in interpreting this result. It essentially says that the q_b path, if chosen by the government, will be less damaging to capital formation if it declines rapidly, thereby becoming a lump-sum tax on wealth. If the q_b is the market value at some date then the inflation is implicitly assumed to not affect the initial value of q_b and hence to have been unanticipated at that time. These caveats make the result less surprising. It is also clear that this analysis' focus on mature firms (a feature it shares with Feldstein, et al.) implies that we are ignoring any disincentive that inflation generates for the formation of new firms.

More intuitively, a rising capital gains tax reduces capital demand since f' rises as $\dot{\tau}_c$ rises. A rising τ_c reduces capital demand since it reduces the future income stream from investment to an investor.

The dependence of f' on q_b is surprisingly ambiguous. This comparative exercise corresponds to examining two firms in the same financial and tax environment except one is "newer" with a basis value closer to its current market value. The firm with the greater q_b/q will demand less capital if ϕ exceeds $\dot{\tau}_c/\tau_c$. This shows that two forces are at work in determining the relationship between q_b and capital demand. As ϕ rises, the distortion becomes more important and since the distortion is present only to the extent q_b differs from zero, it falls most heavily on the new firm. On the other hand, a rising capital gains tax affects f' in proportion to $(q - q_b)q^{-1}$, the tax base of the capital gains tax expressed as a share of firm value, therefore falling more heavily on the firm with the relatively low basis. In the long run, ϕ should exceed $\dot{\tau}_c/\tau_c$ since τ_c is presumably bounded above and below and investors will demand a positive return in the long run. In such a state, the high basis firms will reduce demand for capital more due to capital gains taxation. This is misleading however since there is no presumption for $\phi > \dot{\tau}_c/\tau_c$ always. In fact, if the real net rate of return is negative, the new firm invests more even if τ_c is constant.

The dependence of f' on the τ_c is also ambiguous when taxes and basis values are not constant. An increase in τ_c causes f' to rise if

$$\frac{-\dot{q}_b}{q_b} + \dot{\tau}_c(q - q_b) > 0$$

If τ_c and q_b were constant, a firm whose owners paid a higher capital gains tax will demand less capital, the intuitive result. If τ_c is dropping and the present value of the basis is not dropping too rapidly, then an increase in τ_c will increase the demand for capital. This is due to the value of manipulating the repurchases so as to economize on the tax payments by shareholders. For example, if $q_b = 0$, then the firm will want to reinvest today and distribute the earnings tomorrow if τ_c is falling. The extent of this manipulation is limited by the declining internal marginal efficiency of investment. However, the greater the current τ_c is, the more valuable the manipulation and the more capital retained currently.

The present value of the basis matters if $q_b \neq 0$ since the lower the basis is tomorrow, the greater the tax on realizations, and the greater the tax is on the marginal return to current investment. Therefore, if the present value of the basis is dropping, the firm has less incentive to invest.

Examination of (10) immediately shows that an increase in τ_K will reduce demand for capital if $f' > 0$, the only reasonable case.

Our cost of capital formula should be compared to the usual approach to dealing with capital gains taxation. The impact of capital gains taxation on the overall cost of capital is usually approximated by an averaging formula. For example, King and Fullerton (1984) would account for capital gains taxation in our model by adding

$$\frac{-\dot{E}/E}{-\dot{E}/E + \rho} \tau_c$$

to the other components of their effective tax rate consumption. It is clear from the foregoing that such a procedure would not be an acceptable approximation in this model for short-run purposes. We will see below that it is not for long-run purposes either.

We also note that our cost of capital formula is substantially different from that derived in Constantinides (1983). He assumes that agents randomly liquidate assets, thereby ignoring the realization nature of the capital gains tax. Under this specification of investor behavior (along with the assumption of an insurance market which compensates the investor when asset sales generate particularly large capital gains tax liabilities) he shows that the capital gains tax acts as an accrual tax. We find instead that accrual and realization taxes have qualitatively different impacts on investment incentives. If we are to judge between the two forms of capital taxation, we should not make simplifying assumptions which wash out the differences. While the examination of a stochastic economy is beyond the scope of this paper, this shows that further efforts are needed to determine the impacts of capital gains taxation in a risky world.

