

Discussion Paper No. 648

INCENTIVE COMPATIBILITY TEN YEARS LATER

by

Theodore Groves

and

John O. Ledyard

Northwestern University

and

California Institute of Technology

April 1985

# Incentive Compatibility Ten Years Later\*

Theodore Groves\*\* and John O. Ledyard\*\*\*

## I. INTRODUCTION

## II. RESOURCE ALLOCATION IN CLASSICAL ECONOMIC ENVIRONMENTS

### A. PRIVATE GOODS MODEL

### B. PUBLIC GOODS MODEL

### C. ALLOCATION MECHANISMS

#### 1. Language

#### 2. Outcome function

### D. SELF-INTERESTED BEHAVIOR

### E. PERFORMANCE AND EVALUATION

## III. EFFICIENCY AND INCENTIVE COMPATIBILITY

### A. DOMINANT STRATEGIES

### B. FINITE ECONOMIES AND DOMINANT STRATEGY MECHANISMS

### C. FINITE ECONOMIES AND EFFICIENT DOMINANT STRATEGY MECHANISMS

### D. SUMMARY

#### APPENDIX TO SECTION III: Proof of Theorem 3.2

---

\* We would like to thank Andy Postlewaite, Roy Radner, Stefan Reichelstein, and especially John Roberts for their many helpful comments and efforts to save us from inaccuracies. Remaining errors are our own.

\*\* University of California, San Diego

\*\*\*Northwestern University and California Institute of Technology

#### IV. EFFICIENCY WITHOUT DOMINANT STRATEGIES

##### A. NON-PARAMETRIC MECHANISMS

1. Bayes Equilibrium

2. Nash Equilibrium

a. Specific examples

i. Private goods

ii. Public goods

b. Characterizations

3. Manipulative Nash

4. Other Possibilities

##### B. PARAMETRIC MECHANISMS

1. Direct Information

2. Indirect Information

##### C. SUMMARY

APPENDIX TO SECTION IV: Sketches of Proofs

#### V. EFFICIENCY AND INCENTIVE COMPATIBILITY IN LARGE ECONOMIES

##### A. CONTINUUM ENVIRONMENTS

##### B. LARGE FINITE ECONOMIES

##### C. SUMMARY

APPENDIX TO SECTION V: Sketches of Proofs

#### BIBLIOGRAPHY

## I. INTRODUCTION

Although discussions of the role of private incentives have been included in writings on economics and political economy for over two hundred years--at least as far back as Adam Smith's *Wealth of Nations*--the formal treatment of the subject is a recent development in economics. A seminal paper of the modern era was written in 1972 by Leo Hurwicz, a dozen years after his pathbreaking work on the foundations of decentralized resource allocation mechanisms.<sup>1</sup> In that paper he introduced the concept of incentive compatibility and proved that there cannot exist any informationally decentralized mechanism (or procedure) for resource allocation in private goods economies that simultaneously yields Pareto-efficient allocations and provides sufficient incentives to consumers to honestly reveal their true preferences.<sup>2</sup> There were earlier papers that formally discussed mechanisms for making resource allocation decisions in a manner compatible with individual incentives, such as Vickrey (1961), Groves (1970), and Clarke (1971), but Hurwicz was the first to establish results for a classical full general equilibrium model of an exchange economy. His paper was the major stimulus to the large number of papers that have subsequently appeared. For this reason we begin this retrospective at that date.<sup>3</sup>

The concept of incentive compatibility, introduced by Hurwicz to capture the forces for individual self-interested behavior, has proven to be one of great scope, serving as an organizing principle of considerable power. Perhaps the ~~close~~<sup>close</sup> analogy in economics is the concept of efficiency. For the positivist, notions of self-interested behavior lie at the foundation of all microeconomic theory. Indeed, the only outcomes that can be generally realized in any situation are those that result from individual decision makers following their own interests. For the normativist, relatives of the concept of incentive compatibility may be traced to the "invisible hand" of Adam Smith who claimed that in following individual self-interest the interest of society might be served. Related issues were a central concern in the "Socialist Controversy" which arose over the viability of a socialist society. It was

1. See Hurwicz (1960) and Hurwicz (1972). The later paper continues to be an excellent introduction to the subject of incentives in resource allocation.

2. A precise statement of this theorem is given below in Section III.A.

3. We began this survey in 1982, which may explain the arithmetic in the title.

argued by some that such societies would have to rely on individuals to follow the rules of the system. Some believed this reliance was naive; others did not. These debates led to the modern theory of mechanism design which treats incentive compatibility as a constraint on the choice of procedures able to be used to make group allocation decisions in various economic institutional contexts.

In this paper, we present an organized overview of what is now known about the possibilities for the incentive compatible design of mechanisms. We also indicate some of the major remaining mysteries. However, incentive compatibility questions have been addressed for models of central planning, regulation of monopoly, transfer pricing, and capital budgeting, to name just a few. Therefore, rather than try to survey the entire recent literature on the subject (a book-length task), we have chosen, following Hurwicz (1972), to concentrate instead on incentives in two well-known classical general equilibrium models of resource allocation -- one being the standard private goods pure exchange model, the other a simple public and private goods general equilibrium model. Thus, many papers on incentive compatibility written in the last decade will not be mentioned here. In particular we ignore the large amount of exciting work concerned with design and incentive problems in a partial equilibrium framework [see Myerson (1983) for an excellent introduction] or the work on particular institutions in which information and incentive issues are crucial [see, for example, Milgrom and Weber (1982) and Wilson (1983)].<sup>4</sup> Furthermore, even in the narrow area to which we have constrained ourselves, our survey is undoubtedly incomplete -- rather, it is a personal overview of results comparing private goods economies with those with public goods. Two surveys from differing points of view but covering some of the same results are those of Schmeidler (1982) and Postlewaite (1983).<sup>5</sup>

Our decision to concentrate on the differences between and similarities of the conclusions one may draw concerning incentive compatible design of resource allocation mechanisms in private and

---

4. Special subsets of the general equilibrium environments will be mentioned in subsequent sections. These include environments restricted to quasi-linear preferences and to zero-one choices. Since many of the results known to hold in these special cases do not survive in the more general environments, we most often refer to them as examples only.

5. For some other surveys of the voluminous literature on Incentive Compatible Social Choice, Implementability of Social Choice Rules, etc. see the surveys of Dasgupta, Hammond, and Maskin (1979), Groves (1979), and Laffont and Maskin (1983).

public goods environments allows us both to summarize a large number of contributions, many of which address this central issue, and to show how rigorous analyses of incentive compatibility have deepened and changed the conventional wisdom regarding the possibility for achieving Pareto-efficient allocations via decentralized means (such as competitive markets). That conventional wisdom before 1972, it is fair to say, could be summarized in two statements:

- 1) In classical private goods economies, Pareto-efficiency is consistent with individual self-interest since price taking behavior is reasonable in competitive markets, especially if the number of agents is large.
- 2) In classical public goods economies, Pareto-efficiency is not consistent with individual self-interest since agents will have an incentive to "free ride" on others' provision of public goods (in order to reduce their own share of the burden of providing them).

As we show in the sections below, it is now known that these statements are seriously misleading and obscure some important and subtle distinctions between private and public goods. For the impatient reader, all of the results we detail are summarized at the end of each Section of this paper.<sup>6</sup> To whet the appetite, however, we briefly summarize the five main results which most effectively highlight the differences between private and public goods. The first three hold for both private and public goods environments.

1. In classical (private *and* public goods) economies with a finite number of agents there are no non-parametric mechanisms that simultaneously yield Pareto-efficient allocations and provide individuals agents with incentives to report their true preferences honestly.

Thus, since agents cannot be induced to behave in an incentive compatible manner, the analysis of resource allocation mechanisms requires some prediction of agent behavior.

2. In classical (private *and* public goods) economies with a finite number of agents, there are non-parametric mechanisms that yield Pareto-efficient allocations when all agents follow their self-interest by playing a Nash-equilibrium strategy.

---

6. We have delayed summaries to the end since much of the language used must be precisely defined before it is really understood. These definitions are contained in the body of the paper.

Since the pre-1972 conventional wisdom suggests that price taking behavior in private goods economies with many independent agents is in each agent's interest, one might look to economies with a continuum of agents to find a difference between public and private goods.

3. In classical (private *and* public goods) economies with a continuum of agents, there exist mechanisms that simultaneously yield Pareto-efficient allocations and provide individual agents with incentives to report their true preferences honestly. (Compare to 1 above.)

In large finite but growing economies, we can find a distinction between private and public goods economies for mechanism design.

4. In classical *private goods* economies, there exist mechanisms such that the Nash equilibrium strategy yields an "almost" Pareto-efficient allocation as the outcome and is "almost" equivalent to reporting agents' true preferences, if the economy is "large enough".

The same result does not appear to hold for public goods economies.

5. In classical *public goods* economies, there exist mechanisms such that the Nash equilibrium strategy is "almost" equivalent to reporting agents' true preferences, if the economy is "large enough", but it seems that none of these mechanisms simultaneously yields an "almost" Pareto-efficient allocation, no matter how many (finite number of) agents there are in the economy.<sup>7</sup>

We turn now to a survey of the literature that underlies these and many other facts that have been discovered in the last decade.

---

7. Parts of 5) remain conjecture. This is carefully discussed in Section V.

## II. RESOURCE ALLOCATION MECHANISMS IN CLASSICAL ECONOMIC ENVIRONMENTS

To begin our survey, we first introduce a useful model for organizing the material in this area. This model allows us to standardize notation and to compare and contrast the results of many researchers within a common framework. It is our hope that others, unfamiliar with this area, will also find this to be helpful.

The four primary components of our model are the environment (endowments, preferences, opportunities, etc.), the allocation mechanism (a language and an outcome rule), a reduced form description of self-interested behavior (an example is Nash equilibrium), and a concept of "good" allocations (such as Pareto-efficient, equitable, etc.). The first and the last will be familiar to economists since these components are from standard general equilibrium theory. The second and third will be familiar to game theorists since much of these components comes from standard n-person, non-cooperative game theory. The analysis of incentive compatibility requires all four to be merged into a common framework which we do below.

### A. PRIVATE GOODS MODEL

In the classical model of a private goods economy, there are  $N$  consumers and  $L$  goods. Each consumer is endowed with an amount of each good; denoted by the  $L$  dimensional vector,  $w_i$ . We represent the consumption of the  $i^{\text{th}}$  consumer by the  $L$  dimensional vector  $x_i$ . Each consumer has a neo-classical utility function,  $u_i(x_i)$ , which is assumed to be strictly quasi-concave, monotonic, and  $C^2$  on  $R_+^L$ . We assume that the consumer can only consume bundles  $x_i$  of commodities with non-negative amounts of each commodity. In some cases, we will represent the  $i^{\text{th}}$  utility function as  $u(x_i, y_i)$  where  $y_i$  is the parameter defining the particular utility function from some class of functions. In the tradition of Hurwicz (1960), using the language of mechanism theory, we call  $e_i = (y_i, w_i)$  the characteristic of consumer  $i$  and we call the full vector,  $e = (e_1, \dots, e_N)$ , the environment.



An allocation in this classical environment is a vector of consumption bundles,  $x=(x_1, \dots, x_N)$ . Several of these allocations have special significance for economists. An allocation is feasible<sup>8</sup> for the environment  $e$  if and only if  $x_i \geq 0$  for each  $i$  and  $\sum x_i = \sum w_i$ . An allocation is Pareto-efficient in the environment  $e$  if it is feasible and if there is no other feasible allocation at which every consumer is at least as well off and at least one is better off. Formally,  $x^*$  is Pareto-efficient in  $e$  if and only if (i)  $x^*$  is feasible for  $e$  and (ii) if  $x$  is feasible for  $e$  then  $u_i(x_i) < u_i(x_i^*)$  for at least one  $i$ . An allocation,  $x$ , is Walrasian for  $e$ , (sometimes called a competitive allocation), if (i)  $x$  is feasible and (ii) if there is a vector  $p$  in  $R_+^L$ , a price vector, such that (iia)  $px_i = pw_i$  and (iib) if  $u_i(x_i^*) > u_i(x_i)$  then  $px_i^* > pw_i$ .

The Fundamental Welfare Theorem applies to all environments which we have called classical (see, e.g., Arrow (1951) or Debreu (1959)). If  $e$  satisfies the assumptions we have made then two results hold:

- 1) if  $x$  is Walrasian in  $e$  then  $x$  is Pareto-efficient in  $e$
- 2) if  $x$  is Pareto-efficient in  $e$  then there exists a redistribution of  $w$  such that for the new environment  $e'$ ,  $x$  is Walrasian in  $e'$ . (A redistribution of  $w$  is a vector  $w' = (w_1', \dots, w_N')$  such that  $\sum w_i' = \sum w_i$ .)

It has been received doctrine since the time of Adam Smith (1776) that private-ownership market institutions are efficient under competitive conditions and that it is in the self-interest of the individuals to behave competitively. Stated another way, in private-ownership economies, even if all agents aggressively follow their self-interest, the market will lead them to promote the interests of the whole. The classical welfare theorems stated above provide one of the two necessary steps for a formal statement and proof of this conventional wisdom. Interpreting Pareto-efficiency as "the interest of the whole" we know from these theorems that if individuals do behave competitively they

---

8. This definition of feasibility is standard and includes both individual feasibility and material balance. Recently, for mechanism design problems, some authors [e.g. Myerson (1981) and (1983)] have suggested that incentive compatibility constraints be included in the definition of feasibility to reflect the fact that certain allocations may not be attained because they require the transmission of private information and the holders of that information may have an incentive to dissemble in transmitting it. Optimality is then defined relative to these informational/incentival constraints as a "second-best" concept. In this paper we are interested in the possibility of designing mechanisms that yield "first-best" (sometimes called *expost* optimal) allocations and, thus, we stay with the standard definitions.

will serve this interest. The other step is the demonstration that it is in the self-interest of the consumers to behave competitively. Prior to 1972 most economists believed that fact to be either true or a good enough approximation in an economy with a large number of consumers.

## B. PUBLIC GOODS MODEL

In the classical model of a public goods economy, there are  $N$  consumers,  $L$  private goods, and  $M$  public goods. Each consumer is endowed with an amount of each private good; denoted by the  $L$  dimensional vector,  $w_i$ . We represent the consumption of the  $i^{\text{th}}$  consumer by the  $L-M$  dimensional vector  $(x_i, z)$ . Each consumer has a neo-classical utility function,  $u_i(x_i, z)$ , which is assumed to be strictly quasi-concave, monotonic, and  $C^2$  on  $\mathbb{R}_+^{L+M}$ . We assume that the consumer can only consume bundles  $x_i$  of commodities with non-negative amounts of each commodity. In some cases, we will represent the  $i^{\text{th}}$  utility function as  $u(x_i, z, y_i)$  where  $y_i$  is the parameter defining the particular utility function from some class of functions. We assume that there is no initial endowment of public goods but that a transformation surface defines the rate at which private goods can be used to produce public goods. This surface is denoted by  $T(r, z) = 0$ , where  $r$  is the vector of private goods inputs. We assume for simplicity that  $T(\cdot)$  is linear; that is, if  $T(r, z) = 0$  then  $T(\lambda r, \lambda z) = 0$  for all  $\lambda > 0$ . As above, we call  $e_i = (y_i, w_i)$  the characteristic of consumer  $i$  and we call the full vector,  $e = (e_1, \dots, e_N, T(\cdot))$ , the environment.

An allocation in this classical environment is a vector,  $(x, z) = (x_1, \dots, x_N, z)$ . As in the case of the private goods economy several of these allocations have special significance. An allocation is feasible for the environment  $e$  if and only if (i)  $x_i \geq 0$  for each  $i$  and  $z \geq 0$ , (ii)  $T(r, z) = 0$  and (iii)  $\sum x_i - r = \sum w_i$ . An allocation is Pareto-efficient in the environment  $e$  if it is feasible and if there is no other feasible allocation at which every consumer is at least as well off and at least one is better off. Formally,  $(x^*, z^*)$  is Pareto-efficient in  $e$  if and only if (i)  $(x^*, z^*)$  is feasible for  $e$  and (ii) if  $(x, z)$  is feasible for  $e$  then  $u_i(x_i, z) < u_i(x_i^*, z^*)$  for at least one  $i$ . An allocation,  $x$ , is Lindahl for  $e$  if (i)  $(x, z)$  is feasible and (ii) if there is a vector  $p$  in  $\mathbb{R}_+^L$ , and vectors,  $q_i$ , one for each  $i$ , in  $\mathbb{R}_+^M$  such that

(iia)  $px_i + q_i z = pw_i$ , (iib) if  $u_i(x_i^*, z^*) > u_i(x_i, z)$  then  $px_i^* + q_i z^* > pw_i^*$ , and (iic)  $qz - pr = 0$  where  $r = \Sigma x_i - \Sigma w_i$  and  $q = \Sigma q_i$ .

The Fundamental Welfare Theorem applies to all public goods environments which we have called classical (see, e.g., Foley (1967)). If  $e$  satisfies the assumptions we have made then two results hold:

- 1) if  $x$  is Lindahl in  $e$  then  $x$  is Pareto-efficient in  $e$
- 2) if  $x$  is Pareto-efficient in  $e$  then there exists a redistribution of  $w$  such that for the new environment  $e'$ ,  $x$  is Lindahl in  $e'$ .

It has been received doctrine since Samuelson (1954), that private-ownership market institutions are not efficient when there are public goods since it is not in the self-interest of individuals to behave competitively. Stated another way, in private-ownership economies with public goods, if all agents aggressively follow their own self-interest, decentralized institutions will not lead them to promote the interests of the whole. Although it was never formalized, prior to 1972 most economists believed that in the presence of public goods efficient allocations were impossible to attain with decentralized mechanisms if agents behaved in their own self-interest.

### C. ALLOCATION MECHANISMS

An allocation mechanism is an abstraction of the enormous variety of institutions used to allocate resources; that is, used to choose a specific allocation given the environment. Many abstract models of allocation systems have been proposed since the seminal paper of Hurwicz (1960). We use one in this paper that we have found to be especially useful. It does not explicitly model all the possible communication and decision relationships between every agent in the economy, nor does it explicitly model the sequences and number of iterations necessary to complete the transfer of information. We therefore refer to this as the *normal* form of a mechanism. This description is adequate initially but we will later discuss its limitations. An allocation mechanism (in normal form), then, is simply a language and an outcome rule.

### 1. Language

Let  $M_i$  denote the language (message space) which agent  $i$  can use to communicate. A few specific examples of the types of messages the language might contain are a vector of proposed trades (quantity demanded or supplied), a description of  $i$ 's characteristic, a list of the amounts  $i$  is willing to spend on each good, a description of  $i$ 's cost structure, or a collection of conditional responses to others' proposals. Letting  $M$  be the product space  $M_1 \times \dots \times M_N$ , we call  $M$  the language of the allocation process.

### 2. Outcome function

The other part of an allocation mechanism is a function which associates an allocation with any vector  $m = (m_1, \dots, m_N)$  of messages from the language  $M$ . We denote this outcome function as  $h: M \rightarrow A$  where  $A$  is the set of allocations to be chosen among. Many problems arise if  $h$  is not single-valued. For example, agents may be unable to coordinate actions or a single agent may be unable to evaluate the consequences of his actions even if he knows the actions of others. Mechanisms that are not single-valued are not well-defined. (An example of such a mechanism, called the Competitive Mechanism, can be found below following Theorem 3.1.) For almost all of this paper, we avoid these problems by simply assuming  $h$  is a function. We will point out when this is not assumed.

It is important to note that in order for this description of an outcome function to make sense it is necessary to know something about the class of environments in which the mechanism is operating. In particular we need to know the number of agents or consumers and the type of allocations which will be considered. The space  $A$  will look different for private goods economies than it does for public goods economies. The need for this prior information is not a handicap but it is a limitation which should be noted.

Another point to be noted is that this formulation of an allocation mechanism is more general than might be apparent. Even though we have not explicitly modeled any form of iteration, it is possible to include mechanisms of that type in the same way that a normal form game can sometimes summarize a sequence of moves in an extensive form game. Recognition of this fact is important for

the later discussion of implementation. Models of allocation mechanisms which explicitly allow for the iterative steps in a communication process can be found in Hurwicz (1960), Reiter (1974), Smith (1979), and Smith (1982).

Finally, in some models of mechanisms in the literature, information other than  $m$  appears in the outcome functions. This "extra" information has to be viewed as common knowledge which is known to the designer of the system as well as all the participants prior to the use of the allocation mechanism. Some examples of this can be found in the Optimal Auction literature (see, e.g., Myerson (1981)) where prior beliefs about which environment is the real one are allowed to be used by the outcome rule. Another type of "extra" information commonly used in the outcome function is information about initial endowments, which potentially can be audited in a way that preferences cannot, in order to insure feasibility of the outcomes (see, e.g., Postlewaite and Schmeidler (1979)). Hurwicz (1972) has called outcome rules which use other information in addition to agents' messages, 'parametric outcome functions'. They can be modeled as  $h: E \times M \rightarrow A$  or as  $h: M \times I \rightarrow A$  where  $I$  is a space of common knowledge information about the environment. We will use this formulation later in Section IV.B.

