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INCENTIVE COMPATIBILITY PROBLEMS IN SOVIET-TYPE ECONOMIES

by

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INCENTIVE COMPATIBILITY PROBLEMS IN SOVIET-TYPE ECONOMIES

I. Introduction

In the classic "Soviet-type" or "command" economy, all ultimate wisdom, virtue, and authority resides at the political apex of a complete hierarchy, that reaches down to the lowest unit of production.[1] The purpose of that hierarchy is to realize the wishes (and wisdom) of the central authority through the implementation, in progressively growing detail, of the objectives outlined by the center and the directives issued subsequent to those wishes and objectives. Of course, that implementation requires detailed information of time, place, and circumstance of activity, all of which is continually changing. Hence the hierarchy must incessantly gather (generate), organize, and transmit the requisite information to the appropriate (derivative) authorities to whom responsibility for dealing with a particular piece of information (aspect of reality) has been delegated by the central (ultimate) authority. In principle this detailed information must rise in the hierarchy to a level sufficiently high to encompass within its purview all of the consequences of any decision that might be taken on the basis of that information. Subordinates merely implement, perhaps with elaboration of detail, the decision taken by a superior by following as closely as possible the commands issued by that superior. They are to do as ordered, and refrain from taking any action not ordered. Thus, in a complex social economy where all economic agents are interdependent, any trade off between objectives or among the products of the system can only be made by the highest central authorities. This is the logic of the model behind the Soviet-type or command economy.

Of course, this logic in practice implies an impossible task for, and burden on, the central authorities. As Alec Nove points out,[2]

> "the tasks which the directing organs ought to be carrying out, given the logic of the model, cannot possibly be carried out by them...(C)entralized decision making in a large, modern industrially developed economy sets up an unmanageably large number of micro-economic inter-relationships. None the less, the economy functions, and this is only because the central planners do not in fact take the bulk of detailed decisions. We shall be seeing that many of the essential weaknesses of the Soviet-type economy arise because, within the logic of the model, the central planners should be taking them."

The enormity of the real economic problem necessarily forces a decentralization of decision-making authority. That decentralization, however, is rigidly channelled by the hierarchical structure of the command economy: each decision is dealt with from the particular perspective of the specific functional responsibility of that "node" in the hierarchy actually making the decision. And this decision can only be made on the basis of "local" information and "local" perceptions of the economic situation. A lower level decision-maker necessarily ignors the impact of his decisions on those branches of the hierarchy outside of his direct control: that is the responsibility of others. The inherent logic of hierarchical organization dictates this as the only proper approach. Yet this approach implies that existence of massive "induced externalities", i.e. impacts, not inherent in either technologies or preferences, on the capabilities of, and consequences (payoffs) to, others in the economy that are necessarily ignored in making the decision.[3] These external effects alter the economic environment of other low-level decision makers influencing both their behaviors and the economy-wide consequences of those behaviors. These effects can only, in principle, be dealt with at a sufficiently high level to internalize them all, yet at that level there is necessarily insufficient information, computational capability, and time to act appropriately. The authority to deal with all but the most general economic questions must usually be delegated, with inevitably highly deleterious consequences. This again follows from the "logic of the model."

Of course, in reality, decision-making authority and information flows cannot (for long) be restricted to hierarchical channels, regardless of the logic of the model: that would invite true disaster. The Soviet Model (in Nove's terms) must be tempered by horizontal (negative) feedbacks between agents belonging to very different branches of the complete hierarchy.[4] The absence of a market interface between autonomous (self-directed) economic agents must be compensated for, and the substitute must be consistent with the hierarchical structure of authority if it is to be viable in a command-economy (centrally planned) environment. And, indeed, it seems that there are several such substitutes operating in the Soviet economy, including (though not mutually exclusive, or restricted to) the Second Economy (Grossman 1977, 1979; Ericson 1983b,1984), informal informational flow and feedback through the use of <u>blat</u>, <u>tolkachi</u>, old school ties, etc. (Berliner, 1957; Powell, 1977), the local Party organs (Hough, 1969), and the formal processes of planning, counter-planning, contract formation (Berliner, 1975), etc. While distinct, all of these mechanisms tend to involve direct, personalized interaction between subordinates and superiors and among subordinates of the same or different superiors.[5] Further, the structure of interaction/communication is strongly influenced, if not always determined, by the hierarchical structure of authority in the economy. This interaction seems to be a major and necessary, part of the process of plan implementation, and the key to the preservation of the viability of the system. The behavior of economic agents generating this interaction is thus an important object of investigation/research if we are to understand the functioning of Soviet-type economies. The structure of the problem, however, means that typical economic models revolving around autonomous agent optimization subject to parametrically given constraints comprise an inappropriate tool for studying this behavior. One of the purposes of this paper is to propose an alternative, game theoretic, framework for the modeling and analysis of this interaction, that captures, albeit in highly simplified form, both its hierarchical structure and its personalized, non-anonymous nature.

Another characteristic of hierarchically organized, centrally planned economies that has been widely recognized, yet has not been subject to formal theoretical analysis, is the somewhat surprising stability of economic behavior and outcomes in the face of repeated "reform" of the economic mechanism. In particular, these economies exhibit a highly stable complex of dysfunctional behaviors and and inefficiencies in spite of repeated changes in incentives and constraints specifically aimed at altering them.[6] Such a situation evidently cannot be predicted by the standard model of economic behavior as optimization subject to parametric constraints. The model proposed in this paper was again, in part, developed to address this issue of behavioral stability. It provides a framework within which the stability question can be meaningfully analyzed, though a formal analysis is not undertaken here.

The existence of personalized, hierarchically structured feedbacks and the persistence of dysfunctional behaviors in command economies suggest a research hypothesis. It is that observed outcomes are generated by an equilibrium of the non-cooperative game arising out of the attempt to centrally manage (i.e. use the command mechanism) an organism as complex (i.e. multi-faceted and multi-intentional) as a national economy. In this game the central authorities (or superiors, in general) play a leading, coordinating role with respect to subordinate agents. In particular they control the structure of communication, and to a large extent the use to which communicated information will be put (Kornai's "control sphere", 1971). Subordinates interact in subgames, structured by the activity of the center, which generate the real outcomes of the economy (Kornai's "real sphere", 1971). These subgames involve the communication and use of, unverifiable "local" information and the taking of centrally unobservable actions in the course of carrying out central commands and implementing the plan. Under an assumption of "Bayesian rationality" (Harsanyi, 1967/8) this structure of interaction generates what Myerson calls a Bayesian Game of Incomplete Information with Communication (Myerson, 1983a) or a Generalized Principal Agent Problem (Myerson, 1982). It is in just this framework that I propose to address the two motivating observations of this paper.

This approach suggests a number of far reaching implications for our understanding of how Soviet-type economies function and why they appear so resistant to efficiency oriented change/reform. Observable outcomes in CPE's are, arguably, the consequence of rational (strategic) equilibrium behavior in the non-cooperative "game" generated by comprehensive central planning and the command mechanism. Economic "reform" which has come in all CPE's but Hungary to mean marginal fiddling with the economic "mechanism", merely results in a slight modification of the "payoff structure" of the game without altering its fundamental nature. Thus equilibria, though somewhat changed by such reforms, retain their essential characteristics; "reform" proves counterproductive and hence, rationally, is reversed or further "reformed".[7] This might explain "mini-cycles" of reform of the type repeatedly observed in the Soviet Union: an obvious inefficiency leads to a "reform" aimed at eliminating it -- the inefficiency abates slightly while agents adjust to the new "system" --

performance begins deteriorating in both the original and in new areas as agents learn to take advantage of the new mechanism -- new constraints/adjustments are made in the reform essentially repealing it.[8] Furthermore, it suggests that the only "reform" that has any hope of surviving is one which fundamentally alters the game faced by economic agents. But the game being played is just the working of the "command mechanism" for implementing superiors' preferences/desires in the face of inevitable, yet natural, informational constraints. Hence true reform might involve wholesale abandonment of the command implementation mechanism and, perhaps, of the attempt to impose all but the most general central preferences. Finally, this approach suggests that there can be no "incentive scheme" for a command economy that avoids the dysfunctional behaviors currently observed in CPE's.[9] For given the structure of the game and the actions of other agents, such patterns of behavior are indeed optimal for each individual agent. The inefficiency of the outcomes they generate is a reflection of the induced externalities inherent to a CPE, which cannot be eliminated by slightly perturbing the small set of payoffs affected by any "incentive scheme". Again, to induce efficient behavior, a fundamental change of the system seems to be implied.

Needless to say, such broad issues cannot be resolved in this, or any single, paper. The aim here is rather more modest: to develop a model that addresses, if only partially, the issues raised above from a different perspective and to explore some of its immediate consequences. In Section II of this paper we present the basic structure of the model and argue for its relevance. A few general results dealing with the existence and characterization of equilibria are developed in Section III. Then two special cases of the formal model are analyzed through examples in Section IV. They deal with a pure planning (collective choice under incomplete information) problem and a pure plan implementation (correlated equilibrium) problem respectively, and illustrate the type of analysis possible in this model. Finally the paper concludes (Section V) with a discussion of the power and limitations of this approach, of the next steps that must be taken to extend this analysis, and of some implications for the Soviet economy.

II. The Model

The centrally planned economy modeled here contains n+1 agents, indexed i=0,1,...,n. The central authorities (hereafter "center"), i=0, comprise the apex of the (two-level) administrative hierarchy, where all wisdom and authority reside. They operate exclusively in the "control sphere" of the economy, determining what ultimately is to be done, how it is to be done, and by whom. The outcome of their activity is a comprehensive economic plan, accompanied by detailed allocation and implementation orders to the other agents. The remaining agents, i=1,...,n, (who may be thought of as CPE ministries) preside over all economic activity of the "real sphere". Their actions directly generate all the economic outcomes relevant to the central authorities, and to themselves, and hence it is they who must implement all economic plans.[10]

Ideally, an omniscient center would determine an "optimal" plan, command its implementation in detail, and subordinate agents would precisely and fully carry out the central commands. There are, however, many serious problems with this scenario, not the least of them being the inherent uncertainty in any economic environment, the bounded rationality of all economic agents, and the self-interested behavior of all economic agents. Further, the information needed to be able to plan "optimally" is dispersed among the subordinate agents and many of the actions that must be taken by subordinates to implement any plan are unmonitorable and hence unenforceable by the center.

These circumstances generate at least two critical incentive problems that must be captured in any reasonable model of planning and plan implementation. As succinctly summarized by Myerson (1983, p. 1), whom I paraphrase, they are:

(i) An agent with unverifiable, private information, not available to others, cannot be compelled to reveal that information honestly without the correct incentives. [Adverse Selection Problem]

(ii) An agent controlling some private decision variable that others can't control or monitor, cannot be directed (commanded) to choose any particular decision or action unless he is given the proper incentive to do so. [Moral Hazard Problem]

The structure of this model was specifically chosen to capture these two incentive compatibility (IC) problems as constraints on the choice of plans and actions that can be implemented in a CPE. The hypothesis is that precisely these problems generate the observed stable complex of dysfunctional behaviors of every CPE.

The uncertainty which generates the dispersion, and hence "private" nature of much economically relevant information in the economy, is modeled as a probability space of "states of the world", $(S, \mathcal{X}, P)$. Although no agent knows the true state, s, all agents operating in the real sphere have some private information about it.[11] The information received by agent i in state s, $t_i(s)$, is modeled by a "signal" function, $t_i: S \to T_i$, where T_i is the set of possible signals (i's "information" or "type" space) that agent i might receive, given his position and function in the economy. This information, though unknown to and unverifiable by others, is of global economic relevance as it reveals something about the true capabilities of the economy and the consequences to actions/decisions taken by any of the agents. Let $T \equiv \times_{i=0}^{n} T_i$ be the space of all conceivable information states of the economy, and $t \equiv (t_1,\ldots,t_n) \in T$ be one such realized state.

Each agent i, including i=0, has a set of actions/decisions, A_i, that he alone can take and/or monitor. The set A_i is a "universal strategy set" for agent i in the sense that it contains all the possible private actions/decisions, a_i, that he might carry out in any conceivable state of the world. However, for a given state, s, only some subset of A_i, in general, will be feasible. This feasibility constraint is incorporated, as suggested by Harsanyi (1967/8), in the "payoff" (objective) function of agent i, $u_i(\cdot)$; attempting an infeasible a_i yields an arbitrarily large negative payoff, $u_i = -K$.[12] Let $A \equiv \times_{i=0}^{n} A_i$ be the space of all conceivable joint actions/decisions of agents in the economy, and let $a \equiv (a_0,\ldots,a_n) \in A$ denote one such joint action. The consequences of the functioning of the economy for an agent i are summarized, in the eyes of that agent, by the payoff function, $u_i : A \times S \to \mathbb{R}$, i=0,1,...n. This function is assumed to be a von Neuman-Morgenstern utility indicator which depends, non-trivially, on the full information state of the economy and the entire vector of joint actions/decisions of all of the agents in the economy. This payoff structure models the direct interdependence engendered in the imposition of a complete hierarchical structure on a complex social economy (Grossman, 1963).[13] Agents recognize this interdependence and are assumed to take it into account in choosing the actions and decisions they will carry out.

As both outcomes and the actions of other agents are directly affected by the unverifiable private information of those agents, any agent i must take the possible states of this information into account in order to "rationally" choose his own best actions. Agents are assumed to be "Bayesian rational" in that they evaluate the likelihood of information states according to a distribution conditional on their own private information: $p_i : T_{-i} \times T_i \to [0,1]$, where $T_{-i} \equiv T_o \times \ldots \times T_{i-1} \times T_{i+1} \times \ldots \times T_n$. Hence, $p_i(\cdot|t_i)$ shows agent i's "belief's" about the likelihood of various $t_{-i} \equiv (t_o, t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \in T_{-i}$, conditional on his knowing $t_i^{-1}(t_i) \subset S$, i.e. $\{s | t_i(s) = t_i\} \in \mathcal{S}$. This distribution is used by the agent to determine the expected payoff to any course of action that he might follow.

To summarize, each agent in this model is thus characterized by a 5-tuple $(A_i, T_i, u_i, p_i, t_i)$ of, respectively, possible actions, possible types, preferences, beliefs, and an information signal. The actions of all agents but the center are unverifiable and hence unenforceable by the center. All conceivable actions that are observable and hence enforceable by the central authorities are included in A_0; it is as if the center could directly take them, even though they will actually be carried out by others. In addition, A_0 contains all incentive schemes, normatives, plan parameters, etc. that the center might want to enforce. The center is also assumed different from subordinates in that it has no direct private information, i.e. t_0 is a constant mapping or T_0 is a singleton. This is an assumption that whatever the center knows ex ante is common knowledge to the subordinates, so that central behavior/communication with respect to the subordinate agents can be assumed to be straightforward.[14] Hence, we can dispense with t_0 and T_0 in what follows, and take p_0 to be an unconditional prior distribution on $T = \times_{i=1}^{n} T_i$.

This structure formally generates a non-cooperative Bayesian game of incomplete information (Myerson, 1983a),

$$(1) \qquad \Gamma = (A_0, \{A_i, T_i, u_i, P_i\}_{i=1}^{n}, u_0, P_0).$$

It is, however, incomplete as a model of command economy planning, as it incorporates neither the exchange of information that is the heart of any planning, nor the distinguished role that the center plays in that process. Modeling the process of planning introduces a communication structure into the game, i.e. a language and rules for transfering information. In a strict command economy the center has absolute control over the structure of communication, which essentially consists of information flowing up the hierarchy and commands flowing down. The only information in the economy that there is to be communicated is the complete information state, $t \in T$, so the central authorities naturally demand its transmission. Similarly, the only message that the center can ultimately send down to the agents is a set of action/decision relevant commands, i.e. a complete plan of action for all agents, $a \in A$. Thus the operation of a CPE can be viewed as taking place in six general stages, only the last two of which relate to the real sphere of the economy.[15]

(i) "Nature" draws $t \in T$, $t_i = t_i(s)$, $s \in S$, according to the probability distribution P, with agent i privately receiving signal t_i (his "type").