The last item to note is that $V = (1 - \theta)k$ at all times. This is calculated by substituting (9) into (6) and integrating by parts. Intuitively, this arises because the investment tax credit subsidizes new capital formation, reducing the market value of old capital by θ. It may appear surprising that the firm's valuation is unaffected by the presence of capital gains taxation. This fact is natural in the present model since there are no adjustment costs, implying that the investment good, even investment in place, is a perfect substitute for current good. If $\theta = 0$ and the firm sold at a discount relative to the book value of its capital stock, then demand for it would be unbounded since consumers could acquire one unit of consumption at a cost less than one unit of consumption. If it sold at a premium, then a firm could increase without bound by selling itself and forming the proceeds into a larger firm which could be sold again for more. These arbitrage arguments show that the only wedge between V and k is that induced by θ. If there were adjustment costs, then forward-looking intertemporal considerations would matter and capital gains taxation would affect a firm's value. In this initial consideration of general equilibrium with capital gains taxation by realization, it is appropriate that we abstract from adjustment costs.

4. Equilibrium

We now combine the firms' and consumers' problems to yield an equilibrium. We assume that all firms have the same basis and are subject to the same tax laws. The crucial equations are (4), (8) and (9), which relate the demand for capital with the required rate of return, ϕ, and the definition of ϕ, which relates the required rate of return and the marginal utility of consumption.

Before continuing, we should give some justification for jumping to a general equilibrium analysis. Normally a partial equilibrium approach which holds the required rate of return fixed would suffice to examine the effects of a tax change, with adjustment costs giving rise to nontrivial intertemporal interactions. In this case, however, such an approach would not be strictly valid. The problem arises because of the vintage aspect that the basis value brings. All of our analysis assumes that the investor is indifferent between buying and selling a marginal unit of equity, as does the analysis with accrual taxation. If there were other assets present in the economy, the investor may be driven to a corner since he can sell an asset at basis value q_b, some past price, but a purchase will generate an investment with basis value equal to the current price. This in fact will occur in a multiple vintage or multiple asset portfolio, as shown in Balcer and Judd (1985a). Therefore, to be consistent with the assumption that the investor is not on a corner, we must assume that all assets in his portfolio are identical. It would therefore be inconsistent to assume that the required rate of return is fixed and forces us to consider a general equilibrium analysis.

Define $Q(t)$ to be $q(t)\lambda(t)^{-1}\lambda(0)$. Q is the price of equity at t multiplied by the marginal rate of substitution between equity at t and 0. (2a) implies that $Q = q(t)e^{-\rho t}$ and

$$\text{(11)} \qquad \dot{Q} = Q(\frac{\dot{q}}{q} - \rho) = Q(\frac{(1 - \tau_K)f'(k) + \delta\theta - \dot{\theta}}{1 - \theta} - \rho)$$

The definition of ϕ implies that p obeys the differential equation

$$\text{(12)} \qquad \dot{p} = p(\rho - \phi)$$

$$= p(\rho - \frac{\dot{q}}{q}\frac{1 - \tau_c}{1 - \tau_c + \tau_c q_b q^{-1}} - \frac{\tau_c \dot{q}_b - \dot{\tau}_c(q - q_b)}{q(1 - \tau_c) + \tau_c q_b})$$

$$= p\Big(\rho - \frac{(1 - \tau_K)f'(k) + \delta\theta}{(1 - \theta)\big(1 + \frac{\tau_c}{1 - \tau_c}\frac{q_b e^{-\rho t}}{Q}\big)} - \frac{\tau_c \dot{q}_b e^{-\rho t} - \dot{\tau}_c(Q - q_b e^{-\rho t})}{Q(1 - \tau_c) + \tau_c q_b e^{-\rho t}}\Big)$$

Our equilibrium system is now complete, comprised of (11), (12), and the following material balance condition equating net investment with net output less private and public consumption:

$$\text{(13)} \qquad \dot{k} = f(k) - c(p) - g$$

where $c(p)$ expresses consumption as a function of the marginal utility of consumption. (13) implicitly assumes that any extra revenue is returned lump-sum to consumers, and if g is not covered by revenues, lump-sum taxes are imposed.