#### D. SELF-INTERESTED BEHAVIOR

In order to address issues such as those raised by A. Smith and Samuelson concerning the performance of various ways of allocating resources in the face of self-interested behavior, it is necessary to be more precise about the particular form this behavior takes. Formally, given the allocation mechanism  $h$ , we summarize behavior as a mapping,  $b: E \rightarrow M$ , where the dependence of  $b$  on  $h$  is ignored only notationally. To see what is assumed in any particular  $b$  consider a simple example, leaving others for later. We define a dominant strategy for agent  $i$ , given the class of environments  $E$  and the mechanism  $h$ , as a mapping  $d_i: E_i \rightarrow M_i$  such that, for all  $e$  to be considered (all  $e \in E$ ) and all  $m \in M$ ,  $u(h(m), y_i) \leq u(h(m/d_i(e_i)), e_i)$  where the vector  $(m/s_i) \equiv (m_1, \dots, m_{i-1}, s_i, m_{i+1}, \dots, m_N)$ .

A fundamental, but generally unstated, axiom of non-cooperative behavior is that if an individual has a dominant strategy available, he will use it. Under this axiom, if all agents have dominant strategies  $d_i: E_i \rightarrow M_i$  given  $h$ , we can let  $b(e) = (d_1(e_1), \dots, d_N(e_N))$  for all  $e \in E$ . Thus  $b(\cdot)$  captures the behavioral assumption that dominant strategies will be used. Of course if there are no dominant strategies for some  $i$ , the mapping is not well defined and the axiom is not sufficient to describe the behavior of the agents. In this case, it is necessary to turn to other behavioral assumptions.

When mechanism theory was originally formulated in Hurwicz (1960), the behavioral rules were more explicit and were viewed as prescriptive. For example, the rule might be 'report your marginal cost'. It was assumed that agents would follow the rules. Here the behavioral rule,  $b$ , is viewed as a descriptive phenomenon, since we assume that agents will follow self-interested behavior. The function  $b$  will be our model of that behavior.

### E. PERFORMANCE AND EVALUATION

Given a description of the environment,  $e$ , of the allocation mechanism,  $h$ , and of the assumed behavior,  $b$ , we can summarize the performance of that mechanism in that environment (or class of environments) under that behavior by the mapping  $P: E \rightarrow A$ , where  $P(e; h, b) = h(b(e))$  for all  $e \in E$  is the composition of the mechanism's outcome rule and the behavioral rule. Graphically this is represented in Figure 1 by the commuting diagram of Reiter (1977).

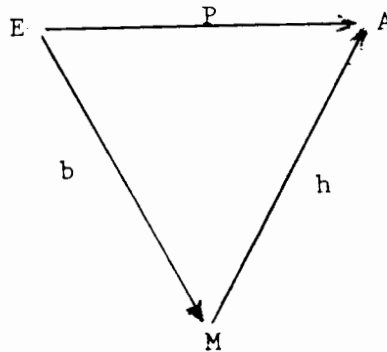


Figure 1

In using the terminology above, we are ignoring other performance characteristics of allocation mechanisms which are important, such as the informational costs and the computational complexities.

We do so in order to concentrate on the incentive aspects of mechanisms.

Once the performance of the mechanism is known, one can then compare that performance to some idealization. For example, it has been traditional to ask whether performance is consistent with Pareto-efficiency. In particular, let  $S(e) = \{\text{allocations in } e \mid \text{allocation is Pareto-efficient in } e\}$ . The question is then, is  $P(e) \subset S(e)$  for all  $e \in E$ ? If the answer is yes, it is sometimes said that the mechanism implements the Pareto correspondence,  $S$ . Notice that the use of the Pareto correspondence is only illustrative. Any correspondence from  $E$  to  $A$  could be considered. Some "ideal performance functions" which have been used in the literature are (1) the Pareto-efficient allocations, (2) the individually rational allocations (i.e., those which leave everyone at least as well off as they were at the initial allocation), (3) the core allocations, (4) the Walrasian allocations, (5) the Lindahl allocations, (6) the allocations which yield the Shapley value, and (7) equitable allocations. We consider some of these in the sections below.

In the past there have been many variations of the basic evaluation question stated above. The original issue in the design of allocation mechanisms (see, e.g., Hurwicz (1960)) was the following: given a class of environments,  $E$ , and a performance criterion,  $P$ , is there a mechanism and a behavioral rule such that the performance of that mechanism under that rule is consistent with the performance criterion over that class of environments. For the purposes of this paper it is important to note that not only were the rules of the mechanism to be prescribed but also the behavior of the agents. In his 1972 paper, Hurwicz raised the incentive issue: suppose we cannot prescribe behavior but instead, as designers, must take it as given. What can we then do? In particular, given a class of environments,  $E$ , a performance criterion,  $S$ , and assumed behavior,  $b$ , does there exist a mechanism,  $h$ , such that  $P(e;h,b) \subset S(e)$  for all  $e \in E$ ? In later work, this continued to be the basic question asked. Sometimes it was extended to ask for a characterization of all such mechanisms; sometimes additional constraints (such as a minimal message space) were placed on the search; and sometimes the designer was allowed to use additional information (such as in the optimal auction literature); but fundamentally the basic question has remained as in Hurwicz (1972).

### III. EFFICIENCY AND STRONG INCENTIVE COMPATIBILITY

Partly because of the known and satisfactory efficiency properties of competitive markets and partly because of the inherent acceptability of the concept of Pareto-efficiency as a minimal welfare criteria, much of the literature on the design and evaluation of allocation mechanisms has adopted the Pareto correspondence as a primary ideal with which to compare performance. In this and the next sections, we survey the state of current knowledge about the consistency of mechanisms with efficiency under various types of behavior. In fact, one of the main unfinished debates in this area of research is over what the appropriate behavioral assumption should be in the analysis of incentive problems.

As we indicated in Section II above, there is wide acceptance of the presumption that if there exist dominant strategies then agents will adopt them. (The only possible violation occurs if agents are able and willing to collude.) With dominant strategies, then, implementation is not an issue since no agent need know anything about the others in order to choose his best message given his information about his characteristic,  $e_i$ . No sophisticated prediction of others' behavior is necessary. The only problem may be one of informational capacity or complexity of calculation which we ignore in this paper even though in experiments in which the dominant strategy is relatively easy to calculate, many subjects still take a few iterations to find the strategy. (An analysis of such an experiment may be found in Coppinger et. al. (1980) and in Cox et. al. (1982).) We summarize in this Section what is known about the efficiency of the performance of resource allocation mechanisms under the assumption that agents will employ dominant strategies if they exist. Later, in Section IV we will discuss what is known for the cases when dominant strategies do not exist.

#### A. DOMINANT STRATEGIES

Although mechanisms for which dominant strategies exist can easily be found, it is not easy to exhibit them if we also require the performance to be efficient. It is now known that in classical economic environments with a finite number of agents: (1) there exist mechanisms that admit dominant strategies for the agents, but (2) there do not exist (non-parametric) mechanisms that



admit dominant strategies and for which the performance is consistent with the Pareto correspondence.<sup>9</sup> We postpone discussion of the situation in large economies in which the number of agents is infinite to Section V.

To explain these results for finite economies we adopt the following language. We call a mechanism  $h$  a dominant strategy mechanism on  $E$  if for all  $e \in E$  and for all  $i$  there is a message  $m_i(e_i)$  such that  $u(h(m/m_i(e_i)), e_i) \geq u(h(m), e_i)$  for all  $m \in M$ . That is, the function  $b_i(e_i) = m_i(e_i)$ , for all  $e_i$ , is a dominant strategy for  $i$ . We will call a mechanism  $h$  an efficient dominant strategy mechanism on  $E$  if it is a dominant strategy mechanism and if  $P(e; h, b) \subset S(e)$  for all  $e \in E$ , where  $S(\cdot)$  is the Pareto correspondence.

## B. FINITE ECONOMIES AND DOMINANT STRATEGY MECHANISMS

The fact that there exist dominant strategy mechanisms on classical environments is easily shown. Let the set of allocations be the set of net trades. That is, in private goods environments let consumption be  $x_i = t_i + w_i$  and  $A = \{t \in R^{NL_i} \mid \sum t_i = 0\}$ . The trivial allocation mechanism, defined by letting  $M_i$  be any non-empty set and  $h(m) = 0$  for all  $m \in M$ , is a dominant strategy mechanism. We call it trivial since any  $m_i$  is a dominant strategy. In public goods environments let  $A$  be the set of net trades in both private and public goods. That is, let  $A = \{(t, s) \in R^{NL+M_i} \mid \sum t_i = 0\}$ . Consumption will be  $(t_i + w_i, s)$  for each  $i$ . If  $h(m) = 0$  for all  $m$  then  $h$  is a dominant strategy mechanism. Clearly these are not very desirable mechanisms. The only "good" thing about them is the existence of dominant strategies.

Non-trivial dominant strategy mechanisms do exist, however, if we restrict further the class of environments to those in which all consumers have quasi-linear utility functions. Such utility functions satisfy the condition that there is a private good,  $l$ , say, such that  $u_i(x_i) = x_{il} + v_i(x_i \setminus x_{il})$  where  $(x_i \setminus x_{il}) \equiv (x_{i2}, \dots, x_{iN})$ , in the private goods only model and  $u_i(x_i, y) = x_{il} + v_i(x_i \setminus x_{il}, y)$  in the

9. Andrew Postlewaite has pointed out that if the mechanism "knows the initial endowments" then (2) is not true if parametric mechanisms are allowed. For example, let  $h(\cdot)$  be the outcome function that gives all the endowments to  $i=1$ . If preferences are monotonic, then there are dominant strategies and the allocation is efficient for this dictatorial procedure. Of course, if  $h$  "does not know the endowments" then (2) is true.

public goods model. Utility functions with this property exhibit no income effect for all goods other than good  $l$ ; that is, the income elasticities of demand for all goods other than good  $l$  are zero. Since mechanisms in these environments are extensively covered in the literature (see, e.g., Groves (1979)), we only briefly indicate what is known to provide a background for the results for the wider class of environments considered in this paper.

In an amazing paper which foreshadowed not only the work in incentives but also the work in incomplete information games and auctions, Vickrey (1961) discovered a particular example of a dominant strategy mechanism for classical private goods environments with quasi-linear utilities. He described his mechanism as follows: "The marketing agency might ask for the reporting of the individual demand and supply curves on the understanding that the subsequent transactions are to be determined as follows: The agency would first aggregate the reported supply and demand curves to determine the equilibrium marginal value, and apply this value to the individual demand and supply curves to determine the amounts to be supplied and purchased by the various individual buyers and sellers. The amount to be paid seller  $S_i$  would, however, somewhat exceed the amount calculated by applying this marginal value to his amount supplied; in effect for the  $r^{\text{th}}$  unit supplied,  $S_i$  would be paid an amount equal to the equilibrium price that would have resulted if  $S_i$  had restricted his supply to  $r$  units, all other purchasers behaving competitively.... An exactly symmetrical method could be simultaneously adopted for dealing with the demand side of the market."(pp.10-12) This mechanism can be easily summarized with the help of Figure 2.

For Vickrey's mechanism, the messages are the "reported demand or supply" functions. The outcome rule determines the amounts of the various goods each should trade, including the "numeraire good"  $x_1$ , as follows. Agents report a demand or supply curve to the marketeer. In figure 2, the curve EE represents the "reported excess supply" of good  $j$ , say, by the agents other than buyer 1. The curve UU represents the "true marginal benefit" to 1 of an extra unit of good  $j$  in terms of the "numeraire good"  $l$ , since we have restricted attention to quasi-linear preferences. Under the standard competitive rules for allocating resources, buyer 1 would be charged the "equilibrium" price for every

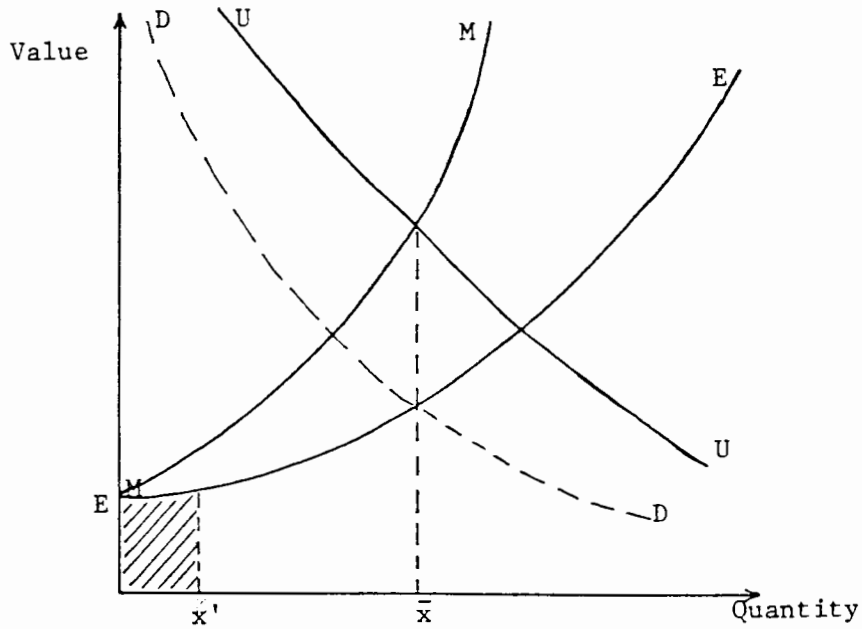


Figure 2

unit of the good. Thus,  $EE$  represents 1's average outlay curve. To calculate his best response to the reports of the others, 1 would calculate  $MM$  (his marginal outlay curve), look at the intersection of  $MM$  and  $UU$ , and then send a "reported" demand function such that it intersects  $EE$  at the same allocation.  $DD$  is such a curve. As one can easily see, 1 has an incentive to understate his demand for the private good  $j$ . This problem does not arise under Vickrey's rules because, under these rules,  $EE$  is converted into the marginal outlay curve for 1 by charging 1 the area under the curve,  $EE$ , for that level of allocation. For example, the charge for  $x'$  is the cross-hatched area. Since  $EE$  is now the marginal outlay curve, 1 wants an allocation at the intersection of  $EE$  and  $UU$ . Obviously if 1 reports  $UU$  as his demand then he will obtain this allocation. Noting that  $UU$  is the appropriate response no matter where the  $EE$  curve lies, one sees that  $UU$  is indeed a dominant strategy.

We can also use figure 2 to describe a dominant strategy mechanism for the public goods environment. Again buyers and sellers use, as their messages, reported demand and supply curves, although in this case these curves are usually called "willingness to pay functions". Since quasi-linear utility functions have been assumed, the demand and willingness-to-pay curves are defined and are the same. (With income effects this would not be true.)  $EE$  now represents the (vertical) summation of the others' supply curves. All else remains as before and, as before, it is in the interest of buyer 1 to report the true willingness to pay function  $UU$ . Groves (1970) later discovered the general class of

these mechanisms and in the public goods environment the Vickrey mechanism becomes the Demand Revelation Mechanism independently discovered by Clarke (1971) and Groves and Loeb (1975).

As with the trivial mechanisms, the allocations produced by the Vickrey mechanism are rarely fully efficient. As Vickrey observed: "The basic drawback to this scheme is, of course, that the marketing agency will be required to make payments to suppliers in an amount that exceeds, in the aggregate, the receipts from purchasers..." (p.13) Clarke and Groves also noted this but were able to adjust the rules so that a surplus would be generated each time. In both the private and public goods cases, the "right" (i.e. Pareto-efficient) level of all non-numeraire commodities would be chosen. However, not all of the numeraire good would be allocated. Thus the allocation would not be fully Pareto-efficient.

While it appears that these Demand Revealing Mechanisms may be better than the Trivial Mechanisms, neither satisfies the criterion of Pareto-efficiency. Vickrey, aware of this problem, went on to remark: "It is tempting to try to modify this scheme in various ways that would reduce or eliminate this cost of operation while still preserving the tendency to optimum allocation of resources. However, it seems that all modifications that do diminish the cost of the scheme either imply the use of some external information as to the true equilibrium price or reintroduce a direct incentive for misrepresentation of the marginal-cost or marginal-value curves." (p.13) Rephrasing, with the advantage of a lot of hindsight, there do not seem to be any mechanisms that do not use specific prior information, that produce efficient allocations and that provide the "appropriate" incentives to reveal correct information. In 1972 Hurwicz formalized and proved this now well known fact for classical private goods economies. We turn to these results now.

### **C. FINITE ECONOMIES AND EFFICIENT DOMINANT STRATEGY MECHANISMS**

In his 1972 paper, Hurwicz considered whether informational decentralization, Pareto-satisfactoriness, and individual incentive compatibility could be combined simultaneously in one mechanism. Informational decentralization was formalized as requiring (1) a non-parametric outcome

function and (2) a behavioral rule which depended for each agent, only on the agent's own characteristic. As stated by Hurwicz: "The requirement of informational decentralization enters through the postulate of 'privacy', which means that no participant, including an enforcement agency if any, has any direct knowledge of others' preferences, endowments, technologies, etc., except possibly the restriction to the *a priori* given classes  $E_i$ ." (pp. 326) Individual incentive compatibility was conceptualized as "no one should find it profitable to 'cheat', where cheating is defined as behavior that can be made to look 'legal' by a misrepresentation of the participant's preferences or endowment, with the proviso that fictitious preferences should be within certain 'plausible' limits." (pp. 323) It later became apparent to researchers in this area that the appropriate formalization of this concept was the requirement that the mechanism be a dominant strategy mechanism. To see this we consider the following more formal model.

In the Hurwicz model an allocation mechanism is an outcome rule and a prescription of behavior in the form of specified "response functions",  $f_i(m^*; e_i) = m_i$  for each  $i$ . These rules instruct each agent which message,  $m_i$ , to send in response to the "previous" joint message,  $m^*$ , of all the others. An outcome is then determined by first looking at "equilibria" of  $f$ : that is, a joint message  $m$  is an equilibrium for  $e$  if and only if  $f_i(m; e_i) = m_i$  for all  $i$ . Next the outcome function  $g(\cdot)$  is applied to the equilibrium message  $m$  to yield the final allocation: that is, the outcome is  $g(m)$  where  $m$  is an equilibrium for  $e$ . Let  $c(e; f)$  be the equilibria for  $e$  under the response rules,  $f$ . Then the result of the mechanism, if all follow the rules, are the allocations  $a = g(c(e; f))$ . If each agent  $i$  acts as if his characteristic is  $e_i^*$  then the outcome is  $g(c(e^*; f)) = a^*$ . If we let  $M = E$  and  $h(m) = g(c(e; f))$ , then  $h$  is an allocation mechanism (as defined above in Section II) that yields the same allocations as the Hurwicz formulation. In this form, with characteristics as messages, these mechanisms are called *Direct Revelation Mechanisms*.<sup>10</sup>

---

10. Although some authors have used the term "Direct Revelation Mechanism" to refer to mechanisms for which messages are characteristics and where reporting the true  $e_i$  is a dominant strategy, we believe that the form of the mechanism (Direct Revelation) should be kept separate from its incentive properties and, therefore, that if reporting  $e_i$  is a dominant strategy then  $(h, E)$  is an *Incentive Compatible Direct Revelation Mechanism*.

We can now formalize the original Hurwicz concept of individual incentive compatibility as follows: the mechanism given by  $(f,g)$  is individually incentive compatible for the class of environments  $E$  if and only if, for all  $i$  and all  $e$  and all  $e_i^* \in E_i$ ,  $u(h(e),e_i) \geq u(h(e/e_i^*),e_i)$ , where  $h(e) = g(c(e;f))$  for all  $e \in E$ . This requires that  $e_i$  be a dominant strategy for  $i$  when  $e_i$  is  $i$ 's characteristic. As it stands this does not seem to require that  $h$  be a dominant strategy mechanism, but it was soon noticed that  $(f,g)$  is individually incentive compatible for  $E$  if and only if  $h$  is a dominant strategy mechanism for  $E$ . It was a short step from this observation to the recognition that if  $h':M \rightarrow A$  is a dominant strategy mechanism for  $E$  then there is a direct revelation version of  $h'$  which is also a dominant strategy mechanism. This insight, due to Gibbard (1973), has been codified as the Revelation Principle, (see Harris and Raviv (1979) and Myerson (1979)), and is straightforward to prove. For some possible drawbacks see Postlewaite and Schmeidler (1983) and Ruppolo (1983). Putting these results together we see that there is an individually incentive compatible mechanism for  $E$  if and only if there is a corresponding dominant strategy mechanism for  $E$ . With these formalizations, we can now turn to the key result.

**THEOREM 3.1 (Hurwicz 1972):** If  $E$  is the classical private goods environments with at least two agents, there is no efficient, dominant strategy, non-parametric mechanism such that  $u(h(b(e),e_i) \geq u(w_i,e_i)$  for all  $i$  and for all  $e \in E$ .

Proof: see Hurwicz (1972).