(ii) The center chooses a "planning procedure" or <u>mechanism</u>, i.e. a procedure for eliciting information and generating plans/commands, and informs all agents of the chosen procedures. Agents are asked to communicate their private information (type) to the center, i.e. t_i for each i.

(iii) Agents respond (noncooperatively) to this elicitation by reporting plausible, and perhaps accurate, information, $\tau_i \in T_i$. Agents respond strategically in that they will manipulate information when it is in their own interest. [Incentive Problem (i) - Adverse Selection]

(iv) The central authorities use the "mechanism" chosen in stage (ii) to generate a comprehensive plan, i.e. a set of commands indicating appropriate actions for all agents, a_i, $i=0,\ldots,n$. Those actions directly controlled by the center, a_0, are publicly announced and become common knowledge (Aumann, 1976). Directives regarding the private actions of any subordinate i, a_i, are privately communicated to i.

(v) Subordinate agents now independently (noncooperatively) and strategically choose the actual actions/decisions they will implement, α_i, and carry them out. [Incentive Problem (ii) - Moral Hazard]. Only α_0 is necessarily as planned, i.e. $\alpha_0 \equiv a_0$.

(vi) Real physical outcomes are generated in the economy as a consequence of the "state of nature", s, and the actions of all the agents, $\alpha \equiv (\alpha_0, a_1, \ldots, \alpha_n)$. This yields (non-transferable) payoffs to each of the agents, $u_i(\alpha, t(s))$, $i=0,1,2,\ldots,n$.

At each stage of this process agents, it is assumed, act as Bayesian-rational optimizers, i.e. they maximize expected utility/payoff through the strategic choice of information reported and final action taken, where the expectation is taken with respect to beliefs about the state of the world (and other agents beliefs) that have been updated on the basis of private information. Economic agents are able to do so consistently as we assume that the structure of the game and its universal spaces $(A, T, \{u_i\}, \{p_i\})$ are common knowledge.

In addition to the general assumptions noted above in building the model, five technical assumptions are made to facilitate the derivation of results.

(A.1) S is compact, metrizable, and $\mathcal{B}$ is P-complete.

(A.2) T_i is finite and $t_i^{-1}(\tau) \in \mathcal{B}$ for all $\tau \in T_i$, $i=1,\ldots,n$. Let $\mathcal{T} = \sigma(t_1,\ldots,t_n) \subset \mathcal{B}$ be the smallest σ-field generated by the maps $\{t_i\}_{i=1}^{n}$.

(A.3) $p_i(\cdot|t_i)$ is a conditional probability distribution on T_{-1}, $i=1,\ldots,n$; $p_0(\cdot)$ is a probability distribution on T.

(A.4) A_i is a compact metric space and $\mathcal{A}_i$ is Borel σ-field, $i = 0,1,\ldots,n$. $\mathcal{A} = \otimes \mathcal{A}_i$.

(A.5) $u_i : u_i : A \times S \to \mathbb{R}$ is $(\mathcal{A} \otimes \mathcal{T})/\mathcal{B}$-measurable, where $\mathcal{B}$ is the real Borel σ-field, and u_i is continuous in $\underline{a}$, $i=0,1,\ldots,n,$.

The assumption that is apparently most restrictive is (A.2) - the finiteness of T_i. For the private information an agent has, his "type", must include what he knows and believes about the information the other agents have, including what they believe about what he knows about what they know, etc... However, Mertens and Zamir (1982) have recently shown that the space of beliefs containing this infinite regression can be approximated arbitrarily closely by a finite space.[16] Thus, in view of the bounded rationality of all economic agents, we can assume with little loss of generality that there are only finitely many information states in which a given agent might find himself. This eliminates any existence problems with respect to conditional probability distributions and the need for any special continuity assumptions regarding the second argument of u_i. Notice that (A.5) implies that, w.l.o.g. $u_i(a,t)$ can be defined on T rather than S -- it cannot vary over elements of $\mathcal{S}$ not contained in $\tilde{\mathcal{T}}$. Finally, notice that there is no necessary consistency among the $\{p_i\}$ or between p_i and P, i=0,1,...,n; agents may have quite different and, indeed, mutually incompatible and/or irrational beliefs.[17] Agents, however, must use those beliefs rationally, as discussed above.

In a command economy, the planning and implementation procedure we are modeling is orchestrated by the central authorities in pursuit of their own interests, interpreted as social interests. That is, the center tries to elicit the true state of the economy, t(s), and issue appropriate commands on the basis of that information, $a = (a_0, a_1, \ldots, a_n)$, so that its expected payoff $U_0 = E\, u_0(a,t)$ is maximized. How commands are related to the elicited information is determined by the <u>mechanism</u> or planning procedure chosen by the center. That mechanism is modeled here in a very "reduced" form.

<u>Definition</u>: A <u>(direct)</u> <u>mechanism</u>, μ, is a vector of distributions over conceivable joint actions of all the agents in the economy, i.e.

(2) $$\mu = (\mu_t)_{t \in T} \quad ; \quad \mu_t : \mathcal{Q} \to \mathbb{R}_+ \quad , \quad \mu_t(A) = \int_A \mu_t(da) = 1,$$

$$\text{and } \mu_t(C) = \int_C \mu_t(da),$$

for all $t \in T$. Let $\mathcal{P}$ be the set of all probability measures on A. Then $\mu \in \mathcal{P}^T$.[18]

A mechanism, μ, is thus a way of choosing a joint action to be commanded as a function of the reported state of the economy, t. If the support of μ_t is a singleton for any report $\underline{t}$, then μ is a function mapping T into A which assigns one precise plan of action to each fixed reported information configuration. However, a given information configuration need not always result in precisely the same plan. The mechanism reflects the infinitely complex process of turning elicited information into a detailed plan. Many fleeting and idiosyncratic factors enter into that process, including the quirks of bureaucratic procedure, the personalities and interaction of individuals in the planning bureau, traditional, historical, and cultural influences, etc.[19] So one might expect the outcome of the process to be stochastic for any given set of information reported. Further, as we shall see, the center often has a positive reason to randomize over plans and commands that it might issue in any state of the economy, in order to preserve proper incentives for subordinate agents. Thus it is natural, at this level of abstraction, to represent the planning mechanism as a distribution over potential commands. If that distribution is degenerate, the mechanism will be called <u>deterministic</u>. A general mechanism is sometimes called <u>stochastic</u>.

The mechanism chosen by the center naturally has a critical impact on the behavior of subordinate agents, as it strongly influences their expectations about the outcomes of economic activity. It gives each agent information about what the center will require of other agents in any state of the economy, and that information can be useful in determining what his best response is. Thus a mechanism, once chosen by the center, generates a Bayesian Communication Game whose "players" are the subordinate economic agents. Formally this game is denoted:

$$(3) \qquad \Gamma_\mu \equiv (A_0, \{\hat{A}_i, T_i, \hat{U}_i, P_i\}, \hat{U}_0, P_0)$$

where $\hat{A}_i = \{(\tau_i, \alpha_i) | \tau_i$ and $\alpha_i : A_i \to A_i$ is ${}_i$-measurable$\}$ and

$$\hat{U}_i(\mu, a_0, \{\tau_j, \alpha_j(a_j)\}_{j=1}^n, t) = \sum_{t \in T_i} P_i(t_{-i}|t_i) \int_A \mu(da|\tau)\ u_i(a_0, \{\alpha_j(a_j)\}_{j=1}^n, t)$$

$i = 1,\ldots,n$, with $\tau \equiv (\tau_1,\ldots\tau_n)$ and all other symbols defined as above. The set $\hat{A}_i$ is the set of participation (reporting, τ, and implementing, α) strategies of the subordinates in the planning and implementation game created by the centrally chosen mechanism, and $\hat{U}_i$ is the expected payoff to i for any vector of chosen participation strategies, given the mechanism μ. A strategy $(\tau_i, \alpha_i) \in \hat{A}_i$ represents a plan by agent i to report τ_i to the center and then to choose his action in A_i as a function of central commands according to α_i, so that he would do $\alpha_i(a_i)$ if the center orders a_i. It is assumed that each agent communicates with the central authorities separately and confidentially, so that agent i's action cannot depend on the directives to the other agents.[20] Notice that, once the mechanism is chosen, the center, i=0, has no further role in this game, though $U_0(\mu)$ can be defined at the solution or equilibrium of this game.

As argued in Myerson (1983a), the natural solution concept for this type of game is that of Bayesian (Nash) Equilibrium. There are, in general, many such equilibria, not all equally desirable. In particular, for planning to be effective, it is desirable that the center elicit true information from subordinates and that subordinates obediently implement plan directives. Such truthful and obedient participation strategies yield to subordinate agents

$$(4) \qquad U_i(\mu|t_i) \equiv \sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i) \int_A \mu(da|t)\ u_i(a,t)\ ,\ i=1,\ldots,n$$

while manipulative behavior by i, given that all $j \neq i$ are truthful and obedient, yields,

$$(5) \qquad V_i(\mu,(\tau_i\,\alpha_i)|t_i) \equiv \sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i) \int_A \mu(da|t_{-i},\tau_i)\ u_i(a_{-i},\alpha_i),t)$$

Again following Myerson (1983a), we say that a mechanism μ is <u>Bayesian Incentive Compatible</u> (hereafter, Incentive Compatible or BIC) if and only if it is a Bayesian equilibrium for all agents to report their information honestly and implement the center's commands when it uses the mechanism μ. Hence μ is called <u>incentive compatible</u> (IC) iff

$$(6) \qquad U_i(\mu|t_i) \geq V_i(\mu,(\tau_i,\alpha_i)|t_i)$$

for all i, all $t_i,\tau_i \in T_i$, and all measurable $\alpha_i : A_i \to A_i$. If the center uses an incentive-compatible mechanism and each agent communicates independently and confidentially with the center, then no agent could ever gain by being the first one to lie or disobey his plan. Conversely we cannot expect all subordinates to participate honestly and obediently in a planning procedure/mechanism unless it is incentive compatible. Hence an incentive compatible mechanism (planning procedure) is one that fully resolves both of the incentive problems noted above.

The central authorities, however, are interested in more than just resolving incentive problems. They want to optimize performance as reflected in central/social preferences, i.e. to solve $\max_{\mu \in T} U_0(\mu)$ where

(7) $$U_0(\mu) \equiv \sum_{t \in T} p_0(t) \int_A \mu(da|t)\, u_0(a,t).$$

This objective function depends crucially on subordinates behaving honestly and obediently at equilibrium, for otherwise the center has lost the ability to plan (i.e. consistently influence) outcomes. Thus the best the center can do is to solve:

(8) $$\max_{\mu} U_0(\mu) \quad \text{s.t.} \quad \mu \in \mathcal{P}^T \text{ and constraint (6).}$$

An <u>optimal mechanism</u> is a mechanism μ^* that solves the problem in equation (8). The whole purpose of a command (CP) economy is the implementation of such an incentive-compatible optimal mechanism. The hypothesis of this paper is that many shortcomings of the command economy result from its failure to do so, though many may also be characteristics of the equilibria of even incentive compatible mechanisms.

This formulation raises the question: How reasonable is it to demand that the equilibria of planning mechanisms be incentive compatible? There will usually still be Bayesian equilibria if μ is not incentive compatible. Further, in general, there may be many other equilibria of the game Γ_μ, even if μ is incentive compatible. There is, however a sense in which these other equilibria for any given μ are irrelevant, as there always exists another incentive compatible mechanism, μ', that generates precisely the same distribution of payoffs to all agents. As Myerson notes, in a different context,[21]

> "... for any given coordination mechanism and for any given Bayesian equilibrium of the induced communication game, there exists an equivalent incentive compatible mechanism, in which every type of every player gets the same expected utility (when all players are honest and obedient) as in the given Bayesian equilibrium of the given mechanism. In this sense there is no loss of generality in assuming that the players communicate with each other through a mediator who first asks each player to reveal all of his private information (his "type"), and who then gives each player only the minimal information needed to guide his action, in such a way that no player has any incentive to lie or cheat. This result has been observed by many writers independently and it is known as the revelation principle."

Thus nothing is lost by considering only incentive compatible mechanisms. In addition, there is a sense in which incentive compatible equilibria are more plausible than other Bayesian equilibria; honest and obedient behavior is a clear "focal point" among strategies (Schelling, 1960), making the attainment of equilibrium more plausible than when agents must equilibrate in manipulations. Finally, we note that "honest and obedient" behavior in a direct mechanism is the only type of strategy consistent with the ideology and teleology of centralized planning and administration. Planners may accept that manipulation exists, and respond to it rationally, but only as an abberation; they will never admit it as a part of the normal functioning of the economy.

The question of reasonableness, however, runs deeper than the technical details of an equilibrium concept. The model itself must be a sufficient reflection of the economic problem addressed to justify the analytic exercise. I want now to argue briefly that the key characteristics of the types of games here used in modeling capture quite well a number of typical and important characteristics of CPE'S. [22]

There are six critical characteristics of these games. The first is the existence of unverifiable local information of global importance, an undeniable fact in any economy. Second, the agents modeled have beliefs about the nature and availability of this information to others. Third, there exist many unobservable local actions which, however, have an impact far beyond the agent undertaking them. Fourth, the center controls the communication/coordination structure, so that any communication between agents must pass through the center. Fifth, the payoffs to any agent depend jointly on the true state of the economy and the actions of all other agents. Finally, the game is inherently non-cooperative, though all actions are coordinated in equilibrium.

These game characteristics fit quite nicely with a number of aspects of CPE's, though the fit is obviously incomplete and inexact. The most restrictive of the game characteristics is perhaps the requirement that all communication be under strict central control. This is an extreme reflection of the very limited range of allowed interactions among agents: communication must be through the hierarchy, vertical interaction predominates, and there is a formal lack of any alternatives. This means that all interaction is highly individualized and personalized, a fact accentuated by the non-anonymous, explicit nature of directives and constraints (e.g. consider the principle of "adres'nost'" and the formal rationing of producers' goods).

Other characteristics of the model are more closely supported by well-known aspects of Soviet-type economies. First is the formal and actual separation of the loci of dispersed information and of the authority to make use of that information. This separation is aggravated by the general unverifiability of real-time information and of many real-time actions of subordinates by those with authority. Further, incentives for those in the real sphere are geared to pleasing superiors and fulfilling plans/commands, and not to generating useful real economic output. This, coupled with the hierarchical segmentation of economic agents, leads to a rather arbitrary relationship between economic actions and the consequences felt by the actor, resulting in a number of dysfunctional behaviors, including manipulation to achieve favorable observable indices of performance and a general lack of action without explicit, conscious direction from above. These effects are at least partially captured by the characteristics of the model. Finally we note that there is always a large set of actions strictly enforceable by the central authorities (superiors), and that rational agents must be assumed to act on the basis of (more or less) systematic beliefs and expectations with respect to the information and actions of others.[23] Hence a Bayesian game of communication seems to reasonably model several important aspects of the functioning of command/centrally planned economies.