A complete analysis of our general equilibrium system, (11)-(13), is beyond the scope of this essay. Some essential features are noted easily, however. If q_b and τ_c are constants, then (12) and (13) yield unique steady state values for p and k. When we impose $qE = V = k(1 - \theta)$ in the steady state, we find that $Q = k(1 - \theta)e^{-\rho t}/E$. Since E decreases exponentially at rate ρ in the steady state, Q will converge to a positive level once we normalize the equity unit.

Given that steady state, we will also find that the local dynamics are uniquely determined. Since the exercises below are local in nature, we see that our comparative dynamic general equilibrium analysis is well-formed. Having developed the necessary general equilibrium model, we next apply it to the important problems.

Using our equilibrium system, (11)-(13), we can next address both positive and normative issues concerning capital income and capital gains taxation.

5. Marginal Investment Impact of Capital Gains Taxation

In this section we analyze the impact a capital gains tax increase has on investment and the path of capital accumulation. In order to focus on the essential features, we assume in this section that $\theta = 0$. We will examine two limiting cases. In both cases, we assume that $\tau_c = 0$ initially and that the economy is in the steady state, i.e., $f'(k) = \rho/(1 - \tau_K)$. We assume that τ_c is then raised immediately and permanently.

The two cases differ in the basis value of equity. First, suppose $q_b = 0$, that is, the base is set at zero. In this case, there are no real effects. To see this, suppose $p(t)$ is unaffected for all t. Then $q(t) = e^{\rho t}q(0)$ by (10) and $V(0)$ is unaffected, implying that $q(0)$ is unaffected. The constancy of p would further imply that $\lambda(q(1 - \tau_c) + \tau_c q_b)^{-1}$ is unaffected, implying that $\lambda(t) = e^{\rho t}q(0)(1-\tau_c p(0))$, which satisfies (2). Therefore, there will be no impact. Since the capital gains tax reduces to a lump-sum tax when the basis is zero, this is expected.

Second, suppose that q_b is $q(0)$, the price of an equity a moment after the announcement. This would be the case if an unanticipated capital gains tax were imposed, but it is decided to tax only those gains which occur after the enactment of the tax. Then (10) simplifies to

$$(14) \qquad q(t) = q(0)\left(\frac{e^{\int_0^t \phi(s)ds} - \tau_c}{1 - \tau_c}\right)$$

Using (14), (12) simplifies to

$$(15) \qquad \dot{p} = p\left(\rho - (1 - \tau_K)f'(k)\left(1 - \tau_c e^{-\rho t} \frac{p(t)}{p(0)}\right)\right)$$

Equations (15) and (13) gives us a two-dimensional equilibrium system in terms of the capital stock and the marginal utility of consumption.[4] Of particular interest is its nonautonomous nature despite the stationary technology, tastes, and tax rates. This nonautonomous structure arises because the price of equity rises relative to the base value, altering the rate of return to holding the asset.

We represent the system (13, 15) in the phase diagram in Figure 1. Note that the steady state, i.e., when $t = \infty$, of our new system is A and is unaffected by the capital gains tax, i.e., there is no additional distortion due to τ_c in the long run. This follows from (15) as long as $p(t)/p(0)$ is finite as t becomes infinite. However, the short run is affected, as Figure 1 displays. At any time t, there will be $\dot{p} = 0$ and $\dot{k} = 0$ loci with stationary point D. At all times, the $\dot{k} = 0$ locus is the locus $f(k) = c(p) - g$. However, the $\dot{p} = 0$ locus moves. In the long run it is the vertical line $f'(k) = \rho/(1 - \tau_K)$. In the short run, because higher p implies a higher effective tax rate, the $\dot{p} = 0$ locus is tilted to the left with the $p = 0$ intercept unaffected. Over time, this tilted $\dot{p} = 0$ locus straightens up, converging to the $f'(k) = \rho/(1 - \tau_K)$ line.

