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EXCESS CAPACITY AND COLLUSION

by

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1. **Introduction**

Recently, there has been a tremendous resurgence of interest in oligopoly theory. The impetus for this resurgence stems from the consideration of dynamic aspects of oligopolistic markets. Two particular topics that have attracted a great deal of attention are the ability of firms to collude in a repeated game setting and the role that the timing of decisions—especially investment decisions—plays in determining oligopolistic outcomes. Friedman (1971) demonstrated that, provided the discount factor is not too low (and the detection lags are not too long), collusive output levels can be supported by a threat of all industry members to permanently revert to the static Nash equilibrium if cheating is ever detected. Subsequent work by Green and Porter (1983), Abreu (1983) and Abreu et al. (1986) extended Friedman's analysis by introducing uncertainty and by developing credible threats that are more appealing than his "grim trigger strategies," either because they accomplish the same objective with less punishment or because they support more collusion. In addition, Brock and Scheinkman (1985) generalized Friedman's analysis by examining the stability of collusive agreements in price-setting games with capacity constraints (rather than quantity-setting games). In all of the above papers, the capital stock is given exogenously at the outset of the game; the choice of the scale of operation is thus not modelled explicitly.

Recent papers by Spence (1977), Dixit (1980), Eaton and Lipsey (1981), Gelman and Salop (1983), Kreps and Scheinkman (1983), and Fudenberg and Tirole (1983) study the influence of the timing of investment decisions on equilibrium outcomes in oligopolistic markets. Dixit, for example, shows that if an incumbent can choose his capital stock before a potential entrant can
commit its resources, then by installing a large enough plant he may be able to deter entry. This will be the case if capital has a low resale value (so that it constitutes a commitment to the market). The large capacity makes it easy and attractive for the incumbent to supply a large quantity to the market if entry were to occur. Prospective entrants realize that they will face an aggressive response on the part of the incumbent, and hence, stay out. Thus, by precommitting in capacity a firm can deter entry. In a similar vein, Kreps and Scheinkman emphasize the importance of precommitment by showing that when firms can choose their capital levels before competing in prices, the outcome of the game will (under certain conditions) be identical to the Cournot outcome. In other words, the Cournot equilibrium can be viewed as the result of price competition between firms provided that prices adjust more easily than plant sizes. Note that in the above two models the authors assume that long-run competition is conducted through capacity (or scale of operation) while short-run competition is waged through prices and/or output, and that collusion does not take place. These features are shared by other papers dealing with the timing of investment decisions.

In this paper, we develop and analyze a model which integrates the essential ideas of both strands of the literature. Our model involves firms choosing a long term capacity level at the outset of the game. We do not allow firms to adjust this capacity level in subsequent periods. Given these initial capacity levels, firms then engage in a repeated game of price competition. Our model should be viewed as an—admittedly extreme—parametrization of a situation in which the technological scale of operation is relatively inflexible in the short run but in which prices are relatively flexible. Throughout this paper, we will assume that firms cannot collude in capacity even though they may be colluding in price. The justification for
this assumption is that investment decisions are much more difficult to coordinate than price or output decisions. Several examples of industries in which firms colluded in price and/or output but not in investment are cited in Scherer (1980, p. 370-71), and include the nitrogenous fertilizer and synthetic fibers industries during the 1960s as well as the plastics and aluminum industries during the 1950s. Furthermore, it is well known that even in cases of overt collusion (such as the German cement cartel) firms find it exceedingly difficult to collude in capacities. Since we are interested in the relationship between excess capacity and the level of collusion that can be sustained in a market, we assume that tacit collusion is the norm and that firms charge the maximum price that can be sustained in a collusive agreement. The agreement is enforced by a threat (by all industry members) to permanently revert to the static Nash equilibrium as soon as anyone is caught cheating. Thus, when a firm contemplates deviating from the agreement it weighs the immediate gains from cheating against the capitalized value of future losses due to retaliation. The maximum sustainable price is defined to be the price that maximizes the "cartel welfare function" subject to the constraint that all firms find it optimal to abide by the agreement. Formally, we characterize the subgame perfect Nash equilibria of a two-stage game in which firms first choose capacity levels and then, in a second stage, the maximum price that can be sustained in a collusive agreement. Equilibrium is calculated by first solving for the maximum sustainable price and profit levels as a function of industry capacity levels. The reduced form payoff functions then allow us to determine the Nash equilibria in capacities. In a duopoly model with constant marginal and average cost of production (up to the capacity constraint) and linear demand we find that all equilibria—except the ones in which firms mimic the static Cournot-Nash equilibrium—involve
excess capacity. The sole purpose of this excess capacity is to punish deviations from the collusive scheme, were these to occur. Furthermore, all equilibria involve capacities at or above their static Cournot-Nash equivalent. We find that the type and number of equilibria in this model depend upon two critical parameters—the cost of capacity and the discount rate. All equilibria are symmetric and are characterized by the level of capacity firms possess, the price they charge and the output they produce.

There are basically three types of equilibria: (a) equilibria in which firms carry considerable excess capacity and charge the monopoly price (henceforth referred to as unconstrained collusive equilibria (UCE)), (b) equilibria in which excess capacity is not sufficient to support the monopoly price but is large enough to support some price above the level that would be charged in the static Nash equilibrium (constrained collusive equilibria (CCE)), and (c) equilibria in which firms carry no excess capacity. In the latter type of equilibrium the price and output levels in each period coincide with their static Cournot-Nash equivalents. We refer to those equilibria as non-collusive equilibria (NCE). Whenever UCE and NCE exist they are unique in the sense that for a given unit cost of capacity and a given interest rate there exists a unique capacity level that supports that type of equilibrium. In contrast, there is generally a continuum of CCE. That is, there exists an interval of equilibrium capacity choices that lead to CCE. In addition, two or three different types of equilibria may coexist for the same parameter values. If, however, attention is restricted to the set of equilibria which are not Pareto dominated, then uniqueness obtains for almost all parameter values.

