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THE ROLE OF EXTERNAL SEARCH IN BILATERAL BARGAINING

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Abstract

We consider the problem of bargaining between two individuals and represent their bargaining strengths in terms of their abilities to search elsewhere for possible alternatives to each other's offers. The joint process of bargaining and search over time is modeled as a noncooperative game in the extensive form with chance moves. We characterize the associated subgame perfect equilibrium and examine how it depends upon the bargainers' relative search abilities.

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1. INTRODUCTION

It seems intuitive that the outcome of a bargaining situation should depend critically on the individuals' relative bargaining "strengths," "positions," or "abilities," and that an individual in a stronger position should be able to attain a superior outcome for himself. It is also clear that in many bargaining situations the individuals may have access to outside alternatives to each other's offers and this fact should also influence the bargaining outcome. Motivated by these two observations, in this paper we represent an individual's bargaining strength in terms of his ability to search for good outside alternatives, characterize the equilibrium outcome of the game of bargaining with search, and examine how it depends upon the bargainers' relative search abilities. We analyze this problem in the context of bargaining between a seller and a buyer, given that both can also search for alternate price offers in the market. For example, one may consider employment negotiations between a firm and a job candidate, and ask how the final contract depends upon the firm's reputation in attracting other candidates vis-a-vis the candidate's marketability in generating job offers from other firms.

The importance of search in bargaining situations has been mentioned recently by Raiffa [11, p. 79] and by Chatterjee and Samuelson [3, p. 850], while Lax [8] has provided illustrative examples suggesting that the value of external search may serve as a "bottom line" in current negotiations. The individual's pure search problem (without details of bargaining) has been well studied in the economics literature that was stimulated by Stigler [17]; see, for example, Kohn and Shavell [7], and surveys by Lippman and McCall [9], and Diamond [6]. The individual's optimal sequential search strategy usually has the "reservation level" property, i.e., he should continue searching until he gets an offer that is at least as good as a predetermined threshold, where the threshold depends upon the individual's search cost and the probability distribution of offers received. This reservation level also represents the optimal expected net value of search.

The bargaining problem, without an explicit consideration of the individuals' outside search possibilities, also has been extensively researched. In the axiomatic approach to bargaining, which was pioneered by Nash [10] and surveyed by Roth [12], the bargaining problem is defined in terms of a set of all possible outcomes that the players can achieve jointly, together with a distinguished point corresponding to the outcome of disagreement. One then stipulates a set of reasonable axioms that a solution to the bargaining problem should satisfy and demonstrates a unique such solution, the Nash's solution being the most celebrated such example. This approach focuses directly on the final outcome, without an explicit consideration of the dynamics of the bargaining process which leads to that outcome. In order to view our problem of bargaining with search in this framework, it may at first seem natural to specify the disagreement point in terms of the best outcome that each individual can achieve entirely on his own by searching outside in case bargaining fails. However, this pure search outcome corresponds to an extreme form of disagreement involving a complete breakdown of negotiations, whereas both individuals may be better off agreeing to continue some form of bargaining dialogue in a more concilliatory tone. Moreover, each individual may do well to search outside even while bargaining, rather than postponing search until after bargaining fails. In fact, we will prove in this paper that the pure search disagreement outcome is Pareto dominated by the equilibrium outcome of a certain noncooperative game in which the two individuals simultaneously search outside and also bargain among themselves, thereby providing a superior disagreement point in the Nash approach. These considerations behoove us to model the joint process of bargaining and search over time as a noncooperative game and to study its (subgame) perfect equilibrium (in the sense of Selten [15]).

The strategic approach to model bargaining as a noncooperative game in the extensive form was proposed in a seminal paper by Rubinstein [13]. He analyzed the problem of dividing a given surplus by using a model in which the two players alternately make and respond to each other's offers through time. He characterized the unique perfect equilibrium outcome, which depends upon the players' time preferences (in terms of their bargaining costs or discount factors) and the order of moves. The Rubinstein approach was further elucidated and extended by Binmore [1] and by Shaked and Sutton [16] in the context of a firm-worker bargaining model. In these models an individual's bargaining strength may be interpreted in terms of his time preference and whether he can make the first move.

In this paper we employ the strategic approach to model the process of bargaining between a seller and a buyer over the price of an indivisible object and represent their bargaining strengths in terms of their abilities to search for better prices in the outside market. The search process is assumed to involve cost and uncertainty about the timings and magnitudes of the outside price offers received. The seller's (buyer's) search ability to discover stochastically higher (lower) prices fast and at low search costs represents his bargaining strength. In addition to searching the market, the two players also bargain among themselves by making and responding to each others' offers through time. Whenever either player receives an outside offer, he may decide to accept it, or make an offer to the other player if he is still available for bargaining, or continue searching outside for better prices. Similarly, whenever a player receives an offer from the other player, he may accept it and terminate the game or he may continue searching. This model of bargaining and search is formulated in Section 2, while Section 3 characterizes the perfect equilibrium outcome of this game as the unique solution of a pair of algebraic equations. The equilibrium strategies are shown to involve each player's establishing an "asking price" (which is the price that he would like to obtain in the outside market) and a "concession price" (the inside price offer that he would be willing to concede to). Section 4 then indicates how the equilibrium outcome depends upon individuals' relative search abilities. It is shown that lower search costs or faster discoveries of stochastically better offers leads to superior outcomes for the individual. The final section concludes with some remarks regarding the proposed rules of the game, the relationship with the Nash bargaining solution and possible extensions.