In addition, there is an absolutely critical characteristic of CPE's that is precisely captured by the model: the dependence of all payoffs on the complete vector of real action and information types. This implies that supplying proper incentives to deal with incentive compatibility problems requires altering real actions and outcomes. In a socialist command economy one cannot buy (second-best) optimal outcomes through financial incentives, e.g. the redistribution of wealth/income; there can be no "separation-theorem" for non-market ecnomies. Utility in such an economy is extremely non-transferable: there is no true "money" and hence financial wealth has very little meaning.[24] In a socialist society "you are what you (are allowed to legally) do": wealth is control over physical materials and processes, and hence well-being depends on position in the hierarchy and its formal powers and capabilities. Thus effective incentives must be tied to direct physical outcomes and movement up the hierarchy, based on superiors' perceptions of those physical outcomes. Incentive problems therefore imply a far more serious real cost in command economies than at the second best "optimum" of a principal-agent problem of a market economy. By a proper choice of action/decision spaces for the agent, this situation can be nicely encompassed by this game-of-incomplete-information model.

The general model presented above is quite complicated and difficult to analyse in detail. Though we present some general existence and characterization results in Section III, much of this paper is devoted to analyzing two special cases, each isolating one of the two general incentive problems outlined above. In Section IV.A we analyze an example of a "pure planning" (collective social choice) problem in which only local information is unverifiable and all local actions are centrally observable and hence enforceable. Thus $A_i = \phi$, $i=1,\ldots,n$, $A = A_0$, and the only incentive problem is one of adverse selection:

$$(9) \qquad \Gamma^P \equiv (A, \{T_i, u_i, p_i\}_{i=1}^n, u_0, p_0)$$

The other incentive problem, moral hazard, is analyzed in isolation in a series of examples in Section IV.B. In a CPE this models a "pure implementation" (coordination) problem in which there is no objective uncertainty $T_i = \{\bar{t}_i\}$, $p_i(\bar{t}_{-i}|\bar{t}_i)$, $= 1$, $i=0, 1,\ldots,n$, but local agents can take actions/decisions that are unmonitorable (except, perhaps, at prohibitive cost). The general communication game then reduces to

$$(10) \qquad \Gamma^I \equiv (A_0, \{A_i, u_i\}, u_0).$$

As in the general case, payoff's and constraints are given by equations (2), (4)-(8), suitably modified to take account of the simplifications of the special cases.

III. Some General Results

The model presented above is largely a reinterpretation of the by now standard model of equilibrium with private information and private actions. Existence and characterization results have been derived for the case where both the set of information states (types) and the set of private actions are finite in number. As our examples in Section IV belong to that case, two of these results are restated below. However, further meaningful development of the model requires that we extend these results to the situation of non-finite spaces of potential private actions. In what follows a general existence result will be proven and the characterization of optimal plans and their implementing instructions begun.

In the standard model, assumption A.4 is replaced by:

(A.4') $\quad A_i$ is a finite set and $\mathcal{A}_i = 2^{A_i}$, $i=0,1,\ldots,n$. $\mathcal{A} = \otimes \mathcal{A}_i$.

Under this assumption a completely standard argument gives

Proposition 1: If (A.1), (A.2), (A.3), (A.4'), (A.5) hold, then the incentive compatibility constraints in Γ define a non-empty closed, convex polyhedron in $\mathbb{R}^{\sum_{i=1}^{n} |T_i||A_i| - n}$, the space of all coordination mechanisms. Hence there exists an optimal coordination mechanism, μ^*.

Thus it is immediate that there also always exist optimal mechanisms in Γ^P and Γ^I, the pure planning and pure implementation games. Further, a rather complete characterization of these optimal mechanisms is possible, as they can be derived as the solution to a finite linear programming problem (Myerson, 1982, Proposition 1, p. 73). This characterization is most thoroughly developed in Myerson (1983), though only for Pareto optimal mechanisms. For centrally, U_0, optimal mechanisms, a modification of this characterization is possible. In this characterization we assume that beliefs are consistent with a common prior and players' "types" are independent random variables in that prior.[25]

Proposition 2: Under the conditions of Proposition 1 an optimal coordination mechanism, μ^*, can be characterized as follows: There exist vectors θ and β such that:

(11.1) $$\theta_i(\tau_i|t_i) \geq 0 \ , \ \beta_i(\alpha_i|a_i,\tau_i,t_i) \geq 0 \text{ for all } i \in \{1,\ldots,n\},$$
$$\forall t_i \in T_i,\ \forall \tau_i \in T_i,\ \forall a_i \in A_i,\ \forall \alpha_i \in A_i;$$

(11.2) $$\sum_{\alpha_i \in A_i} \beta_i(\alpha_i|a_i,\tau_i,t_i) = 1 \qquad \forall i,\ \forall a_i \in A_i,\ \forall \tau_i \in T_i,\ \forall t_i \in T_i;$$

(11.3) $$\sum_{t_{-i} \in T_{-i}} \ \sum_{a \in A} \ \sum_{\alpha_i \in A_i} p(t_{-i}|t_i)\, \mu^*(a|t_{-i},\tau_i)\, \beta_i(\alpha_i|a_i,\tau_i,t_i)\, u_i(a_{-i},\alpha_i,t) =$$
$$= \max_{\alpha_i : A_i \to A_i} V_i(\mu,(\alpha_i,\tau_i)|t) \qquad \forall i,\ \forall \tau_i \in T_i,\ \forall t_i \in T_i;$$

(11.4) $$\theta_i(\tau_i|t_i)[U_i(\mu|t_i) - \max_{\alpha_i : A_i \to A_i} V_i(\mu,(\alpha_i,\tau_i)|t_i)] = 0,\ \forall i, \forall \tau_i, t_i \in T_i;$$

(11.5) $$\sum_{a \in A} \mu(a|t) \sum_{i=0}^{n} V_i(a,t,\theta,\beta) = \max_{a \in A} \sum_{i=0}^{n} V_i(a,t,\theta,\beta),\ \forall t \in T$$

where $V_0(a,t,\theta,\beta) = u_0(a,t)$ and, for $i=1,\ldots,n$

$$(11.6) \quad V_i(a,t,\theta,\beta) = [\sum_{\tau_i \in T_i} \theta_i(\tau_i|t_i)\, u_i(a,t) - \sum_{\tau_i \in T_i} \theta_i(t_i|\tau_i) \times$$

$$\times \sum_{\alpha_i \in A_i} \beta_i(\alpha_i|a_i,t_i,\tau_i)\, u_i(a_{-i},\alpha_i,t_{-i},\tau_i)]/p_i^*(t_i)$$

with p_i^* the common knowledge marginal probability that agent i is of "type" t_i, i.e. $p^*(t) = \prod_{i=1}^{n} p_i^*(t) = p_0(t)$.

When beliefs are consistent and independently drawn from a common prior, this characterization shows an optimal mechanism as one maximizing, for each state of the economy, t, the sum of what Myerson calls virtual utilities of the participants in the planned economy. The virtual utility of the center is just its actual utility, the true optimand of the planning problem. The virtual utility of subordinates (11.6) reflects the extent to which their self-interested behavior threatens the achievement of central objectives. For any given plan-information configuration, it is a positive multiple of actual utility, the multiple being the "shadow price" of preventing deviations from that state minus a weighted average of the agent's payoffs to optimally deviating by reporting that given information state when some other state holds, the weights again being related to the "shadow prices" of the respective deviations.[26] Thus an optimal mechanism compromises central objectives only to the extent necessary to preserve incentive compatibility of the planning procedure, and respects subordinates' objectives only to the extent that they jeopardize the achievement of central goals. It manipulates the outcomes of the planning procedure in such a way that no subordinate can do better, given his information and beliefs, than to report truthfully, and then obediently implement the resulting plan (11.4). And, in so doing, it maximizes the sum of the "values" of the agents in the economy (including the center) (11.5) in terms of central objectives/utility. In addition to this intuition on the qualitative nature of optimal mechanisms, the characterization of Proposition 2 can be used to solve for optimal planning mechanisms in simplified special cases, e.g. our examples in Section IV.

When the space of potential actions (plans) is not finite, the questions of existence and characterization become more delicate. However, the assumptions made above, particularly the continuity of $u_i(\cdot,t)$, $\forall i$, are sufficient to guarantee existence. We might also hope for a dual characterization of the center's optimizing (planning) problem, as in the finite-actions case, but that has so far eluded us. We have only a few very weak characterization results that will be presented following the demonstration of the existence of optimal coordination mechanisms (optimal plans and commands) in our model.

Along with existence, one practical question that must be addressed is: What can be expected to happen in the absence of an incentive compatible planning mechanism? Is there an equilibrium pattern of non-cooperative behavior for the agents in this game? The answer obviously depends on the types of strategies permitted. It is affirmative for distributional strategies, introduced by Milgrom, Weber (1980, p. 14):

Definition: A distributional strategy for agent i is a probability measure, μ_i, on the subsets of $T_i \times A_i$ for which the marginal distribution on T_i is $p_i(t_i)$, i's prior probability of being in information state t_i.[27]

This answer is given in:

Proposition 3: Under assumption (A.1) through (A.5) there exists a Bayesian

Nash equilibrium of Γ in distributional strategies.

This proposition is an immediate consequence of Theorem 2 (p. 20) of Milgrom and Weber (1980) as both information and payoffs are continuous and the action spaces are compact. It shows that, when the center attempts no information gathering or coordination, there does exist an equilibrium configuration of individual behaviors,

$$\{(\mu_0,\mu_1,\ldots,\mu_n)|\mu_0\in \mathcal{P}_0,\mu_i:T_i\to \mathcal{P}_i\ ,\quad \mathcal{P}_i \equiv \text{Set of Probability Measures on } A_i\}$$

such that for each $i=0,1,\ldots,n$, and all $\hat{\mu}_i \in \mathcal{P}_i$,

$$\text{(12)}\qquad \sum_T p_i(t|t_i)\int_A u_i(a,t)\mu_0(da_0)\mu_1(da_1|t_1)\ldots\mu_i(da_i|t_i)\ldots\mu_n(da_n|t_n)$$

$$\geq \sum_T p_i(t|t_i)\int_A u_i(a,t)\mu_0(da_0)\ldots\hat{\mu}_i(da_i|t_i)\ldots\mu_n(da_n|t_n).$$

As an immediate consequence of Proposition 3 we have:

<u>Corollary</u>: There exists a Bayesian Incentive Compatible mechanism for Γ, i.e. there exists a $\mu \in \mathcal{P}^T$ such that Γ_μ has an equilibrium in which subordinates' strategies are honest and obedient.

<u>Proof</u>: Define $\mu(da|t) := \mu_0(da_0)\mu_1(da_1|t_1)\ldots\mu_n(da_n|t_n)$. The incentive compatibility constraints for agent i, $i = 1,\ldots,n$, become:

$$\sum_T p_i(t_i|t_i)\int_A u_i(a,t)\mu(da|t) \geq \sum_T p_i(t|t_i)\int_A u_i(a_{-i},\alpha_i,t)\mu(da|t_{-i},\tau_i)$$

for all $(\tau_i, \alpha_i(\cdot)) \in \hat{A}_i$ (3). Now notice that $\int_A u_i(a_{-i}, \bar{a}_i, t)\mu(da|t_{-i}, \tau_i)$

$$= \int_A u_i(a,t) \prod_{j\neq i} \mu_j(da_j|t_j)\cdot[\mu_i(d\alpha_i^{-1}(\bar{a}_i)|\tau_i)] = \int_A u_i(a,t) \prod_{j\neq i} \mu_j(da_j|t_j)\hat{\mu}_i(da_i|t_i).$$

Hence (12) implies incentive compatibility. Q.E.D.

Thus we know that the set of incentive compatible mechanisms for the general planning and implementation problem is non-empty. It still remains to be shown that an optimal mechanism exists in this class.

In our special cases of pure planning, Γ^P, and pure implementation, Γ^I, it is much easier to find and interpret such incentive compatible mechanisms. When all real actions are verifiable and enforceable, and hence subordinates only transmit information, any imposed mechanism, i.e. one independent of the information supplied by subordinates s.t. $\mu_t = \bar{\mu}$ $\forall t$, is weakly incentive compatible: $U_i(\mu|t) \equiv V_i(\mu, \tau_i|t_i)$, $\forall i$, τ_i. When there is no private information, but agents have unverifiable actions, then any Nash equilibrium of the non-cooperative game without central coordination is trivially an incentive compatible mechanism; subordinates are told to do what they would have done without a plan. When these two types of mechanisms are combined in the general problem we have the Bayesian Nash equilibrium in distributional strategies imposed for all information reports of the agents. Hence the subordinates learn nothing from the mechanism and cannot manipulate it. The best they can do is what they would have done without the plan, and that is just what the plan tells them to do. Thus in all cases the set of incentive compatible mechanisms is non-empty, though these mechanisms are generally far from optimal.

Before proving existence of optimal mechanisms, we should ask what will happen if the center attempts coordination, but does so unsuccessfully, i.e. uses a non-incentive-compatible mechanism. In that case we might suppose that subordinates will attempt to do the best they can, given the information transmitted by the mechanism and the best (manipulative) strategies of the other agents. But what should be considered the strategy space for this behavior? If each A_i is finite then $\hat{A}_i$ is compact so a Bayesian Nash Equilibrium always exists in Γ_μ (equation 3) and $\hat{A}$ is an acceptable strategy space. This is sufficient for our examples in Section 4. But A_i more generally compact is not sufficient to insure that result. It would seem possible to define a set of more general distributional strategies as functions dependent on the informational content of the mechanism, and show that a Nash equilibrium exists in such strategies. But we do not pursue that development in this paper. Finally, we note that it is difficult to see how any such manipulative Nash equilibria might be established as there is no focal point such as the plan is for incentive compatible equilibria.

As argued above (p. 18) an optimal mechanism, μ^*, is a vector of distributions, $(\mu_t^*)_{t \in T}$, such that:

$$(13.1) \qquad U_0(\mu^*) \geq U_0(\mu) \quad \text{for all } \mu \in \mathcal{P}^T \text{ such that}$$

$$(13.2) \qquad U_i(\mu|t_i) \geq V_i(\mu,(\alpha_i,\tau_i)|t_i) \text{ for all } i,\ \alpha_i,\ \tau_i \text{ and } t_i,$$

where U_i is defined in equation (4), V_i -- in equation (5) and U_0 -- in equation (7). It is one of a class of vectors of measures among which subordinates implicitly choose with their choice of strategy. Constraint (13.2) insures that the mechanism intended by the center is the one actually chosen by the subordinates when their individually best strategies are honest and obedient, i.e. when the mechanism is incentive compatible. We have already shown that the set of such mechanisms is non-empty (Corollary to Proposition 3); we need to show that it always contains a U_0-maximal element.

Theorem: Under assumptions (A.1) through (A.5), there exists a solution to (13), i.e. an optimal planning and implementation mechanism, μ^*.

Proof:

(i) Endow $\mathcal{P}^T$ with the topology of weak convergence of measures (weak*). Then $\mathcal{P}^T$ is a compact subset of a complete metric space, and U_0 is continuous on $\mathcal{P}^T$. If the set of incentive compatible mechanisms, i.e. $\mathcal{P}^0 = \{\mu | (13.2) \text{ holds}\}$, can be shown to be non-empty and closed, then by the Weierstrass Theorem there exists a $\mu^* \in \mathcal{P}^0$ solving (13).

(ii) By the corollary to Proposition 3, $\mathcal{P}^0 \neq \emptyset$.