Exogenous changes in the cost of capacity and the interest rate affect the collusive price level. When capital is relatively cheap firms find it
optimal to carry a great deal of excess capacity in order to deter cheating. In this case, an UCE exists. This is also true if the interest rate is low since even minor threats of retaliation would then deter players from cheating, so that the monopoly price is sustainable. In fact, when capacity is cheap or the interest rate low, the UCE is the unique equilibrium of our two-stage game. Increasing the cost of capacity or the interest rate creates equilibria of the CCE and NCE variety. In order to perform meaningful comparative statics, we restrict our analysis to the set of equilibria that are Pareto undominated (see Section 4). An increase in the cost of capacity decreases a firm’s willingness to expand, while an increase in the interest rate leads it to discount the future more heavily. Thus, increasing either parameter lowers the degree of collusion that can be sustained in equilibrium (in moving us from an UCE to a CCE). As the cost of capacity or the interest rate rise further, the level of collusion will continue to fall until the equilibrium becomes noncollusive. We thus find that decreases in the level of collusion are always accompanied by decreases in the amount of excess capacity carried by the industry.

The paper divides into five sections. In Section 1 we compare our results with the theoretical and empirical literature on the relationship between excess capacity and collusion. In Section 2 we present the model and define equilibria. In Section 3, we solve for the equilibrium in the price subgame (with fixed capacities), and in Section 4 we discuss the equilibrium of the full game. We offer some concluding remarks in Section 5.

2. Excess Capacity and Collusion

In our model increases in the level of collusion (due to changes in an exogenous parameter such as the interest rate) are always accompanied by increases in the levels of industry capacity and excess capacity. This is
somewhat surprising since one of the well-known tenets of traditional oligopoly theory holds that greater levels of excess capacity weaken collusive agreements. It is argued that firms which carry a great deal of excess capacity have a strong incentive to cheat because they can capture a large share of the market by undercutting the collusive price. Firms with little or no excess capacity have no incentive to undercut since it is technologically infeasible (or extremely costly) for them to increase production. Thus, as the amount of excess capacity grows any collusive agreement is weakened.6 This argument, however, is incomplete: it ignores the effect of excess capacity on the ability of firms to retaliate when cheating occurs. After all, when the level of excess capacity is substantial the threat of retaliation looms large in the eyes of a potential cheater since firms can (and will) easily dump a large amount of output on the market to punish any chiseler. Similarly, what the level of excess capacity is relatively small a cheater need not worry very much about retaliation since the industry cannot cheaply expand production by any significant amount. Our results indicate that the retaliation effect tends to dominate the traditional chiseling effect and that excess capacity plays a prominent role in supporting collusive agreements.

The empirical evidence on the relationship between excess capacity and collusion is weak.7 At best one can say that there exist conflicting views and that most of the evidence cited in support of the traditional wisdom derives from case studies. The rayon, cement and heavy electrical equipment industries are examples of industries in which collusive agreements broke down in the presence of high levels of excess capacity.8 In each of these instances, however, excess capacity rose because a sudden reduction in demand made it difficult for firms to earn profits even at the collusive price.
Producers began to cut prices in the hope of surviving the recession by driving others out of the market. We believe that it was the decline in demand which led to overcapacity and price wars and that any conclusion that excess capacity resulted in the dissolution of a collusive agreement is unwarranted.

As an example of an industry in which behavior is consistent with the predictions of our model we point to the steel industry in the 1950s and early 1960s. In this industry capacity utilization was rarely above 85% and often went below 75%. Yet, prices remained high during this period in spite of changes in demand. It is also well-known that members of OPEC carried high levels of excess capacity during the time period that the cartel was strongest. Perhaps the most convincing piece of evidence in favor of our model concerns the United States' primary aluminum ingot market in the mid-1950s and 1960s. For this industry, Rosenbaum (1985) presents evidence that the price-cost margin was positively and significantly related to industry excess capacity (as a percentage of total capacity).

2. The Model

Consider a market shielded from entry in which two firms produce a homogeneous product and engage in the following two-stage infinite horizon noncooperative game: in stage one (at time zero) each player simultaneously and independently purchases and installs capacity at a cost of $c$ per unit. Capacity is infinitely lived, does not depreciate and can only be bought at time zero. In stage two (time periods one and beyond) firms compete in prices and produce output to order. We assume that no plant can be pushed beyond its capacity limits. Capacity thus serves as a proxy for the scale of production by placing an upper bound on any firm's output level. Throughout this paper we assume that the industry cannot collude in capacity even though it may be
colluding in price.

It is well-known that in static or finite horizon models in which the component games have a unique Nash equilibrium, collusive outcomes cannot emerge in equilibrium as the result of a noncooperative game played by profit maximizing firms. The basic insight of the literature on repeated games is that if a market situation is repeated infinitely, the industry may settle at a collusive price even if firms are not explicitly colluding. Thus, in order to ensure that collusive outcomes may arise, we assume that the price game is repeated infinitely. In addition, since we are interested in the relationship between excess capacity and the degree of collusion, we assume that tacit collusion is the norm and that firms charge the maximum price sustainable in any collusive agreement.

The types of strategies that support collusion in supernormal games were sketched briefly in the introduction. In this paper we restrict attention to collusive agreements enforced by "grim trigger strategies." These strategies specify that firms remain at the collusive point unless someone cheats. If at any time anyone is detected cheating, players revert to the static Nash equilibrium and remain there forever. Firms will cheat if and only if their immediate gains from cheating dominate the capitalized value of losses due to retaliation. We assume that when the industry chooses a collusive price vector it is aware of the problems inherent in maintaining a collusive agreement. Thus, it always chooses the price vector which maximizes the "cartel welfare function" subject to the constraint that no cheating is ever induced.

To summarize and formalize the model presented thus far let $x_i^0$ denote the profits earned by firm $i$ at the cartel point $(p_1, p_2)$; $x_i^C$ the profits earned by firm $i$ when cheating optimally against $(p_1, p_2)$; $x_i^P$ the profits earned by
firm $i$ in the static Nash equilibrium, and $r$ the interest rate. The net gains from cheating are given by:

$$p_{i} = (\pi_{i}^{C} - \pi_{i}^{E}) = \frac{1}{r}(s_{i}^{C} - s_{i}^{N})$$

(In (1) and below, we suppress the argument $p_{1}, p_{2}$, and $r$, of $\pi_{i}^{C}$ and $\pi_{i}^{N}$.)

