Discussion Paper No. 668

Reputation Effect in Equilibrium Search and Bargaining - A Stigma Theory of Unemployment Duration

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March 1985

(Revised October 1985)

ABSTRACT

Empirical studies of unemployment have shown that both the wage and the probability of leaving unemployment (the hazard rate) are negatively correlated with unemployment duration. Since traditional search models do not yield this result, it has been suggested that selectivity problems, due to the workers' unobserved characteristics, are responsible for this phenomenon. While selectivity alone may generate some decline in the hazard rate, it is not sufficient to explain true duration dependence or to give more insight into the relationship between the hazard rate and the wage. In this paper, unobserved characteristics produce a true duration dependence via reputation effects that weakens the workers' bargaining positions. A labor market in which workers are searching for jobs and firms are searching for workers is presented. Each worker is either a "good match" for a particular firm or a "bad match". Upon meeting a worker, the firm checks the quality of the match, and also negotiates over a wage. If the worker is a good match, his wage is determined according to the Nash bargaining solution. If the worker is a bad match, negotiations are called off. This creates a bad reputation for the worker by increasing the probability that he is a lower type worker. As a result, the worker's position in subsequent bargaining attempts with other firms worsens. It is shown that both the hazard rate and the wage decline with the length of time a worker is unemployed (for a given productivity level). In addition, the model provides closed form solutions for the wages and the equilibrium values from search for the workers and the firms.

1. Introduction

Empirical studies of unemployment have shown that both the wage and the probability of leaving unemployment (the hazard rate) are negatively correlated with unemployment duration.[1] The traditional explanation for unemployment, based upon search theory (Stigler 1961), does not yield this result. In the case of infinite horizon with a stationary distribution of wage offers (Mortensen 1970, Lippman and McCall 1976), the optimal reservation wage is fixed, and therefore, there is no duration dependence in both wage and the probability in unemployment spells. In cases of shrinking horizon (Gronau 1971) or limited budget for search, the reservation wage falls while the probability of leaving unemployment rises with time, again, counter to the evidence.

In this paper, both the wage and the hazard rate decline due to a reputation effect that occurs when the worker is failing to get a job. Failing to get a job reveals some information about the worker's unobserved characteristics via a learning process that takes place during the process of search and bargaining. The bad reputation weakens the worker's position in subsequent bargaining attempts with other firms. The paper investigates this effect under different informational and contractual structures.

The solution developed in previous studies for this problem is simple, namely, heterogeneity, or "unobserved characteristics". Different workers face different distributions of wage offers, and thus, have different reservation wages and different probabilities of leaving unemployment. Through time, workers with higher probabilities of leaving unemployment will leave first. Because of this selectivity problem, as time goes by, the sample of length of unemployment spells consists of more and more unemployed workers with lower hazard rates. The reservation wage can go either way, depending upon the specific assumption one makes. If this effect is strong enough to dominate other effects, such as the increase in the hazard rate due to a finite horizon or budget constraints, the average hazard rate of a given population will decline with the length of unemployment.

Any model that contains some heterogeneity has this selectivity problem. However, this explanation alone is not sufficient to describe the behavior of the hazard rate and the wage together. It is also unable to show whether a true duration dependence exists in these variables. For this purposes, I suggest distinguishing between two possible informational problems as follows:

1) "The researcher's problem" - the market operates under full information, i.e., the type of each individual is public information. However, the researcher cannot observe the true type of each individual.

2) Market under incomplete information - the type of each individual is not known to all agents in the market. The assumption on the knowledge of the researcher is not crucial for this case.

The first structure produces the simplest selectivity problem. Each worker's decisions can be described according to one of the traditional search models, and the observed decline in the hazard rate is the result of only selectivety problem. However, under the second structure, some duration dependence exist, because firms may learn about workers' characteristics, and use this information to their advantage.

Several studies have tried to check whether or not a true duration dependence exists. For example, Flinn and Heckman (1982) have found that "stigma", or "occurrence dependence", exists for the hazard rate.[2] When they control for exogenous differences, the hazard rate still exhibits a negative dependence in time (for the maximum likelihood specification). They also provided a simple, partial equilibrium model of stigma, based on Becker (1981). I use this idea to provide an equilibrium model with stigma of unemployed workers.

When constructing an equilibrium search model, one first has to define what happens upon a meeting between a buyer and a seller. This meeting creates a bargaining situation as, due to search costs, the combined value the two agents obtain from the transaction is bigger than the sum of values each of them obtain upon separation. With respect to the solution for this bargaining problem, the literature can be divided into two branches. The first branch consists of models in which firms set prices, while buyers have to decide whether to buy or to keep searching. This solution to the bargaining problem may give the firms "too much power" in the sense that they may capture all the gains from trade. Indeed, the first model that investigated equilibrium search (Diamond 1971), obtained the monopolistic price as the only equilibrium price. Different writers have used different approaches to overcome this problem. For example, Burdett and Judd (1982) assumed that consumers may sample more than one firm at a time, and Butters (1977) assumed that firms may advertise. Both of these assumptions reduce the bargaining power of the firms.

The other branch of literature includes models that explicitly use specific solution concepts for the bargaining problem. Diamond and Maskin (1979), and Mortensen (1978, 1982), used the Nash bargaining solution (or "split the difference") to investigate whether agents choose the optimal amount of search. Rubinstein and Wolinsky (1985) described the bargaining process as an extensive form game, and achieved a unique perfect equilibrium for the market based upon the analysis of this game.

It appears that the difference in the bargaining solution which the above models use is the result of different assumptions about market conditions. In a market for a small, inexpensive and standardized good, for which substitution is easy to find, the bargaining is over "too small" a cake to justify complicated and time consuming negotiations. However, if the good is relatively expensive, with some unique characteristics, a more complicated bargaining process may take place. The real estate market, the market for cars, and the job market are examples of the last type.

Since I deal with the job market, I use an approach similar to that of Mortensen and Diamond and Maskin. I describe a stigma theory of unemployemnt duration that is based upon an information acquisition that occurs during the process of search and bargaining. These processes may convey information in several ways.[3] For example, consider an unemployed worker who has been searching for a job for a long time. In this case, a new firm he arrives at may suspect that other firms already found him to be "bad". Therefore, the firm may not even bother to take a closer look at his file, or it may attempt to use the weak bargaining position of this worker by offering him a low wage.

This idea is modeled as follows. Workers arrive at firms and bargain over wages. Each firm does not know the true productivity of a particular worker, but it can find this out by checking (testing) the worker. Because there is some correlation among firms, if one firm finds the worker unfit to its needs, the chances that other firms may find him unfit increase. Thus, the worker's history, or at least the length of time he has been unemployed, is valuable information for the firm. Two cases are considered: a symmetric information case, in which the firms can observe the exact history of the workers, and an asymmetric information case, in which the firms can observe only the length of time a given worker has been unemployed.

In the symmetric case, I use the Nash bargaining solution to show that the bargaining position of the worker is worsening with the number of failures he has accumulated. Therefore, a worker with more failures gets a lower wage, conditional on the event that is now less likely to occur, that he will be hired. The model describes the learning process by the firm, and shows that both the hazard rate and the wage are declining with the length of unemployment.

In the case of asymmetric information, I use Myerson's (1984) solution for the Nash bargaining solution to describe the outcome of the bargaining situation. With regard to the hazard rate and the wage, this model yields results that are similar to those of the complete information case. The analysis of the incomplete information, and the introduction of a learning process about workers' abilities are the major differences between this paper and the works by Mortensen and Diamond and Maskin.

The paper is organized as follows. In section 2, a basic model of learning in a process of search and bargaining is introduced. Section 3 describes a very simple strategic model to highlight some of the results of section 2. In section 4, an asymmetric information model, in which firms can observe only the length of time each worker has been unemployed is described. Concluding remarks are given in section 5.

2. A model of bargaining and search with learning

Consider the following labor market. There are many unemployed workers and many firms with job openings. Each worker's utility is linear in money and he draws no utility from leisure. He is either a "good match" for a particular firm or a "bad match". In the case of a good match, the value of the worker to the firm is 1 dollar per period of employment, and in the case of a bad match the value is 0. For a worker, all firms are identical ex-ante (i.e., before the quality of the match has been realized) and they have linear utility in money.

The matching process between firms and workers is as follows. Each period a machine randomly and costlessly assigns workers to firms. Workers, therefore, have no cost of searching for open jobs, but can search only one firm per period. When a worker and a firm meet, they cannot observe the true value of the match. Instead, they both have some beliefs about the probability of a good match. In addition, the firm can costlessly check the true quality of the match. The check fully reveals whether or not the worker is a good match or not. If the firm does not want to check the worker or the worker does not want to be checked, they break off negotiation. In this sense, the check-up is similar to a learning process about the match quality (Jovanovic 1979) and cannot be avoided.

Each period M new workers and M new firms enter the market. Thus, each period the population of unemployed workers consists of the M workers who just entered and all the "old" workers who did not find a good match. In the same way, the population of firms is composed of the M new firms and all the "old" firms that did not find a good match. We assume that the number of "old" firms is equal to the number of "old" workers, so that the total number of workers and firms is identical each period. Note that exit from the market occurs in pairs, and only when a "good match" happens.

A match is valuable in two ways. Before the check-up takes place, it yields the value of time it takes to find another match. After a "successful" check-up, it has the extra value of a good realization. In both cases, therefore, the match may yield some "cake" to be divided between the parties. Thus, a solution concept is needed in order to select an equilibrium. Now, a solution for this kind of bargaining problem may be complicated. At each point in time, each bargainer has to decide, simultaneously, whether to continue negotiating with his current partner or not, and what to say (accept/reject his partner's offer, to give a counter offer, to do nothing, etc.). In this work we separate these decisions in the following way. First, each partner's share in the "cake" (if there is any) is determined by some rule. Given this rule, each side has to say whether he accepts it or not. If either of them refuses to sign the contract, they separate. We use, whenever it is possible, the general Nash bargaining solution to determine the allocation of the cake. The threat point is the utility each player can achieve by returning to the market. In turn, this utility level is affected by the solution concept that exists in the market. It appears, therefore, that the market conditions and the solution for the bargaining between the firm and the worker are closely related. One of the tasks in the paper is to explore this relationship.

I would like to distinguish between two kinds of contracts;

1) A pre-check contract - the bargaining is done before the check-up. A wage is determined according to the expected payoff of each player. The worker will get this wage if he is found to be a good match. If he is a bad match,

no production and payments take place, and the parties separate.

2) A post-check contract - the firm decides whether to check the worker or not before the bargaining takes place. If it checks the worker and finds him to be a good match, they bargain over the wage (if the worker wants to). If the worker is a bad match, they separate.

We describe now the solution for the bargaining game under the pre-check contract (the post-check contract is described in section 4). Let us first introduce some notation and additional properties of the system.

Let N be the set of non-negative integers and let $p: N \times N \to [a,b]$ $0<a<b<1$ be the probability that the worker is a good match given that he was checked k times and was found to be good j times.

To see how p(k,j) evolves, consider a worker who just enters the system and is found to be a bad match for some firm. Suppose that when he arrives at a new firm, he cannot hide this information. How does this fact change his position in the bargaining situation he faces now? Mortensen (1978, 1981), Diamond and Maskin (1979), and Rubinstein and Wolinsky (1985) all assume that there is no correlation between the quality of two different matches. In their models, therefore, no information is acquired during the process of searching and bargaining. On the other hand, if there is a perfect correlation between matches, no search process can take place. After one stage either the workers find jobs or they find out that they are not able to do so.

It is plausible, therefore, to assume that the correlation is somewhere in between these two extreme cases. The fact that one firm finds the worker unfit to its needs does not mean he is "bad" for another, but merely shows that his probability of being a good match is now smaller. This discussion suggests the following relations

$p(k+1,j) < p(k,j)$

$p(k,j+1) > p(k,j)$

We assume the function p is common knowledge.

It is shown in theorem 2.2 that the above relations hold in general for a learning process with sampling from a Bernoulli distribution. At this stage, however, it is useful to give the following example.