2. THE SEARCH-BARGAINING GAME

Consider the seller of an indivisible object as player 1 and the buyer as player 2. At time $t = 0$, the seller offers to sell the object at price P_{10} and, simultaneously, the buyer offers to purchase it at price P_{20}. If $P_{10} < P_{20}$, the trade concludes immediately, suppose at the price $(P_{10} + P_{20})/2$, corresponding to equal sharing of the surplus. If $P_{10} > P_{20}$, the disagreement leads the two players to start searching for better prices in the market while agreeing to "keep in touch."

The search activity requires effort and involves uncertainty about the timings and magnitudes of the outside offers received. Let C_i denote the search (or waiting) cost rate per unit time for player i, i = 1,2. Suppose that the two players receive outside offers according to independent Poisson processes with intensities λ_1 and λ_2, respectively. Consequently, the time intervals between outside offers received by player i are independent exponentially distributed random variables, each with the mean $1/\lambda_i$, and a typical one may be denoted as T_i, i = 1,2. Finally, suppose that the magnitudes of the outside offers received by player i are independent identically distributed nonnegative square integrable random variables, denoted typically as Q_i, each with a continuous distribution function $F_i(\cdot)$, i = 1,2.

Thus, player i incurs the expected search cost C_i/λ_i to receive an outside offer Q_i drawn from the distribution F_i. Lower C_i, higher λ_i and better F_i (in the sense of stochastic dominance) represent a superior search ability of player i in discovering good offers fast and at low cost. The model parameters (C_i, λ_i, F_i), i = 1,2, which characterize the players' search abilities, are exogenously specified and assumed to be common knowledge. Thus, we do not consider the players' problem of selecting their search effort levels which determine (C_i, λ_i, F_i), i = 1,2, although later on we will provide the relevant comparative statics. For now the players' search is assumed to involve merely waiting to receive outside offers.

As they receive outside offers, the two players bargain among themselves by making and responding to each other's offers over time. As to the players' bargaining decisions, we stipulate that each player makes a move whenever he acquires new information, which is in the form of an offer, either from the outside market or from the other player. In particular, whenever a player receives an outside offer, he may decide to accept it (thereby leaving the other player to search thereafter on his own), or he may "return to the bargaining table" and make an offer to the other player (if the latter is still available for bargaining), or he may continue searching for better offers. Similarly, whenever a player receives an offer from the other player, he may either accept it and terminate the game or he may reject it, in which case both players continue searching for better offers. The process ends as soon as both players have accepted offers, either from each other or from the outside market. Since the players' search abilities are known and the planning horizon is infinite, we may assume, without loss of generality (as, for example, in DeGroot [5, Ch. 13]) that once an offer is rejected it will not be recalled. Similarly, given the common knowledge assumption about the players' characteristics, we have assumed that bargaining in a given information state at any time instant simply involves an offer by one player and its acceptance or rejection (but not an instantaneous counter offer) by the other player, as in Rubinstein [13]. Finally, note that in our partial equilibrium setting, the two players bargain only among themselves but not with outside market offers.