The last condition in the theorem, which has come to be called *individual rationality*, requires that the mechanism allow each participant a no-trade option. One particularly interesting example of a mechanism satisfying this condition is the Competitive Mechanism. It can be defined as a direct revelation mechanism as follows. (There are other possible representations, but this is the easiest one with which to work.) The message of any agent  $i$  is that agent's characteristic and the outcome function picks net trades. Thus,  $h:M \rightarrow A$  is defined as follows. Given a characteristic  $e_i$ , let  $D(p,e_i) = \{x \in R^L \mid u(x+w_i,e_i) > u(x+w_i,e_i) \Rightarrow px' > px = pw_i\}$  be the demand correspondence for agent  $i$  with characteristic  $e_i$  where  $p \in R_+^L$  the space of all prices  $p$ . Let  $C(e) = \{p \in R_+^L \mid \sum D(p,e_i) = 0\}$

denote the set of competitive equilibrium prices for the environment  $e$ . Then,  $h$  assigns the net trade  $D(C(e), e_i)$  to the agent with the reported characteristic  $e_i$ . Now, since the competitive mechanism is also efficient, by Hurwicz's theorem it cannot be a dominant strategy mechanism; that is, it is not individually incentive compatible in the sense that all agents have an incentive to correctly report their true characteristics.

This theorem of Hurwicz (1972) thus provided a formal proof of the Vickrey hunch and, simultaneously, established that a search to find an efficient dominant strategy mechanism, which was also individually rational, was doomed to failure. Left undecided was whether or not removal of the requirement of individual rationality would allow discovery of an efficient, dominant strategy mechanism. Theorem 3.4 below resolves this question negatively. Also left open was the size of the subset of  $E$  for which incentives and efficiency were incompatible. This was partially answered in Ledyard (1977) as "most".

Even though the Hurwicz impossibility theorem established that the conventional wisdom for classical private goods environments was incorrect if there were a finite number of agents, few were surprised to find that his result was also valid for classical public goods environments. In Ledyard and Roberts (1974), a diagram used by Malinvaud (1971), who attributed it to Kolm was adopted and with a modification of the Hurwicz proof the following theorem was shown.

**THEOREM 3.2 (Ledyard/Roberts 1974):** If  $E$  is the set of classical public goods environments with at least two agents, there is no efficient, dominant strategy, non-parametric mechanism such that  $u(h(b(e), e_i)) \geq u(w_i, 0, e_i)$  for all  $i$  and for all  $e \in E$ .

**Proof:** We have included a proof of this Theorem 3.2 in Appendix to Section III since the Ledyard and Roberts (1974) paper is relatively inaccessible.

Again this left open the question of the existence of an efficient, dominant strategy mechanism if individual rationality were not required but this gap was soon filled. Hurwicz (1975) showed that when the number of agents is at least three there is no mechanism with a "smooth" outcome function  $h$  that both is efficient and admits a dominant strategy.

A somewhat indirect but, in the end, more wide ranging theorem was obtained in a sequence of papers dealing with the class of Groves mechanisms described earlier. First, Green and Laffont (1977) established that if utilities are restricted to be quasi-linear but allowed to be non-concave then the only dominant strategy mechanisms that choose an efficient level of the public good are Groves mechanisms. Walker (1978) has shown that even if utilities are restricted to the class of concave, quasi-linear functions, this characterization remains valid. Finally, Green and Laffont (1978) and Walker (1980) showed that there is no Groves mechanism which "balances the transfers" over the whole class  $E'$ , the subset of classical public goods environments with quasi-linear utility functions. (A mechanism is said to "balance the transfers" if the final allocations produced by the mechanism satisfy the balance condition  $\sum x_i - r = \sum w_i$ .) Since balanced transfers are a necessary condition for efficiency, this collection of papers (see also Holmstrom (1979) and Makowski and Ostroy (1984)) established

**THEOREM 3.3:** If  $E$  is the space of classical public goods environments with at least two agents, there is no efficient, dominant strategy, non-parametric mechanism.

Proof: Follows from Green and Laffont (1977) & (1978), Walker (1980), and Holmstrom (1979).

Finally, a unifying result has been established by Hurwicz and Walker (1983) for all classical economies, both private and public. In fact, they went even further and proved that the failure of existence of efficient dominant strategy mechanisms is "generic" on a large set of classical economies with quasi-linear preferences and more than two agents.

#### **D. SUMMARY**

Combining all the results in the previous sections, it is relatively easy to summarize the state of knowledge concerning efficient, incentive compatible mechanisms in:

**THEOREM 3.4:** In classical environments, both private and public, with a finite set of agents greater than one,



there exist non-parametric, dominant strategy mechanisms but,

there do not exist non-parametric, efficient, dominant strategy mechanisms.

The net effect of the research in this area has been to verify Hurwicz's conjecture (which we first heard in 1967) that informational decentralization, welfare maximization, and incentive compatibility are unattainable simultaneously.

### APPENDIX TO SECTION III

Included in this appendix is a slightly modified version of the proof of Theorem 3.2 found on pages 7-9 of Ledyard and Roberts (1974).

*Proof of Theorem 3.2:*

The economy we construct has two identical consumers, one private good,  $x$ , and one public good,  $z$ , that can be produced from the private good under constant returns to scale. By a choice of units, the transformation of private into public good is one-for-one, that is, the production relation  $g(\cdot)$  is given by  $z = g(x) = x$ . Each consumer holds one unit of private good and has preferences that are given by the indifference map in Figure 3.

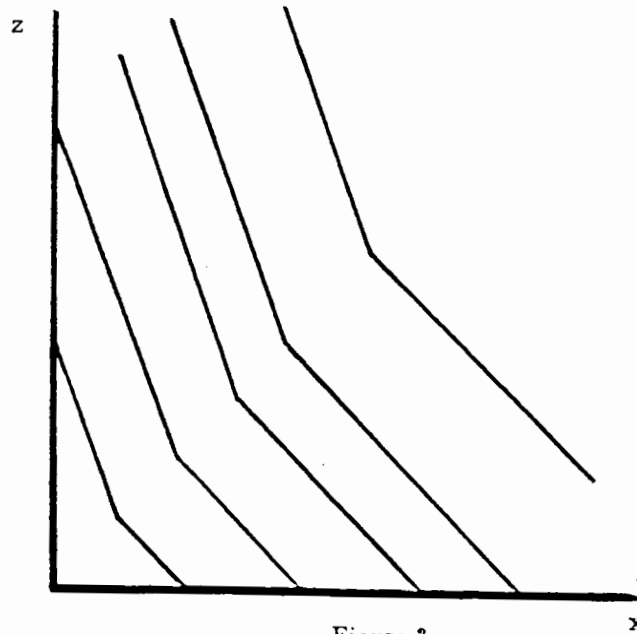


Figure 3

For  $z < x$ , the indifference curves have slope of -1, while for  $z > x$ , the slopes are -3.

It is convenient to represent this economy graphically, in Figure 4, by means of an analogue of the Edgeworth box diagram. This construction was used by Malinvaud (1971), who attributed it to Kolm. The equilateral triangle in Figure 4 has height 2. Since the sum of the distances from any point in the triangle to the three sides is a constant, and since a feasible allocation  $(x_1, x_2, z)$  in this economy satisfies  $x_1 + x_2 + z = 2 = w_1 + w_2$ , there is a one-to-one correspondence between points in the triangle and the feasible allocations: using the point B as the origin for the first agent and C as that

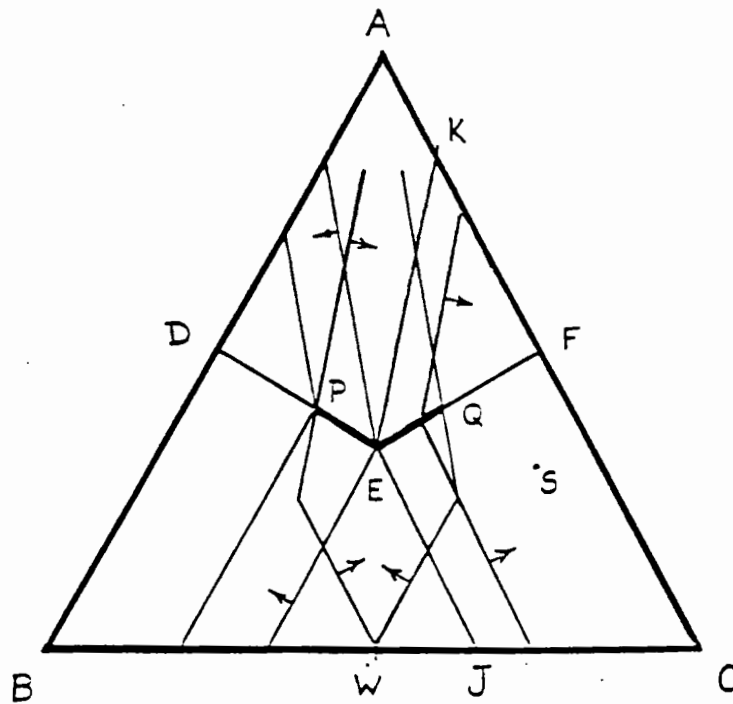


Figure 4

for the second, a point such as S corresponds to an allocation where  $z$  is the distance from S to BC,  $x_1$  is the distance from S to AB and  $x_2$  is the distance from S to AC. The initial position (1,1,0) is then the point W on BC. Sample indifference curves for the two agents are shown. Pareto optima correspond to "double tangencies", and thus the Pareto optima are the points along DEF.

The points on PEQ are the Pareto optima in this economy that are preferred or indifferent for each agent to the initial allocation, W. We refer to the set of Pareto optima that are individually rational as the contract curve.

Any mechanism that selects allocations on the contract curve must select some point on PEQ if the agents reveal their true preferences. Suppose that outcome were on the segment PE. Then, if the second consumer reveals his true preferences, the first agent will be better off if he can, by misrepresenting his preferences, shift the apparent contract curve into the region to the right of JEK.

Clearly he can do this. For example, he can use the strategy that can be rationalized as being the true response of an agent with preferences given by straight line indifference curves with slope -3. This is illustrated in Figure 5, where the apparent contract curve is now GT. Since the final allocation must be on GT, it is not individually incentive compatible for the first agent to reveal his true preferences (i.e. the strategies of telling the truth,  $m_i^*$ , do not constitute a Nash equilibrium).

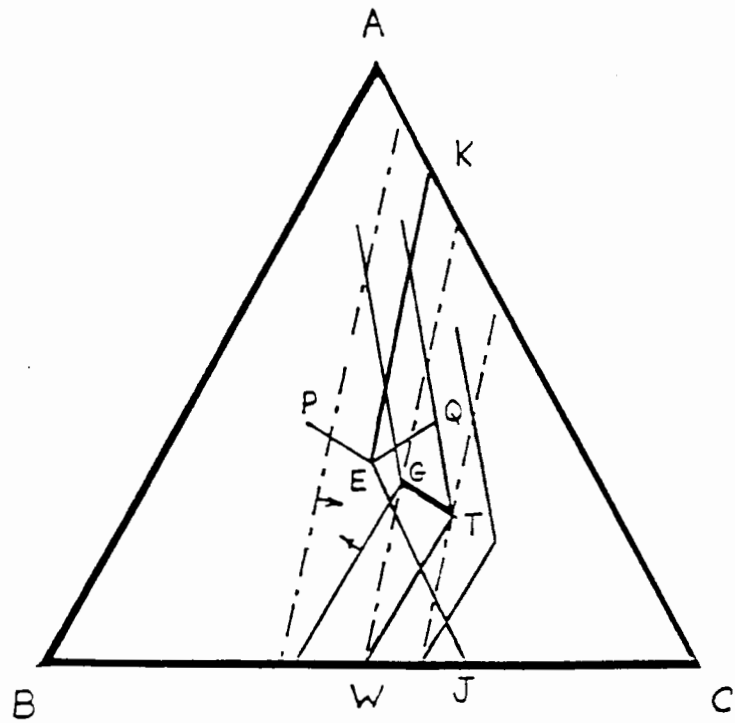


Figure 5

This result is, of course, what one would have expected: it ought not to be any easier to obtain incentive compatibility with public goods than in their absence--the case examined in Hurwicz (1972).

#### IV. EFFICIENCY WITHOUT DOMINANT STRATEGIES

In this section, we retain the requirement that the mechanism's performance be consistent with the Pareto-correspondence. But, as we found we must, we give up the requirement that there exist dominant strategies. This immediately opens up the question of which of the many other equilibrium concepts should be used as a behavioral rule. This is really an empirical question since, in designing a mechanism  $h$ , one must predict how a group of  $N$  individuals with characteristics  $e_i, i=1, \dots, N$ , will behave when confronted with the mechanism. Which (equilibrium) message will result when  $h$  is implemented?<sup>11</sup> To answer this question one needs to know more about the mechanism than its normal form. We need to know, for example, how many iterations of information transmission are allowed. What is the stopping rule? Is communication through a central "computer", is contact random, or must one search out information? As those familiar with experiments will point out, the outcome function alone is insufficient to describe an institution as it might be actually implemented.

Two extreme examples will illustrate this point. One conceivable implementation of a mechanism,  $h: M \rightarrow A$ , is as a sealed bid auction, a one-iteration process. Under this implementation, each agent is required to send  $m_i$  without knowledge of the others' messages. Upon receipt of all  $m_i$ , the "auctioneer" announces the allocation,  $h(m)$ . A second possible implementation of the same mechanism,  $h$ , is as an iterative procedure with an endogenously determined number of iterations. In this implementation, each agent sends  $m_i$  without knowledge of the others' messages. If, for every  $i$ ,  $m_i$  exactly matches the previous  $m_i$ , the process stops and the "auctioneer" announces  $h(m)$ . If, for at least one  $i$ ,  $m_i$  is different from  $i$ 's previous message then another iteration occurs. We can obviously also conceive of innumerable other implementations where the stopping rule for the iterative procedure is to stop after  $T$  iterations unless all  $m_i$  match the previous messages at some prior iteration. (See Smith (1977), (1979), and (1982) for some examples and a discussion.) Although each of these extensive forms of the mechanism,  $h$ , may be represented by the same normal form, it seems

11. In the incentive literature the word "implement" has come to mean that it is possible to match some desired performance rule,  $P: E \rightarrow A$ , with a mechanism  $h: M \rightarrow A$  under some behavior  $b: E \rightarrow M$ . That is,  $h$  is said to *b-implement*  $P$  on  $E$  if  $h\{b(e)\} \subseteq P(e)$  for all  $e \in E$ . When we speak of implementing a mechanism we mean actually using the mechanism to determine some allocation.

unlikely that agents' behavior will be the same under each form. That is, even if  $e$  were the same, we would expect the final allocation to be different if a one-iteration process were used than if an endogenous iteration process were used.

Although it is an unsettled empirical issue how agents will behave in each case, one can still point to several models which are adequate as first attempts to explore the issue. We suggest that the 'model' best suited to analyze the one-iteration implementation is the one common in modeling auctions--the incomplete information game model with common knowledge and a Nash (Bayes) equilibrium as the behavior rule. This model has the additional advantage of being normatively pleasing in that (Bayesian) agents should play this way (see Myerson (1984)). For the endogenous iteration model, the natural normative choice of behavior, (how the agents *should* behave), would be a Nash (Bayes) equilibrium of the repeated, incomplete information game with the number of stages endogenous. Characterization of these games remains an unsolved problem and thus, in place of that natural choice, we turn to the Nash equilibrium of the 'complete information' game. We do not suggest that each agent knows all of  $e$  when they compute  $m_i$ , just as in real markets no auctioneer knows the excess demand function when equilibrium prices are calculated. We do suggest, however, that the Complete Information Nash game-theoretic equilibrium messages may be the possible "equilibrium" of the iterative process, i.e. the stationary messages, just as the demand-equal-supply price is thought of as the "equilibrium" of some unspecified market dynamic process. We have misgivings with each of these models as representations of actual behavior, although there is some evidence that each may be reasonably accurate. (See Smith (1979) for a discussion of some experimental evidence.)

#### A. NON-PARAMETRIC MECHANISMS

Non-parametric mechanisms are those for which the message space  $M$  is the same for all environments  $e \in E$  and the outcome function,  $h: M \rightarrow A$ , is a function of the joint message  $m$  only; that is, those in which the designer cannot incorporate any information other than that received from the agents. (See Hurwicz 1972, p. 310.) A non-parametric mechanism is said to be efficient on  $E$

under the behavior,  $b$ , if  $P(e) = h(b(e;h))$  is Pareto-efficient in  $e$  for all  $e \in E$ . In this section we explore the existence and the characterization of non-parametric mechanisms that are efficient on classical environments under various types of behavior.

### 1. Bayes equilibrium

As indicated above, if a mechanism is implemented as a one-iteration sealed-bid auction, a reasonable candidate for the description of behavior is Bayes equilibrium behavior based on some common knowledge prior beliefs. A precise formulation of this behavioral postulate follows.

Given the class of classical environments  $E$ , consider a given a non-parametric mechanism  $h$ . The Bayesian behavioral rule is specified by first assuming that each agent has a prior density on  $E$ , say  $q_i(e)$ . The vector of priors  $q = (q_1, \dots, q_N)$  is assumed to be common knowledge. Letting  $d_i: E_i \rightarrow M_i$  denote a decision rule for  $i$ , the vector of decision rules  $d = (d_1, \dots, d_N)$  is called a Bayes-equilibrium if and only if, for each  $i$  and for each  $e_i \in E_i$

$$\int_{E_{\sim i}} u(d(e); e_i) q_i(e_i | e_{\sim i}) de_{\sim i} \geq \int_{E_{\sim i}} u(d(e); m_i; e_i) q_i(e_i | e_i) de_{\sim i} \text{ for all } m_i \in M_i$$

where  $E_{\sim i} \equiv \prod_{j \neq i} E_j$ . A result, based on the Revelation Principle, is that if  $d$  is a Bayes equilibrium for the mechanism  $h$ , then another direct revelation mechanism,  $F: M \rightarrow A$ , can be defined where  $M_i = E_i$  for each  $i$  and  $F(e) = h(d(e))$  such that the identity map  $I(e) = e$  is then a Bayes equilibrium for the mechanism  $F$ . Thus, if it is possible to find an efficient non-parametric mechanism under Bayes equilibrium behavior then it must be possible to find a direct revelation, efficient non-parametric mechanism under Bayes equilibrium behavior.

We can now ask whether there are efficient, non-parametric mechanisms for classical environments under the Bayes equilibrium behavioral assumption. The answer is basically no as can be seen from the following theorem.

**THEOREM 4.1** (Ledyard 1978 and 1979): Given any vector of priors  $q$ , the direct revelation mechanism  $h$  has the identity map,  $I: E \rightarrow E$ , as a Bayes equilibrium if and only if  $h$  is a dominant

strategy mechanism in  $e$  for almost every  $e \in E$  with respect to  $q$ .

Proof: See Ledyard (1978).

Thus if  $h$  were an efficient non-parametric mechanism on  $E$  under Bayes behavior, there would be an efficient direct revelation mechanism,  $F$ , with the identity map  $I(\cdot)$  as a Bayes-equilibrium. This in turn implies that  $F$  is an efficient dominant strategy mechanism on almost all of  $E$ . But by the results of section III this is impossible on classical environments. Therefore, there can be no efficient, non-parametric mechanisms on classical environments under Bayes-equilibrium behavior. The use by the agents of the additional information on the prior distribution,  $q$ , does not help. Two facts should help in understanding this result. Requiring  $h(d(e))$  to be efficient for all  $e$  (a form of *ex post* efficiency) is much stronger than requiring *ex ante* efficiency in expected utility, as is usually done in the optimal auction literature. Also, in this theorem  $h$  is not allowed to depend on  $q$  as is customarily the case in that literature. (We analyze that case later in Section IV.B.) Therefore, the impossibility result should not be too surprising.

To summarize, we state:

**THEOREM 4.2:** In classical environments with a finite set of agents, there are no efficient, Bayes equilibrium, non-parametric mechanisms.

Proof: Follows from the theorems of Section III and Theorem 4.1.

## 2. Nash-equilibrium

Having not had much luck with one-iteration implementations of mechanisms, we consider next an idealization of the behavior expected in an infinite iteration implementation of a mechanism.

Given a mechanism defined by the language  $E$  and outcome function  $h$ , we define the Nash behavioral rule  $b^N: E \rightarrow M$  as follows: For all  $e \in E$ , and all  $i$

$$u(h(b^N(e)); e_i) \geq u(h(b^N(e)/m_i); e_i) \text{ for all } m_i \in M_i.$$

As with Bayes-equilibria, there can be a problem of too many equilibria; however, this will not be an issue in our analysis.



Now we can ask the main question: Are there any efficient, non-parametric mechanisms on classical environments under Nash behavior? The answer is yes. In addition, there are several results which characterize a wide class of such mechanisms. First, we discuss five specific mechanisms; then we turn to the characterizations.

*2.a. Specific examples*

In the face of the pessimism expressed in the literature in the search for efficient, incentive sensitive mechanisms in the early seventies, we were somewhat surprised to discover a mechanism to allocate public goods in classical environments whose Nash equilibria were Pareto-efficient. (See Groves-Ledyard (1977).) Soon many more such mechanisms were found.