Firm $i$ cheats if $Z_{i} > 0$. Let $\Omega$ denote the set of prices that can be supported in a collusive agreement:

$$\Omega = \{(p_{1}, p_{2}) : Z_{1} < 0 \text{ and } Z_{2} < 0\}$$

Finally, if we let $F(s_{1}^{C}, s_{2}^{C})$ denote the cartel welfare function (with $F_{1} > 0$, $F_{2} > 0$) then the optimal sustainable price vector is given by the solution to the following maximization problem:

$$\max_{(p_{1}, p_{2})} F(s_{1}^{C}, s_{2}^{C}), \text{ subject to } (p_{1}, p_{2}) \in \Omega$$

The solution to (3) depends on the capacities chosen by firms in the first stage of the game. Let $\pi_{1}^{C}(K_{1}, K_{2})$ and $\pi_{2}^{C}(K_{1}, K_{2})$ represent the cartel profits evaluated at the price vector that solves (3) and let $p_{1}^{C}(K_{1}, K_{2})$ and $p_{2}^{C}(K_{1}, K_{2})$ denote those prices. We are now in a position to define equilibrium in the two-stage game (for a fixed value of the interest rate $r$):

Definition: $(K_{1}^{*}, K_{2}^{*}, p_{1}^{*}, p_{2}^{*})$ is an equilibrium of the two-stage game if:

(a) $(p_{1}^{*}, p_{2}^{*}) = (p_{1}^{C}(K_{1}^{*}, K_{2}^{*}), p_{2}^{C}(K_{1}^{*}, K_{2}^{*}))$, and

(b) $\pi_{1}^{C}(K_{1}^{*}, K_{2}^{*}) - cK_{1} > \pi_{1}^{C}(K_{1}, K_{2}^{*}) - cK_{1}$, for all $K_{1}$
\[ n(x_1, x_2) - x_1^* + x_2^* - x_2^* - x_2^* = 0, \text{ for all } K_2 \]

(a) simply states that \( p_1^* \) and \( p_2^* \) solve (3) given \( K_1^* \) and \( K_2^* \). The conditions in (b) guarantee that \((K_1^*, K_2^*)\) constitutes a Nash equilibrium in capacities.

3. Equilibrium in the Price Subgames

In this section we compute the price vector that solves (3), as a function of the capacities in the industry. Let \( D(p) \) denote the market demand curve and \( P(x) \) its inverse. \( P(0) \), the choke price, is assumed finite. In addition, we assume that \( P(x) \) is strictly positive on some bounded interval \([0, \tilde{x}]\), on which it is twice continuously differentiable and strictly increasing (for \( x > \tilde{x}, P(x) \equiv 0 \)). Moreover, we assume that the revenue function \( xP(x) \) is single peaked, attaining a unique maximum at \( x_m \) and that this function is strictly concave on \([0, x_m]\). Each firm can produce output at zero cost as long as capacity is not exceeded. These assumptions imply the existence of a unique pure-strategy Cournot-Nash equilibrium and allow us to characterize, in Theorems 1, the (possibly mixed strategy) Bertrand-Nash equilibria which serve as a threat point for the collusive agreements.

In price-setting games firms may choose to charge different prices. If they do, we assume that customers first buy from the cheapest supplier. When the lowest priced supplier cannot satisfy all demand at that price, some customers will be left for the remaining firm. How much this firm will actually sell depends upon the pool of customers that remains to be served.

We make the following simplifying assumption: the low priced firm serves the consumers with the highest reservation prices. Thus, if \( p_i < p_j \) firm \( j \) faces a contingent demand of:
\( q(p) = \max(0, D(p) - K) \)

and earns the following profits

\[ \tau_j(p_1, p_2 | K_1, K_2) = p_j \min(\max(0, D(p_j) - K_j), K_j) \]

Firm i's profits are given by

\[ \tau_i(p_1, p_2 | K_1, K_2) = p_i \min(D(p_i), K_i) \]

When \( p_1 = p_2 \), firms share the market in some appropriate fashion (to be made precise below).

In order to solve program (i), we must calculate \( \pi^c, \pi^o \), and \( \pi^N \) for both firms. This task is simplified by Theorem 2, which proves that in any collusive agreement, firms must charge identical prices. First, we must introduce some additional notation. Let \( B(x) = \max[p(D(p) - x)] \), and let \( v_i(K_1, K_2) \) denote firm i's minimax payoff. That is,

\[ v_i(K_1, K_2) = \inf_{p_1} \sup_{p_2} \tau_i(p_1, p_2 | K_1, K_2) \]

Finally, define \( R(x) = \max \{ xP(x) + x \} \) so that \( R(x) \) is the Cournot best reply function. Some elementary calculations yield

\[ v_i(K_1, K_2) = \begin{cases} 
B(K_i) & \text{if } K_1 > R(K_1) \\
K_1P(K_1 - K_i) & \text{if } K_1 < R(K_1)
\end{cases} \]

From Kreps and Scheinkman (1985), we can now extract the following theorem which relates expected profits in the static-Nash equilibrium to the
minmax payoffs (observe that firms may use mixed strategies in equilibrium).

Theorem: For each pair \((K_1, K_2)\) with \(K_1 < K_2\), there exists a unique pair of static-Nash equilibrium payoffs \(N(K_1, K_2)\) satisfying:

(a) If \(K_2 < R(K_1)\), \(N(K_1, K_2) = K_1 + K_2 = v_1(K_1, K_2)\)

(b) If \(K_2 > R(K_1)\) and \(K_1 < D(0)\),

\[ N(K_1, K_2) = v_2(K_1, K_2) \]

(c) If \(K_1 > D(0)\), \(N(K_1, K_2) = v_1(K_1, K_2) = 0\).