(iii) As I and T are finite sets, the constraint consists of a finite intersection of sets of the form:

$$\mathcal{M}_{i,t}^{\tau_i} \equiv \{\mu \in \mathcal{P}^T | U_i(\mu|t_i) \geq V_i(\mu,(\alpha_i,\tau_i)|t_i) \text{ for fixed } i,t,\tau_i \text{ and for all } \alpha_i \in \mathcal{Q}_i/\mathcal{Q}_i\}.$$

Notice that $V_i(\mu,(\alpha_i,\tau_i)|t_i) \equiv U_i(\hat{\mu} \circ \bar{\alpha}_i^{-1}|t_i)$ where $\hat{\mu}$ is the vector of measures with t_i (and only the given t_i) replaced by τ_i and $\bar{\alpha}_i = Id \times \alpha_i : A_{-i} \times A_i \to A_{-i} \times A_i$. Hence, if we can show that each $\mathcal{M}_{i,t}^{\tau_i}$ is closed in the topology of weak convergence of measures then $\mathcal{P}^0$ must also be closed and therefore compact.

(iv) To simplify notation, fix i and t and w.l.o.g. let $\tau_i = t_i$ so that $\mathcal{M}_{i,t}^{\tau_i} \equiv \mathcal{M}$ and $U_i(\mu|t_i) \equiv U(\mu)$. Let $\{\mu^n\}_{n=1}^{\infty}$ be a sequence of measures such that

(a) $\mu^n \in \mathcal{M}$ for all n, and

(b) $\mu^n \to \mu$ in the topology of weak convergence of measures.

We need to show that $\mu \in \mathcal{M}$. Notice that (a) implies $U(\mu^n) \geq U(\mu^n \circ \bar{\alpha}^{-1})$ for all n and all measurable actions α of agent i. For closure of $\mathcal{M}$, we must show that $U(\mu) \geq U(\mu \circ \bar{\alpha}^{-1})$ for the μ in (b) and any measurable $\alpha: A_i \to A_i$. Clearly if α is a continuous function, the desired inequality holds in the limit by the definition of weak convergence of measures (Hildenbrand, 1974, p. 48). Further, all α are Lusin μ-measurable (Castaing, Valadier, 1977, p. 60-62) so that, by Lusin's Theorem (Rudin, 1966, p. 53) we know:

<u>Lemma</u>: For all $\varepsilon > 0$ and for any measurable $\alpha: A_i \to A_i$, there exists a continuous function $\alpha_\varepsilon: A_i \to A_i$ and a set $N_\varepsilon \in A_i$ such that $\alpha_\varepsilon\big|_{A_i \setminus N_\varepsilon} = \alpha\big|_{A_i \setminus N_\varepsilon}$ and $\mu(A_{-i} \times N_\varepsilon) < \varepsilon$.

(v) Choose a sequence of continuous α_k each satisfying the above lemma, such that $\varepsilon_k \underset{k\to\infty}{\downarrow} 0$, i.e. $\alpha_k \underset{k\to\infty}{\to} \alpha$ in measure and pointwise. Notice that for each k, $U(\mu^n) \to U(\mu)$ and $U(\mu^n \circ \bar{\alpha}_k^{-1}) \to U(\mu \circ \bar{\alpha}_k^{-1})$. Therefore $U(\mu) \geq U(\mu \circ \bar{\alpha}_k^{-1})$ for all k. we now need only show that the inequality is preserved as $k \to \infty$. From (4) and (5) we have (remember: i,t, and $\tau_i = t_i$ are fixed!) $U(\mu) = \int_A u(a)d\mu$ and $U(\mu \circ \bar{\alpha}_k^{-1}) = \int_A u(a_{-i}, \alpha_k(a_i))d\mu$. Look at

$U(\mu) - U(\mu \circ \bar{\alpha}_k^{-1}) \geq 0$ as k gets larger. In the limit

$$\int_A [u(a) - u(a_{-i}, \alpha(a_i))]d\mu =$$

$$= \int_A [u(a) - u(a_{-i}, \alpha_k(a_i)) + u(a_{-i}, \alpha_k(a_i)) - u(a_{-i}, \alpha(a_i))]d\mu$$

$$= \int_A [u(a) - u(a_{-i}, \alpha_k(a_i))]d\mu + \int_A [u(a_{-i}, \alpha_k(a_i)) - u(a_{-i}, \alpha(a_i))]d\mu$$

$$\geq 0 + -K \cdot \mu(A_{-i} \times N_\varepsilon) \underset{k \to \infty}{\to} 0,$$

where $K = \sup_{a,a'} |u(a_{-i}, a_i) - u(a_{-i}, a_i')| < \infty$ as A is compact and u is continuous. Hence $\mu \in \mathcal{M}$ so that $\mathcal{M}$ and $\mathcal{P}^0$ are closed as required. Q.E.D.

Though we have shown an optimal planning and implementation mechanism to exist under our maintained hypotheses, we have no easy way to compute or characterize it. As Myerson (1982, p. 75) points out, "...it is only in the finite case that the principal's problem reduces to a linear programming problem." Thus we cannot hope for an easily calculated characterization such as that given in Proposition 2. However, as formulated in equation (13), the central problem does appear to be one of linear programming in some linear function spaces, although without the usual Slater-like condition. So far we are unable to correctly formulate the problem and its dual, or even to properly topologize the relevant spaces. That effort is the subject of continuing research.

Thus there is relatively little we can currently say about the characteristics of optimal planning mechanisms. However, there appears to be one salient feature that seems important for understanding the functioning of CPE's: optimal mechanisms generally seem to be stochastic whenever some incentive compatibility constraints are binding. Though the statement needs to be made more precise to be formally correct, the intuition behind it is quite simple. The distribution over commanded actions as a function of reported types must punish (in expected value terms) deviations by any subordinate to locally more desirable information reports and implemented actions, while bolstering through randomization over the actions of others, that agent's expected payoff to truthful and obedient (but locally undesirable) action. In the case where all action spaces are finite, and utilities concave, this is apparently what the maximization of the sum of virtual utilities does. In general, the existence of an optimal deterministic mechanism can imply the absence of true incentive compatiblity problems, with some exceptions noted below.

To develop this observation further we begin with an analysis of the simplified problems, Γ^I and Γ^P respectively.

Proposition 4: Any incentive compatible deterministic mechanism in Γ^I implements a Nash equilibrium in pure strategies.

Proof: The conditions for incentive compatibility in Γ^I are:

$$\int_A \mu(da)u_i(a) \geq \int_A \mu(da)u_i(a_{-i}, \alpha_i) \quad \forall i,\ a_i \in A_i,\ \alpha_i \in \mathcal{Q}_i/\mathcal{Q}_i \tag{14}$$

If μ is deterministic, it places unit mass at some a^*, so (14) becomes

$$u_i(a^*) \geq u_i(a^*_{-i}, \alpha_i)\ \forall i,\ \alpha_i \in A_i$$

which is just the definition of a Nash equilibrium in pure strategies. Q.E.D.

As an optimal mechanism must be incentive compatible, this shows that an optimal mechanism cannot be deterministic in the face of a true moral hazard problem.

This is also frequently the case when faced with a pure adverse selection (planning) problem Γ^P. To see why this might be, let $a^0(t) \in \underset{A}{\text{argmax}}\ U_0(a,t)$, $a^i(t) \in \underset{A}{\text{argmax}}\ U_i(a|t_i)$ for each $t = (t_1,\ldots,t_n)$. Clearly, if $a^0(t) = a^i(t)$, $\forall i$, $\forall t$, there can be no incentive problems. Further, if $t^i = (t_{-i},\tau_i)$ for $\tau_i \in T_i$, then we say that there is <u>no adverse selection problem</u> if $U_i(a^0(t)|t_i) \geq U_i(a^0(t^i)|t_i)$ for all i and all t^i attainable by agent i. A special case of this is when the center has a unique action, optimal for all states. In either case the center is safe in choosing its optimal action for each reported information state. The only interesting case arises when there exists at least one j, t and t^j such that

$$(15) \qquad U_j(a^0(t)|t_j) < U_j(a^0(t^j)|t_j)$$

so that at least one incentive constraint is binding, and the central choice is sensitive to the information received. Here we say a true adverse selection problem exists, so that the center can no longer choose the actions it desires. This is clearly seen in the first-order conditions that must hold for an optimum (Myerson, 1983, Theorem 1, p. 29):

$$(16.1) \qquad \theta_i(\tau_i|t_i)[U_i(\mu|t_i) - V_i(\mu,\tau_i|t_i)] = 0 \quad \forall i,t_i,\tau_i;$$

$$(16.2) \qquad \int_A \mu(da|t) \sum_{i=1}^{n} v_i(a,t,\theta) = \max_{a\in A} \sum_{i=1}^{n} v_i(a,t,\theta)$$

where

$$v_0(a,t,\theta) = u_0(a,t), \text{ for } i = 1,\ldots,n,$$

$$(16.3) \qquad v_i(a,t,\theta) = [\sum_{\tau_i \in T_i} \theta_i(\tau_i|t_i]u_i(a,t) - \sum_{\tau_i \in T_i} [\theta_i(t_i|\tau_i)u_i(a,t_{-i},\tau_i)]/p_i^*(t_i)$$

and

$$(16.4) \qquad p_0(t) = p^*(t) = \prod_{i=1}^{n} p_i^*(t_i), \ \theta_i(\tau_i|t_i) \geq 0 \ \forall i,t_i,\tau_i.$$

The concessions needed are evident in the fact that the choice of mechanism must maximize the sum of the virtual utilities of all agents, and not just the utility of the center.

There are in general two approaches, which may be combined, that the center might follow to achieve the needed compromise with subordinate interests. The first is to compromise in the choice of a plan, a^*, so that $U_i(a^*(t)|t_i) \geq U_i(a^*(t_{-i},\tau_i)|t_i)$, $\forall i,t,\tau_i$. The best such $a^*(t)_{t \in T}$, would solve $\sum_{i=1}^{n} v_i(a^*(t),t,\theta) = \max_{a \in A} \sum_{i=1}^{n} v_i(a,t,\theta)$, $\forall t$, and would be the outcome of a deterministic mechanism. I know no general result that rules out this possibility, or provides reasonable sufficient conditions under which such a deterministic mechanism can always be found.[28] However, considering the alternative leads us to believe that those conditions might be quite restrictive. The second approach involves randomization among a number of alternatives that the center might find preferable to any compromise action. Such an approach makes use of the "punishment intuition" given above: by sometimes telling the other agents to carry out suboptimal (for the true state) actions that severely punish a given deviation by any agent, the center may be able to get truthful revelation and thus its best outcome, even when most of the time that outcome is less desirable for the agent than that of the given deviation. In the very special case where A is finite and $a^0: T \to A$ maps onto A, it is clear that only this second approach is possible in the face of a true adverse selection problem: as there does not exist an a^*, optimal incentive compatibility can only be achieved by randomizing among some of the actions optimal in particular states. In general, for a finite A, it is only by coincidence that a deterministic mechanism, $a^*: T \to A$, can be found, though an optimal randomized strategy may require using some $a \in A$ that would not be optimal for any t, were t known to the center.

For a general A, it is much more likely that a deterministic, "compromise" mechanism can be found, particularly if there is sufficient "consistency" among the preferences of the agents.[29] However, there are a number of complicating factors that must be taken into account. These factors largely revolve around the "risk-aversion" of each agent in each state of the world, i.e. the concavity of $u_i(\cdot,t)$. If all agents are risk neutral, then there is clearly no difference in their eyes between a deterministic compromise plan and a plan chosen from an appropriately randomized mechanism. When risk aversion is introduced agents have a natural preference for deterministic plans or mechanisms with minimally dispersed payoffs. On the other hand, from the center's perspective, randomization might be fruitfully used to prevent (punish) deviations by more risk averse "types", (i.e. separate types by risk aversion). Further, it is well known that, with concave utilities, the incentive compatibility constraints (6) define a non-convex set of actions (plans/deterministic mechanisms).[30] Thus the appropriate (u_0-optimal) compromise, $(a^*(t))_{t \in T}$ may not be incentive compatible, though its expected payoff's could be generated by an appropriate randomization. Hence, even with risk aversion, the optimal mechanism may be stochastic in the case of a general A. Whether it is or not will also depend on the comparative degrees of risk aversion among agents (including the center) and among the types of any given agent. A further factor is whether the center must give up too much to achieve a deterministic compromise plan, for the center is indifferent to imposing the costs of risk bearing on agents, as long as that does not aggravate incentive compatibility problems. And if that, indeed, solves incentive compatibility problems, so much the better. Finally, we should note that if subordinates' reports can be used to accurately (with some probability) deduce that one or more agent is lying, then a deterministic mechanism built around taking "punishing" actions can frequently be found.

When we turn to the general situation, Γ, involving both moral hazard and adverse selection, the problem becomes much less tractible. Though we have as yet no general result, we feel that the intuition preceeding Proposition 4 is generally valid: randomization is necessary to deal with moral hazard and will often be so for adverse selection. As long as there are gains to unenforceable coordination, which must itself be properly informed, the center must randomize to prevent manipulation in the face of true incentive compatibility problems. Ongoing research is aimed at clarifying this intuition.

We are now in a position to begin applying these general results to a pair of simple examples. The examples are largely meant to illustrate the use of this type of analysis to illuminate some observed regularities of Soviet-type economies. The examples are highly stylized and make no claim to completeness. However, they do seem to capture some interesting features of the functioning of CPE's. The primary importance of our general results is to assure us that the analysis of such models is not a vacuous exercise.

IV. A. <u>A Pure Planning Example</u>

The first example deals with an extremely simple production planning problem of the "team-theory" genre involving one center and two subordinates or agents. It deals with planning the use of centrally allocated inputs when both technologies and preferences of subordinates are stochastic, with the knowledge of their realized values private and dispersed among the subordinates. This information may be elicited from agents prior to planning, and it is assumed that any plan of action (allocation and subsequent production) is fully enforceable subject to only physical feasibility constraints.[31] Thus we are in Γ^P where the only incentive problem is one of eliciting truthful reports from the subordinates.

To keep matters simple we assume that there are only two possible information states ('types') for each agent ($t_i \in \{0,1\}$, $i=1,2$), and hence only four decision-relevant events in $\mathcal{S}$. It is also assumed that beliefs are identical and uniform. The problem the center faces is to properly allocate a fixed amount of an outside resource, x, among the producing agents, where each agent also uses the output of the other as an intermediate input. The objective of the center is to maximize the value of net output of the team (sector), which is a function of the technologies possessed by subordinates. Agents posess a Leontief production technology with coefficients that depend on the state of the economy: when $t_i = 0$ production by agent i is more materials-intensive, and requires relatively more of the centrally allocated resource than when $t_i = 1$. Agent preferences also depend on the realized state: when $t_j = 0$ agent i desires to minimize his required level of activity, while when $t_j = 1$, i wants to maximize gross output.[32] Formally this yields the following model:

$$(17) \qquad u_0(a,t) = q' T_t y_{at} \text{ and } u_i(a,t) = (\pi_t - \gamma^i) y_{iat}$$

where $q' = (q_1, q_2)$; $y_{at} = (y_{1at}, y_{2at})$; $y_{iat} = \min_j \{\frac{x_{ji}}{c_{jit}}\}$, $\forall i, \forall t$; $t = (t_1, t_2)$; $x_{1i} \equiv a_i$, $\sum_i a_i = x$; $x_{2i} \leq y_{kat}$, $i \neq k$; $y_{iat} \geq 0$ $\forall i,t$; and

$$T_t = \begin{bmatrix} 1 & -c_{12t} \\ -c_{22t} & 1 \end{bmatrix}.$$

The symbols used have the following interpretation:

q_i - per-unit value to center of net output of agent i;

y_{iat} - gross output of i in state t when a implemented;

x_{ji} - availability of input j to agent i;

c_{jit} - input coefficient for input j in sector i in state t;

π_t^i - per-unit value to i of own gross output in state t;

γ^i - per-unit "cost" to i of own activity level;

T_t - net output operator for whole sector — transforms gross into efficient net output;

a - sequence of "actions" enforced by the center and generating outputs y.