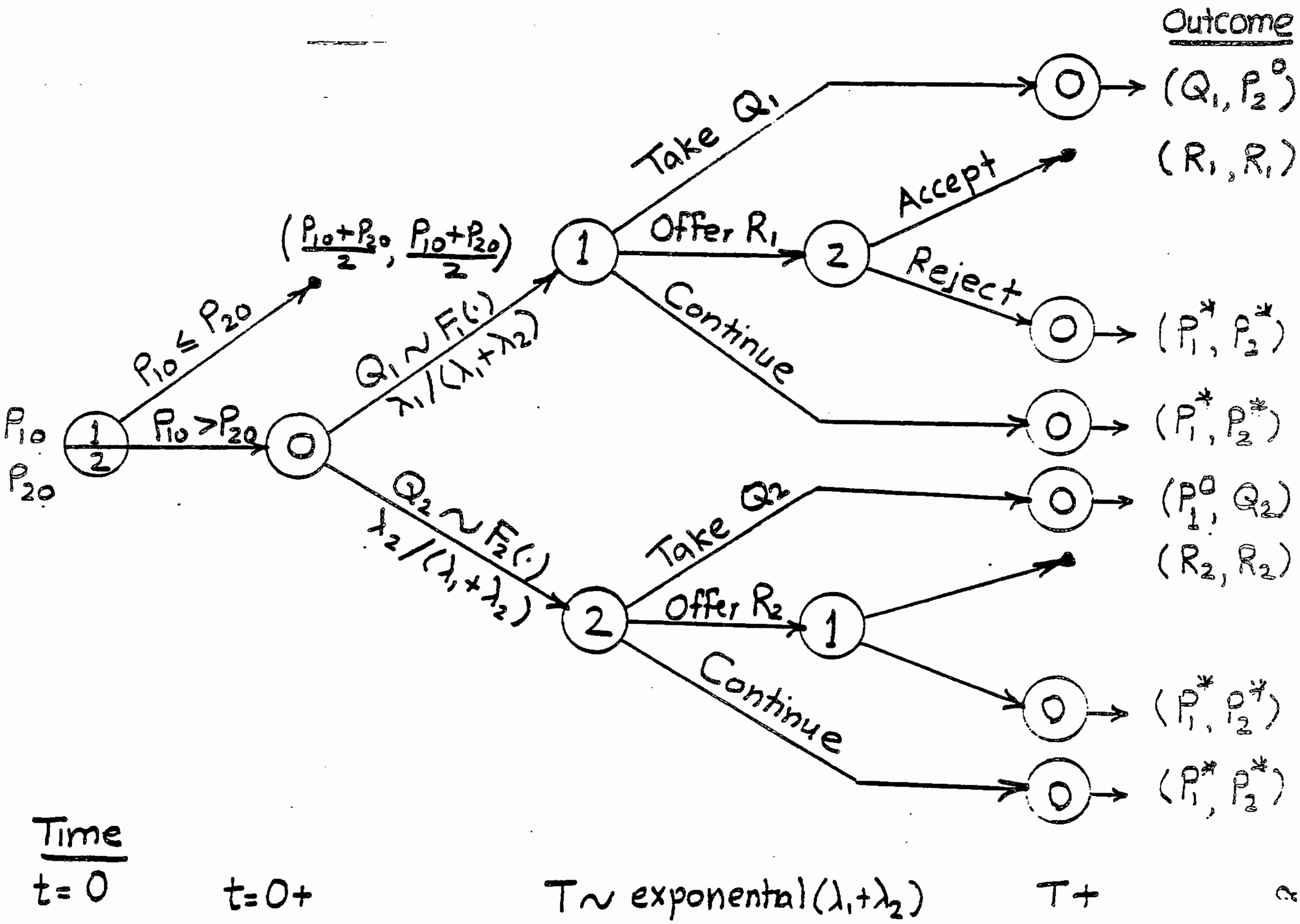
With this specification of the search and bargaining phases, we may describe their joint evolution over time $t \in (0,\infty)$ as follows. As the players start searching at $t = 0+$, one of them, say player i, receives first an outside offer Q_i from the market (denoted as player 0) at a random time $T = \operatorname{Min}\{T_1, T_2\}$. It is easy to check that T is exponentially distributed with parameter $(\lambda_1 + \lambda_2)$ and that $P\{T = T_i\} = \lambda_i/(\lambda_1 + \lambda_2)$ is the probability that player i is the first one to receive an outside offer. Player i may decide to accept the outside offer Q_i, or make an offer, say R_i, to the other player $j \neq i$, or continue searching. Player i's acceptance of Q_i terminates bargaining and from then on player j faces the pure search problem of waiting for an appropriate outside offer Q_j. If, however, player i decides to offer R_i to player j, then player j may either accept it and the game ends with trade at price R_i, or player j may reject R_i and the game continues. Of course, the game also continues if player i decides not to accept Q_i nor to make an offer to player j. The continuation of the search-bargaining game over (T,∞), is a probabilistic replica of the original game over $(0,\infty)$, due to the stochastic structure (specifically, the strong Markov property) of our model, the infinite time horizon (which ensures stationarity) and the common knowledge assumption (which eliminates the players' learning more about each other and the market). Figure 1 summarizes the first stage of the game evolution over $(0,T]$ and also shows the players' equilibrium outcomes P_i^* and P_i^0, $i = 1,2$ in the subgame over (T,∞), which will be defined and computed in the next section.

In addition to capturing the players' bargaining strengths in terms of their search abilities, our model of bargaining with search has also introduced in a natural way two types of chance moves suggested by Binmore [1] as generalizations of Rubinstein's [13] bargaining model, namely, the time between successive offers is an (exponentially distributed) random variable T and the player i who makes the next offer is selected (with probability $\lambda_i/(\lambda_1 + \lambda_2)$) by a random device.