Example 1 Suppose that each worker can be one of two types (for each firm). Type "a" has productivity 1 with probability a and productivity 0 with probability 1-a, and type "b" has productivity 1 with probability b and productivity 0 with probability 1-b, where $b > a$. Let E be the event that a firm will meet a worker of type b, and let ξ be the prior probability that the firm will meet a worker of type b. We assume that firms are identical ex-ante, so that ξ is equal across firms.

Each firm updates its beliefs according to Bayes' rule, i.e.,

$$\Pr[\, E^c \mid k=0,\ j=0 \,] = 1-\xi$$

$$\Pr[\, E \mid k=0,\ j=0 \,] = \xi$$

$$\Pr[\, E \mid k=1,\ j=0 \,] = \frac{\xi(1-b)}{\xi(1-b)+(1-\xi)(1-a)}$$

and, in general, if the firm see n failures with no success, it has the following posterior

$$\Pr[\, E \mid k=n,\ j=0 \,] = \frac{\xi(1-b)^n}{\xi(1-b)^n+(1-\xi)(1-a)^n}$$

Therefore, the probability of a given worker to be "good" after a sequence of failures evolves as follows

$$p(0,0) = \xi b + (1-\xi)a$$

$$p(1,0) = \frac{\xi(1-b)}{\xi(1-b)+(1-\xi)(1-a)} b + \frac{(1-\xi)(1-a)}{\xi(1-b)+(1-\xi)(1-a)} a$$

$$p(n,0) = \frac{\xi(1-b)^n}{\xi(1-b)^n+(1-\xi)(1-a)^n} b + \frac{(1-\xi)(1-a)^n}{\xi(1-b)^n+(1-\xi)(1-a)^n} a$$

It may be seen that $p(n,0) > p(n+1,0)$ in this example.

The probability of success is determined by both the number of checks and the number of successes. However, under the pre-check contract one state variable may be eliminated. Indeed, if this contract holds, then the workers obey it. Since a worker with one more success may have higher value from search, it is not clear whether he would like to obey the contract, or some enforceability is needed. It can be seen that under the most reasonable conditions, the worker would not like to break the contract. However, to eliminate any possible complication, we make the following assumption

Assumption 1 The contract is enforceable.

Furthermore, let us assume that each new entrant enters the system with $k=0$ and $j=0$. Using these assumptions, it is not possible to find a worker with $j \geqslant 1$ who is staying in the market. Thus, it is possible to write p as depending on k alone, i.e., $p:N \to [a,b]$ where $p(k) > p(k+1)$.

For the purpose of this section, we use an additional assumption, a "decreasing learning" assumption, as follows

Assumption 2 $p(k) - p(k+1) \geqslant p(k+1) - p(k+2)$ for every k.

It is easy to see that our example satisfies this property.

In addition, let us define the following

v_f = the expected profits of a firm from being in the market, before meeting a partner.

$v_f(k)$ = The expected utility of a firm that has a partner with k checks.

$v_o(k)$ = the expected utility of a worker of type k from "search"

δ = the discount factor (common to all workers and firms), $0 < \delta < 1$.

The discounted market values, $\delta v_o(k)$ and δv_f represent the alternative values that each party can obtain upon a separation. therefore, these values represent the disagreement outcome in a Nash bargaining solution. It can be seen that the match yields a non-trivial set of individually rational outcomes (i.e., individually rational outcomes that are not the disagreement outcome only) if $1-\delta v_o(k)-\delta v_f > 0$. In this case, the set of individually rational and Pareto optimal outcomes can be written as the set S,

$S = \{w \mid 1-\delta v_f \geq w \geq \delta v_o(k)\}$. The Nash solution selects one outcome out of this set.

In general the set of individually rational and Pareto optimal outcomes may contain only the disagreement outcome. For example, consider the case where check-up involves some positive costs, say $\varepsilon > 0$. Then, $S \neq \phi$ iff $p(1 - \delta v_f - \delta v_o(k)) - \varepsilon \geq 0$. But, for sufficiently small p's this inequality may not hold. As is shown later, in our case this situation cannot happen. If $v_o(k)$ and v_f have unique solutions, then $0 \leq v_o(k) + v_f \leq 1$ because the the "cake" cannot exceed 1 and cannot fall below 0 (this is true for any k). Therefore, the Nash solution can be applied to every bargaining situation with every type of workers, and it yields a wage that solves the following problem (see Roth (1979) page 15)

$$2.1)\quad \max_w \{[p(1-w) + (1-p)\delta v_f] - \delta v_f\}^{\gamma_1} \{[pw + (1-p)\delta v_o(k+1)] - \delta v_o(k)\}^{\gamma_2}$$

where $p(1-w) + (1-p)\delta v_f$ represents the firm's utility from the contract, $pw + (1-p)\delta v_o(k+1)$ represents the worker's utility, and γ_1, γ_2 are, respectively, the weights that are assigned to the firms and to the workers, $\gamma_1 > 0$, $\gamma_2 > 0$.

This problem is equivalent to the following

$$2.2)\quad \max_{w}\ (1-w-\delta v_f)^{\gamma_1}\ \{pw + (1-p)\delta v_o(k+1) - \delta v_o(k)\}^{\gamma_2}$$

The first-order condition for a solution is

$$2.3)\quad \frac{p(1-w-\delta v_f)}{\gamma} = pw + (1-p)\delta v_o(k+1) - \delta v_o(k)$$

where $\gamma = \frac{\gamma_1}{\gamma_2}$ is the real bargaining power of the firm.

By virtue of equation 2.3 we can solve for the wage

$$2.4)\quad w(k) = \frac{1-\delta v_f}{\gamma+1} + \frac{\gamma\delta}{(\gamma+1)p}\,[v_o(k) - (1-p)v_o(k+1)]$$

Equation 2.4 gives w as a function of v_f, v_o and p. In turn, v_f and v_o depend on w. One of the main tasks in this paper is to explore these relations. Let us assume, for the moment, that v_f has a unique solution. Then, $v_o(k)$ will also have a unique solution. To see this, note that given an equilibrium wage in the market, w(k), $v_o(k)$ can be written as follows

$$2.5)\quad v_o(k) = p(k)\,w(k) + (1-p(k))\delta v_o(k+1)$$

Using (2.4) we may obtain

$$2.6)\ v_o(k) = \frac{1}{\gamma+1}\,[p(k)(1-\delta v_f) + \gamma\delta v_o(k) + (1-p(k))\delta v_o(k+1)]$$

Rearranging terms and solving for $v_o(k)$ to obtain

$$2.7)\ v_o(k) = \frac{1}{\gamma+1-\gamma\delta}\,[p(k)(1-\delta v_f) + (1-p(k))\delta v_o(k+1)]$$

Equation 2.7 is a difference equation. It is obvious that given a pre-determined value $v_o(n) = c$ for some $n\in\{k,k+1,......\}$ (where $c\in R$ is a prescribed constant), equation 2.7 has a unique solution, $\Psi^n(k,c)$, that may be obtained by solving it recursively. Our theory does provide us with such a condition. The solution for (2.7) must be bounded, since the value of the worker for the firm cannot exceed 1 while his alternative is at least 0. Therefore, we are interested in the following result

<u>Proposition 2.1</u> There exists a unique bounded solution, $\Psi(k)$, for equation 7 given by the following

$$2.8)\ \Psi(k) = \frac{\beta}{\alpha}\sum_{i=k}^{\infty} p(i)(\delta/\alpha)^{i-k}\Big\{\frac{1}{1-p(k-1)}\prod_{j=k}^{i}[1-p(j-1)]\Big\}$$

where $\alpha = \gamma+1-\gamma\delta$, $\beta = 1-\delta v_f$.

<u>Proof</u> We first show that (2.7) has a unique solution. Let U be the complete space of bounded functions in the sup norm mapping the non-negative integers into the real. For any $u\in U$ define

$$(Tu)(k) = \frac{1}{\gamma+1-\gamma\delta}\,[p(k)(1-\delta\, v_f) + (1-p(k))\delta u(k+1)]$$

It can be seen that T maps U into itself. Indeed, if $\|u\|_\infty \leqslant M$, then $\|Tu\|_\infty \leqslant M+1-\delta v_f$. Also, T is monotone (i.e., if $u \geqslant w$ then $Tu \geqslant Tw$) and, for any constant r, $T(u+r) \leqslant Tu + \delta r$. This follows from the fact that $\frac{(1-p(k))\delta}{\gamma+1-\gamma\delta} \leqslant (1-p(k))\delta < \delta$. It follows, therefore, that T is a contraction mapping in the sup norm on U of modulus δ and T has a unique fixed point, $\Psi(\cdot)$ which solve (2.7).

Now, we want to show that equation 2.8 is indeed a solution for equation 2.7. We show first that (2.8) is well defined. Let

$$\Psi^m(k) = \frac{\beta}{\alpha}\sum_{i=k}^{m} p(i)(\delta/\alpha)^{i-k}\prod_{j=k-1}^{i}[1-p(j)], \quad m\in\left(k,k+1,\ldots..\right).$$

Now, $\Psi^m(k)$ is monotonically increasing in m and is bounded. Indeed

$$0 \leqslant \Psi^m(k) < \Psi(k) < \frac{\beta}{\alpha}\sum_{i=k}^{\infty}\left(\prod_{j=k-1}^{i}[1-p(j)]\right) < \frac{\beta}{\alpha}\sum_{i=k}^{\infty}\left(\prod_{j=k-1}^{i}[1-a]\right) = \frac{\beta}{\alpha a} < \infty$$

Therefore, $\lim_{m\to\infty}\Psi^m(k)$ exists, and equals $\Psi(k)$ as is defined by (2.8). It is easy to check that (2.8) indeed satisfies (2.7), so it is a solution. By the previous part of the proposition we know that it is the unique bounded solution. Q.E.D.

Proposition 1 means that any other solution for equation 2.7, (if it exists), is not bounded. In particular, if Ψ^* is such a solution, then $\Psi^*(k) \xrightarrow[k\to\infty]{} \infty$.[4] Moreover, if $\Psi^n(k,c)$ is the solution for equation 10 when $v_o(n) = c$, then $\lim_{n\to\infty}\Psi^n(k,c) = \Psi(k)$ independent of c.[5] It follows, therefore, that (2.8) is the appropriate solution for $v_o(k)$ to use.

It can be seen that (2.8) satisfies the property $v_o(k) \leq 1-\delta v_f$ which is required for the problem to be well defined (the set S to be non-void). This follows from the facts that $\alpha > 1$, $\delta/\alpha < 1$, and $\sum_{l=k}^{\infty} p(k) \prod_{j=k}^{i} [1-p(j-1)] = 1$ (this is the probability that the worker will find a good match).

Equation 2.8 in proposition 1 gives the value of $v_o(k)$ when v_f is given. In turn, given a sequence of market values for each type of workers, v_f has a unique solution. To show this, note that the utility of the firm given it met a worker of type k, and given an equilibrium wage w(k), can be written as follows

$$2.9) \quad v_f(k) = p(k)[1-w^b(k)] + [1-p(k)]\delta v_f$$

To see how v_f is calculated, note that in general, firms expect to get different payoffs from different types of workers. Since the number of workers of type k in the market changes stochastically (due to the stochastic nature of the outcome of the match), firms may face, in different period, different probabilities to meet each type of workers. This may complicate the analysis in several ways. First, if firms cannot observe the true realization of probabilities, and if they have an access only to their own history of meeting with workers, each firm will have a different v_f according to its own history. Another possibility is that all firms know the true realization of probabilities. In this case, given a realization of probabilities and given the transition law, firms can evaluate their expected value from staying in the market without a specific partner (which is now equal across firms).

At this point, however, I would like to concentrate on the effect of the learning process, and therefore, I simplify as follows.

Assumption 3 Firms cannot observe the true realization in the market, and cannot learn from their history, i.e., they "forget".

Further suppose that the system is at a steady state distribution (the conditions for which are stated later), and all firms know this distribution. Let θ_k be the probability that a worker with k checks will be drawn at random from the population of workers, and let $\theta = (\theta_0, \theta_1, \ldots\ldots)$. Then, the firm's expected utility from being in the market without a specific partner is

$$2.10)\quad v_f = E_\theta\left\{ \sum_{k=0}^{\infty} \theta_k v_f(k) \right\}$$

This means that v_f is independent of k or of time trends. Accordingly, v_o is independent of time. We can now show the following

Proposition 2.2 Let $\{v_o(k)\}_{k=0}^{\infty}$, $v_o(k) \in [0,1]$ for every k, be given. Then, v_f has a unique solution.