To complete the model we make number of specific assumptins:

(i) q_i, γ_i, x are independent of t,

(18) (ii) $\pi_t^i = \gamma^i \pm \delta_i$ as $t_j = \{^1_0, \ \delta_i > 0$,

(iii) $\frac{c_{1it}}{c_{2it}}\Big|_{t_i = 0} > \frac{c_{1it}}{c_{2it}}\Big|_{t_i = 1}$ and $\sum_j c_{jit}\Big|_{t_i = 0} > \sum_j c_{jit}\Big|_{t_i = 1}$

Assumption (i) is inessential, and only made to avoid further burdening an already complicated example. Assumptions (ii) and (iii), however, carry the weight of the argument — they are essential to the existence of incentive compatibility problems in this example. Finally, it is assumed that the allocation of inside inputs is carried out subsequent to the allocation of x, and hence can only be a constraint if x_i is insufficient to produce enough y_i for the production needs implied by x_j, i.e. $x_i < c_{1it}\left(\frac{x_j}{c_{ijt}}\right) c_{2it}$. In that case x_j cannot be fully used. However, y is always fully used, either in production or by the center to generate social value. Notice that the space of potential actions here is extremely complicated and hence is described only implicitly through its consequent allocation decisions and production outcomes. We drastically simplify by assuming that only the outside-resource allocation requires planning, and hence the mechanism is a distribution over x_1 vectors conditional on reported types. To keep the analysis within a finite framework we will limit the domain of any mechanism to only four points — the optimal allocation vectors for the four possible states of the world, x_{it}, for t = 00, 01, 10, 11.[33]

The activity vectors in the potential support of any mechanism are just those that would be chosen by a fully informed center. In any known state t the center faces a trivial linear programming problem such as that depicted in Figure 1. The solution generally involves maximizing the output of the agent whose product is most desired, taking into account both its direct value, q_i, and the product's ease of, and implicit value in, production, and only producing enough to the other good to provide necessary inputs. The agent whose output is maximized will be called the "leading producer". The "social value" of output i is easily seen to be

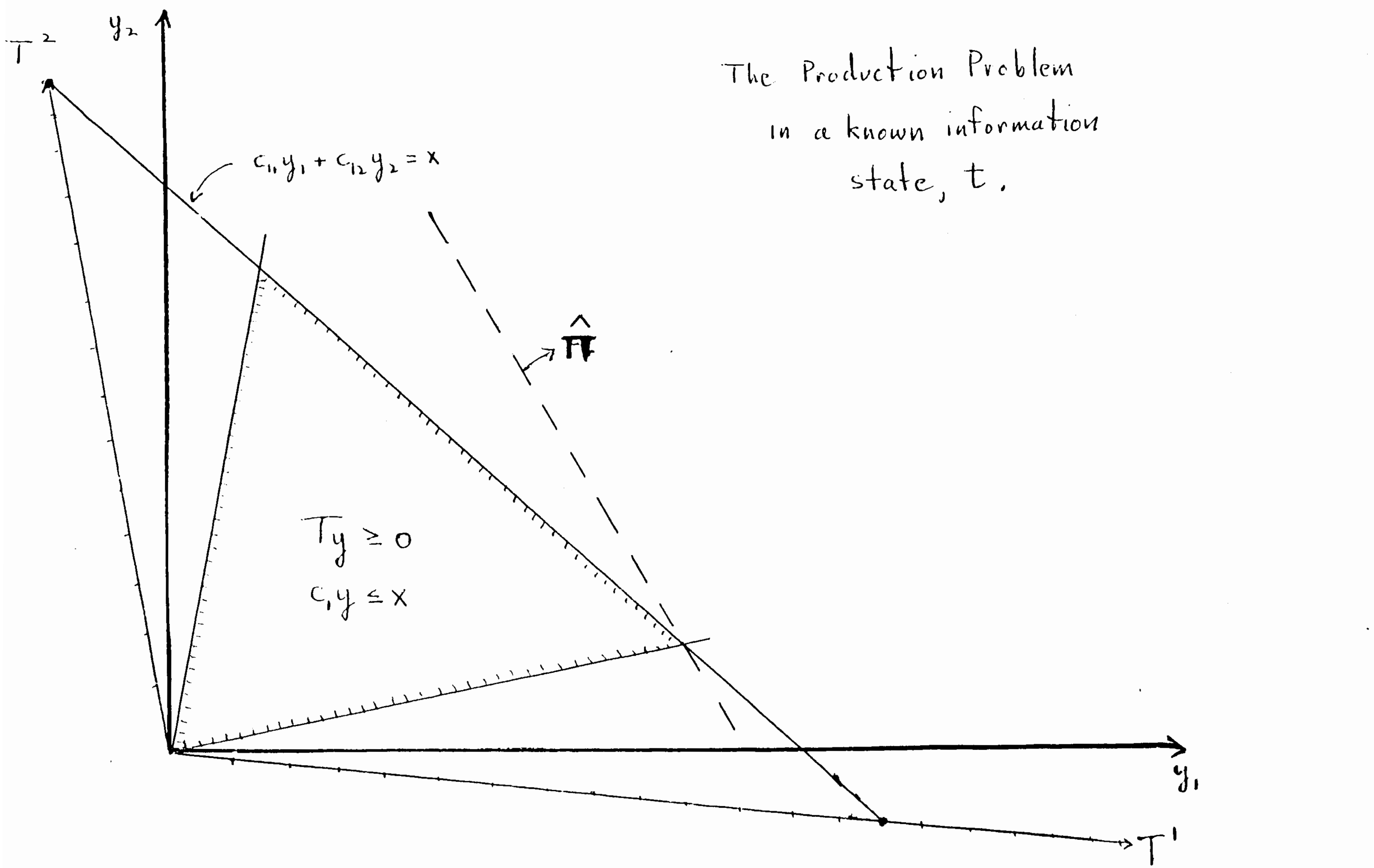
$$\hat{\pi}_{it} = \frac{q_i}{c_{1it} + c_{ijt}c_{2it}} .$$

Feasibility requires that net output be non-negative, $T_t y \geq 0$, and that $\sum x_i \leq x$. Therefore the optimal fully-informed action in any state t is (see Figure 1):

$$\hat{\pi}_{1t} > \hat{\pi}_{2t} \Rightarrow \begin{cases} x_{11t} = c_{11t}y_{1t}, \quad x_{12t}y_{2t} = x - x_{11t}, \\ y_{1t} = \dfrac{x}{c_{11t} + c_{12t}c_{21t}} \text{ and } y_{2t} = \dfrac{c_{21t}x}{c_{11t} + c_{12t}c_{21t}} , \end{cases}$$

(19)

$$\hat{\pi}_{1t} < \hat{\pi}_{2t} \Rightarrow \begin{cases} c_{11t} = c_{11t}y_{it} = x - x_{12t}, \quad x_{12t} = c_{12t}y_{21}, \\ y_{1t} = \dfrac{c_{22t}x}{c_{12t} + c_{21t}c_{22t}} \text{ and } y_{2t} = \dfrac{x}{c_{12t} + c_{21t}c_{22t}}. \end{cases}$$



The Production Problem
in a known information
state, t.
T^2
y_2
$c_{11}y_1 + c_{12}y_2 = x$
$\hat{\Pi}$
$Ty \geq 0$
$c_1 y \leq x$
y_1
T^1

Figure 1

Clearly the center could do no better than implement the technologically feasible plan $y_t = (y_{1t}, y_{2t})$ in each state t. However, attempting to do so would give agents an incentive to distort their information reports.

Proposition 5: The deterministic mechanism of (18) is not incentive compatible.

Proof: We can see this immediately from the incentive constraints (6). Here

$$\mu(y_t|t) = 1,\ \mu(y_{t'}|t) = 0\ \forall t,\ t' \neq t;$$

$$U_i(\mu|t_i) = 1/2\ (\pi^i_{t_i0} - \gamma^i)y_{t_i0} + 1/2(\pi^i_{t_i1} - \gamma^i)y_{t_i0},\ t_i = 0,\ 1;$$

$$V_i(\mu,\tau_i|t_i) = 1/2(\pi^i_{t_i0} - \gamma^i)y_{\tau_i0} + 1/2(\pi^i_{t_i1} - \gamma^i)y_{\tau_i1}, \tau_i \neq t_i.$$

Clearly the first term of each expression is negative and the second positive, and $\pi^i_{\cdot}$ is independent of t_i (18ii), so that any deviation reducing $y_{\cdot 0}$ and/or increasing $y_{\cdot 1}$ will violate the incentive constraints. It is easy to see that for $t_i = 0$, if i is the leading producer ($\hat{\pi}_{it} > \hat{\pi}_{jt}$), then claiming $\tau_i = 1$ yields $V_i(\mu,1|0) > U_i(\mu|0)$.[34] Similarly, if $t_i = 1$ and j is the leading producer, then claiming $\tau_i = 0$ improves the local payoff. Q.E.D.

The immediate implication of this results is that "naive planning," i.e. using a deterministic mechanism based on the assumption of agent truthtelling, cannot be optimal for the center. Such a mechanism merely sets up a noncooperative manipulation game, Γ_μ, for which agents might be expected to find some Bayesian Nash equilibrirum. Given each agent's interest in minimizing his level of activity (output) in some states (i.e. when the other agent's type is zero), this equilibrium is bound to be bad for the center. This will in general be true for any mechanism that is not incentive compatible, as such a mechanism merely fixes the payoff to agent announcements and carries no useful information about the other agent's behavior. Thus, to have any true influence in this model, the center must implement an incentive compatible mechanism.

Of all incentive compatible mechanisms two clearly stand out, the optimal mechanism, μ^*, and what we will call the uninformed mechanisms. The latter are defined as any mechanisms that assign a fixed allocation-production plan to each agent, independent of that (or any other) agent's report. The best such mechanism is one solving

$$(20) \qquad \max_{a \in A} \sum_t p_t u_0(a,t) \equiv \max_{(x_1,x_2)} \sum_t p_t (q' T_t y)$$

$$\text{s.t. } T_t y \geq 0,\ \forall t, \text{ and } x_1 + x_2 = x.$$

It is clearly incentive compatible as no agent's report can affect any action. In this simple example with only four possible allocations it is easily computable by comparing $\sum_t p_{0t} V_t(z_{t'})$ for each $t' = 00, 01, 10, 11$ where V_t is the value to the center in state t of the fixed allocation $z_{t'}$, where $z_t \equiv (x_{11t}, x_{12t})$ (equation (19)). The optimal uninformed allocation plan clearly depends on the prior probabilities of the center p_{0t}, and on the loss in each state associated with non-optimal allocations, i.e. $V_t(z_t) - V_t(z_{t'}) > 0$, $t' \neq t$. There seems little that can be said in general, but some potentially interesting observations will be made in the context of specific numerical examples below.

Similarly, there is little that can be said beyond the general results of Section III about the optimal mechanism with out resorting to specific numerical values.

However, for this example, it seems clear that the optimal mechanism must be stochastic. This is because, in any state but '11', there is at least one agent whose interests are diametrically opposed to those of the center, i.e. that agent values his output negatively. Though the center wants production out of that agent, it must shift the burden of production onto the other agent in those states with sufficient probability to equalize the expected return to truthtelling and to the optimal deviation.

These and other conclusions are best illustrated in a pair of numerical examples capturing all the assumptions made above. They differ in the relative value the center places on the net output of the subordinates, and in the efficiency advantage that the second subordinate has in production.

Ex. a) $q = (1.5, 1)$ $\gamma = (1, 1.5)$ $x = 10$

$t = 00$: $\pi_t = (.8, 1)$ $c_{1t} = (.7, .1)$ $c_{2t} = (.5, .1)$

$t = 01$: $\pi_t = (1.5, 1)$ $c_{1t} = (7, .1)$ $c_{2t} = (.3, .2)$

$t = 10$: $\pi_t = (.8, 2)$ $c_{1t} = (.4, .3)$ $c_{2t} = (.5, .1)$

$t = 11$: $\pi_t = (1.5, 2)$ $c_{1t} = (.4, .3)$ $c_{2t} = (.3, .2)$

Ex. b) $q = (1.5, 1.5)$; $t_2 = 0$: $c_{22t} = .2$; $t_2 = 1$: $c_{22t} = .3$.

In example (a) the center values the output of agent 1 significantly higher, yet agent 2 is noticeably more productive.[35] In example (b) outputs are equally valuable, but agent 2, is less efficient than in example (a). The optimal allocations and their consequences are catalogued in Table I. Agent 1 is clearly the leading producer in (a) while, agent 2 is leading in (b). Yet

Table I

Centrally Optimal Allocations and Payoffs

Case (a)

State	x_1	x_2	y_1	y_2	$(Ty)_1$	$(Ty)_2$	u_1	u_2	u_0
00	9.3	.7	13.33	1.3	13.2	0	- .7	- 2.7	19.8
01	3.2	6.8	4.5	22.75	0	22.3	2.3	-11.4	22.3
10	7.3	2.7	18.2	5.45	17.6	0	- 3.6	2.7	26.5
11	8.2	1.8	20.4	6.1	19.2	0	10.2	3.1	28.8

Case (b)

State	x_1	x_2	y_1	y_2	$(Ty)_1$	$(Ty)_2$	u_1	u_2	u_0
00	2.2	7.8	3.1	15.6	0	15.3	- .6	- 7.8	23
01	4.1	5.9	5.9	19.6	0	19	2.9	- 9.8	28.5
10	7.3	2.7	18.2	5.5	17.1	0	- 3.6	2.7	25.6
11	2.9	7.1	7.1	23.8	0	21.7	3.6	11.9	32.5

Table II

Case (a) — Allocation

State	Agent	z_{00}	z_{01}	z_{10}^{**}	z_{11}
00	u_0	19.8^*	18	19.2	19.4
	u_1	- 2.7	- .9	-2.1^0	- 2.3
	u_2	$-.7^0$	- 6.8	- 2.7	- 1.8
01	u_0	20.2	22.3^*	20.9	20.6
	u_1	6.7	2.3	5.2	5.8^0
	u_2	-1.1^0	-11.4	- 4.5	- 3.1
10	u_0	6.5	21.1	26.5^*	17.8
	u_1	$-.9^0$	- 1.6	- 3.6	- 2.4
	u_2	.7	6.8	2.7^0	1.8
11	u_0	10.4	25.5	28.2	28.8^*
	u_1	3.7	4	9.1	10.2^0
	u_2	1.1	11.4	4.5^0	3.1
u_0 expected value		14.2	21.66	23.7	21.67

case (b)

State	Agent	z_{00}	z_{01}^{**}	z_{10}	z_{11}
00	u_0	23^*	22.1	20.6	22.7
	u_1	$-.6^0$	- 1.2	- 2.1	- .8
	u_2	- 7.8	-5.9^0	- 2.7	- 7.1
01	u_0	15.2	28.5^*	23.6	19.8
	u_1	1.6	2.9^0	5.2	2
	u_2	-5.2^0	- 9.8	- 4.5	- 6.8
10	u_0	24.5	24.9	25.6^*	24.6
	u_1	-1.1^0	- 2.1	- 3.6	- 4.1
	u_2	7.8	5.9	2.7	7.1^0
11	u_0	24.9	31.4	28.6	32.5^*
	u_1	2.7	5.1^0	9.1	3.6
	u_2	9.1	9.8	4.5	11.9^0
u_0 expected value		21.9	26.7	24.6	24.9

Symbols

* - payoff to optimal central decision under full information.
** - optimal uninformed allocation.
0 - payoff to agent's most desired allocation, assuming other agent reports truthfully.
z_t - allocation (x_1,x_2) optimal for center in state t.

there always exists a state such that the other agent becomes the primary producer of direct socal (central) value due to its productivity advantage.