3. THE EQUILIBRIUM

In this section we wish to characterize the perfect equilibrium of our search-bargaining game. For this purpose we may define a strategy for each player i as a decision rule that specifies at each decision point, as a function of the entire past history of the game, which offers R_j from the other player and Q_i from the outside market to accept, what offers R_i

Figure 1: The Search-Bargaining Process



to make to the other player and when to continue searching and bargaining for better offers. A pair of strategies yields an outcome in terms of the prices that the seller expects to receive and the buyer expects to pay, net of the search costs involved, in the game over the time interval $(0,\infty)$. A pair of strategies is then said to constitute a perfect equilibrium if from any time t onwards neither player has any incentive to deviate unilaterally from continuing the originally planned strategy in the subgame over $[t,\infty)$. Although these definitions can be formalized as in Rubinstein [13], we omit the notational details and instead follow a more intuitive and direct approach in the spirit of Shaked and Sutton [16]. Specifically, we employ a dynamic programming argument to characterize simultaneously the perfect equilibrium outcome and the strategies which yield that outcome.

Let P_1^* and P_2^* respectively denote the optimal prices (net of search costs) that the seller and the buyer can expect to receive and pay in a perfect equilibrium of the search-bargaining game over the time interval $(0,\infty)$. Now consider the (random) time T at which one of the players, say player i, first receives an outside offer Q_i. Then he may choose to take Q_i, or make an offer R_i to the other player j, or continue searching. For each decision we will compute the players' perfect equilibrium outcomes in the resulting subgame over (T,∞).

If player i accepts Q_i, then player j faces the pure search problem over (T,∞). Let P_j^0 denote the optimal price (including the search costs) that player j can expect by searching on his own, given that player i is <u>not</u> available for bargaining. Adapting the well-known results from the individual search theory (see, for example, DeGroot [5, ch. 13]) with the given assumptions, we know that the optimal pure search strategy exists for player j, that it has the reservation level property and that P_1^0 and P_2^0 are in fact the pure search "reservation prices" which can be obtained as the unique solutions of the following respective equations.

$$P_1^0 = E[\max(Q_1,P_1^0)] - C_1/\lambda_1 \tag{1}$$

$$P_2^0 = E[\min(Q_2,P_2^0)] + C_2/\lambda_2. \tag{2}$$

It is instructive to review the derivation of these equations since a similar technique will also enable us to characterize the optimal prices P_1^* and P_2^* when both players are available for bargaining. Consider, for example, the seller who searches on his own over (T,∞) and expects to get P_1^0 as the maximum price net of the search costs. When he receives an offer Q_1 at time $T_1 > T$, he may either take it or continue searching; in the latter case he is in the same situation as at time T and hence can expect at most P_1^0 over (T_1,∞). Therefore, at time T_1 he should accept Q_1 if and only if $Q_1 > P_1^0$ and thus receives $\max(Q_1,P_1^0)$ after having paid the search cost $C_1(T_1 - T)$. The expected value of this net price computed at T must therefore be the best price P_1^0 that he can expect over (T,∞). This yields equation (1) and similar derivation yields equation (2). Alternatively, suppose each player wishes to optimize search within the class of strategies that have the reservation level property. For example, suppose the seller wishes to determine a reservation price P_1 which would maximize his expected profit, say $\pi_1(P_1)$, from searching until he receives an offer which is at least as high as P_1. Then

$$\pi_1(P_1) = [\int_{P_1}^{\infty} q\lambda_1 dF_1(q) - C_1]/\{\lambda_1[1 - F_1(P_1)]\}. \tag{3}$$

Maximizing $\pi_1(P_1)$ with respect to P_1 then yields $P_1^0 = \pi_1(P_1^0)$ as the condition that optimal P_1^0 must satisfy, i.e., the optimal reservation price equals the optimum expected net price obtainable by employing that reservation price strategy. Rearranging $P_1^0 = \pi_1(P_1^0)$ then yields

$$(4) \qquad \int_{P_1^0}^{\infty}(q - P_1^0)dF_1(q) = C_1/\lambda_1.$$

Similarly, for the buyer, we have

$$(5) \qquad \int_0^{P_2^0}(P_2^0 - q)dF_2(q) = C_2/\lambda_2.$$

Thus each player's pure search reservation price exactly balances the expected benefits and costs of search. Finally, equations (4) and (5) can be seen to be equivalent to (1) and (2).

Thus, given that player i first receives an outside offer Q_i at time T, if he decides to accept it, the resulting equilibrium outcomes for the two players over $[T,\infty)$ will be Q_i and P_j^0. If player i does not accept Q_i he may choose to make an offer R_i to player j who may then accept it or reject it. Now in a perfect equilibrium, player j will accept R_i only if it is at least as good as what he can expect in equilibrium in the subgame over (T,∞) which results from rejecting that offer. But this subgame over (T,∞) is identical with the original game over $(0,\infty)$ and therefore player j expects to get P_j^* over (T,∞). Hence, if player i chooses to make an offer R_i to player j the best offer he can make which player j "cannot refuse" is $R_i = P_j^*$, yielding (P_j^*,P_j^*) as the outcome at time T. Finally, if at time T player i does not accept Q_i nor make an offer P_j^* to player j, the game continues as before and the players continue to expect (P_1^*,P_2^*) as the perfect equilibrium outcome in the subgame over (T,∞).