*2.a.i. Private goods environments*

Many of the mechanisms discovered to date that are efficient under Nash behavior (in private goods environments), have the additional property that they select Walrasian allocations. Much of the work in this area has been summarized by Schmeidler (1982) who also provided one of the first examples (Schmeidler (1980)) of a mechanism whose Nash equilibrium allocations are Walrasian in classical environments and are, therefore, efficient. A slightly later version has the additional desirable property that its Nash equilibria are also strong Nash equilibria. This mechanism is described as follows. The message space is given by

$$M_i = \{(p, t) \in \mathbb{R}_{++}^L \times \mathbb{R}^L \mid p \cdot t = 0 \text{ and } \sum_1 p_i = 1\} \text{ and } M = \prod_1 M_i.$$

The outcome rule,  $h = (h_1, \dots, h_N)$ , is then defined for each  $m = (m_1, \dots, m_N)$  by

$$T_i = \{k \mid p_k = p_i\} \text{ and } h_i(m) = t_i - \sum_{k \in T_i, k \neq i} \frac{t_k}{\# T_i} \text{ for all } i.$$

**THEOREM 4.3** (Schmeidler 1980): In classical environments with initial endowments,  $w_i$ , that are positive in each coordinate, with utility functions that have continuous partial derivatives, and with at least three agents,  $N \geq 3$ , (i) every Nash equilibrium is a strong equilibrium and (ii) the set of Nash equilibrium allocations is the set of Walrasian allocations.

Proof: See Schmeidler (1980).

One problem with the Schmeidler mechanism however, is that the outcome rule is not continuous. Thus small variations in messages can cause large jumps in the allocation. If only Nash equilibria were assumed to be implemented this would not cause difficulties; however, as we indicated above, Nash equilibria are plausible as a model of the probable outcomes only if a number of iterations occur. Since we would expect to see terminal messages close (but not necessarily equal) to Nash equilibria, any discontinuity in the outcome rule, especially near Nash equilibria, means it is difficult to approximate the eventual outcome, even though the messages were almost "right". If the outcome rule were continuous, one would know that if the messages are close to Nash equilibria then the allocations will be close to Nash equilibrium allocations. Because of the fallibility of information transmission it is highly desirable to have outcome functions which are continuous.

Hurwicz (1979a) has exhibited an allocation mechanism with the desired features.

Let the message space be given by:  $M_i = \{(p,t) \in R_{++}^{L-1} \times R^1 \times R^{L-1} \mid p \gg 0\}$ . The outcome function is then defined by:  $h(m) = (h_1(m), \dots, h_N(m))$  where  $h_i(m) = (r_i(m), s_i(m))$  and  $s_i(m) = t_i - t_{-i}$ , where  $t_{-i} \equiv \{\sum_{j \neq i} t_j\} / (N-1)$ , and  $r_i(m) = -p_{-i} s_i(m) - L_i(m) + S_i(m)$ , where  $p_{-i} \equiv \{\sum_{j \neq i} p_j\} / (N-1)$ ,  $L_i(m) = (p_i - p_{-i})^2$ , and  $S_i(m) = p_{-i} t_{-i} + p_{-i}^2 + \{\sum_{j \neq i} p_j^2\} / (N-1) - \{\sum_{j \neq i} (2p_j - t_j) \sum_{k \neq i, j} p_k\} / (N-1)(N-2)$ .

**THEOREM 4.4** (Hurwicz 1979a): In classical private goods environments such that all consumers preferences are strictly increasing in good 1 and with at least three agents,  $N \geq 3$ , the set of Nash equilibria allocations is equal to the set of Walrasian equilibrium allocations.

Proof: See Hurwicz (1979a).

Several remarks about the above two mechanisms are in order. First, each requires at least three traders. Hurwicz (1976), however, did define an efficient Nash mechanism for environments with only two traders. The outcome rule for that mechanism is not, however, balanced (i.e. the outcome function does not satisfy the condition  $\sum h_i(m) \leq 0$ ) nor is it individually rational. (See also Reichelstein (1984).) Second, the dimension of the message space used in the above mechanisms is  $2N(L-1)$ . It is known that the minimal dimension needed to obtain Walrasian allocations under

prescribed behavior is  $N(L-1)$ . See, for example, Mount and Reiter (1974). Thus, an open question of interest is whether it possible to design a Nash efficient mechanism with the dimension of  $M$  being  $N(L-1)$ .

Finally, neither of the above mechanisms is feasible in all environments,  $e$ , at all messages,  $m$ , in the sense that  $h(m)$  may not be a feasible allocation for some message, environment pair  $(m,e)$ . But, as we will see shortly, no mechanism exists that is balanced, non-parametric, feasible, and non-trivial (that is, which has a non-zero outcome for some  $m$ ).

*2.a.ii. Public goods*

There are at least three specific mechanisms which are designed to allocate public goods in classical environments. The first, by Groves and Ledyard (1977), is defined as follows: The message space is  $M_i = R^L$  for all  $i$  and  $M = \prod_i M_i$ . The allocation rule is  $h = (h_1(m), \dots, h_N(m), y(m))$  where  $y(m)$  is the chosen levels of public goods and  $h_i(m)$  is the amount of the numeraire good to be traded by  $i$ .

$$y(m) = \sum_i m_i$$

$$h_i = \frac{1}{N}y(m) - \frac{\gamma}{2} \left\{ \frac{N}{N-1} (m_i - \mu_{\sim i})^2 - \sigma(m_{\sim i}) \right\}$$

where  $\gamma > 0$  is an arbitrary constant,  $\mu_{\sim i} \equiv \frac{1}{N-1} \sum_{j \neq i} m_j$ , and  $\sigma(m_{\sim i}) = \frac{1}{N-2} \sum_{j \neq i} (m_j - \mu_{\sim i})^2$ .

**THEOREM 4.5 (Groves/Ledyard 1977):** In classical public goods environments with at least 3 agents, the mechanism defined above is an efficient Nash mechanism.

Proof: (1)  $\sum h_i(m) = y(m)$  for all  $m$  and thus the mechanism is balanced.

(2) At Nash equilibria,  $(u_{iy}/u_{ix}) - \{(1/N) + (N/(N-1))(m_i - \mu_{\sim i})\} = 0$ . Summing over all  $i$  implies  $\sum (u_{iy}/u_{ix}) - 1 = 0$ , the Samuleson-Lindahl necessary conditions for efficiency. QED

In an interesting article, Bergstrom, Simon, and Titus (1983) show that this mechanism will have a large number of Nash equilibrium messages, on the order of  $2^{N-1}$ . Each will yield an efficient allocation but, as they point out, multiple equilibria may create problems for our justification of Nash

behavior. In particular, as they state: "(i)f there are multiple equilibria with differing distributions of utility, then individuals may have an incentive to falsify their preferences in order to drive the adjustment process to a preferred equilibrium." (p. 167) As we discuss below, in Section IV.A.3, there is no commonly accepted model of self-interested individual behavior of an agent confronted with an adjustment process. Until there is such a model, the implementability of this mechanism remains an open question.

Another property of this mechanism is that Nash equilibrium allocations may leave a consumer worse off than at his initial endowment; that is, there may be consumers who would be better off not participating. In a mechanism by Hurwicz (1979a) this is avoided. His mechanism has the message space  $M_i = \{(y_i, p_i) \in R^M \times R^M\}$  and  $M = \prod_i M_i$ . The allocation rule is

$h(m) = (h_1(m), \dots, h_N(m), y(m))$  where

$$y(m) = (1/N)\sum y_i \text{ and}$$

$$h_i(m) = -\left[(1/N) + p_{i+1} - p_{i+2}\right]y(m) - p_i(y_i - y_{i-1})^2 - p_{i-1}(y_{i-1} - y_{i-2})^2$$

where  $N+1 = 1$  and  $N+2 = 2$ .

**THEOREM 4.6** (Hurwicz 1979a): In classical public goods environments with utility functions that are strictly increasing in the numeraire good and with at least three agents, the set of Nash equilibrium allocations of the above mechanism are equivalent to the Lindahl allocations. Therefore it is an efficient Nash mechanism, with individually rational allocations.

Proof: (1) At Nash equilibria  $p_i(y_i - y_{i-1}) = 0$  for all  $i$ . Therefore, (2)  $h_i(m) = r_i(m)y(m)$  where  $r_i(m) = (1/N) + p_{i+1} - p_{i+2}$ . Also, (3)  $(u_{iy} / u_{ix}) - r_i - 2p_i(y_i - y_{i-1}) = 0$  for all  $i$ . And, from (1), (4)  $(u_{iy} / u_{ix}) = r_i$ . QED

An unfortunate property of this mechanism is its large message space.  $M$  is a  $2NM$  dimensional space whereas the Quadratic mechanism of Groves-Ledyard uses only an  $NM$  dimensional space for  $M$ . However, Walker (1981) discovered another mechanism which selects Lindahl allocations and which uses a smaller space for messages than does the mechanism of Hurwicz. Let  $M_i = R^M$  and let  $M = \prod_i M_i$ . Let  $y(m) = \sum m_i$  and let  $h_i(m) = [(1/N) + m_{i+1} - m_{i-1}]y(m)$  for each  $i$ .

THEOREM 4.7 (Walker 1981): In classical public goods environments with utility functions that are monotonic in the numeraire good and with at least three agents, the set of Nash equilibrium allocations is equivalent to the set of Lindahl allocations.

Proof: (1)  $y(m) = \sum h_i(m)$ , or balancedness, for all  $m$ .

(2) At a Nash equilibrium,  $h_i(m) = r_i(m)y(m)$  where  $r_i(m) = (1/N) - m_{i+1} - m_{i-1}$ .

(3) At a Nash equilibrium,  $(u_{iy}/u_{ix}) - r_i(m) = 0$ . QED

To this point we know that, for classical environments, there exist continuous, balanced, non-parametric, individually-rational, Nash efficient mechanisms. There are also other mechanisms satisfying some, but not all, of these conditions. Unfortunately, although the equilibrium allocations are individually feasible, none of the above specific mechanisms are necessarily individually feasible at non-equilibrium messages. We say unfortunately for the same reason that we desired continuous outcome rules; in case of small errors in communication, implementation may require that non-equilibrium messages be used to compute the allocation. If this happens it is very desirable to know that whatever allocation is chosen, it will be feasible for all agents. The next theorem due to Hurwicz, Maskin, and Postlewaite sharpens some of the limits of mechanism design.

THEOREM 4.8 (Hurwicz/Maskin/Postlewaite 1982): If a non-parametric outcome function is an efficient Nash mechanism and is individually feasible then the message space for  $i$  must depend on  $w_i$ ,  $i$ 's initial endowment.

Proof: Suppose  $h$  is an efficient, individually-feasible Nash mechanism. Suppose for  $e$  it allocates  $h(b^N(e)) \neq 0$ . This will be true if  $w$  is not Pareto-efficient. Let  $\tilde{m} = b^N(e; h)$ . There is an  $i$  and a  $k$  such that  $h_{ik}(\tilde{m}) = a < 0$ . Consider the environment  $e'$  which is derived from  $e$  by lowering  $w_{ik}$  to  $c$  where  $0 < c < -a$ . Then  $h$  is not individually feasible in  $e'$ . QED

Allowing  $M_i$  to depend on  $w_i$  is formally equivalent to parameterizing the outcome function by  $w_i$ . Therefore, non-parametric mechanisms, i.e. those with non-parametric outcome functions and non-parametric message spaces, cannot be both individually feasible and Nash efficient. This result is actually deeper: non-trivial, non-parametric mechanisms cannot be individually feasible.

To summarize the results in this section, we first recap some terminology. A mechanism is continuous if the outcome function  $h: E \rightarrow A$  is continuous in an appropriate topology. A mechanism is balanced when allocating private goods if  $\sum_i h_i(m) = 0$  for all  $m$  and balanced when allocating public goods if  $T_i \left[ \sum_i h_{ip}(m), h_y(m) \right]_i = 0$  where  $h_{ip}(m)$  is the net addition to (or reduction in)  $i$ 's endowment of private goods in the allocation  $h(m)$  and  $h_y(m)$  is the public good allocation. A mechanism is individually feasible if  $h_{ip}(m) \geq -w_i$  for all  $i$  and  $m$  and  $e$ . A mechanism is non-parametric if it is independent of  $e$ . A mechanism is efficient Nash on  $E$  if  $h(b^N(e;h))$  are Pareto-efficient allocations for all  $e \in E$ .

We have learned that:

THEOREM 4.9: In classical environments with at least two agents,

- (a) there exist continuous, balanced, non-parametric, efficient Nash mechanisms,
- (b) there do not exist (even with two agents) individually feasible, efficient Nash mechanisms.

### 2.b. Characterizations

All five specific mechanisms displayed in the previous section have desirable as well as undesirable properties. We touched on message size. Undiscussed were complexity, stability, and coalitional manipulability, to name just a few issues. Viewed from the perspective of mechanism design, before effort is spent on further analysis of these five mechanisms, it would be nice to know how many others there are. That is, we would like to characterize the class of all efficient Nash mechanisms on classical environments. Although there have been several interesting papers written in this area, the characterizations remain incomplete.

In an interesting exposition of the Groves/Ledyard Quadratic mechanism, Brock (1980) presents a method of generating an enormous class of efficient Nash mechanisms for public goods environments. His systematic approach also highlights what is needed to design such mechanisms. In particular, suppose the message spaces  $M_i$  and functions  $y=y(m)$  and  $T_i(m)=T_i$  for all  $i$  must satisfy (as is required for efficiency) balancedness (i.e. (1)  $\sum_i T_i(m) = qy(m)$  for all  $m$ ) and be such that Nash

equilibrium allocations are efficient; that is, if  $u_{iy}(y, w_i - T_i)(dy/dm_i) - u_{ix_i}(y, w_i - T_i)(dT_i/dm_i) = 0$  for all  $i$ , then the Samuelson-Lindahl condition,  $\sum_i [u_{iy}/u_{ix_i}] = q$ , must hold. It is easy to see that this condition is satisfied if and only if (2)  $\sum_i [(dT_i/dm_i)/(dy/dm_i)] = q$  for all  $m$ . Now, as Brock showed, equations (1) and (2) can be used to generate innumerable efficient Nash mechanisms. For example, let  $M_i = R^K$  and let  $y(m) = \sum_i m_i$ . Then the functions,  $T_i(m)$ , must satisfy  $\sum_i T_i(m) = q \sum_i m_i$  and  $\sum_i dT_i/dm_i = q$  for all  $m$  and  $i$ . Suppose we try a series of polynomials for  $T_i$ . First consider  $T_i = a_i + b_i m$ . It is required that  $\sum a_i + \sum b_i m = q \sum m_i$ . Therefore,  $\sum a_i = 0$  and  $\sum_{ij} b_{ij} m_j = q \sum_j m_j$ , for all  $m$ . The latter is possible if and only if  $b_{ij} = b_j$  for all  $i$  and  $\sum b_j = q$ . If we were to require symmetry in the mechanism then  $a_i = 0$  for all  $i$  and  $b_i = (1/N)q$  for each  $i$ . Therefore, taxes for all agents are equal, i.e.  $T_i(m) = (1/N)q \sum_j m_j$ . It is easy to see that this mechanism satisfies the requirements; its Nash equilibria allocations are Pareto-efficient. The difficulty with this particularly simple mechanism of equal taxation is that Nash equilibria under the mechanism rarely exist. As in Hammond (1979), equilibria for this mechanism will exist if and only if there are fair Lindahl equilibria; that is, Lindahl equilibria such that all  $i$  have the same marginal rate of substitution. In most classical environments such Lindahl equilibria simply do not exist. (Note: If the message spaces  $M_i$  are compact then Nash equilibria may exist but will almost always be boundary points; that is, most  $m_i$  will be at the lower bound and only one agent  $i$  will effectively determine the allocation which will then not be Pareto-efficient since the Samuelson-Lindahl condition will not hold.)

Because of the existence problem with linear functions  $T_i$ , let us consider quadratic ones instead; that is, suppose  $T_i = a_i + b_i + m' C^i m$  for all  $m$  and  $i$ . Now, for symmetric functions  $T_i$ , it must be true that  $\sum a_i = 0$ ,  $b_{ij} = b_j$ ,  $b_j = \frac{1}{N}q$  and  $m' (\sum C^i) m = 0$  for all  $m$  to ensure balancedness. To ensure the Samuelson-Lindahl condition we also need  $\sum b_i + 2(\sum_i C_i^i) m = q$  and, therefore,

$$(\sum_i C_i^i) m = 0.$$

It is straightforward, if somewhat tedious, to show that the quadratic rules of Groves-Ledyard satisfy these restrictions. One can obviously proceed in a similar way to cubics and higher order polynomials. Since polynomials approximate most functions, one should be able to characterize all efficient Nash mechanisms this way. This, however, remains to be done.

The approach of Brock can also be used to construct mechanisms that generate Lindahl allocations at their Nash equilibria. If the joint message  $m$  is a Nash equilibrium that produces a Lindahl allocation under a given mechanism, then the tax share for each agent  $i$ ,  $T_i(m)$  must equal  $q_i(m_{-i})y(m)$  for Lindahl prices  $q_i(m_{-i})$  that may depend on the messages  $m_j$  of other agents, but not on agent  $i$ 's own message. The Lindahl prices  $q_i(m_{-i})$  by definition also sum to  $q$ . Thus, in place of the balancedness (1) and Samuelson-Lindahl conditions (2) above, we have the conditions:

(3)  $T_i(m) = q_i(m_{-i})y(m)$  and (4)  $\sum q_i(m_{-i}) = q$ . It is easy to see that (3) and (4) imply (1) and (2).

Suppose, then, that polynomial functions of  $m_{-i}$  are constructed for  $q_i(m_{-i})$ . In the simplest case, that of constant functions,  $q_i = a_i$  and the mechanism would pick at a Nash equilibrium those Lindahl allocations for which the marginal rates of substitution  $u_{iy}/u_{ix_i} = a_i$ . However, for any given environment  $e$ , for a pre-specified set of  $a_i$ 's, such Lindahl allocations would not likely exist.

Turning to the linear functions,  $q_i(m_{-i}) = a_i + b_{ij}m_j$  with  $b_{ii} = 0$ , for the symmetric case we need  $a_i = (1/N)q$  and  $\sum b_{ij} = 0$ . There are many such  $b_{ij}$ ; Walker (1981) found one particularly simple structure, where  $b_{ij} = 1$  if  $j = i+1$ ,  $b_{ij} = -1$  if  $j = i-2$ , and  $b_{ij} = 0$  otherwise. It is interesting to note that the form of  $q_i$  is independent of the form of  $y(m)$ . As long as  $dy/dm_i \neq 0$  for all  $m$  and  $i$ , any function  $y(m)$  will do. We also know from Hurwicz (1979) that the form  $T_i = q_i(m_{-i})y(m)$  is not necessary if  $h$  is to generate Lindahl allocations. His mechanism, as defined above, has the form  $T_i(m) = q_i(p_{-i})y(r) + R_i(r,p)$  where  $m = (r,p)$ . We note, however, that in equilibrium  $R_i(r,p) = 0$ , leaving the form  $T_i = q_i(m_{-i})y(m)$ . It remains an open question whether all mechanisms whose Nash allocations are Lindahl have essentially only this structure.

Turning now to private goods environments and proceeding as above we immediately run into a problem. As with Brock's approach for public goods mechanisms, we look for functions  $x_i(m)$  and



$T_i(m)$  for all  $i$  such that  $\sum x_i(m) = 0$ ,  $\sum T_i(m) = 0$ , and the Nash equilibrium allocation,  $(w_j - T_j, x_j)_j$  is Pareto-efficient. These functions must then satisfy  $\sum x_i(m) = 0$  and  $\sum T_i(m) = 0$  (the balancedness condition) and if  $(du/dx_i)(dx_i/dm_i) - (du/dy)(dT_i/dm_i) = 0$ , for all  $i$ , then there is a  $P$  such that  $(du/dx_i)(du/dy_i) = P$  for all  $i$ . Given balancedness we therefore require that  $(dT_i(m)/dm_i)/(dx_i(m)/dm_i) = P(m)$  for all  $i$  and all  $m$ . Looking first at  $x_i(m)$  suppose that  $M_i = \mathbb{R}^{L-1}$  and  $x_i(m) = m_i$ , a proposed trade. This is not balanced since, in general,  $\sum m_i \neq 0$ . To balance this one can subtract the average surplus,  $\sum m_i/N$ , and get  $x_i(m) = m_i - [\sum m_i/N]$ . We can rewrite this as  $x_i(m) = ((N-1)/N)(m_i - r_{\sim i})$  where  $r_{\sim i} = \sum_{h \neq i} m_h / (N-1)$ . Rescaling  $m$  gives  $x_i(m) = m_i - r_{\sim i}$  which is Hurwicz's rule. Now given this rule for  $x_i(m)$ , we can take the approach of Brock to characterize  $T_i$ . Suppose  $T_i$  is a polynomial,  $T_i = a_i + b_i m + \dots$ . Then we require that  $\sum a_i = 0$ ,  $\sum b_i = 0$ ,  $b_{ij} = P(m)$  for all  $m$ . Thus  $b_{ij} = b^*$  for all  $i$ . If we also require symmetry then, in addition,  $a_i = 0$  for all  $i$ , and  $b_{ij} = -\frac{1}{N-1} b^*$  for all  $i \neq j$ . Thus,  $T_i(m) = b^*(m_i - r_{\sim i}) = b^* x_i(m)$ . The problem with these rules is now obvious; one needs to know the Walrasian price to know  $b^*$  if one wishes an equilibrium to exist, but the designer does not have that information. In fact, it is impossible to find any  $T_i(m)$  that do the job we wish. The message space  $M$  is simply not big enough. As we indicated in the last section if Nash allocations are to be Walrasian and  $h$  is to be differentiable then  $M$  must have dimension at least  $N(L-1)$ . In the public goods model we assumed that the vector of public goods prices was known to the designer and that there was only one private good which also had a known price equal to unity. That still left an incentive problem in those models. If relative prices were known in a private goods environment, however, there would then be no incentive problem. Let  $x_i(m) = m_i$ ,  $T_i(m) = -p m_i$ , and  $h_i(m) = (x_i(m), T_i(m))$ . If  $p$  is indeed a Walrasian relative price for  $e$  then this is an efficient Nash mechanism. The mechanism is of course parametric as the relative price  $p$  will depend on the environment  $e$  and assuming that it is known is assuming away the incentive problem entirely. Presumably one can also design mechanisms, for public goods environments, that not only choose the level of public goods but also choose prices and the level of private goods. This remains to be done, however.