Proof: The only part of this theorem not covered by Kreps and Scheinkman is \(N(K_1, K_2) = v_1(K_1, K_2)\) in (b). We prove this assertion in two steps. First, assume \(K_1 < R(K_2)\). By charging \(s(K_2) \equiv P(K_2 + R(K_2))\), a feasible action in equilibrium, firm one earns at least:

\[ v_1(K_1, K_2) = \min (K_1, D(M(K_2))) - K_2 = v_1(K_1, K_2). \]

Thus \(N(K_1, K_2) = v_1(K_1, K_2)\). Now turn to the case in which \(K_1 < R(K_2)\). We then have \(v_1(K_1, K_2) = K_1 P(K_1, K_2)\) whereas \(v_1(K_1, K_2) = K_1 P(K_1, K_2) = v_1(K_1, K_2)\). Here we used the fact that \(p\), the lowest point in the support of the mixed strategy equilibrium distribution, must exceed \(P(K_1, K_2)\). If this were not the case, profits would be increasing over the interval \([P(K_1, K_2)]\) as each firm would be selling at capacity independently of whether the other firm underpriced it or not. 

We are now ready to establish the following result.

Corollary: Let \(p_1^0\) denote the price charged by firm 1 in a collusive agreement. Then if \(p_2^0 \neq p_2^0\) it follows that either \(N_1 < N_1\) or \(N_2 < N_2\).
Proof: If \( p_1^c \neq p_2^c \), say \( p_1^c < p_2^c \), then firm two earns at most his minmax profits:

\[
\pi_2^c = \pi_2(p_1^c, p_2^c | K_1, K_2) \leq \sup_{p_2} \pi_2(p_1^c, p_2 | K_1, K_2) = \sup_{p_2 \in \mathcal{P}} (0, p_2 | K_1, K_2) = \nu_2(K_1, K_2)
\]

Theorem 1 implies \( \nu_2(K_1, K_2) < \nu_2^N(K_1, K_2) \), yielding the desired result. A similar argument can be applied for the case \( p_2^c < p_1^c \), completing the proof. \( \square \)

We can now state:

**Theorem 2**: If \((p_1, p_2) \in \Omega\), then \( p_1 = p_2 \).

Proof: Suppose to the contrary that \((p_1, p_2) \in \Omega\) satisfies \( p_1 < p_2 \). From the Corollary to Theorem 1 this implies that either \( \pi_1^c < \pi_1^N \) or \( \pi_1^c < \pi_2^N \). We can immediately rule out the case \( \pi_1^c < \pi_1^N \), since \( p_1 < p_2 \) implies \( \pi_1^{ch} > \pi_1^N \) (the low priced firm can gain from cheating by raising its price slightly), contradicting \( Z_1 < 0 \).

Assume then that \( \pi_1^c < \pi_2^N \). Since \( Z_1 \) is nonpositive by assumption, we must have \( \pi_1^{ch} = \pi_1^N \). Now if \( \pi_1^c = \pi_1^N \), we will reach our desired contradiction since (1) \( \pi_1^{ch} > \pi_1^N \) would imply \( Z_1 > 0 \), and (ii) \( \pi_1^{ch} = \pi_1^N \) would imply that capacities are such that we are in the pure strategy region of Theorem 1, i.e., \( K_1 \in R(K_2) \) and \( K_2 \in R(K_1) \), and that \( p_1 = p_2 = P(K_1 + K_2) \), i.e., that both firms are selling at capacity. Thus, \( \pi_1^c > \pi_2^N \).

To complete the proof let \( p \) denote the lowest price in the support of the Bertrand-Nash equilibrium distributions implicit in Theorem 1. Kreps and Scheinkman show that

\[
\pi_1^N = p \min(D(p), K_1) \quad \text{and} \quad \pi_1^N = p \min(D(p), K_1).
\]
Since \( x_i^N > x_i^N \), it must be that \( p_i > p \). Firm \( j \) can cheat on the agreement by charging a price \( \hat{p} \) such that \( p < \hat{p} < p_i \). This yields:

\[
\hat{q}_j = p \min(D(p),K_j) > \hat{p} \min(D(\hat{p}),K_j) = \hat{q}_j^N
\]

contradicting \( \hat{q}_j = q_j^N < q_j^N \). \( \square \)

Our consumer allocation rule (see equation (4) above) did not specify how market shares are determined when firms charge identical prices. The most natural assumption is that consumers sort in such a way that each firm's sales volume is proportional to the size of its plant. However, this sharing rule loses its intuitive appeal when plants become too large. For example, when each firm has a plant large enough to serve the entire market, it seems natural to assume that sales become independent of capacities. Also, in case (b) of Theorem 1 Nash profits and prices are independent of \( K_2 \), and thus there is no real sense in which further increasing \( K_2 \) makes firm 2 any larger. In order to adequately deal with these problems, we suggest a slightly different sharing rule:

\[
\delta_i = \frac{\min(K_i,1)}{\min(K_i,1) + \min(K_j,1)}
\]

where \( \delta_i \) denotes firm \( i \)'s market share. This sharing rule has two nice properties. First, sales are proportional to capacity when capacity matters. There is a significant body of evidence that suggests such a rule in setting output quotas (see Brander and Harris (1984) and Osborne and Pitchik (1983)). Second, when a firm becomes large enough to supply the entire market, market shares become independent of that firm's capacity.\(^{13}\)
The use of sharing rule (5) and the fact that both firms must charge the same price at any point in $Q$ greatly simplifies the characterization of the solution to program (3). First of all, observe that for any $p$ in $Q$ collusive profits are given by

$$\pi^c(K_1, K_2, p) = p \min(S_1(p), K_1)$$

Both profits functions are increasing over the interval $[0, p^m]$ and decreasing over the interval $[p^m, \tilde{p}(0)]$, where $p^m = \max\{P(x^m), P(K_1 + K_2)\}$ is the price that a monopolist with capacity $K_1 + K_2$ and no cost of production would charge if capacity costs were sunk. Because these functions are single-peaked and reach their maximus at the same price $p^a$, the solution to program (3) is independent of $H(\cdot, \cdot)$. Moreover, since optimal cheating is accomplished by undercutting the collusive price by an arbitrarily small amount, cheating profits are given by

$$\pi^c_p(K_1, K_2, p) = \sup_{y < p} \{ \min(D(y), K_1) \} = p \min(D(p), K_1)$$