Therefore player i's equilibrium strategy at time T involves comparing his equilibrium outcomes Q_i, P_j^* and P_i^* in the subgame over (T,∞) and accepting Q_i or offering P_j^* to player j or continuing search in expectation of P_i^*, depending upon whichever is the best outcome (i.e., the maximum if player i is the seller and the minimum if he is the buyer). Notice that player i takes Q_i if and only if Q_i is better than both P_i^* and P_j^*, so that we may define

$$(6) \qquad P_1^{**} = \max(P_1^*, P_2^*) \text{ and } P_2^{**} = \min(P_1^*, P_2^*)$$

and call P_i^{**} as player i's "asking price" which he seeks to obtain in the outside market, given that player j is still available for bargaining. (Recall that P_i^0 is player i's "reservation price" which he seeks to obtain given that player j is not available). On the other hand, in perfect equilibrium, player i will concede to an inside offer of P_i^* from player j, and hence P_i^* may be called player i's "concession price."

In order to compute P_i^*, $i = 1,2$, notice that player i who receives an outside offer Q_i first at time T takes it if it is better than P_i^{**}, in which case player j expects to receive P_j^0. If Q_i is worse than P_i^{**}, then player i will either offer P_j^* to player j or continue searching and in either case player j will get P_j^*. Thus, conditional upon the seller's receiving an outside offer first, his expected price is $E[\max(Q_1, P_1^{**})]$ and the buyer's expected price is $P_2^*F_1(P_1^{**}) + P_2^0[1 - F_1(P_1^{**})]$. Similarly, conditional upon the buyer's receiving an outside offer first, the seller's expected price is $P_1^*[1 - F_2(P_2^{**})] + P_1^0F_2(P_2^{**})$ and the buyer's expected price is $E[\min(Q_2, P_2^{**})]$. Finally, at time $t = 0+$ the perfect equilibrium outcome (P_1^*, P_2^*) of the search-bargaining game over $(0,\infty)$ must equal the expected equilibrium prices in the subgame starting at T, adjusted for the search costs over (0,T) and the uncertainty about which player i receives an outside offer first. This leads to the following two equations which the perfect equilibrium outcome (P_1^*, P_2^*) must satisfy.

$$P_1^* = \frac{\lambda_1}{\lambda_1 + \lambda_2} E[\max(Q_1, P_1^{**})] + \frac{\lambda_2}{\lambda_1 + \lambda_2}\{P_1^*[1 - F_2(P_2^{**})] + P_1^0 F_2(P_2^{**})]\} - \frac{C_1}{\lambda_1 + \lambda_2}$$

$$P_2^* = \frac{\lambda_1}{\lambda_1 + \lambda_2}\{P_2^* F_1(P_1^{**}) + P_2^0[1 - F_1(P_1^{**})]\} + \frac{\lambda_2}{\lambda_1 + \lambda_2} E[\min(Q_2, P_2^{**})] + \frac{C_2}{\lambda_1 + \lambda_2}$$

These may be simplified to the following equations, which may be seen to be analogous to equations (1) and (2) for the pure search problem.

$$P_1^* = E[\max(Q_1, P_1^{**})] - \frac{\lambda_2}{\lambda_1} F_2(P_2^{**})[P_1^* - P_1^0] - \frac{C_1}{\lambda_1} \tag{7}$$

$$P_2^* = E[\min(Q_2, P_2^{**})] + \frac{\lambda_1}{\lambda_2}[1 - F_1(P_1^{**})][P_2^0 - P_2^*] + \frac{C_2}{\lambda_2} \tag{8}$$

In order to analyze these equations consider, for example, the solution P_1^* of equation (7) for a given value of P_2^*. If we have $P_1^* > P_2^*$, then equation (7) satisfied by P_1^* reduces to

$$P_1^* = E[\max(Q_1, P_1^*)] - \frac{\lambda_2}{\lambda_1} F_2(P_2^*)[P_1^* - P_1^0] - \frac{C_1}{\lambda_1} \tag{9}$$

and it can be verified that $P_1^* = P_1^0$ must be the unique solution, where P_1^0 solves equation (1). If, on the other hand, $P_1^* < P_2^*$ then P_1^* must satisfy

$$P_1^* + \frac{\lambda_2}{\lambda_1} F_2(P_1^*)[P_1^* - P_1^0] = E[\max(Q_1, P_2^*)] - \frac{C_1}{\lambda_1} \tag{10}$$

and the solution P_1^* can be seen to be increasing in P_2^* although at a rate less than one. Thus, for $P_2^* < P_1^0$ we have $P_1^* = P_1^0$ as the solution of (7), and for $P_2^* > P_1^0$ we have P_1^* increasing in P_2^*.