Proof Substituting (2.4) into (2.9) we may obtain

$$2.11)\quad v_f(k) = \frac{1}{\gamma+1}\left(p(k)(1-\delta v_f) + \gamma\delta[v_o(k) - (1-p(k))v_o(k+1)]\right).$$

Let $A(k) = \frac{\gamma\delta}{\gamma+1}[v_o(k) - (1-p(k))v_o(k+1)]$,

Since $v_o(k) \in [0,1]$ for every k, then $|A(k)| < 1$ for every k. Substitute (2.11) into (2.10) yields

$$2.12)\quad v_f = E_\theta\Big(\sum_{k=0}^{\infty}\theta_k\Big[\frac{p(k)(1-\delta v_f)}{\gamma+1} + A(k)\Big]\Big) =$$
$$= (1-\delta v_f)E_\theta\sum_{k=0}^{\infty}\frac{\theta_k p(k)}{\gamma+1} + E_\theta\sum_{k=0}^{\infty}\theta_k A(k).$$

Now, $\frac{1}{\gamma+\delta}\sum_{k=0}^{\infty}\theta_k p(k) < \sum_{k=0}^{\infty}\theta_k$, and $\sum_{k=0}^{\infty}\theta_k|A(k)| < \sum_{k=0}^{\infty}\theta_k = 1$.

Therefore, the expectation in (12) is finite, and the unique solution for v_f (after rearranging (12)) can be written as follows

$$2.13)\quad v_f = \frac{1}{1+\delta}E_\theta\Big(\sum_{k=0}^{\infty}\theta_k\Big[\frac{p(k)}{\gamma+1} + A(k)\Big]\Big).\quad \text{Q.E.D.}$$

For any value of v_f we have unique values for $v_o(k)$, and for every sequence $\{v_o(k)\}$ we have a unique solution for v_f. Therefore, we want to show the following

<u>Theorem 2.1</u> There exists unique equilibrium values $\left(v_f^*, \{v_o^*(k)\}_{k=0}^{\infty}\right)$.

<u>Proof</u> Equations (2.8) and (2.10) form a system of equations that determined v_f^* and $\{v_o^*(k)\}$. From equation (2.10) we obtained equation (2.13). Now, using equation (2.8) we may write

$A(k) = (1-\delta v_f)[B(k)-(1-p(k))B(k+1)]$

where

$$B(k) = \frac{\gamma\delta}{(\gamma+1)\alpha}\Big[\sum_{i=k}^{\infty}p(i)(\delta/\alpha)^{i-k}\Big\{\frac{1}{1-p(k-1)}\prod_{j=k}^{i}[1-p(j-1)]\Big\}.$$

It follows that $0 < B(k) < 1$ for every k. Now, substituting $B(k)$ into (13) yields

$$v_f = \frac{1}{1+\delta}E_\Theta\Big\{\sum_{k=0}^{\infty}\frac{p(k)\theta_k}{\gamma+1}\Big\} + \frac{1-\delta v_f}{1+\delta}E_\Theta\Big\{\sum_{k=0}^{\infty}\theta_k[B(k)-(1-p(k))B(k+1)]\Big\}$$

Let $D(k) = E_\Theta \left\{ \sum_{k=0}^{\infty} \theta_k [B(k) - (1-p(k))B(k+1)] \right\}$

Since $|B(k)| < 1$ then $D(k)$ is finite, and the unique solution for v_f is

$$v_f^* = \frac{1}{1+\delta+D(k)} \left[E_\Theta \left(\sum_{k=0}^{\infty} \frac{\theta_k p(k)}{\gamma+1} + D(k) \right) \right]$$

By proposition 1 we now know that given this solution, $v_0(k)$ has a unique solution for every $k \in \{0,1,2,.....\}$, (which can be achieved by substituting v_f^* into (8)), Q.E.D.

Since each worker is checked every period, k can be identified with t (the number of periods the worker has been in the process). Using this, we can state the main results with regard to the hazard rate and the wage. It has been shown, in example 1, that the hazard rate declines with k. This property, although not general, holds in many cases, since the expected value of the posterior distribution tends to fall with "bad" realizations as compared to the expected value of the prior. Since the hazard rate in this model is the expected value of the probabilities of success (with respect to their distribution), it should decline with the number of failures. Although this result is not new, we state it for sampling from the Bernoulli distribution as follows

Theorem 2.2 Suppose that X is a random sample from a Bernoulli distribution with unknown value of the parameter W. Suppose also that the prior distribution of W, H(W), has a non-degenerate support. Let $P(X=1|H)$ represents the probability of success. Then, 2.14) $P[X=1|H(W)] < P[X=1|H(W|X'=1)]$.

<u>Proof</u> The probability of success is given as follows

2.15) $$P(X=1|H) = E_H(W) = \int_0^1 WdH(W).$$

Using Bayes' rule, we may obtain

$$dH(W|X'=1) = WdH(W)/ \int_0^1 WdH(W)$$

Therefore,

2.16) $$P[X=1|H(W|X'=1)] = \int_0^1 W^2dH(W) \ / \int_0^1 WdH(W).$$

From (15) and (16) it follows that (14) holds if

2.17) $$E(W^2) = \int_0^1 W^2dH(W) > [\int_0^1 WdH(W)]^2 = [E(W)]^2$$

Clearly, (17) holds if the variance is non-zero. But this is true because the support is non-degenerate. Q.E.D.

If a success is defined as failing to get a job, then inequality 14 means a decreasing hazard rate. In this case, an increase in the probability of success means an increase in the probability of staying unemployed, i.e., a decrease in the hazard rate.

Note that the special features of this model make the hazard rate independent of the worker's decision. The decline in the hazard rate is, therefore, a purely statistical property. This stands in contrast with most search models, in which the hazard rate is determined by the reservation wage chosen by the worker. Jensen and Vishwanath (1985) describe a partial equilibrium model with declining hazard rate based upon this approach. Any analysis of this kind, as well as in any generalization of our model to include the above relationship, is complicated by the fact that the worker may take into account the effects of being unemployed. In this case, the hazard rate may increase as a result of a decrease in the reservation wage. This level of complication is beyond the scope of the paper.

The second empirical implication, the decline in the reservation wage, is given by the following

Theorem 2.3 Under assumption 2 ("decreasing learning"), w is strictly decreasing in k (for $0 < \gamma < \infty$) .

Proof From (2.4) we have

$$2.18)\quad w = \frac{1}{\gamma+1} \left[1-\delta v_f + \frac{\gamma\delta}{p(k)}[v_o(k) - (1-p(k))v_o(k+1)]\right]$$

By rearranging equation 2.7 we can obtain

$$2.19)\quad (1-p(k))v_o(k+1) = v_o(k) \left[\frac{\gamma+1-\gamma\delta}{\delta} \right] - \frac{1}{\delta} (1-\delta v_f)p(k)$$

Substituting (2.19) into (2.18) yields

$$w(k) = \frac{1-\delta v_f}{\gamma + 1} + \frac{\gamma\delta}{(\gamma+1)p(k)}[v_o(k) - \frac{\gamma+1-\gamma\delta}{\delta} v_o(k) + \frac{1}{\delta}(1-\delta v_f)p(k)] =$$

$$= 1 - \delta v_f + \gamma(\delta-1) \frac{v_o(k)}{p(k)}$$

From (2.8) we have

$$\frac{v_o(k)}{p(k)} = \frac{\beta}{\alpha}\left[1 + \frac{\delta(1-p(k))p(k+1)}{\alpha p(k)} + \frac{\delta^2}{\alpha^2}\frac{(1-p(k))(1-p(k+1))p(k+2)}{p(k)} + \ldots\right]$$

It can be seen that assumption 2 implies, in our model, that $\frac{p(k+1)}{p(k)} < \frac{p(k+2)}{p(k+1)}$, and, also, $\frac{p(k+i)}{p(k)} < \frac{p(k+i+1)}{p(k+1)}$ for every i. Therefore, term by term, $v_0(k+1)/p(k+1)$ is bigger than $v_0(k)/p(k)$. It thus follows that $v_0(k)/p(k)$ is strictly increasing in k (strictly decreasing in p(k)). Since $\gamma(\delta-1) < 0$, then $\gamma(\delta-1)v_0(k)/p(k)$ is strictly decreasing in k, and, therefore, w(k) is strictly decreasing in k, Q.E.D.

Theorem 2.3 gives the intuitive result that the worker's wage falls as the probability of his being good falls. It is important to notice that the fall in wage does not reflect the fact that the worker's value to the firm falls. It reflects the fact that ex-ante (before the check), a worker with a smaller number of checks is considered to be "better" than a worker with a higher number of checks. Therefore, the decrease in wage reflects a worsening in the bargaining position of the worker as the number of checks he has been through increases.

As example 1 demonstrates, our results may be generated by exogenous unobserved characteristics. Theorem 2.3 shows that this exogenous heterogeneity, combined with a special informational structure, creates an endogenous effect on the wage. Thus, workers with the same exogenous characteristics may get different wages because of different "luck" (i.e., because they have different history). As a result of this interaction, it is necessary to investigate both the hazard rate and the wage when testing which model is correct.

What looks surprising in the above result is the fact that we have to use the assumption that $p(k)-p(k+1) \geqslant p(k+1)-p(k+2)$. If the threat point is decreasing in k, why is it not sufficient to have any decreasing p? The reason for this dependence is the kind of contract we are using. In the pre-checked agreement, the value each partner may get in the market after the check is part of the "cake". Since $(1-p(k))\delta v_o(k+1)$ is decreasing in k, the cake shrinks as k increases. This hurts the worker more than it hurts the firm. Therefore, if $v_o(k+1)$ decreases "too fast", the wage may goes up in order to compensate the worker.

To illustrate this point, consider the following example:

Example 2 Let $p(0)=p(1)=0.5$ and $p(k)=0$ for all $k \geqslant 2$ (the argument will go through for $p(2)$ positive but close enough to 0, and for $p(0) > p(1)$ but both are sufficiently bigger than $p(2)$). This implies that $v_o(2) = 0$, so that

$$w(1) = 0.5(1-\delta v_f) + \delta v_o(1)$$

Also,

$$v_o(1) = 0.5w(1) = 0.25(1-\delta v_f) + 0.5\delta v_o(1) \implies v_o(1) = \frac{1-\delta v_f}{2(2-\delta)}.$$

This enables us to solve for w(1) as follows

$$w(1) = (1-\delta v_f)(0.5 + \frac{1}{2}(2-\delta)).$$

Doing the same for w(0) and $v_o(0)$ yields

$$v_o(0) = 0.5w(0) + 0.5\delta v_o(1) = \frac{2-\delta^2}{2(2-\delta)}(1-\delta v_f)$$

$$w(0) = 0.5(1-\delta v_f) + \frac{\delta(1-\delta v_f)}{2(2-\delta)} \left[\frac{2-\delta^2}{2-\delta} - \frac{\delta}{2} \right].$$

Now, to see that w(0) may be smaller than w(1) delete one from the other to obtain

$$w(0) - w(1) = \frac{\delta(1-\delta v_f)}{2(2-\delta)} \left[\frac{2-\delta^2}{2-\delta} - 1.5\delta \right]$$

this expression is negative whenever $\delta^2 - 6\delta + 4 < 0$, i.e., for $\delta > 0.77$ (δ is smaller than 1).

3. A simple strategic model where firms have all the bargaining power.

In section 2 we show that the wage is strictly decreasing whenever neither side has all the bargaining power, i.e., whenever $0 < \gamma < \infty$. It can be seen from equation 2.4 that $\gamma \to \infty \implies w(k) \to 0$ for every k. In this situation, firms have all the bargaining power, and they take advantage of it.