With this phase representation at each t, we can determine the short-run effect of an increase in τ_c. If p(0) rose or did not change, then the (k(0), p(0)) point would lie above the $\dot{p} = 0$ locus at all times and p would rise, sending it above the $\dot{k} = 0$ locus, implying that p and k would forever rise since they would always lie above the $\dot{p} = 0$ and $\dot{k} = 0$ loci. Therefore, p must fall from A to a point such as B at $t = 0$. However, as the $\dot{p} = 0$ locus straightens up, the economy moves from B back to A along the path C. The net

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[4]In both cases examined in this section, the assumption of a constant q_b causes our general equilibrium system to be reducible to a two-dimensional system. This will not be the case if the basis value is affected by the real evolution of the system such as when the basis is raised at death.

result is that the capital gains tax would cause investment to fall in the short run, but later rise until the capital stock attained its initial value.

Note that an accrual capital gains tax (or, equivalently, a corporate income tax) would affect the rate of return and distort investment in a very different fashion. Examination of a permanent and unanticipated increase in an accrual tax (as in Judd (1985)) shows that the capital stock decreases monotonically to a lower steady-state level, a dynamic response much different than that displayed here for a realization tax. This is further evidence that accrual tax approximations of realization taxes are not reliable.

The crucial feature eliminating any impact by τ_c on the steady state is that the basis value of equity disappears relative to its market value. If the basis increased but not at the same rate as equity rose in value, as it would for example if capital gains were allowed to pass through untaxed at the time a parent dies and leaves the equity to an heir, fluctuations in q_b would not vanish relative to market value and a long-run distortion would result. Our cost of capital formula displays this effect since a constant τ_c will affect f' as long as $(\phi q_b - \dot{q}_b)/q$ does not converge to zero in the long run.

While this exercise certainly ignores many considerations relevant to the analysis of capital gains taxation, it does point out the need for an equilibrium analysis in order to understand capital gains taxation. For example, the averaging techniques commonly employed (e.g., see King and Fullerton) would seriously overestimate the long-run impact of capital gains taxation on the cost of capital in <u>this</u> model. Those techniques also have little relation to our cost of capital formula and therefore have questionable applicability even to short-run considerations. While the net effect of adding more real world elements is ambiguous, it is clearly important to model the realization nature of exsting capital gains taxation.

6. Efficiency Costs of Corporate and Capital Gains Taxation

Policymakers are not only interested in the positive effects of capital gains taxation on the economy, but also in whether it is a relatively efficient way of raising revenue. We can examine this normative issue to some extent in our model. In this section, we examine the impact on welfare and revenue of marginal increases in τ_c and τ_K.

Our procedure will be to begin with an economy in the steady state corresponding to a combination of taxes, shock it at the margin by an unanticipated and permanent increase in one of the taxes, and then compare the welfare cost per dollar of revenue raised for each tax where any revenues are rebated lump-sum to consumers. We will thereby be computing the excess burden of such tax changes.

We first record some obvious and essentially trivial results. Recall that if the basis value of equity is zero, then a capital gains tax of τ_c is equivalent to an expropriation of that fraction of the current capital stock. Therefore, if the capital gains tax is increased in the steady state and a zero basis value is used then the capital gains tax is a lump-sum tax, imposing no loss of efficiency on the economy. This is a well-known fact. Furthermore, if we are in a steady state where the basis value of equity has become essentially zero relative to the market value, an increase in the capital gains tax will also be a lump-sum tax. That result is a product of the long-run nature of our model, the least realistic feature of our analysis since it makes excessive use of the absence of asset trading.

Therefore, in order to focus on the distortionary effects of capital gains tax, we eliminate the portion of a capital gains tax which is a tax on pure rents by assuming also that the basis value for equity under this new tax is the market value of equity at the time of the tax increase. Without loss of generality, we can also assume that there is initially no capital gains tax which assigns a zero basis value to equity since the present value of revenue from the old tax depends only on the current value of the capital stock, which is not affected by capital income or capital gains tax changes.

These arguments lead us to focus on the case where $\tau_c = 0$ initially. We assume that the economy has achieved the steady state corresponding to (possibly positive) capital income tax of τ_K. To examine the effects of capital gains taxation, we assume an unanticipated increase in τ_c of ε and set $q_b = q(0)$, the more interesting case. Define $x_\varepsilon(t)$ as the rate of change in the value of x at time t as τ_c is increased from 0 by ε, for x = p,k. Similarly, let $X_\varepsilon(s) = \mathcal{L}[x_\varepsilon](s) \equiv \int_0^\infty e^{-st} x_\varepsilon(t)\, dt$, the Laplace transform of $x_\varepsilon(t)$, for x = p,k, X = P,K. Laplace transforms are natural for our purposes since they are just discounted sums, and we will be interested in the present value of the change in government revenue and the change in the total lifetime discounted utility of the representative agent.