In an important paper Hurwicz took an entirely different approach to the characterization of efficient Nash mechanisms. He was able to demonstrate that, under fairly reasonable conditions, if one wants to design a mechanism whose Nash equilibrium allocations were Pareto-efficient and individually rational on the classical private goods environments then those Nash equilibrium allocations must coincide with the Walrasian allocations. A similar result obtains for public goods environments. That is, remarkably, any mechanism designed to produce individually rational, efficient, Nash equilibria would have to yield Walrasian or Lindahl allocations. Attempts to obtain other allocations would be fruitless. Similar results can be found in Schmeidler (1982a).

The precise nature of this amazing result is as follows:

**THEOREM 4.10 (Hurwicz 1979):** Given an allocation mechanism  $h$ , suppose that the Nash equilibria allocations,  $h(b^N(e;h))$ , are contained in the set of individually rational, Pareto-efficient allocations for  $e$  for each  $e$  in  $E$ .

(A) If  $b^N(e;h)$  is non-empty and upper semicontinuous on  $E$  then, for each  $e \in E$ , the Walrasian allocations of  $e$  are contained in  $h(b^N(e;h))$ , the Nash equilibrium allocations.

(B) Define  $B_i(m) = h_i(m;M_i) - R_+^L$ . (This is the set of consumptions  $i$  can unilaterally get to from the message  $m$ .) If  $B_i(m)$  is convex for all  $i$  and all  $m \in M$ , then for all  $e \in E$  the allocations in  $h(b^N(e;h))$  which are interior are also Walrasian. (An allocation is interior if, for all  $i$ ,  $x_i + w_i \gg 0$ .)

Proof: See Hurwicz (1979).

Continuity of the Nash equilibrium correspondence is a very desirable property for an allocation mechanism for the same reason that continuity of  $h$  is desirable. Small errors in observation or calculation will, then, not lead to large perturbations in allocations. Convexity of the sets  $B_i(m)$  is desirable because it is sufficient to ensure that the best response functions of the  $i$  are upper semicontinuous which in turn is used to get upper semicontinuity of the Nash correspondence. Thus both properties required by Hurwicz are reasonable.<sup>12</sup> They are satisfied by all five examples we have

12. In Reichelstein (1984b) it is shown that without the convexity requirement there are many more implementable choice rules; in particular, the one that selects individually rational Pareto-optima is fully implementable (Corollary to Proposition 3.1).

presented in this section. Of these mechanisms, only the quadratic rules of Groves-Ledyard do not have Walrasian or Lindahl allocations as Nash outcomes. The reason is that the mechanism does not satisfy the requirement of individually rationality.

Summarizing, individually rational, efficient Nash mechanisms with continuity and minimal message spaces are those which produce Walrasian or Lindahl allocations. In private goods environments the appropriate mechanisms are those of the following general form:

Let  $M_i = R^{L-1} \times R^{k_i}$  where  $k_i \geq L-1$ . Let  $m_i = (s_i, p_i)$ , and define  $x_i(m) = s_i - v_{\sim i}$  where  $v_{\sim i} = \sum_{j \neq i} \frac{1}{N-1} s_j$  and  $t_i(m) = f_i(m_{\sim i})x_i(m) + T_i(s_{\sim i}, p)$ , where  $\min_p T_i(s_{\sim i}, p) = 0$  for all  $s_{\sim i}$ .

In public goods environments, assuming relative output prices are known, the appropriate mechanisms are those of the following general form:

Let  $M_i = R^K$  and define  $y(m) = \sum m_i$  and  $T_i(m) = f_i(m_{\sim i})y(m)$  where  $\sum f_i(m_{\sim i}) = q$ . If relative prices also need to be determined by the mechanism then a larger message space will be needed.

If we forego individual rationality other mechanisms become available such as the quadratic rules for public goods. And, if we are willing to forego continuity we may be able to reduce the size of the message space. Full characterizations, though, remain to be done.<sup>13</sup>

### 3. Manipulative Equilibria

The assumption that behavior is modeled by Bayes equilibrium or Nash equilibrium, is by far the most common in the literature. However, it has been argued by some that the assumption of Nash behavior views participants to be somewhat naive. Thus it might be expected that cleverer agents would find out that they could improve on their allocation with more sophisticated play. Since we always assume that players in these games are at least as clever as the modelers, this observation raises some serious issues for mechanism design which must be considered.

13. In a related paper, Aghion (1984) has shown that the set of continuous mechanisms whose equilibria are inefficient is dense in the set of continuous mechanisms. This, however, still leaves more than enough candidates whose equilibria are efficient.

To illustrate this problem, consider a 2 person environment with the mechanism,  $h$ , which is known to be an efficient Nash mechanism. That is,  $h(b^N(e;h))$  is Pareto-efficient in  $e$  for all  $e \in E$ . Now consider the indirect utility function of  $m$  given by  $v_i(m) = u(h(m), e_i)$  for each  $i$ . Graphically one can plot the indifference curves for  $i = 1$  as in Figure 6.

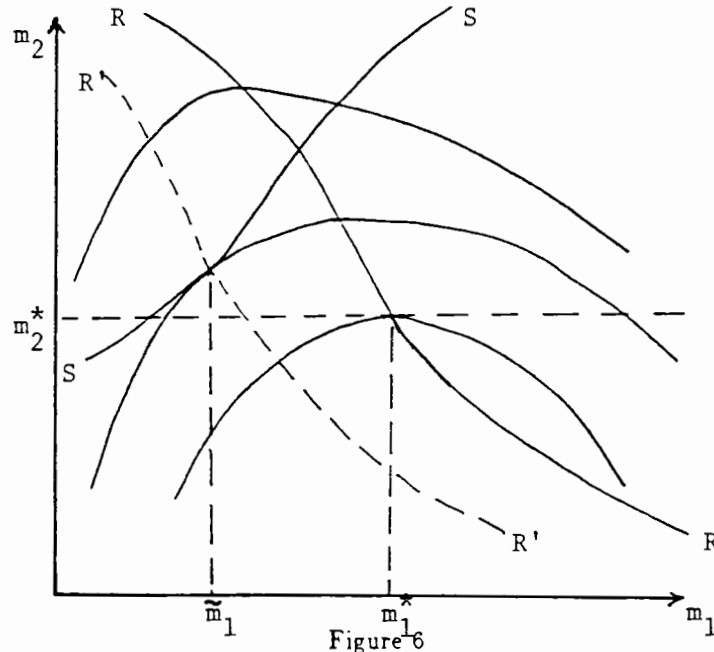


Figure 6

Although it may not be the fully rational game-theoretic equilibrium, one way to think of a single agent's behavior in the infinite iteration process is as a myopic maximizer at each iteration. If we let  $m_2 = m_2^*$  then we can find the  $m_1$  which maximizes  $v_1$  given  $m_2^*$ . This is  $m_1^*$  in the diagram. In fact, we can plot all such best replies as the locus  $RR$  in Figure 6. Similarly, we can plot 2's best responses as the locus  $SS$ . The intersection of the  $SS$  and  $RR$  curves gives us the Nash equilibrium messages, the only stationary points given this myopic behavior. But 1 has available a better strategy if 1 can identify the function  $SS$ . If 2 will indeed respond as predicted by the Nash assumption then if 1 chooses  $\bar{m}_1$  to maximize  $v(m)$  subject to the joint strategy  $(m_1, m_2)$  lying on  $SS$ , 1 will be better off. Of course, it is possible for 1 to disguise this manipulative behavior. Instead of choosing a single message 1 can pretend to have preferences which yield the pseudo-response curve  $R'R'$ . That is, 1 chooses  $e'_1$  to maximize his indirect utility  $u(h(b^N(e/e'_1); e_1))$ . Then, if 2 behaves myopically according to  $SS$  and 1 behaves myopically according to the response function for  $e'_1$ , the same result will occur as if 1 were to choose  $\bar{m}_1$ .<sup>14</sup>

14. Economists will note that these are old concepts in the literature. The Nash equilibrium was proposed by Cournot in modeling oligopoly. The sophisticated response was proposed by Stackleberg to model leader-

If all agents attempt this level of sophisticated behavior in our general model of resource allocation, an appropriate equilibrium concept for this situation would be what Hurwicz (1975) called the *Manipulative Nash Equilibrium*. Formally,  $m^*$  is a Manipulative Nash Equilibrium (MNE) if  $m^* = b^N(\tilde{e}, h)$  where  $u[h(b^N(\tilde{e}, h)); e_i] \geq u[h(b^N(\tilde{e}/e'_i, h)); e_i]$  for all  $e'_i \in E_i$  and  $b^N(\cdot, h)$  is the (naive) Nash behavior rule given the mechanism  $h$ . *Manipulative Nash Behavior*  $b^M(e; h)$  then is defined as the mapping from environments  $e$  to the MNE for  $e$ . This concept may be interpreted as the equilibrium joint message that would result if each agent behaves during the iterations as if he is a (naive) Nash player but strategically chooses the personal characteristic  $\tilde{e}_i$  that generates the best Nash response for him given that the others are following Nash behavior as well. Alternatively, we could imagine that the given mechanism  $h$  is implemented as a direct revelation mechanism  $h'$  in which each agent is asked for his characteristic  $\tilde{e}_i$  and the allocation then calculated is that which would be given by the original mechanism  $h$  if the joint message  $\tilde{m}$  is the (naive) Nash equilibrium for the stated characteristics  $\tilde{e}$ .

The implications of manipulative Nash behavior for efficient mechanism design are negative as established by Hurwicz in the following theorem:

**THEOREM 4.11 (Hurwicz 1975):** There are no mechanisms,  $h$ , such that the Manipulative Nash allocations,  $h[b^M(e; h)]$ , are Pareto-efficient on the classical environments.

Proof: See Hurwicz (1975).

A corollary to this theorem is that even if a mechanism is an efficient Nash mechanism its manipulative Nash allocations are not Pareto-efficient. Thus, even if a mechanism is designed to effectively channel the incentives under (naive or non-manipulative) Nash behavior, if agents are sophisticated and adopt manipulative Nash behavior, the effort will be unsuccessful. [See Thomson (1984) for some results concerning likely outcomes under manipulative Nash behavior in environments with quasi-linear preferences.]

---

follower price-setting behavior.

Although it may appear that no mechanism can prevent sophisticated manipulation from leading to inefficiency, one should note that the definition of manipulative behavior above is based on the assumption that the underlying naive behavior is Nash. One can generalize the above notion of manipulative behavior by considering other naive models. For example, let  $b(e;h)$  be an arbitrary model of behavior for the mechanism  $h$ . If this model is correct, then the outcome will be  $h(b(e;h))$ . But clearly, one can also use this model to compute an optimal manipulation. We call  $b^M(e;h,b)$  the manipulative behavior model given  $h$  and  $b$ , if  $b^M: E \rightarrow A$  where  $b^M(e;h,b) = h(b(e^*;h,b))$  and  $u_i[h(b(e^*;h,b));e_i] \geq u_i[h(b(e_i^*/e_i^*;b));e_i]$  for all  $i$  and  $e_i^* \in E_i$ . That is,  $e^*$  is a Nash equilibrium of the direct revelation mechanism  $f(e^*;h,b) = h(b(e^*;h,b))$ . This generalizes our previous definition in the sense that  $b = b^N$ , or Nash behavior, is only one possible  $b$ . To see that the assumed behavior  $b$  is important, consider the following theorem which stands in direct contrast to Hurwicz's result.

**THEOREM 4.12 (Reichelstein 1982):** If  $h$  is an efficient Nash mechanism and if the postulated behavior is  $b(e;h) = [b_1(e_1;h), \dots, b_N(e_N;h)]$  where  $b_i(\cdot;h): E_i \rightarrow M_i$  is an onto map then the allocations  $h(b^M(e;h,b))$  are Pareto-efficient.

Proof: See Reichelstein (1982).

(Note that the postulated naive behavior cannot be Nash behavior!) This theorem asserts that if each agent postulates that all others' naive behavior depends only on their own characteristic  $e_i$ , then the manipulative equilibrium allocation will be efficient if  $h$  is a Nash efficient mechanism. Notice that if all  $i$  follow the postulated naive behavior  $b$  and do not manipulate then  $h(b(e;h))$  cannot be efficient over all  $E$ . The main problem with this result is that it doesn't make sense to us to assume that sophisticated behavior is Nash when naive behavior is not. The players' postulate about each other's naive behavior should be consistent with the assumed level of sophistication.

It is important to note that with arbitrary outcome function-behavior rule pairs  $(h,b)$  the analysis becomes simply that of direct revelation mechanisms. That is, given  $(h,b)$  define  $f(e) = h(b(e))$ . Any question about the manipulative performance of  $h$ , given the assumption that the naive behavior is  $b$ , is equivalent to the same question about the Nash equilibrium behavior of the

mechanism with  $M = E$  and  $h'(m) = f(m)$ . For example,  $(h, b)$  has efficient manipulative equilibria if and only if  $f$  is an efficient (direct revelation) Nash mechanism.

An interesting result that follows from the above is:

**THEOREM 4.13:** There exist individually rational direct revelation mechanisms for classical environments such that their Nash equilibria are Pareto-efficient. However, at the Nash equilibria, the agents' (equilibrium) messages  $e_i^* = m_i^*$  are not, in general, coincident with their true characteristics  $e_i$ .

Proof: Follows from Theorem 4.12 (Reichelstein), Theorem 3.1 (Hurwicz), and Theorem 3.2 (Ledyard/Roberts).

In view of these results, it is important to ask how likely manipulative behavior is. For one shot implementations of mechanisms it is hard to find any rationalizations for manipulative behavior (as defined here); it's plausibility surely depends on some form of iterative implementation of a mechanism. However, when a mechanism is iteratively implemented, many aspects that are not included in a normal form description of the mechanism become important. As Smith (1982) has emphasized for experimental economics, detailed instructions must be specified to implement any specific mechanism. Suppose, for example, that one wished to use the Walker mechanism to make a public goods choice. The instructions would have to specify:

(1) *the language of communication*. If there is a single public good, then the message any agent sends to the experimenter (or the 'center' or the 'auctioneer') is a single real number, say  $m_i$ . Then, the experimenter will return a message to each agent. In this particular mechanism, this message may be either all others' responses, or just the responses  $(m_{i-1}, m_{i+1})$ , or a number, say  $q_i$ , where  $q_i = \frac{1}{N}q + m_{i-1} - m_{i+1}$ . Each of these responses by the center represents, in principle, a different institution.

(2) *the timing and addresses of communication*. Each agent must know when to communicate their response and when to expect to receive a response. They must also know to whom to send a message

and from whom to receive one. In the above centralized institution, communication proceeds in orderly iterations where each agent sends a message  $m_i$  to the experimenter who, after receiving all responses, sends a (possibly different) message simultaneously to each agent. It is this step which begins to identify what each agent knows, other than  $e_i$ , at each step of the process.

(3) *a stopping rule*. The communication process must stop sometime and the nature and timing of its cessation must be specified. One possible example, in this case, is to state that the iterative process described in (2) will end when either (a) every agent matches their previous message two times in a row or (b) when, say, thirty iterations have elapsed, whichever occurs first. It should be obvious that if (b) is deleted one would have a radically different process: different stopping rules may lead to dramatically different outcomes.

(4) *an outcome rule*. Each agent should know what action is to be implemented after the communication ceases. One rule for the Walker mechanism could be as follows. If (3a) is the reason communication ended then the experimenter takes the last response of each agent and computes as follows: the public goods level chosen is  $y = \sum m_i$ , and each  $i$  pays  $T_i = (\frac{1}{N}q + m_{i-1} - m_{i+1})y$ . This is simply the rule  $(x_1, \dots, x_N, y) = h(m)$ . If (3b) is the reason communication ended then let  $(x_1, \dots, x_N, y) = 0$ . That is if there is no agreement, the *status quo* is the implemented allocation.

If the experimenter or the mechanism designer did not specify all of the above components of the process, the mechanism  $h$  could not produce a choice. Thus the entire process is necessary. It is also important since it is highly unlikely that, in practice, different versions of (2) and (3) will produce the same allocation even if (1) and (4) are identical versions of the same mechanism. This means that the behavior,  $b(e;h)$ , may be different depending on the, generally unspecified, components of the process. The normal form of the mechanism may be an insufficient description for a thorough analysis of the performance of designed mechanisms. Thus, a deeper analysis of manipulative behavior must depend on a rigorous analysis of behavior when details of the extensive form are specified.<sup>15</sup> In particular, given the process, (1) to (4) above, each agent is faced with a complex

15. It should be noted that in his initial papers in this area, Hurwicz carefully specified the iterative process of communication. Many of those models of resource allocation mechanisms were complete processes in the sense



sequential incomplete information game. Manipulation can be achieved only through the sequence of messages one sends and the real issue is whether or not the outcome attained is near to the normal form Nash equilibrium.<sup>16</sup> This is both an empirical and a theoretical issue; empirical in the sense that what we want to know is how agents will actually behave when confronted with a new mechanism and theoretical in the sense that we want to know how agents should behave when confronted with a new mechanism. It is our view that when this type of analysis is done it will be discovered that the specifics of (1) to (4) will be very important and that there are processes in which sophisticated manipulation is virtually impossible because of the informational requirements of such a strategy. Of course, these remain open questions.

#### *4. Other possibilities*

The design and evaluation of efficient mechanisms has been carried out for other types of presumed behavior. We include two of the more common in this section for completeness.

The first is Maximin behavior. The main results in this area are due to Thomson (1979). Maximin behavior results from an agent's hypothesis that, for each message he chooses, the other players will jointly choose their messages to minimize his payoff given his message. Under this hypothesis, the agent then chooses a message which maximizes this minimum payoff. Thomson has shown that for the subset of environments with quasi-linear preferences (those analysed by Vickrey and Groves) there exists an efficient maximin mechanism. Results for other environments are unknown. The maximin behavioral rule arises naturally in the context of 2 person zero sum games but does not seem to be compelling in the N person non-zero sum games that we consider in this paper. Only if the individuals are infinitely risk averse or extremely paranoid should they be expected

---

that (1), (2), and (4) were explicitly specified. The stopping rules (3) were implicit but, since only "equilibrium" messages were considered, they were probably something like (a) above. Actually, the initial specifications of Hurwicz went further as each agent was also told which response to make given the message of the center and their own characteristic. It was the realization that the designer could not guarantee that the specified rules would be followed, since the designer did not know the particular  $e_i$  of each agent, that led to the formulation of the incentive problem as we have presented it in this paper.

16. Indeed the fact that the Complete Information Nash equilibria are not in general Manipulative Nash equilibria leads us to suspect that a detailed analysis of the incomplete information, repeated game will show that the outcome will not be near to the outcome attained at the normal form Nash equilibrium messages.

to behave as required by the maximin hypothesis.

The other behavioral rule rests on an assumption of myopia that arises naturally in the context of planning procedures. The research in this area for private goods environments dates to the debate over the relative merits of socialist planning and free markets. See the planning models of Malinvaud (1967), Weitzman (1970), and Heal (1973) and the survey of Hurwicz (1973) for a summary of this literature. For public goods environments, the initial literature consists of papers by Malinvaud (1971) and by Dreze and de la Vallee Poussin (1971). Robert's paper in this volume provides an excellent summary of the extensive literature that followed from these original papers. Our remarks here are intended merely to provide a bridge between our paper and his. The general structure of these planning procedures bears a close resemblance to the processes described in the previous section on manipulation. The main formal difference is that, in general, the outcome rule of a planning procedure depends on the entire sequence of messages sent, not just on the last message.

Formally, a (discrete time) planning procedure is a language,  $M_i$ , for each  $i$ , a state space,  $S$ , and an explicit iterative process of communication. This iterative process is modeled as

$$(1) \quad s(t+1) = f(s(t), m(t)), \text{ for dates } t = 1, 2, \dots$$

The final allocation determined by the process is given by an outcome rule,  $h(s) = a$ , and a stopping rule which defines that state  $s(t)$  to which  $h$  is to be applied. If  $f(\cdot)$  is defined so that  $s(t+1) = m(t)$  then this is an allocation mechanism with  $h$  as the outcome function. The generalized form (1) allows planning procedures which are indirect control devices; that is, instead of directly determining the outcome through  $h$ , the agents directly control  $s$  and indirectly control the outcome through the cumulative effect of  $m$  on  $s$ . This allows for smoother but possibly less rapid convergence to the desired allocation.