To illustrate this extreme case, and to show some of the complications in dealing with reputation effects in the market, consider the following strategic model. the model is a modification of Diamond (1971) and can be regarded as a simplification of Rubinstein (1982). Suppose firms uses a "take it or leave it" strategy, i.e., each firm, upon meeting a worker, offers a wage, and the worker may only respond by saying "accept" ("a") or "do not accept" ("na"). The firm's strategy is, therefore, a mapping from the workers' types into wages, i.e., $g\colon N \to [0,1]$. Each worker's strategy is a mapping from wages to {"accept", "do not accept"}, i.e., $f_k\colon [0,1] \to \{a, na\}$. Such a game, between a firm and a worker of type k, is described in figure 1. In this figure, the firm offers a wage, $w \in [0,1]$, and the worker reacts by saying "a" or "na". If the worker says "na", they both get their market value. If he says "a", they get (w, 1-w) with probability p(k) and their post-check market value with probability 1-p(k). We refer to this game as a "small game".

We consider only symmetric strategies in the sense that identical agents have identical strategies. Therefore, the strategies in the market can be defined as a collection of strategies of all the small games, i.e., the firms' market strategy is a function g as above, and the workers' market strategy is a sequence f, $f = \{f_k\}_{k=0}^{\infty}$. Given g and f, the market values of the workers and the firms can be calculated as follows:

$$3.1)\quad v_o(k,g,f) = \begin{cases} p(k)w'+[1-p(k)]\delta v_o(k+1,g,f) & \text{if } g(k)=w' \text{ and } f_k(w')=a \\ \delta v_o(k,g,f) & \text{if } g(k)=w' \text{ and } f_k(w')=na \end{cases}$$

$$3.2)\quad v_f(k,g,f) = \begin{cases} p(k)(1-w')+[1-p(k)]\delta v_f(g,f) & \text{if } g(k)=w' \text{ and } f_k(w')=a \\ \delta v_f(g,f) & \text{if } g(k)=w' \text{ and } f_k(w')=na \end{cases}$$

The set of strategies (g*,f*) form an equilibrium (Nash) if

3.3) For every g and k, $v_f(k,g^*,f^*) \geq v_f(k,g,f^*)$.

3.4) For every f and k, $v_o(k,g^*,f^*) \geq v_o(k,g^*,f)$.

Conditions 3.3 and 3.4 mean that the strategies derived from g* and f* form a Nash equilibrium in every small game. However, it can be seen that any wage may be supported by Nash equilibrium strategies. Therefore, in order to eliminate "unreasonable" equilibria, we use the notion of Subgame Perfect equilibrium (Selten 1975). The strategies (g*,f*) form a Subgame Perfect equilibrium if they describe a Subgame Perfect equilibrium in every small game. This occurs whenever strategies (g*,f*) satisfy condition 3.3 together with the following

3.5) For every k, if $f_k(w') = a$ then

$p(k)w' + [1-p(k)]\delta v_o(k+1,g^*,f^*) \geq \delta v_o(k,g^*,f^*)$, and if $f_k(w') = na$, then the inequality is reversed.

Condition 5 means that the worker reacts optimally from every subgame (i.e., after any wage offer by the firm).

One may think that this model yields a decreasing wage as follows. A worker with k failures is willing to accept any wage offer that equals or is greater than $\delta v_o(k)$. Therefore, the firm's optimal strategy is to offer a wage that satisfies $w(k) = \delta v_o(k)$. Since workers with lower number of failures have higher market value, their wage will be higher. This argument fails here, as the following proposition shows

Proposition 3.1 The unique set of market strategies which induces a Subgame Perfect equilibrium strategies in all of the small games that are described by figure 1 is the set $(*) = \{g^*, f^*\}$ given by

$g^*(k) = 0$ for each k

$f^*_k(w) = a$ for each k and for each $w \geq 0$.[6]

Proof Consider any small game with a worker of type k. Given any wage $w \in [0,1]$, the worker's best response is

$$3.6) \quad f_k(w) = \begin{cases} a & \text{if } pw + (1-p)\delta v_o(k+1) \geq \delta v_o(k) \\ na & \text{otherwise} \end{cases}$$

Given this response, the firm's best strategy is to offer the lowest wage possible. Let $\bar{w}(k)$ be the wage that satisfies

3.7) $p\bar{w} + (1-p)\delta v_o(k+1) = \delta v_o(k)$

then, as w is bounded below by 0, the lowest wage the firm may offer is $w^*(k) = \text{Max}\ \{0, \bar{w}(k)\}$

Therefore, the best strategy for the firm is

3.8) $g(k) = w^*$

Now, (3.6) and (3.8) describe strategies that are Subgame Perfect equilibrium in any small game with any k. It remains, therefore, to show that (3.6) and (3.8) coincide with (*).

Given (3.8), we may have two cases. If $w^*(k) > 0$, then, from (3.7), $v_o(k) = \delta v_o(k) \Longrightarrow v_o(k) = 0$. If $w^*(k) = 0$, then, $v_o(k) = [1-p(k)]\delta v_o(k+1)$. The unique bounded solution for this difference equation is $v_o(k) = 0$. By the claim below, we also know that $v_o(k+1) = 0$. This implies $w^*(k) = 0$ for every k and therefore, the strategy in (6) now becomes $f_k(w) = a$ if $w \geqslant 0$, i.e., (6) and (8) coincide with (*), Q.E.D.

We still have to show the following

<u>Claim</u> If, for every $k \in N$ $g(k) = w^*$, then $v_o(k+1) = 0$.

<u>Proof</u> Suppose not. Then, there exists $k \in N$ for which $v_o(k+1) > 0$ (note that $v_o(k+1)$ cannot be negative). This, combined with the fact that $v_o(k) = 0$ implies that $\bar{w}(k) < 0$, i.e., $w^*(k) = 0$. In either case, therefore, $w^*(k) = 0$ and thus, it is zero for every k. But, having offered only zero implies that the value from search is zero (in any situation) and hence, $v_o(k+1) = 0$, a contradiction, Q.E.D

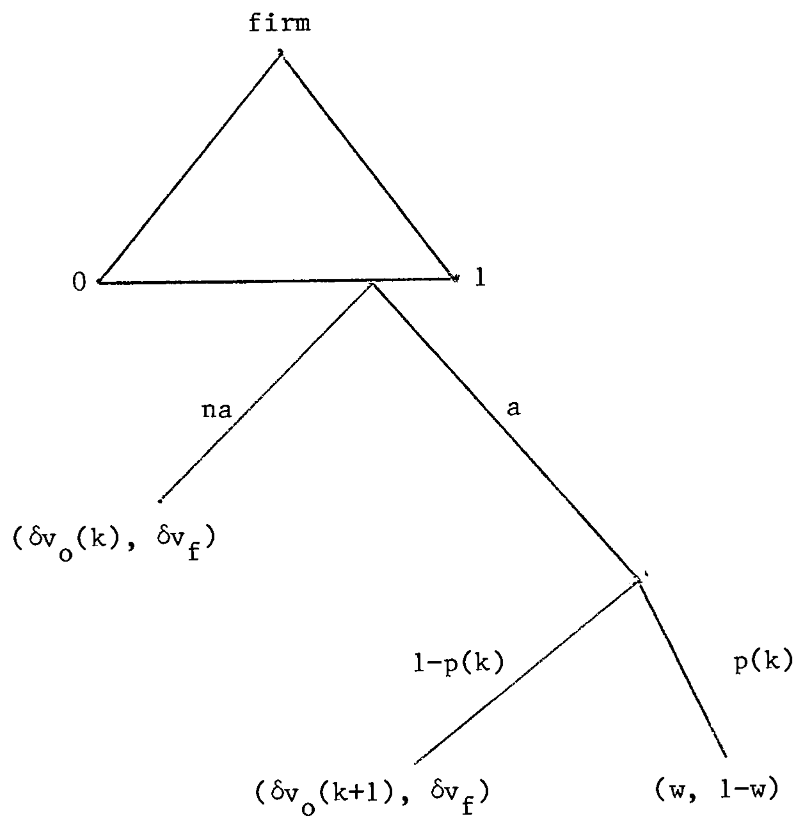
This result, a wage of zero to every worker, is similar to Diamond's

(1971) result. In his model, the only equilibrium is the monopolistic price, since allowing only firms to submit offers give them all the bargaining power. It is interesting that this result does not depend on the probability of success. When firms have all the bargaining power, workers' positions cannot be any worse after a check, since their part of the cake is always zero.

The analysis for the case where $\gamma = 0$ (workers have all the bargaining power) is very similar. By looking at equation (2.4) alone, we can only see that $w(k) = 1-\delta v_f$. However, by letting the workers to have the possibility to offer the wage in the above game, we obtain $1-\delta v_f = 1$. Indeed, if we only change the role of each player, it is obvious that proposition 3.1 will yield w=1 for each k. If all workers are offering w=1, firms will accept this wage and their value from being in the market will be 0.

These extreme cases raise the following issue. If workers get nothing out of search, why would they participate in the game? In particular, if we assume that search involves some direct costs, workers will not search whenever their bargaining position is "too bad". Since it is hard to imagine such a market equilibrium, workers probably get more than the costs of search out of the bargaining game. However, in the case of workers with bad reputation, such equilibrium may exist. Their bargaining situation may be so bad that it will not be worthwhile searching. Consequently, bad reputation may generate another effect, namely, a drop out of the labor market. Although it is not difficult to describe such an effect in our framework, the discussion of this issue is deferred to future research.

Figure 1



4. A post check contract when the outcome of the check is private information

We turn now to investigate the post-check contract. Under this contract, the firm and the worker bargain over a wage after the result of the check is known. This bargaining problem is different from the pre-check problem in two ways. First, the worker's productivity is now known (with certainty); second, the alternative the worker is facing is his market value after a successful test, as opposed to staying with the same history in the pre-check contract.

The fact that no agreement is made prior to the check raises the interesting problem of information structure, namely, whether or not the result of the test is private information. This problem is redundant in the pre-check contract, because of the enforceability assumption. By this assumption, if a worker leaves a firm, it is because he failed the test. Therefore, knowing that a worker has been t periods in the market (and, thus, has been checked t times), is equivalent to knowing that he has failed t times.

In the post-check contract it is not possible to make such an assumption because no deal is made before the result of the test is realized. Therefore, the disagreement outcome (for the worker) is to continue searching after a success (failure) occurs. Three kinds of informational structures are possible. First, we may assume that the result of the test is public knowledge. In this framework, information remains symmetric (provided that it is symmetric in the first period), but it contains two state variables, the number of periods in the market and the number of successes, as opposed to just one state in the pre-check contract. The second possibility is to assume that firms can observe nothing upon meeting workers. This makes our model uninteresting.

We will examine the remaining possibility, which assumes that firms can observe the length of unemployment spells, but not the results of the tests, or how many tests the worker has taken. In this set-up, if a firm meets a worker that has been t period in the market, it knows that his type, (k,j), belong to the set V, $V = \{(k,j) \mid k \in \{0,1,\ldots\ldots,t\},\ j \in \{0,1,\ldots\ldots,k\}\}$, where (t,j) represents a worker that was checked k times, and gathered j successes (note that the number of possible types in period t is equal $\sum_{i=0}^{t} (i+1)$). Therefore, a worker is characterized by a triple (t,k,j). However, it follows from proposition A-1 in the appendix that the move of not taking the test is not profitable in any way. Hence, we can disregard it and treat the worker's type as a tuple (t,j).

Since workers know their own history while firms know only parts of it, we are dealing with a case of bargaining under incomplete information. However, this asymmetry does not exist at the time the worker enters the system. At this time, he is of type (0,0), and this fact is public information. Therefore, we are dealing with a situation that involves both bargaining under incomplete information and bargaining under complete information. This, combined with the fact that we use the Nash bargaining solution for the symmetric case, makes the Myerson (1984) solution for the bargaining problem very attractive. Myerson (1984) proposed an axiomatic solution for the bargaining problem under incomplete information that is a generalization of the Nash solution and coincides with the Nash solution in case where information is symmetric. In the following, we use this solution.