(5), (7) and Theorem 1 provide the necessary information to describe $Q$, the set of sustainable prices, for any given value of $K_1$, $K_2$ and $r$. If $Q$ is empty, we assume that firms resort to randomization, and revert to the static Nash equilibrium described in Theorem 1. If $p^m \in Q$, then $p^m$ solves (3). This is because both $\pi^c_1$ and $\pi^c_2$ attain their maximus at $p^c = p^m$. Finally, if $Q$ is nonempty and $p^m \notin Q$, $Q$ will consist of an interval to the left of $p^m$ (when $p^m$ is not sustainable, then no price above $p^m$ is sustainable either). The solution to program (3) then coincides with the right endpoint of this interval. In order to calculate the maximal sustainable price as a function
of $K_1, K_2$ and $r$ we need an analytic expression for $\xi^N_i$, the static Bertrand-Nash equilibrium profits. Theorem 1 provides this information, except in case (b) where only a lower bound on $\xi^N_i$ is given. In general, an analytic expression for $\xi^N_i$ is not available. Furthermore, even if such an expression were available, determining the maximal sustainable price would be difficult, as the equations $Z_k = 0$ are highly nonlinear. For the remainder of the paper, we will confine attention to the analytically tractable case of linear demand, which can be written as

$$ (8) \quad \xi(p) = 1 - g $$

after a suitable choice of units. The static Bertrand-Nash equilibrium profits for the linear demand case were derived in Davidson and Denecker (1983), Kreps and Scheinkman (1983), and Osborne and Pitchik (1986). Their results are collected in Theorem 3, which is the analogue of Theorem 1 for the linear demand case (note that when the equilibrium occurs in mixed strategies, the equilibrium distribution functions are not given, since they are not needed here. The interested reader is referred to any of the above papers.)

**Theorem 3:** For each pair $(K_1, K_2)$ with $K_1 < K_2$, the static pricing game with capacity constraints has a unique static Nash equilibrium:

(a) If $K_1 > 1$ the equilibrium is in pure strategies, both firms charge $p = 0$ and earn zero profits.

(b) If $K_2 < (1/2)(1 - K_1)$ the equilibrium is in pure strategies, both firms charge $p = 1 - K_1 - K_2$ and profits are given by $\xi^N_i = K_i(1 - K_1 - K_2)$ for $i = 1, 2$.

(c) If $(1/2)(1 - K_1) < K_2 < (1/2)[1 + \sqrt{K_1(2 - K_1)}]$, the equilibrium is in
mixed strategies, and profits are given by 
\[ \pi_1^N = \frac{K_1}{4K_2}(1 - K_1)^2 \]
(d) if \( K_2 > (1/2)[1 + \sqrt{K_1(2 - K_1)}] \) and \( K < 1 \) the equilibrium is in mixed strategies, profits are given by 
\[ \pi_2^N = (1/4)(1 - K_1)^2 \]
and 
\[ \pi_1^N = \frac{K_1}{2}[1 - \sqrt{K_1(2 - K_1)}]. \]

Some important features of the static Bertrand-Masch equilibrium are worth noting:

1. Profits are continuous in \( K_1 \) and \( K_2 \).
2. The solution approaches the monopoly solution for firm 2 as \( K_1 \) approaches zero and \( K_2 \) remains constant.
3. If we set \( K_1 = K_2 = k \) and let \( K \) vary, then at one endpoint of the mixed strategy region the Cournot solution appears, while at the other endpoint the Bertrand solution emerges.
4. When \( K_2 \) becomes sufficiently large (cases a and d) profits become independent of \( K_2 \).
5. In case b firms are selling at capacity and thus cannot increase sales by lowering prices.

Theorem 3 calculates the maximum sustainable price as a function of \( K_1 \), \( K_2 \) and \( r \). The theorem is more easily interpreted when the following properties are kept in mind:

1. The equilibrium price and profit functions take on different functional forms in different parts of the parameter space.
2. In all four cases there exists a critical interest rate \( r \) such that if \( r < r \) the monopoly price (calculated on the assumption that all capital costs are sunk) is sustainable. Intuitively, when the
interest rate is low firms do not discount the future heavily and
the threat of retaliation is sufficient to keep them from chiseling
on the agreement.

3. In all four cases there exists a critical interest rate \( \bar{r} \) such that if
\( r > \bar{r} \) then either no price above the static Bertrand-Nash level is
sustainable\(^{14}\) or \( \Delta \) is empty. When the interest rate is high firms
discount the future heavily. The immediate gains from cheating then
donate the capitalized value of the losses due to retaliation.

4. For \( \bar{r} < r < \bar{r} \) there exists a price \( p^*(K_1, K_2, r) \) that describes the
maximum sustainable price in any collusive agreement. \( p^* \) is a
decreasing function of \( r \). Thus, as the interest rate rises it
becomes more difficult to support any collusive agreement.

5. The level of collusion that can be sustained is a decreasing
function of the difference between \( K_2 \) and \( K_1 \). In other words, when
the industry becomes more symmetric higher prices can be
supported. The driving force behind this result is as follows: as
\( K_2 \) rises firm two's share of collusive profits rise and firm one's
drop. It then becomes more difficult to keep firm one from
cheating.