These responses to a change in the capital gains tax can be determined by differentiating the equilibrium conditions. When we differentiate our equilibrium system, (13), (15), with respect to ε, and apply Laplace transform methods, we find[4]

$$(16)\quad \begin{pmatrix} \dfrac{P_\varepsilon(s)}{p} \\[2ex] \dfrac{K_\varepsilon(s)}{F} \end{pmatrix} = (sI - J)^{-1} \left\{ \mathcal{L}\left[\begin{pmatrix} \rho e^{-\rho t} \\ 0 \end{pmatrix}\right](s) + \begin{pmatrix} \dfrac{p_\varepsilon(0)}{p} \\[2ex] 0 \end{pmatrix} \right\}$$

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[4]See Judd (1985) for details on this linearization and the following calculations concerning utility and revenue. $\mathcal{L}$ is the Laplace transform operator.

$$= \begin{pmatrix} s - f' & \frac{(1-\tau_K)f'}{1-\theta} & \frac{\theta_L}{\sigma\theta_K}f' \\ -\beta\theta_c & & s \end{pmatrix} \begin{pmatrix} \frac{\rho}{\rho+s} + \frac{p_\varepsilon(0)}{p} \\ 0 \end{pmatrix} /[(s-\mu)(s-\zeta)]$$

where J is the Jacobian of the sytem (13,15) with respect to p and k evaluated at the initial steady state. Also, θ_c denotes the share of net output which goes to private consumption and μ and ζ are the positive and negative eignevalues of J, respectively. Stability of equilibrium implies that k_ε and p_ε are bounded in t. Therefore, $P_\varepsilon(\mu)$ and $K_\varepsilon(\mu)$ are both finite. This is possible only if the zero in the denominator of (16) arising when $s = \mu$ is offset by a zero in the denominator. This occurs only if

$$\frac{p_\varepsilon(0)}{p} = -\frac{\rho}{\rho+\mu} \tag{17}$$

giving us our endogenous change in p at t = 0. This formula verifies quantitatively the drop in p suggested in our earlier discussion and displayed in Figure 1.

With the solution for p_ε and K_ε in (16) and (17), we can compute the revenue and welfare impact of increasing τ_c from zero to ε. The change in revenue, dR, and the change in real income, $(dU/d\varepsilon)/u'(c(0))$, expressed in terms of consumption at t = 0, are given by

$$\frac{dR}{d\varepsilon} = \int_0^\infty e^{-\rho t}\left(\tau_K f'(k)k_\varepsilon - \theta(\dot{k}_\varepsilon + \delta k) + \rho e^{-\rho t}(1 - e^{-\rho t})k\right)dt$$

$$= \left(\frac{\tau_K f' - \theta(\rho+\delta)}{\rho+\mu} \frac{\beta\theta_c f'}{(\rho-\zeta)\theta_K} + \frac{1}{2}\right) k$$

$$\frac{dU/d\varepsilon}{u_1} = (f' - \rho)K_\varepsilon(\rho) = (f' - \rho)\frac{\beta\theta_c F}{2(\rho + \mu)(\rho - \zeta)}$$

$$= \frac{(f' - \rho)\beta\theta_c f' k}{2(\rho + \mu)(\rho - \zeta)\theta_K}$$

where we use the fact that realized gains at t equal $q\dot{E} = \rho k$ when $\varepsilon = 0$, that is, when there is no tax imposed. (See Judd (1985) for the derivation of these formulas.)