Let us describe the game between a worker of type (t,j) and a firm when they meet. Assume that the worker's market value, $v_o(t,j)$, and the firm's market value, Ev_f, are given. Also, let the expected payoff for the firm from reaching an agreement with a worker of type (t,j) be $x(t,j)$, and the expected payoff for a worker of type (t,j) be $z(t,j)$ (where x and z are given by the Myerson solution). The firm does not have to make any decision prior to the test, since the test involves no costs and no commitments. However, after the result of the test is known, the firm has to decide whether it wants to negotiate with the worker (a), or does not (na). At this stage, the firm knows only the fact that the worker has been t+1 periods in the market (i.e., has opportunities to take t+1 tests). It does not know the actual type of the worker, but it has some posterior probability distribution over the set of possible types. Clearly, the firm does not want to negotiate with a worker that fails the test. Therefore, to simplify notations, we assume that the worker and the firm must separate in such an event.

The worker has to decide whether to take the test (y) or not (ny). In addition, given that the firm agrees to negotiate with him, a successful worker has to decide whether he wants to stay and negotiate (s), or does not (ns). This decision may not be redundant, since his position in the market is better now than what it was prior to the test.

To summarize, the set of strategies for the firm in any such a game is $\{a,na\}$, and the set of strategies for the worker is $\{(y,s),(y,ns),(n,s),(n,ns)\}$. The labor market consists of all possible games of this kind. Therefore, to describe equilibrium or the market values for the workers and the firms, it is necessary to describe what happens in every possible game. It is useful to write the strategies in the following manner. Let $g:N \to \{a,na\}$ be a list of strategies that the firm has in all possible games (with all possible types it may face), and let G be the space of all such functions. For example, $g(t) = a$ means that the firm agrees to negotiate with a worker of type t upon his success. Here we are using the assumption that firms are identical, so that they behave the same in any situation. In addition, let $f\colon N\times N \to \{(y,s),(y,ns),(n,s),(n,ns)\}$ be a list of strategies that any type of workers (t,j) is choosing in "his" game, i.e., $f(t,j) = (y,s)$ means that the strategy of a worker of type (t,j) is (y,s). Let F represent the space of all such functions. The worker type is his type prior to the test.

We are now in a position to write the market values of the firms and the workers. These values can be described as follows

4.1) $v_o[(t,j),z,f,g] =$

$$\begin{cases} p(t,j)z(t+1,j+1) + [1-p(t,j)]\delta v_o(t+1,j) & \text{if } f(t,j)=(y,s),\ g(t+1)=a \\ p(t,j)\delta v_o(t+1,j+1) + [1-p(t,j)]\delta v_o(t+1,j) & \text{if } f(t,j)=(y,ns),\ g(t+1)=a \\ & \text{or if } f(t,j)=(y,s),\ g(t+1)=na \\ \delta v_o[(t+1,j),z,x,f,g] & \text{if } f(t,j)=n \end{cases}$$

$$4.2)\ v_f[(t,j),x,g,f] = \begin{cases} x(t,j) & \text{if } f(t,j)=(y,s),\ g(t+1)=a,\ \text{"good"} \\ \delta Ev_f & \text{otherwise} \end{cases}$$

where Ev_f is calculated as follows

$$Ev_f = E_\theta\Big\{ \sum_{t=0}^{\infty} \theta_t \sum_{i=0}^{t} q(i|t)\big(p(t,i)v_f[(t,i),x,g,f] + [1-p(t,i)]\delta Ev_f\big)\Big\}$$

$$4.3)\quad Ev_f = \lambda E_\theta\{ \sum_{t=0}^{\infty} \theta_t \sum_{i=0}^{t} q(i|t)\big(p(t,i)v_f[(t,i),x,g,f]\big)\}$$

where $\lambda = [\ 1 - \delta E_\theta\{ \sum_{t=0}^{\infty} \theta_t \sum_{i=0}^{t} q(i|t)(1-p(t,i))\}\ ]^{-1}$

and $q(i|t) = q(i|t,f,g)$ is the probability that a worker of type t has i successes (this probability is calculated, given the players strategies, according to Bayes rule).

The term $v_f[(t,j),x,g,f]$, the expected profit for the firm from meeting a worker of type (t,j), has no relevence for the firm's decision problem, because the firm cannot observe j. Instead, the expected profit from meeting a worker of type t is relevent, and it is given by the following

$$4.4)\quad \bar{v}_f[t,x,g,f] = \sum_{i=0}^{t} q(i|t)\big(p(t,i)v_f[(t,i),x,g,f] + [1-p(t,i)]\delta Ev_f\big)$$

An equilibrium is a set of strategies $\{g^*, f^*\}$ such that

a) For every $g \in G$, and for every t

$\bar{v}_f[t,x,g^*,f^*] \geqslant \bar{v}_f[t,x,g,f^*]$

b) For every (t,j) and for every $f \in F$

$v_o[(t,j),z,g^*,f^*] \geqslant v_o[(t,j),z,g^*,f]$

Note that q is not introduced explicitly into the definition of the equilibrium because we are not defining the formation of beliefs in off-equilibrium paths. In any particular game, the firm cannot observe any deviation of the worker from his equilibrium strategies in previous games. Also, the other firms will not be able to observe him deviating in this game. Therefore, we are able to calculate q according to Bayes rule in any situation.

It should be emphasized, however, that q depends on the solution concept for the bargaining problem also. As is shown later, the solution for the bargaining problem may admit some probability of separation (for some types of workers), as part of the revealing mechanism that is used. Therefore, whenever we specify equilibrium strategies, we have to specify a q which is evaluated according to Bayes rule, taking into account the probability of separation that is given by the solution for the bargaining problem.

In the complete information case, the only reasonable equilibrium is for both sides to agree to negotiate if the worker is found good. This kind of equilibrium looks plausible under incomplete information too, because the value of the match cannot be higher anywhere else, and the solution for the bargaining problem is individually rational. This means that the "cake" exists, and under the proposed mechanism for allocation, both sides agree to cooperate. Therefore, we proceed as follows. First, we show that strategies (*) that are defined by

$$(g^*, f^*) = \begin{cases} g(t) = a & \text{for every } t \in N \\ f(t,j) = (y,s) & \text{for every } (t,j) \in N \times N \end{cases}$$

where x and z are given by the Myerson solution, and q is given by

$$q(i \mid t, f^*, g^*) = \begin{cases} 1 & \text{if } i=0 \\ 0 & \text{otherwise} \end{cases} \qquad t=0,1,2,\ldots\ldots$$

are, indeed, equilibrium strategies. Second, we describe the properties of this equilibrium (equilibrium "(*)"), and show that it is the only "reasonable" equilibrium.

Consider a worker who just enters the market and meets with a firm. His type is (0,0), and this information is public knowledge. If he succeeds in the test, his type will be (1,1), and this fact (as well as the result of the test) is known both to the firm and to the worker himself. Therefore, given market values for every k and j, they are facing a symmetric bargaining problem, and the wage is determined according to the Nash solution, i.e., it is the solution for the following problem (we use in this chapter the Nash symmetric solution, i.e., $\gamma_1 = \gamma_2 = 1$)

$$4.5) \quad \max_{w} \; [1 - w - \delta Ev_f] \, [w - \delta v_o(1,1)]$$

which is

$$4.6) \quad w(0,0) = \frac{1}{2} [1 - \delta Ev_f + \delta v_o(1,1)]$$

Under equilibrium (*), this is the solution for the wage, and both sides agree to this. Therefore, a worker who has been 1 period in the market will be recognized as a worker of type (1,0) (i.e., the firm assign probability 1 to the worker to be of this type). Hence, the problem remains that of a bargaining under complete information, and the wage is given by

$$4.7) \quad w(1,0) = \frac{1}{2} [1 - \delta Ev_f + \delta v_o(2,1)]$$

Under equilibrium (*), this situation holds in general. When the firm sees a worker of type t, it makes the conjucture that he has failed all his tests because, otherwise, he would not have been in the market. Therefore, the firm belives that a worker of type t is, in fact, a worker of type (t,0), and this expectation is fulfilled by the workers' behavior. Hence, the equilibrium wage for a worker of type t is given as follows

4.8) $w(t,0) = \frac{1}{2}\,[1 - \delta Ev_f + \delta v_o(t+1,1)]$

To show that strategies (*) indeed form an equilibrium, and to investigate some of its properties, we have to describe the properties of the term $v_o(t+1,1)$, the market value of a successful worker. This value is the result of a separation after a success, which is an off-equilibrium path. Therefore, this worker is facing a bargaining situation under incomplete information in which the firm he is bargain with assign 0 probability to him being of type (t+1,1) (or any other type different from (t+1,0)).

For example, let us describe the simplest of such cases, the bargaining situation that a worker of type (1,1) is facing upon meeting a firm. After a success, his type is (2,2), but the firm believes that his type is (2,1). Can he signal his type by any way? Note that he cannot reveal himself by a mechanism that depends upon the result of the test, since no negotiation is allowed prior to the test. However, his market value, $\delta v_o(2,2)$, is higher than the market value of the "bad" type, $\delta v_o(2,1)$, because, even under the same wage (which is feasible for him), the worker has a higher probability of success. Therefore, the (2,2) type can reveal himself via a mechanism that puts some positive probability on the disagreement outcome (which is worth more for him than for the (2,1) type). In particular, let (π,w) be the probability of breaking-off of negotiations and a wage for a worker who is claiming to be of (2,2) type. If (π,w) satisfies

$$\pi\delta v_o(2,2) + (1-\pi)w \geq w(1,0)$$
$$\pi\delta v_o(2,1) + (1-\pi)w \leq w(1,0)$$

then the worker reveals himself, and get a higher expected payoff. Such a pair, (π,w), is possible to find since $\delta v_o(2,2) > \delta v_o(2,1)$. Therefore, we may expect that a worker of type (2,2) will do better (will get a higher wage) in the bargaining, as compared to a worker of type (2,1). Indeed, it is shown in the appendix that the Myerson solution yields a higher wage for type (2,2). The wage is given by (A-1) as follows

$$4.9)\quad v_o(t,j) = \tfrac{1}{2}p(t,j)[1-\delta Ev_f+\delta v_o(t+1,j+1) + [1-p(t,j)]\delta v_o(t+1,j).$$

Using (4.9) we can show the following

<u>Proposition 4.1</u> For every $Ev_f \in [0,1]$, there exists a unique bounded solution for equation 4.9.

<u>Proof</u> Let $U = \{u \mid u:N\times N \to R,\ \|u\|_\infty \leq 1-\delta Ev_f\}$, and, for every $u \in U$ define

$$(Tu)(t,j) = \frac{1}{2}\, p(t,j)[1-\delta\, Ev_f + \delta u(t+1,j+1)] + [1-p(t,j)]\delta u(t+1,j)$$

It can be seen that T maps U into itself. Indeed

$$\|Tu\|_\infty \leq \frac{1}{2}\, p(t,j)[1-\delta Ev_f + \delta(1-\delta Ev_f)] + [1-p(t,j)]\delta(1-\delta Ev_f) = (1-\delta Ev_f)[\delta(1-\frac{1}{2}\, p) + \frac{1}{2}\, p].$$

Now, $\delta(1-\frac{1}{2}\, p) + \frac{1}{2}\, p \leq 1$. To see this, notice that this expression is positive and increasing in δ, and for $\delta=1$ it equals 1. Therefore, $\|Tu\|_\infty \leqslant 1-\delta Ev_f$. Also, T is monotone and, for any constant r, $T(u+r) \leqslant Tu + \delta r$. It follows, therefore, that T is a contraction mapping in the sup norm on U of modulus δ, and T has a unique fixed point. Moreover, since this solution is bounded by $1-\delta Ev_f$, then the bargaining problem is well defined. Q.E.D.

Proposition 4.1 guarantees that, for every market value of the firm, there exist unique market values for every types of workers. In turn, we want to show the following

Proposition 4.2 Let $\{v_o(t,1)\}_{t=0}^{\infty}$, $v_o(t,1)\in[0,1]$ for every t, be given. Then Ev_f has a unique solution.