By a similar argument, we may verify that for given values of $P_1^* < P_2^0$, the solution P_2^* of equation (8) is increasing in P_1^* at a rate less than one and satisfies

$$(11) \qquad P_2^* - \frac{\lambda_1}{\lambda_2}[1 - F_1(P_2^*)][P_2^0 - P_2^*] = E[\min(Q_2, P_1^*)] + \frac{C_2}{\lambda_2}$$

while $P_1^* > P_2^0$ reduces equation (8) to

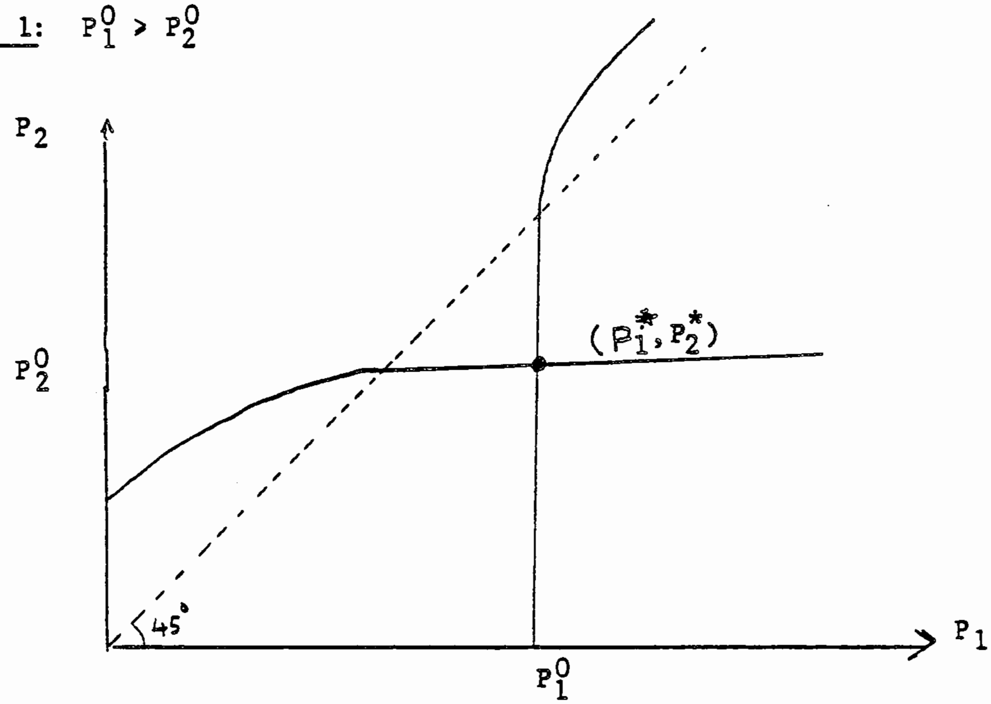
$$(12) \qquad P_2^* = E[\min(Q_2, P_2^*)] + \frac{\lambda_1}{\lambda_2}[1 - F_1(P_1^*)][P_2^0 - P_2^*] + \frac{C_2}{\lambda_2}$$

which has the unique solution $P_2^* = P_2^0$ satisfying equation (2).

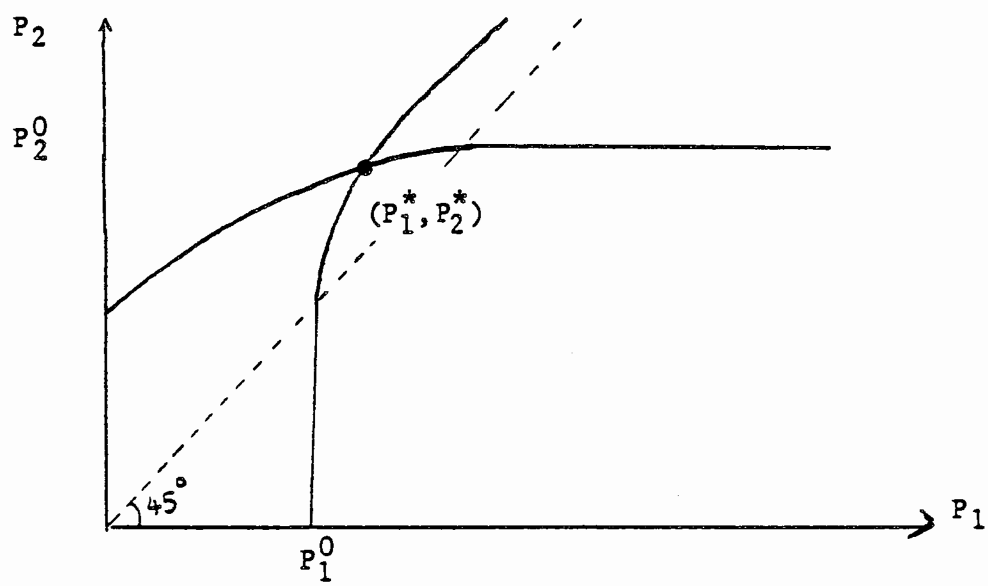
Thus, we can plot equations (7) and (8) as in Figure 2 and obtain a unique solution (P_1^*, P_2^*) depending upon the relative magnitudes of P_1^0 and P_2^0, the players' pure search reservation levels. It can be seen that if $P_1^0 > P_2^0$, then $P_i^{**} = P_i^* = P_i^0$, $i = 1,2$. In this case, the seller expects to get a higher market price than what the buyer expects to pay, neither player has anything to gain from bargaining and each player simply searches the market on his own for his optimal reservation price. If, on the other hand, $P_1^0 < P_2^0$, then we have $P_1^0 < P_1^* < P_2^* < P_2^0$, so that $P_i^{**} = P_j^*$. In that case, each player searches outside for a price which is at least as good as what the other player is willing to settle for. In any case, we have $P_1^* > P_1^0$ and $P_2^* < P_2^0$, so that both players are better off playing the search-bargaining game than altogether abandoning negotiations to conduct independent search. We summarize the main results of this section as Proposition 1.