In the Dreze-de la Vallee Poussin and Malinvaud (DVM) mechanisms,  $m_i$  is a vector of marginal rates of substitution, or marginal willingnesses to pay and  $s$  is the "proposed" allocation. (Although in the original papers (1) is in 'continuous' form; that is,  $ds/dt = f(s, m)$ , we consider here the discrete time versions.) In these models the appropriate response rule of each  $i$  is specified as

$$(2) \quad m_i(t) = g(s(t), e_i),$$

the assumption being that each agent will follow the rules. Of course it was realized that agents might not and the incentive properties of the procedures were analyzed in those papers under different behavioral assumptions. The basic approach is to assume that, at each iteration, an agent is only concerned with the current increase in utility. This is similar to assuming that the agent thinks the current iteration is the last and that  $h(s(t+1))$  will be implemented. One can analyze this myopic, or local, behavior in the same way that we have analyzed the global model. We give two examples and refer the reader to Roberts (1984) for others.

In their 1971 paper Dreze and de la Vallee Poussin presented a planning procedure with the property that if agents adopt local maximin behavior then they will report their true marginal rates of substitution and the procedure will then converge to a Pareto-efficient allocation. That is, in classical public goods environments with a finite set of agents, there exists an efficient, local maximin procedure. Roberts (1979) has shown that if agents use local Nash behavior (that is,  $m(t)$  is a Nash equilibrium for the local game) then the same DVM procedure will converge to Pareto-efficient allocations, although at a slower rate. That is, in classical public goods environments with a finite set of agents, there exists an efficient, local Nash procedure. One drawback of the local Nash assumption is that it is not clear how agents are to determine these Nash responses since the procedure does not explicitly allow for a sequence of iterations of messages before  $m(t)$  is determined. Thus the justification we used for relying on the (global) Nash assumption is missing for the local Nash assumption. If agents look ahead, instead of behaving myopically, we are in a similar situation to that described in the last section on manipulative equilibria of mechanisms.

The form of the planning procedure (1) provides the agents with a dynamic game where the strategies are  $m_i(\cdot)$ , functions of  $t$  from 0 to  $T$ , the transition equations are given by (1), and the payoffs are  $u(h(s(T)), e_i)$  for each  $i$ . If  $i$  knew the strategies of the others,  $i$  could compute a best response strategy by solving an optimal control problem, but as in the case of the manipulation of allocation mechanisms, each agent only knows their own characteristic and the sequence of reports

from the center. This is thus a very complicated sequential incomplete information game. Truchon (1984) has shown, for quasi-linear preferences, that there is a discrete time process whose Perfect Nash equilibrium converges to an efficient allocation. Roberts, in this volume, has also begun an analysis of this problem but with essentially negative results. He states "the results that we do get, however, suggest that the informational savings involved in adopting an iterative procedure can be realized only at a cost of lost efficiency." In other words, there may be no planning procedures whose sequential equilibrium allocations are Pareto-efficient. See Roberts's paper for a discussion of this and possible future research.

## B. PARAMETRIC MECHANISMS

To this point we have seen that if we restrict ourselves to non-parametric outcome functions then it is impossible to design efficient, dominant strategy mechanisms if the class of environments is reasonably large. It is, however, possible to design efficient Nash mechanisms but, again, if the class of environments is reasonably large then the outcome rules for these mechanisms cannot be fully feasible. A logical next step in the development of the theory of allocation mechanisms is to explore what can be accomplished if we allow parametric outcome functions. If information about the environment  $e$ , or about the class of environments  $E$ , can be used in the outcome function, even though that information was not transmitted as a message by any agent, then we say that function is parametric. We denote such functions by  $h(m, I(e))$  where  $I(e)$  denotes that information about  $e$  which is to be used by  $h$ . This information usually is used in one of two forms; either direct information about the specific environment  $e$  or indirect information about the range of possible  $e$  in  $E$  in the form of a probability measure on  $E$ . Let us look at each of these in turn.

### *1. Direct information*

One of the drawbacks of many of the mechanisms whose Nash equilibrium allocations are Pareto-efficient is that, at non-Nash equilibrium messages, the outcome rule may compute an allocation which is not feasible for some  $i$ , even if that rule is continuous and balanced. (See Theorem

4.8 above.) If such a mechanism were actually used in, say, an iterative process which was terminated prior to the attainment of a full Nash equilibrium, then it is possible that some agents would be unable to survive on the indicated allocation. Hurwicz, Maskin, and Postlewaite (1982) were able to overcome this problem with the use of parametric outcome rules. They incorporated direct information about the initial endowments into the outcome function  $h$  by allowing the admissible strategies to depend on the initial endowment  $w$ . The function  $h$  maps strategies into actions as follows:  $h:S(e) \rightarrow A$  where  $S(e) = \prod_i S_i(e_i)$  and  $S_i(e_i) = M_i \times [0, w_i]$ . This form of parametric function possesses two desirable features. First, it retains much informational decentralization since no agent need know the characteristics of the others. Because of this feature, Hurwicz-Maskin-Postlewaite called these outcome functions decentralized parametric. Second, the center need only be able to verify that each agent has at least as much initial endowment as reported. Endowments are, in principle, capable of being audited; in this case, however, agents need only produce the claimed allocation. Ignoring the costs of that auditing procedure, this class of parametric functions seems to be a natural candidate for consideration in our search for efficient mechanisms.

In their paper Hurwicz-Maskin-Postlewaite consider this class of mechanisms in detail and provide a list of possibilities. Although they consider both private and public goods environments, let us restrict our attention, for now, to classical private goods environments. The results for public goods environments are the same as those below if we replace the identifier "Walrasian" with "Lindahl". The first result follows from a clever example and a theorem of Maskin (1977) on necessary conditions for implementability when endowments are known by the designer.

**THEOREM 4.14 (Hurwicz/Maskin/Postlewaite 1982):** There is no decentralized parametric outcome function such that  $h(b^N(e;h)) = W(e)$  on the class of classical private goods environments.

**Proof:** See Hurwicz, Maskin, Postlewaite (1982).<sup>17</sup>

17. They actually prove more: even if the designer knows the initial endowments, the Walrasian correspondence is not implementable since it is not monotonic. A performance correspondence,  $P(e)$ , is said to be implementable (in Nash equilibria on  $E$ ) if and only if there is an outcome function  $h$  such that  $h(b^N(e;h)) \subseteq P(e)$  for all  $e$  in  $E$ . A performance correspondence,  $P(e)$ , is monotonic if, given  $a \in P(e)$ , then for any  $e' \in E$  such that whenever  $u(z, e_i) \leq u(a, e_i)$  then  $u(z, e_i') \leq u(a, e_i')$ ,  $a \in P(e')$ .

The difficulty in producing the desired mechanism arises in those environments in which the Walrasian allocation is on the boundary of the feasible set of allocations. Hurwicz, Maskin, and Postlewaite show, however, that it is possible to adjust for this anomaly and to produce a decentralized parametric mechanism that is feasible at all messages and that selects efficient Walrasian allocations when these are interior and efficient allocations otherwise. Before presenting their mechanism, consider the following useful performance correspondence. The Constrained Walrasian correspondence,  $CW(e)$ , is defined as follows:

$$CW(e) = \{(x_1, \dots, x_N) \mid \text{there is } p \text{ such that } \sum_i CD_i(p) = 0, x_i \in CD_i(p) \text{ for all } i\}$$

$$\text{where } CD_i(p) = \{x_i \mid px_i = pw_i, u(x_i, e_i) \geq u(z, e_i) \text{ for all } z \text{ such that } pz = pw_i \text{ and } z \leq \sum_h w_h\}.$$

Note that these are the market equilibrium allocations when individual demand choices are constrained so that agent  $i$ 's demand does not require more of any commodity than is available to the entire economy. Letting  $P(e)$  be the Pareto-efficient allocations for  $e$  and  $IP(e)$  be those  $a$  in  $P(e)$  such that  $u(a, e_i) \geq u(w_i, e_i)$  for all  $i$ , i.e. the individually rational allocations, then we can describe the relationships between these various performance correspondences as follows:

$$W(e) \subseteq CW(e) \subseteq IP(e) \subseteq P(e) \text{ for all } e \in E.$$

Note that  $W(e) = CW(e)$  whenever either  $a \in W(e)$  implies that  $a_i > 0$  for all  $i$ , or, less restrictively, whenever  $a_i < w$  for all  $i$ . Given these concepts we can state

**THEOREM 4.15 (Hurwicz/Maskin/Postlewaite 1982):** There is a decentralized parametric allocation mechanism, with finite dimensional messages, for which  $h(b^N(e;h)) = CW(e)$  on the classical private goods environments.

Proof: See Appendix to Section IV for a sketch of the proof.

The mechanism described in the proof of this theorem is such that its Nash equilibrium allocations are constrained Walrasian allocations as long as endowment misrepresentations can only be less than the true endowment. That is, endowments can be withheld. Hurwicz, Maskin, and Postlewaite also provide a proof of the above theorem if withheld endowments must be destroyed.

Such would be the case if, for example, endowments can not only be required to be shown but also can be found if they are withheld. In practice, the former is similar to the requirement that a buyer demonstrate a sufficient bank balance prior to purchase. the latter is similar to an IRS tax audit. The former is clearly less expensive than the latter.

The HMP theorem implies the following simple corollary.

COROLLARY:(a) There exist decentralized parametric, feasible Nash efficient allocation mechanisms on the class of classical private goods environments.

(b) There exist decentralized parametric, feasible Nash efficient allocation mechanisms whose allocations are individually rational.

Also, a slight modification of Hurwicz's theorem characterizing Nash efficient mechanisms establishes

THEOREM 4.16 (Hurwicz 1979): If  $h$  is a continuous, decentralized parametric outcome function such that the Nash equilibrium allocations are individually rational and Pareto-efficient, then the constrained Walrasian allocations are contained in the Nash equilibrium allocations. That is,  $CW(e)$  is the smallest individually rational, Pareto-efficient performance correspondence which is implementable with a decentralized mechanism.

Proof: See Hurwicz (1979).

One drawback of the mechanism used to demonstrate existence of Nash efficiency above is that the function,  $h(s,v)$ , (defined in Appendix IV by equations (1.1) to (1.3)), is not continuous in either  $s$  or  $v$ . There is, however, a mechanism due to Postlewaite (as reported by Schmeidler (1982)) that is continuous in  $s$  and can be used to establish similar results.

THEOREM 4.17 (Postlewaite and Wettstein 1983): There exist decentralizable parametric, feasible Nash efficient mechanisms,  $h(s,v)$ , that are continuous in  $s$ .

Proof: See Appendix IV for a sketch.

It remains an open question whether there exist continuous (in all  $m$ ) decentralized parametric feasible Nash efficient allocation mechanisms for the classical private goods economies.

## 2. Indirect information

In his seminal article of 1972, Hurwicz considered briefly a model of the design problem which is a little different than those we have considered to now. In his model, there was a welfare function on outcomes,  $W$ , so that the welfare associated with the mechanism  $h$ , given the behavior  $b$ , in the environment  $e$  could be expressed as  $w = W(h(b(e;h)),e)$ . A variant of this model allowed resources to be expended in the operation of the mechanism. Thus, for this variant, if  $r(h,e)$  is the cost of operating mechanism  $h$  in the environment  $e$ ,  $w = W(h(b(h;e)-r(h,e),e) = w(h,b,e)$ . The designer's problem was then recognized to be the statistical decision problem to maximize  $w$  over the set  $E$  by choosing  $h$ . If a probability measure over  $E$  were available to the designer, the designer might choose  $h$  to maximize expected welfare. This important observation by Hurwicz foreshadowed the literature on optimal auction design and provides the basis for the inclusion of what we call indirect information into the design of the allocation mechanism. This information will be in the form of a probability measure on  $E$  which is common knowledge to all agents and to the designer. This is indirect information in that it is not directly auditable since, just as in the case of preferences, probability beliefs are non-observable and may only be indirectly inferred from evidence about actions. This inability to audit beliefs raises serious questions about the efficacy of mechanisms that are designed assuming knowledge of those beliefs. However, since the literature in this area is extensive and others do not share these doubts, we summarize the results under the assumption that the mechanism can indeed use indirect parametric information.

Suppose that  $P(E)$  is the set of probability measures on  $E$ . Consider parametric mechanisms,  $h(m_1, \dots, m_N, p)$  where  $p \in P(E)$ . In the language of incomplete information games, we assume that  $p$  is common knowledge to the mechanism designer, the mechanism operator and all the agents. At issue is the same question we have addressed all along: given some behavior that is consistent with the agents' incentives, are there any mechanisms of this type such that some specified performance



results? A very natural assumption of reduced form behavior in this type of mechanism, given the common knowledge assumption, is that of Bayes equilibrium. This was defined earlier in section IV.A.1 as follows:

A strategy for  $i$  is a function  $d_i: E_i \rightarrow M_i$ . A Bayes equilibrium, given  $h$  and  $p$ , is a vector of strategies  $(d_1, \dots, d_N)$  such that for every  $i$  and every  $e_i$ ,  $d_i(e_i, p)$  solves

$$\text{Max}_{d_i} \int_{E_{-i}} u(h((m_1(e_1), \dots, m_N(e_N))/m_i, p), e_i) dp(e/e_i) \text{ where } E_{-i} \equiv \prod_{j \neq i} E_j.$$

Given a Bayes equilibrium,  $d$ , the outcome is  $a = h(d(e, p), p)$  for each  $e$  in  $E$ . Of course, multiple Bayes equilibria<sup>18</sup> can be a problem in the same way as multiple Nash equilibria, although for our purposes there is no difficulty. For example, if we want  $h(d(e, p), p)$  to be efficient and there are multiple equilibria, let  $D(e, p)$  be the set of equilibria. Then we simply ask if  $h(d, p)$  is efficient for all  $d \in D(e, p)$ .

One question which immediately suggests itself is whether there is any indirect information mechanism which is also efficient at Bayes equilibria, in the sense that  $h(d(e, p), p)$  is Pareto-efficient for all  $e$  in  $E$ . There are at best some partial answers. In classical public goods environments, if we further restrict preferences to be quasi-linear, there exists an efficient parametric mechanism for a subset of possible probability measures.

**THEOREM 4.18** (d'Aspremont/Gerard-Varet 1979): If  $E$  is the set of classical public goods environments with quasi-linear preferences and  $p$  is a probability measure on  $E$  such that  $e_i$  is distributed independently of  $e_{-i}$  for all  $i$ , then there exists a direct-revelation mechanism  $h: (M, p) \rightarrow A$  such that  $e = d(e, p)$  and  $h(e, p)$  is Pareto-efficient for all  $e$  in  $E$ .

Proof: See Appendix IV for a sketch of the proof. This theorem was also independently discovered by Arrow (1979).

18. It is interesting that there are few examples of multiple Bayes equilibria in resource allocation models. The apparent example in Laffont and Maskin (1981) is, unfortunately, not a valid one. See, however, Postlewaite and Schmeidler (1983).

It remains an open question, as far as we are aware, whether or not there exist parametric mechanisms that are efficient at all Bayes equilibria for all classical environments although Postlewaite and Schmeidler (1984) have made some progress under some information structures.

### C. SUMMARY

Combining the results in the previous sections, we can summarize the state of knowledge concerning the possibilities for the design of efficient, incentive-sensitive mechanisms in finite environments.

**THEOREM 4.19:** In classical environments (both private and public goods) with at least two agents:

- (a) There do not exist non-parametric, efficient Bayes mechanisms (Section IV.A.1).
- (b) It is an open question whether there exist parametric, efficient Bayes mechanisms (Section IV.B.2).
- (c) For environments with at least three agents there exist continuous, balanced, non-parametric, efficient Nash mechanisms (Section IV.A.2).
- (d) There do not exist non-parametric, individually feasible, non-trivial efficient Nash mechanisms (Section IV.A.2).
- (e) There exist decentralized parametric, feasible, efficient Nash mechanisms (Section IV.B.1).
- (f) It is an open question whether there exist continuous, parametric, feasible, efficient Nash mechanisms (Section IV.B.1).
- (g) There do not exist non-parametric mechanisms whose Manipulative Nash allocations are Pareto-efficient (Section IV.A.3).
- (h) There exist non-parametric planning procedures that generate efficient allocations under either local maximin behavior or local Nash behavior (Section IV.A.4).
- (i) It is an open question whether there exist planning procedures that generate efficient allocations under sequential Bayes behavior (Section IV.A.4).

Finally, we should emphasize the open questions raised by the concept of manipulative equilibrium, by Roberts's (1984) work in this volume on planning procedures, and by our justification of the assumption of Nash equilibrium behavior. There is not a consensus normal or extensive form model of rational behavior for agents in iterative processes.

## APPENDIX TO SECTION IV: SKETCHES OF PROOFS

We include in this appendix sketches of proofs of a few of the major theorems of Section IV.

*Sketch of Proof of Hurwicz, Maskin, Postlewaite Theorem 4.15:*

HMP define a decentralizable parametric allocation mechanism which satisfies the conclusion of the theorem in two steps. First an auxiliary function is given which determines outcomes, given reported endowments  $w$ ,  $h(s_1, \dots, s_N; w) = a$ . This auxiliary function is defined to be feasible, given  $w$ , for all messages  $s$ , and so that  $h(b^N(e; h), w)$  is Pareto-efficient in the environment with preferences as in  $e$  and endowments  $w$ . Let  $s_i = (x_i, p)$  and  $S_i = \{(p, x) \in P^1 \times R^L : px = pw_i\}$ , where  $x_i$  denotes the total consumption (not net trade) of the  $i^{\text{th}}$  agent.

The outcome function  $h$ , for total holdings, is defined as follows:

1. If there exist  $i, j, k \in N$  such that  $p_i, p_j, p_k$  are distinct, then

$$h_i(s) = \left[ \left[ \sum_{r \in N} \left[ \left[ x_r^i \right] \right] \right] \right] w, \text{ for all } i \in N.$$

2. If there exist only two distinct announced prices  $p'$  and  $p''$ , and at least two agents announce each  $p'$  and  $p''$ , then  $h_i(s) = w_i$  for every  $i \in N$ : i.e. there are no trades.

3. If there is a  $p$  such that  $p_i = p$  for all  $i \in N$  (unanimous agreement on the announced price), and when

- 3.1.  $\sum_i x_i \neq w$ , then  $h_i(s) = w_i$ , for all  $i \in N$ ;

- 3.2.  $\sum_i x_i = w$ , then  $h_i(s) = x_i$ , for all  $i \in N$ .

4. If there is a  $p$  and an agent  $m \in N$  such that  $p_m \neq p$  but  $p_j = p$  for all  $j \neq m$ , then

$$h_m(s) = \lfloor pw_m / px_m \rfloor x_m$$

- 4.1.  $\lfloor pw_m / px_m \rfloor x_m \leq w$ ;

$$h_j(s) = \frac{1}{N-1} [w - h_m(s)] \text{ for } j \neq m$$

- 4.2.  $h_i(s) = w_i$ , for all  $i \in N$ , if  $\lfloor pw_m / px_m \rfloor x_m > w$ .

It can now be shown that  $h(b^N(e/w; h), w) = CW(e/w)$  for all  $e \in E$  and all  $w \geq 0$ .

The next step in the proof is to provide a mechanism which yields the correct reported endowment as a Nash equilibrium. Let  $m_i = (v_i, x_i, p_i)$  where  $x_i \in \mathbb{R}^L$ ,  $p_i \in \mathbb{R}^L$ , and  $v_i \in \mathbb{R}_+^{NL}$ . Note  $v_i = (v_{i1}, \dots, v_{iN})$  where  $v_{ij}$  can be interpreted as  $i$ 's report about  $j$ 's endowment. Let  $M_i = \{m_i\}$ ,  $M = \prod_i M_i$ . For all  $i, j$  it is required that  $v_{ii} \leq w_i$ . The outcome function,  $h(v, s)$ , is defined as follows:

(a) If there is a  $\tilde{v}$  such that  $v_i = \tilde{v}$  for all  $i$  then

$$(1.1) \quad h(m) = h(v, s) \text{ as defined previously.}$$

If not, then let

$$A(m) = \{i \in N \mid v_{ii} \geq v_{ij}, \text{ for all } j \neq i, j \in N\},$$

$$v(m) = \sum_i v_{ii}; \quad B_i(m) = \sum_{j, j \neq i, k, k \neq i} [v_{ij} - v_{jk}], \quad B_i(m) = \sum_j [v_{ii} - v_{ij}], \text{ for all } i.$$

Thus

(b) If  $A(m) = \emptyset$ , (the empty set), but there is no  $\tilde{v}$  such that  $v_i = \tilde{v}$  for all  $i \in N$ , then  $\sum_j B_j > 0$  and

we set

$$(1.2) \quad h_i(m) = \frac{B_i(m)}{\sum_j B_j(m)} v(m) - v_{ii}, \text{ for all } i \in N.$$

And

(c) If  $A(m) \neq \emptyset$ , then  $\sum_j G_j(m) > 0$  and we set

$$(1.3) \quad h_i(m) = \begin{cases} \frac{G_i(m)}{\sum_j G_j(m)} v(m) - v_{ii} & \text{for } i \in A(m) \\ -V_{ii} & \text{for } i \notin A(m) \end{cases}$$

*Sketch of Proof of Postlewaite Theorem 4.17:*

Let  $s_i \equiv (p_i, z_i, r_i) \in S_i \times \mathbb{R}^L$  where  $S_i \equiv \{(p_i, z_i) \in \mathbb{R}_+^L \times \mathbb{R}^L \mid p_i z_i = 0 \text{ and } \sum_{k=1}^L p_{ik} = 1\}$ . Given messages

$s_1, \dots, s_N$  and endowments  $w_1, \dots, w_N$ , define:  $a_i \equiv \sum_{j,k=i} |p_j - p_k|$ ,  $a \equiv \sum_{i=1}^N a_i$ ,  $b_i = \frac{a_i}{a}$  if  $a > 0$  and  $b_i = 1$

if  $a = 0$ ,  $\bar{p} = \sum_{i=1}^N b_i p_i$ . Define  $x_i$  to be the closest point to  $z_i$  in  $\{z \mid \bar{p} z = 0, z + w_i \geq 0\}$ . Finally, let

$h_i'(s_1, \dots, s_N; w) = r^* \cdot r_i \cdot x_i + (r^* \cdot r_i - 1)w_i$  where  $r^* \equiv \max\{r \in \mathbb{R} \mid r \cdot r_i \leq 1, \forall$

$i, \text{ and } \sum_i r \cdot r_i (x_i + w_i) \leq \sum_i w_i\}$ .