To measure the efficiency cost of a tax increase relative to the revenue gains, we examine the ratio of the welfare gain to the revenue change. It is denoted $MDWL_{cg}$ and equals

$$MDWL_{cg} = \frac{(f' - \rho)\beta\theta_c f'}{2(\tau_K f' - \theta(\rho + \delta))\beta\theta_c f' + (\rho + \mu)(\rho - \zeta)\theta_K}$$

The alternative way of raising revenue in this model is to raise τ_K, the tax on corporate income. To determine the better of the two tax instruments at the margin, we next examine a marginal increase in τ_K around the initial steady state, determine its efficiency in raising revenue, and then compare the two tax instruments. The efficiency index for a marginal increase in the corporate income tax is

$$MDWL_K = \frac{(f' - \rho)\beta\theta_c f'}{(\tau_K f' - \theta(\rho + \delta))\beta\theta_c f' + \mu\theta_K(\rho - \zeta)(1 - \theta)}$$

where $MDWL_K$ is the marginal deadweight loss of capital income taxation. (See the Appendix for a review of the relevant calculations.)

In order to determine which of these marginal tax increases is better, we compute the ratio of the two marginal deadweight loss measures:

$$D \equiv \frac{MDWL_{cg}}{MDWL_K} = \frac{(\tau_K f' - \theta(\rho + \delta))\beta\theta_c f' + \mu\theta_K(\rho - \zeta)(1 - \theta)}{2(\tau_K f' - \theta(\rho + \delta))\beta\theta_c f' + (\rho + \mu)(\rho - \zeta)\theta_K} \tag{18}$$

It is clearly difficult to determine if this ratio of marginal deadweight losses is greater or less than unity. However, if the level of current taxes is low, then we find it to be less than one. If τ_K and θ are both nearly zero, then D reduces to $\mu/(\rho + \mu)$ which is clearly less than one since ρ and μ are both positive. Therefore, if tax rates are low, then a marginal increase in the capital gains tax is less distortionary than a capital income tax increase. We can be a little more precise about the comparative efficiency if we use parameterizations considered representative of the U.S. See Judd (1985) for a discussion of these parameterizations and values of μ/ρ. Since $\rho < \mu < 3\rho$ for these parameterizations, they indicate that capital gains taxation is at least a half to three-fourths as distorting as capital income taxation when we assume factor substitutability and intertemporal consumption elasticities considered reasonable.

It is also clear that when the initial corporate income tax rate is large, capital income taxation will be less distortionary. This is true since μ often exceeds 2ρ, implying that the denominator of D is more likely to become zero before the numerator does as τ_K increases. The relative inefficiency of capital gains taxation when the basis is set at the current market value and taxation is high is not surprising. First, any distortion to the capital stock is very costly. The capital gains tax distorts capital accumulation only in intermediate run but it is in the intermediate run when it begins to generate revenue since initially equity values do not exceed the basis value. In contrast, the corporate income tax generates revenues immediately.

These results indicate that an unambiguous ranking of the two taxes is not possible, but depend on the basis value assigned to shares at the time of the capital gains tax increase and the existing tax rates. We have not engaged in any exhaustive study of the relative desirability of these two taxes since our model abstracts from a large number of important elements. Howver, this work does show that the use of accrual equivalents to represent the normantive impacts of realization taxation would be misleading. Future work which carefully respects the differences between accrual and realization taxation is necessary to gain any reliably robust results concerning desirable reforms.