Figure 2: The Equilibrium

Case 1: $P_1^0 > P_2^0$



Case 2: $P_1^0 < P_2^0$



Proposition 1. The search-bargaining game has a unique perfect equilibrium outcome (P_1^*, P_2^*), which is given by the solution of equations (7) and (8), where (P_1^{**}, P_2^{**}) are defined in (6) and (P_1^0, P_2^0) are the players' pure search "reservation prices" obtained as the solutions of (1) and (2). We either have (a) $P_1^* = P_1^0 > P_2^0 = P_2^*$ and the search bargaining game reduces to two disjoint individual search problems, or (b) $P_1^0 < P_1^* < P_2^* < P_2^0$, in which case P_j^* is player i's "asking price" that he searches for in the market and P_i^* is his "concession price" that he will accept if offered by player j. In either case, the search-bargaining game equilibrium outcome (P_1^*, P_2^*) is Pareto superior to the pure search outcome (P_1^0, P_2^0).

Finally, given the equilibrium (P_1^*, P_2^*) of the search bargaining game over $(0,\infty)$, it is clear that at time $t = 0$ the two players simultaneously offer $P_{10} = P_1^*$ and $P_{20} = P_2^*$. If $P_1^* > P_2^*$ (which happens when $P_1^0 > P_2^0$), they have a perpetual disagreement resulting in independent searches. If, however, $P_1^* < P_2^*$, then our rules of the game specify concluding the trade at $t = 0$ at price $(P_1^* + P_2^*)/2$. Thus, given the perfect information structure of the game, either the bargaining goes on forever or the trade takes place instantaneously.

4. SEARCH ABILITY AS THE BARGAINING STRENGTH

Our model is intended to capture a player's bargaining strength in terms of his ability to receive good outside offers fast and at low search costs. In this section we provide the straightforward comparative static results which show that in equilibrium a stronger player can expect a superior outcome for himself.

Consider, for example, the seller with search characteristics (C_1, λ_1, F_1). Lower values of C_1, higher values of λ_1 and pointwise smaller distribution function $F_1(\cdot)$ represent a seller in a stronger bargaining position. From the individual search theory, it is well-known that P_1^0 is decreasing in C_1 and $F_1(\cdot)$ and increasing in λ_1. (This follows from the fact that the left side of equation (4) is decreasing in P_1^0 and $F_1(\cdot)$.) Similarly, the buyer's pure search outcome P_2^0 is increasing in C_2 and $F_2(\cdot)$ and decreasing in λ_2. Consequently, if $P_1^0 > P_2^0$, we are in Case 1 of Figure 2 with $P_1^* = P_1^0$ and $P_2^* = P_2^0$, and it is clear than an increase in the seller's (buyer's) search ability increases (decreases) his equilibrium price $P_1^*(P_2^*)$, without, however, affecting the other player's outcome.

If $P_1^0 \leq P_2^0$, we are in Case 2 of Figure 2 and (P_1^*,P_2^*) is the solution of equations (10) and (11). From equation (10) it is clear that a decrease in C_1, an increase in λ_1, or a stochastic increase in Q_1 (i.e., a pointwise decrease in $F_1(\cdot)$) increases the right side directly and decreases the left side through an increase in P_1^0. Since the left side of (10) is increasing in P_1^*, it follows that with an increase in the seller's search ability, the new equilibrium point (P_1^*,P_2^*) will be higher. By a similar analysis of equation (11) it follows that a decrease in C_2, an increase in λ_2 or a stochastic decrease in Q_2 decreases the buyer's equilibrium price P_2^*, while the seller's price P_1^* may decrease or remain the same. Thus we have

Proposition 2. In equilibrium of the search-bargaining game, an increase in a player's bargaining strength (in terms of a favorable stochastic shift in his market price distribution or an increase in the offer arrival rate or a decrease in the search cost rate) improves his outcome without improving the other player's outcome.