Proof Notice that, in equilibrium, $q(i|t) = 1$ if $i=0$, and 0 otherwise. Also,

$$4.10)\quad v_f(t,0) = 1- w(t,0) = \frac{1}{2}\,[1+ \delta Ev_f - \delta v_o(t+1,1)].$$

Substituting these relations into (4.3) we obtain

$$Ev_f = \lambda E_\theta\Big\{ \sum_{t=0}^{\infty} \theta_t p(t,0)\, \frac{1}{2}\,[1+ \delta Ev_f - \delta v_o(t+1,1)]\Big\}.$$

By rearranging terms it follows

$$4.11)\quad Ev_f = \lambda' E_\theta\Big\{ \sum_t \theta_t p(t,0)\, \frac{1}{2}\,[1-\delta\, v_o(t+1,1)]\Big\}$$

where $\lambda' = \dfrac{\lambda}{1-\lambda\alpha(\delta/2)}$, $\quad \alpha = E_\theta\Big\{ \sum_t \theta_t p(t,0)\Big\}$.

By substituting for λ we may obtain

$$\lambda' = \frac{1}{(1/\lambda) - (\delta/2)\alpha} = [1 - \delta E_\theta\{ \sum_t \theta_t(1-p(t,0))\} - (\delta/2)\alpha]^{-1}$$

Now, $E_\theta\{ \sum_t \theta_t(1-p(t,0))\} = E_\theta\{\sum_t \theta_t\} - \alpha = 1-\alpha$. Therefore $\lambda' = \frac{1}{1-\delta+(\delta/2)\alpha}$. It is easy to see that $\alpha = E_\theta\{ \sum_t \theta_t p(t,0)\} \leqslant E_\theta\{ \sum_t \theta_t\} = 1$. Hence, λ' is positive and bounded. Also, since $0 < 1-\delta v_o(t+1,1) < 1$, then $E_\theta\{ \sum_t \theta_t p(t,0) \frac{1}{2} [1- \delta v_o(t+1,1)]\} < 1$, and, therefore, Ev_f has a well defined, unique solution. Q.E.D.

We can now show the following

Theorem 4.1 There exist unique equilibrium values $(Ev_f^*, \{v_o^*(t,j)\}_{t,j=0}^{\infty})$.

Proof Substituting (4.11) into (4.9) we obtain

$$4.12)\ v_o(t,j) = \frac{1}{2} p(t,j)[1- \lambda' E_\theta\{ \sum_t \theta_t p(t,0) \frac{1}{2} [1- \delta v_o(t+1,1)]\} + \frac{1}{2} p(t,j)\delta v_o(t+1,j+1) + [1-p(t,j)]\delta v_o(t+1,j)$$

Let U be the complete space of bounded function u, $u:N\times N \to R$, and define the operator T on (4.12) as usual. It can be seen that (Tu) maps U into itself, and that it is monotonic. Now, for any constant r,

$$T(u+r) = (Tu) + [\frac{0.25p(t,j)\alpha}{1-\delta+(\delta/2)\alpha} \delta + \frac{1}{2} p(t,j)\delta + [1-p(t,j)]\delta]\ r \leqslant (Tu) + \delta r$$

where the inequality follows from the fact that $\frac{\alpha/2}{1-\delta+(\delta/2)\alpha} \leqslant 1$. Indeed, this expression is increasing in δ, and at $\delta=1$ it equals 1. Therefore, Tu is a contraction mapping in the sup norm, and $v_o(t,j)$ has a unique solution for every $(t,j) \in N\times N$. By virtue of proposition 4.2, this implies that Ev_f has a unique solution too. Q.E.D.

Notice that, if it is always optimal for the worker to take the test, strategies $\{g^*,f^*\}$ indeed form an equilibrium, since $\delta v_o(t,j) < 1-\delta Ev_f$ for every (t,j), and the solution for the bargaining game is individual rational. We show in proposition 4.3 that it is always optimal to take the test. It remains, therefore, to show that the wage is decreasing in the number of failures. By looking at equation 4.8, it is clear that it suffices to show the following

Theorem 4.2 v_o is strictly decreasing in t.

Proof We show first that v_o is decreasing in t. Let U and T be as is defined in proposition 4.1. For any $u \in U$, $T^n u$ converges to v_o uniformly as $n \to \infty$. It suffices, therefore, to show that T maps decreasing functions into decreasing functions. Suppose that $u \in U$ is decreasing. Now

$$4.13)\ (Tu)(t,j) - (Tu)(t+1,j) = p(t,j)\,\frac{1}{2}\,[1-\delta Ev_f+\delta\,u(t+1,j+1)] + [1-p(t,j)]\delta u(t+1,j) - p(t+1,j)\,\frac{1}{2}\,[1-\delta Ev_f+\delta u(t+2,j+1)] - [1-p(t+1,j)]\delta u(t+2,j) =$$

$$p(t+1,j)\frac{\delta}{2}[u(t+1,j+1) - u(t+2,j+1)] + [p(t,j)-p(t+1,j)]\frac{1}{2}[1-\delta Ev_f+\delta u(t+1,j+1)] + [1-p(t,j)][\delta u(t+1,j)-\delta u(t+2,j)] - [p(t,j)-p(t+1,j)]\delta u(t+2,j) > 0.$$

To see how inequality (4.13) is obtained, notice that all the terms in (4.13) are non-negative, except the last term $-[p(t,j)-p(t+1,j)]\delta u(t+2,j)$. However, from the fact that u is decreasing and bounded by $1-\delta Ev_f$ it follows

$$\frac{1}{2}\,[1-\delta Ev_f+\delta u(t+1,j+1) \geq \frac{1}{2}\,[1-\delta Ev_f+\delta u(t+2,j+1)] > \delta u(t+2,j).$$

This implies that $[p(t,j)-p(t+1,j)][\,\frac{1}{2} - \frac{\delta}{2}Ev_f+ \frac{\delta}{2}u(t+1,j+1) - \delta u(t+2,j)] > 0$, and inequality 4.13 now follows.

To see that v_o is strictly decreasing, notice that (4.9) and (4.13), combined with the result that v_o is decreasing, yield the following inequalities

$$v_o(t,j) - v_o(t+1,j) \geq [p(t,j)-p(t+1,j)][\frac{1}{2} - \frac{\delta}{2} Ev_f + \frac{\delta}{2} v_o(t+1,j+1) - \delta v_0(t+2,j)] > 0$$

The last inequality results from the following

$$\frac{1}{2} [1-\delta Ev_f + \delta v_o(t+1,j+1)] - \delta v_o(t+2,j) \geq$$
$$\frac{1}{2}(1-\delta Ev_f) + \frac{\delta}{2} v_o(t+2,j+2) - \delta v_o(t+2,j) \geq$$
$$\frac{1}{2}(1-\delta Ev_f) + \frac{\delta}{2} v_o(t+2,j) - \delta v_o(t+2,j) = \frac{1}{2} [1-\delta Ev_f - \delta v_o(t+2,j)] > 0$$

where the last inequality in this chain of inequalities follows from the fact that $v_o(t+2,j) \leq 1-\delta Ev_f$, and $\delta < 1$. Therefore, v_o is decreasing in t (for every j). Q.E.D.

we can now show that the wage is strictly decreasing;

Corollary 4.1 The equilibrium wage, $w(\cdot,0)$, is strictly decreasing.

Proof The corollary follows immediately from (4.8) and theorem 4.2. Q.E.D.

It is interesting to compare corollary 4.1 with the same result for the pre-check contract (theorem 2.3). Although we obtain a strictly decreasing wage in both, the result of corollary 4.1 is stronger, in the sense that it does not depend upon any restriction on the process of the probability of success p, while theorem 2.2 requires a "decreasing learning". As discussed in chapter 2, the "decreasing learning" assumption is neccessary because the failure in a test has two effects (in the pre-check contract). It decreases the market values of workers, and causes the "cake" to shrink. These two effects affect the wage differently; the decrease in the market value decreases the wage, while the shrinking cake tends to increase the wage. In the post-check contract, however, the cake is not shrinking because the result of the test (and, thus, the true productivity of the worker) is known before the bargaining takes place. Therefore, the worker's market value is the only variable that affects the wage. Hence, corollary 4.1 follows from the fact that the workers market values decrease as the probability of success decreases.

It remains to be shown that the strategies we have examined form an equilibrium. This property is shown in proposition A-1 in the appendix. Note that this equilibrium is the only "reasonable" equilibrium in our model.

5. Conclusions

The decline in the wage and in the hazard rate is strong and persistent across various sets of data. It has been suggested in this paper that selectivity problems alone are not responsible for this phenomenon. Instead, under most reasonable conditions, selectivity may lead to a learning process about the workers' unobserved characteristics. This learning process enhances the effects of selectivity on the wage and the hazard rate, and creates some time dependence effects on these variable that cannot be explained as a selecivity problem.

In addition, as compared to the selectivity-type arguments, the model provides a more complete description of the relationship between the behavior of the wage and the hazard rate over time. It also helps in identifying the effects of other variables, such as previous spells of unemployment, on the wage and the hazard rate.

To keep the model tractable, several issues have been omitted. Below is a partial list of such issues, with some suggestions of how to incorporate them into the model:

1) Search versus non-participation in the market - if search involves some costs, workers with low probability of success will stop searching, because the payoff from search is lower than its costs. Similarly, if the test involves some costs, firms will refuse to check workers with many failures. Both situations create a drop-out of the market of workers with bad reputations. The timing of the drop-out depends upon the exogenous variables (the bargaining ability of the workers, the discount rate, etc.).

2) With a perfect test that has only two outcomes, success and failure, the hazard rate has no "true" time dependence in our model. However, if the test is not perfect, in the sense that it can tell the true productivity of the workers only in a probabilistic manner, then it may produce some time dependence. In particular, firms will demand higher scores from workers with worse reputations.

Footnotes

1) There exist many empirical studies that show the decline in the hazard rate (see, for example, Nickell 1979 and Lancaster 1979). The decrease in the wage is shown by Kasper (1967). It should be mentioned, however, that not many empirical studies on the behavior of the wage exist.

2) Note, however, that the technique used in their paper is controversial.

3) For example, consider one of the solutions for the symmetric uniform trading problem proposed by Chatterjee and Samuelson (1983), or Myerson and Satherthwaite (1983). In these kinds of solutions, different types of bargainers have different probability of separation. Therefore, the probability that is attached to a bargainer after he has been to a bargaining situation is different, in general, than the probability before. It is not clear, however, that these solutions are consistent with a market equilibrium.

4) To see this, use equation 10 to obtain

$$v(k) = \frac{\alpha}{\delta[1-p(k-1)]} v(k-1) - \frac{\beta p(k-1)}{\delta[1-p(k-1)]} =$$

$$= \frac{-\beta p(k-1)}{\delta[1-p(k-1)]} - \frac{\alpha}{\delta^2[1-p(k-1)][1-p(k-2)]} [\beta p(k-2) - \alpha v(k-2)]$$

continuing recursively we may obtain

$$\text{i) } v(k) = -\beta \sum_{i=1}^{k} \left[\frac{\alpha^{i-1} p(k-i)}{\delta^i \Pi_{j=1}^{i}[1-p(k-j)]}\right] + \frac{\alpha^k}{\delta^k \Pi_{i=1}^{k}[1-p(k-i)]} v(o)$$

Let $A(i) = \dfrac{1}{\delta^i \ \Pi_{j=1}^{i} [1-p(k-j)]}$. If we use (11) for v(0), (i) becomes

$$\text{ii)}\ \tilde{v}(k) = -\beta \sum_{i=1}^{k} A(i)\alpha^{i-1} p(k-i) + A(k)\beta\alpha^{k-1} \sum_{i=0}^{\infty} p(i)(\delta/\alpha)^{i} \prod_{j=0}^{i} [1-p(j-1)]$$

Now, the term for i=k in the left sum is $-A(k)\beta\alpha^{k-1}p(0)$ which equal minus the terms for i=0 in the right sum. Accordingly, the term for i=k-1 in the left sum equals minus the term for i=1 in the right sum, etc. Hence, (ii) can be written as follows

$$\tilde{v}(k) = A(k)\alpha^{k-1}\beta \sum_{i=k-1}^{\infty} p(i+1)(\delta/\alpha)^{i-(k-1)} \prod_{j=k-1}^{i} [1-p(j)].$$

Suppose now that v(0) is different from v(0) as is defined by (11). In particular, suppose that $|\Psi(0) - \Psi^*(0)| = \varepsilon$, $\varepsilon > 0$. Then,

$\Psi^*(k) = \tilde{v}(k) + A(k)\alpha^k \varepsilon$. Now, $A(k)\alpha^k > \dfrac{\alpha^k}{\delta^k} \xrightarrow[k\to\infty]{} \infty$ since $\alpha > 1$ and $\delta < 1$.