In each case the equilibrium is completely characterized by the
boundaries of the parameter region, the values of \( r, \bar{r} \) and \( p^*(K_1, K_2, r) \). To
facilitate the statement of the theorem, we introduce the following notation:

\[
\begin{align*}
\alpha &= 1 - \sqrt{K_1(2 - K_1)}, \\
\gamma &= 2(K_1 + \min(K_2, 1)) - 1 \\
\delta &= \frac{1}{1 + r(1 + r - r(K_1 + K_2))}, \\
\phi &= 2(1 + r - r(K_1 + \min(K_2, 1)))
\end{align*}
\]
\[ \delta = \frac{1}{K_2(1+r)^2(1-K_1)^2(K_1 + K_2)}, \quad \phi = 8\pi(1+r)(K_1 + K_2) \]

**Theorem 4:** For each triple \((K_1, K_2, r)\) with \(K_1 < K_2\) and for every \(F(\cdot, \cdot)\) satisfying the property \(F_1, F_2 > 0\) there is a unique solution to program (3) with sharing rule (5), given by:

\[ p = \begin{cases} 1/2 & \text{if } r < \bar{r} \\ p^c(K_1, K_2, r) & \text{if } \bar{r} \leq r < \bar{r} \end{cases} \]

and when \(r > \bar{r}\) the equilibrium is as in Theorem 3. Moreover,

(a) if \(K_1 > 1\) then \(r = \bar{r} = 1\)

(b) if \(K_1 < \frac{\bar{r}}{2}(1 - K_1)\) then

- if \(K_2 > \frac{1}{2} - K_1\)
  \[ \bar{r} = \frac{z}{1 - K_1 - K_2}; \quad p^c(K_1, K_2, r) = \frac{z + K_2}{1 + r} \]
- if \(K_2 < \frac{1}{2} - K_1\) the static Nash equilibrium price is equal to the monopoly price. That is, \(\bar{r} = \bar{r} = 0\).

(c) if \(\frac{1}{2}(1 - K_1) < K_2 < \frac{1}{2}(1 + \sqrt{K_1(2 - K_1)}\) then

\[ \bar{r} = \begin{cases} \frac{K_1}{K_2} & \text{if } K_1 > \frac{1}{2} \\ \frac{1}{K_2}[K_2 - (1 - K_1)^2(K_1 + K_2)] & \text{if } K_1 < \frac{1}{2} \end{cases} \]

\[ \bar{r} = \frac{K_1}{K_2} \]
\[ p^c(K_1, K_2, r) = \frac{1}{2} \{ 0 + \sqrt{\delta^2 - 8} \} \]

(d) if \( K_2 > \frac{1}{2} \{ 1 + \sqrt{K_1(2 - K_1)} \} \) and \( K_1 < 1 \) then

\[ r = \begin{cases} 0, (z + 1)(1 - z)/z - 1) & \text{if } K_1 \leq \frac{1}{2} \\ \max \{ 0, K_1(1 - z(1 + z))/\min(K_2, 1) \} & \text{if } K_1 > \frac{1}{2} \end{cases} \]

\[ \tilde{r} = \max \{ 0, \frac{1 - \alpha - \left[ a^2 + 2a(K_1 + \min(K_2, 1) - 1) \right]^{1/2}}{K_1 + \min(K_2, 1) - (1 - \alpha) + \left[ a^2 + 2a(K_1 + \min(K_2, 1) - 1) \right]^{1/2}} \} \]

\[ p^c(K_1, K_2, r) = \frac{1}{4(1 + r)} [ 4 + \sqrt{8^2 - 8} ] \]

Theorem 4 and equations (5) and (6) can be used to obtain the equilibrium profits in the price subgame (as a function of \( K_1 \) and \( K_2 \)). These reduced form payoff functions allow us to compute the subgame perfect Nash equilibria of the full two-stage game.

4. Equilibrium in the Two-Stage Game

How much capacity should firm 1 purchase at time zero when it expects firm \( j \) to install capacity \( K_j^* \)? To answer this question, observe that for any value of \( K_j \) its profits are given by:

\[ v_1(K_1 | K_j) = \inf_{K_2} v(K_1, K_2) - cK_1 - p^c \min(S_1(1 - p^c), K_1) - cK_1 \]

where \( p^c \) as in Theorem 4. Let \( \overline{v}_1(K_j) \) denote the value of \( K_j \) that maximizes \( v_1(K_1 | K_j) \). \( \overline{K}_j(K_j) \) is called firm 1's reaction function or best reply function. Firm \( j \)'s reaction function is obtained in a similar way. \( (K_1^*, K_2^*) \) then constitute a Nash equilibrium in capacities if and only if \( K_j(K_j^*) = K_j^* \).
A typical reaction function for firm 2, when the cost of capacity and the interest rate are low, is shown in Figure 1. The reaction function lies completely above the 45° line except at the point (1, 1). Thus, (1, 1) is the unique equilibrium for this case. From Theorem 4 we know that when the interest rate is low (i.e., \( r < 1 \)) both firms will charge the monopoly price.

The reaction function in Figure 1 has an unusual shape; its slope changes sign at point a and it includes concave, convex and linear segments. These properties are most pronounced when the cost of capacity and the interest rate are low and thus, we will focus mostly on this case. When capacity is cheap and firm ignores, for the most part, the cost of capacity. It thus chooses a capacity level that will support collusion and provide it with a large share of collusive profits. Let us examine firm two's best reply to a value such as \( K_1 \) in Figure 1. Since \( r \) is low the monopoly price will be sustainable at \( K_2 = K_1 \). As \( K_2 \) rises above \( K_1 \), firm two's profits improve (because its share of collusive profits increases). At the same time, however, firm one's share falls, thereby making that firm more inclined to cheat. Eventually firm one becomes indifferent between cheating and remaining at the collusive point.

Further increases in \( K_2 \) must then be accompanied by a reduction in the collusive price. From this point on, increases in \( K_2 \) increase firm two's profits because its share of collusive profits rises yet, at the same time, lower its profits due to the fall in the maximum sustainable price. The optimal value of \( K_2 \) is the value that just balances these two countervailing forces. That value is denoted by \( K_2^*(K_1) \).

Suppose now that \( K_1 \) increases to \( K_1^* \). Clearly, at the point \((K_1^*, K_2(K_1^*))\) higher prices are sustainable. (Since firm one's share of collusive profits rises, its temptation to cheat declines. Thus, the collusive price can increase without inducing firm one to cheat.) This implies that the optimal
value of $K_2$ rises along with $K_1$ and explains why the reaction function is upward sloping beyond point a. It also provides an explanation for why the reaction function lies above the 45° line at all points except (1,1) (at (1,1) firm two cannot increase its share of collusive profits by increasing capacity. Moreover (1,1) is the only symmetric point with that property).