5. REMARKS

We have considered the problem of bargaining between two individuals who can also search outside for alternatives to each other's offers and we have viewed their search abilities as bargaining strengths. We have modeled the joint process of bargaining and search as a noncooperative game in the extensive form and we have characterized the unique perfect equilibrium outcome and strategies. We have shown that a player with a stronger search ability can expect a better outcome for himself.

In addition to modeling an individual's bargaining strength, our analysis may be viewed as an attempt to combine theories of bargaining and search. In order to incorporate the search possiblity in the axiomatic approach to bargaining, we have shown that (P_1^*, P_2^*) is Pareto superior to, and hence a "more appropriate" disagreement point than (P_1^0, P_2^0), i.e., both players should agree to play the proposed search-bargaining game than threatening to abandon bargaining in favor of pure search. (In this connection, see Binmore, Rubinstein and Wolinksy [2] for a general discussion of the choice of the Nash model specification in economic applications). With (P_1^*, P_2^*) as the disagreement point, if $P_1^0 > P_2^0$, we have $P_1^* = P_1^0$ and $P_2^* = P_2^0$, yielding (P_1^0, P_2^0) as the Nash bargaining solution corresponding to indpendent searches by the players who have nothing to gain from mutual negotiations. If, however, $P_1^0 < P_2^0$, a gain from trade exists and the Nash bargaining solution yields $((P_1^* + P_2^*)/2, (P_1^* + P_2^*)/2)$ as the outcome. In either case, the Nash solution with (P_1^*, P_2^*) as the disagreement point coincides with the equilibrium outcome of our game at $t = 0$. Implementation of (P_1^*, P_2^*) as the disagreement point corresponds to the players' agreeing to conduct bargaining and search simultaneously, rather than as two disjoint phases and this has led naturally to a strategic model of bargaining with search.

As is true of all strategic models, the reasonableness of (P_1^*, P_2^*) as the equilibrium outcome depends upon the reasonableness of the proposed rules of the game involving bargaining and search. Although any proposed set of rules of the game may be questioned, we feel feel that, given our problem structure, ours are natural and reasonable. For example, one may criticize that in our model, player i who receives an outside offer first has the right to make an offer to the other player and hence in equilibrium can "drive him" (up or down) to P_j^*, regardless of how inferior player i's outside offer is. In this connection, player i's right to make an offer first is natural because he is the first one to know about the outside alternative available to him and hence can make the first move. Moreoever, he does not need to inform the other player about the size of his outside offer and therefore has an advantage in selecting the offer to be made. Note also that, in spite of this advantage, the proposer cannot drive the other player all the way to P_j^0, but only to P_j^*. Even though a player i with a relatively high λ_i but inferior F_i may seem to have an unfair advantage (in terms of being able to make an offer first and hence extract a maximum surplus), his expected outcome P_i^* will be inferior, since it depends not only on λ_i but also on F_i. Similarly, given the perfect information assumption, we have ruled out any moves by the players (such as counter offers) unless new information becomes available. As long as no new information is exchanged, any sequence of moves and countermoves by the players would collapse, in equilibrium, to an offer by one player and its acceptance or rejection by the other player. Finally, it is clear that by adopting a different set of rules of the game we will get a different equilibrium outcome. However, based on our analysis of alternate rules, we conjecture that (P_1^*,P_2^*) may be the best that the players can hope for and hence represents an optimum degree of disagreement. In any case, in this paper, our main objective has been to propose a game of bargaining and search whose equilibrium outcome is superior to the independent search outcome that appeared reasonable as a disagreement point at first blush.

As to possible extensions of this paper, we first note that ours has been a partial equilibrium analysis involving bargaining between two players who can search for, but do not bargain with, external alternatives. A market equilibrium analysis would involve modelling the process of search and bargaining among all sellers and buyers. This approach has been adopted recently in an important paper by Rubinstein and Wolinsky [14]. They propose an elaborate set of rules to embed Rubinstein's bargaining model into a market setting with matching between given numbers of identical sellers and buyers, and show that the equilibrium does not resemble the competitive solution even in the limit as the market becomes "frictionless." The corresponding extension of our model to a market equilibrium analysis would involve an infinite number of hetergeneous sellers and buyers with endogenously determined arrival rates and price distributions.

As another possible extension in our setting of bilateral exchange, one may consider the possibility of incomplete information about the players' bargaining strengths. An excellent review of the relevant literature and a comprehensive bibliography on the problem of bargaining under incomplete information may be found in Cramton [4].

We hope to report at a later date on both of these important but difficult extensions of the present paper.

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