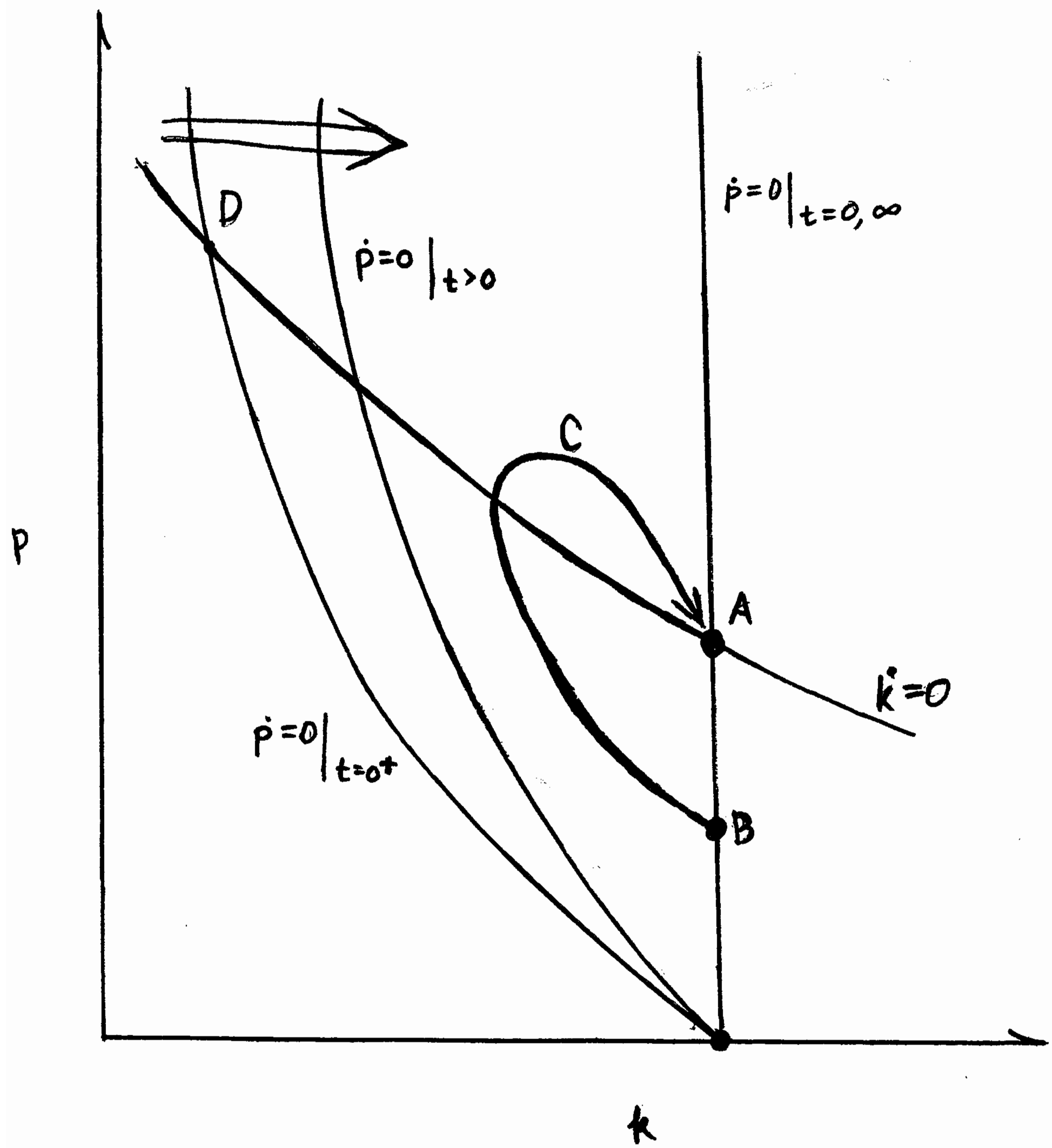
6. Conclusions

We have studied a highly stylized general equilibrium model of capital gains taxation. In this model we are however able to address both positive and normative questions concerning capital gains taxation. We first developed a cost of capital formula for value-maximizing firms. We found that the parameters of capital gains tax affect the cost of capital in ways substantially different from the impact of an accrual tax such as a corporate income tax. In fact, an increase in the current capital gains tax may reduce the current cost of capital and raise investment if the capital gains tax rate is dropping over time. In general, we show that simple averaging rules do not capture the impact of capital gains taxation on the cost of capital because they cannot capture the dynamic elements unique to capital gains taxation.

We next compared the efficiency of capital gains taxation relative to an accrual tax. First we find that if a zero basis value is assigned to equity when the tax is imposed, the tax is essentially a tax on current wealth, a lump-sum tax. In this case there are no positive impacts on equlibrium and the excess burden of the revenue is zero. In the second and more realistic case of a substantial basis value for equity, we find that substantial positive effects will be induced and the welfare cost of the extra revenue will be nontrivial. However, we find the disinvestment initially induced by the capital gains tax will be only temporary, and there will be no steady-state distortion of the capital stock if the basis value of equity does not increase. Furthermore, we find that capital gains taxation is more efficient than capital income taxation if the existing tax rates are low, even if equity is assigned a basis value equal to its value at the time of the capital gains tax. However, if the existing taxes and the basis values are high, the capital gains tax is more distortionary, even though a capital gains tax will distort the long-run capital stock whereas the capital gains tax won't.