Table II contains the matrix of payoffs to each agent (including the center) for each allocation in each state in both examples. These are the numbers that determine the outcome of any possible mechanism. In particular, they allow us to numerically solve the linear programming problem (8) for an optimal mechanism. But first there are a number of observations to be made.

(i) The naive deterministic mechanism, $\mu^0(z_t|t) = 1$, $\mu^0(z_{t'}|t) = 0$, (indicated by superscript ^{0}) is clearly not incentive compatible; in every state at least one subordinate has an incentive to lie.

(ii) If the naive mechanism is implemented, by Bayesian game $\Gamma_\mu{}^0$ yields the following equilibria (easily calculated from the payoff matrices):

Case (a): Agent 1 reports truthfully and Agent 2 always claims state 0 holds. Expected payoffs are : $\hat{U}_0 \simeq 23.675$, $\hat{U}_1 \simeq 2.375$, $\hat{U}_2 \simeq 1.35$.

Case (b): There is no pure strategy equilibrium. One mixed strategy equilibrium is as follows: Both agents ignore their own information, agent 1 reports 0 with probability .6 and 1 with probability .4, and agent 2 reports 0 with probability $\approx$ .1781 and 1 with probability $\approx$ .8219. Payoffs are:
$\hat{U}_0 \simeq 25.46$, $\hat{U}_1 \simeq 1.08$, $\hat{U}_2 \simeq .52$.

(iii) The optimal uninformed allocation (indicated by superscript **) has an interesting nature: the exogeneous resource is allocated as if the leading producer were efficient in its use and the supporting producer inefficient. That allocation maximizes (among the four available allocations) the availability of the resource to the supporting producer without reversing output priority. It is thus a rather conservative plan, aimed at avoiding endogeneous intermediate product bottlenecks. The expected payoffs to the agents under this mechanism are readily calculated to be:

Case (a): $U_0^{**} \simeq 23.7$, $U_1^{**} \simeq 2.15$, $U_2^{**} = 0$.

Case (b): $U_0^{**} \simeq 26.7$, $U_1^{**} \simeq 1.175$, $U_2^{**} = 0$.

To determine the fully optimal mechanism we must solve a 16-variable, 8-constraint linear programming problem where four of the constraints define incentive compatibility. Each variable, $\mu_{t't}$, represents the probability of assigning allocation $z_{t'}$ when t is reported. The optimal solution is as follows (states numbered consecutively 1 to 4):

Case (a): $\mu_{11} \simeq .345$, $\mu_{21} \simeq .655$, $\mu_{12} \simeq .79$, $\mu_{22} \simeq .21$, $\mu_{33} = 1$, $\mu_{44} = 1$.

Case (b): $\mu_{11} \simeq .35$, $\mu_{21} \simeq .65$, $\mu_{22} = 1$, $\mu_{13} \simeq .9999$, $\mu_{43} \simeq .0001$, $\mu_{24} \simeq .57$, $\mu_{44} \simeq .43$.

In case (a) the binding incentive constraints are that agent 1 not report being efficient when he is not and that agent 2 not report being inefficient when he is efficient. In case (b) the only non-binding incentive constraint is that an inefficient agent 2 not report being efficient. The expected payoffs to all agents under the centrally optimal mechanism are:

Case (a): $U_0 \simeq 23.8$, $U_1 \simeq 2.7$, $U_2 \simeq -.54$.

Case (b): $U_0 \simeq 26.8$, $U_1 \simeq 1.3$, $U_2 \simeq .53$.

This demonstrates the superiority of the optimal mechanism in the eyes of the center, and shows that both subordinates also prefer it to ininformed optimization by the center in case (a), but not in case (b). It should also be noted that the optimal uninformed mechanism is superior to the naive mechanism in the eyes of the center, while subordinates' attitudes are mixed. In general, one would expect the subordinates to do better in the latter case, at least to the extent that their preferences differ from those of the center, though there would seem to be nothing we can say for sure.[36] Finally, we should note that, as anticipated, optimality requires randomization in order to insure incentive compatibility.

Two further aspects, that will not be elaborated here, have been studied in the context of this example. One involved complicating the model by requiring that the center allocate both x and the intermediate product flows between subordinate producers ex-ante, rather than just allocating x and allowing the latter to adjust to realized technology as above. This yielded results qualitatively similar to those above, with one exception: the optimal uninformed allocatin turned out to be z_{11}, that based on the most optimistic assumption about technology in both producers (Stalinist "overtautness"). The second aspect dealt with the issue of limited collusion among subordinates, so that each knows the full information state, yet must choose his report to the center independently and confidentially. Here the only incentive compatible mechanisms are imposed (independent of agent reports) or are randomizations among the Nash equilibria in each of the four full-information games: the truth is only reported when the center restricts itself to not using that information. Outcomes in this situation are less favorable for the center and generally more favorable for the agents than under the other mechanisms studied above. The present model does not seem rich enough to investigate collusion more thoroughly, so the matter was not pursued further.

To summarize, we have presented a simple activity analysis model in which there is a true adverse selection problem. If the planner ignors this problem, agents will manipulate the planning procedure to their own advantage, and a simple BNE results. The planner can deal with the problem in two general ways: (i) impose a plan (ignore subordinate information) or (ii) elicit information in an incentive compatible manner. The optimal uninformed plan is optimistic with respect to priority sectors and pessimistic with respect to intermediate producers. It is better for the center than naive non-incentive compatible planning, but is naturally inferior to the optimal incentive compatible elicitation of information. The latter requires a stochastic mechanism and is thus itself inferior to the expected payoff to full information. This illustrates the loss caused by the existence of an adverse selection problem. These results also show how it is possible for planning to yield uncoordinated looking outcomes, eg. non-specialization, excess inventories, "letting partners down", etc. For if the planning is not incentive compatible, effective coordination breaks down (allocations not consistent with technology, excess output from non-priority producer, etc.), and even an optimal mechanism must sometimes introduce slack into the system (with a known non-optimal allocation) in order to preserve incentives for truthtelling. This further illustrates the irreduceable economic cost to unverifiable, dispersed, yet globally payoff-relevant information in the economy.

IV. B. A Pure Implementation Example

The second class of examples addresses the pure moral hazard problem: there is no private information, but subordinates in order to implement the plan must carry out unverifiable and unenforceable actions.[37] Many examples of such actions suggest themselves from Soviet experience: choice of detailed assortment; choice of component quality and characteristics; use of lubricants and abrasives; timing, size, and sequencing of production runs; prophylactic maintenance of equipment; material-intensity of production; use of local investment funds (nature & details of capital construction); storage and handling of machinery and materials; etc.[38] The incentive compatibility problem here is to insure that subordinates choose actions that are the best attainable for the center, i.e. that they follow the central recommendation/ command even though it cannot be enforced. We will explore this problem in a stylized production model similar to that of Section IV.A.

It is a simple model of the coordination of directly unobservable production activity involving the use of a unique local resource with high opportunity cost in each of two producers. Here the center takes no real actions but merely plans and commands the actions of agents in the "real sphere" of the economy. Subordinates engage in centrally desired production activity using their local resource and some of the output of the other producer, make other local uses of the resource, and allocate the centrally desired output between the center, and the other user of that product. The center observes none of these decisions, but only the value of the aggregate net output of the two interdependent producers. It issues coordinating instructions in an effort to maximize that value. The incentive problem arises as the producers have different interests. Each desires a high sectoral (joint) output, but each would rather that the other produce it as each has a highly desirable use for its own local resource. This partial conflict of interests creates a coordination problem, Γ^I, amenable to solution through incentive-compatible planning.

Before formalizing this model in an analytic example, it might be helpful to look at some of the practical situations it addresses. One such situation is that of a branch of machine building required to introduce some new complex of machinery for use elsewhere in the economy. Let the branch consist of two subordinate enterprises, each capable of working out only a part of this complex. One enterprise is "upstream" in carrying out the production process that yields a high valued output to the center/ministry. While fully cooperating in the production of this new process has its rewards for both subordinates, each can do better for itself by producing a close, but inferior (obsolescent), product which allows general plan overfulfillment according to more observable indices and some slack for the future. If both enterprises cooperate, then a high quality output results. If only one enterprise works on the project then the new product appears but it is of inferior quality and higher cost to that enterprise. The other enterprise does significantly better according to centrally observable plan parameters. In this case the poor quality of the production process will only become evident much later, or perhaps not at all if its use merely reduces the productivity increase at its users. However, if neither enterprise puts an honest effort into the project, it will be clear that nothing new has resulted, and both will be punished by the central authorities.

A similar situation arises with respect to a recent change in investment priorities of Soviet planners.[39] To increase the productivity of investment they have ordered a shift from financing new projects/construction to investing in the renovation and reequiping of existing enterprises. However, as Boris Rumer (1982) notes, the central authorities cannot distinguish whether the use of the allocated funds by the enterprise has been for renovation or expansion (new construction).[40] This creates a serious incentive problem as it is in the interest of both enterprises and construction organizations to undertake expansion rather than renovation -- it is easier, cheaper, and less disruptive of other plan performance for both. However, if enough enterprises attempt to use their investment funds for expansion rather than renovation, bottlenecks develop with respect to those resources and capacities peculiar to new contruction, and all such projects will fall behind schedule tying up significant amounts of resources, and no ones investment plans will be fulfilled. In a stylized model of an industrial branch with two enterprises this situation is reflected in a payoff structure that awards expansion if the other renovates as ordered, but punishes a joint deviation to expansion.

A third characteristic situation involves the choice of a proper output assortment in a branch of industry. All enterprises want to produce "easy" or "advantageous" (vygodnye) products, but others that are difficult and unrewarding to produce are just as needed elsewhere in the economy. Thus it is advantageous to ignore those other products, but all enterprises in the branch will suffer if no one produces them. This is also typical of the final situation we mention here: the use of locally generated (through the enterprise incentive funds) investment funds. These funds can be used either in ways that allow the enterprise to better pursue the objectives of its branch supervisors (maintenance, retooling, petty mechanization) or in ways that add to managerial slack and the "quality of life" in the enterprise[41] without furthering intra-branch cooperation. As the center can observe only aggregate outcomes, it is advantageous for any one enterprise to pursue the latter as long as others are pursuing branch interests. However, if too few enterprises pursue branch interests, the entire branch suffers from its plan failure.

Each of these situations generates a noncooperative game between subordinates, in which there are gains to proper coordination, particularly from the perspective of the center. Assuming that there are only two subordinates, we can write a characteristic payoff matrix as follows:

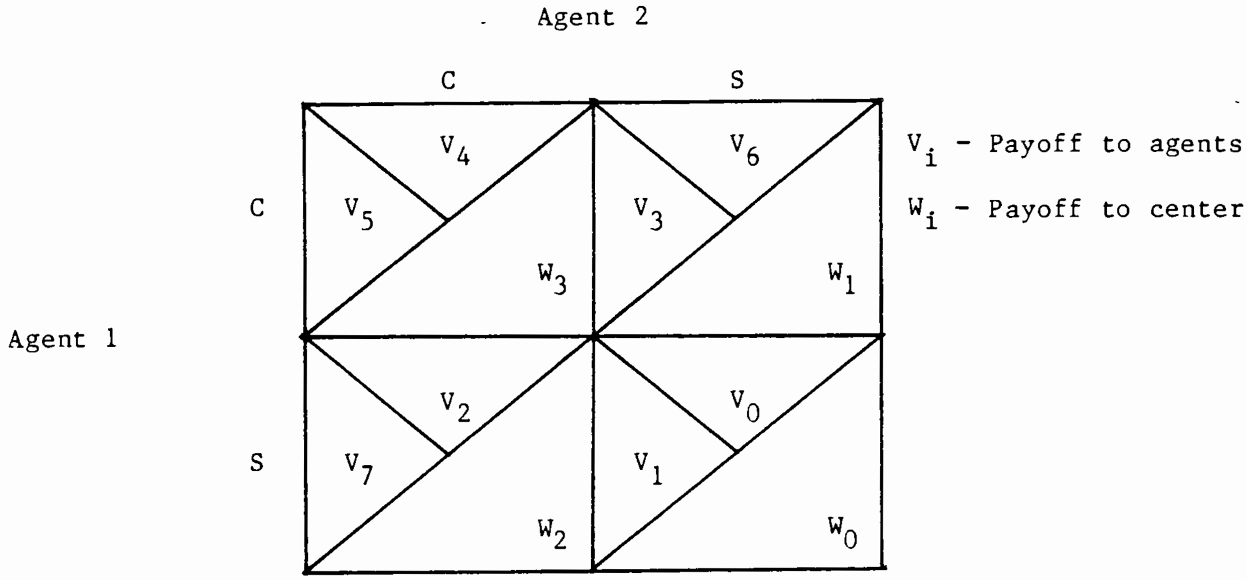


Figure 2

where i > j implies $V_i \geq V_j$ with strict inequality when j is odd and $W_i > W_j$, and the strategies of each agent are generically designated as cooperative (C) [e.g. renovate, choose proper assortment] and self-serving (S) [e.g. expand, 'easy' assortment]. Notice that the central payoffs are irrelevant to the strategic decisions of the agents. There are typically three Nash equilibria in such a game: two pure strategy - (S,C), (C,S), and one mixed strategy depending on the sizes of the relative payoffs. This is the set of outcomes that might be expected in the absence of central coordination.