Therefore, if $\Psi^*(0) > \Psi(0)$, then $\Psi^*(k) \xrightarrow[k\to\infty]{} \infty$ and $\Psi^*(k) \xrightarrow[k\to\infty]{} -\infty$ if $\Psi^*(0) < \Psi(k)$.

5) To see this, let $c \in R$ be a prescribed constant, and consider the solution for (10) given that $v_o(n) = c$, for some $n \in \{k+1, k+2, \ldots\ldots\}$. By solving recursively, it can be seen that this solution may be written as follows

$$\text{i)}\ \Psi^n(k) = \frac{\beta}{\alpha} \sum_{i=k}^{n-1} p(i)(\delta/\alpha)^{i-k} \prod_{j=k-1}^{i} [1-p(j)] + \left(\frac{\beta}{\alpha}(\delta/\alpha)^{n-k} \prod_{j=1}^{n-k} [1-p(n-j)] \right)c$$

Now, $\frac{\delta}{\alpha} < 1$ and $1-p(n-j) < 1$ for each n and j. Therefore,

$$\lim_{n\to\infty} \frac{\beta}{\alpha}(\delta/\alpha)^{n-k} \prod_{j=1}^{n-k} [1-p(n-j)] = 0 \text{ and, hence, } \lim_{n\to\infty} \Psi^n(k) = \Psi(k) \text{ for every } c \in R,$$

6) In fact, strategies (*) are not the unique Subgame Perfect equilibrium, but the unique Perfect equilibrium. Indeed, if we allow the workers to decide whether or not to participate in the game, another Subgame Perfect equilibrium is for the workers not to participate. This equilibrium is not "Perfect" in our model since, if there is any probability that the firms offer more then 0, the workers will participate. However, under a more general model, where search involves some costs, this equilibrium is the only one possible (see discussion at the end of the section).

Appendix

A-1. The Myerson solution

Let us use the example in Myerson (1984, p. 479) to solve for the wage for a worker of type (2,2). To do so, we modify the problem as follows. Suppose two players, player 1 (the worker) and player 2 (the firm), can jointly carry on a transaction that costs \$1. This transaction is commonly known to be worth $1-\delta Ev_f$ to player 2, but its value to player 1 depends on his type, which is unknown to player 2. If 1's type is h, then the transaction is worth $1-\delta v_o(2,1)$ to him, and player 2 assigns a subjective probability $1-\varepsilon$ to this event. If 1's type is b, then the transaction is worth $1-\delta v_o(2,2)$ to him, and player 2 assigns probability ε to this event.

Myerson (1984) uses the concept of Bayesian bargaining problem to describe this bargaining situation. Formally, a two-person Bayesian bargaining problem Γ is an object of the form (see Myerson (1984) p. 462))

$$\Gamma = (D, d_0, T_1, T_2, u_1, u_2, p_1, p_2)$$

where D is the set of collective decisions or feasible outcomes available to the two players if they cooperate, $d_0 \in D$ is the conflict outcome, T_i is the set of possible types for player i, u_i is the payoff function for player i, and each p_i is a function that specifies the conditional probability distribution that each type of player i would assess over the other player's possible types.

To formally model our problem, we let $T_1 = (1h, 1b)$, $T_2 = (2)$, $D=(d_o, d_1, d_2)$, $p(1h) = 1-\varepsilon$, $p(1b) = \varepsilon$ with utility functions as follows

(u_1, u_2)	d_0	d_1	d_2
$t_1 = 1h$	$\{0,0\}$	$\{-\delta v_o(2,1),\ 1-\delta Ev_f\}$	$\{1-\delta v_o(2,1),\ -\delta Ev_f\}$
$t_1 = 1b$	$\{0,0\}$	$\{-\delta v_o(2,2),\ 1-\delta Ev_f\}$	$\{1-\delta v_o(2,2),\ -\delta Ev_f\}$

The decision options in D are interpreted as follows: d_0 is the decision not to carry on the transaction; d_1 is the decision to carry it on at 1's expense; and d_2 is the decision to carry it on at 2's expense. There is no need to consider intermediate financing options, because they can be represented by "randomized" strategies (both players are risk neutral).

Given a mechanism π, let $\pi(i,h)$ be the probability of choosing action d_i given that player 1 is of type h, and let $\pi(i,b)$ be the probability of choosing action d_i if 1's type is b. The incentive-compatible choice mechanisms are those satisfying the following inequalities

$$-\delta v_o(2,1)\pi(1,h) + [1-\delta v_o(2,1)]\pi(2,h) \geqslant -\delta v_o(2,1)\pi(1,b) + [1-\delta v_o(2,1)]\pi(2,b)$$
$$-\delta v_o(2,2)\pi(1,b) + [1-\delta v_o(2,2)]\pi(2,b) \geqslant -\delta v_o(2,2)\pi(1,h) + [1-\delta v_o(2,2)]\pi(2,b)$$
$$\pi(0,h) + \pi(1,h) + \pi(2,h) = 1, \qquad \pi(0,b) + \pi(1,b) + \pi(2,b) = 1$$

and all $\pi(i,j) \geqslant 0$. The first inequality says that player 1 should not want to claim to be type b if he is really type h; the second inequality says that 1 should not want to claim to be h if he is really b.

The incentive feasible set is the set of allocation vectors $U = (U_1^h, U_1^b, U_2)$ such that

$$U_1^h = -\delta v_o(2,1)\pi(1,h) + [1-\delta v_o(2,1)]\pi(2,h)$$
$$U_1^b = -\delta v_o(2,2)\pi(1,b) + [1-\delta v_o(2,2)]\pi(2,b)$$
$$U_2 = (1-\varepsilon)[(1-\delta Ev_f)\pi(1,h)-\delta Ev_f\pi(2,h)] + \varepsilon[(1-\delta Ev_f)\pi(1,b)-\delta Ev_f\pi(2,b)]$$

It can be seen that, for sufficiently small ε, the set of incentive-efficient utility allocations satisfying individual rationality is a triangle in R^3_+ with extreme points as follows

$$\{\ 1-\delta v_o(2,1)-\delta Ev_f,\ 1-\delta v_o(2,2)-\delta Ev_f,\ 0\ \}$$
$$\{\ \delta v_o(2,2) - \delta v_o(2,1),\ 0,\ 1-\delta Ev_f-\delta v_o(2,2)\}$$
$$\{\ 0,\ 0,\ (1-\varepsilon)[1-\delta Ev_f-\delta v_o(2,1)]\ \}$$

The first of these allocations is implemented by having player 1 pay δEv_f and player 2 pay $1-\delta Ev_f$ independently of the state, or by using the mechanism π_1 where

$\pi_1(d_1|t_1) = \delta Ev_f$, $\quad \pi_1(d_2|t_1) = 1-\delta Ev_f$ for every t_1.

The second of these allocations is implemented by having player 1 pay $1-\delta v_o(2,2)$ and player 2 pay $\delta v_o(2,2)$ independently of the state, or by using π_2 where

$\pi_2(d_1|t_1) = 1-\delta v_o(2,2)$, $\quad \pi_2(d_2|t_1) = \delta v_o(2,2)$, for every t_1.

The third of these allocation is implemented by having player 1 pay $1-\delta v_o(2,1)$ and player 2 pay $\delta v_o(2,1)$ if 1's type is h, and by not carry on the transaction if 1's type b; or by using the mechanism π_3 where

$$\pi_3(d_1|1h) = 1-\delta v_o(2,1), \qquad \pi_3(d_2|1h) = \delta v_o(2,1), \qquad \pi_3(d_0|1b) = 1.0$$

Notice that π_1 is the best feasible mechanism for both types of player 1, and, for sufficiently small ε, π_3 is the best feasible mechanism for player 2. Thus, a random dictatorship would implement the mechanism $\pi_4 = 0.5\pi_1 + 0.5\pi_3$, that is

$$\pi_4(d_1|h) = \tfrac{1}{2}[1-\delta v_o(2,1)+\delta Ev_f], \qquad \pi_4(d_2|h) = \tfrac{1}{2}[1+\delta v_o(2,1)-\delta Ev_f],$$
$$\pi_4(d_1|b) = 0.5\delta Ev_f, \qquad \pi_4(d_2|b) = 0.5(1-\delta Ev_f), \qquad \pi_4(d_0|b) = 0.5$$

This mechanism yields the following utilites

$$U_1(\pi_4|h) = 0.5[1- \delta v_o(2,1)- \delta Ev_f]$$
$$U_1(\pi_4|b) = 0.5[1- \delta v_o(2,2)- \delta Ev_f]$$
$$U_2(\pi_4) = 0.5(1-\varepsilon)[1- \delta Ev_f- \delta v_o(2,1)]$$

In order to compare these payoffs to the wages, we have to add the values we deleted from the utilities in the beginning. It can be seen that

$$U_1(\pi_4|h) + \delta v_o(2,1) = \frac{1}{2}[1+ \delta v_o(2,1)- \delta Ev_f] = w(1,0)$$
$$U_2(\pi_4) + \delta Ev_f = \frac{1}{2}[1+ \delta Ev_f- \delta v_o(2,1)] = 1- w(1,0).$$

Thus, the firm and the "bad" worker get the same payoff as they get under the Nash solution (see equation 4.6). Also, it can be seen that the (2,2) type recieves a higher payoff. Indeed

$$U_1(\pi_4|b) + \delta v_o(2,2) = \frac{1}{2}[1+ \delta v_o(2,2)- \delta Ev_f] > w(1,0)$$

The solution for the bargaining problem with t types of player 1 may be obtained by investigating the above solution. Notice that the best feasible outcome for all types of player 1 is

$$\{1-\delta v_o(t,1)-\delta Ev_f,\ 1-\delta v_o(t,2)-\delta Ev_f,\ \ldots\ldots,\ 1-\delta v_o(t,t)-\delta Ev_f,\ 0\ \}$$

which is implemented by π_1 (or by having player 1 pay δEv_f and player 2 pay $1-\delta Ev_f$ independently of the state). Also, the best feasible outcome for the firm (for ε sufficiently small) is

$$\{ 0, 0, \ldots\ldots, 0, (1-\varepsilon)[1- \delta Ev_f - \delta v_o(t,1)] \}$$

which is implemented by π_3, where $\delta v_o(t,1)$ is replacing $\delta v_o(2,1)$, and $\pi(d_0 | \text{any type of worker different than } (t,1)) = 1.0$. Thus, a random dictatorship would implement the mechanism π_4, with the appropriate modifications.

It can be seen that mechanism π_4 gives the worker a wage of $1-\delta Ev_f$ with probability 0.5, and the conflict outcome with probability 0.5 if he claims to be of a type different than (t,0). Therefore, the value of a worker of type (t,j) from breaking off negotiations can be written as follows

$$\text{A-1)}\quad v_o(t,j) = \frac{1}{2}\, p(t,j)(1-\delta Ev_f) + \frac{1}{2}\, p(t,j)\delta v_o(t+1,j+1) + [1-p(t,j)]\delta v_o(t+1,j)$$

Note that A-1 equals equation 4.9 in section 4.

A-2. The complete game, where workers have the option not to take the test

Let us describe the game between a worker of type (t,k,j) and a firm when they meet. Assume that the worker's market value, $v_0(t,k,j)$, and the firm's market value, Ev_f, are given. Also, let the expected payoff for the firm from reaching an agreement with a worker of type (t,k,j) be x(t,k,j), and the expected payoff for a worker of type (t,k,j) be z(t,k,j). Notice that, although the probability of success, p(k,j), does not depend directly on the number of period the worker has been searching (t), the payoff that the worker may get does depend on t, since the number of possible types of worker changes with t (and, thus, the payoff that each type is getting may change too).