In region oa the reaction function is downward sloping. In this region $K_1$ is small and thus $K_2$ must be small if collusion is to be sustained (otherwise firm one’s share of collusive profits is so small that it cheats). Theorem 4, case (b), shows that when $K_1$ and $K_2$ are both small the static Nash and collusive equilibria coincide; each firm sells at capacity and charges the market clearing price. Thus, $K_2(K_1,K_2) = K_2(1 - K_1 - K_2)$ which reaches a maximum at $K_2 = (1/2)(1 - K_1)$. When the cost of capacity is small enough to be ignored, this is also the profit maximizing value of $K_2$. As $K_1$ increases $(1/2)(1 - K_1)$ falls, explaining the negative slope in region oa. This property is thus inherited from the static Cournot-Nash reaction functions.

Increases in the interest rate or the cost of capacity affect firm two’s reaction function in a similar manner. An increase in the interest rate reduces the capitalized value of the losses due to retaliation, and thus makes it more difficult to support collusion. The industry must then become more symmetric if collusive prices are to continue to characterize equilibrium. Thus, the optimal value of $K_2$ falls as $r$ rises (except in region ca). An increase in the cost of capacity also causes the reaction function to shift down because it becomes more costly for a firm to increase its share of collusive profits. In addition, increases in $r$ or $c$ cause the sign reversal of the slope of the reaction function to occur later (point a moves down and to the right).
Figure 1 exhibits a unique equilibrium in which both firms charge the monopoly price. However, because increases in c and/or r cause the reaction function to shift down toward the 45° line, additional equilibria will be created when these parameter values become sufficiently high. For any given \( r \) (c) there exists a critical value of \( c \) (r) denoted \( \tilde{c}(r) \) such that if \( c > \tilde{c}(r) \) (r > \( \hat{r} \)) the linear segment of the reaction function--(bc)--coincides with the 45° line, creating a continuum of additional equilibria. In this type of equilibrium, firms charge a price above the static Bertrand-Nash level but below the monopoly price. Thus, we refer to it as a constrained collusive equilibrium (CCE). All equilibria in which firms charge the monopoly price will be called unconstrained collusive equilibria (UCCE). Once \( c \) (r) reaches \( \tilde{c}(r) \), further increases in c (r) expand the set of CCE by shifting additional portions of the reaction function down to the 45° line. Reaction functions for three different values of c are depicted in Figure 2. Lower reaction functions correspond to higher costs of capacity.

Referring to the same figure, a third type of equilibrium emerges when c and/or r rise even further. This type appears whenever the reaction function's downward sloping portion reaches the 45° line before its slope changes sign. For any given \( r \) (c) there thus exists a critical value of \( c \) (r) denoted \( \hat{c}(r) \) such that if \( c < \hat{c}(r) \) (r > \( \hat{r} \)) such an equilibrium will be present. In these equilibria no price above the static Bertrand-Nash level is supportable. We will therefore refer to them as noncollusive equilibria (NCE). When a NCE exists it is unique in the sense that for any given value of \( r \) and c there exists a unique capacity level that supports an equilibrium of that type.

To summarize, three types of equilibria may occur: unconstrained collusive equilibria in which the monopoly price is charged, constrained...
collusive equilibria in which a price above the static Bertrand-Nash level but below the monopoly level is charged, and noncollusive equilibria in which no price above the static Bertrand-Nash level is supportable. Equilibria of the UCE type are characterized by high levels of capacity and excess capacity while equilibria of the NCE variety are characterized by low levels of capacity and no excess capacity. In the UCE case the excess capacity provides firms with a powerful weapon for retaliation and thus facilitates collusion. Whenever equilibria of the UCE and NCE variety exist they are unique.

Constrained collusive equilibria are characterized by smaller capacity and excess capacity levels than UCE. In addition, there is a continuum of CCE whenever they are present. Raising \( r \) or \( c \) creates more equilibria of the CCE variety when the level of \( c \) is not too high. However, for large values of \( c \) increases in \( c \) and \( r \) tend to reduce the number of CCE.

In Figure 3 the values of \( c \) and \( r \) which are consistent with equilibria of each type are shown. It is clear from the figure that more than one type of equilibrium may be present at any one time. On the other hand, equilibria can always be Pareto ranked. If we restrict our attention to the set of undominated equilibria, we are left with unique capacity levels for most values of the parameters. Figure 4 illustrates, for given values of \( c \) and \( r \), the price charged in the Pareto superior equilibrium. As is evident from this figure, increases in \( c \) or \( r \) may, ceteris paribus, reduce the level of collusion and hence of equilibrium prices. Such reductions are always accompanied by a fall in the levels of capacity and excess capacity. In other words, lower levels of excess capacity coincide with lower levels of collusion.

At this point, it is instructive to reflect on what properties (if any) of the linear demand example carry over to a more general setting. In Theorem
5 of the Appendix, it is proven that in any symmetric equilibrium of the two-stage game in which firms do not implement the static Cournot-Nash equilibrium, there is excess capacity. Moreover, it is clear that the comparative static properties of our model (the relationship between the degree of collusion and the amount of excess capacity held by an industry) will generalize. On the other hand, we have been unable to generalize the result that all equilibria must involve capacity choices no lower than the static Cournot-Nash level (taking into account the cost of capacity).

6. Conclusion

In this paper we presented a model of oligopolistic competition in which firms' capital decisions play a crucial role in determining the level of collusion that can be supported in equilibrium. We found that if firms precommit in capacity and then compete in prices the level of collusion will be positively related to the amount of excess capacity producers choose to carry in equilibrium.

In our model collusive agreements are supported by credible threats of retaliation against cheaters. Since there are immediate gains from cheating on such agreements, and since all retaliation occurs in the future, the rate at which firms discount future profits plays a critical role in determining the maximum level of collusion that can be sustained. Excess capacity also plays a critical role by limiting the damage an industry can inflict on a cheater. Thus, as the interest rate falls or the level of excess capacity grows, collusive agreements become easier to support. These are the economic forces that drive our main results.

When the interest rate is low and/or capacity is cheap firms choose a scale of operation large enough to support the monopoly price. As the interest rate or the cost of capital rise, it becomes too costly for firms to
carry enough excess capacity to support the monopoly price. Equilibrium capacities and the level of collusion thus fall. If either of these two parameters rise further, it may eventually become impossible to support any collusion at all.
Footnotes

1The component games of the ensuing supergame are just Edgeworth's price-setting games with capacity constraints. This game has been analyzed in detail by Kreps and Scheinkman (1983), Davidson and Denecker (1983), and Osborne and Pitchik (1986).