Coordination in this model where no action is enforceable (Γ^I) must consist of recommending actions that are chosen in such a way that agents find it in their own interest to implement them. Thus a <u>mechanism</u> in this model is a common knowledge distribution over possible plans from which are drawn the actions each agent is ordered to implement (a correlated equilibrium: Aumann, 1974). As argued in Section III, for a mechanism to have any impact here it must be incentive compatible. Further, any Nash equilibrium can be implemented by an incentive compatible mechanism. It must be stochastic to be optimal as the pure strategy Nash equilibria are undesireable (Proposition 4 above), and it must (weakly) improve on every agents expected payoff, as it would otherwise fail to be incentive compatible. These points are immediate consequences of the programming problem that determines an optimal mechanism.

$$\max \sum_{i=0}^{3} \mu_i W_i \qquad \text{s.t.} \qquad \sum_{i=0}^{3} \mu_i = 1$$

(21) $$IC_1: \quad (V_5 - V_2)\mu_3 + (V_3 - V_1)\mu_1 \geq 0; \; (V_7 - V_5)\mu_2 + (V_1 - V_3)\mu_0 \geq 0$$

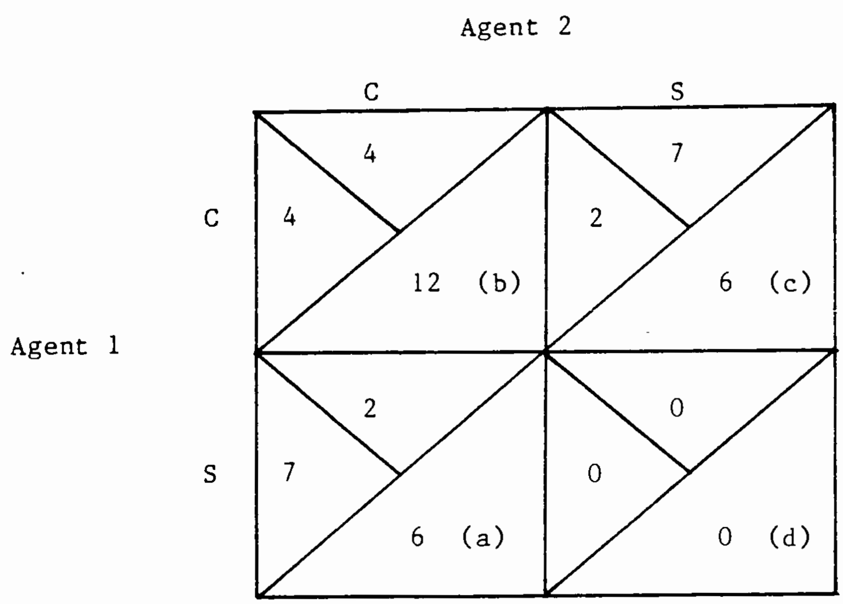
$$IC_2: \quad (V_4 - V_6)\mu_3 + (V_2 - V_0)\mu_2 \geq 0; \; (V_6 - V_4)\mu_1 + (V_0 - V_2)\mu_0 \geq 0$$

To illustrate these results we return to the structural model outlined at the beginning of this section, and put it in analytic form. As indicated there, the center takes no real action, and only observes and cares about the aggregate value of net output: $u_0(a) = q'Ty(a)$, $a = (a_1, a_2)$. Subordinates choose actions a_i, $i = 1, 2$, where $a_i \in A_i(a_j)$, $i \neq j$, and $A_i(a_j) = \{y_i, z_i, \ell_i) | c_{1i}y_i + \ell_i \leq L_i, \; c_{2i}y_i \leq z_j, \; z_i \leq y_i, \text{ and } z_i \geq c_{2j}z_j\}$, with z_i = that part of output shipped to other producer as intermediate input, ℓ_i = use of local resource for branch-oriented production, and L_i is the availability of the local resource. The objective is to maximize $u_i(a_1, a_2)$ where $a_i = (y_i, z_i, \ell_i)$. Assuming locally efficient and consistent behavior, the model can be reformulated as one of choosing output levels, with payoffs defined on the joint outputs. This gives $\hat{u}_0(y) = q'T_y$ and $\hat{u}_i(y_1, y_2)$ with $\frac{\partial \hat{u}_i}{\partial y_j} > 0$, $\frac{\partial \hat{u}_i}{\partial y_i} < 0$, $\frac{\partial^2 \hat{u}_i}{\partial y_i \partial y_j} < 0$ for $y_j >$ some $\bar{y}_j$, $\frac{\partial^2 \hat{u}_i}{\partial y_i^2} < 0$, and $\frac{\partial^2 \hat{u}_i}{\partial y_j^2} > 0$. The feasible choice set becomes $Y = \{(y_1 y_2) | Ty \geq 0,\ y_i \leq (L_i - \ell_i^*)/c_{1i}\}$ where ℓ_i^* is the optimal local use of the local resource.[42]

This general situation is depicted in Figure 3, where we look at net outputs so that $Ty = \hat{y} \geq 0$. The indifference curves of agent 1 are indicated by solid level curves and agent 2 by dashed curves. Such level curves could be generated by payoff functions to the agents of the form: $\hat{u}_i(\hat{y}_i, \hat{y}_j) = [\alpha\hat{y}_j + \beta\hat{y}_i](1 + \hat{y}_i)^{-1}$. Clearly the best outcome for agent 1 is (a) (Figure 3), where he only produces enough centrally desired output to support agent 2's provision to the center. Similarly (c) is the most desired outcome for agent 2, while (b) is most desireable from the perspective of the branch authorities. The totally non-cooperative (d) is least desireable for all.

These outcomes can be nicely summarized in a payoff matrix like that of Figure 2. for clarity we make an inessential assumption of payoff symmetry. As indifference curves and constraints are linear, we need only consider vertices.

(22)



The pure strategy Nash equilibria are (a) and (c), where agents 2 and 1 are, respectively, the "leading sector" in the eyes of the center. Notice,

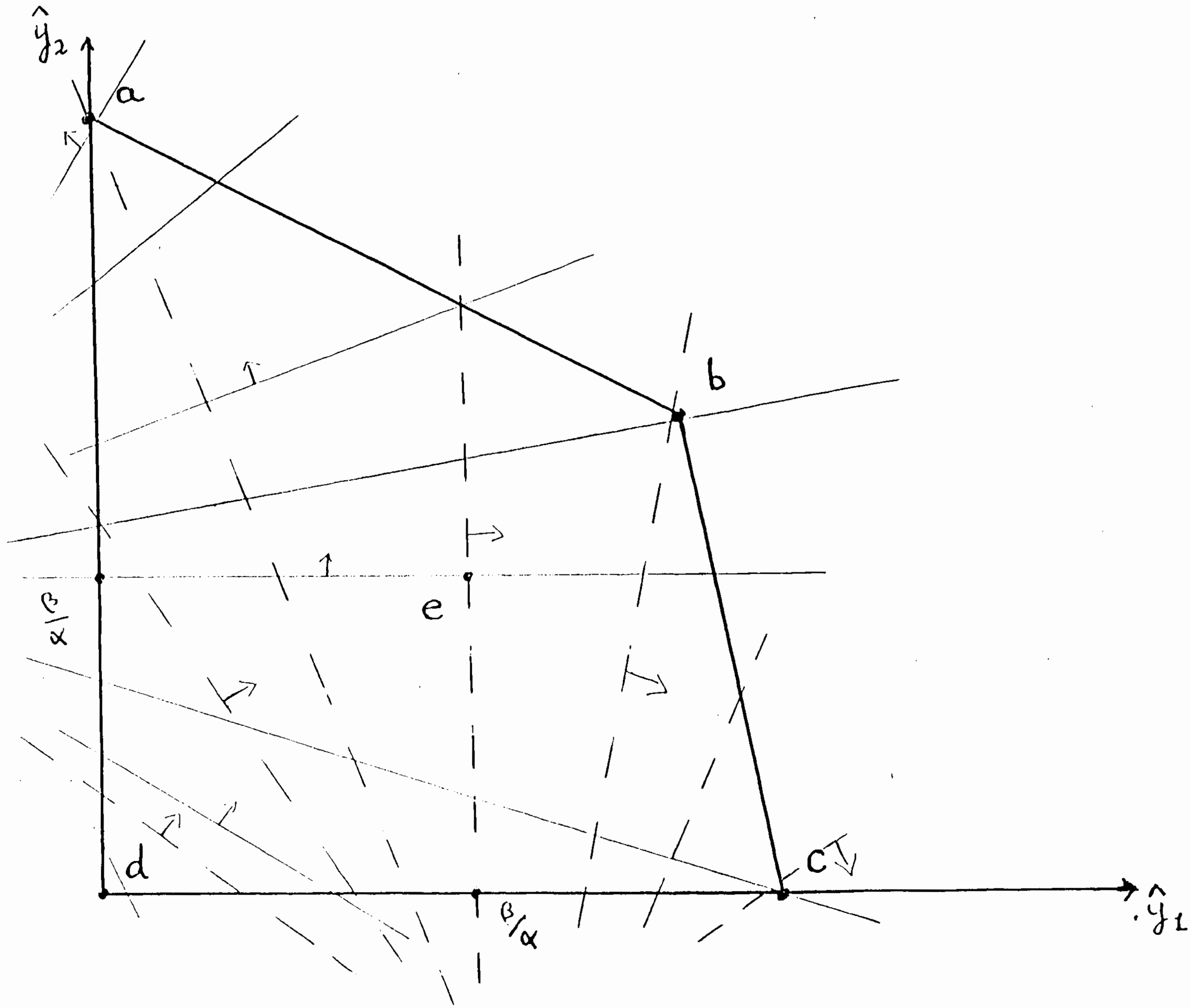


Figure 3

however, that this "leader" is a Stackelberg follower in the strategic interaction, receiving lower utility than the passive leader. The mixed strategy Nash (Cournot) equilibrium (by linearity of level curves, payoff equibalent to (e) in Figure 3) here is for each to choose C with probability .4 and S - .6. Each subordinate here receives an expected payoff of 2.8 which is an improvement only for the Stackelberg follower, while the center gets 4.8, decidedly worse than in the pure strategy equilibria. Any of these equilibria could be implemented as a central plan, yet clearly none is optimal.

The optimal mechanism, derived in problem (21), is the following: $\mu_a^* = .375$, $\mu_b^* = .25$, $\mu_c^* = .375$, $\mu_d^* = 0$. It yields payoffs to the agents of: $U_0^* = 7.5$, $U_1^* = 4.375$, $U_2^* = 4.375$. Notice that this mechanism is stochastic, and puts the maximum weight possible on (b), given the incentive compatibility constraints. While this is the best that the center can hope to do, there do exist incentive compatible mechanisms that are better for both subordinates, e.g. $\mu_a = \mu_c = .5$, $\mu_b = \mu_d = 0$. This mechanism implements optimal collusion between the subordinates at a significant cost in potential value to the center ($u_0 = 6$, $u_1 = u_2 = 4.5$), and as such might be thought of as a consequence of Second Economy interaction in the production sector.[43] This is the best joint outcome for the agents, though any one would rather be a Stackelberg leader were the other to acquiese in that. Finally we note that there is a continuum of efficient mechanisms between these corresponding to raising μ_b from 0 to .25 with a corresponding reduction of μ_a and μ_c by $\mu_b/2$ each.

To summarize, we have seen here that non-incentive-compatible mechanisms cannot influence subordinate behavior -- agents will ingore such instructions with respect to their unverifiable actions. Any Nash equilibrium of the game is incentive compatible, and can be ordered, and the only incentive compatible deterministic mechanisms are the pure strategy Nash equilibria. Any incentive compatible mechanism including the optimal mechanism, must improve (weakly) on all agents payoffs. Further, the optimal mechanism must be stochastic if there is any true incentive problem. Finally, the example illustrates that collusion among subordinates is undesireable for the center when the mechanism is incentive compatible, but may be desireable for all when the mechanism is not. Some implications of these results for our understanding of Soviet-type economies will be drawn in the conclusion.

This example readily generalizes in a number of different directions which we do not pursue here. Asymmetry can be introduced without qualitative impact. If the payoff to joint cooperation is greater than 4.5, then the optimal coordination mechanism can actually generate agent utilities above the collusive payoff [e.g. if the subordinate payoff to (c,c) in (22) is (5,5), then $\mu_a^* = \mu_b^* = \mu_c^* = 1/3$ gives $U_0^* = 8$, $U_1^* = U_2^* = 4.67 > 4.5$]. It is also easy to extend these results to more than two subordinate agents, though the elaboration of an analytic structural model becomes extremely complicated. Thus the analysis seems generally applicable to interaction of the type Γ^I, pure moral hazard without exogeneous uncertainty. The true test of these results, however, is how robust they are to the simultaneous introduction of adverse selection problems.

V. Conclusion

In this paper, we have only begun the investigation of a potentially far-reaching hypothesis about the functioning of Soviet-type economies. This hypothesis might be briefly formulated as follows. The structure of the command economy (CPE) is such that each agent is forced to play a series of compartmentalized "games" with known "opponents" who have private information and actions that vitally affect his performance. While the structure of each game is an invariant of the system, the payoffs to agents can be manipulated to a limited extent by the central authorities, who attempt to elicit operational information and subsequently coordinate the activities of the agents in the pursuit of "central" interests. Agents naturally manipulate both the information they provide and the details of their plan implementation behavior, whenever that is advantageous. What we then observe in command economies is an equilibrium outcome of these types of interactions in such games.

Our model is a first attempt to formalize this hypothesis and hence to provide the beginnings of an alternative paradigm for economic behavior in CPE's. It introduces a class of models which allow (i) non-anonymous interaction among agents, generating (ii) endogeneously determined (by others' "types" and actions) preferences, and (iii) essentially involving communication (planning) between superiors and subordinates. These models explicitly analyse two important sources of dysfuntional behavior in organizations in general and CPE's in particular: adverse selection and moral hazard. Due to these problems, the models possess equilibria that are suboptimal relative to the standard full-information, full-control benchmark, and distinguish those that are the best possible from the perspective of the central authorities. This provides two approaches, within the context of these models, to the explanation of observed outcomes: (i) some inefficiencies may be the natural cost of achieving incentive compatibility; (ii) observed inefficiencies may be the consequence of a lack of incentive compatibility in planning. Thus this approach provides a new way to categorize/structure our knowledge about behavior and outcomes in CPE's, one that we believe could be fruitfully applied to the data.

Our first explorations of this framework in the context of examples have illustrated these general points and have provided some conclusions suggestive of future possibilities. In particular, they may throw some light on the failure of the "Kosygin" and succeeding "reforms" in the Soviet economy. For those reforms, broadly speaking, attempted to enhance subordinate input into planning and increase the scope for subordinate decision-making. In our model that involves increasing mechanism depedence on subordinate reports and expanding the A_i. Without incentive compatible elicitation, however, our examples indicate that it is better for the center to ignore agent reports. Further, they show that unless coordination of unmonitored activities increases (weakly) the reward of subordinates over their best non-cooperative equilibrium responses, it cannot be incentive compatible and can have no positive effect. Also, if coordination is not incentive compatible then expanding subordinate discretion increases the opportunities for unauthorized cooperation among subordinates, which may improve overall economic performance even if not in ways desired by the center. Finally, the characterization of optimal (incentive compatible) mechanisms in these examples as stochastic shows how difficult it is to implement such a mechanism, supporting the contention that the "reform" mechanism is not incentive compatible. Thus we see here a number of possible economic reasons for the "reforms" to be ignored in practice and eventually themselves "reformed" away (Schroeder, 1979, 1982) and for the probably rapid growth of the Soviet Second Economy.

Of course, these results are merely suggestive as the model is highly stylized and as yet largely undeveloped. In particular, a number of technical problems must be solved and the institutional structure of the models must be made more specific for the model to become adequate to the evidence. One important technical problem relates to the existence and characterization of equilibrium behavior in the face of a non-incentive-compatible mechanism, when the space of actions is non-finite. The primary questions that must be resolved are: What constitutes "reasonable" manipulation by subordinates? How much can they learn from their plan, and how can they use it? What is a reasonable equilibrium concept in such manipulation games? These are closely related to the choice of an appropriate topology for general $\hat{A}_i$ (3) and to the appropriate formulation of the dual programming problem to (8). Solving these problems would allow as complete a formal characterization of the optimal mechanism in our most general case as that in Proposition 2 for the finite case. Doing so would also allow us to evaluate the costs of achieving incentive compatibility and of failing to do so, and to allocate those costs among (types of) agents. This would, we hope, elucidate the nature, necessity, and degree of bottlenecks in Soviet-type economies, particularly when the institutional context is more fully specified than in the above examples.

There are also a number of technical questions that need to be resolved in the examples of Section IV in order to fully exploit such examples. First, what is lost be restricting A, in Section IV.A, to only 4 extreme points? Perhaps nothing, due to linearity of preferences and constraints, but that must be formally verified. Next, under what conditions can a deterministic mechanism be optimal in adverse selection problems? What restrictions does that place on agent preferences, potential actions? How does this relate to the "severity" of the incentive problem? Though some speculative discussion is contained in Section III, we know no formal answer to these questions.