To see what the chain of decisions and events is in this game, it is useful to use figure 4.1, that describes a particular game between a firm and a worker with type (t,k,j) prior to the test. Since the test changes the worker's type and position in the market, he has to decide whether he wants to take the test (y) or not (n). This decision may be important since, by not taking the test, the worker is facing a new situation as he becomes a worker of type (t+1,k,j), as opposed to a worker of type (t,k,j). A worker of type (t+1,k,j) may get a higher payoff than a worker of type (t,k,j) since the set of possible types of workers is different and, as a result, he may have "better" opportunities to cheat.

The sets of strategies that are available to the firms and to the workers are exactly the same as the sets that are given in the game of section 4. The extension of $v_o(t,j)$ to $v_o(t,k,j)$ is obvious. Now, to show that it is never profitable for a worker not to take a test, it suffices to show that, for every (t,k,j), $v_o(t,k,j) = v_o(t+1,k,j)$ (given that firms always want to negotiate). Indeed, if this equality holds, then the worker's market value does not change when k and j remain the same, and the worker cannot get a higher wage. Therefore, we want to show the following

<u>Proposition A-1</u> Given Ev_f, $v_o(t,k,j) = v_o(t+1,k,j,)$ for every (t,k,j).

<u>Proof</u> As before, it can be seen that $j > 0$ implies $v_o(t,k,j) > v_o(t,k,0)$. Therefore, if the worker takes the test and succeeds, his payoff will be according to mechanism π_4. Hence, if we include the option not to take a test, the worker's market value can be written as follows

$$\text{A-2)}\quad v_o(t,k,j) = \text{Max}\left\{\delta v_o(t+1,k,j),\quad \frac{1}{2}\,p(k,j)(1-\delta Ev_f) + \frac{1}{2}\,p(k,j)\delta v_o(t+1,k+1,j+1) + [1-p(k,j)]\delta v_o(t+1,k+1,j)\right\}$$

Now, let $U = \{u \mid u: N\times N\times N \to R, \|u\|_\infty \leq 1-\delta E v_f\}$, and define the operator T as usual. Following the same line as in the proof of proposition 4.1 we can show that v_o has a unique bounded solution. We show now that for every $\varepsilon>0$, $|v_o(t,k,j) - v_o(t+1,k,j)| < \varepsilon$. First assume that, for any t

A-3) $v_o(t,k,j) = \frac{1}{2}p(k,j)[1-\delta E v_f+\delta v_o(t+1,k+1,j+1)] + [1-p(k,j)]\delta v_o(t+1,k+1,j)$

In this case it can be seen that

$$\Delta(t,j) = \frac{\delta}{2} p(k,j)\Delta(t+1,j+1) + [1-p(k,j)]\delta\Delta(t+1,j)$$

where $\Delta(t,j) = v_o(t,k,j) - v_o(t+1,k,j)$. W.L.O.G. we may assume that $\Delta(t+1,j+1) \geq \Delta(t+1,j)$. Hence, $\Delta(t,j) \leq [1- \frac{1}{2}p(k,j)]\delta\Delta(t+1,j+1) < \delta\Delta(t+1,j+1)$. Continuing this way we may write $\Delta(t,j) < \delta^n\Delta(t+n,j+n)$. Since $|\Delta(t+n,j+n)| < 1$, then $|\Delta(t,j)| < \delta^n$ and, for every $\varepsilon>0$ we can find sufficiently big n such that $|\Delta(t,j)| < \delta^n < \varepsilon$. Therefore, $v_o(t,k,j) = v_o(t+1,k,j)$ if 4.15 holds for every t. Whenever A-3 does not hold, i.e., $v_o(t,k,j) = \delta v_o(t+1,k,j)$, there has to be the first n for which

$$v_o(t,k,j) = \delta^n\{ \frac{p(k,j)}{2} [1-\delta E v_f+ \delta v_o(t+n,k+n,j+1)] + [1-p(k,j)]\delta v_o(t+n,k+n,j)\}$$

(otherwise, $v_o(t,k,j) = 0$). The same is true about $v_o(t+1,k,j)$. Therefore, we can express the difference $\Delta(t,k)$ as we did previously. Q.E.D.

Proposition A-1 has two implications. First, it implies that strategies (*) indeed form an equilibrium, because they yield higher payoffs than the strategy of not taking the test (by the individually rational property, given that the worker has taken the test and succeeded, it is optimal to negotiate). Second, given the result that the workers always want to take tests, we may consider the number of periods and the number of tests as a single state variable.

A-3. Conditions for Steady State.

The system we have described consists of the following. Each period M new workers and M new firms are entering, and they can drop out only in pairs. The probability that a pair will drop out is $p(t)$, where t is the number of periods the worker has been into the system.

The way the workers' number of each generation evolves can be described by a "Branching Process" (see Karlin and Taylor (1975) chapter 8). Let ξ be the number of new workers each worker will generate in the next period (including himself). Then, $p(\xi=1/\ k=i) = 1-p(i)$ and $p(\xi=0/\ k=i) = p(i)$. Since $p(i) \geqslant a$ for each $i \in N$, each generation will be extinct (i.e., each worker will find a firm) after a finite number of periods with probability 1.

However, we are interested in the (steady state) probability of drawing a worker of type i from the population of workers. For this, we use the following notation

X_i^t = the number of workers with i checks in period t.

Also let $Y^t = \sum_{i=0}^{t} X_i^t$ be the total number of workers in period t. As before, θ_i^t is the probability of drawing a worker of type i in time t from the "pool" of workers. Now,

$$\theta_i^t = \frac{X_i^t}{Y^t} \quad \text{and} \quad \theta_i = \lim_{t\to\infty} \frac{X_i^t}{Y^t} = \frac{X_i}{Y} \quad \text{where} \quad Y = \sum_{i=0}^{\infty} X_i$$

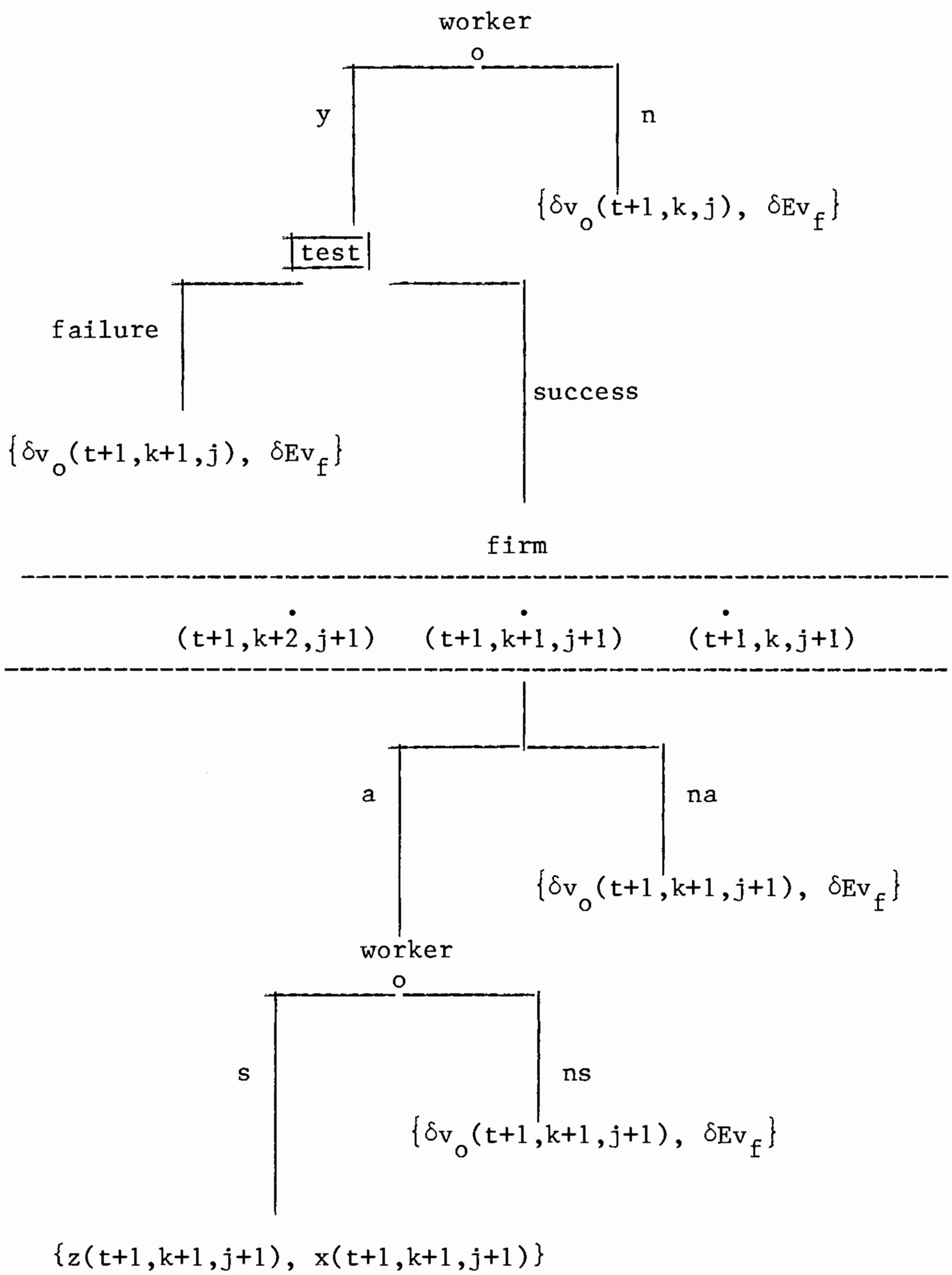
This follows from the fact that X_i^t does not depend on t for $t > i$. Note that X_i depends on the previous realization of X_{i-1} only. given that $X_{i-1} = x'$, X_i has a binomial distribution with parameters x' and $1-p(i-1)$. Therefore, starting from M and p(0), we are able to calculate the unconditional distribution of every X_i.

From this we can calculate the distribution function of y and θ_i. The distribution of θ_i is given by a quotient of two random variables, X_i and Y. Since $X_0=M$ deterministically, $Y \geqslant M > 0$, and θ_i is well defined. If $Y = \infty$, let $\theta_i = 0$ for every i, and let $\theta_\infty = 1$. This event, $Y = \infty$, is not desirable for our purposes, but it is not likely to occur, i.e., $Y < \infty$ a.s. To show this, it suffices to show that the (unconditional) probability of the event $Y < \infty$ is 1 a.s. Now, the unconditional distribution of X_i is Binomial with parameters M and $q_i = \prod_{j=0}^{i-1} [1-p(j)]$. Let Z_i be Binomial distributed with parameters M and $(a)^i$, where a is the lower bound of the support of the distribution of p(i), and let $Z = \sum_{i=0}^{\infty} Z_i$. Clearly,

$p(X_i=0) \geqslant p(Z_i=0) = [1-(1-(a)^i)]^M$ and, therefore, $p(Y < \infty) \geqslant p(Z < \infty)$. Hence, it suffices to show $p(Z < \infty) = 1$ a.s. Since Z is finite for every finite number of periods, it suffices to show that, for every $\varepsilon > 0$, there exists i such that $1 - \prod_{j=0}^{\infty} [1-(1-a)^{i+j}]^M < \varepsilon$, i.e., for each $\varepsilon > 0$ there exists i s.t. $\| \sum_{j=0}^{\infty} \ln[1-(1-a)^{i+j}]^M \| < \varepsilon$. Now, $1 - (1-a)^{i+j})^M$ is increasing in j, and, for each ε there exists i s.t.

$$\| \ln[1-(1-a)^{i+j}]^M \| < \frac{\varepsilon}{2^j} \implies \| \sum_{j=0}^{\infty} \ln[1-(1-a)^{i+j}]^M \| < \varepsilon.$$

Figur 4.1


worker
o
y
n
{δv_o(t+1,k,j), δEv_f}
test
failure
success
{δv_o(t+1,k+1,j), δEv_f}
firm
(t+1,k+2,j+1)
(t+1,k+1,j+1)
(t+1,k,j+1)
a
na
{δv_o(t+1,k+1,j+1), δEv_f}
worker
o
s
ns
{δv_o(t+1,k+1,j+1), δEv_f}
{z(t+1,k+1,j+1), x(t+1,k+1,j+1)}

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