2The results of our model will approximate closely those of a model in which firms have limited flexibility in adjusting their capital levels after the initial capacity choice, and wide flexibility in adjusting their prices over time (flexibility, of course, is not an absolute concept; it is defined relative to the discount factor). For example, one might parameterize flexibility by assuming that there is an upper bound $\Delta K$ to the amount with which the capital stock can be adjusted from one period to the next, and similarly for price ($\Delta P$). If $\Delta K$ is small and $\Delta P$ is large (with respect to the discount factor), the equilibria of the resulting model will be "close" to ours (see Benoit and Krishna (1985)).

3In his description of the German cartels, Scherer (1980, pp. 370-71) writes: "In Germany during the 1920s and 1930s, shares were allocated on the basis of production capacity. Cartel members therefore raced to increase their sales quotas by building more capacity... Even when market shares are not linked formally to capacity, a cartel member's bargaining power depends upon its fighting reserves—the amount of output it can dump on the market, depressing the market, if others hold out for unacceptably high quotas."

4Our model is a synthesis of Kreps and Scheinkman (1983) and Brock and Scheinkman (1985). We extend the Brock and Scheinkman paper by allowing capacity constraints to be determined endogenously and we extend the Kreps and Scheinkman analysis by allowing the price game to be repeated infinitely so that collusive outcomes can be supported in equilibrium.

5We also assume a constant marginal cost of capacity for ease of computation; the results would be similar, however, for a more general cost function.

6See Scherer (1980), especially pages 209-211.

7See, for example, Esposito and Esposito (1974) and Mann, Meehan and Ramsay (1979).

8See Scherer (1980), pages 209-211.

9See the U.S. Congress, Senate Subcommittee on Antitrust and Monopoly Report No. 1387 (1938), pp. 45-51, 81-85.
See Scherer (1980).

See, for example, Friedman (1977) and Benoit and Krishna (1984).

Abreu (1983) has shown that in quantity-setting games firms may do better (i.e., support more collusion) when they use strategies other than grim trigger strategies. He does so by proving that an optimal punishment scheme exists (optimal in the sense that it supports the highest level of collusive profits), and that such a scheme requires firms to produce above the Cournot-Nash output level when punishing. Although Abreu's punishments seem attractive, they are quite difficult to characterize because in asymmetric games, they require non-stationary punishment paths (the two-phase punishment schemes which are easily computed for symmetric games work only if the discount factor is sufficiently high). Thus, it is highly unlikely that such strategies would ever be implemented in the real world. On the other hand, standard trigger strategies require only simple calculations and are easily understood by industry participants. It is more readily imagined that firms will use these simple punishments to support tacit agreements.

There are two other reasons why we make use of grim trigger strategies in our analysis. (i) It is easily proved that our punishments are optimal over regions (a) and (b) of Theorem 1. Thus, our analysis differs from Abreu's only over regions (c) and (d). (ii) Abreu's proofs depend on the continuity of payoff functions. Since payoff functions are discontinuous in price games it is unclear whether his results generalize.

Two papers that focus on the sharing rule and its effects on collusion are Osborne and Pitchik (1983) and Brander and Harris (1984). Both models are static and do not provide an explanation of how collusion is supported.

When $K_1$ and $K_2$ are such that the static Bertrand-Nash equilibrium occurs in mixed strategies (case c and d of Theorem 3), this statement should be interpreted as follows: firms charge a price below the monopoly price, but high enough so that each earns profits above the static Bertrand-Nash level.

Appendix

Let \( R_C(x) = \arg \max_z \{ z(P(z + x) - \frac{c}{r} - f) \} \) be the Cournot-Nash best reply function in a game where the marginal (and average) cost of output, including per period capital cost, is \( c/r \). Denote the solution to the equation \( x = R_C(x) \) by \( \tilde{x}(c) \). \( \tilde{x}(c) \) is thus the static Nash equilibrium in a Cournot game with constant marginal cost \( c/r \). Finally, let \( \hat{x}(c) = R_C(0) \), the static monopoly capacity. We may now state.\(^{15}\)

**Theorem 5:** In any symmetric equilibrium of the two-stage game involving capacity choices of \( k \) and a collusive price of \( p \), \( 2k > \tilde{x}(c) \). Moreover, if \( (k,p) \neq (\hat{x}(c), P(2\tilde{x}(c))) \), then \( D(p) < 2k \).

**Proof:** If \( 2k < \tilde{x}(c) \), the most lucrative equilibrium in the subgame \((k,k)\) yields firms profits of \( P(2k) - c/r)k \). If firm 1 expands capacity to \( R_C(k) \), it will earn at least its minmax profits in the subgame \((k, R_C(k))\), namely \( P(k + R_C(k)) - c/r)R_C(k) \), which exceeds \( P(2k) - c/r)k \). This proves \( 2k > \tilde{x}(c) \). To prove the second part of the theorem, observe that if \( k = \hat{x}(c) \) and \( p > P(2\hat{x}(c)) \) we are done. Since feasibility requires \( D(p) < 2k \), we are left with the case \( k > \hat{x}(c) \) and \( p = P(2k) \). In that case, firms earn, per period, \( P(2k) - c/r)k \) which is strictly less than \( P(k + R_C(k)) - c/r)R_C(k) \) by the definition of \( R_C(*) \). The latter payoff is the payoff a firm would receive if rather than choosing \( k \) it chose \( R_C(k) \), and if it were subsequently minmaxed in the subgame \((k, R_C(k))\). Since any collusive equilibrium in that subgame must yield that firm at least its minmax profits, it would choose to deviate to the capacity level \( R_C(k) \), contradicting the fact that \((k,p)\) was an equilibrium. \( \square \)
References


Figure 1
Figure 4

\[ p = p^n \rightarrow \text{noncollusive equilibrium (nce)} \]

\[ p = p^m = \text{unconstrained collusive equilibrium (uce)} \]