Thirdly, what is the precise analytic form of the optimal uninformed mechanism in Section IV.A? Knowing this would allow us to draw general conclusions about its tautness and relation to the fully optimal (IC) mechanism. Finally, more work is needed on the elaboration of the structural model behind the payoff matrices in Section IV.B, and on the types of subordinate collusion that might occur there. Resolving these questions would provide a basis for itegrating both types of examples.

Institutional specification needs to be aimed at addressing specific kinds of behaviors that are difficult to explain with the standard model of optimization subject to parametric constraints. In particular, the practices of storming; materials and labor hoarding, hiding capacity, and over-ordering (strakhovka); simulating and/or exaggerating performance (ochkovtiratel'stvo); the use of tolkachi and blat; frequent central changes of subordinate plans and enterprise interconnections; working to the observable plan, regardless of social cost; insensitivity to financial levers; etc. In view of the stability of these behaviors in the face of repeated reform, the hope is that a tractible structural model can be built to capture a whole complex of such behaviors as an equilibrium phenomenon. Some early steps in that direction were made in the examples of Section IV. Further progress will require a more detailed interpretation of the structure of such examples and bringing them together so that both incentive problems can interact in the agents' behavior. This is important as most stable dysfunctional behaviors involve both manipulation of reported information and manipulative interpretation and implementation of commands. The central authorities must be convinced that you are doing the best you can for them even as you "feather your own nest" and insure yourself against their future demands.

Why should that be? What is it that makes incentive compatibility problems endemic to Soviet-type economies? Without pretending to provide a complete answer, let us conclude by indicating one possible line of argument suggested by the preceding work. Most incentive compatibility problems in command economies are created, not by the essential economic issues faced, but by the economic system. That is, the system generates a set of non-cooperative games of incomplete information between its participants, and these games have only "bad" equilibria. By "bad" equilibria we mean those that are Pareto-inefficient and non-optimal from the center's perspective. Why that might be the case cannot be dealt with in any detail here, but a partial answer might be built on the following observations on characteristics of CPE's.

1. Agent payoffs are unrelated to use-values and/or demands. Further, they can only very partially represent central preferences which themselves are generally unrelated to use-values. (Nove, 1980).
2. Agent behavior is primarily motivated by pleasing superiors in the hierarchical structure, so there is no necessary relationship between incentives and true economic consequences. There is, and can be, no "bottom line" -- the system is ultimately irresponsible (Kushnirsky, 1983).
3. Rationalization of the hierarchical structure, e.g. eliminating "wasteful competition" and "unnecessary duplication", compartmentalizes and isolates units generating massive "induced externality" problems (Wolf, 1979, and Granick, 1982).
4. The system is highly illiquid, and structurally rigid, preventing a proper response to uncertainty: it is impossible to mobilize the means to deal with a fleeting problem or opportunity, even if the economic "cost" of failing to do so is extraordinarily high (Grossman, 1975).

5. Subordinates have no legal alternatives to their assigned interactions with others. Hence there is no way for subordinates to quickly rearrange their economic interactions at their own volition and/or in response to a changing situation.

Therefore each agent is stuck playing a small, compartmentalized game, which he cannot leave or refuse to play, and in which his payoffs are largely independent of the broader economic consequences of his actions. In this light, it does not seem surprising that resulting equilibria must be seriously inefficient. Further, it seems plausible that for any economic reform to improve efficiency that reform must change the nature of the game, and not just perturb payoffs. Is that possible without abandoning the command economy?

APPENDIX

To close the proof of Proposition 5, we show that:

<u>Lemma</u>: $\hat{\pi}_{11} > \hat{\pi}_{21} \Rightarrow V_1(\mu,1|0) > U_1(\mu|0)$.

<u>Proof</u>:
$$U_1(\mu|0) = 1/2(\pi^1_{00} - \gamma^1)y_{1,00} + 1/2(\pi^1_{01} - \gamma^1)y_{1,01},$$
$$V_1(\mu,1|0) = 1/2(\pi^1_{00} - \gamma^1)y'_{1,10} + 1/2(\pi^1_{01} - \gamma^1)y'_{1,11},$$

where $y_{1,t}$ is the output in state t for allocation z_t and $y'_{1,t}$ is the output in state t for allocation z_{t1}. We can write, dropping the identifier 1,

$$(23) \qquad 2(U - V) = (\pi_{00} - \gamma)[y_{00} - y'_{10}] + (\pi_{01} - \gamma)[y_{01} - y'_{11}].$$

If the expression in (23) can be shown to be negative, we are done. Notice that $-\delta = (\pi_{00} - \gamma) < 0 < \pi_{01} - \gamma) = \delta$ by (18ii). By (18iii):

$$(24) \quad \begin{aligned} &(a) \quad y_{00} - y'_{10} = \frac{x}{c_{11,00}} [(c_{11,00} + c_{12,00}c_{21,00})^{-1} - (c_{11,10} + c_{12,10}c_{21,10})^{-1}] < 0, \\ &(b) \quad y_{01} - y'_{11} = \frac{x}{c_{11,01}} [(c_{11,01} + c_{12,01}c_{21,01})^{-1} - (c_{11,11} + c_{12,11}c_{21,11})^{-1}] < 0, \end{aligned}$$

and the terms in parentheses in (b) are smaller than their counterparts in (a). Therefore combining (23) and (24) yields:

$$(25) \qquad 2(U - V) = \frac{\delta x}{c_{11}} [D^{-1}_{00} - D^{-1}_{10} - D^{-1}_{01} + D^{-1}_{11}] < 0,$$

where $D_t = c_{11t} + c_{12t}c_{21t}$. Q.E.D.

An identical arguement can be made showing that $\hat{\pi}_{11} > \hat{\pi}_{21}$ implies $V_2(\mu,0|1) > U_2(\mu|1)$, and symmetric arguements can be made when 2 is the leading producer.

FOOTNOTES

1. On the nature of the Soviet-type, "'command' centralized directive planning" system, see G. Grossman (1962, 1963) and A. Nove (1980). On the nature of the "complete hierarchcy" see Montias, Rose-Ackerman, (1981).

2. This logic is beautifully illustrated by Alec Nove in his very balanced discussion of The Soviet Economic System (1980). The quotes are from p. 37, 62.

3. These are called "derived externalities" by Charles Wolf (1979). Again Nove (1980) contains an enlightening discussion, cf. pp. 77-81. Also see Granick (1981), and David A. Dyker (1983), esp. pp. 21,26-50. The existence of such externalities is central to all the phenomena he studies.

4. Ericson (1982,1983a) has formally demonstrated this necessity in an obviously special model of resource allocation under uncertainty in a command economy.

5. This should be contrasted with the highly impersonal, frequently anonymous interaction characteristic of market intermediation among agents.

6. This is a widely noted phenomenon in the Soviet general and economic press. Western commentary on this phenomenon can be found in D.A. Dyker (1981, 1983), J. Berliner (1975), G. Schroeder (1979, 1983), A. Nove (1980), and Gregory & Stuart (1982).

7. This pattern is thoroughly documented in the Schroeder papers cited above in footnote 6.

8. Examples of such cycles can be found in each of the following "reforms": (i) the "New Soviet Incentive Mechanism," (ii) "decentralized investment," controlled by the enterprise, (iii) the new "Basic Methodology" for investment, (iv) the establishment of long-term ties for supply, and (v) the use of wholesale trade for distribution of producers products.

9. An "incentive scheme" is usually understood as a way of associating a personal financial reward/transfer for managers to the observed performance of the activity they manage. Miller and Murrell (1982, 1984) cogently argue that no optimal incentive scheme, i.e. one inducing managers to take decisions optimal from the central perspective, can exist in reasonable environments. We would argue further that such incentives can have at best only a marginal impact on behavior and must leave the general thrust of dysfunctional behavior unaffected. In a socialist society, as argued below, well-being depends on the entire vector of outcomes, with personal monetary wealth playing only a negligable role.

10. Needless to say, this is a modeling simplification made to allow analysis of a two, rather than multi, level hierarchy. Ministries are actually entirely in the control sphere. Only the lowest level subordinates actually produce or exchange anything.

11. Any public or centrally/generally verifiable information has been excluded from S. States thus describe relevant information that cannot be obtained or verified, except perhaps at prohibitive cost, by any agent in the economy. Certain aspects, however, may be revealed costlessly to certain agents.

12. This outcome may be the result of both a direct impact of the functioning of the economy on this agent's welfare and a punishment imposed by the central authorities, who may realize from results that the infeasible was not achieved. Of course, this violates technical assumption (A.5) below; if the A_i depend on s then (A.5) must be weakened to upper semi-continuity in a.

13. Goals are imposed from above, direct administered coordination is attempted, horizontal initiative/interaction is discouraged and frequently punished, and the separating effect of markets is eliminated in CPE's, inducing massive externalities. The model here attempts, if crudely, to capture these aspects.

14. This assumption can be justified on a number of grounds, the most straightforward of which is that it simplifies analysis in early modeling attempts. It might also be justified on grounds of the ideology of central planning: the center has nothing to hide and can in fact improve subordinate cooperation by informing them of its precise situation -- all will then cooperate in the "building of Socialism." It is, however, a restrictive assumption that I hope to dispence with in later work. See Myerson (1983b) for some of the difficulties involved.

15. Thus introducing communication generates a game in what might be called "pseudo-extensive" form. We reduce the game to normal form before undertaking any real analysis.

16. This is an oversimplification of a deep result that is most clearly explained in Myerson (1983a) pp. 7-13.

17. While this is generally so, some characterization results are most clearly stated for restricted, consistent beliefs. In the examples of Section IV we impose independence and consistency requirements.

18. Clearly a "direct mechanism" is a special case. One could easily imagine, as does Myerson (1982), more general communication/message spaces. However, the ideology of central planning dictates the direct communication of relevant information up, and of the actual plans/commands down, the hierarchy. Furthermore, the revelation principle (below, p. 19) agrues that there is no loss in doing so.

19. Notice that we are making a fairly strong "neo-classical" assumption here: that only final outcomes, and not the process of their generation, count. Though questionable, this assumption is standard in economic literature.

20. This description closely follows Myerson (1983a). While the assumption of private communication of commands may seem overly strong, it must be remembered that a_i only relates to actions unobservable and unmonitorable by others. The type of major commands relating to observable actions are all contained in a_0, and hence are common knowledge. The commands in a_i might relate to input quality norms, the treatment and maintenance of equipment, the fine tuning of the use of investment (or other allocated) funds, the precise scheduling and sequencing of production activity, etc.

21. Myerson (1983a), p.23. He then proceeds to give an heueristic proof of this principle. For a more formal statement and proof, see Myerson (1982), Proposition 2, pp. 73-5.

22. The latter are abviously heavily influenced by my experience with and understanding of the economy of the Soviet Union, though I believe them to be of more general relevance.

23. Of course there are also actions (indeed, most) that are partially verifiable. We have not attempted to model these in the interest of tractibility.

24. By a "true money" I mean "generalized command over goods and services at any time." The lack of availability, general "sellers' market," and massive documentation required for any transaction, among many other characteristics, illustrate this lack of money. In fact, the nature of a command economy makes the existence of a true money impossible. For some of the argument, see Grossman (1963, 1966).

25. There is little loss in generality here as Myerson has shown that a general problem is probability equivalent to one in which beliefs are consistent with such an independent common prior. See Myerson (1983), Section 3.

26. Notice that $\theta_i(\tau_i|t_i) = \sum_{\alpha_i : A_i \to A_i} \theta_i^0(\alpha_i(\cdot),\tau_i|t_i)$ where θ_i^0 is the Lagrange multiplier ("shadow price") on the constraint that i not be tempted to report τ_i and implement $\alpha_i(\cdot)$ when t_i is true, and $\beta_i(\alpha_i|a_i,\tau_i,t_i)$ is the conditional probability that i will choose action α_i given that he is in state t_i, has reported state τ_i, and was told to implement a_i. Hence

$$\beta_i(\alpha_i|a_i,\tau_i,t_i)\theta_i(\tau_i|t_i) = \sum_{\{\alpha_i(a_i)=\alpha_i\}} \theta_i^0(\alpha_i(\cdot),\tau_i|t_i)$$

for all i, a_i, α_i, t_i, τ_i, and satisfies (6.3) (even when $\theta_i = 0$). See Myerson (1983), p. 42, 43.

27. As argued by Milgrom and Weber (p. 39) these p_i need not be derived from the same information structure p(t) and hence need not be mutually consistent. All that is needed is that each $p_i(t)$ be a continuous information structure, which is implied by (A.2) [Proposition 3, p. 20, Milgrom, Weber (1980)].

28. When agents are risk neutral and the action space is non-finite and connected then it seems such actions can always be found. Myerson (1983a) has an example. The literature on optimal bidding in auctions also contains such examples.

29. We do not yet have a formalization of the required "consistency." Intuitively it means that meeting the IC constraint of one agent doesn't automatically jeopardize that of another. If agents are playing a (different) zero-sum game for each state, then such consistency would be lacking.

30. Prescott and Townsend (1982, 1984). When all agents are risk-neutral this set is convex.

31. This assumption is clearly unrealistic, and is made only to eliminate moral hazard considerations. We further ignore the feedback from physical constraints to the knowledge that the center has about the true state: any information revealed by physical outcomes cannot, by assumption, be used against the agent. The assumption behind this is that there is enough uncertainty in outcomes in the real world that the center could never really determine that the cause of any disruption was lying by the subordinates.

32. This captures the idea that payoffs/rewards to economic activity frequently depend on factors not only outside of the control of the agent, but beyond his knowledge. Here specifically, whether the reward for activity exceeds its local cost depends on factors about which the enterprise can know nothing.

33. More precisely, the support is the complex of actions which would yield y_t if the true state were known to be t. If t is not the state when that complex of actions is undertaken, the realized output will naturally be less than y_t. We will, however, continue to refer to that complex of actions as y_t.

34. Detailed computations are in the Appendix. We define i as a <u>leading/priority</u> producer if $\hat{\pi}_{it} > \hat{\pi}_{jt}$ for at least three of the four states. In general there will be a leading producer in our framework.

35. Here productivity is measured in terms of materials use, i.e. lower input coefficients implies higher productivity.

36. A non-incentive compatible mechanism only alters the payoff matrix faced by agents and doesn't convey any information about the true state or the other agents' actions. Thus the outcome of the "reporting game" between subordinates will be some Bayesian Nash equilibrium of this game, with altered payoffs. Central preferences can have no influence on this outcome.

37. This is an unrealistic assumption as one would anticipate that outcomes would reveal actions in a world without uncertainty. It is made solely for analytic clarity.

38. Even casual perusal of the Soviet economic press will yield these and other cases where uncontrolled/unplanned choices made by producers has caused major disruption elsewhere in the economy, while being desirable from the perspective of those producers.

39. See the various papers in Part I of the J.E.C., U.S. Congress (1983), and in particular Robert Leggett (1983). He details the inability of the central authorities to distinguish investment in expansion, renovation, and technological reequipment.

40. Rumer (1982) estimates that over 60% of all funds officially used for renovation actually goes into new construction/expansion.

41. Many such measures, as housing, recreation, daycare, pollution control, etc. are desired by the local government within whose jurisdiction the enterprise physically resides.

42. The ℓ_i's will in general be a function of the y_i's, though for the sake of a tractible analysis we ignore that dependence here. More generally, we want assumptions such that the branch feasible set remains convex.

43. On the Soviet second economy see Grossman (1977, 1979). On its potential role in the production sector see Ericson (1983b, 1984).

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