While we have covered only a few special cases, they arguably illustrate robust phenomena. Allowing the basis value to grow in an uneven pattern, as it would if capital gains could be passed through estates without taxation, would imply a long-run distortion from capital gains taxation, presumably making it a less desirable source of revenue compared to capital income taxation. As long as the basis value responds with a lag to equity value, the undershooting of capital stock in response to a capital gains tax increase is still likely. While the resolution of these and other conjectures await further analysis, we have here a structure capable of addressing these important issues in tax policy.

Figure 1: Response to Capital Gains Tax Increase



Appendix

This Appendix indicates some of the steps in the derivation of $MDWL_K$. If τ_K is increased by ε in the steady state, differentiation of (13, 15) will yield differential equations representing the impact of that increase on the time paths of p and k. In this case, the Laplace transforms will obey

$$\begin{pmatrix} P_\varepsilon(s)/p \\ K_\varepsilon(s)/F \end{pmatrix} = (sI - J)^{-1}\left[\begin{pmatrix} f'/((1-\theta)s) \\ 0 \end{pmatrix} + \begin{pmatrix} p_\varepsilon(0)/p \\ 0 \end{pmatrix} \right]$$

The change in p at t = 0 relative to ε will have to be

$$p_\varepsilon(0)/p = -f'/(\mu(1-\theta))$$

to ensure stability of the dynamic system. Using this p response and our solution for the Laplace transform of K shows that

$$K_\varepsilon(\rho) = \frac{-\beta\theta_c f'(\mu-\rho)}{\mu\rho(\rho-\mu)(\rho-\zeta)(1-\theta)} = \frac{\beta\theta_c f'}{\mu\rho(\rho-\zeta)(1-\theta)}$$

The impact on discounted revenue will be

$$\frac{dR}{d\varepsilon} = [(\tau_K f' - \theta(\rho-\delta))\frac{\beta\theta_c F f'}{\mu\rho(\rho-\zeta)(1-\theta)} + \frac{f'k}{\rho}]$$

The fall in real income is found to be

$$\frac{dU/d\varepsilon}{u_1} = \frac{(f'-\rho)\beta\theta_c f' F}{\mu\rho(\rho-\zeta)(1-\theta)}$$

Combining these elements yield the formula for $MDWL_K$ indicated in the text.

References

Balcer, Yves, and Kenneth L. Judd, "Optimal Consumption Rules and Portfolio Management with Duration-Dependent Returns," CMSEMS Discussion Paper No. 673, September, 1985, Northwestern University (a).

Balcer, Yves and Kenneth L. Judd, "The Efficiency Cost of Capital Taxation in a Life-Cycle Model: Capital Gains Vs. Corporate," mimeo, October 1985, Northwestern University (b).

Brock, William A., and Stephen Turnovsky, "The Analysis of Macroeconomic Policies in Perfect Foresight Equilibrium," International Economic Review (1981).

Constantinides, George M., "Capital Market Equilibrium with Personal Tax," Econometrica 51 (May, 1983), 611-636.

Constantinides, George M. and Myron S. Scholes, "Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing," Journal of Finance 35 (1980), 439-452.

Feldstein, Martin, Joel Slemrod and Shlomo Yitzhaki, "The Effects of Taxation on the Selling of Corporate Stock and the Realizations of Capital Gains," QJE 94 (1980), 777-791.

Judd, Kenneth L., "Short-run analysis of Fiscal Policy in a Simple Perfect Foresight Model," JPE (1985).

_____________, "Exercises in Voodoo Economics," mimeo (1981) and CMSEMS Discussion Paper No. 557, June, 1983, Northwestern University.

King, Mervyn and Donald Fullerton, The Taxation of Income from Capital, University of Chicago Press, 1984.

Kovenock, Daniel J. and Michael Rothschild, "Capital Gains Taxation in an Economy with an 'Austrian Sector'," J. Pub. Econ. 21 (1983), 215-256.

Stiglitz, Joseph E., "Some Aspects of the Taxation of Capital Gains", Journal of Public Economics 21 (July, 1983), 257-294.