Now, in equation (1.1) use  $h'(s; w)$  in place of the function  $h(s, v)$ . The function now defined by

(1.1'), (1.2) and (1.3) satisfies the conclusion of the theorem.

*Sketch of Proof of d'Aspremont/Gerard-Varet Theorem 4.18:*

The appropriate mechanism is a Groves mechanism with transfers arranged so that they balance, (i.e. the transfers sum to zero). The mechanism is a direct revelation mechanism and chooses

$y = y(m_1, \dots, m_n)$  and  $t_i = t_i(m_1, \dots, m_n)$  as follows. Remembering that  $u(x_i, y, e_i) = u(y, e_i) + x_i$  for all  $i$ ,  $y(m_1, \dots, m_n)$  maximizes  $W(y, m) = \sum_i u(y, m_i)$ . Let  $x_i(m) = w_i - t_i(m, p)$ , where

$t_i(m, p) = - \int_{E_{-i}} u(y(m), e_j) dp(e_j/m_i) - \frac{1}{n-1} \sum_{k \neq i} \int_{E_{-k}} u(y(m), m_j) dp(e_j/m_k)$ . Thus

$t_i(m, p) = -g(m_i, p) - \frac{1}{n-1} \sum_{k \neq i} h(m_k, p)$ . Given any true value,  $e_i^*$ , agent  $i$  wants to choose  $m_i$  to

maximize  $u(y(d(e)/m_i), e_i^*) - \int_{E_{-i}} u(y(d(e)/m_i), e_j) dp(e_j/e_i)$ . It is easy to verify that  $e_i = e_i^*$  solves this

problem.

## V. LARGE ECONOMIES AND EFFICIENT DOMINANT STRATEGY MECHANISMS

Hurwicz (1972) was well aware that both the pessimistic impossibility results of section III and the need to consider non-dominant strategies might disappear if there were a large number of traders. In particular he said "In answering this question" about incentive compatible and efficient mechanisms "the crucial distinction is whether the economy is atomistic or not." We turn now to an exploration of this observation. The main question of interest is whether or not some type of approximation to the design of efficient dominant strategy mechanisms is possible when there are a large number of agents. A second question of interest is whether there is any difference in the answers for private and public goods. The standard approach is to first consider environments with a continuum, or a countable infinity, of agents and then to "pass back" the results, using continuity, to large but finite economies. We follow that approach here.

### A. CONTINUUM ENVIRONMENTS

It has long been conventional wisdom that, in private goods environments, if there are a large number of consumers then price taking behavior is incentive compatible. We have been unable to find a formal statement and proof of this insight in the literature although there is an implicit understanding of it in a paper by Roberts and Postlewaite (1976). Hammond (1979) proved a theorem utilizing a model with a continuum of agents that was developed by Aumann (1966). We present a slightly modified version of this theorem.

**THEOREM 5.1:** In private goods environments with an atomless continuum of agents, the Competitive Mechanism defined above in Section II.C is an efficient, dominant strategy mechanism.

Proof: See Appendix V for a sketch of the proof.

This result should surprise no one. The fact that a similar result obtains in classical public goods environments with a continuum of agents should surprise many since this runs counter to Downs's (1957) and others' intuition. To see why, we first present a specific direct revelation version of an allocation mechanism for public goods. The message of any agent is that agent's characteristic.

The outcome function picks a level of public goods and a vector of net trades in private goods. Given a vector of announced characteristics  $e'$ , let  $P(e')$  be the set of Pareto-efficient allocations for  $e'$ .

(Remember that these announced characteristics may well be different from the true characteristics comprising the true environment,  $e$ .) Let  $F(e')$  be those allocations in  $P(e')$  such that

$$px_i = pw_i - \frac{1}{N}qy \text{ where } (p,q) \text{ are the prices that support the Pareto-efficient allocation } (x_1, \dots, x_N, y).$$

Then  $h(e') = F(e')$  is called the Fair-Efficient Mechanism (FEM), since all agents "pay" an equal cost share of the public good. It does not necessarily select Lindahl allocations and is, therefore, different from the Privately Fair Lindahl Mechanism of Hammond (1979) which rarely has an equilibrium.

(See Groves/Ledyard (1977) and (1980)). Fair-Efficient allocations exist unless  $\frac{1}{N}qy > pw_i$  for some  $i$ ; that is, unless the proportional tax bankrupts some agent. The surprising result is:

**THEOREM 5.2:** In public goods environments with an atomless continuum of agents, the Fair-Efficient mechanism is an efficient dominant strategy mechanism.

Proof: See Appendix V for a sketch of the proof.

It would appear that, in continuum economies as in finite economies, there is fundamentally no difference between private and public good environments with respect to the possibility of the design of efficient, dominant-strategy mechanisms. However, as noted by Hammond (1979) and Downs (1957), the reason truth is dominant in the Fair-Efficient mechanism is that changes in  $d$  have absolutely no effect on the level of public good received or on the level of taxes paid. Thus, any  $d'$  such that  $x(v,d) = x(v,d')$  is a dominant strategy. There is no incentive either to lie or to tell the truth. Muench and Walker (1979) also noted this phenomenon for some versions of the Quadratic Mechanism. In the private goods case this is not true for the competitive mechanism. Thus there appears to be a subtle difference in the type of result, in spite of the superficial similarities. This difference is most easily highlighted by considering large finite economies.

## B. LARGE FINITE ECONOMIES



To discover what happens in large finite economies, we consider limiting results as the number of agents approaches infinity. We already know that it is impossible to design efficient, dominant strategy mechanisms in finite economies, even if they are large. However, if there is continuity as the number of agents grows, then the existence of efficient, dominant strategy mechanisms in large economies should give us some hope that in large finite economies we can have mechanisms which are "almost" efficient, dominant strategy mechanisms.

### 1. Limiting Incentive Compatibility

Two papers have addressed this issue by considering the potential gain from misrepresentation. In the first, by Roberts and Postlewaite (1976), a definition of "almost" dominant strategy is given for private goods economies. In particular, they defined a mechanism to be limiting incentive compatible if for any  $\epsilon > 0$ , and any utility function representing an agent's preferences, in sufficiently large economies the gain from using some characteristic other than the truth is less than  $\epsilon$ .

Endowing the set of measures which have compact support on  $D$ ,  $\{v\}$ , with the topology of weak convergence, we can talk about large finite economies which are "close" to atomless environments. Letting  $C(v)$  be the set of competitive equilibrium prices for the environment  $v$ , we have the following major result:

**THEOREM 5.3 (Roberts/Postlewaite 1976):** On the class of classical private goods environments. let  $v_k \rightarrow v'$  where  $v'$  is an atomless measure. If  $C(v)$  is continuous<sup>19</sup> at  $v'$  then for any  $\epsilon > 0$  and any utility function there is a  $k^*$  such that  $k > k^*$  implies that  $u(h(v_k), d) > u(h(v_k/d'), d) - \epsilon$  for any agent  $d$  in  $v_k$ . (That is, the gains from misrepresentation are arbitrarily small.)

Proof: See Roberts/Postlewaite (1976).

In the other paper to consider large environments, Roberts (1977) adapted the previous definition of limiting incentive compatibility to public goods environments and looked at the performance of several general classes of mechanisms. The paper contains several impossibility

---

19. For most atomless  $v'$ ,  $C(v)$  is continuous at  $v'$ .

results; we present one.

**THEOREM 5.4 (Roberts 1977):** Let  $v_k$  be an expanding sequence of public goods environments (the number of agents increases) with one private good,<sup>20</sup> and let  $h(v) = (x(v,d), y(v))$  be an allocation mechanism such that  $h$  is individually rational (i.e.  $u(h(v),d) \geq u(w(d),0)$ ), such that  $y(v)$  is uniformly continuous on the sequence  $\{v_k\}$ , and such that  $y(v_k) \rightarrow y^*$ . If  $x(v_k,d) \rightarrow x^* < w$ , then  $h$  cannot be limiting incentive compatible for the sequence  $\{v_k\}$ .

**Proof:** The proof of this theorem consists of showing that the misrepresentation of acting as if one receives no utility from the public good yields a gain that "is bounded away from zero unless the agent's implicit tax goes to zero". For details, see Roberts (1977), p. 367.

This theorem would seem to point to a key difference between private and public goods. But, the Fair-Efficient mechanism we used earlier in the limit economy is not individually rational and, therefore, is not subject to the conclusion of this theorem. In fact it can be shown that that mechanism is limiting incentive compatible.

**THEOREM 5.5:** Consider the class of environments characterized by quasi-linear preferences, one private and one public good, and crowding in the production of the public good so that the optimal quantity of the public good is bounded above by  $y^*$  finite. Let  $v_k$  be an expanding sequence of public goods environments in this class. There exists an allocation mechanism for this class of environments that is limiting incentive compatible for the sequence  $\{v_k\}$ .

**Proof:** See Appendix V for a sketch of the proof.

It would seem at this point that there is absolutely no difference, from a mechanism design point of view, between public and private goods, since all five of the following facts apply to both classical private goods environments and classical public goods environments.

- (1) In finite environments there exists at least one dominant strategy mechanism.
- (2) In finite environments there do not exist efficient, dominant strategy mechanisms, if the

---

20. There does not seem to be anything special about 1 private good.

class of environments is large enough.

(3) In finite environments there exist efficient Nash mechanisms.

(4) In atomless environments there exists at least one efficient, dominant strategy mechanism.

(5) There exists at least one mechanism which is limiting incentive compatible, unless individual rationality is required.

What then, if anything, accounts for the conventional wisdom that the incentives in allocating private goods are fundamentally different from those in allocating public goods? Consider an alternative approach.

## *2. Nash Equilibrium Behavior*

We have just seen that in large finite environments there are mechanisms such that almost all agents have a small incentive to free ride. It is possible, however, that the combined effects are magnified; that is, that although each misrepresents a little, all together the effect is large. Therefore, rather than assume that all but one agent behaves truthfully, as was done in the definition of limiting incentive compatibility, let us consider what occurs if all misrepresent. Since there will be no dominant strategies, we must consider other behavior. We assume Nash equilibrium behavior.

In the private goods environment, consider the competitive mechanism. If the economy is replicated, as  $N$  grows the aggregate excess demand curve of everyone except some one agent flattens out. As it does, the best misrepresentation for that particular agent is an offer curve that intersects this excess demand of the others in a point nearer and nearer to the intersection of that curve with his true offer curve. In the limit his best strategy is the truth. Thus, not only is the gain from misrepresentation near zero in large environments, but the Nash equilibrium strategies are near the true preferences.

**THEOREM 5.6:** In classical private goods environments, with enough continuity, there is a (direct revelation) mechanism which is "almost" an efficient dominant strategy mechanism if the economy is large enough. (That is, the Nash equilibrium strategies are almost dominant strategies and the Nash

equilibrium outcomes are almost Pareto-efficient.)

Proof: Follows from the above argument. See Appendix V for a sketch of the proof.

In public goods environments this theorem does not seem to hold. Consider the Fair-Efficient Mechanism presented in Section V.A. For each agent,  $d$ , let  $y'(d)$  solve  $\frac{\max}{y} u(y,d) - f(y)$ , and then let  $d^*$  solve  $\frac{\max}{d} y'(d)$ . If  $y'(d) < y'(d^*)$  then it will be in  $d$ 's interest to send the misrepresentation,  $d'$  where  $u(y,d') = 0$  for all  $y$ . It will be in  $d^*$ 's interest to send the misrepresentation,  $d'^*$  where  $u(y,d'^*) = u(y,d^*)$  for all  $y$ , if all others send  $d'$ . Thus the Nash equilibrium outcome will be  $y = y'(d^*)$  and each  $d$  will pay  $f(y'(d^*))$ . As the economy grows, these remain the appropriate misrepresentations, and the outcomes and the allocations remain bounded away from efficiency. If we consider the limit of these environments as  $N$  grows, we see that although the gain from these "free riding" strategies goes to zero, there is no loss from following them even in the limit. That is, even in the limit environment with an infinite number of agents these are "reasonable" strategies; nothing is lost by following them; they are optimal. In fact, they are almost dominant. (In the limit economy, since no agent can change either taxes or the level of public good, almost all misrepresentations are as good as the truth.)

The above behavior, as one passes to the continuum, does not occur, however, in all allocation mechanisms for public goods. In a recent paper, Muench (1983) examines the implications of Manipulative Nash Behavior in large finite economies for the Quadratic Mechanism of Groves and Ledyard. He shows that, as the environment is replicated "there are (local) Manipulative Nash equilibria" (of the symmetric Nash equilibrium) "arbitrarily close to Pareto-efficient allocations" and "the allocations in the limit involve dividing the cost of the public good equally among all consumer." (Muench, 1983 p.1) In our language, the Manipulative Nash equilibrium allocations converge to the Fair-Efficient allocations as the economy is successively replicated. As we pointed out in Section IV, considering manipulative Nash behavior in a mechanism  $h(m)$  is equivalent to considering Nash behavior in the direct revelation mechanism  $h[b^N(e;h)] = H(e)$ . Thus Muench has, in effect, shown

that there is a mechanism,  $H$ , whose Nash equilibrium allocations are approximately efficient in large finite replica economies, with enough continuity.

It would seem that we now have a result for public goods environments like Theorem 5.5 above. But Muench also proves that, in the limit, the Nash equilibria of  $H(e)$  involve misrepresentations. This should be evident since the Nash equilibrium allocations of the Quadratic Mechanism,  $h$ , are not fair even in the limit. Thus, convergence of the Nash allocations of  $H$  to fair efficient allocations implies that the Nash equilibria of  $H$ ,  $b^N(e;H)$ , do not converge to  $e$ . Therefore, they cannot be approximate dominant strategies.

To summarize, and also to provide a contrast with the private goods environments, we state two propositions.

**THEOREM 5.7:** In classical public goods environments, (a) there is a (direct revelation) mechanism whose Nash equilibria are almost dominant strategies if the economy is large enough and (b) there is a (direct revelation) mechanism whose Nash equilibria are almost efficient if the economy is large enough.

Proof: Summarizes above discussion.

**CONJECTURE 5.8:** In classical public goods environments, there is no mechanism that, in large economies, is "almost" an efficient, dominant strategy mechanism.

If this conjecture is correct, it is the first fact that differentiates private goods environments from public goods environments.

It should be noted that both direct revelation mechanisms--the competitive mechanism and the manipulative Nash version of the Quadratic Mechanism--do not produce efficient allocations in finite environments; only in a limiting sense are they efficient. Yet we know from above that there are other mechanisms that are efficient Nash in all finite environments. Can we find one that, in the limit, is almost an efficient dominant strategy mechanism? For now our answer is that we don't know. For the mechanisms like those in Section IV, the message space is smaller than the space of

classical environments. It seems thus unlikely to us that dominant strategies exist even in the limit.

We can carry this analysis a step further. In private goods economies, the Nash efficient mechanisms discussed in Section IV have the additional property that they select Walrasian allocations. That is,  $h[b^N(e;h)] \subseteq W(e)$ . We know that  $W(\cdot)$  is simply the outcome function of the competitive process which is almost a dominant strategy mechanism in large environments. Therefore, as the private goods environment grows larger the Manipulative Nash equilibria of those Nash efficient mechanisms converge to a dominant strategy, the true  $e_i$ .

**COROLLARY 5.9:** In large finite private goods environments there are Nash efficient mechanisms with the property that truth is almost a Manipulative Nash equilibrium. That is, it is almost a dominant strategy to employ Nash behavior (according to one's true characteristics) in one's message responses, if everyone else employs Nash behavior (according to some arbitrary characteristic).

A similar result does not seem to be valid in public goods environments. From our work and from that of Muench (1983) and Muench and Walker (1979) and (1983), we know that the Quadratic Mechanism is an efficient Nash mechanism that in the limit is an efficient, but not a dominant strategy, mechanism. [The manipulative Nash equilibria are not efficient in finite economies but are in the limit.] However, the Manipulative Nash equilibria converge neither to the true characteristics nor to a dominant strategy. Thus an analogous result to that of Corollary 5.8 will not hold for the Quadratic mechanism. Almost the same conclusions can be reached for any mechanism whose Nash equilibrium allocations are Lindahl. The only difference is that the manipulative Nash equilibria converge to a dominant strategy (which is to act as if one gets no utility from public goods) but are never efficient even in the limit.

**CONJECTURE 5.10:** In public goods environments, there are no mechanisms with the property that, in large economies, truthful Nash behavior is almost a dominant strategy, or truth is almost a Manipulative Nash Equilibrium.

Again we have a subtle but important distinction between private and public goods environments if Conjectures 5.8 and 5.10 can be verified. This work remains to be done.

### C. SUMMARY

Combining the results in the previous section, we can summarize the state of knowledge concerning the possibilities for the design of efficient, incentive-sensitive mechanisms in "large" economies as follows:

THEOREM 5.11: (a) In classical environments (both public and private) with a continuum of agents, there exist non-parametric, efficient, dominant strategy mechanisms. (Section V.A)

(b) In classical environments (both public and private) there are mechanisms which are efficient and limiting incentive compatible, if individual rationality is not required.<sup>21</sup> (Section V.B.1)

(c) In classical private goods environments, with enough continuity, there exists a mechanism which is "almost" an efficient, dominant strategy mechanism if the economy is "large enough". (Section V.B.2)

(d) In classical public goods environments, with enough continuity, there exists a mechanism whose Nash equilibrium allocations are "almost" efficient if the economy is "large enough" and there exists a mechanism whose Nash equilibrium strategies are "almost" dominant strategies if the economy is "large enough". The two known mechanisms are not the same. (Section V.B.2)

(e) (Conjecture) In classical public goods environments, with enough continuity, there do not exist mechanisms which are "almost" efficient, dominant strategy mechanisms even in "very large" economies. (Section V.B.2)

(f) In classical private goods environments, there are efficient Nash mechanisms for which truth is "almost" a Manipulative Nash equilibrium if the economy is "large enough". (Section V.B.2)

(g) (Conjecture) In classical public goods environments, there are no efficient Nash mechanisms such that truth is "almost" a Manipulative Nash equilibrium even if the economy is "large enough".

---

21. This has been proven in the case of environments with public goods only and quasi-linear preferences. The more general statement is conjecture at this point.

(Section V.B.2)

With these results, we close our survey.



## APPENDIX TO SECTION V: SKETCHES OF PROOFS

We include in this appendix sketches of proofs of a few of the theorems of Section V.

### *Sketch of Proof of Theorem 5.1:*

The environment is modeled as a measure on a set of possible characteristics. Let  $D$  be a set of characteristics -- endowments and preferences -- and let  $\nu$  be a measure on that set such that  $\nu(D) = 1$ . We say that  $\nu$  is atomless if  $\nu(\{d\}) = 0$  for all  $d \in D$ . [If  $\nu$  is a finite environment, represented by, say,  $e = (e_1, \dots, e_N)$ , then  $\nu(\{d\}) = 1/N$  if  $d = e_i$  for some  $i$  and  $\nu(\{d\}) = 0$  otherwise.]

Given an environment  $\nu$ , let  $\nu/(d, d')$  be the same environment with  $d$  replaced by  $d'$ . If  $\nu$  is atomless then for the competitive mechanism the set of equilibrium prices  $C(\nu) = C(\nu/(d, d'))$ . Thus if some atomless agent reports  $d'$  instead of  $d$ , there is no effect on the equilibrium price. It follows easily that an agent's best response is the true characteristic no matter what  $\nu$  is. Thus the competitive mechanism is a truth dominant mechanism in classical private goods environments with a non-atomistic measure of agents. That the competitive mechanism is efficient is already known.

### *Sketch of Proof of Theorem 5.2:*

As in the private goods model, we let  $\nu$  be the measure on characteristics which describes the environment. The Fair-Efficient mechanism  $h(\nu)$  selects a public goods level,  $y(\nu)$ , and a net trade in private goods for each agent,  $x(\nu, d)$ , in such a way that there are prices  $p(\nu)$  and  $q(\nu)$  such that

$$(1) \text{ if } z(\nu) = \int x(\nu, d) d\nu \text{ then } (y(\nu), z(\nu)) \text{ solves}$$

$$\max p(\nu)z + q(\nu)y \text{ subject to } (z, y) \in Y,$$

$$(2) x(\nu, d) \text{ solves } \max u(y(\nu), x; d) \text{ subject to } p(\nu)x + \frac{1}{N}q(\nu)y(\nu) \leq p(\nu)w(d) + m(\nu, d) \text{ where}$$

$$\int m(\nu, d) d\nu = p(\nu)z(\nu) + q(\nu)y(\nu), \text{ and } \int d\nu = N,$$

$$(3) \int \left\{ \frac{\partial u(y(\nu), w + x(\nu, d), d)}{\partial u(y(\nu), w + x(\nu, d), d)} \frac{\partial y}{\partial x_1} \right\} d\nu = q(\nu).$$

It remains to show that this mechanism,  $h(\nu)$ , is an efficient, dominant strategy mechanism if  $\nu$  is atomless. It is obviously efficient if truth is a dominant strategy. To see that it is a dominant

strategy mechanism, consider how each of the above three relations change as one atomless agent replaces  $d$  with  $d'$ . First, none of  $z(v)$ ,  $p(v)$ ,  $q(v)$ , or  $y(v)$  change. Thus the only change which the agent can effect is in  $h(v,d)$ , but it is then optimal to send the true  $d'$ . Thus, sending the true  $d$  is a dominant response.

*Sketch of Proof of Theorem 5.5:*

To understand the proof, consider a simplified set of environments; those with quasi-linear preferences, one private and one public good, and crowding in production so that the optimal quantity of the public good  $y_k \rightarrow y^*$  finite, as the number of agents  $k$  gets large.

Let  $N$  be the number of agents in the economy. Assume that  $g(y)$  is the amount of private good needed to produce  $y$ , with  $g(y) = Nf(y)$ , and  $g'(y) = Nf'(y)$ . In this case the Fair-Efficient mechanism introduced earlier is given by:  $x(v,d) = -\frac{1}{N}g(y(v)) = -f(y)$ , and  $y(v)$  solves

$$(*) \quad \int \frac{\partial u(y,d)}{\partial y} dv = g'(y) = Nf'(y).$$

To see that this is indeed limiting incentive compatible assume that  $v$  is finite and consider an agent's decision as to which characteristic to report. If  $d$  reports  $d'$  then  $y = y(v/d')$  and  $x = x(v/d',d') = -\frac{1}{N}g(y(v/d')) = -f'(y(v/d'))$ . Therefore, this agent should 'select' his best  $y'$  and then choose  $d'$  such that  $y(v/d') = y'$ . By best we mean that  $y'$  which maximizes  $u(y,d) - f'(y)$ . As  $N$  gets large  $y'$  does not change, because of the crowding assumption. Given  $y'$ , let us see how to calculate  $d'$ . Remember that  $y(v)$  solves (\*). Suppose that  $y' < y(v)$  and that one cannot claim that  $y$  is a public bad. Then the most one can gain by misrepresenting is by sending  $d'$  such that  $u(y,d') = 0$  for all  $y$ ; that is, to claim to have no interest in the public good. The new public good level  $y(v/d')$  solves  $\int \frac{\partial u(y,d)}{\partial y} dv - v(d) \frac{\partial u(y,d)}{\partial y} = g(y)$ . It is easy to see that as  $N$  grows  $y(v/d') \rightarrow y(v)$ , since  $v(d) \rightarrow 0$ . It follows that the incentive to misrepresent,  $u(y(v/d'),d) - u(y(v),d) - [f(y(v/d')) - f(y(v))]$ , goes to 0. Basically, as  $N$  grows large all agents' ability to manipulate  $y$  grows small and therefore the gain grows small. Thus, this mechanism is

limiting incentive compatible.

*Sketch of Proof of Theorem 5.6:*

Let  $x(p, e)$  solve  $\max u(x, e)$  subject to  $px = pw$ . (Assume  $w_1, \dots, w_N$  are known.) Let  $e^* = (e^*_1, \dots, e^*_N)$

denote true preferences. Let  $E = E_1 \times \dots \times E_N$  be such that (1)  $u \in C^4$ , (2) there exists a unique

competitive equilibrium price system, and (3)  $\frac{\partial \sum x(p, e_i)}{\partial p} < 0$  at that equilibrium. At the  $k^{\text{th}}$

replicate misrepresentation Nash equilibrium

(1)  $e^k$  is the equilibrium where if two agents,  $i$  and  $j$ , are the same type then  $e^{k_i} = e^{k_j}$

(2)  $p^k$  solves  $\sum x(p, e^{k_i}) - w_i = 0$ .

(3) for all  $i$ ,  $\nabla_x u(v, e^*_i) \cdot [k \sum_h x_{hp}(p^k, e^{k_h}) - x_{ip}(p^k, e^{k_i})] p^{e_i^k} = 0$

where  $v = -\sum_h x_h(p^k, e^{k_h}) + x_i(p^k, e^{k_i}) = x_i(p^k, e^{k_i})$ .

From (2),  $p_{e_i} = \frac{\partial x_i(p^k, e^{k_i}) / \partial e_i}{k \sum_h \frac{\partial x_h(p^k, e^{k_h})}{\partial p}}$ .

Let  $e^k = \hat{e}$  where  $\hat{e} \neq e^*$  (true characteristic) and  $p^k = \hat{p}$ .

From (3),  $\nabla_x u(v^k, e^*_i) \cdot \left[ \frac{\partial x_i}{\partial e_i} - \frac{\partial x_i}{\partial p} \frac{\partial p^k}{\partial e_i} \right] = 0$ .

Therefore, if  $\lim_{k \rightarrow \infty} \nabla_x u(v^k, e^*_i) \cdot \left[ \frac{\partial x_i}{\partial e_i} - \frac{\partial x_i}{\partial p} \frac{\partial p^k}{\partial e_i} \right] = 0$  then  $\nabla_x u(x_i(\hat{p}, \hat{e}), e^*_i) \cdot \frac{\partial x_i(\hat{p}, \hat{e})}{\partial e_i} = 0$ .

But this will be true if and only if  $x(\hat{p}, e^*_i) = x(\hat{p}, \hat{e})$  for all  $i$ . Therefore,  $\hat{p} = p^*$  and  $x_i = x_i(p^*, e^*_i)$ .

## BIBLIOGRAPHY

- P. Aghion, "On the Generic Inefficiency of Differential Market Games," unpublished mimeo, CEPREMAP, Paris, 1984.
- K. J. Arrow, "An Extension of the Basic Theorems of Classical Welfare Economics," in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. J. Neyman, pp. 507-532, University of California Press, Berkeley (1951).
- K. J. Arrow, "The Property Rights Doctrine and Demand Revelation under Incomplete Information," in *Economics and Human Welfare*, Academic Press (1979).
- R. Aumann, "Existence of Competitive Equilibria in Markets with a Continuum of Traders," *Econometrica*, vol. 34, pp. 1-17 (1966).
- T. Bergstrom, C. Simon, and J. Titus, "Counting Groves-Ledyard Equilibria via Degree Theory," *Journal of Mathematical Economics*, vol. 12, pp. 167-184 (1983).
- W. Brock, "The Design of Mechanisms for Efficient Allocation of Public Goods," in *Quantitative Economics and Development*, ed. M. Nerlove, L. Klein, & S. Tsiang, Academic Press, New York (1980).
- E. H. Clarke, "Multipart Pricing of Public Goods," *Public Choice*, vol. 8, pp. 19-33 (1971).
- V. Coppinger, V. Smith, and J. Titus, "Incentives and Behavior in English, Dutch and Sealed-Bid Auctions," *Economic Inquiry*, vol. 18, pp. 1-22 (1980).
- J. Cox, B. Roberson, and V. Smith, "Theory and Behavior of Single Object Auctions," in *Research in Experimental Economics*, ed. V. Smith, vol. 2, pp. 1-43, JAI Press, Inc., Greenwich, Connecticut (1982).
- C. D'Aspremont and L. A. Gerard-Varet, "On Bayesian Incentive Compatible Mechanisms," in

*Aggregation and Revelation of Preferences*, ed. J.-J. Laffont, pp. 269-288, North-Holland, Amsterdam (1979).

P. Dasgupta, P. Hammond, and E. Maskin, "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," *Review of Economic Studies*, vol. 46, pp. 185-216 (1979).

G. Debreu, *Theory of Value*, John Wiley & Sons, Inc., New York (1959).

A. Downs, *An Economic Theory of Democracy*, Harper and Row, New York (1957).

J. Dreze and D. de la Vallee-Poussin, "A Tatonnement Process for Public Goods," *Review of Economic Studies*, vol. 38, pp. 133-150 (1971).

D. Foley, "Resource Allocation and the Public Sector," *Yale Economic Essays*, vol. 7 (1967).

D. Foley, "Lindahl's Solution and the Core of an Economy with Public Goods," *Econometrica*, vol. 38, pp. 66-72 (1970).

A. Gibbard, "Manipulation of Voting Schemes: A General Result," *Econometrica*, vol. 41, pp. 587-602 (1973).

J. Green and J.-J. Laffont, "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, vol. 45, pp. 427-438 (1977).

J. Green and J.-J. Laffont, *Incentives in Public Decision Making*, North Holland, Amsterdam (1978).

T. Groves, "The Allocation of Resources under Uncertainty," Ph.D. Dissertation, University of California, Berkeley, 1970.

T. Groves, "Incentives in Teams," *Econometrica*, vol. 41, pp. 617-663 (1973).

T. Groves, "Efficient Collective Choice With Compensation," in *Aggregation and Revelation of*

*Preferences*, ed. J.-J. Laffont, pp. 37-59, North-Holland. Amsterdam (1979).

T. Groves and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to the 'Free Rider' Problem," *Econometrica*, vol. 45, pp. 783-809 (1977).

T. Groves and J. Ledyard, "The Existence of Efficient and Incentive Compatible Equilibria with Public Goods," *Econometrica*, vol. 48, pp. 1487-1506 (1980).

T. Groves and M. Loeb, "Incentives and Public Inputs," *Journal of Public Economics*, vol. 4, pp. 211-226 (1975).

P. Hammond, "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies*, vol. 46, pp. 263-282 (1979).

M. Harris and A. Raviv, "Optimal Incentive Contracts with Imperfect Information," *Journal of Economic Theory*, pp. 231-259 (1979).

M. Harris and R. Townsend, "Resource Allocation under Asymmetric Information," *Econometrica*, vol. 49, pp. 33-64 (1981).

B. Holmstrom, "Groves Schemes on Restricted Domains," *Econometrica*, vol. 47, pp. 1137-1144 (1979).

L. Hurwicz, "Optimality and Informational Efficiency in Resource Allocation Processes," in *Mathematical Methods in the Social Sciences, 1959*, ed. K. Arrow, S. Karlin, and P. Suppes, pp. 27-46, Stanford University Press, Stanford, California (1960).

L. Hurwicz, "On Informationally Decentralized Systems," in *Decision and Organization: A Volume in Honor of Jacob Marschak*, ed. R. Radner and C. B. McGuire, pp. 297-336, North-Holland, Amsterdam (1972).

L. Hurwicz, "The Design of Mechanisms for Resource Allocation," *American Economic Review*, vol.

63. pp. 1-30 (1973).

- L. Hurwicz. "On The Existence of Allocation Systems Whose Manipulative Nash Equilibria are Pareto-Optimal." Paper given at Third World Congress of the Econometric Society in Toronto, 1975.
- L. Hurwicz. "On Allocations Attainable Through Nash-Equilibria," in *Aggregation and Revelation of Preferences*, ed. J.-J. Laffont, pp. 397-419, North-Holland, Amsterdam (1979).
- L. Hurwicz. "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies*, vol. 46, pp. 217-225 (1979a).
- L. Hurwicz. "On the Incentive Problem in the Design of Non-Wasteful Resource Allocation Systems," in *Studies in Economic Theory and Practice*, ed. N. Assorodobraj-Kula, C. Bobrowski, H. Hagemeyer, W. Kula, and J. Los, pp. 93-106, North-Holland, Amsterdam (1981).
- L. Hurwicz, E. Maskin, and A. Postlewaite. "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," unpublished mimeo, 1982.
- L. Hurwicz and M. Walker. "On the Generic Non-Optimality of Dominant-Strategy Allocation Mechanisms, with an Application to Pure Exchange Economies," unpublished mimeo, 1983.
- J.-J. Laffont and E. Maskin. "The Theory of Incentives: An Overview," in *Advances in Economic Theory: Invited Papers for the Fourth World Congress of the Econometric Society at Aix-en-Provence. September 1980*, ed. W. Hildenbrand, pp. 31-94, Cambridge University Press, Cambridge (1982).
- O. Lange and F. M. Taylor, *On the Economic Theory of Socialism*, Lippincot, New York (1938).
- J. Ledyard, "Incentive Compatible Behavior in Core Selecting Organizations," *Econometrica*, vol. 45, pp. 1607-1621 (1977).

- J. Ledyard, "Incomplete Information and Incentive Compatibility," *Journal of Economic Theory*, vol. 18, pp. 171-189 (1978).
- J. Ledyard, "Dominant Strategy Mechanisms and Incomplete Information," in *Aggregation and Revelation of Preferences*, ed. J.-J. Laffont, pp. 309-320. North-Holland, Amsterdam (1979).
- J. Ledyard and J. Roberts, "On the Incentive Problem with Public Goods," Discussion Paper # 116, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1974.
- A. Lerner, *The Economics of Control*, Macmillan, New York (1944).
- L. Makowski and J. Ostroy, "Individual Appropriation of the Gains to Trade and Incentive Compatibility," unpublished mimeo, 1984.
- L. Makowski and J. Ostroy, "Vickrey-Clarke-Groves Mechanisms and Perfect Competition," unpublished mimeo, 1984.
- E. Malinvaud, "Decentralized Procedures for Planning," in *Activity Analysis in the Theory of Growth and Planning*, ed. E. Malinvaud and M. O. L. Bacharach, pp. 170-208, Macmillan, London (1967).
- E. Malinvaud, "A Planning Approach to the Public Goods Problem," *Swedish Journal of Economics*, vol. 1, pp. 96-111 (1971).
- E. Maskin, "Nash Equilibrium and Welfare Optimality," unpublished mimeo, 1977.
- P. Milgrom and R. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica*, vol. 50, pp. 1089-1122 (1982).
- K. Mount and S. Reiter, "The Informational Size of Message Spaces," *Journal of Economic Theory*, vol. 8, pp. 161-191 (1974).



T. Muench, "Optimality, Lying, and Income Distribution in Large Economies using Groves-Ledyard Mechanisms," unpublished mimeo, 1983.

T. Muench and M. Walker, "Identifying the Free Rider Problem," in *Aggregation and Revelation of Preferences*, ed. J.-J. Laffont, pp. 61-90, North-Holland, Amsterdam (1979).

T. Muench and M. Walker, "Are Groves-Ledyard Equilibria Attainable?," *Review of Economic Studies*, vol. 50, pp. 393-396 (1983).

R. Myerson, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, vol. 47, pp. 61-73 (1979).

R. Myerson, "Optimal Auction Design," *Mathematics of Operations Research*, vol. 6, pp. 58-73 (1981).

R. Myerson, "Bayesian Equilibrium and Incentive Compatibility: An Introduction," Discussion Paper # 548, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1983.

A. Postlewaite, "Implementation via Nash Equilibria in Economic Environments," unpublished mimeo, 1983. forthcoming in *Social Goals and Social Organization: A Volume in Memory of Elisha Pazner*, ed. L. Hurwicz, D. Schmeidler, and H. Sonnenschein

A. Postlewaite and D. Schmeidler, "Notes on Optimality and Feasibility of Informationally Decentralized Allocation Mechanisms," in *Game Theory and Related Topics*, ed. O. Moeschlin and D. Pallaschke, North-Holland, Amsterdam (1979).

A. Postlewaite and D. Schmeidler, "Revelation and Implementation under Differential Information," unpublished mimeo, 1983.

A. Postlewaite and D. Schmeidler, "Differential Information and Strategic Behavior in Economic

Environments: A General Equilibrium Approach," in *this volume* (1984).

A. Postlewaite and D. Wettstein, "Implementing Constrained Walrasian Equilibria Continuously,"

CARESS Discussion Paper 83-24, University of Pennsylvania, Philadelphia, 1983.

S. Rassenti, V. Smith, and R. Bulfin, "A Combinatorial Auction Mechanism for Airport Time Slot

Allocation," *Bell Journal of Economics*, vol. 13, pp. 402-417 (1982).

S. Reichelstein, "Implementation in Nash-Equilibrium Strategies," in *Incentives and Information in*

*Economic Organizations*, pp. 61-77, Ph.D. Dissertation, Northwestern University (1982).

S. Reichelstein, "Incentive Compatibility and Informational Requirements," Discussion Paper # IP-34,

University of California, Berkeley, 1983.

S. Reichelstein, "Smooth versus Discontinuous Mechanisms," Working Paper # EAP-3, University of

California, Berkeley, 1984a.

S. Reichelstein, "A Note on Allocations Attainable through Nash Equilibria," *Journal of Economic*

*Theory*, vol. 32, pp. 384-390 (1984b).

S. Reiter, "Informational Efficiency of Iterative Processes and the Size of the Message Spaces,"

*Journal of Economic Theory*, vol. 8, pp. 193-205 (1974).

S. Reiter, "Information and Performance in the (New) Welfare Economics," *American Economic*

*Review*, vol. 67, pp. 226-234 (1977).

J. Roberts, "The Incentives for the Correct Revelation of Preferences and the Number of Consumers,"

*Journal of Public Economics*, vol. 6, pp. 359-374 (1976).

J. Roberts, "Strategic Behavior in the MDP Planning Procedure," in *Aggregation and Revelation of*

*Preferences*, ed. J.-J. Laffont, pp. 353-362, North-Holland, Amsterdam (1979).

- J. Roberts, "Incentives, Information, and Iterative Planning," in *this volume* (1984).
- J. Roberts and A. Postlewaite, "The Incentives for Price-Taking Behavior in Large Economies," *Econometrica*, vol. 44, pp. 115-128 (1976).
- P. Samuelson, "The Pure Theory of Public Expenditures," *Review of Economics and Statistics*, vol. 36, pp. 387-389 (1954).
- P. Samuelson, "Diagrammatic Exposition of a Theory of Public Expenditures," *Review of Economics and Statistics*, vol. 37, pp. 350-356 (1955).
- M. Satterthwaite, "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," *Journal of Economic Theory*, vol. 10, pp. 187-217 (1975).
- M. Satterthwaite and H. Sonnenschein, "Strategy-Proof Allocation Mechanisms at Differentiable Points," *Review of Economic Studies*, vol. 48, pp. 587-597 (1981).
- D. Schmeidler, "Walrasian Analysis via Strategic Outcome Functions," *Econometrica*, vol. 48, pp. 1585-1593 (1980).
- D. Schmeidler, "A Condition Guaranteeing that the Nash Allocation is Walrasian," *Journal of Economic Theory*, vol. 28, pp. 376-378 (1982).
- D. Schmeidler, "Economic Analysis via Strategy Outcome Functions: A Survey of Recent Results," in *Games, Economic Dynamics, and Time Series Analysis*, pp. 59-66, Physica-Verlag, Wien (1982).
- V. Smith, "The Principle of Unanimity and Voluntary Consent in Social Choice," *Journal of Political Economy*, vol. 85, pp. 1125-1139 (1977).
- V. Smith, "Incentive Compatible Experimental Processes for the Provision of Public Goods," in *Research in Experimental Economics*, ed. V. Smith, vol. 1, JAI Press, Greenwich, Connecticut

(1979).

V. Smith, "Experiments with a Decentralized Mechanism for Public Goods Decisions," *American Economic Review*, vol. 70, pp. 584-589 (1980).

V. Smith, "Microeconomic Systems as an Experimental Science," *American Economic Review*, vol. 72, pp. 923-955 (1982).

W. Thomson, "Maximin Strategies and Elicitation of Preferences," in *Aggregation and Revelation of Preferences*, ed. J.-J. Laffont, pp. 245-260. North-Holland, Amsterdam (1979).

M. Truchon, "Non-Myopic Strategic Behavior in the MDP Planning Procedure," *Econometrica*, vol. 52, pp. 1179-1190 (1984).

W. Vickrey, "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, vol. 16, pp. 8-37 (1961).

M. Walker, "A Note on the Characterization of Mechanisms for the Revelation of Preferences," *Econometrica*, vol. 46, pp. 147-152 (1978).

M. Walker, "On the Impossibility of a Dominant Strategy Mechanism to Optimally Decide Public Questions," *Econometrica*, vol. 48, pp. 1521-1540 (1980).

M. Walker, "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica*, vol. 49, pp. 65-71 (1981).

M. Weitzman, "Iterative Multi-Level Planning with Production Targets," *Econometrica*, vol. 38, pp. 50-65 (1970).

R. Wilson, "Competitive Exchange," *Econometrica*, vol. 49, pp. 65-73 (1978).

R. Wilson, "Double Auctions," in *this volume* (1983).